Cooperative and Noncooperative Spectrum Sensing Techniques Using Welch's Periodogram in Cognitive Radios

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***Abstract*—Radio spectrum deployment is growing considerably. With this respect finding unutilized frequency channels for new applications has become much more challenging. The problem can be solved by letting unlicensed systems to dynamically use unexploited licensed bands. This kind of flexible spectrum usage requires telecommunication systems to be equipped by an ability to specify unoccupied parts of radio spectrum. One method to identify temporarily unused parts of radio spectrum is spectrum sensing. In this paper, we focus on spectrum sensing using Welch's periodogram. In particular, we generalize and apply the theoretical analysis of the energy detection to the Welch's periodogram. Furthermore, we extend our study to cooperative spectrum sensing. The results indicate that the cooperation between two radios provides the highest cooperation gain.**

I. Introduction

Existing policies for radio resource allocation for different communication applications are indeed inflexible due to the fact that most of the radio spectrum is usually allocated to specific services. These service providers are referred to as primary or licensed users. The deployment of radio frequencies for different applications is agreed internationally, since radio waves can cause harmful interference to other wireless systems. The need for radio spectrum is growing considerably, while number of novel technologies also increases rapidly. This kind of development has caused a clear lack of unallocated frequency channels [1]. This has led to several discussions about new approaches for spectrum sharing, due to the fact that either finding unoccupied channels or improving the way by which they are deployed has become much more challenging. The Federal Communication Commission (FCC) Spectrum Policy Task Force (SPTF) has noticed considerable fluctuations in the licensed bands exploitation in time and space [1]. It can be assumed that spectrum scarcity results from inefficient spectrum usage, as well as lack of available radio frequencies. The problem can be solved by allowing secondary users' transmissions in under-utilized bands. Basically, secondary users are not allowed to cause any kind of interference to the primary users. Earlier approaches limit unlicensed users to low powers in order to minimize the probability for interference. Such approaches have been used, for example, in ultra-wideband (UWB) [2].

However, nowadays it is possible to use more sophisticated methods, which can avoid interference by the use of other techniques than transmit power control. Cognitive radios [3] use spectrum more efficiently by adapting radio channel utilization based on the state of environment. Cognitive radios seek and exploit unused radio resources without causing interference to the incumbent users [4]. Avoiding interference to the licensed users demands techniques for identifying channel usage. To be capable to sense very weak signals, cognitive radios must have significantly better sensitivity than conventional radios [5]. Spectrum opportunities can be discovered, for instance, using database, beacons or spectrum sensing [6]. Spectrum sensing can be done, for example, by energy [7] or feature [8] detection. The Welch's method [9] is energy detection based method which attempts to improve the statistical properties of the periodogram method. The Welch method as also the other modified methods decrease the variance of the estimated spectrum at expense of increasing the bias and, hence, decreasing resolution [10]. The idea of the Welch's periodogram is to divide the data sequence into segments in order to reduce the large fluctuations of the periodogram. In the Welch's method these data segments are also allowed to overlap, which is a feature that distinguishes it from some other modified periodograms (such as Bartlett or Blackman-Tukey methods) [10].

In order to improve the performance of the spectrum sensing, several authors have proposed cooperation among secondary users (SU) [5], [2]. The performance improvement of the cooperative spectrum sensing comes from the fact that the probability that all users see deep fades is very low. Also the interference to the primary user (PU) is decreased. The cost for that, however, is the increased complexity and requirement for wider control channel due to increased control traffic between the nodes. The delays due to the combining and relaying processes reduce the time of the data transmission. In addition, independence and trust issues - such as malicious and malfunctioning nodes - can affect to the performance improvement of the cooperation [11]. In an experimental study in the Berkeley Wireless Research Centre [12] it was shown that the probability of detection monotonically increases as the separation between two cooperating radio increases. This is due to the fact that shadowing correlation degrades the performance of the cooperative sensing when SUs are close to each other [2].

The difference between hard and soft information combining strategy has been investigated in e.g., [13] and [11]. In a hard information combining strategy each sensing node performs local hypothesis test and reports a binary value indicating whether it believes that channel is occupied or not. The channel is decided to be free only if all of the nodes agree that it is free. Experimental study on the hard decision combining [12] concluded that the biggest cooperation gain is attained when moving from single radio to two cooperating radios. In the soft information combining, each node sends the full observation and a likelihood ratio test is used for making the decision whether the channel is occupied or not. In [13] it was concluded that soft combining clearly outperforms hard combining in terms of probability of missed opportunity, whereas, in [11] the difference between the two combining methods was reported to be small. This can be explained by the fact that in [13] the radios are assumed to be tightly synchronized and therefore they can collectively overcome the lower bound on the SNR, below which detection is not reliable. In order to assume tightly synchronised radios, a central controller is required. Thus, the practical assumption depends on the considered network topology.

