# Performance of Spectrum Sensing Using Welch's Periodogram in Rayleigh Fading Channel

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*Abstract*—**In this paper we present theoretical performance evaluation of spectrum sensing with energy detection using Welch's periodogram for cognitive radio systems. We generalize the theoretical expressions for the probability of detection and the probability of false alarm of energy detection in Rayleigh fading channel to the case of Welch's periodogram. We verify the theoretical results by simulations both in single node and cooperative sensing scenarios. In particular, cooperation is crucial in fading environment. Protection of primary systems from harmful interference is the key requisite for the introduction of cognitive radio systems into the future spectrum regulatory framework if the systems are deployed on the same spectrum bands. The primary user's concern is how often it could be susceptible to potential interference from the cognitive radio system, which, as we show, is dependent on the probability of detection. Therefore, performance evaluation and in particular the probability of detection is critical in assessing the potential capabilities of the future cognitive radio systems.**

## I. INTRODUCTION

The radio frequency spectrum is a limited natural resource. The use of radio spectrum requires certain level of administration to protect the wireless systems from harmful interference, as has been demonstrated during the past 100 years. During the decades, spectrum bands have been allocated to different services, such as mobile, fixed, broadcast, fixed satellite and mobile satellite services, along with their appearance. Each country has an administration that manages spectrum use in its area but international cooperation, e.g. at the International Telecommunication Union (ITU), is needed since radio waves propagate over national borders. More information on the use of radio frequencies is given in [1].

The challenge in today's wireless telecommunication market is the difficulty for new services to enter the market as acquiring access to spectrum is difficult. Cognitive radio systems with the capabilities to obtain knowledge and dynamically adjust their performance to the radio operational environment and learn from the results, offer a potential technical approach for the challenge. In fact, the work towards the international introduction of cognitive radio techniques into the spectrum regulatory framework that governs the use of radio spectrum has been started. While administrator in some countries, such as the Federal Communications Commission (FCC) in the US, promote cognitive radio techniques, the introduction of cognitive radio techniques in the global scale requires still much effort. For example in Europe the spectrum regulatory framework is more scattered and currently cognitive radio discussions are in the starting point.

Important step on the global scale for the possible deployment of cognitive radio systems is the next World Radiocommunication Conference of the ITU in 2011 (WRC-11) that will consider regulatory measures and their relevance to enable the introduction of software-defined radio and cognitive radio systems. If the cognitive radio systems used the spectrum belonging to a primary user owning the rights of use for the spectrum, the primary user should be well protected from harmful interference. Therefore, finding suitable performance metrics and evaluation of the cognitive radio techniques' performance with the metrics are crucial for the potential introduction of future cognitive radio systems. The detection of primary users is thus critical and if it is done with spectrum sensing techniques, the reliability and performance evaluation of the spectrum sensing techniques are important.

Typically, the performance of spectrum sensing is evaluated with the probability of detection and probability of false alarm that constitute to the receiver operating characteristics (ROC) [2]. From the primary user's point of view, the probability of detection is critical as it determines how often primary user is susceptible to potential interference from the cognitive radio system. This is because the time between failures in detecting the presence of primary user depends on the probability of detection. Therefore, we are interested in the probability of detection as a measure for spectrum sensing performance.

The fundamental results on the theoretical ROC for spectrum sensing using energy detection in additive white Gaussian noise (AWGN) and Rayleigh and Nakagami fading channels were presented in [3]. The theoretical performance analysis of energy detection presented in [3] was generalized to Welch's periodogram [4] in AWGN channel and verified by simulations in [5]. In fading environment spectrum sensing becomes more challenging due to the uncertainty from radio wave propagation. To obtain satisfactory performance, cooperation between several cognitive radio nodes is needed as proposed in [6]. In [6] theoretical performance evaluation was presented for cooperative sensing in fading channels. The ROC for autocorrelation based spectrum sensing was evaluated in Rayleigh fading channels with different diversity techniques in [7]. The influence of Rayleigh fading on cyclostationary

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feature detection was studied in [8]. In [9] the performance of cooperative spectrum sensing in Rayleigh fading channels was derived by taking into account the errors in the feedback channel. The performance of cooperative spectrum sensing in fading channels using linear combination of local statistics from individual cognitive radios was derived in [10].

