Digital Signal Processing

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Contents

***Abstract*—This manual provides a simple introduction to digital signal processing.**

1 Software Installation

Run the following commands

```
sudo apt-get update
sudo apt-get install libffi-dev libsndfile1 python3
    -scipy python3-numpy python3-matplotlib
sudo pip install cffi pysoundfile
```

2 Digital Filter

2.1 Download the sound file from

```
wget https://github.com/AvinashNayak27/
    digital/blob/master/codes/Sound_Noise.
    wav
```

2.2 You will find a spectrogram at https://academo.org/demos/spectrum-analyzer. Upload the sound file that you downloaded in Problem 2.1 in the spectrogram and play. Observe the spectrogram. What do you find?
Solution: There are a lot of yellow lines between 440 Hz to 5.1 KHz. These represent the synthesizer key tones. Also, the key strokes are audible along with background noise.

2.3 Write the python code for removal of out of band noise and execute the code.
Solution:

```
import soundfile as sf
from scipy import signal

#read .wav file
input_signal,fs = sf.read('Sound_Noise.wav'
    )

#sampling frequency of Input signal
sampl_freq=fs

#order of the filter
order=4

#cutoff frquency 4kHz
cutoff_freq=4000.0

#digital frequency
Wn=2*cutoff_freq/sampl_freq

# b and a are numerator and denominator
    polynomials respectively
b, a = signal.butter(order,Wn, 'low')

#filter the input signal with butterworth filter
output_signal = signal.filtfilt(b, a,
    input_signal)
#output_signal = signal.lfilter(b, a,
    input_signal)

#write the output signal into .wav file
sf.write('Sound_With_ReducedNoise.wav',
    output_signal, fs)
```

2.4 The output of the python script in Problem 2.3 is the audio file Sound_With_ReducedNoise.wav. Play the file in the spectrogram in Problem 2.2. What do you observe?
Solution: The key strokes as well as background noise is subdued in the audio.

Also, the signal is blank for frequencies above 5.1 kHz.

3 Difference Equation

3.1 Let

$$x(n) = \left\{ \underset{\uparrow}{1}, 2, 3, 4, 2, 1 \right\} \tag{3.1}$$

Sketch $x(n)$.

3.2 Let

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2),$$
$$y(n) = 0, n < 0 \tag{3.2}$$

Sketch $y(n)$.

Solution: The following code yields Fig. 3.2.

```
wget https://github.com/AvinashNayak27/
    digital/blob/master/codes/xnyn.py
```

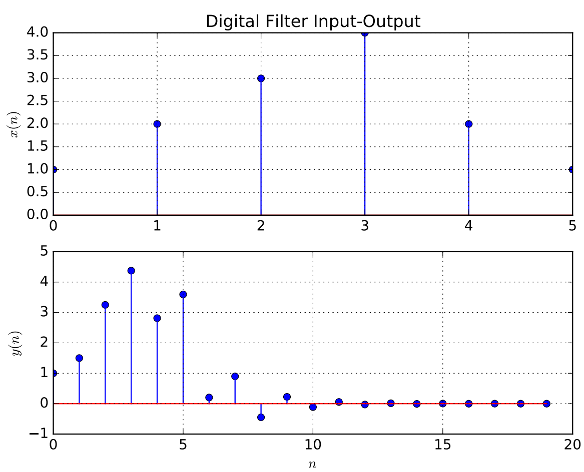

Fig. 3.2

3.3 Repeat the above exercise using a C code.

Solution: Download the C code from the below link,

```
wget https://github.com/AvinashNayak27/
    digital/blob/master/codes/3.3.1.c
```

Then run the follwing command in terminal

```
gcc 3.3.1.c
./a.out
```

Then for the plot **??** download the python file from the below link,

```
wget https://github.com/AvinashNayak27/
    digital/blob/master/codes/3.3.2.py
```

