Lecture #4 - Computer Architecture

AMath 483/583 - Spring 2016

Announcements

- Homework #1 Technical Issues —> Extension to **Monday, 25 April 5:00pm**
- Good Homework #1 Strategy
    - [1] Jupyter Notebook —> Requested functions / scripts / modules
    - [2] Write directly to target. Write many tests.
- Primary Resources for this lecture on Syllabus

Binary

- Decimal numeral system (base 10)

$$31.415 = 3 \times (10^1) + 1 \times (10^0) + 4 \times (10^{-1}) + 1 \times (10^{-2}) + 5 \times (10^{-3})$$

- Binary system (base 2)

$$\begin{aligned} 110.110_2 &= 1 \times (2^2) + 1 \times (2^1) + 0 \times (2^0) + 1 \times (2^{-1}) + 1 \times (2^{-2}) + 0 \times (2^{-3}) \\ &= 6.75_{10} \end{aligned}$$

- Hexadecimal system (base 16)

$$2a4.8f_{16} = 2 \times (16^2) + 10 \times (16^1) + 4 \times (16^0) + 8 \times (16^{-1}) + 15 \times (16^{-2})$$

Storing Data on a Computer

- Computer memory divided into “bytes”: 1 byte = 8 bits
- 1 byte = 256 different values
- `int` = 4 bytes (32 bits)
- `long` = 8 bytes (64 bits)
- 0 + 0 = 0
  0 + 1 = 1 + 0 = 1
  1 + 1 = 10

8-bit integers

`00000000` = 0

`00000001` = 1

`00000010` = 2

`00000011` = 3

…

`11111111` = 255

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  1 + 1 = 10

8-bit signed integers

`10000000`	= -128
`10000001`	= -127
`10000010`	= -126
	…
`11111110`	= -2
`11111111`	= -1
`00000000`	= 0
`00000001`	= 1
`00000010`	= 2
	…
`01111111`	= 127

Storing Data on a Computer

- What about floating point numbers?
- Base 10 proposal: integer part + fractional part

```
00003.14159  (pi)
00000.000314 (pi / 10000)
31415.90000  (pi * 10000)
```

- Disadvantages:
  - precision depends on size of number
  - many wasted bits
  - limited range (science requires large and small numbers)

Storing Data on a Computer

- Solution: use scientific notation

  `0.2345e-18` = 0.2345 x 10^(18) =
  0.0000000000000000002345

- Mantissa = 0.2345,   Exponent = -18

- Mantissa = 0.10110,   Exponent = -11011

$$\begin{aligned} 0.101101 &= 1(2^{-1}) + 0(2^{-2}) + 1(2^{-3}) + 1(2^{-4}) + 0(2^{-5}) + 1(2^{-6}) \\ &= 0.703125_{10} \\ -11011 &= -1(2^4) + 1(2^3) + 0(2^2) + 1(2^1) + 1(2^0) \\ &= -27_{10} \end{aligned}$$

- So the number is:

$$0.703125 \times 10^{-27}$$

Storing Data on a Computer

- double = 8 bytes (64 bits)
  - 53 bit (signed)mantissa, 11 bit (signed)exponent
  - 52 bits of significant figures / precision

$$2^{-52} \approx 2.2 \times 10^{-16}$$

roughly 15 digits of precision

Floating Point Operations

- Often, digits of precision are lost in operations (add, mul). Two reasons:
  - non-exact binary representation

    `0.1 (base 10) = 0.0001100110011…`

  - numbers with different scales (exponents)

$$0.123 \times 10^{18} + 0.456 \times 10^{-12}$$

Quick Python Demo

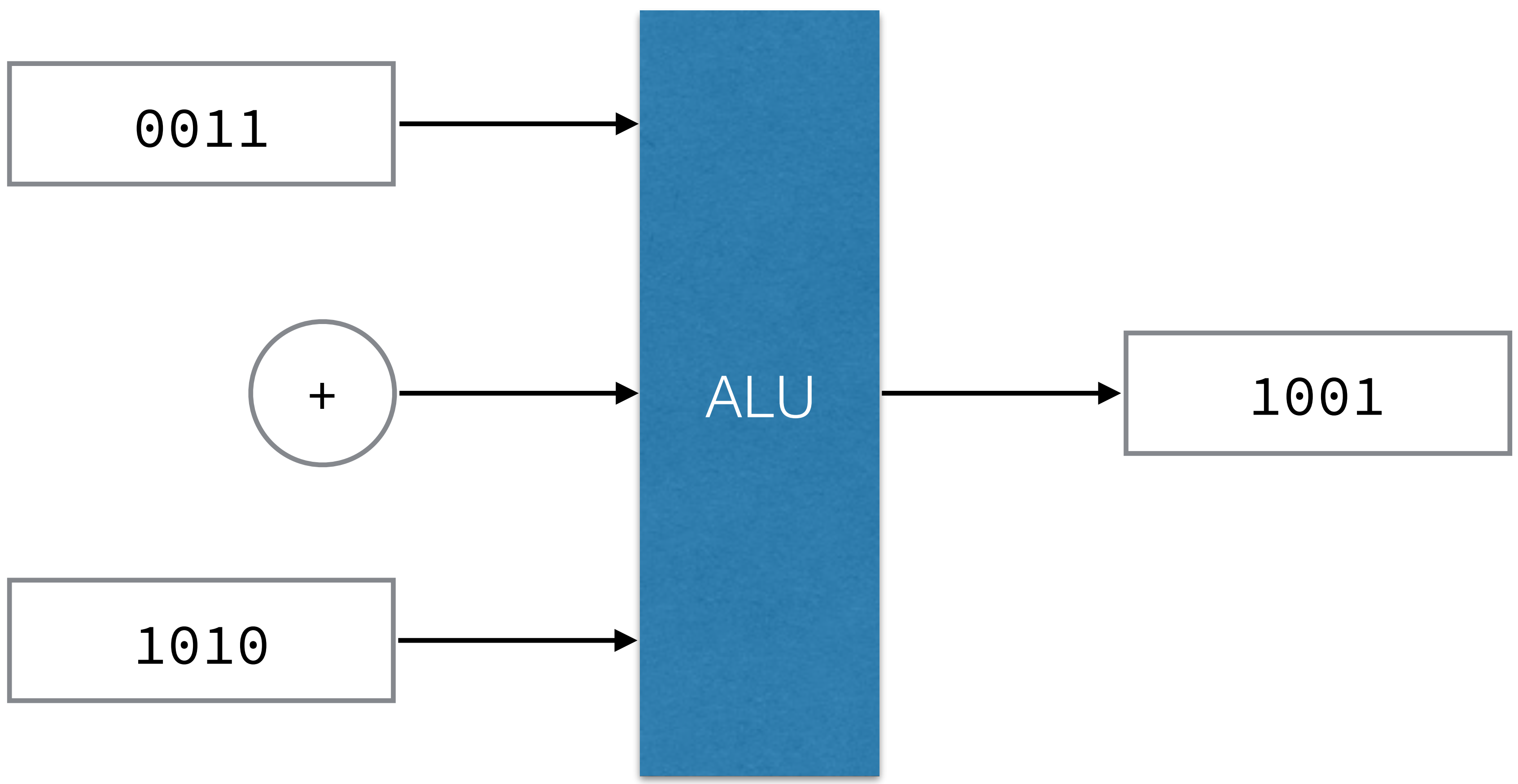
Floating Point Operations Error

CPU

- Carries out arithmetic instructions on input data
- Components:
  - control unit - directs CPU to carry out instructions
  - arithmetic logic unit - bitwise operations
  - memory management unit - maps data to location in RAM, cache, etc.

CPU

- arithmetic logic unit - two data inputs and an operation input with corresponding output