In [14] and [15] the performance of traditional energy detection has been analyzed. In this paper we generalize the analysis and apply those to Welch's periodogram. In addition we extend our study to cooperative spectrum sensing. The rest of the paper is organized as follows. In section II, we define the system model. In section III spectrum sensing analysis are presented. In section IV contains results and section V concludes the paper.

II. System model

The simulation model is presented in Fig.1. The primary user sends quadrature phase shift keying (QPSK) symbols on a 1 MHz frequency channel with a carrier frequency of 4 MHz over a complex additive white Gaussian noise (AWGN) channel. Symbols are sent at the symbol rate of R_{S} = 500 ksymbols/s. First, the noise has been added to the RF input signal. Then received signal has been downconverted to baseband. The Welch periodogram is used for detection by alerting when received signal energy exceeds the detection threshold. The number of frequency bins to be averaged around the zero frequency is denoted by L. The input signal is segmented in time domain and the number of segments is denoted by M.

III. Spectrum sensing

The goal of the spectrum sensing is to decide between the two hypothesis, namely

$$x(t) = \begin{cases} n(t) & , H_0 \\ hs(t) + n(t) & , H_1 \end{cases} \tag{1}$$

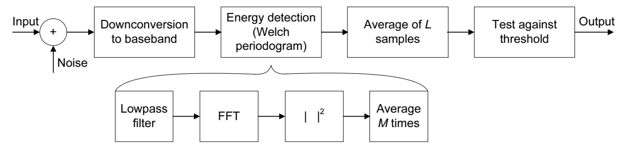

Fig. 1. Block diagram of the simulation model.

where $x(t)$ is the complex signal received by the cognitive radio, $s(t)$ is the transmitted signal of the primary user, $n(t)$ is the AWGN and h is the complex amplitude gain of the ideal channel, H_0 stands for the hypothesis: no signal transmitted, and H_1 stands for the hypothesis: signal transmitted [2].

For a conventional energy detector, the output of the integrator, which serves as decision statistic, follows the chi-square distribution [14], [15]. The probability of false alarm (P_{f}) can be computed using central chi-square (or gamma) PDF with N degrees of freedom [14]

$$P_{\mathrm{f}} = P\{Y > \lambda | H_0\} = \frac{\Gamma(N/2, \frac{\lambda}{2\sigma^2})}{\Gamma(N/2)}, \tag{2}$$

where $\Gamma(.,.)$ and $\Gamma(.)$ are the incomplete and complete gamma function respectively, N is the number of degrees of freedom, σ^2 is noise variance, Y is a decision statistic, and λ is the decision threshold. On the other hand, the probability of detection (P_{d}) can be computed using noncentral chi-square PDF with N degrees of freedom [14]

$$P_{\mathrm{d}} = P\{Y > \lambda | H_1\} = Q_{N/2}\left(\sqrt{\frac{s^2}{\sigma^2}}, \sqrt{\frac{\lambda}{\sigma^2}}\right), \tag{3}$$

where $Q_{N/2}(.,.)$ is Marcum Q-function. The parameter $s^2 = \sum_{i=0}^{N/2} A_i^2$ is called the noncentrality parameter of the distribution and A_i is the signal amplitude.

The theoretical power density spectrum curve of linearly modulated signal, when the sequence of information symbols is uncorrelated, is given by [16]

$$\Phi(f) = \sigma_i^2 A^2 T \mathrm{sinc}^2(\pi f T) + A^2 \mu_i^2 \delta(f), \tag{4}$$

where f denotes frequency bins, σ_i^2 is variance of information symbol, μ_i is information symbols mean, $\delta(f)$ is Dirac's delta function and T is symbol length. When information symbols have zero mean, i.e., $\mu_i = 0$ the discrete frequency components vanish. This condition is satisfied when the information symbols are equally likely and symmetrically positioned in the complex plane as in the QPSK and quadrature amplitude modulation (QAM) signal cases.

In the calculations of theoretical P_d and P_f we assume that the noncentrality parameter s^2 is a constant. In reality, it is a random variable, but if the degrees of freedom is large, s^2 approaches a constant value. For simplifying the analysis, we do not take into account an aliasing phenomenon and neither the possible overlapping in Welch's periodogram.

