

AI1110 Assignment 2

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Question 4

Using properties of determinants prove that:

$$\begin{vmatrix} x & x(x^2 + 1) & x + 1 \\ y & y(y^2 + 1) & y + 1 \\ z & z(z^2 + 1) & z + 1 \end{vmatrix} = (x - y)(y - z)(z - x)(x + y + z)$$

Solution:

$$M = \begin{vmatrix} x & x(x^2 + 1) & x + 1 \\ y & y(y^2 + 1) & y + 1 \\ z & z(z^2 + 1) & z + 1 \end{vmatrix}$$

Using Split property of determinant at column 3 we get

$$M = \begin{vmatrix} x & x(x^2 + 1) & x \\ y & y(y^2 + 1) & y \\ z & z(z^2 + 1) & z \end{vmatrix} + \begin{vmatrix} x & x(x^2 + 1) & 1 \\ y & y(y^2 + 1) & 1 \\ z & z(z^2 + 1) & 1 \end{vmatrix}$$

As 1st and 3rd columns of 1st determinant are same it's value becomes zero then

$$M = \begin{vmatrix} x & x^3 + x & 1 \\ y & y^3 + y & 1 \\ z & z^3 + z & 1 \end{vmatrix}$$

Using Split property of determinant at column 2 we get

$$M = \begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix} + \begin{vmatrix} x & x & 1 \\ y & y & 1 \\ z & z & 1 \end{vmatrix}$$

Similarly as 1st and 2nd columns of

2nd determinant are same it's value becomes zero then

$$M = \begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix}$$

Using row transformation properties i.e changing row1 to (row1-row2) and row2 to (row2-row3) we get

$$M = \begin{vmatrix} x - y & x^3 - y^3 & 0 \\ y - z & y^3 - z^3 & 0 \\ z & z^3 & 1 \end{vmatrix}$$

Evaluating the determinant of matrix at (3,3) position we get value of determinant as

$$\begin{aligned} &= (y^3 - z^3)(x - y) - (y - z)(x^3 - y^3) \\ &= (y - z)(y^2 + z^2 + y.z)(x - y) - (y - z)(x^2 + y^2 + x.y) \\ &= (y - z)(x - y)[y^2 + z^2 + y.z - x^2 - y^2 - x.y] \\ &= (y - z)(x - y)[(z - x)(z + x) + y(z - x)] \\ &= (y - z)(x - y)(z - x)[z + x + y] \\ &\text{By rearranging the terms we get the value of determinant as} \\ &= (x - y)(y - z)(z - x)(x + y + z) \\ &= \text{R.H.S} \end{aligned}$$

Hence proved!

The following is a result of c code which

takes inputs for x,y,z and checks whether both LHS and RHS are equal

```
PROBLEMS    OUTPUT    TERMINAL    DEBUG CONSOLE

avinashnayak@AVINASHs-MacBook-Air folder % gcc main.c
avinashnayak@AVINASHs-MacBook-Air folder % ./a.out
4 9 0
LHS of given equation: 2340
RHS of given equation: 2340
LHS=RHS
Hence proved!
avinashnayak@AVINASHs-MacBook-Air folder % ./a.out
4 5 5
LHS of given equation: 0
RHS of given equation: 0
LHS=RHS
Hence proved!
```