- "Execution pipeline" - control unit and ALU managing tasks

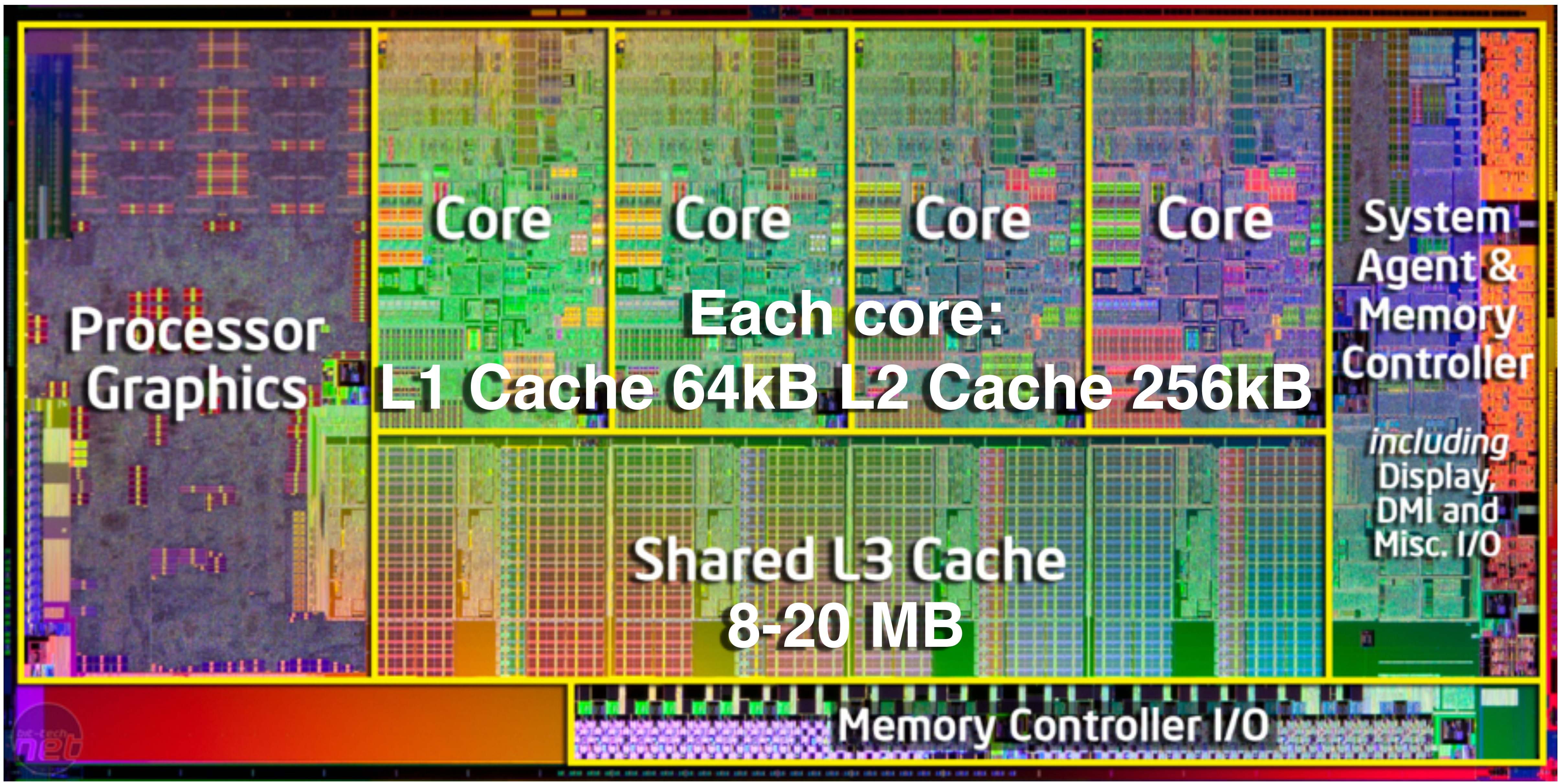
CPU

- memory management unit - manages registers and RAM addresses
  - registers - 32-bit / 64-bit storage units for the processor / ALU to retrieve and store data
    - data, address, instruction, stack pointer, …

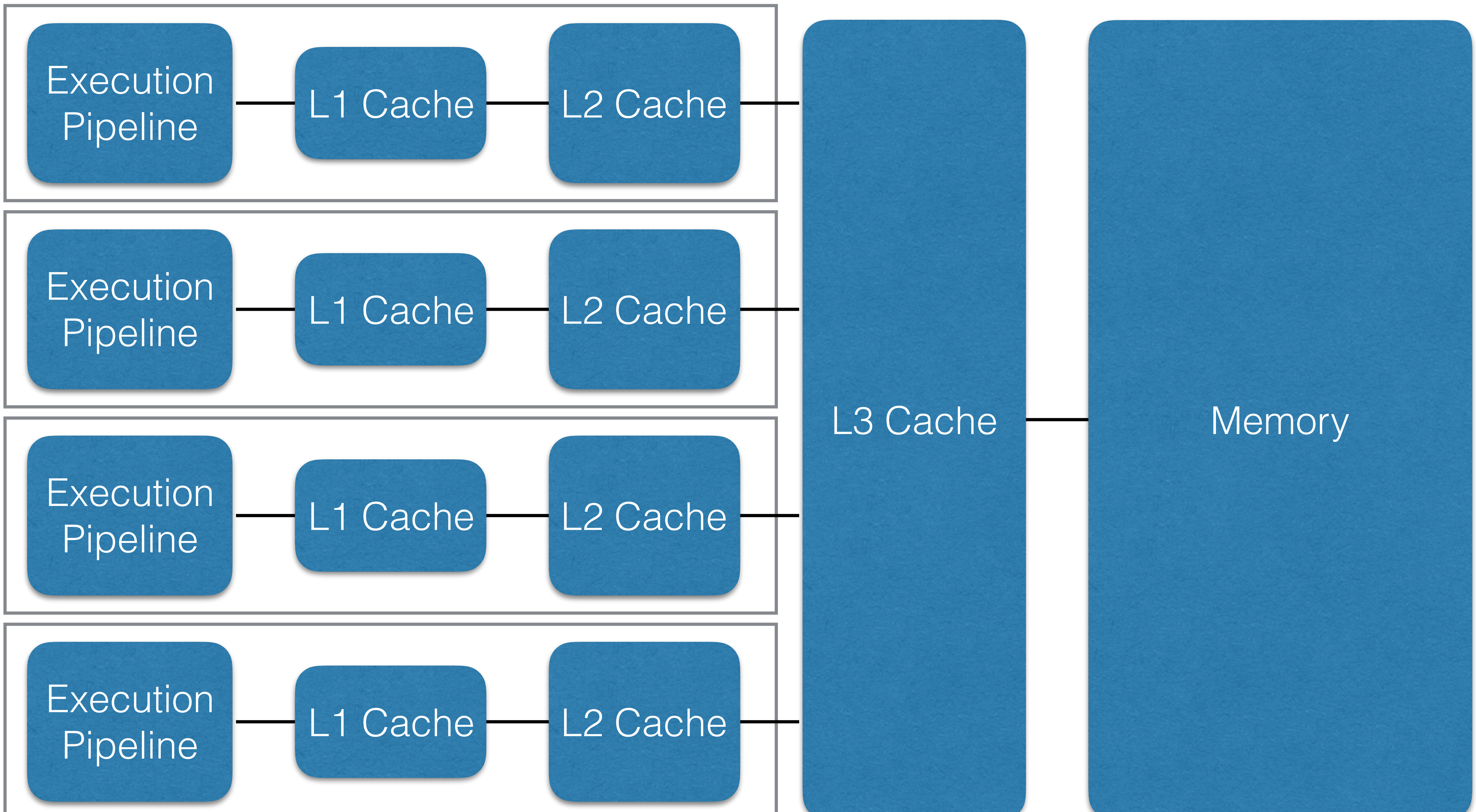
Memory

- RAM - Random Access Memory
  - where your program and data live
- Registers - where your CPU can interact with data
- Cache - an “in-between” space where data from RAM is moved “closer” to the CPU for performance

Intel Sandy Bridge CPU



Memory Layout



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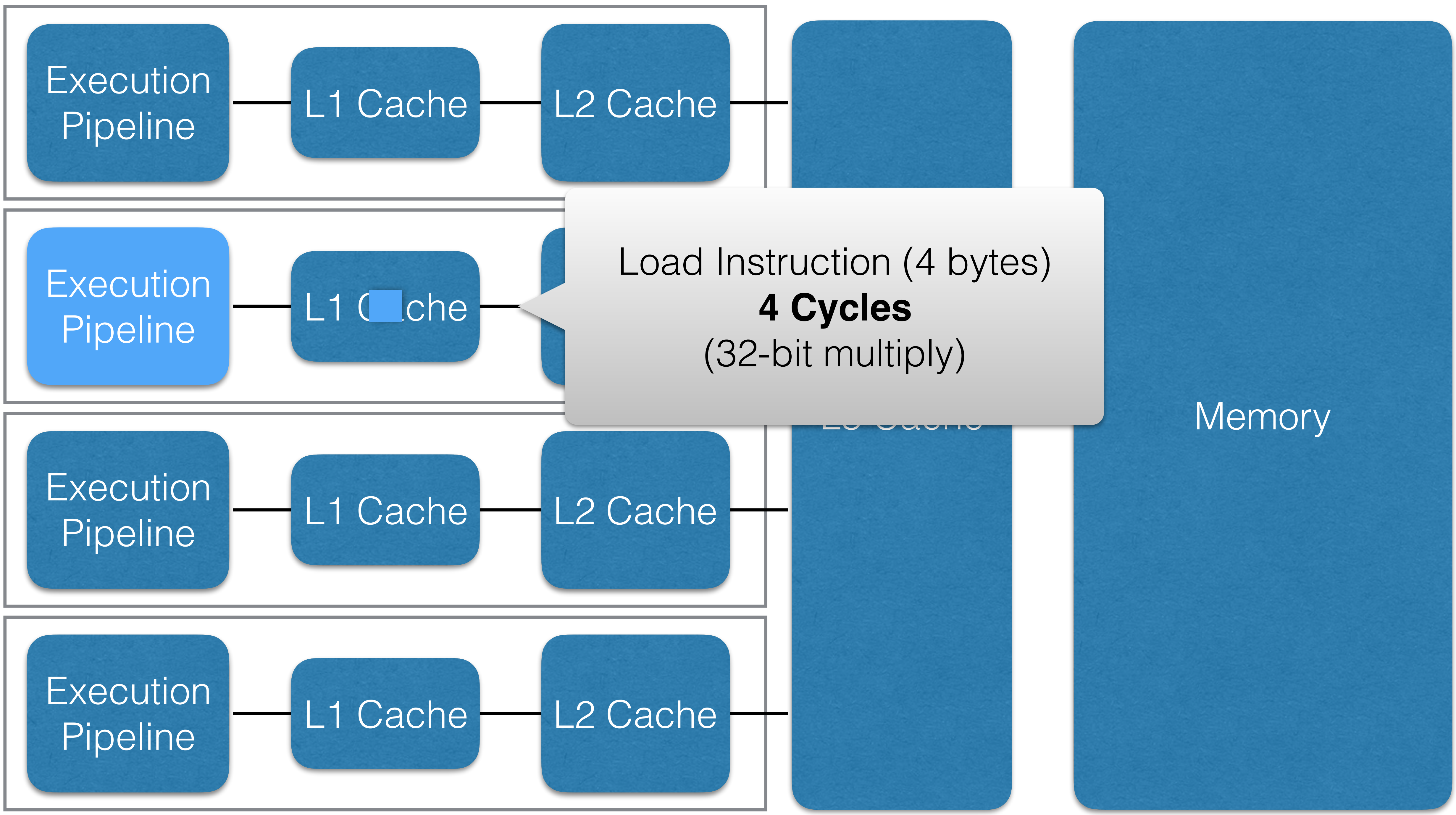