Applying the analysis into the Welch's periodogram, we can see that (2) and (3) are valid for all frequency bins independently because the frequency bins are orthogonal, the additive noise is white and Gaussian, and a rectangular window is used. Using (4), we can calculate the energy of different frequency bins. Thus, the noncentrality parameter becomes

$$s^2 = M \sum_{l=-L/2}^{L/2} A^2 T(\mathrm{sinc}(\pi f_l T))^2, \tag{5}$$

where L is the number of frequency bins to be averaged around the zero frequency and $f_l = l/(TN_{FFT})$ are the corresponding frequencies. If L is small enough compared to total number of frequency bins i.e., $|f_l| << 1/T$, we note that sinc^2-function remains almost a constant. Thus (5) reduces to $s^2 \approx LMA^2T$. Finally, for Welch's periodogram, (2) and (3) can be rewritten as

$$P_\mathrm{f} = P\{Y > \lambda|H_0\} = \frac{\Gamma(LM, \frac{\lambda}{2\sigma^2})}{\Gamma(LM)} \tag{6}$$

$$P_\mathrm{d} = P\{Y > \lambda|H_1\} = Q_{LM}\left(\sqrt{\frac{LMA^2T}{\sigma^2}}, \sqrt{\frac{\lambda}{\sigma^2}}\right), \tag{7}$$

where the product LM corresponds to $N/2$.

In the cooperative sensing simulations, hard decision combining, is considered and the decisions from cooperating radios are combined using simple OR-rule: if one of the cooperating radios detects a primary user, the decision is made that a primary user is present. The joint probability for detection Q_d and false alarm Q_f can therefore be given as [14]

$$Q_\mathrm{f} = 1 - (1 - P_\mathrm{f})^{n_\mathrm{CR}} \tag{8}$$

$$Q_\mathrm{d} = 1 - (1 - P_\mathrm{d})^{n_\mathrm{CR}}, \tag{9}$$

where n_CR denotes the number of cooperating radios and probabilities of false alarm P_f and detection P_d are calculated using (6) and (7), respectively.

IV. Results

The analysis of the Welch's periodogram are verified by Monte Carlo simulations. Each simulation scenario is repeated 10^5 times. A complex AWGN channel and QPSK signal are used in the simulations. The SNR = E/N_0 values are -7 and 3 dB. Used FFT lengths N_FFT, which correspond to the segment length and rectangular window, are 512 or 1024. When we do not use overlapping, block lengths N_b are 205 and 410 symbols for FFT sizes 512 and 1024, respectively. And correspondingly when using overlapping N_b is 116 or 231. In the analysis and simulations we use T = 20.

Figs. 2 – 4 present theoretical and simulated receiver operating characteristic (ROC) curves for the Welch periodogram. The theoretical ROC curve is obtained using equations (6) and (7). In Fig. 2, there are theoretical and simulated ROC-curves for one and eight segments. The used SNR values are -7 dB and -2 dB. N_FFT is 1024. In this case the number of frequency bins to be averaged around the zero frequency L is 1. Now we compare two cases. In the first case we use only one segment which corresponds to periodogram. In the second case we use 8 non-overlapping segments. It can be seen that the performance is better when using eight non-overlapping

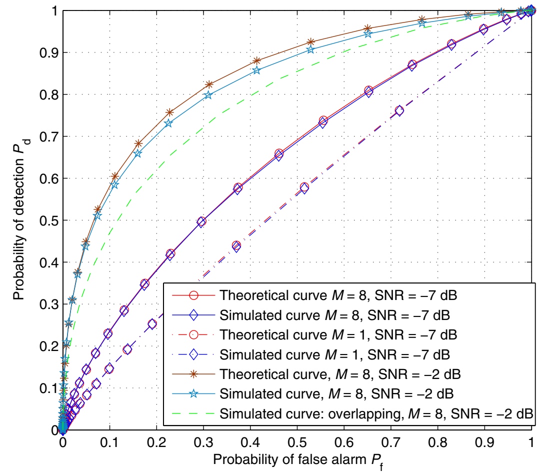

Fig. 2. One and eight segments of length N_FFT = 1024.

