COMPLEXITY: Exercise No. 1

1. Prove or disprove:

   (a) $(2n)! = O(n!^2)$.

   (b) $f(n) = O(n) \Rightarrow 2^{f(n)} = O(2^n)$.

   (c) There is a function f such that $f(n) = O(n^{1+\epsilon})$ for every $\epsilon > 0$ but $f(n) = \omega(n)$.

2. Describe explicitly (i.e., write down the transition function δ) a 1-tape DTM, which given an input $w \in \{0, 1\}^*$, converts it to the reversed string, without using additional space (that is, you are allowed to use only the part of the tape containing w, and the output is to be written there too).

3. Give a NDTM that accepts the *complement* of L_{pal} with space $O(\log n)$ and time $O(n)$.

4. Show that any 1-tape DTM which at any step may move to the left, to the right or stay, can be simulated by a regular 1-tape DTM (where the head has to move at any step).

5. Consider a 1-tape DTM that after each step can move its head either one step right or jump to the first position in the tape (but it can not move one step left). Show that such a machine can simulate an ordinary 1-tape DTM whose time complexity is $t(n)$, in $O(t(n)^2)$ time.