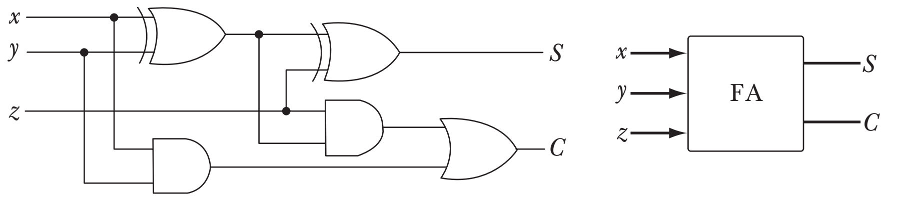


(a) Logic diagram (b) Block diagram

Figure 1-18 Full-adder circuit.

1-6 Flip-Flops

The digital circuits considered thus far have been combinational, where the outputs at any given time are entirely dependent on the inputs that are present at that time. Although every digital system is likely to have a combinational circuit, most systems encountered in practice also include storage elements, which require that the system be described in terms of sequential circuits. The most common type of sequential circuit is the synchronous type. Synchronous sequential circuits employ signals that affect the storage elements only at discrete instants of time. Synchronization is achieved by a timing device called a clock pulse generator that produces a periodic train of *clock pulses*. The clock pulses are distributed throughout the system in such a way that storage elements are affected only with the arrival of the synchronization pulse. Clocked synchronous sequential circuits are the type most frequently encountered in practice. They seldom manifest instability problems and their timing is easily broken down into independent discrete steps, each of which may be considered separately.

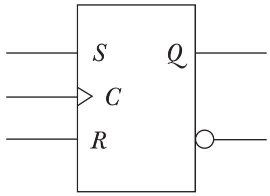
clocked sequential circuit

The storage elements employed in clocked sequential circuits are called flip-flops. A flip-flop is a binary cell capable of storing one bit of information. It has two outputs, one for the normal value and one for the complement value of the bit stored in it. A flip-flop maintains a binary state until directed by a clock pulse to switch states. The difference among various types of flip-flops is in the number of inputs they possess and in the manner in which the inputs affect the binary state. The most common types of flip-flops are presented below.

SR Flip-Flop

The graphic symbol of the *SR* flip-flop is shown in Fig. 1-19(a). It has three inputs, labeled *S* (for set), *R* (for reset), and *C* (for clock). It has an output *Q* and sometimes the flip-flop has a complemented output, which is indicated with a small circle at the other output terminal. There is an arrowhead-shaped symbol in front of the letter *C* to designate a *dynamic input*. The dynamic indicator symbol denotes the fact that the flip-flop responds to a positive transition (from 0 to 1) of the input clock signal.

The operation of the *SR* flip-flop is as follows. If there is no signal at the clock input *C*, the output of the circuit cannot change irrespective of the values at inputs



(a) Graphic symbol

S	R	$Q(t+1)$	
0	0	$Q(t)$	No change
0	1	0	Clear to 0
1	0	1	Set to 1
1	1	?	Indeterminate

(b) Characteristic table

Figure 1-19 *SR* flip-flop.

S and *R*. Only when the clock signal changes from 0 to 1 can the output be affected according to the values in inputs *S* and *R*. If $S = 1$ and $R = 0$ when *C* changes from 0 to 1, output *Q* is set to 1. If $S = 0$ and $R = 1$ when *C* changes from 0 to 1, output *Q* is cleared to 0. If both *S* and *R* are 0 during the clock transition, the output does not change. When both *S* and *R* are equal to 1, the output is unpredictable and may go to either 0 or 1, depending on internal timing delays that occur within the circuit.