Then run the command

```
python3 3.3.2.py
```

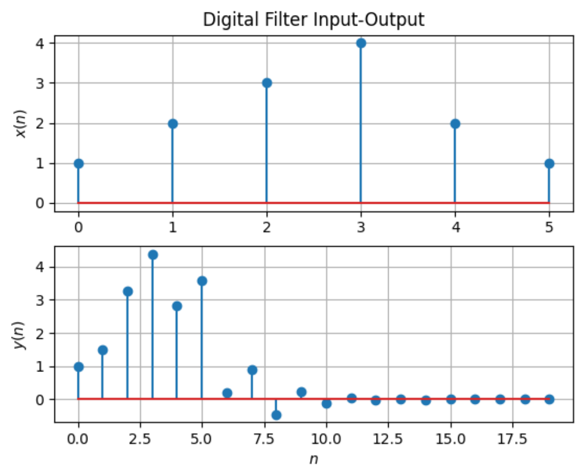

Fig. 3.3: Plot using C code

4 Z-transform

4.1 The Z-transform of $x(n)$ is defined as

$$X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{4.1}$$

Show that

$$\mathcal{Z}\{x(n-1)\} = z^{-1}X(z) \tag{4.2}$$

and find

$$\mathcal{Z}\{x(n-k)\} \tag{4.3}$$

Solution: Given that,

$$X(z) = \mathcal{Z}\{x(n)\} \tag{4.4}$$
$$= \sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{4.5}$$

So,

$$\mathcal{Z}\{x(n-1)\} = \sum_{n=-\infty}^{\infty} x(n-1)z^{-n} \tag{4.6}$$

Take $k = n - 1$,

$$= \sum_{k=-\infty}^{\infty} x(k) z^{-(k+1)} \tag{4.7}$$

$$= z^{-1} \sum_{k=-\infty}^{\infty} x(k) z^{-k} \tag{4.8}$$

$$= z^{-1} \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{4.9}$$

$$= z^{-1} X(z) \tag{4.10}$$

resulting in (4.2) and similarly following the above steps you will get,

$$\mathcal{Z}\{x(n-k)\} = z^{-k} X(n) \tag{4.11}$$

Hence proved.

4.2 Obtain $X(z)$ for $x(n)$ defined in problem 3.1. **Solution:** Now we will find Z transform of the signal $x(n)$,from (**??**),

$$\mathcal{Z}\{x(n)\} = \sum_{n=0}^{5} x(n) z^{-n} \tag{4.12}$$

$$= 1z^0 + 2z^{-1} + 3z^{-2} + 4z^{-3} + 2z^{-4} + 1z^{-5} \tag{4.13}$$

$$= 1 + 2z^{-1} + 3z^{-2} + 4z^{-3} + 2z^{-4} + z^{-5} \tag{4.14}$$

4.3 Find

$$H(z) = \frac{Y(z)}{X(z)} \tag{4.15}$$

from (3.2) assuming that the Z-transform is a linear operation.
Solution: Applying (4.11) in (3.2),

$$Y(z) + \frac{1}{2} z^{-1} Y(z) = X(z) + z^{-2} X(z) \tag{4.16}$$

$$\implies \frac{Y(z)}{X(z)} = \frac{1 + z^{-2}}{1 + \frac{1}{2} z^{-1}} \tag{4.17}$$

Solution: Now we will rewrite (3.2),

$$y(n) + \frac{1}{2} y(n-1) = x(n) + x(n-2) \tag{4.18}$$

Now since Z-transform is a linear operator we can write that,

$$Y(n) + \frac{1}{2} Y(n-1) = X(n) + X(n-2) \tag{4.19}$$

From (4.11),

$$Y(n) + \frac{z^{-1}}{2} Y(n) = X(n) + z^{-2} X(n) \tag{4.20}$$

$$\implies \frac{Y(n)}{X(n)} = \frac{1 + z^{-2}}{1 + \frac{z^{-1}}{2}} \tag{4.21}$$

4.4 Find the Z transform of

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases} \tag{4.22}$$

and show that the Z-transform of

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{4.23}$$

is

$$U(z) = \frac{1}{1 - z^{-1}}, \quad |z| > 1 \tag{4.24}$$

Solution: The Z-transform of δn is,

$$\mathcal{Z}\{\delta n\} = \sum_{n=-\infty}^{\infty} \delta(n) z^{-n} \tag{4.25}$$

$$= \delta(0) z^0 + 0 \ \text{(Using (4.22))} \tag{4.26}$$

$$= 1 \tag{4.27}$$

and the Z-transform of unit-step function $u(n)$ is,

$$U(n) = \sum_{n=-\infty}^{\infty} u(n) z^{-n} \tag{4.28}$$

$$= 0 + \sum_{n=0}^{\infty} 1.z^{-n} \tag{4.29}$$

$$= 1 + z^{-1} + z^{-2} + \ldots \tag{4.30}$$

Above is a infinite geometric series with z^{-1} as common ratio , so we can write it as

$$U(n) = \frac{1}{1 - z^{-1}} \because |z| > 1 \tag{4.31}$$

4.5 Show that

$$a^n u(n) \overset{\mathcal{Z}}{\rightleftharpoons} \frac{1}{1 - az^{-1}} \quad |z| > |a| \tag{4.32}$$