In this paper we extend the theoretical performance analysis of energy detection from [3] to Welch's periodogram [4] in Rayleigh fading channel. We present the theoretical probability of detection and probability of false alarm of Welch's periodogram in Rayleigh fading channels and verify the performance with simulations. The results are also extended to cooperative sensing. The fading is assumed to be slow compared to the observation window of Welch's periodogram, i.e. corresponding to a snap-shot method.

The rest of this paper is organized as follows. In Section II we present the theoretical expressions for the probability of detection and the probability of false alarm for energy detection in Rayleigh fading channel. In Section III we present the considered system model and extend the theoretical expressions of energy detection from Section II to Welch's periodogram. The results including analytical and simulated ROC are presented in Section IV. Finally, conclusions are drawn in Section V.

## II. PERFORMANCE OF SPECTRUM SENSING

### A. Performance measures

The performance of spectrum sensing is typically characterized with ROC that captures the relations of the probability of detection and the probability of false alarm that are interrelated. The probability of false alarm describes how efficiently the spectrum opportunities can be perceived. The probability of detection measures how well the cognitive radio system notices the presence of primary systems. The probability of detection is a critical performance measure because the sensing methods to be deployed in the future cognitive radio systems should protect the primary users from harmful interference if they are deployed on the same spectrum bands.

From the primary user's point of view the critical performance measure is the time between the potential appearance of sources for harmful interference that correspond to failing in detecting the presence of primary user. To fulfill the requirements set by the primary user, the time between failures in detection should be kept low. Following the radar literature [2] where the time between false alarms (i.e. time between detecting a target when there is no target) is critical, we can derive the time between failures in detection $T_{\mathrm{fd}}$ from the probability of detection $P_{\mathrm{d}}$ as

$$T_{\mathrm{fd}} = \frac{T_{\mathrm{dec}}}{1 - P_{\mathrm{d}}}, \tag{1}$$

where $T_{\mathrm{dec}}$ denotes the time between sensing decisions in periodic spectrum sensing. $T_{\mathrm{dec}}$ depends on the primary user's tolerance to harmful interference.

### B. Energy detection

In spectrum sensing using energy detection, the received signal is filtered, squared and integrated. In fading channel, the output from the integrator, i.e. the decision variable, follows conditional chi-square distribution that is conditioned on the channel state. The theoretical probabilities of detection and false alarm for energy detection in AWGN, Nakagami and Rayleigh fading channels were derived in [3].

The probability of false alarm $P_{\mathrm{f}}$ in AWGN and Rayleigh fading channels can be computed from the central chi-square distribution with $N$ degrees of freedom as [3]

$$P_{\mathrm{f}} = \frac{\Gamma\left(N/2, \frac{\lambda}{2\sigma^2}\right)}{\Gamma(N/2)}, \tag{2}$$

where $\Gamma(.,.)$ and $\Gamma(.)$ are the incomplete and complete gamma functions, respectively, $N$ is the number of degrees of freedom, $\sigma^2$ is the variance of the I component of the complex AWGN, and $\lambda$ is the decision threshold.

According to [3], the probability of detection $P_{\mathrm{d}}$ in AWGN channel can be calculated from the noncentral chi-square distribution with $N$ degrees of freedom as

$$P_{\mathrm{d}} = Q_{N/2}\left(\sqrt{\frac{s^2}{\sigma^2}}, \sqrt{\frac{\lambda}{\sigma^2}}\right), \tag{3}$$

where $Q_{N/2}(.,.)$ is the Marcum Q-function and $s^2$ is the noncentrality parameter of the distribution of the detector output.