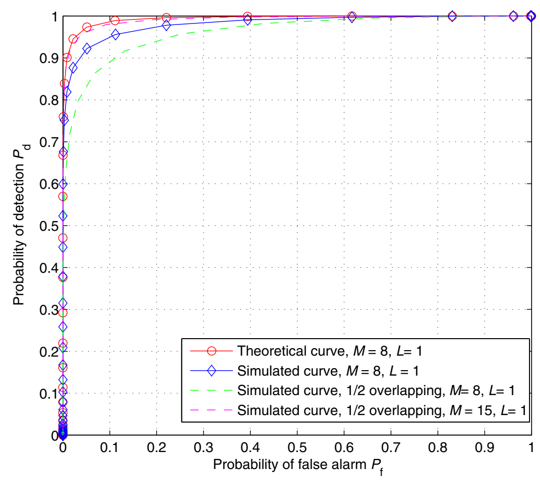

Fig. 3. Eight segments of length N_FFT = 1024.

segments compared to one segment case. Also one overlapping case is presented with eight segments and block length N_b of 231 samples. In non-overlapping case N_b is 410. The segments are overlapping on each other by half of the N_FFT samples. We can notice that when using overlapping the performance is almost the same as without overlapping but now the length of the block can be much smaller. Fig. 3 shows ROC-curves when N_FFT = 1024, SNR = 3 dB, $L = 1$ and $M = 8$. We have also simulated case where M = 15 and N_b = 410, i.e. the block length corresponds to case when we do not use overlapping. We clearly see performance improvement when we compare overlapping case with 15 segments to non-overlapping case with eight segments. Fig. 4 presents overlapping and non-overlapping cases when SNR = -7 dB, N_FFT = 512, L is 1 or 10, and in overlapping case M is 8 or 15. When using L = 1, we can see that performance is worse compared to the case when L = 10. In addition, simulations show that we can achieve small gain using overlapping compared to the non-overlapping case. However, even with the averaging and overlapping the

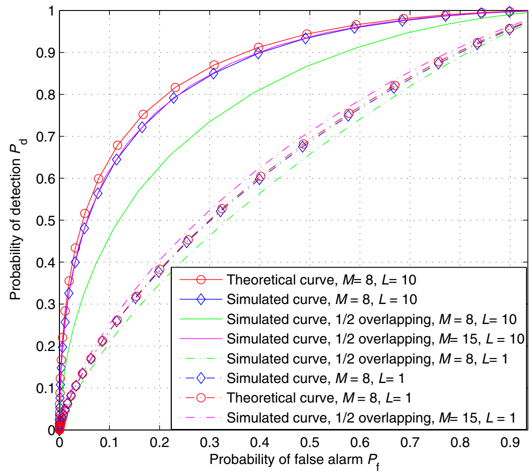

Fig. 4. One, eight and fifteen segments of length N_{FFT} = 512.

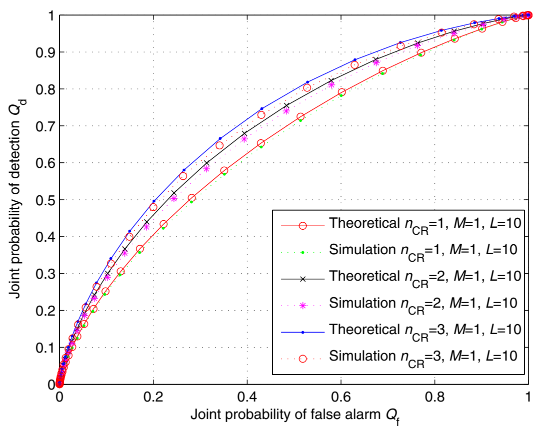

Fig. 5. One segment of length N_{FFT} = 512.

probability of detection is low with low probabilities of false alarm, especially with low SNR values.

In the following, the results for cooperative sensing are presented. The probabilities of false alarm and detection are simulated for a number of cooperating cognitive radios n_{CR} of two and three. In the cooperative sensing simulations the E/N_0 = -7dB and the block length is $M{\cdot}N_{\mathrm{FFT}}$, thus, no overlapping is considered. The case of single cognitive radio is also shown for comparison. In Fig. 5 and Fig. 6, one segment is considered and the FFT lengths are N_{FFT} = 512 and N_{FFT} = 1024. The theoretical curves calculated using (6), (7), (8) and (9) are also shown as a reference. It can be noted that there is only 1 % of difference between theory and simulation in Fig. 5 and the difference is even smaller in Fig. 6. Otherwise the length of FFT does not have significant influence on the results and from both Fig. 5 and Fig. 6 it can be seen that by increasing the number of cooperating radios, both the probability of false alarm and the probability of detection are increased. With 20 % probability of false alarm the increase in probability of detection is 5 and 8 percentage units for two and three cooperating radios, respectively.