Solution: The Z- transform will be

$$\mathcal{Z}\{a^n u(n)\} = \sum_{n=0}^{\infty} a^n z^{-n} \tag{4.33}$$

$$= 1 + \frac{a}{z} + \left(\frac{a}{z}\right)^2 + \ldots. \tag{4.34}$$

Above is a infinite geometric series with first term 1 and common ratio as $\frac{a}{z}$ and it can be written as,

$$\mathcal{Z}\{a^n u(n)\} = \frac{1}{1 - \frac{a}{z}} \because |a| < |z| \tag{4.35}$$

Therefore,

$$a^n u(n) \overset{\mathcal{Z}}{\rightleftharpoons} \frac{1}{1 - az^{-1}} \quad |z| > |a| \tag{4.36}$$

4.6 Let

$$H\left(e^{j\omega}\right) = H\left(z = e^{j\omega}\right). \tag{4.37}$$

Plot $\left|H\left(e^{j\omega}\right)\right|$. Comment. $H(e^{j\omega})$ is known as the *Discret Time Fourier Transform* (DTFT) of $h(n)$.

Solution: Download the code for the plot 4.6 from the link below

```
wget https://github.com/AvinashNayak27/
    digital/blob/master/codes/dtft.py
```

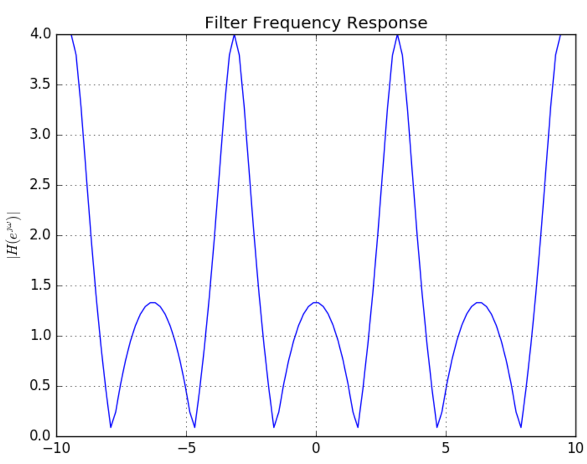

Fig. 4.6: $|H(e^{j\omega})|$

Now using (4.17), we will find $\left|H\left(e^{j\omega}\right)\right|$,

$$H\left(e^{j\omega}\right) = \frac{1 + e^{-2j\omega}}{1 + \frac{e^{-j\omega}}{2}} \tag{4.38}$$

$$\implies \left|H\left(e^{j\omega}\right)\right| = \frac{\left|1 + e^{-2j\omega}\right|}{\left|1 + \frac{e^{-j\omega}}{2}\right|} \tag{4.39}$$

$$= \frac{\left|1 + e^{2j\omega}\right|}{\left|e^{2j\omega} + \frac{e^{j\omega}}{2}\right|} \tag{4.40}$$

$$= \frac{|1 + \cos 2\omega + j \sin 2\omega|}{\left|e^{j\omega} + \frac{1}{2}\right|} \tag{4.41}$$

$$= \frac{\left|4\cos^2(\omega) + 4j\sin(\omega)\cos(\omega)\right|}{|2e^{j\omega} + 1|} \tag{4.42}$$

$$= \frac{|4\cos(\omega)|\,|\cos(\omega) + j\sin(\omega)|}{|2\cos(\omega) + 1 + 2j\sin(\omega)|} \tag{4.43}$$

$$\therefore \left|H\left(e^{j\omega}\right)\right| = \frac{|4\cos(\omega)|}{\sqrt{5 + 4\cos(\omega)}} \tag{4.44}$$

