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Grover algorithm

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Abstract

An unsorted database contains N records, of which just one satisfies a particular property. The problem is to identify that one record. Any classical algorithm, deterministic or probabilistic, will clearly take O (N) steps since on the average it will have to examine a large fraction of the N records. Quantum mechanical systems can do several operations simultaneously due to their wave like properties. Therefore O($\sqrt{N}$) step quantum mechanical algorithm for identifying that record. It is within a constant factor of the fastest possible quantum mechanical algorithm.

1 Introduction

Searching large databases is an important problem with broad applications. The Grover search algorithm provides a powerful method for quantum computers to perform searches with a quadratic speedup in the number of required database queries over classical computers. It is an optimal search algorithm for a quantum computer , and has further applications as a subroutine for other quantum algorithms.ou have likely heard that one of the many advantages a quantum computer has over a classical computer is its superior speed searching databases. Grover's algorithm demonstrates this capability. This algorithm can speed up an unstructured search problem quadratically, but its uses extend beyond that; it can serve as a general trick or subroutine to obtain quadratic run time improvements for a variety of other algorithms. This is called the amplitude amplification trick.

The Grover search algorithm has 4 stages: **initialization**, **oracle**, **amplification**, and **measurement**. The **initialization** stage creates an equal superposition of all states. The **oracle** stage marks the solution(s) by flipping the sign of that state's amplitude. The **amplification** stage performs a reflection about the mean, thus increasing the amplitude of the marked state. Finally, the algorithm output is measured. For a search database of size N, the single-shot probability of measuring the correct answer is maximized to near-unity by repeating the oracle and amplification stages O($\sqrt{N}$) times. By comparison, a classical search algorithm will get the correct answer after an average of N/2 queries of the oracle. For large databases, this quadratic speed up represents a significant advantage for quantum computers

1.1 Unstructured Search

Unstructured data (or unstructured information) is information that either does not have a predefined data model or is not organized in a pre-defined manner. Unstructured information is typically text-heavy, but may contain data such as dates, numbers, and facts as well.Suppose you are given a large list of N items. Among these items there is one item with a unique property that

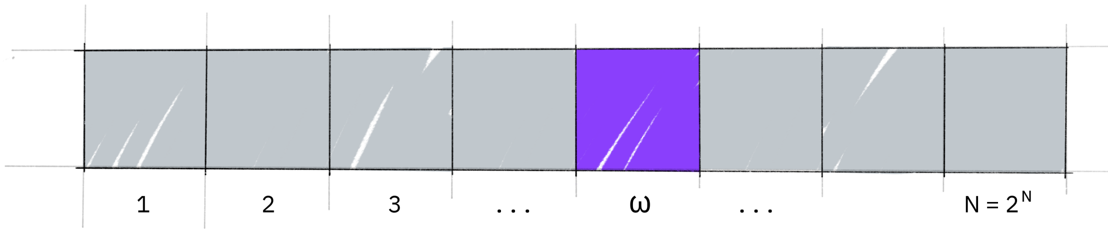


Figure 1: A large list.

we wish to locate; we will call this one the winner w. Think of each item in the list as a box of a particular color. Say all items in the list are gray except the winner w, which is purple.On a quantum computer, we can find the marked item in roughly O($\sqrt{N}$) steps with Grover's amplitude amplification trick howerver using classical computation, one would have to check on average N/2 of these boxes, and in the worst case, all N of them

1.2 The Oracle

Suppose we are supplied with a quantum oracle with the ability to recognize solutions to the search problem. This recognition is signalled by making use of an oracle qubit. More precisely, the oracle is a unitary operator, O, defined by its action on the computation basic

