b. Write the output of the following: (4)

```
public class ObjComp
{
   public static void main(String [] args )
   {
         int result = 0;
         ObjComp oc = new ObjComp();
         Object o = oc;
         if (o == oc)
{
                 result = 1;
                 System.out.println("result = " + result);
         }
         if (o != oc)
{
                 result = result + 10;
                 System.out.println("result = " + result);
         }
         if (o.equals(oc) )
         {
                 result = result + 100;
                 System.out.println("result = " + result;
         }
         if (oc.equals(o) )
         {
                 result = result + 1000;
                 System.out.println("result = " + result);
         }

}}
```

lil 19/5/16 Evening

[This question paper contains 6 printed pages.]

Sr. No. of Question Paper : 6917 **FC-2** **Your Roll No...............**

Unique Paper Code : 32375201

Name of the Paper : Introductory Probability

Name of the Course : **B.Sc. (Hons) Statistics / B.A. (Hons.) / B.Com. (Hons.) under CBCS – GE**

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Section A is compulsory.
3. Attempt any **five** questions, selecting at least **two** questions from each of the sections B and C.
4. Use of simple calculator is allowed.

SECTION – A

1. Answer the following :

(i) If X and Y are independent random variables, then Cov(X, Y) = ______. (1)

(ii) Let F(1) = 0.5, F(2) = 0.4, F(3) = 0.7 and F(4) = 1.0. Can F(x) serve as distribution function of r.v. with the range 1, 2, 3 and 4. State the reason. (1)

(iii) A and B are events such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$ and $P(A') = \frac{2}{3}$. Find P(B). (1)

P.T.O.

(iv) If Cov(X, Y) = –30 and mean of X and Y is 12 and 8 respectively, then E(XY) = ______ . (1)

(v) Define almost sure convergence. (1)

(vi) Name the discrete distributions for which mean is equal to variance and mean is less than variance. (2)

(vii) If X is a Poisson variate with m.g.f. $M_X(t) = e^{3(e^t-1)}$, then $P(X = 2) =$ ______ . (2)

(viii) If X and Y are independent r.v.'s with $\mu_X = 3$, $\mu_Y = 5$, $\sigma_X^2 = 8$ and $\sigma_Y^2 = 12$, then obtain mean and variance of U = X + 3Y. (2)

(ix) If A and B are mutually exclusive events with P(A) = 0.37, and P(B) = 0.44, then $P(B')$=____ and $P(A \cap B')$ = ______ . (2)

(x) What is the smallest value of k in Chebyshev's theorem for which the probability that a random variable will take on a value between $\mu - k\sigma$ and $\mu + k\sigma$ is atleast 0.99 ? (2)

SECTION – B

2. (a) State Bayes theorem. A seed packaging company purchases 40% of their bean seeds from supplier A and 60% from supplier B and mixes these seeds together. Bean seeds from supplier A have an 85% germination rate and those from supplier B have a 75% germination rate.

   (i) Find the probability that a seed selected at random from the mixed seeds will germinate.

   (ii) Given that a seed germinates, find the probability that the seed was purchased from supplier A.

(b) A biology professor has two graduate assistants helping her with her research. The probability that the older of the two assistants will be absent on any given day is 0.08, the probability that the younger of the two will be absent on any given day is 0.05, and the probability that they will both be absent on any given day is 0.02. Find the probabilities that

(i) Either or both of the graduates assistants will be absent on any given day;

(ii) At least one of the two graduate assistants will not be absent on any given day;

(iii) Only one of the two graduate assistants will be absent on any given day. (6,6)

3. (a) Consider a function :

$$f(x) = \begin{cases} \frac{x}{2} & \text{for } 0 < x \le 1 \\ \frac{1}{2} & \text{for } 1 < x \le 2 \\ \frac{3-x}{2} & \text{for } 2 \le x < 3 \\ 0 & \text{elsewhere} \end{cases}$$

(i) Show that f(x) is the probability density function of X.

(ii) Determine distribution function F(x). Hence evaluate $P(X > 2.5)$.

(iii) Compute $P(1.5 \le X \le 2.5)$.

(b) If X has the distribution function :

$$F(x) = \begin{cases} 0 & \text{for } x < -1 \\ \frac{1}{4} & \text{for } -1 \le x < 1 \\ \frac{1}{2} & \text{for } 1 \le x < 3 \\ \frac{3}{4} & \text{for } 3 \le x < 5 \\ 1 & \text{for } x \ge 5 \end{cases}$$

(i) Obtain p.m.f. of X.

(ii) Compute $P(X \le 3)$ and $P(-0.4 < X < 4)$.

(iii) Compute mean and variance of X. (8,4)

4. (a) Given that X has the probability mass function $f(x) = \frac{1}{8}\binom{3}{x}$ for x = 0, 1, 2, and 3, find the moment generating function of random variable X and hence find its mean and variance.

(b) Let the probability density function of the random variable Y be given by

$$f(y) = \begin{cases} k(y+1), & \text{for } 2 < y < 4 \\ 0 & \text{elsewhere.} \end{cases}$$

(i) Obtain the value of k.

(ii) Find $E[(3Y+2)^2]$ and $Var(3Y + 5)$. (5,7)

SECTION – C

5. (a) State Lindeberg-Levy central limit theorem. Let $X_1, X_2, \ldots, X_n$ be the sequence of independent random variables with distribution defined as $P\left(X_k = \pm k^{\alpha}\right) = 1/2$. Examine whether the central limit theorem holds.

(b) State Chebyshev's inequality. If X is the number scored in a throw of a pair of die, show that the Chebyshev's inequality gives $P(|X - \mu| \geq 2.5) \leq 0.47$, where μ is the mean of X, while the actual probability is 1/3. (6,6)

6. (a) In the instant lottery with 20% winning tickets, if X is equal to the number of winning tickets out of 8 tickets purchased, what is the distribution of X ? Also find the probability of purchasing 2 winning tickets ?

(b) Let X be a r.v. following binomial distribution with parameters $n = 8$ and $p = 0.4$. Derive the m.g.f. of X and hence obtain the m.g.f. of $Y = X - 3$.

(c) If X is a Poisson variate with parameter λ such that $P(X = 1) = P(X = 2)$ then find $P(X = 3)$. (4,4,4)

7. (a) If X is a r.v. having a geometric distribution then show that $P(Y = t | X \geq k) = P(X = t)$, where $Y = X - k$. Which property of geometric distribution does it depict ?

(b) The random variables X_i (i = 1, 2, 3, 4) are independent and identically distributed with p.d.f.

$$f(x) = \frac{1}{\sqrt{48\pi}} \exp\left(-\frac{(x-2)^2}{48}\right).$$

Obtain the distribution of (i) $V = \frac{1}{4}\sum_{i=1}^{4} X_i$, and (ii) $W = 2X_1 - 3X_2 + X_3$.

(7,5)

8. (a) A random variable X has a Gamma distribution and its p.d.f. is given by

$$f(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}, \quad \text{for } x > 0$$

Prove that the rth moment of x about the origin is

$$\mu'_r = \frac{\beta^r\Gamma(\alpha+r)}{\Gamma(\alpha)}.$$

Hence find its mean and variance.

(b) If X is Uniformly distributed with mean 2 and variance 1/3. Find $P(X \leq 2.5)$.

(c) Define hypergeometric distribution. Find its mean. How does it differ from binomial distribution ? (4,4,4)