

Maths Assignment

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B.E. MECHANICAL

1. The maximum weight that an elevator in an apartment complex can accommodate is 800kg. The average adult weight be about 70 kgs with a variance of 200. What is the probability that the in safely reaches the ground when there are 10 adults in the lift?

Solution:

Given :

- The maximum weight that an elevator can accommodate is 800kg. The average adult weight be about 70 kgs with a variance of 200.
- It is given that there are 10 adults in the elevator whose each adult's average weight is 70kgs.
- So, $10 \times 70 = 700$

And the maximum weight an elevator can accommodate is 800kg.

- Still 100 kgs of weight is left.

So yes, the elevator can reach the ground safely.

Given mean = 70

variance = 200

hence mean for 10 adults = $10 \times 70 = 700$

variance for 10 adults = $10 \times 200 = 2000$

therefore standard deviation $SD = \sqrt{2000} = 44.72$

If the weight > 800 kg causes the elevator to "unsafely" reach the ground, then we can find the upper tail of our normal distribution:

$P(\text{Weight of 10 adults} > 800 \text{ kg})$.

$Z\text{-score} = (X - \mu) / SD = (800 - 700) / 44.72 = 2.24$

Hence $P(Z < 2.24)$, using z table we get 0.9875 or 98.75%

Hence it is safe to reach the ground when there are 10 adults in the lift.

2. The life of a 60-watt light bulb in hours is known to be norm distributed with $\sigma = 25$ hours. Create 5 different random samples of 100 bulbs each which has a mean life of $\bar{x} \sim 1000$ hours and perform one-way ANOVA with state it.

Solution:

The total sample size is $N = 5 * 100 = 500$

Therefore, the total degrees of freedom are

$$df_{\text{total}} = 500 - 1 = 499$$

The between groups degrees of freedom are

$$df_{\text{between}} = 5 - 1 = 4$$

And the within-groups degrees of freedom are

$$df_{\text{within}} = df_{\text{total}} - df_{\text{between}} = 499 - 4 = 495$$

$$\sum X_{ij} = 499712$$

$$(\sum X_{ij})^2 = 499691630$$

$$SS_{\text{total}} = (\sum X_{ij})^2 - 1/N(\sum X_{ij})^2 = 267464.112$$

$$SS_{\text{within}} = 266084.42$$

$$SS_{\text{between}} = 1379.692$$

$$MS_{\text{between}} = SS_{\text{between}} / df_{\text{between}} = 1379.692 / 4 = 344.923$$

$$MS_{\text{within}} = SS_{\text{within}} / df_{\text{within}} = 266084.42 / 495 = 537.544$$

$$F = MS_{\text{between}} / MS_{\text{within}} = 344.923 / 537.544 = 0.642$$

The following null and alternative hypotheses need to be tested:

$$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$$

H_1 : Not all means are equal.

The above hypotheses will be tested using an F-ratio for a One-Way ANOVA.

Based on the information provided, the significance level is $\alpha = 0.05$, and the degrees of freedom are $df_1 = 4$ and $df_2 = 495$, therefore, the rejection region for this F-test is $R = \{F: F > F_c = 2.39\}$.

Test Statistics

$$F = MS_{\text{between}} / MS_{\text{within}} = 344.923 / 537.544 = 0.642$$

Since it is observed that $F = 0.642 < 2.39 = F_c$, it is then concluded that the null hypothesis is not rejected. Therefore, there is not enough evidence to claim that not all 5 population means are equal, at the $\alpha = 0.05$ significance level.

Using the P-value approach: The p-value is $p=0.633$ and since $p=0.633 \geq 0.05$, it is concluded that the null hypothesis is not rejected. Therefore, there is not enough evidence to claim that not all 5 population means are equal, at the $\alpha=0.05$ significance level.

3. Fifteen trainees in a technical program are randomly assigned to three different types of instructional approaches, all of which are concerned with developing specified level of skill in computer-assisted design. The achievement test scores at the conclusion of the instructional unit are reported in Table along with the mean Performance score associated with each instructional approach. Use the analysis of variance procedure to test the null hypothesis that the three-sample means were obtained from the same population, using the 5 percent level of significance for the test.

Instrumental method	Test scores					Total scores	Mean test scores
A1	86	79	81	70	84	400	80
A2	90	76	88	82	89	425	85
A3	82	68	73	71	81	375	75

Solution:

We have 15 students A1, A2, A3

$\alpha=0.05$

$H_0: \mu_1=\mu_2=\mu_3$

H_1 : at least one of the means is different.

From excel ANOVA calculations

Anova: Single Factor

SUMMARY

Groups	Count	Sum	Average	Variance
Row 1	5	400	80	38.5
Row 2	5	425	85	35
Row 3	5	375	75	38.5

ANOVA

Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	250	2	125	3.348214	0.069909	3.885294
Within Groups	448	12	37.33333			
Total	698	14				

The F value calculated was 3.34. This is less than the stated critical value (F_{crit}) of 3.88, and the probability of obtaining this result by chance (P-value) was calculated as 0.0699 (6.99% to three significant figures). We conclude that there was not a significant difference in means and the three-sample means were obtained from the same population, since $P > 0.05$.

Alternative method by manual calculations:

$n = 5$ replications

$a = 3$ treatment

alpha, $\alpha = 0.05$

overall mean $\bar{\mu} = \frac{380+85+75}{3} = 80$

$SS_{treatment} = 5 \times [(80-80)^2 + (85-80)^2 + (75-80)^2]$

$SS_{treatment} = 250$

$SS_{total} =$

$[(86-80)^2 + (79-80)^2 + (81-80)^2 + (70-80)^2 + (84-80)^2 + (90-80)^2 + (76-80)^2 + (88-80)^2 + (82-80)^2 + (89-80)^2 + (82-80)^2 + (68-80)^2 + (73-80)^2 + (71-80)^2 + (81-80)^2]$

$SS_{total} = 698$

$SSE = SS_{total} - SS_{treatment}$

$SSE = 448$

$MS_{treatment} = \frac{SS_{treatment}}{(a-1)} = 125$

$MSE = \frac{SSE}{a(n-1)} = 37.333$

$F_o = \frac{MS_{treatment}}{MSE}$

$F_o = 3.348$

critical value = $F(a, a-1, a(n-1)) = F(0.05, 2, 12) = 3.89$

since $F_o < 3.89$ hence null hypothesis can not be rejected

Hence we conclude that the three sample means were obtained from the same population.