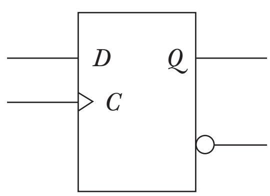
The characteristic table shown in Fig. 1-19(b) summarizes the operation of the *SR* flip-flop in tabular form. The *S* and *R* columns give the binary values of the two inputs. $Q(t)$ is the binary state of the *Q* output at a given time (referred to as *present state*). $Q(t + 1)$ is the binary state of the *Q* output after the occurrence of a clock transition (referred to as next state). If $S = R = 0$, a clock transition produces no change of state [i.e., $Q(t + 1) = Q(t)$]. If $S = 0$ and $R = 1$, the flip-flop goes to the 0 (clear) state. If $S = 1$ and $R = 0$, the flip-flop goes to the 1 (set) state. The *SR* flip-flop should not be pulsed when $S = R = 1$ since it produces an indeterminate next state. This indeterminate condition makes the *SR* flip-flop difficult to manage and therefore it is seldom used in practice.

D Flip-Flop

The *D* (data) flip-flop is a slight modification of the *SR* flip-flop. An *SR* flip-flop is converted to a *D* flip-flop by inserting an inverter between *S* and *R* and assigning the symbol *D* to the single input. The *D* input is sampled during the occurrence of a clock transition from 0 to 1. If $D = 1$, the output of the flip-flop goes to the 1 state, but if $D = 0$, the output of the flip-flop goes to the 0 state.

The graphic symbol and characteristic table of the *D* flip-flop are shown in Fig. 1-20. From the characteristic table we note that the next state $Q(t + 1)$ is

Figure 1-20 *D* flip-flop



(a) Graphic symbol

D	$Q(t+1)$	
0	0	Clear to 0
1	1	Set to 1

(b) Characteristic table

determined from the D input. The relationship can be expressed by a characteristic equation:

$$Q(t + 1) = D$$

This means that the Q output of the flip-flop receives its value from the D input every time that the clock signal goes through a transition from 0 to 1.

Note that no input condition exists that will leave the state of the D flip-flop unchanged. Although a D flip-flop has the advantage of having only one input (excluding C), it has the disadvantage that its characteristic table does not have a "no change" condition $Q(t + 1) = Q(t)$. The "no change" condition can be accomplished either by disabling the clock signal or by feeding the output back into the input, so that clock pulses keep the state of the flip-flop unchanged.

JK Flip-Flop

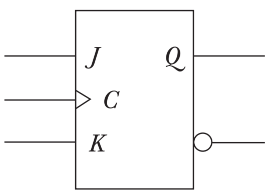
A JK flip-flop is a refinement of the SR flip-flop in that the indeterminate condition of the SR type is defined in the JK type. Inputs J and K behave like inputs S and R to set and clear the flip-flop, respectively. When inputs J and K are both equal to 1, a clock transition switches the outputs of the flip-flop to their complement state.

The graphic symbol and characteristic table of the JK flip-flop are shown in Fig. 1-21. The J input is equivalent to the S (set) input of the SR flip-flop, and the K input is equivalent to the R (clear) input. Instead of the indeterminate condition, the JK flip-flop has a complement condition $Q(t + 1) = Q'(t)$ when both J and K are equal to 1.

T Flip-Flop

Another type of flip-flop found in textbooks is the T (toggle) flip-flop. This flip-flop, shown in Fig. 1-22, is obtained from a JK type when inputs J and K are connected to provide a single input designated by T. The T flip-flop therefore has only two conditions. When $T = 0$ ($J = K = 0$) a clock transition does not change the state of

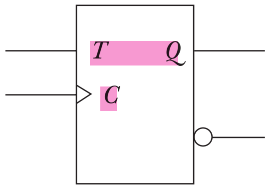
Figure 1-21 *JK* flip-flop



(a) Graphic symbol

J	K	$Q(t + 1)$	
0	0	$Q(t)$	No change
0	1	0	Clear to 0
1	0	1	Set to 1
1	1	$Q'(t)$	Complement

