Assignment 5

Malothu Avinash
AI21BTECH11018

June 3, 2022

Outline

Question

Using price's theorem,Derive $\mathrm{E}(\mathrm{x}^2 \cdot y^2) = E(x^2) \cdot E(y^2) + 2E(xy)$

Answer

From price's theorem

$$\frac{\partial^n I(\mu)}{\partial \mu^n} = E\left(\frac{\partial^{2n} g(x,y)}{\partial^n x \cdot \partial^n y}\right) \tag{1}$$

$$\text{Let } n = 1 \text{ and } g(x,y) = x^2 y^2 \tag{2}$$

$$\frac{\partial I(\mu)}{\partial \mu} = E\left(\frac{\partial^2 (x^2 y^2)}{\partial x \cdot \partial y}\right) \tag{3}$$

$$\frac{\partial I(\mu)}{\partial \mu} = 4E(xy) \tag{4}$$

$$\frac{\partial I(\mu)}{\partial \mu} = 4\mu \tag{5}$$

$$I(\mu) = 2\mu^2 + I(0) \tag{6}$$

$$As\ I(\mu) = E(x^2 \cdot y^2) \tag{7}$$

$$at\ \mu = 0\ random\ variables\ x\ and\ y\ are\ independent \tag{8}$$

$$\implies I(0) = E(x^2) \cdot E(y^2) \tag{9}$$

$$FromI(\mu) = 2\mu^2 + I(0) \tag{10}$$

$$E(x^2 \cdot y^2) = 2[E(x \cdot y)]^2 + E(x^2) \cdot E(y^2) \tag{11}$$

$$\therefore E(x^2 \cdot y^2) = E(x^2) \cdot E(y^2) + 2[E(x \cdot y)]^2 \tag{12}$$

$$\tag{13}$$