Assignment 2

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Outline

Question

2 Answer

Question

Using properties of determinants prove that:

$$\begin{vmatrix} x & x(x^2+1) & x+1 \\ y & y(y^2+1) & y+1 \\ z & z(z^2+1) & z+1 \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)$$
 (1)

Answer

$$M = \begin{vmatrix} x & x(x^2+1) & x+1 \\ y & y(y^2+1) & y+1 \\ z & z(z^2+1) & z+1 \end{vmatrix}$$
 (2)

Using Split property of determinant at column 3 we get

$$M = \begin{vmatrix} x & x(x^2+1) & x \\ y & y(y^2+1) & y \\ z & z(z^2+1) & z \end{vmatrix} + \begin{vmatrix} x & x(x^2+1) & 1 \\ y & y(y^2+1) & 1 \\ z & z(z^2+1) & 1 \end{vmatrix}$$
(3)

As 1st and 3rd coloumns of 1st determinant are same it's value becomes zero then



$$M = \begin{vmatrix} x & x^3 + x & 1 \\ y & y^3 + y & 1 \\ z & z^3 + z & 1 \end{vmatrix}$$
 (4)

Using Split property of determinant at column 2 we get

$$M = \begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix} + \begin{vmatrix} x & x & 1 \\ y & y & 1 \\ z & z & 1 \end{vmatrix}$$
 (5)

Similarly as 1^{st} and 2^{nd} coloumns of 2^{nd} determinant are same it's value becomes zero then



$$M = \begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix}$$
 (6)

Using row transformation properties i.e changing row1 to (row1-row2) and row2 to (row2-row3) we get

$$M = \begin{vmatrix} x - y & x^3 - y^3 & 0 \\ y - z & y^3 - z^3 & 0 \\ z & z^3 & 1 \end{vmatrix}$$
 (7)

Evaluating the determinant of matrix at (3,3) position we get value of determinant as

$$= (y^3 - z^3)(x - y) - (y - z)(x^3 - y^3)$$
 (8)

$$= (y-z)(y^2+z^2+y.z)(x-y) - (y-z)(x^2+y^2+x.y)$$
 (9)

$$= (y-z)(x-y)[y^2+z^2+y.z-x^2-y^2-x.y]$$
 (10)

$$= (y-z)(x-y)[(z-x)(z+x) + y(z-x)]$$
 (11)

$$= (y-z)(x-y)(z-x)[z+x+y]$$
 (12)

$$= (x - y)(y - z)(z - x)(x + y + z)$$
(13)

$$= R.H.S \tag{14}$$

Hence proved!



Code Output:

The following is a result of c code which takes inputs for x,y,z and checks whether both LHS and RHS are equal

```
PROBLEMS
           OUTPUT
                     TERMINAL
                                 DEBUG CONSOLE
avinashnavak@AVINASHs-MacBook-Air folder % gcc main.c
avinashnayak@AVINASHs-MacBook-Air folder % ./a.out
490
LHS of given equation: 2340
RHS of given equation: 2340
LHS=RHS
Hence proved!
avinashnayak@AVINASHs-MacBook-Air folder % ./a.out
4 5 5
LHS of given equation: 0
RHS of given equation: 0
LHS=RHS
Hence proved!
```