$$|x\rangle\,|q\rangle \rightarrow |x\rangle\,|x \oplus f(x)\rangle$$

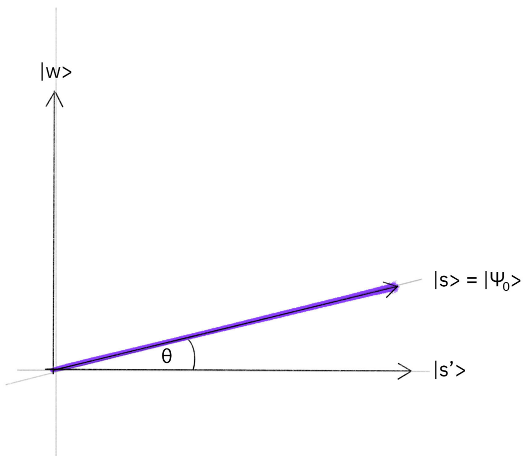
where $|x\rangle$ is the index register, $\oplus$ denotes addition modulo 2, and the oracle qubit $|q\rangle$ is a single qubit which change sign when $|x\rangle$ is solution to search problem, initial state of $|q\rangle$ is $\frac{(|0\rangle-|1\rangle)}{\sqrt{2}}$ For the examples in this textbook, our 'database' is comprised of all the possible computational basis states our qubits can be in. For example, if we have 3 qubits, our list is the states $|000\rangle$,$|001\rangle$,... $|111\rangle$ (i.e the states $|0\rangle \rightarrow |7\rangle$)
Grover's algorithm solves oracles that add a negative phase to the solution states. I.e. for any state $|x\rangle$ in the computational basis:

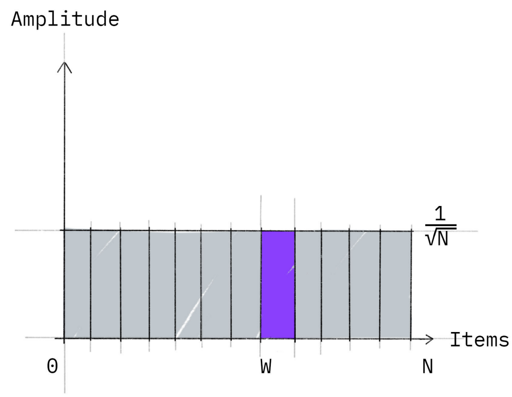
1.3 Amplitude Amplification

If at this point we were to measure in the standard basis $|x\rangle$, this superposition would collapse, according to the fifth quantum law, to any one of the basis states with the same probability of $\frac{1}{N} = \frac{1}{2^n}$. Our chances of guessing the right value w is therefore 1 in 2n, as could be expected. Hence, on average we would need to try about $\frac{N}{2} = 2^{n-1}$ times to guess the correct item.

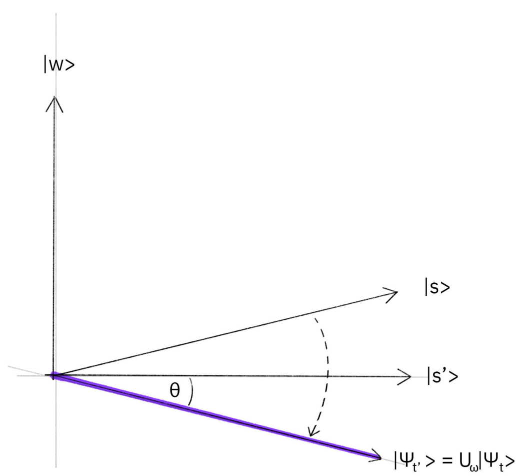
Enter the procedure called amplitude amplification, which is how a quantum computer significantly enhances this probability. This procedure stretches out (amplifies) the amplitude of the marked item, which shrinks the other items' amplitude, so that measuring the final state will return the right item with near-certainty. This algorithm has a nice 2 Dimension geometry interpretation in terms of two reflections operator. We need to consider only two-state one winner —w¿ state and the other is a uniform superposition $|s\rangle$ of states. These two vectors are span two-dimension plans in the herbert space of C^N. They are not quite perpendicular to each other because $|w\rangle$ is occur in a superposition of $|s\rangle$with an amplitude of $N^{\frac{-1}{2}}$ as well. Now take another state $|s'\rangle$ which is perpendicular to the state $|w\rangle$. This obtained from $|s'\rangle$ by removing the state $|w\rangle$ from it.

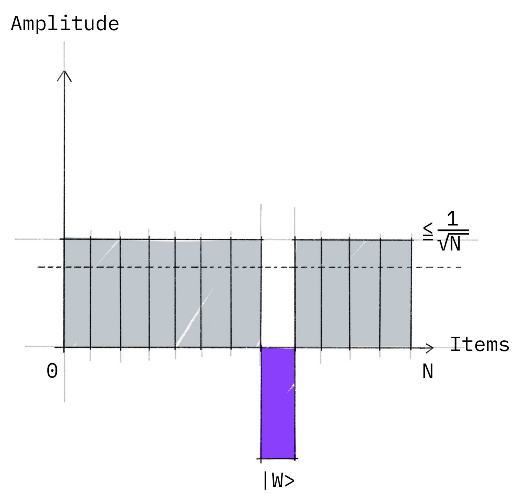
STEP 1: The amplitude amplification procedure starts out in the uniform superposition $|s\rangle$, which is easily constructed from $|s\rangle = H^{\oplus n} |0\rangle^n$.





