AI1110 Assignment 2

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Question 4

Using properties of determinants prove that:

$$\begin{vmatrix} x & x(x^2+1) & x+1 \\ y & y(y^2+1) & y+1 \\ z & z(z^2+1) & z+1 \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)$$

Solution: Given Matrix, let M

$$\mathrm{M}=\begin{vmatrix} x & x(x^2+1) & x+1 \\ y & y(y^2+1) & y+1 \\ z & z(z^2+1) & z+1 \end{vmatrix}$$

Using Split property of determinant at column 3 we get

$$\mathrm{M}=\begin{vmatrix} x & x(x^2+1) & x \\ y & y(y^2+1) & y \\ z & z(z^2+1) & z \end{vmatrix} + \begin{vmatrix} x & x(x^2+1) & 1 \\ y & y(y^2+1) & 1 \\ z & z(z^2+1) & 1 \end{vmatrix}$$

As 1^{st} and 3^{rd} coloumns of 1^{st} determinant are same it's value becomes zero then

$$\mathrm{M}=\begin{vmatrix} x & x^3+x & 1 \\ y & y^3+y & 1 \\ z & z^3+z & 1 \end{vmatrix}$$

Using Split property of determinant at column 2 we get

$$\mathrm{M}=\begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix} + \begin{vmatrix} x & x & 1 \\ y & y & 1 \\ z & z & 1 \end{vmatrix}$$

Similarly as 1^{st} and 2^{nd} coloumns of 2^{nd} determinant are same it's value becomes zero then

$$\mathrm{M}=\begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix}$$

Using row transformation properties i.e changing row1 to (row1-row2) and row2 to (row2-row3) we get

$$\mathrm{M}=\begin{vmatrix} x-y & x^3-y^3 & 0 \\ y-z & y^3-z^3 & 0 \\ z & z^3 & 1 \end{vmatrix}$$

Evaluating the determinant of matrix at (3,3) position we get value of determinant as

$$= (y^3-z^3)(x-y)-(y-z)(x^3-y^3)$$

$$= (y-z)(y^2+z^2+y.z)(x-y)-(y-z)(x^2+y^2+x.y)$$

$$= (y-z)(x-y)[y^2+z^2+y.z-x^2-y^2-x.y]$$

$$= (y-z)(x-y)[(z-x)(z+x)+y(z-x)]$$

$$= (y-z)(x-y)(z-x)[z+x+y]$$

By rearranging the terms we get the value of determinant as

=(x-y)(y-z)(z-x)(x+y+z)

=R.H.S

Hence proved!

THe following is a result of c code which takes inputs for x,y,z and checks whether both LHS and RHS are equal