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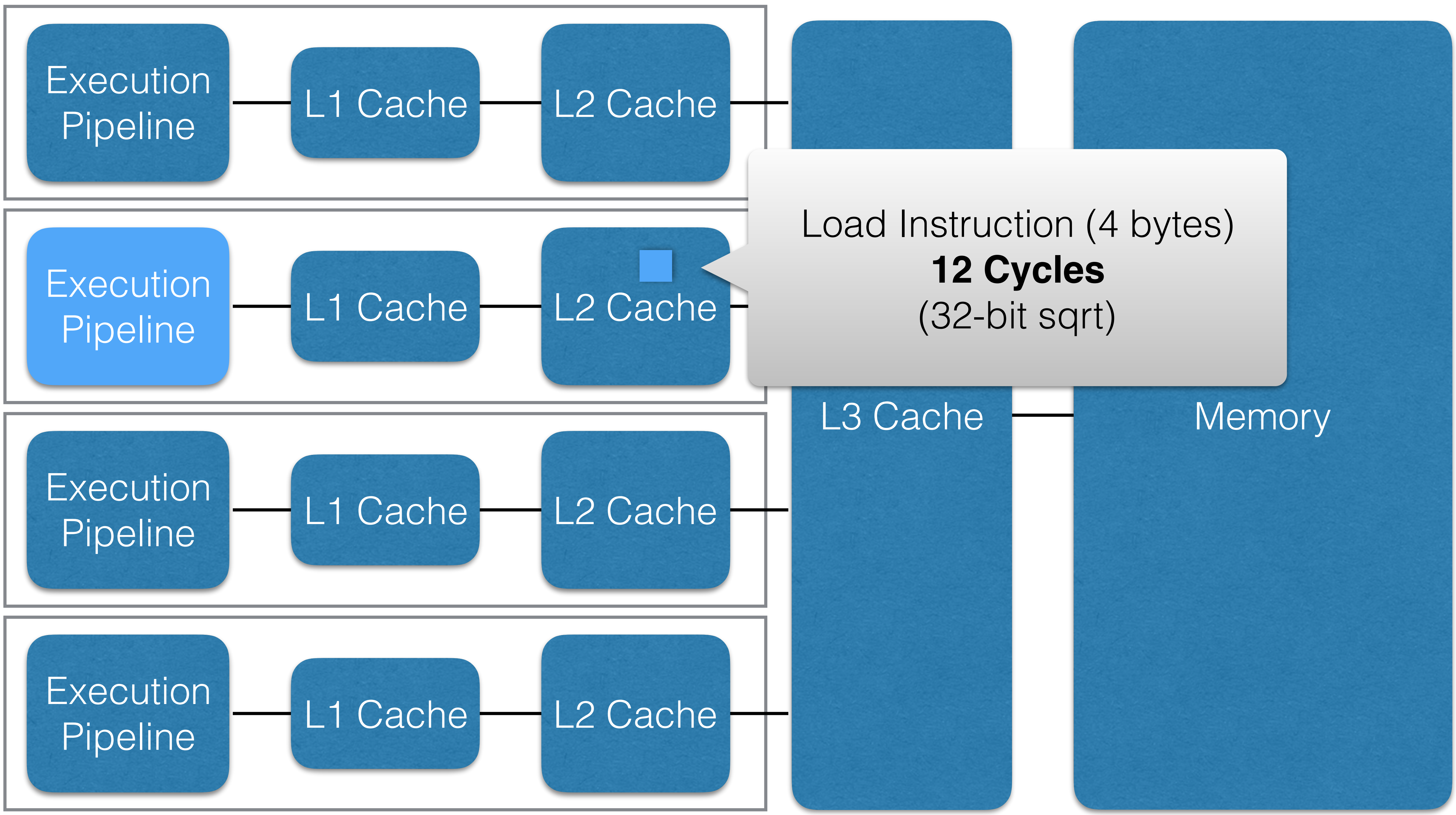

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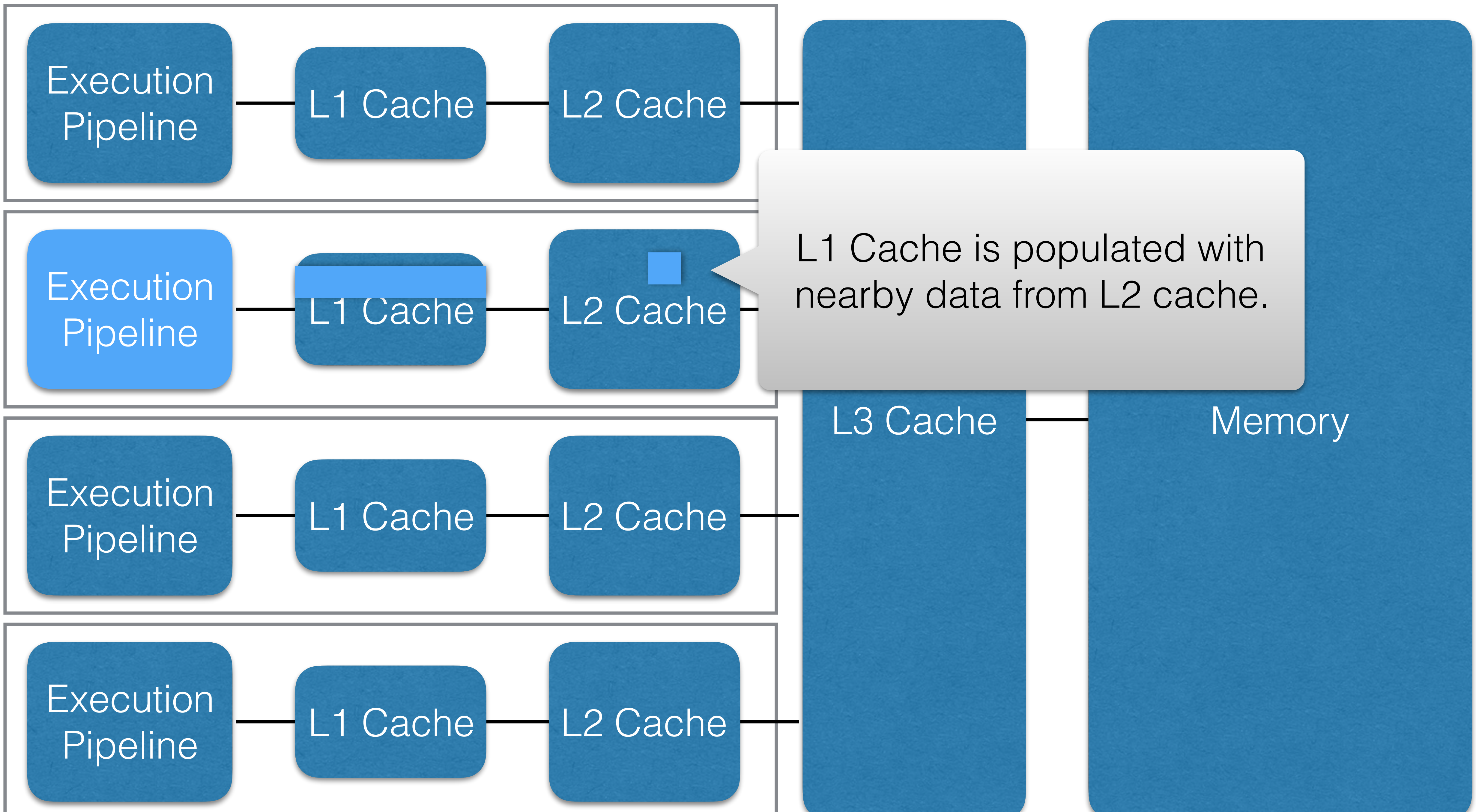

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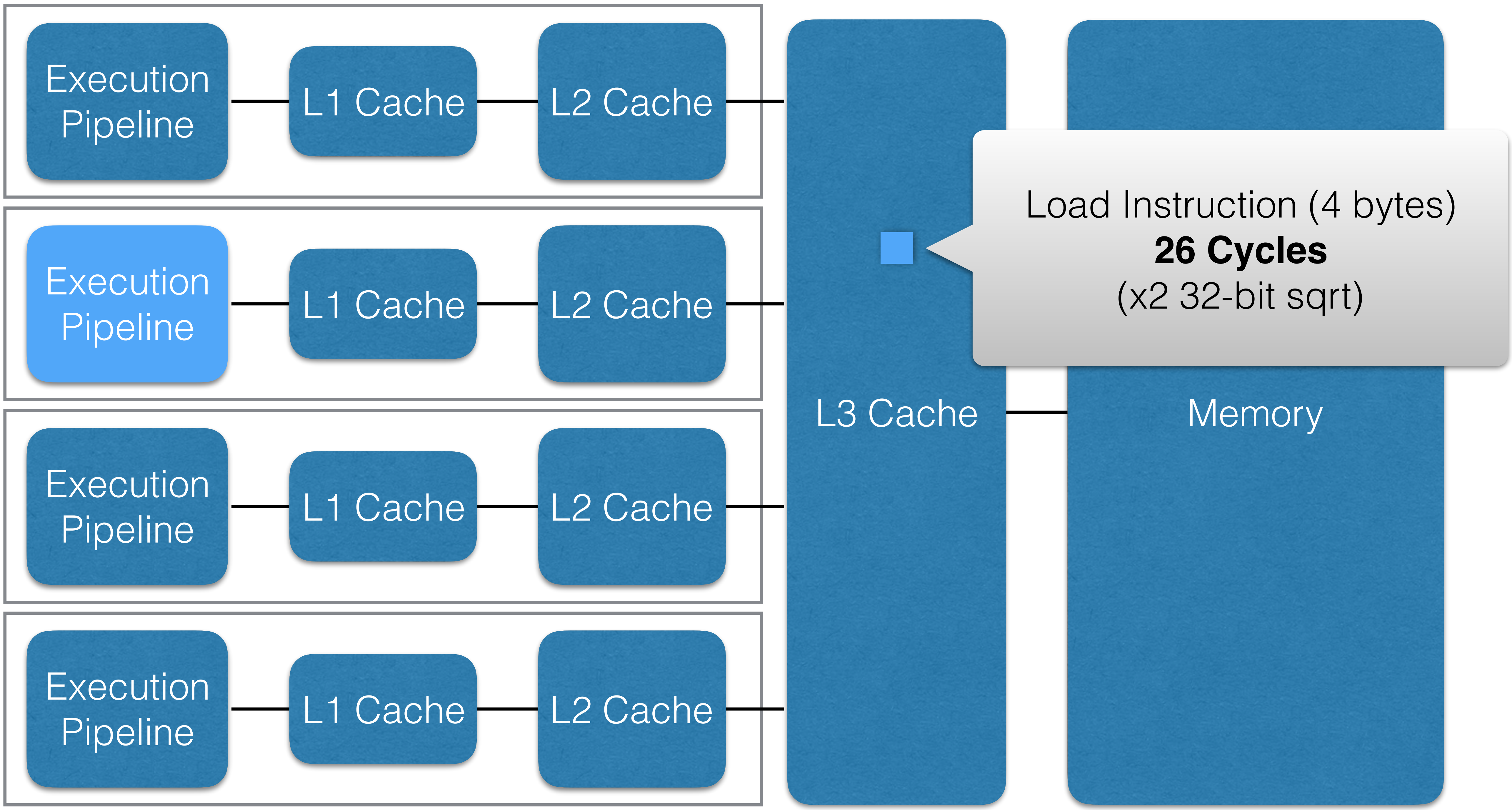

