MATHEMATICS

SECTION A

February 6, 2024

1 Vectors

1. Using vectors, find the value of x such that the four points **A** $(x, 5, -1)$,**B** $(3, 2, 1)$,**C** $(4, 5, 5)$ and **D** $(4, 2, -2)$ are coplanar.

2 Matrices

2. If x, y, z are different and $\Delta = \begin{vmatrix} x & x^2 & x^3 - 1 \\ y & y^2 & y^3 - 1 \\ z & z^2 & z^3 - 1 \end{vmatrix} = 0$,then using properties of determinants, show that $xyz = 1$.

3 Probability

3. In answering a question on a multiple choice questions test with four choices in each questions, out of which only one is correct, a student either guesses or copies or knows the answer. The probability that he makes a guess is $\frac{1}{4}$ and the probability the he copies is also $\frac{1}{4}$. The probability that the answer is correct, given that he copied it is $\frac{3}{4}$. Find the probabiliy that he knows the answer to the question, given that he correctly answered it.

4 Algebra

4. Prove that :

$$\sin^{-1}\frac{4}{5} + \tan^{-1}\frac{5}{12} + \cos^{-1}\frac{63}{65} = \frac{\pi}{2}$$

5 Differentiation

5. If $y = 5e^{7x} + 6e^{-7x}$, show that $\frac{d^2y}{dx^2} = 49y$.

6. Differentiate $\tan^{-1}\frac{3x-x^3}{1-3x^2}, |x| < \frac{1}{\sqrt{3}}$ w.r.t. $\tan^{-1}\frac{x}{\sqrt{1-x^2}}$.

6 Integration

7. Integrate:

$$\frac{e^x}{\sqrt{5 - 4e^x - e^{2x}}}$$

with respect to x.

7 Linear forms

8. Find the coordinates of the foot Q of the perpendicular drawn from the point $P(1,3,4)$ to the plane $2x-y+z+3=0$. Find the distance PQ and the image of P treating the planes as a mirror.

8 Geometry

9. Find the co-ordinates of the point, where the line $\frac{x+2}{1}=\frac{y-5}{3}=\frac{z+1}{5}$ cuts the yz-plane.

9 Function

10. Show that the function f in $A=R-\{\frac{2}{3}\}$ defined as $f(x)=\frac{4x+3}{6x-4}$ is one-one and onto. Hence, find f^{-1}.