The probability of detection in Rayleigh fading channel is obtained from (3) by averaging over the fading distribution [3]. Then, the probability of detection becomes

$$\begin{aligned} P_{\mathrm{d}} &= e^{-\frac{\lambda}{2\sigma^2}} \sum_{i=0}^{N/2-2} \frac{\left(\frac{\lambda}{2\sigma^2}\right)^i}{i!} + \left(\frac{2\sigma^2 + s^2}{s^2}\right)^{N/2-1} \\ &\times \left[ e^{-\frac{\lambda}{2\sigma^2 + s^2}} - e^{-\frac{\lambda}{2\sigma^2}} \sum_{i=0}^{N/2-2} \frac{\left(\frac{\lambda s^2}{2\sigma^2(2\sigma^2 + s^2)}\right)^i}{i!} \right]. \end{aligned} \tag{4}$$

### C. Cooperative detection

Fading environment significantly influences the probability of detection as the propagation path between the primary user and the cognitive radio node might experience a deep fade during the spectrum sensing. To guarantee high enough probability of detection acceptable to the primary users, cooperation between cognitive radio nodes will be needed.

There are different approaches and rules for combining the sensing results from several cognitive radio nodes, such as AND, OR and majority rules. Here we use the OR rule where a decision on the presence of the primary user is made if one of the cognitive radio nodes detects the primary user. The theoretical joint probabilities of false alarm and detection for $n$ cooperative nodes can be calculated from [6]

$$Q_{\mathrm{f}} = 1-\left(1-P_{\mathrm{f}}\right)^{n} \tag{5}$$

$$Q_{\mathrm{d}} = 1-\left(1-P_{\mathrm{d}}\right)^{n}, \tag{6}$$

where the probability of false alarm $P_{\mathrm{f}}$ and the probability of detection $P_{\mathrm{d}}$ are obtained from (2) and (4) for the energy detection in Rayleigh fading channel.

## III. WELCH'S PERIODOGRAM

### A. System model

We use the system model presented in Fig. 1 for evaluating the performance of spectrum sensing using Welch's periodogram [4] in Rayleigh fading channel. We are interested in the ROC including the probability of detection and the probability of false alarm in both single node and cooperative sensing scenarios. In the considered system model, the primary user transmits quadrature phase shift keying (QPSK) symbols over a 1 MHz bandwidth. The data is transmitted over a Rayleigh fading channel where AWGN is summed.

At the cognitive radio receiver, the received signal is first converted down to baseband. Energy detection with Welch's periodogram is used for detecting the presence of the primary user's signal. In Welch's periodogram the downconverted signal is first lowpass filtered. Then, the signal is divided into $M$ nonoverlapping or overlapping segments and the segments are processed with fast Fourier transform (FFT). After FFT the samples are squared and averaged over the $M$ segments. $L$ samples are taken from the output of Welch's periodogram around the assumed frequency of the baseband signal and an average over the $L$ samples is taken. Finally, the decision on the presence or absence of the primary user's signal is done by comparing output from the detector with a threshold. The difference of Welch's periodogram compared to the traditional energy detection comes from the segmenting of data before the FFT operation.

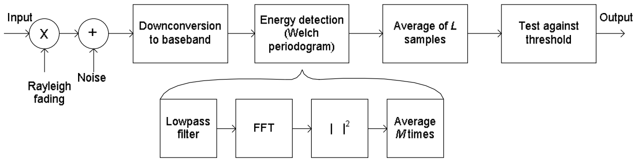


Figure 1. Block diagram of system model.