Since $\left|H(e^{j\omega})\right|$ is function of cosine we can say it is periodic.And from the plot 4.6 we can say that it is symmetric about $\omega = 0$ (even function) and it is periodic with period 2π.You can find the same from the theoritical expression $\left|H\left(e^{j\omega}\right)\right|$,

$$H(e^{j\omega}) = H(e^{j(-\omega)}) \text{ (cos is an even function)} \tag{4.45}$$

And to find period, the period of $|\cos(\omega)|$ is π and the period of $\sqrt{5 + 4\cos(\omega)}$ is 2π. So the period of division of both will be,

$$lcm(\pi, 2\pi) = 2\pi \tag{4.46}$$

This gives us the period of $\left|H(e^{j\omega})\right|$ as 2π

4.7 Express $h(n)$ in terms of $H\left(e^{j\omega}\right)$.

Solution: We know that

$$H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} h(k)e^{-j\omega k} \tag{4.47}$$

$$\implies \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})e^{j\omega n}d\omega \tag{4.48}$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi}\sum_{k=-\infty}^{\infty} h(k)e^{-j\omega k}e^{j\omega n}d\omega \tag{4.49}$$

$$= \frac{1}{2\pi}\sum_{k=-\infty}^{\infty} h(k)\int_{-\pi}^{\pi} e^{j\omega(n-k)}d\omega \tag{4.50}$$

$$= \frac{1}{2\pi}\left\{\sum_{k\neq n} h(k)\left.\frac{e^{j\omega(n-k)}}{j(n-k)}\right]_{-\pi}^{\pi} + h(n).2\pi\right\} \tag{4.51}$$

$$= \frac{0 + 2\pi h(n)}{2\pi} \tag{4.52}$$

$$= h(n) \tag{4.53}$$

$$\therefore h(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})e^{j\omega n}d\omega \tag{4.54}$$

5 Impulse Response

5.1 Using long division, find

$$h(n), \quad n < 5 \tag{5.1}$$

for H(z) in (4.17).
Solution: $H(z)$ is given by

$$H(z) = \frac{1 + z^{-2}}{1 + \frac{1}{2}z^{-1}} = \frac{2 + 2z^{-2}}{2 + z^{-1}} \tag{5.2}$$

$$2z^{-1} - 4 \tag{5.3}$$

$$z^{-1} + 2 \;\big)\; 2z^{-2} + 2 \tag{5.4}$$

$$2z^{-2} + 4z^{-1} \tag{5.5}$$

$$-4z^{-1} + 2 \tag{5.6}$$

$$-\,4z^{-1} - 8 \tag{5.7}$$

$$10 \tag{5.8}$$

So,

$$H(z) = 2z^{-1} - 4 + \frac{10}{z^{-1} + 2} \tag{5.9}$$

$$= 2z^{-1} - 4 + \frac{5}{\frac{1}{2}z^{-1} + 1} \tag{5.10}$$

$$= 2z^{-1} - 4 + 5\sum_{n=0}^{\infty}\left(-\frac{z^{-1}}{2}\right)^{n} \tag{5.11}$$

$$= 1 - \frac{1}{2}z^{-1} + \sum_{n=2}^{\infty}\left(-\frac{1}{2}\right)^{n} z^{-n} \tag{5.12}$$

So,h(n) will be given by

$$h(n) = \begin{cases} 5\times\left(-\frac{1}{2}\right)^{n} & n \geq 2 \\ \left(-\frac{1}{2}\right)^{n} & 2 > n \geq 0 \\ 0 & n < 0 \end{cases} \tag{5.13}$$

5.2 Find an expression for $h(n)$ using $H(z)$, given that

$$h(n) \overset{\mathcal{Z}}{\rightleftharpoons} H(z) \tag{5.14}$$

and there is a one to one relationship between $h(n)$ and $H(z)$. $h(n)$ is known as the *impulse response* of the system defined by (3.2).
Solution: From (4.17),

$$H(z) = \frac{1}{1 + \frac{1}{2}z^{-1}} + \frac{z^{-2}}{1 + \frac{1}{2}z^{-1}} \tag{5.15}$$

$$\implies h(n) = \left(-\frac{1}{2}\right)^{n} u(n) + \left(-\frac{1}{2}\right)^{n-2} u(n-2) \tag{5.16}$$

using (4.32) and (**??**).