(b) Characteristic table



(a) Graphic symbol

T	$Q(t+1)$	
0	$Q(t)$	No change
1	$Q'(t)$	Complement

(b) Characteristic table

Figure 1-22 T flip-flop

the flip-flop. When $T = 1$ ($J = K = 1$) a clock transition complements the state of the flip-flop. These conditions can be expressed by a characteristic equation:

$$Q(t + 1) = Q(t) \oplus T$$

Edge-Triggered Flip-Flops

The most common type of flip-flop used to synchronize the state change during a clock pulse transition is the edge-triggered flip-flop. In this type of flip-flop, output transitions occur at a specific level of the clock pulse. When the pulse input level exceeds this threshold level, the inputs are locked out so that the flip-flop is unresponsive to further changes in inputs until the clock pulse returns to 0 and another pulse occurs. Some edge-triggered flip-flops cause a transition on the rising edge of the clock signal (positive-edge transition), and others cause a transition on the falling edge (negative-edge transition).

clock pulses

Figure 1-23(a) shows the clock pulse signal in a positive-edge-triggered D flip-flop. The value in the D input is transferred to the Q output when the clock makes a positive transition. The output cannot change when the clock is in the 1 level, in the 0 level, or in a transition from the 1 level to the 0 level. The effective positive clock transition includes a minimum time called the setup time in which the D input must remain at a constant value before the transition, and a definite time called the *hold time* in which the D input must not change after the positive transition. The effective positive transition is usually a very small fraction of the total period of the clock pulse.

Figure 1-23(b) shows the corresponding graphic symbol and timing diagram for a negative-edge-triggered D flip-flop. The graphic symbol includes a negation small circle in front of the dynamic indicator at the C input. This denotes a negative-edge-triggered behavior. In this case the flip-flop responds to a transition from the 1 level to the 0 level of the clock signal.

master-slave flip-flop

Another type of flip-flop used in some systems is the master-slave flip-flop. This type of circuit consists of two flip-flops. The first is the master, which responds to the positive level of the clock, and the second is the slave, which responds to the negative level of the clock. The result is that the output changes

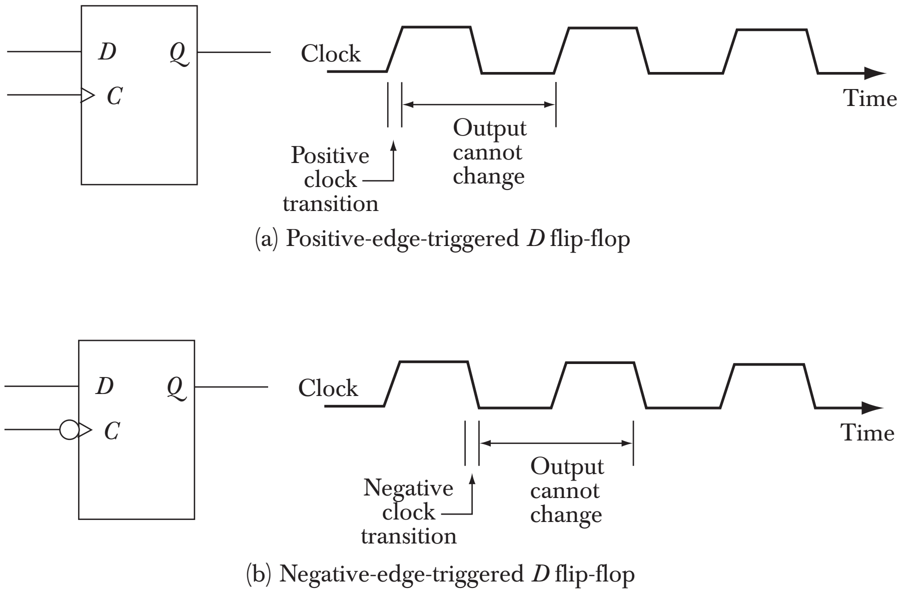


Figure 1-23 Edge-triggered flip-flop.

during the 1-to-0 transition of the clock signal. The trend is away from the use of master-slave flip-flops and toward edge-triggered flip-flops.

Flip-flops available in integrated circuit packages will sometimes provide special input terminals for setting or clearing the flip-flop asynchronously. These inputs are usually called "preset" and "clear." They affect the flip-flop on a negative level of the input signal without the need of a clock pulse. These inputs are useful for bringing the flip-flops to an initial state prior to its clocked operation.