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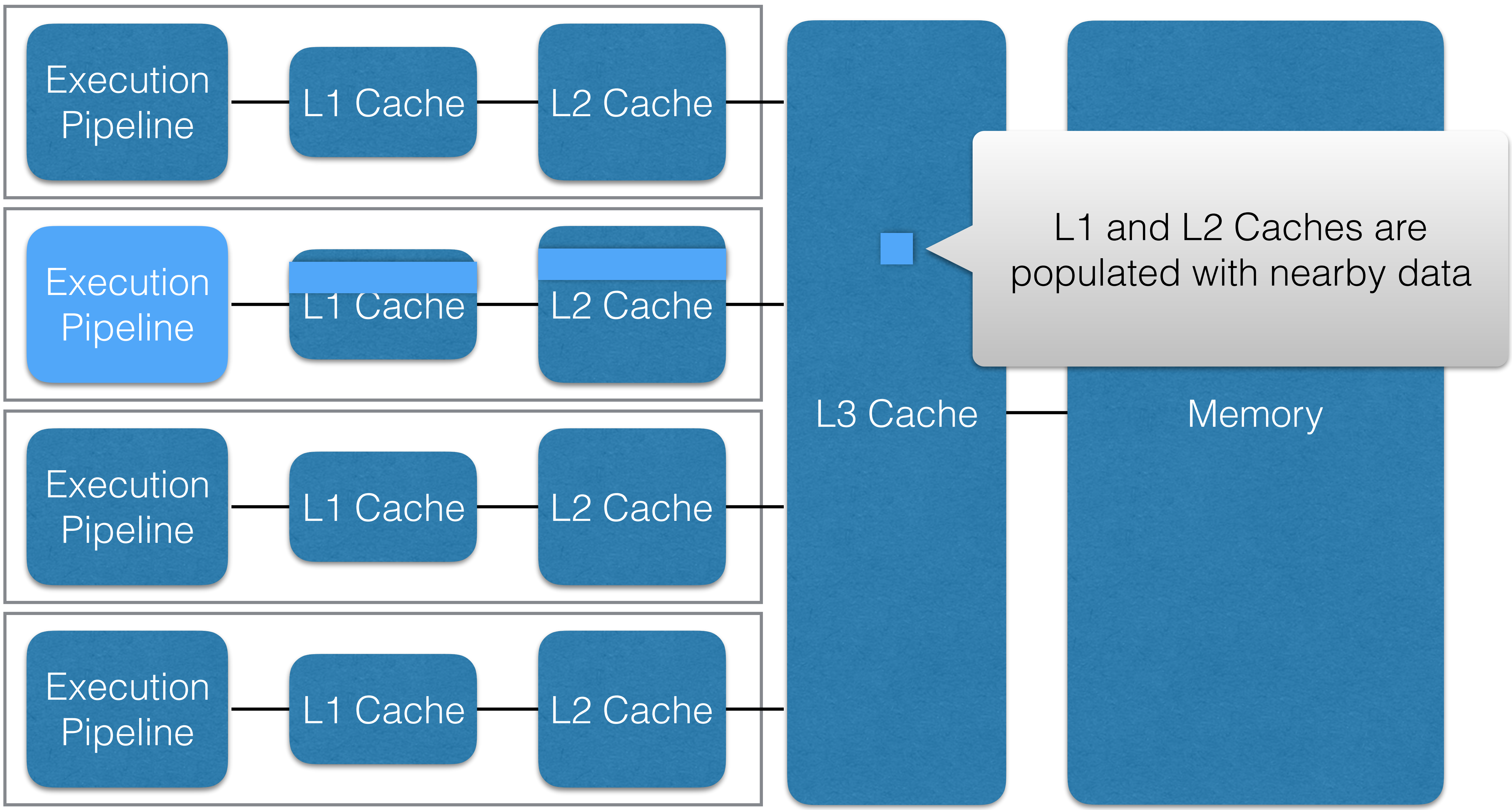

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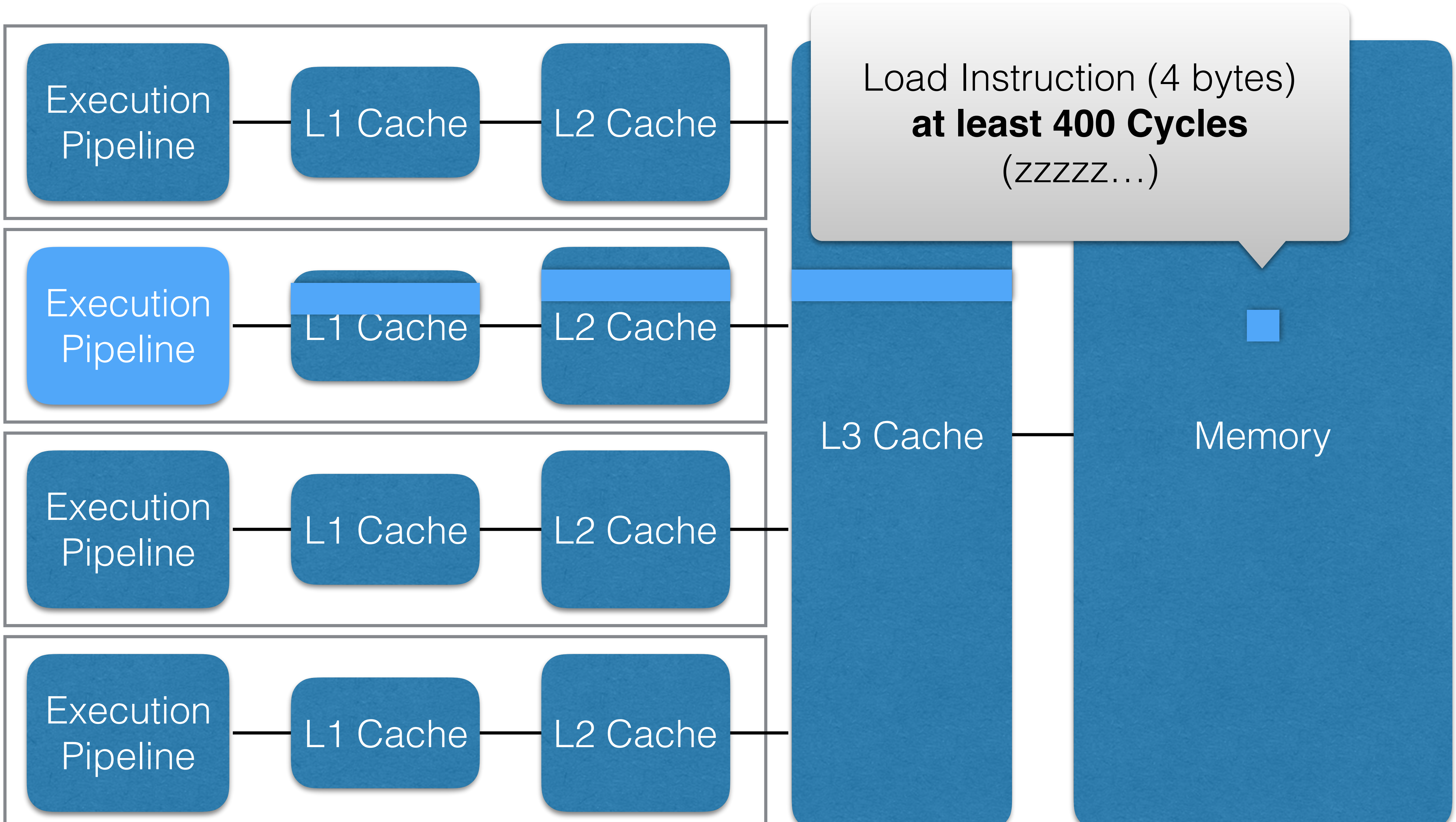

Moral of the Story

Try to keep the data you need as close as possible.