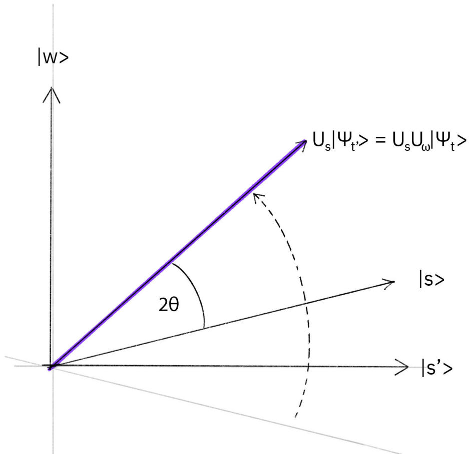
STEP 2: We apply the oracle reflection U_f to the state $|s\rangle$.

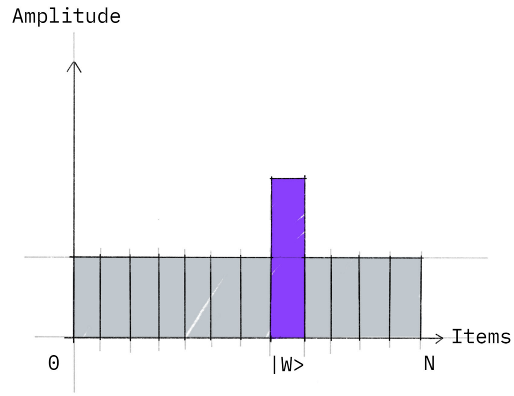




STEP 3: We now apply an additional reflection U_s about the state $|s\rangle$: $U_s = 2|s\rangle\langle s| - 1$.This transformation maps the state to $U_s U_f |s\rangle$ and completes the transformation.

Two reflections always correspond to a rotation. The transformation UsUf rotates the initial state —s⟩ closer towards the winner —w⟩. The action of the reflection Us in the amplitude bar diagram can be understood as a reflection about the average amplitude. Since the average amplitude has been lowered by the first reflection, this transformation boosts the negative amplitude of —w⟩ to roughly three times its original value, while it decreases the other amplitudes. We then go to step 2 to repeat the application. This procedure will be repeated several times to zero in on the





winner.
After t steps we will be in the state $|\psi_t\rangle$ where: $|\psi_t\rangle = (|U_s\rangle\,|U_f\rangle)^t\,|s\rangle$.

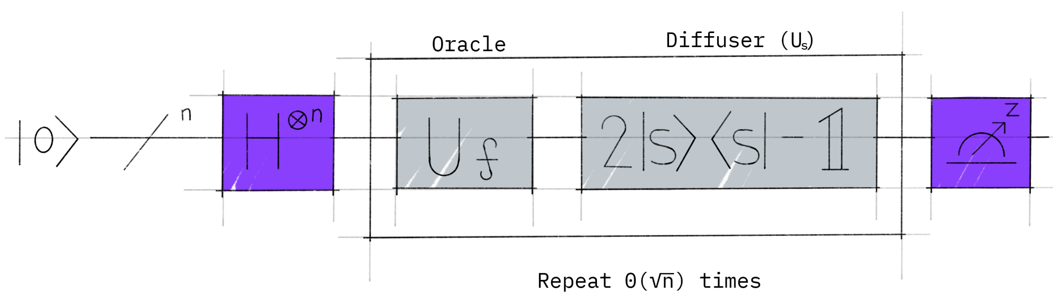


Figure 2: Full grovers circuit.

After repeating this operation $\sqrt{N}$ we get get winner state will emerge out. If we left for long time Then the probability of finding operator is a decrease

1.4 Result

Then the probability of finding operator is a decrease I have Implemented a Grover circuit with a database for 3 qubits it gives a result with an accuracy of around 80% and for the real device (ibm-lagos) its accuracy comes down to 40 %. This error is coming due to large circuit formation, which introduces an error in the result. After transpiling the circuit, we can see the size circuit which is big. I putting githublink for notebook.
This is my link for: Jupyter Notebook.