5.3 Sketch $h(n)$. Is it bounded? Justify theoretically.
Solution: The following code plots Fig. 5.3.

```
wget https://github.com/AvinashNayak27/
    digital/blob/master/codes/hn.py
```

on simplfying we get h(n) as

$$\begin{cases} 5\times\left(-\frac{1}{2}\right)^{n} & n \geq 2 \\ \left(-\frac{1}{2}\right)^{n} & 2 > n \geq 0 \\ 0 & n < 0 \end{cases} \tag{5.17}$$

$$\because 5\times\left(-\frac{1}{2}\right)^{n} \to 0 \quad \text{for} \quad n \to \infty \tag{5.18}$$

So, we can conclude that h(n) is bounded.

5.4 Convergent? Justify using the ratio test.

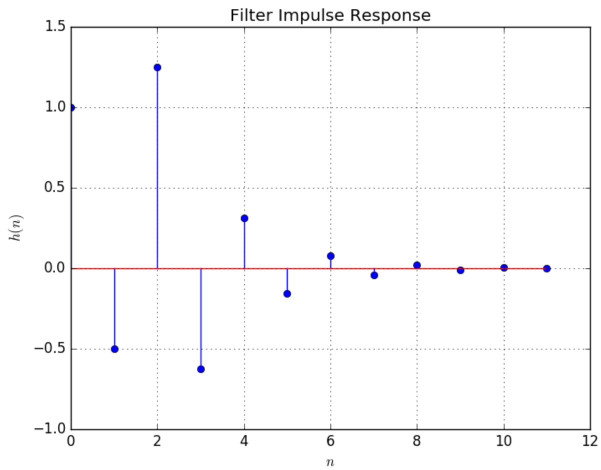

Fig. 5.3: $h(n)$ wrt n

Solution: We can say a given real sequence $\{x_n\}$ is convergent if

$$\lim_{n\to\infty}\left|\frac{x_{n+1}}{x_n}\right| < 1 \tag{5.19}$$

This is known as Ratio test.
In this case the limit will become,

$$\lim_{n\to\infty}\left|\frac{h\left(n+1\right)}{h\left(n\right)}\right| = \lim_{n\to\infty}\left|\frac{5\left(\frac{-1}{2}\right)^{n+1}}{5\left(\frac{-1}{2}\right)^{n}}\right| \tag{5.20}$$

$$= \lim_{n\to\infty}\left|\frac{-1}{2}\right| \tag{5.21}$$

$$= \frac{1}{2} \tag{5.22}$$

As $\frac{1}{2} < 1$, from root test we can say that $h\left(n\right)$ is convergent.

5.5 The system with $h(n)$ is defined to be stable if

$$\sum_{n=-\infty}^{\infty} h(n) < \infty \tag{5.23}$$

Is the system defined by (3.2) stable for the impulse response in (5.14)?
Solution: For system of 3.2 ,$h(n)$ is defined in (5.13) So,

$$\sum_{n=-\infty}^{\infty} h(n) = \sum_{n=2}^{\infty} 5\times\left(-\frac{1}{2}\right)^n + \sum_{n=0}^{1}\left(-\frac{1}{2}\right)^n + \sum_{n=-\infty}^{-1} 0 \tag{5.24}$$

$$= 5\times\frac{1}{6} + \frac{1}{2} \tag{5.25}$$

$$= \frac{4}{3} \tag{5.26}$$

Since the sum is finite so the system is stable for impulsive response

5.6 Verify the above result using a python code.
Solution: The above result is verified using the below python code

```
wget https://github.com/AvinashNayak27/
    digital/blob/master/codes/hnverify.py
```

5.7 Compute and sketch $h(n)$ using

$$h(n) + \frac{1}{2}h(n-1) = \delta(n) + \delta(n-2), \tag{5.27}$$

This is the definition of $h(n)$.
Solution: The following code plots Fig. 5.7. Note that this is the same as Fig. **??**.

```
wget https://github.com/AvinashNayak27/
    digital/blob/master/codes/hndef.py
```