Excitation Tables

The characteristic tables of flip-flops specify the next state when the inputs and the present state are known. During the design of sequential circuits we usually know the required transition from present state to next state and wish to find the flip-flop input conditions that will cause the required transition. For this reason we need a table that lists the required input combinations for a given change of state. Such a table is called a flip-flop excitation table.

Table 1-3 lists the excitation tables for the four types of flip-flops. Each table consists of two columns, $Q(t)$ and $Q(t+1)$, and a column for each input to show how the required transition is achieved. There are four possible transitions from present state $Q(t)$ to next state $Q(t+1)$. The required input conditions for each of these transitions are derived from the information available in the characteristic tables. The symbol $\times$ in the tables represents a don't-care condition; that is, it does not matter whether the input to the flip-flop is 0 or 1.

TABLE 1-3 Excitation Table for Four Flip-Flops

SR flip-flop			
$Q(t)$	$Q(t+1)$	S	R
0	0	0	×
0	1	1	0
1	0	0	1
1	1	×	0

D flip-flop		
$Q(t)$	$Q(t+1)$	D
0	0	0
0	1	1
1	0	0
1	1	1

JK flip-flop			
$Q(t)$	$Q(t+1)$	J	K
0	0	0	×
0	1	1	×
1	0	×	1
1	1	×	0

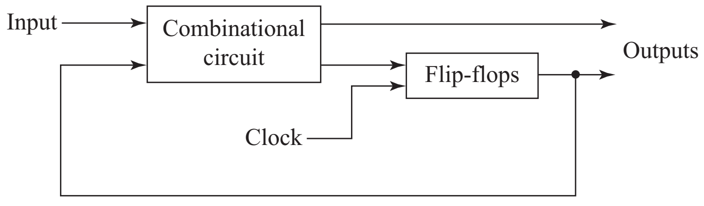
T flip-flop		
$Q(t)$	$Q(t+1)$	T
0	0	0
0	1	1
1	0	1
1	1	0

The reason for the don't-care conditions in the excitation tables is that there are two ways of achieving the required transition. For example, in a *JK* flip-flop, a transition from present state of 0 to a next state of 0 can be achieved by having inputs J and K equal to 0 (to obtain no change) or by letting $J = 0$ and $K = 1$ to clear the flip-flop (although it is already cleared). In both cases J must be 0, but K is 0 in the first case and 1 in the second. Since the required transition will occur in either case, we mark the K input with a don't-care × and let the designer choose either 0 or 1 for the K input, whichever is more convenient.

1-7 Sequential Circuits

A sequential circuit is an interconnection of flip-flops and gates. The gates by themselves constitute a combinational circuit, but when included with the flip-flops, the overall circuit is classified as a sequential circuit. The block diagram of a clocked sequential circuit is shown in Fig. 1-24. It consists of a combinational

Figure 1-24 Block diagram of a clocked synchronous sequential circuit.

