Assignment 2

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Outline

Question

Using properties of determinants prove that:

$$\begin{vmatrix} x & x(x^2+1) & x+1 \\ y & y(y^2+1) & y+1 \\ z & z(z^2+1) & z+1 \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z) \qquad (1)$$

Answer

$$M = \begin{vmatrix} x & x(x^2+1) & x+1 \\ y & y(y^2+1) & y+1 \\ z & z(z^2+1) & z+1 \end{vmatrix} \tag{2}$$

Using Split property of determinant at column 3 we get

$$M = \begin{vmatrix} x & x(x^2+1) & x \\ y & y(y^2+1) & y \\ z & z(z^2+1) & z \end{vmatrix} + \begin{vmatrix} x & x(x^2+1) & 1 \\ y & y(y^2+1) & 1 \\ z & z(z^2+1) & 1 \end{vmatrix} \tag{3}$$

As 1st and 3rd coloumns of 1st determinant are same it's value becomes zero then

$$M = \begin{vmatrix} x & x^3 + x & 1 \\ y & y^3 + y & 1 \\ z & z^3 + z & 1 \end{vmatrix} \tag{4}$$

Using Split property of determinant at column 2 we get

$$M = \begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix} + \begin{vmatrix} x & x & 1 \\ y & y & 1 \\ z & z & 1 \end{vmatrix} \tag{5}$$

Similarly as 1st and 2nd coloumns of 2nd determinant are same it's value becomes zero then

$$M = \begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix} \tag{6}$$

Using row transformation properties i.e changing row1 to (row1-row2) and row2 to (row2-row3) we get

$$M = \begin{vmatrix} x - y & x^3 - y^3 & 0 \\ y - z & y^3 - z^3 & 0 \\ z & z^3 & 1 \end{vmatrix} \tag{7}$$

Evaluating the determinant of matrix at (3,3) position we get value of determinant as

$$= (y^3 - z^3)(x - y) - (y - z)(x^3 - y^3) \tag{8}$$

$$= (y - z)(y^2 + z^2 + y.z)(x - y) - (y - z)(x^2 + y^2 + x.y) \tag{9}$$

$$= (y - z)(x - y)[y^2 + z^2 + y.z - x^2 - y^2 - x.y] \tag{10}$$

$$= (y - z)(x - y)[(z - x)(z + x) + y(z - x)] \tag{11}$$

$$= (y - z)(x - y)(z - x)[z + x + y] \tag{12}$$

$$= (x - y)(y - z)(z - x)(x + y + z) \tag{13}$$

$$= R.H.S \tag{14}$$

Hence proved!

Code Output:

The following is a result of c code which takes inputs for x,y,z and checks whether both LHS and RHS are equal