### B. Receiver operating characteristics

Next we derive the analytical receiver operating characteristics for Welch's periodogram in Rayleigh fading channel. We can rewrite the probability of false alarm and the probability of detection for the energy detection presented in (2) and (4) to Welch's periodogram [4] following the analysis presented in [5] for the AWGN channel. In the case of Welch's periodogram, the number of degrees of freedom $N$ in (2) is replaced by $2LM$, where $L$ is the number of frequency bins used for averaging around the assumed frequency of the baseband signal and $M$ is the number of segments over which averaging is done. The probability of false alarm for Welch periodogram can then be analytically calculated from

$$P_{\mathrm{f}} = \frac{\Gamma\left(LM, \dfrac{\lambda}{2\sigma^{2}}\right)}{\Gamma\left(LM\right)}. \tag{7}$$

The probability of detection for the energy detection in (4) can be rewritten in the case of Welch's periodogram by replacing $N$ with $2LM$ and approximating the noncentraility parameter $s^2$ with $LMA^2T$, where $A$ is the signal amplitude and $T$ is the symbol length. Then, the probability of detection for Welch's periodogram in Rayleigh fading channel becomes

$$P_{\mathrm{d}} = e^{-\frac{\lambda}{2\sigma^{2}}} \sum_{i=0}^{LM-2} \frac{\left(\dfrac{\lambda}{2\sigma^{2}}\right)^{i}}{i!} + \left(\frac{2\sigma^{2}+LMA^{2}T}{LMA^{2}T}\right)^{LM-1} \times \left[ e^{-\frac{\lambda}{2\sigma^{2}+LMA^{2}T}} - e^{-\frac{\lambda}{2\sigma^{2}}} \sum_{i=0}^{LM-2} \frac{\left(\dfrac{\lambda LMA^{2}T}{2\sigma^{2}(2\sigma^{2}+LMA^{2}T)}\right)^{i}}{i!} \right]. \tag{8}$$

For cooperative sensing using Welch's periodogram with the OR rule, the theoretical joint probabilities of false alarm and detection for $n$ cooperative nodes can be calculated from (5) and (6) where the probability of false alarm $P_{\mathrm{f}}$ and the probability of detection $P_{\mathrm{d}}$ are obtained from (7) and (8), respectively.

## IV. RESULTS

The analytical results for the performance of Welch's periodogram in Rayleigh fading channel derived in Section III are next verified with Monte Carlo simulations. We are interested in the ROC of Welch's periodogram that captures the relations of the probability of detection and the probability of false alarm and consider both single node sensing and cooperative sensing.

In the simulations complex QPSK signals are transmitted with symbol rate $R_{\mathrm{s}} = 500$ ksymbols/s over a one tap Rayleigh fading channel. The fading is assumed to be slow compared to the observation interval of the sensing method. Thus, the channel is assumed to remain constant during the transmission of the data block but it varies randomly between the transmissions of consecutive data blocks. Independent complex AWGN samples are added to the signal after fading. Welch's periodogram is used to detect the presence of the primary user signals. The number of frequency bins averaged around the assumed frequency of the baseband signals $L$ is equal to 1 or 10. The number of segments $M$ to which the received signal samples are divided before FFT is equal to 1 or 8. We use nonoverlapping segments. Welch's periodogram uses an FFT of size 512 or 1024 which at the same time is equivalent to the segment length and the length of the rectangular window. The block length is 410 symbols and the symbol length $T$ is 20. Thus, 8200 samples are taken from from the primary user's signal. The product of the FFT size and $M$ corresponds to the number of samples used for processing in Welch's periodogram, which varies from 512 to 8192 with different combinations of input parameters. Signal-to-noise ratio (SNR) is $\frac{E}{N_0} = \frac{A^2 T}{2\sigma^2}$ where the noise variance $\sigma^2$ is 0.5.

To quantify the different combinations of the probability of detection $P_d$ and the probability of false alarm $P_f$, we use a sliding threshold $\lambda$. The threshold is slid between the smallest and largest values of the output of the detector with a small step size and thus all combinations of $P_d$ and $P_f$ can be captured.

Figs. 2-6 present the ROC of Welch's periodogram in AWGN and Rayleigh fading channels for single node and cooperative sensing with different parameter values. For single sensing node case, the theoretical ROC for Welch's periodogram in Rayleigh fading channels is calculated from (7) and (8), and in AWGN from (2) and (3) with the same parameter changes as in the Rayleigh fading (see Section III B or [5]). The theoretical ROC for cooperative spectrum sensing in Rayleigh fading channel using the OR decision rule is obtained by inserting (7) and (8) into (5) and (6).