The Call Stack

- Stores info about the active functions and subroutines in a program.

```
int foo(float x)
{
    float y = 2;
    float a = bar(x,y);
    int b = floor(a);
    return b;
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float bar(float x, float y)
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    double z = x + y;
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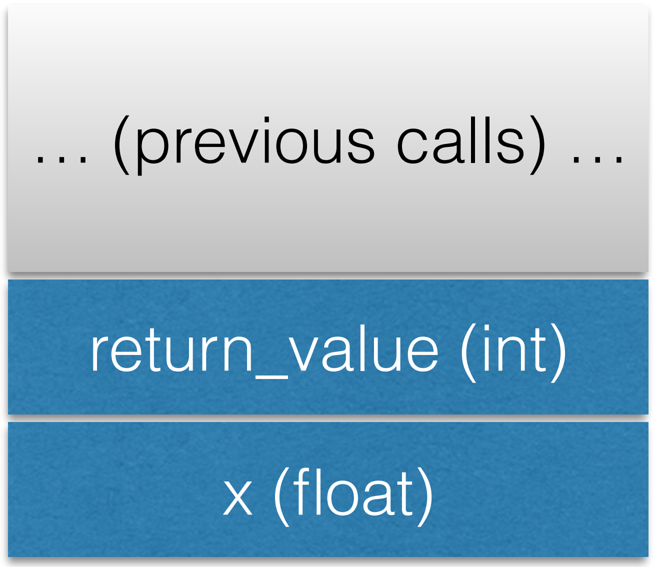

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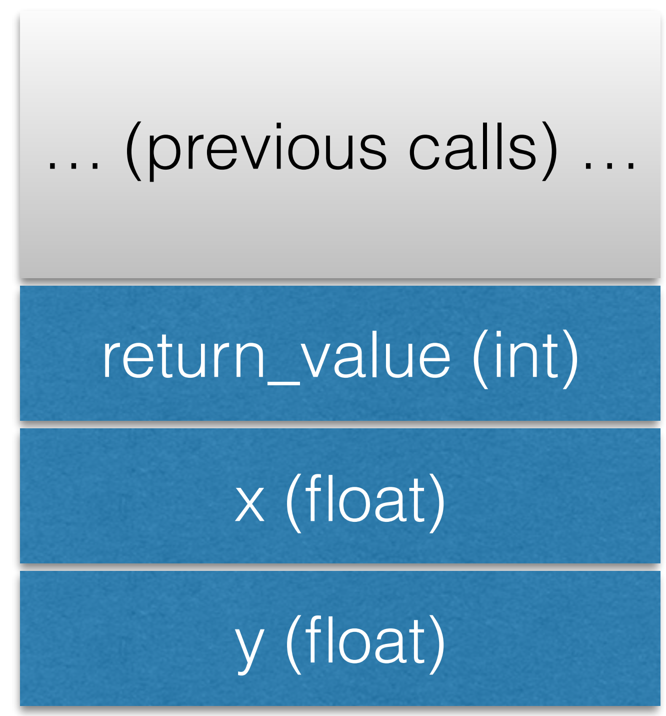

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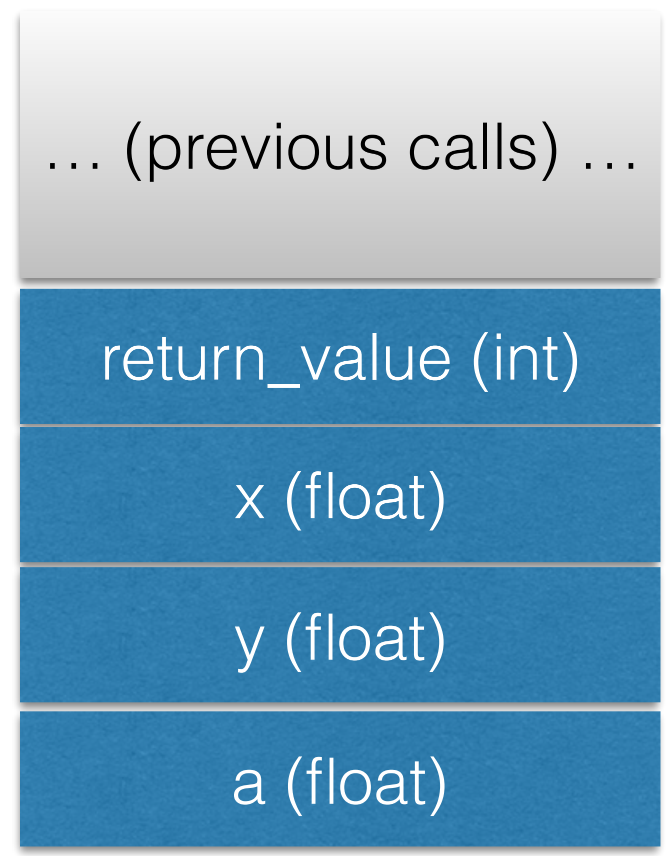

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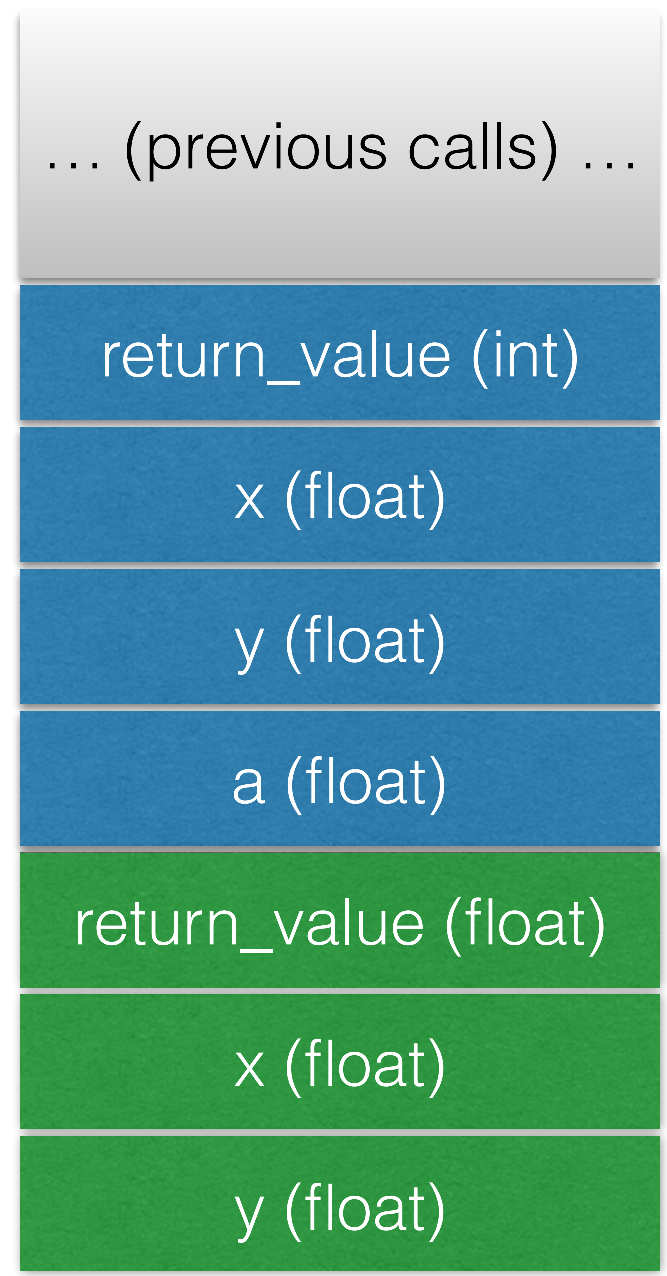