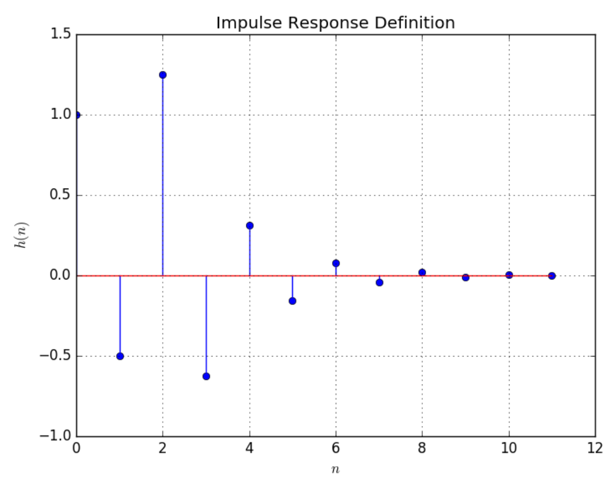

Fig. 5.7: $h(n)$ from the definition

5.8 Compute

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) \tag{5.28}$$

Comment. The operation in (5.28) is known as *convolution*.

Solution: The following code plots Fig. 5.8. Note that this is the same as $y(n)$ in Fig. **??**.

```
wget https://github.com/AvinashNayak27/
    digital/blob/master/codes/yndef.py
```

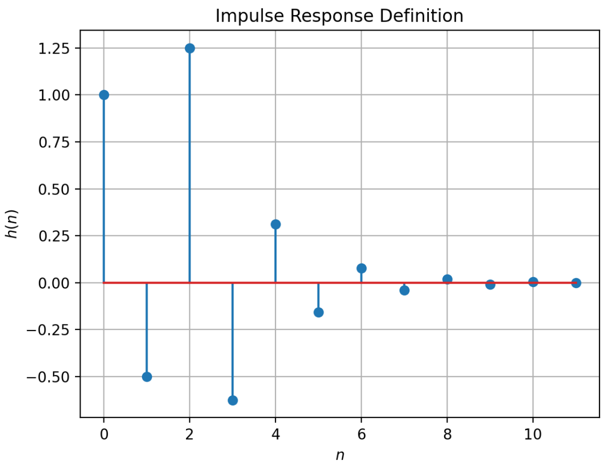

Fig. 5.8: $y(n)$ from the definition of convolution

5.9 Express the above convolution using a Toeplitz matrix.
Solution:

```
wget https://github.com/AvinashNayak27/
    digital/blob/master/codes/ynconv.py
```

From (5.28),we express $y(n)$ as

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) \tag{5.29}$$

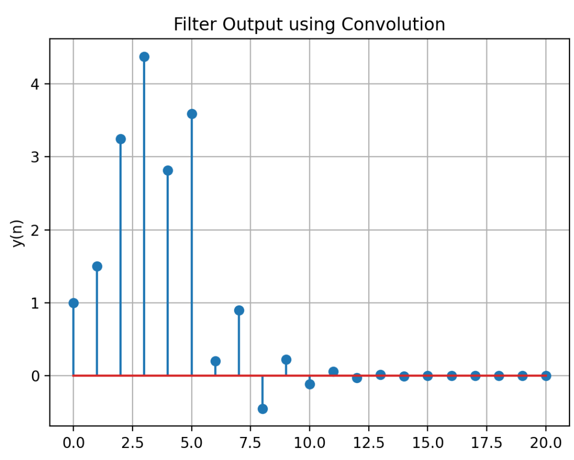

Fig. 5.9: Convolution of $x(n)$ and $h(n)$ using toeplitz matrix

To understand how we can use a Toeplitz matrix, we will see what we are doing in (5.28)

$$y(0) = x(0)\, h(0) \tag{5.30}$$

$$y(1) = x(0)\, h(1) + x(1)\, h(0) \tag{5.31}$$

$$y(2) = x(0)\, h(2) + x(1)\, h(1) + x(2)\, h(0) \tag{5.32}$$

$$.$$
$$.$$

The same thing can be written as,

$$y(0) = \begin{pmatrix} h(0) & 0 & 0 & . & . & .0 \end{pmatrix} \begin{pmatrix} x(0) \\ x(1) \\ x(2) \\ . \\ . \\ x(5) \end{pmatrix} \tag{5.33}$$

$$y(1) = \begin{pmatrix} h(1) & h(0) & 0 & 0 & . & . & .0 \end{pmatrix} \begin{pmatrix} x(0) \\ x(1) \\ x(2) \\ . \\ . \\ x(5) \end{pmatrix} \tag{5.34}$$

$$y(2) = \begin{pmatrix} h(2) & h(1) & h(0) & 0 & . & .0 \end{pmatrix} \begin{pmatrix} x(0) \\ x(1) \\ x(2) \\ . \\ . \\ x(5) \end{pmatrix} \tag{5.35}$$