In Fig. 2 the theoretical and simulated ROC for single sensing node are given in AWGN and Rayleigh fading channels with SNR equal to 2 dB or -3dB, $M$ equal to 8, $L$ equal to 1 and FFT size equal to 1024. The simulations verify the theoretical results. The performance in Rayleigh fading channel is severely degraded as compared to the AWGN case which shows that single node sensing is not reliable in fading environment due to uncertainty of radio wave propagation. Fig. 3 shows the same results for the single sensing node as Fig. 2 but now $M$ is equal to 1 and $L$ equal to 10. The results are close to those of Fig. 2 and follow the theoretical results. In Fig. 4 the same parameters are used as in Fig. 3 but now FFT size is changed from 1024 to 512. This change only slightly degrades the performance of Welch's periodogram.

Next we consider cooperative spectrum sensing. Figs. 5 and 6 show the theoretical and simulated results for two and three cooperative nodes, respectively, using the same parameter values as Fig. 2. The theoretical results for AWGN are shown for comparison to show how significantly Rayleigh fading degrades the performance. In fading environment, cooperation is therefore necessary to obtain reliable sensing results.

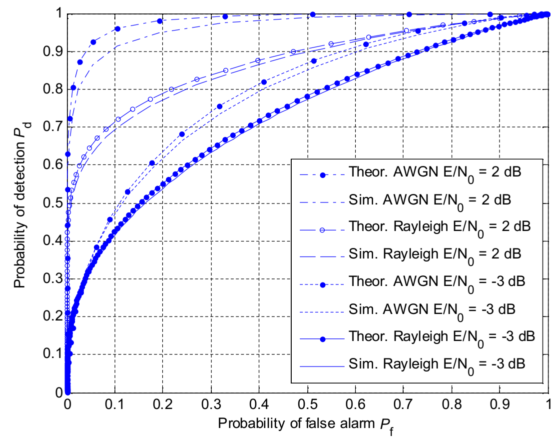


Figure 2. ROC for single sensing node in AWGN and Rayleigh fading channels with SNR = 2 dB or -3dB, $M = 8$, $L = 1$, and FFT size = 1024.

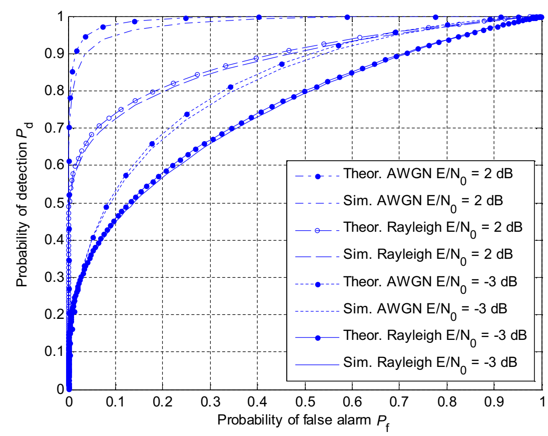


Figure 3. ROC for single sensing node in AWGN and Rayleigh fading channels with SNR = 2 dB or -3dB, $M = 1$, $L = 10$, and FFT size = 1024.

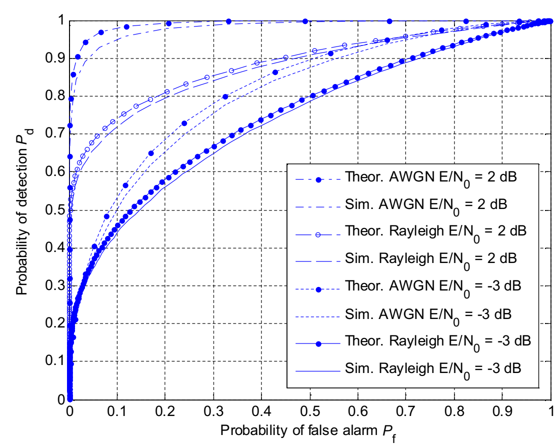


Figure 4. ROC for single sensing node in AWGN and Rayleigh fading channels with SNR = 2 dB or -3dB, $M = 1$, $L = 10$, and FFT size = 512.