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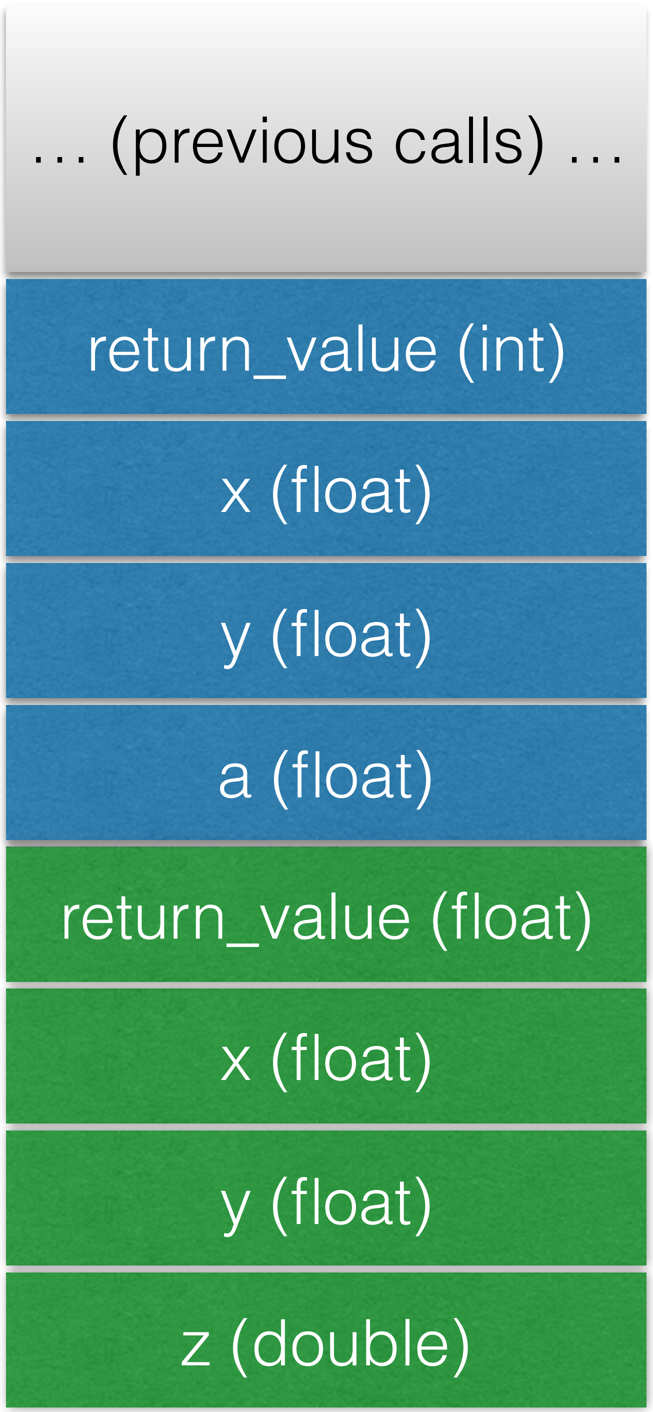

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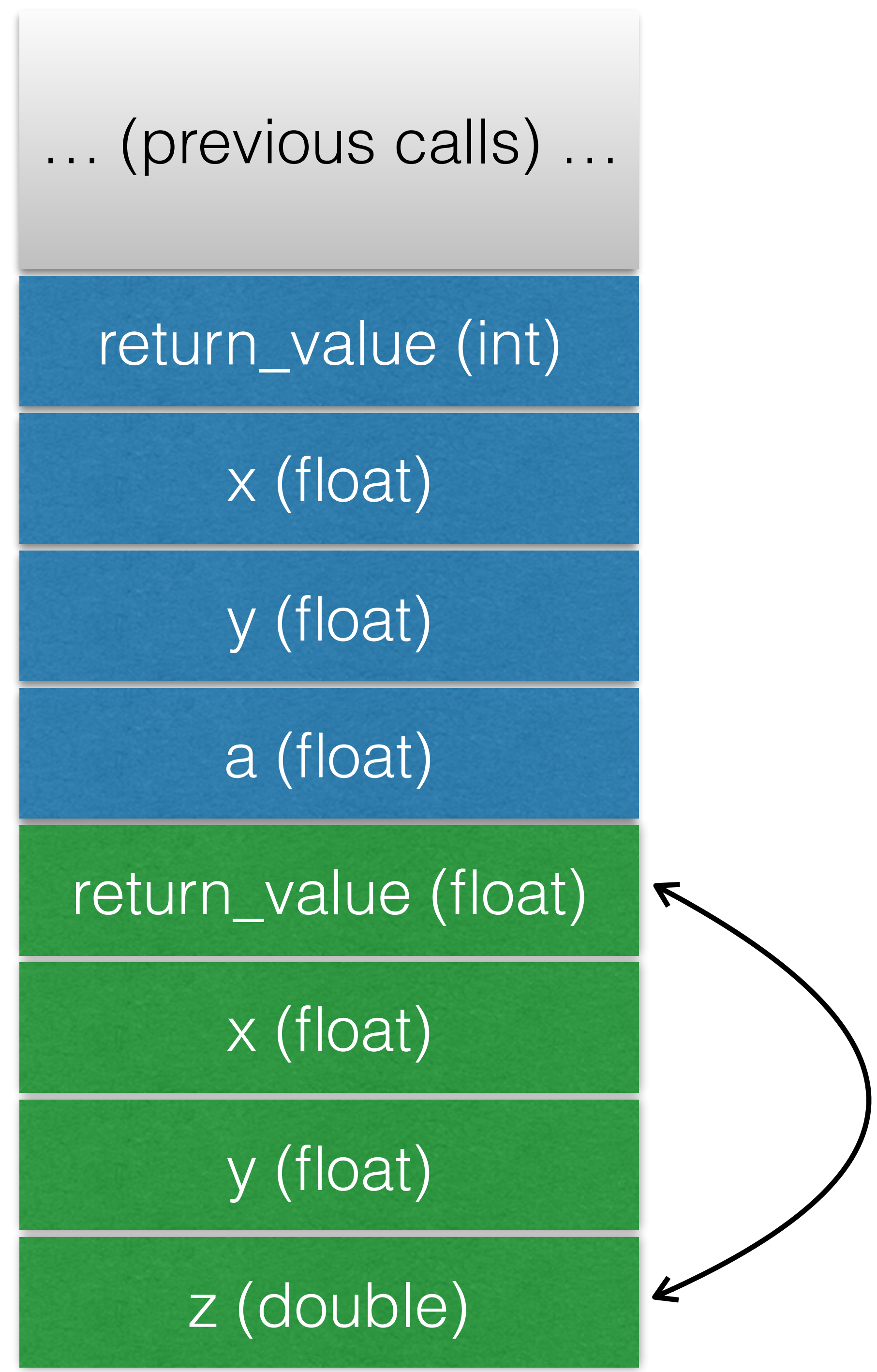

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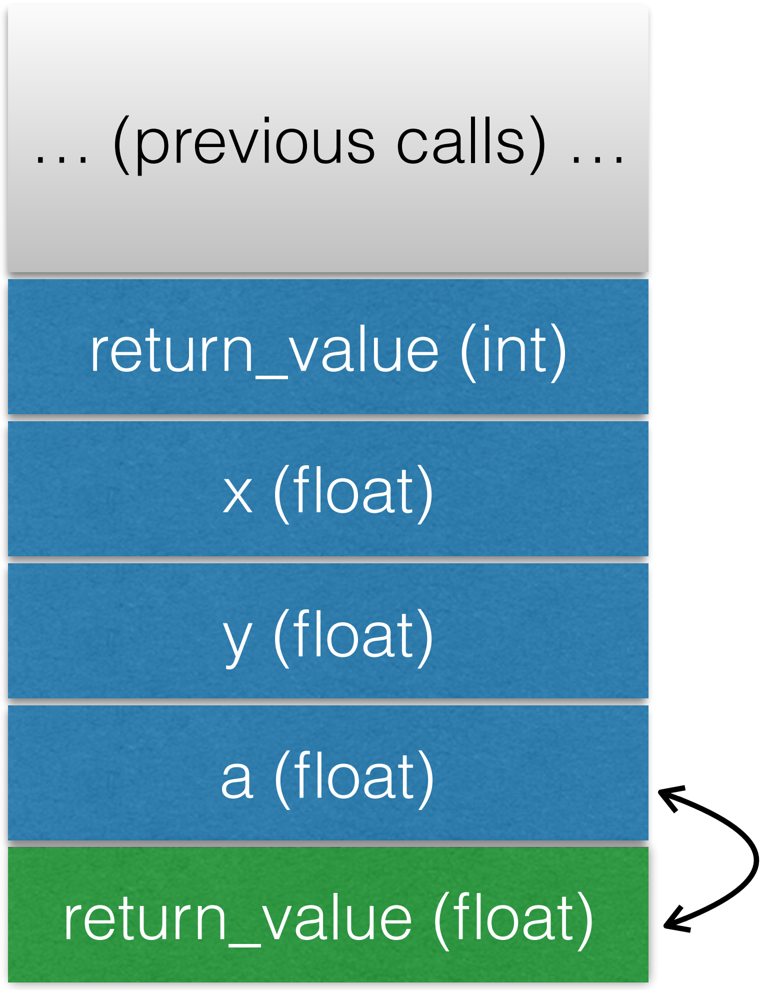

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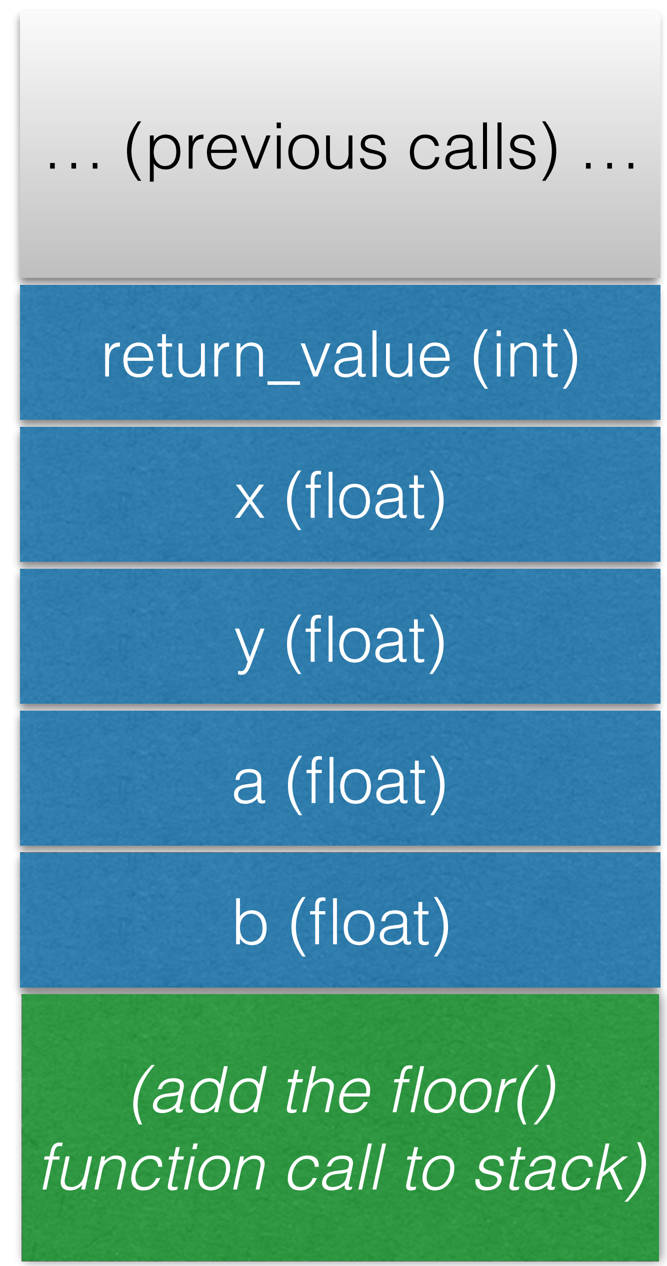

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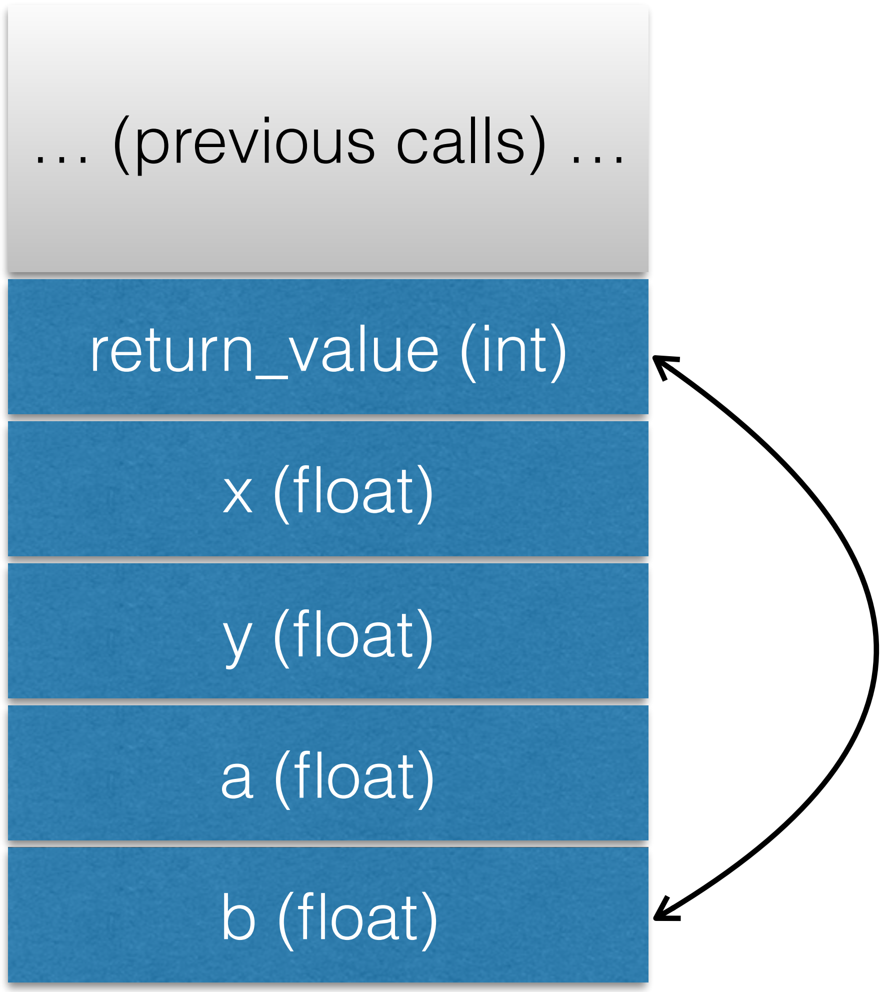

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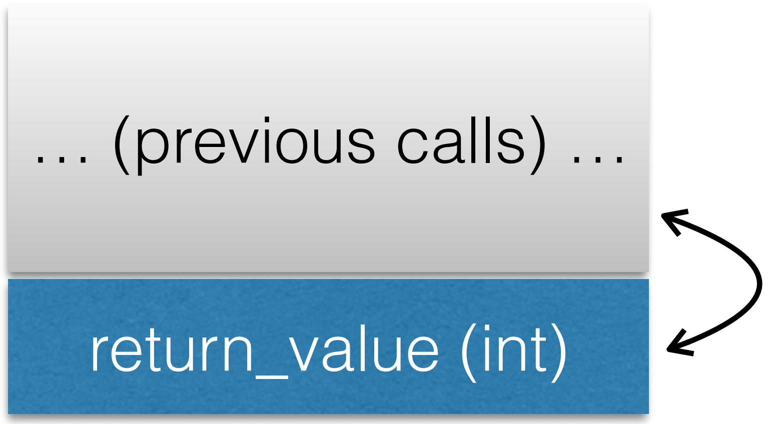
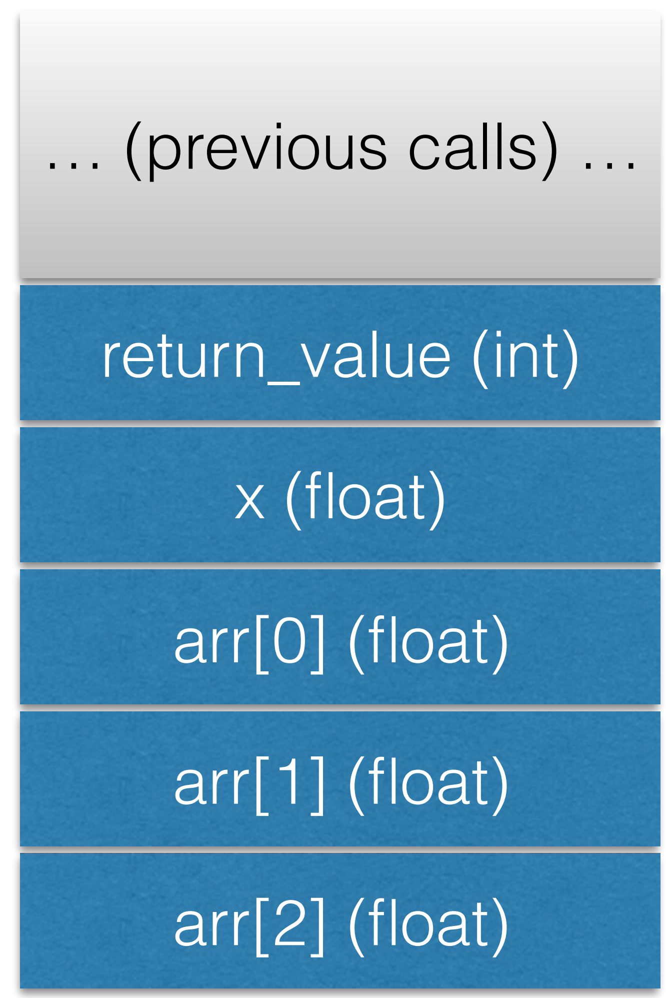


The Heap

- Allocating space for arrays on the stack:
