This question paper contains 7 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2999**

Unique Paper Code : **32375201** **IC**

Name of the Paper : **Introductory Probability**

Name of the Course : **Statistics : GE for Honours**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Section I is compulsory.

Attempt any *five* questions,

selecting at least *two* questions from each of the

Sections II and III. Use of simple calculator is allowed.

Section I

1. (*a*) If $P(A) = 0.3$, $P(B) = 0.2$ and $P(A \cap B) = 0.1$, then obtain $P(\bar{A} \cap \bar{B})$.

(*b*) State Kolmogorov's three axioms of probability.

(c) If X and Y are independent events and $E(X) = 10$, $E(Y) = 20$, then find $E(XY)$.

(d) Can, for some random variable X, $P(\mu_X - 2\sigma_X \leq X \leq \mu_X + 2\sigma_X) = 0.9545$?

(e) If $M_X(t) = (0.9 + (0.1)e^t)^{100}$, then obtain $E(X^2)$.

(f) If $E(X) = 1$, $E(X^2) = 4$ and $M_X(t) = e^{\alpha + \beta t + \gamma t^2}$, then obtain the values of α, β and γ.

(g) X and Y are independent normal variates with means 1, 2 and variances 25 and 36 respectively. Compute $E(e^{\alpha X + \beta Y})$.

(h) If $E(X) = \mu$ and $Var(X) = \sigma^2$ and using Chebychev's inequality we get $E(|X - \mu \leq k\sigma|) \geq 1 - a$, then obtain the value of a.

(i) If $\sigma_X^2 = 25$, $\sigma_Y^2 = 36$ and $Cov(X, Y) = 10$, then obtain $Var(2X - 3Y)$.

(j) State Demoivre Laplace central limit theorem.

(1,1,1,1,1,2,2,2,2,2)

Section II

2. (a) A box contains 20 fuses, in which five are defective. If three of the fuses are selected at random and are removed from the box in succession, then what is the probability that all are defective ?

(b) A coin is tossed three times and the eight possible outcomes are assumed to be equally likely. If A be the event that a head occurs on each of the first two tosses, B is the event that a tail occurs on the third toss and C is the event that exactly two tails in the three tosses, then show that :

(i) A and B are independent events.

(ii) B and C are dependent events. 6,6

3. (a) Two socks are selected at random and are removed in succession from a drawer containing five brown socks and three green socks. If random variable W represents the number of brown socks selected, then find probability distribution of W and E(W).

(*b*) Find the cumulative distribution function(cdf) of random variable X whose probability density function(pdf) is given by :

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 \le x < 2 \\ 0, & \text{otherwise} \end{cases}$$

Also sketch the graph of pdf and cdf. 6,6

4. (*a*) Three random variables X, Y and Z have variances $\sigma_X^2 = 1, \sigma_Y^2 = 5, \sigma_Z^2 = 2$ and Cov(X, Y) = −2, Cov(Y, Z) = −1 and Cov(Z, X) = 1 respectively. Find Cov(U, V) where U = 3X − Y + 2Z and V = X − 3Y + Z.

(*b*) Find mean and variance of random variable X whose probability density function is : 6,6

$$f(x) = \begin{cases} \dfrac{4}{\pi\left(1 + x^2\right)}, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Section III

5. (*a*) An item is produced in large numbers. The machine is known to produce 5% defectives. A quality control inspector is examining the items by taking them at random. What is the probability that at least 4 items are examined in order to get 2 defectives ?

(*b*) If the probability is 0.75 that an applicant for a driver's license will pass the road test on any given try. What is the probability that an applicant will finally pass the test on the fourth try ?

(*c*) If two percent of the books bound at a certain bindery have defective bindings, then use the Poisson approximation to the binomial distribution to determine the probability that five of 400 books bound by this bindery will have defective bindings. 4,4,4

6. (a) If X has the discrete uniform distribution $f(x) = 1/k$, for $x = 1, 2, \ldots\ldots; k$, then show that moment generating function is given by :

$$M_x(t) = E(e^{tX}) = \frac{e^t(1 - e^{kt})}{k(1 - e^t)}.$$ Also find $M'_x(0), M''_x(0)$ and hence compute mean and variance of random variable X.

(b) Let $b(x; n, \theta) = \binom{n}{x}\theta^x(1-\theta)^{n-x}$ and $B(x; n, \theta) = \sum_{k=0}^{x} b(k, n, \theta)$. Show that :

(i) $b(x; n, \theta) = b(n - x; n, 1 - \theta)$

(ii) $b(x; n, \theta) = B(x; n, \theta) - B(x - 1; n, \theta)$

(iii) $b(x; n, \theta) = B(n - x; n, 1 - \theta) - B(n - x - 1; n, 1 - \theta)$

(iv) $B(x; n, \theta) = 1 - B(n - x - 1; n, 1 - \theta)$. 6,6

7. (a) Show that $\alpha = \mu\left[\frac{\mu(1-\mu)}{\sigma^2} - 1\right]$ and $\beta = (1-\mu)\left[\frac{\mu(1-\mu)}{\sigma^2} - 1\right]$, where (α, β) are two parameters of the beta type-I distribution and (μ, σ^2) are the mean and variance of the distribution respectively.

(b) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{2}\int_0^\infty e^{-\frac{1}{2}z^2}\,dz$ and hence evaluate $\Gamma\left(\frac{1}{2}\right)$. 6,6

8. (a) The random variables $X_i (i = 1, 2, 3, 4)$ are independently and identically distributed with probability density function $f(x) = \frac{1}{\sqrt{32\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-7}{4}\right)^2\right]$. Obtain the probability density function of $U = \frac{1}{4}\sum_{i=1}^{4} X_i$ and $V = X_1 - 2X_2 + 3X_3 - 4X_4$.

(b) The amount of cosmic radiations to which a person is exposed when flying by JET across the US is a random variable having normal distribution with a mean of 4.35 units and a standard deviation of 0.59 units. What is the probability that a person will be exposed to more than 5.53 units of cosmic radiations on such a flight ? 6,6

[Use $e^{-8} = 0.00035$, $e^{-4} = 0.01832$ and for standard normal distribution $P(-1 < Z < 1) = 0.6628$, $P(-2 < Z < 2) = 0.9544$ and $P(-3 < Z < 3) = 0.9973$]