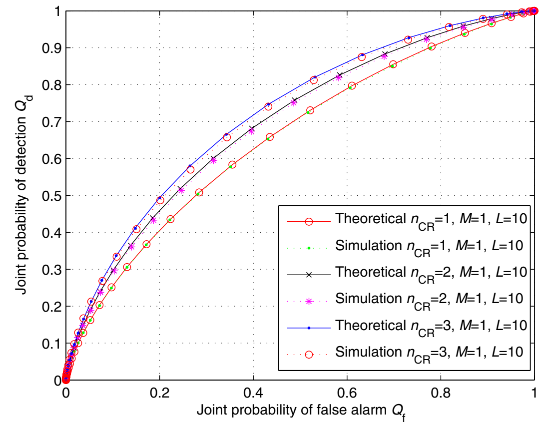

Fig. 6. One segment of length N_{FFT} = 1024.

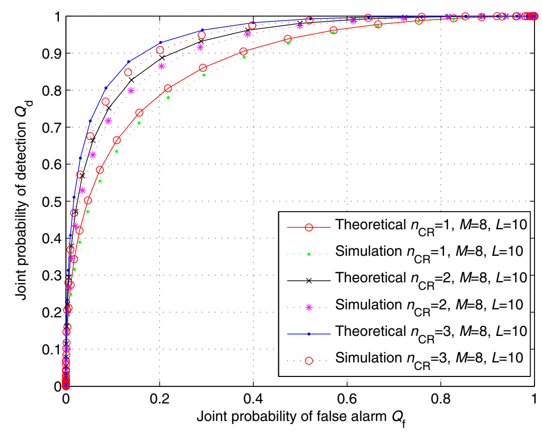

Fig. 7. Eight non-overlapping segments of length N_{FFT} = 512.

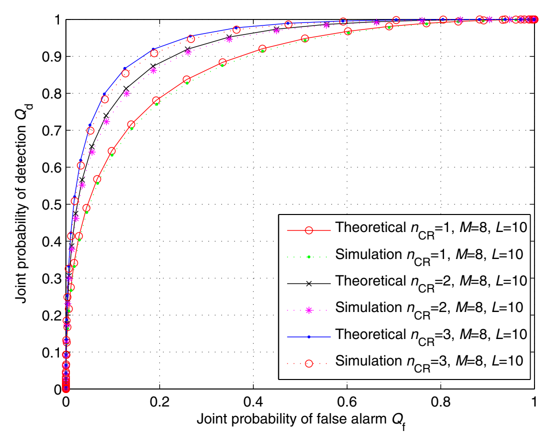

Fig. 8. Eight non-overlapping segments of length N_{FFT} = 1024.

In Fig. 7 and Fig. 8 eight non-overlapping segments are considered with lengths of N_{FFT} = 512 and N_{FFT} = 1024, respectively. From Fig. 7 it can be noted that the difference between theory and simulations is approximately three percentage units whereas in Fig. 8 it is about one percentage unit. It can also be seen from these figures that the cooperation gain becomes more significant than in the one segment periodogram. With

20 % probability of false alarm the increase in probability of detection is 10 percentage units and 13 percentage units for two and three cooperating radios, respectively. This supports the result from the experimental study in [12] indicating that the largest increase in probability of the detection is observed when moving from single radio to two cooperating radios.

V. Conclusion

In this paper, we studied Welch's periodogram from a spectrum sensing and cognitive radio perspective. We generalized and applied the previous theoretical analysis of the energy detection to the Welch's periodogram. Furthermore, we extended our study to cooperative spectrum sensing. The simulations show that Welch's periodogram signal detection method operates well for narrowband signals. Simulations confirm that Welch's periodogram enhances the performance of the periodogram method. The main limitations of the periodogram method yield from the variance. The periodogram is an inconsistent spectral estimator which means that it continues to fluctuate around the true PSD with a nonzero variance. This effect cannot be eliminated even if the length of the processed sample increases without a bound. Furthermore, the fact that the periodogram values are uncorrelated for large number of the processed samples makes the periodogram exhibit an erratic behavior. From the results on cooperative sensing presented in this paper it can be concluded that the highest increase in probability of detection is observed when moving from single cognitive radio to two cooperating radios, however, adding phenomena such as shadowing, multipath fading, or hidden terminal problem to the simulation model would add the unreliability of the sensing information and could therefore lead into results favoring more extensive cooperation between users. This would lead to a trade-off between reliable sensing information and the costs caused by more extensive cooperation - such as complexity and increased signaling. Adding shadowing would bring up another trade-off on the distance between the cooperating radios. Decreasing the distance would lead to lower delays; however, correlation of shadowing - caused by two radios being blocked by the same object - would degrade the performance of cooperative sensing when radios are close to each other. This would lead to the development of algorithms for finding the optimal radios for cooperation from the candidates. Before mentioned problems are not covered in this paper, however, they are interesting topics and subjects for future research.

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