  - if the size is known at compile time then the complier knows how much space to make on the stack
  - deleted once function returns

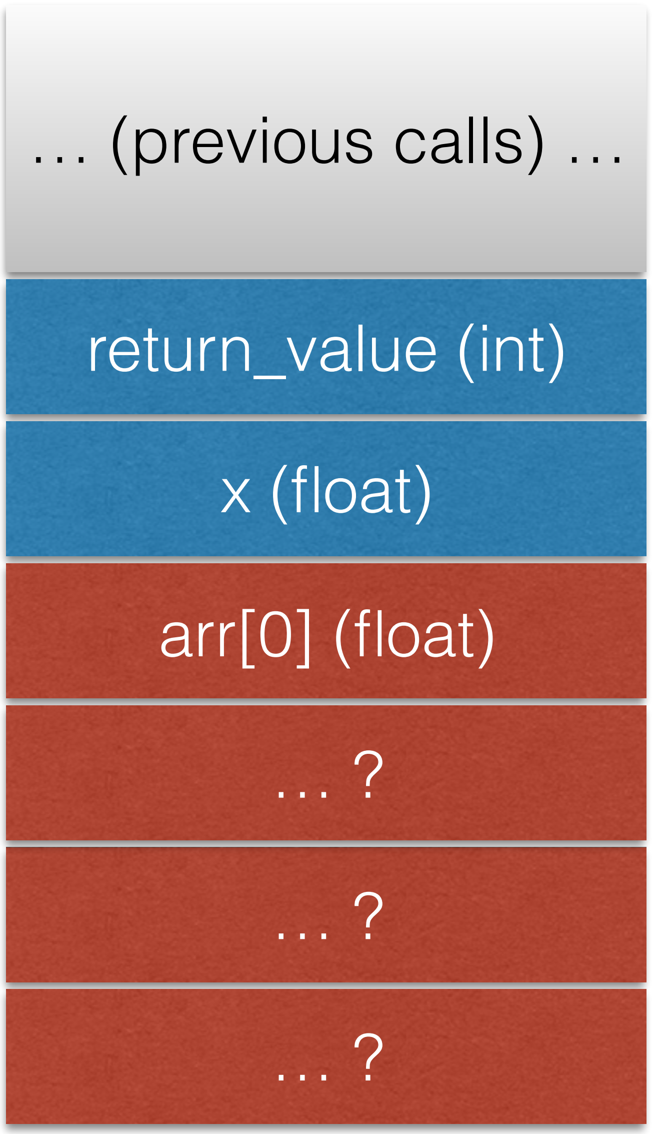
```
int foo(float x)
{
    float arr[3];
    ...
}
```



The Heap

- Allocating space for arrays on the stack:
    - but what if the array size is not known at compile time?

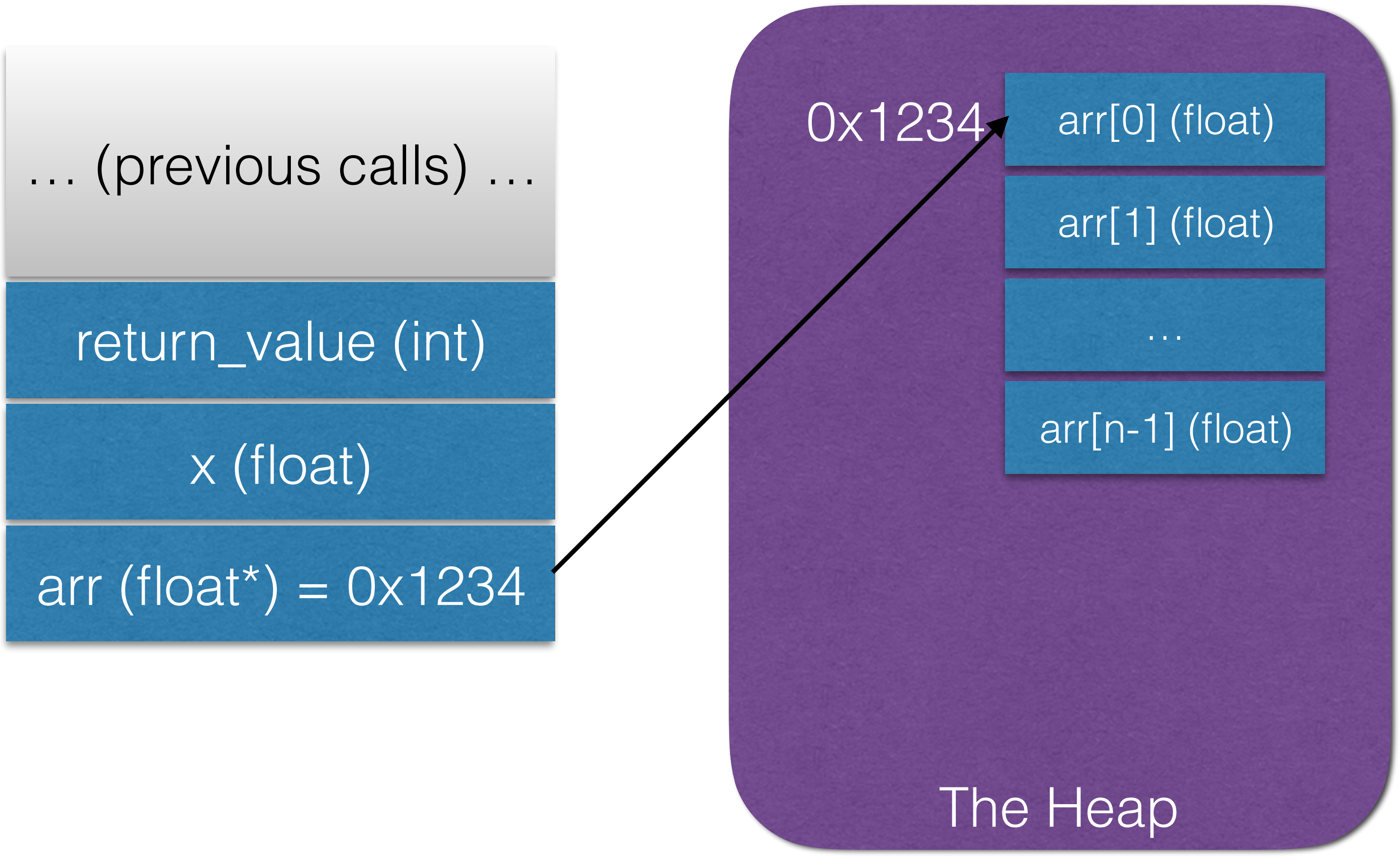
```
int foo(float x, size_t n)
{
    float arr[n]; // error
    ...
}
```



The Heap

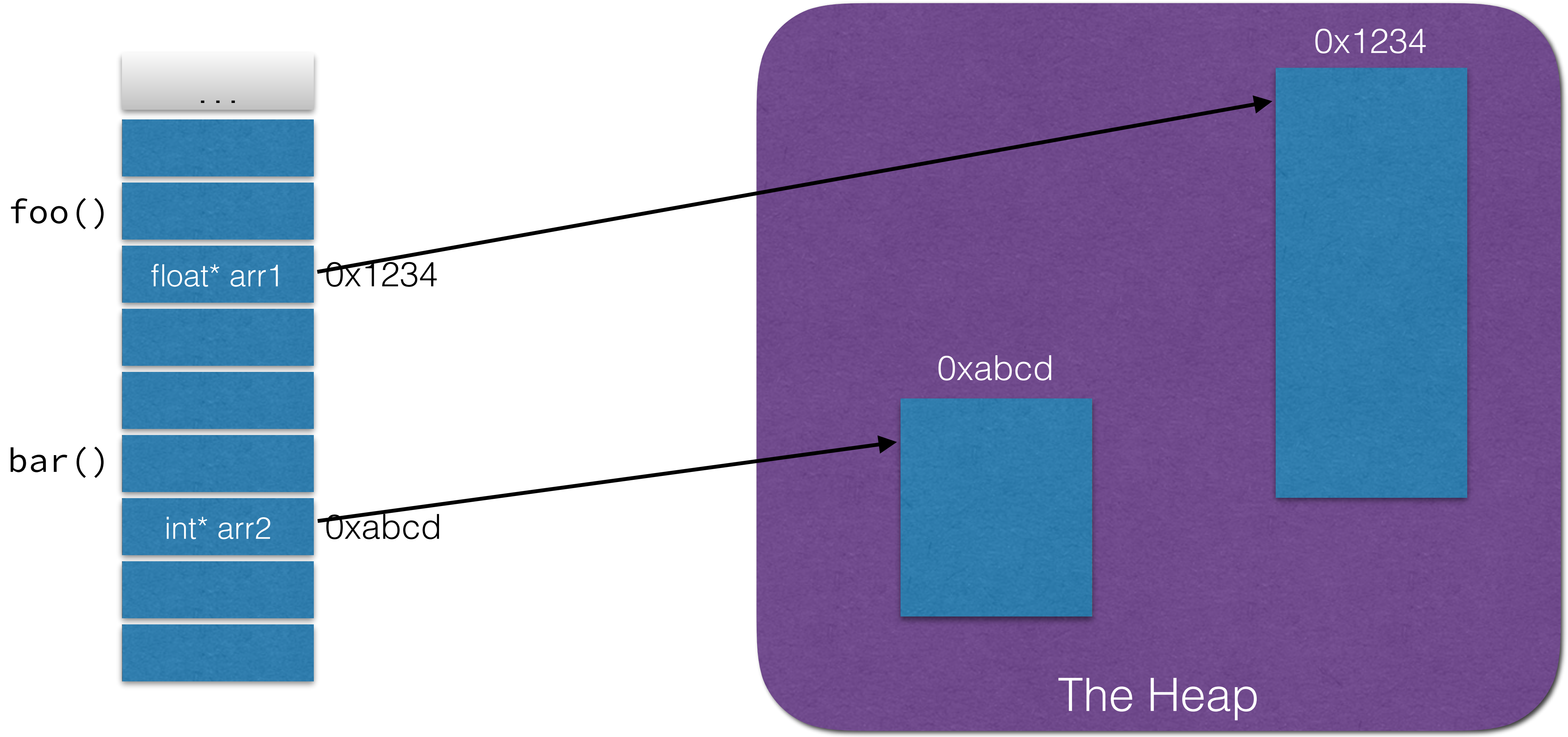
- Instead, program allocates memory separate area called *the heap*.
    - The stack contains the “address” of that location in memory
    - OS tries to find requested *contiguous* memory during runtime