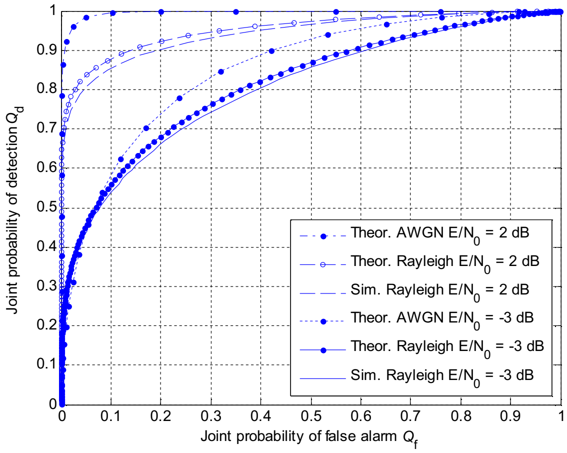


Figure 5. ROC for two cooperative sensing nodes in Rayleigh fading channel with SNR = 2 dB or -3dB, $M = 8$, $L = 1$, and FFT size = 1024.

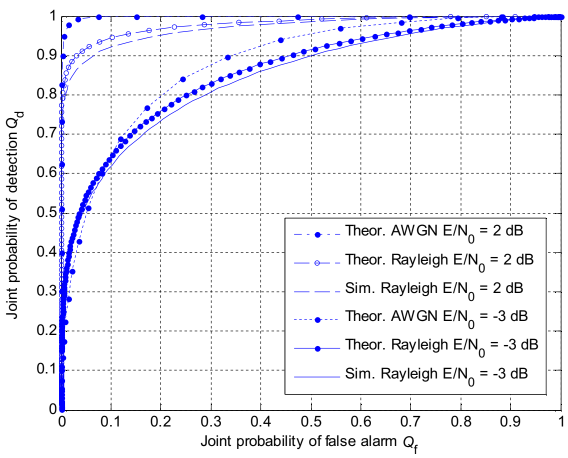


Figure 6. ROC for three cooperative sensing nodes in Rayleigh fading channel with SNR = 2 dB or -3dB, $M = 8$, $L = 1$, and FFT size = 1024.

## V. Conclusion

In this paper, we have evaluated the performance of spectrum sensing using Welch's periodogram in Rayleigh fading channels for cognitive radio systems. The performance measures considered were the receiver operating characteristics that quantify the relations of the probability of detection and the probability of false alarm.

The probability of detection will be a critical performance measure for spectrum sensing. In particular, the introduction of cognitive radio techniques into the future spectrum regulatory framework requires taking the primary user system's view point if the systems are to be deployed on the same spectrum bands. Then it is critical how often the primary user of the spectrum tolerates failures in detection by the cognitive radio system, i.e. sources of potential interference to the primary user. For this we predict that the time between failures in detection becomes the crucial parameter. The time between failures in detection sets the requirements for the performance of spectrum sensing techniques in terms of probability of detection. This is because the time between failures in detection depends on the probability of detection that should be made very high.

We have derived theoretical expressions for the probability of detection and the probability false alarm for Welch's periodogam in Rayleigh fading channel from the general results of energy detection in Rayleigh fading. We have also verified the theoretical expressions with simulations. The sensing performance in Rayleigh fading channel is significantly lower compared to the AWGN channel. We have demonstrated the benefit from cooperation between cognitive radio nodes to improve the sensing performance. In fading environment, cooperation will be crucial to obtain sensing performance that is acceptable for the primary user.

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