$$.$$
$$.$$

Using Toeplitz matrix of $h(n)$ we can simplify it as,

$$y(n) = \begin{pmatrix} h(0) & 0 & 0 & . & . & .0 \\ h(1) & h(0) & 0 & . & . & .0 \\ h(2) & h(1) & h(0) & . & . & .0 \\ & & .. & & & \\ & & .. & & & \\ 0 & 0 & 0 & . & . & .h(m-1) \end{pmatrix} \begin{pmatrix} x(0) \\ x(1) \\ x(2) \\ . \\ . \\ x(5) \end{pmatrix} \tag{5.36}$$

Now from (3.1) we will take n

$$x(n) = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 2 \\ 1 \end{pmatrix} \tag{5.37}$$

And from (5.13) we will take some values of n,

$$h(n) = \begin{pmatrix} 1 \\ -0.5 \\ 1.25 \\ . \\ . \end{pmatrix} \tag{5.38}$$

Now using (5.36),

$$y(n) = x(n) * h(n) \tag{5.39}$$

$$= \begin{pmatrix} 1 & 0 & 0 & . & . & .0 \\ -0.5 & 1 & 0 & . & . & .0 \\ 1.25 & -0.5 & 1 & . & . & .0 \\ & & .. & & & \\ & & .. & & & \\ 0 & 0 & 0 & . & . & . \end{pmatrix} \begin{pmatrix} x(0) \\ x(1) \\ x(2) \\ . \\ . \\ x(5) \end{pmatrix} \tag{5.40}$$

$$= \begin{pmatrix} 1 \\ 1.5 \\ 3.25 \\ . \\ . \\ . \end{pmatrix} \tag{5.41}$$

The above equation (5.41) is the convolution of $x(n)$ and $h(n)$

5.10 Show that

$$y(n) = \sum_{k=-\infty}^{\infty} x(n-k)h(k) \tag{5.42}$$

Solution: From (5.28),

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) \tag{5.43}$$

Replacing $n-k$ with a,we get

$$y(n) = \sum_{n-a=-\infty}^{\infty} x(n-a)h(a) \tag{5.44}$$

$$= \sum_{-a=-\infty}^{\infty} x(n-a)h(a) \tag{5.45}$$

$$= \sum_{a=-\infty}^{\infty} x(n-a)h(a) \tag{5.46}$$

6 DFT

6.1 Compute

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \dots, N-1 \tag{6.1}$$

and $H(k)$ using $h(n)$.

Solution: Download the below python code for the plot 6.1,

```
wget https://github.com/AvinashNayak27/
digital/blob/master/codes/dft.py
```

And run the following command in the terminal,

```
python3 dft.py
```

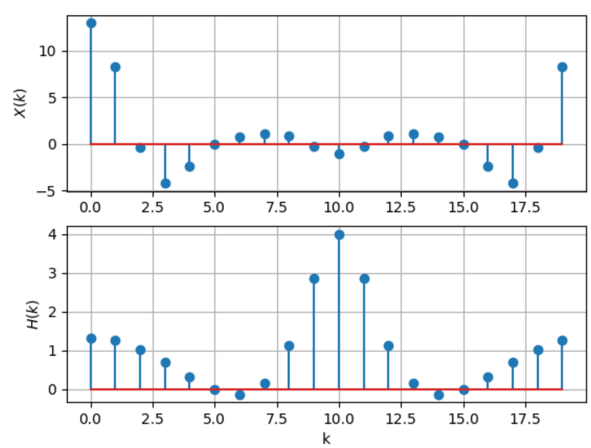

Fig. 6.1: Plot of real part of Discrete Fourier Transforms of $x(n)$ and $h(n)$

6.2 Compute

$$Y(k) = X(k)H(k) \tag{6.2}$$

Solution: Download the below python code for the plot 6.2,

```
wget https://github.com/AvinashNayak27/
    digital/blob/master/codes/yK.py
```