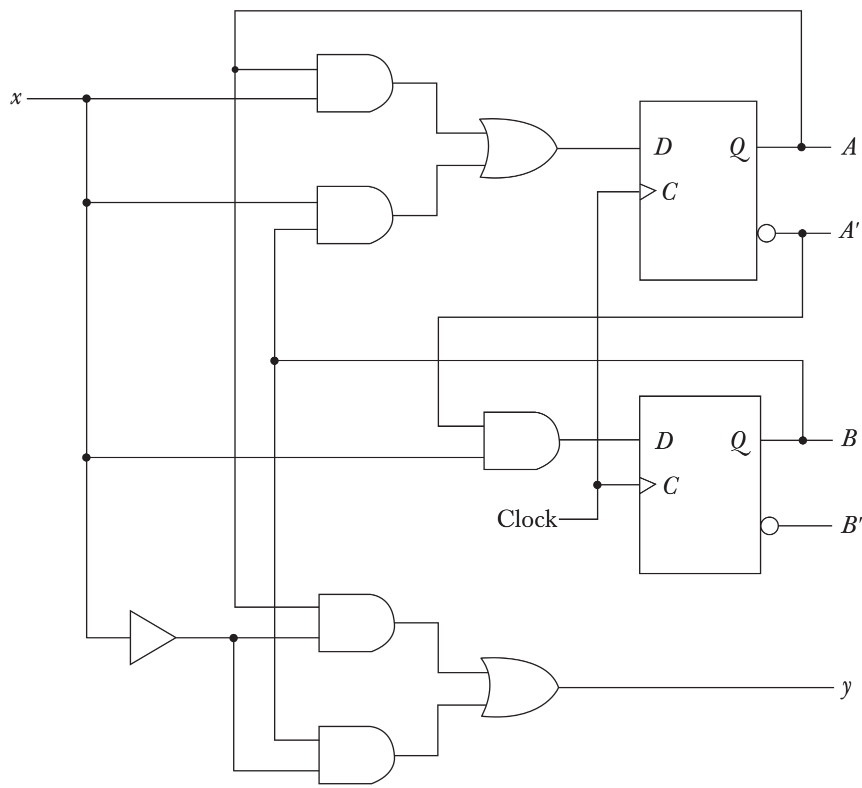


circuit and a number of clocked flip-flops. In general, any number or type of flip-flops may be included. As shown in the diagram, the combinational circuit block receives binary signals from external inputs and from the outputs of flip-flops. The outputs of the combinational circuit go to external outputs and to inputs of flip-flops. The gates in the combinational circuit determine the binary value to be stored in the flip-flops after each clock transition. The outputs of flip-flops, in turn, are applied to the combinational circuit inputs and determine the circuit's behavior. This process demonstrates that the external outputs of a sequential circuit are functions of both external inputs and the present state of the flip-flops. Moreover, the next state of flip-flops is also a function of their present state and external inputs. Thus a sequential circuit is specified by a time sequence of external inputs, external outputs, and internal flip-flop binary states.

Flip-Flop Input Equations

An example of a sequential circuit is shown in Fig. 1-25. It has one input variable x, one output variable y, and two clocked D flip-flops. The AND gates, OR gates, and inverter form the combinational logic part of the circuit. The interconnections

Figure 1-25 Example of a sequential circuit



among the gates in the combinational circuit can be specified by a set of Boolean expressions. The part of the combinational circuit that generates the inputs to flip-flops are described by a set of Boolean expressions called flip-flop input equations. We adopt the convention of using the flip-flop input symbol to denote the input equation variable name and a subscript to designate the symbol chosen for the output of the flip-flop. Thus, in Fig. 1-25, we have two input equations, designated D_A and D_B. The first letter in each symbol denotes the D input of a D flip-flop. The subscript letter is the symbol name of the flip-flop. The input equations are Boolean functions for flip-flop input variables and can be derived by inspection of the circuit. Since the output of the OR gate is connected to the D input of flip-flop A, we write the first input equation as

input equation

$$D_A = Ax + Bx$$

where A and B are the outputs of the two flip-flops and x is the external input. The second input equation is derived from the single AND gate whose output is connected to the D input of flip-flop B:

$$D_B = A'x$$

The sequential circuit also has an external output, which is a function of the input variable and the state of the flip-flops. This output can be specified algebraically by the expression

$$y = Ax' + Bx'$$

From this example we note that a flip-flop input equation is a Boolean expression for a combinational circuit. The subscripted variable is a binary variable name for the output of a combinational circuit. This output is always connected to a flip-flop input.

State Table

The behavior of a sequential circuit is determined from the inputs, the outputs, and the state of its flip-flops. Both the outputs and the next state are a function of the inputs and the present state. A sequential circuit is specified by a state table that relates outputs and next states as a function of inputs and present states. In clocked sequential circuits, the transition from present state to next state is activated by the presence of a clock signal.

present state

The state table for the circuit of Fig. 1-25 is shown in Table 1-4. The table consists of four sections, labeled *present state, input, next state,* and *output*. The present-state section shows the states of flip-flops A and B at any given time t. The input section gives a value of x for each possible present state. The next-state section shows the states of the flip-flops one clock period later at time $t + 1$. The output section gives the value of y for each present state and input condition.

next state