MFD :

$$H_0 \sim \mathcal{N}(0, \cancel{x}\ \sigma_w^2) \rightarrow \frac{1}{\sqrt{2\pi\sigma_w^2}} e^{\frac{-x^2}{2\sigma_w^2}}.$$

$$H_1 \sim \mathcal{N}(z(n), \sigma^2)$$

Derivation of MFD :

$$H_0 : x(n) = w(n)$$

$$H_1 : x(n) = s(n) + w(n) \rightarrow \mathcal{N}(0, \sigma^2)$$

Primary user signal.

Let there be N independent signals,

~~H~~ H_0 : Primary user absent.

$$\prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-x(i)^2}{2\sigma^2}\right)$$

H_1 : Primary user present.

Signal Received is: $$\prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-|x(i) - s(i)|^2}{2\sigma^2}\right)$$

Priori probability:

$$P(H_1) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^N \exp\left(-\left(\frac{|\bar{x}-\bar{s}|^2}{2\sigma^2}\right)\right)$$

$$P(H_0) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^N \exp\left(\frac{-|\bar{x}|^2}{2\sigma^2}\right)$$

Here N independant signal present. So probabil is product of individual.

$\bar{x}$ & $\bar{s}$ → Vectors containing N signals

Likelihood Ratio Test:

$$\delta(x) = \frac{P(H_1)}{P(H_0)}$$

Threshold

$$\delta(x) = \begin{cases} \geq \gamma & H_1 \text{ (signal present)} \\ < \gamma & H_0 \text{ (signal absent)} \end{cases}$$

Tedious!

Log Likelihood ratio test is done

For H_1:

$$\ln(\gamma(x)) = \left(\frac{-1}{2\sigma^2}\right)\left(\|\bar{x}-\bar{s}\|^2 - \|\bar{x}\|^2\right) \geq \ln\gamma$$

$$\Rightarrow \|\bar{x}\|^2 - \|\bar{x}-\bar{s}\|^2 \geq \ln(\gamma)\cdot 2\sigma^2$$

$$\|x\|^2 - \|x\|^2 - \|s\|^2 + 2\bar{s}^T\bar{x} \geq \ln(\gamma)\, 2\sigma^2$$

$$\therefore \bar{s}^T\bar{x} \geq \frac{2\sigma^2\ln(\gamma) + \|s\|^2}{2} = \gamma'$$

$$\therefore \text{If } \bar{s}^T\bar{x} \geq \gamma' \rightarrow H_1$$

$$< \gamma' \rightarrow H_0$$

For H_0,

$$\bar{s}^T \bar{x} = s(1)x(1) + s(2)w(2) + \dots, s(N)w(N).$$

Mean is 0

Variance $\|\bar{s}\|^2 \sigma^2$.

Probability given H_0

$$P_{FA} = P\left(\bar{s}^T \bar{x} \geq \gamma' \mid H_0\right)$$

$$\boxed{P_{FA} = Q\left(\frac{\gamma'}{\sigma \|s\|}\right)}$$

$$\boxed{\gamma' = \sigma \|s\| \cdot Q^{-1}(P_{FA})} \longrightarrow 1$$

For H_1,

9 $$P_d = P\left(\bar{s}^T\bar{x} \geq \gamma' \mid H_1\right)$$

10 Here $\bar{x} = \bar{s} + \bar{w}$

11 $\therefore \bar{s}^T\bar{x}$ is with mean $\|s\|^2$ & variance

12 $\sigma^2\|s\|^2$

1

$$P_d = Q\left[\frac{\gamma' - \|\bar{s}\|^2}{\sigma\|s\|}\right] \quad -(2)$$

2

3

4 From (1) & (2).

5 $$P_d = Q\left(Q^{-1}(P_{FA}) - \underbrace{\sqrt{\frac{\varepsilon}{\sigma^2}}}_{SNR}\right)$$

6

7