Then run the following command in the terminal,

```
python3 yK.py
```

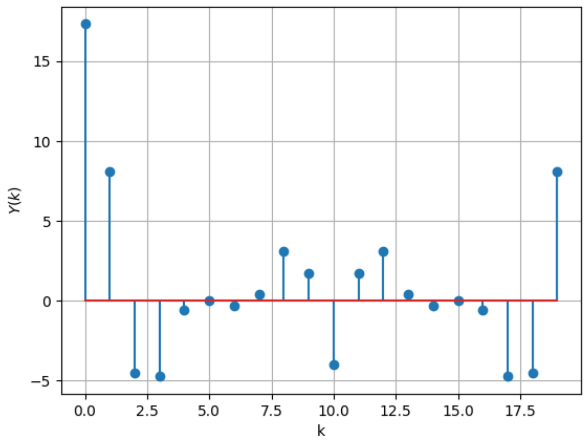

Fig. 6.2: $Y(k)$ as the product of $X(k)$ and $H(k)$

6.3 Compute

$$y(n) = \frac{1}{N}\sum_{k=0}^{N-1} Y(k)\cdot e^{\mathrm{j}2\pi kn/N}, \quad n = 0, 1, \ldots, N-1 \tag{6.3}$$

Solution: Download the below python code for the plot **??**,

```
wget https://github.com/AvinashNayak27/
    digital/blob/master/codes/yndft.py
```

Then run the following command,

```
python3 yndft.py
```

6.4 Repeat the previous exercise by computing $X(k), H(k)$ and $y(n)$ through FFT and IFFT.
Solution: Download the below python code for the plot **??**,

```
wget https://github.com/AvinashNayak27/
    digital/blob/master/codes/ynIIFT.py
```

Then run the following command,

```
python3 ynIIFT.py
```

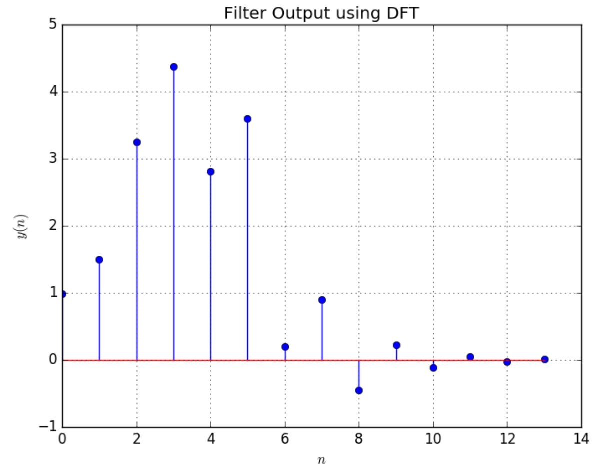

Fig. 6.3: $y(n)$ using IDFT and difference equation

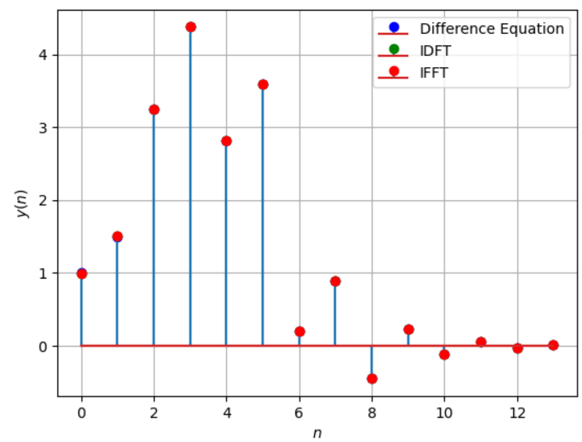

Fig. 6.4: The plot of $y(n)$ using IFFT

7 Exercises

Answer the following questions by looking at the python code in Problem 2.3.

7.1 The command

```
output_signal = signal.lfilter(b, a,
    input_signal)
```

in Problem 2.3 is executed through the following difference equation

$$\sum_{m=0}^{M} a(m)\, y(n-m) = \sum_{k=0}^{N} b(k)\, x(n-k) \tag{7.1}$$

where the input signal is $x(n)$ and the output signal is $y(n)$ with initial values all 0. Replace **signal.filtfilt** with your own routine and verify.

7.2 Repeat all the exercises in the previous sections for the above a and b.

7.3 What is the sampling frequency of the input signal?

Solution: Sampling frequency(fs)=44.1kHZ.

7.4 What is type, order and cutoff-frequency of the above butterworth filter

Solution: The given butterworth filter is low pass with order=2 and cutoff-frequency=4kHz.

7.5 Modifying the code with different input parameters and to get the best possible output.