```
int foo(float x, size_t n)
{
    float* arr = malloc(
      n*sizeof(float));
}
```



The Heap

- Caution: heap allocations can stick around after function calls (unlike stack allocations)
  - "Garbage collection" = automatic heap cleaning
  - C does not do garbage collecting - manually "free" heap allocations (Python does GC)



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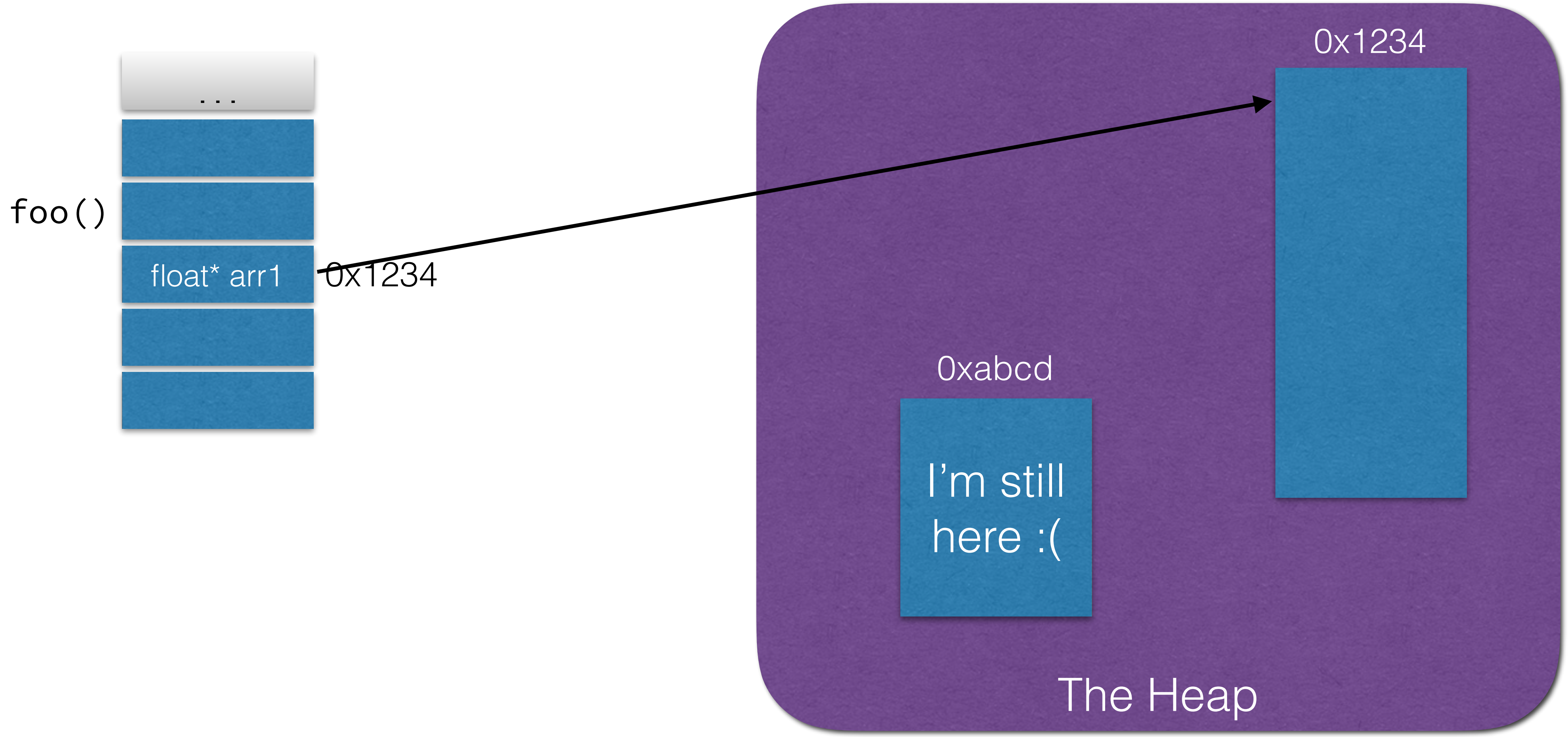

Stack and Heap

- Next week dive into C
    - spend almost two weeks on it —> plenty of time
- Memory management more direct than in Python

Demo

Navigating the stack in Python using `pdb`