Assignment 8

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Outline

Question

The process x(t) is cyclostationary with period τ, mean $\eta(t)$ and correlation R(t_1, t_2).
Show that if R$(t+\tau, t) \rightarrow \eta^2(\tau)$ as $\tau \rightarrow \infty$, then

$$\lim_{c\to\infty} \int_{-c}^{c} x(t)\, dt = \frac{1}{\tau} \int_{0}^{\tau} \eta(t)\, dt$$

Answer

The process x(t)=v(t-θ) is stationary with mean $\bar{\eta}$ and covariance $\bar{c}(\tau)$ given by

$$\bar{\eta} = \frac{1}{\tau}\int_0^\tau \eta(t)\,dt \tag{1}$$

$$\bar{c}(\tau) = \frac{1}{\tau}\int_0^\tau c(t+\tau, t)\,dt \tag{2}$$

$$If\ \ R(t+\tau, t) \to \eta^2(t)\ as\ t \to \infty then \tag{3}$$

$$C(t+\tau, t) \to 0\ as\ t \to \infty \tag{4}$$

$$hence \tag{5}$$

$$\bar{c}(\tau) \to 0\ as\ \tau \to \infty \tag{6}$$

This shows that $\bar{c}(\tau)$ is ergodic therefore

$$\frac{1}{2c}\int_{-c}^{c}\bar{x}(t)\,dt = \frac{1}{2c}\int_{-c+\theta}^{c+\theta}\bar{x}(t)dt \tag{7}$$

$$\rightarrow \bar{\eta} \tag{8}$$

$$\textit{this yields} \tag{9}$$

$$\lim_{c\to\infty}\int_{-c}^{c}x(t)\,dt = \frac{1}{\tau}\int_{0}^{\tau}\eta(t)\,dt \tag{10}$$

$$\textit{Hence proved} \tag{11}$$