ASSIGNMENT-1

1) Find the space complexity of the following.

a) Counting bits of binary representation of given number.

Algorithm:

```
CountBits (n)
{
  int count=0;
  while(n>0){
     n=n/2;
     count++;
  }
  return count;
}
```

Fpart	Vpart	Space
count	–	1
a	n	n/2
		$S(P) = \left(\frac{n}{2}+1\right)$

b) Matrix Multiplication

```
Algo.mat_mul (a,b,n)
{
 for(i=0;i<n;i++)
 {
   for(j=0;j<n;j++)
   {
     c[i,j]=0;
     for(k=0; k<n;k++)
     {  c[i,j]=c[i,j]+a[i,k]+b[k,j];
     }
   }
 }
```

Fpart	Vpart	Space
i,j,k	–	3
n	–	1
	a	n^2
	b	n^2
	c	n^2
		$S(P) = 3n^2+4$

eg-5:

```
Algo mat_mul (a, b, n)
{
  for (i=0; i<n; i++)
  {
   count++; - n
   for (j=0; j<n; j++)
   {
    count++; - n^2
    c[i,j]=0;
    count++; - n^2
    for (k=0; k<n; k++)
    {
     count++; - n^3
        c[i,j]=c[i]
     count++; - n^3
    }
    count++; - n^2
   }
   count++; - n
  }
  count++; - 1
}
```

No. of steps $= 2n^3 + 3n^2 + 2n + 1$

b) Binary search

```
Binary search(int arr[], int size, int target){
{
 count=0;
 while(left <= right){
    count++;                                  n
    int mid = left+(right-left)/2;
    count++;                                  n
    if(arr[mid] == target)
        return mid;
    if(arr[mid] < target)
        left = mid+1
    else
        right = mid-1;
 }
 count++;                                     1
 ~~return -1~~
 count++;                                     1
 return -1;
```

~~No.~~ Time complexity = $2n+2$

eg: 4 Binary representation of an number

Algorithm: 1

```
Binary(n)
// Input: positive decimal integer n
// Output: The number of binary digits
count ← 1
while n>1 do
    count ← count+1
    n ← [n/2]
return count
```

Step-1: Input's size n

Step-2: Basic operation ~~Comparision~~ Division

Step-3: Check wher the number of times the basic operation executed depends only on the size of an input

Step-4: Set up a Sum

$$\sum_{K=0}^{\log_2 n} 1$$

$$\log_2 n - 0 + 1$$

$$= \log_2 n + 1$$

$\frac{n}{2^K}$ $0 < K < 6$

$2^K = n$

$\log_2 2^K = \log n$

$K \log_2 2 = \log n$

$K = \log_2 n$

eg:5 - Fibonacci series

Algo:

```
//input: positive integer n
//output: The nth fibonacci number

if n = 0 then
    return 0

else if n = 1 then
    return 1

else
    a = 0
    b ← 1
    for i from 2 to n do
        temp ← a + b
        a ← b
        b ← temp

    return b.
```

1: Input size n

2: Basic operation Addition

3: Check whether the no. of times the basic operations executed depends only on the size of an input.

4: Set up a sum

$$C(n) = \sum_{i=2}^{n} 1 \quad \Big| \quad \text{upper} - \text{lower} + 1$$

$$= n - 2 + 1$$

$$= n - 1$$

$$\sum_{i=2}^{n} 3$$

$$3 \sum_{i=2}^{n} 1 = 3(n-2+1)$$

$$= 3(n-1)$$

$$= 3n - 3$$

11.) Check the following using Asymtotic Notation

a.) $2n^4+2 \notin \Omega(n^2)$

$T(n) = 2n^4+2$

$G(n) = n^2$

Condition: $2n^4+2 >= cn^2$, $n \geq n_0$

c	n_0	n	$2n^4+2 \geq cn^2$	T/F
1	1	1	$4 >= 1$	T
1	1	2	$18 >= 4$	T

The Condition is satisfied

b.) $2n^4+2 \in O(n^2)$

$T(n) = 2n^4+2$

$G(n) = n^2$

Condition: $2n^4+2 <= cn^2$, $n > n_0$

c	n_0	n	$2n^4+2 \leq cn^2$	T/F
1	1	1	$4 \leq 1$	F
2	1	1	$4 \leq 2$	F
3	1	1	$4 \leq 3$	F
4	1	1	$4 \leq 4$	T

The condition is satisfied

5)

a.) $n(n+1)/2 \in O(n^3)$

$t(n) = \frac{n(n+1)}{2}$

$g(n) = n^3$

$\lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{n^3}$

$\frac{1}{2} \lim \left(\frac{n^2+n}{n^3}\right)$

$\frac{1}{2} \lim \left(\frac{1}{n} + \frac{1}{n^2}\right)$

$\frac{1}{2} \left[\frac{1}{\infty} + \frac{1}{\infty}\right]$

$\frac{1}{2} [0+0] = 0$

b.) $n(n+1)/2 \in O(n^2)$

$t(n) = \frac{n(n+1)}{2}$

$g(n) = n^2$

$\lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{n^2}$

$\frac{1}{2} \lim \left(\frac{n^2+n}{n^2}\right)$

$\frac{1}{2} \lim \left(1 + \frac{1}{n}\right)$

$\cancel{\frac{1}{2} \lim_{x\to\infty}}$

$\frac{1}{2} \left(1 + \frac{1}{\infty}\right)$

$= \frac{1}{2} \in O(n^2)$

c.) $n(n+1)/2 \in \Theta(n^3)$

$\lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{n^3}$

$\frac{1}{2} \cancel{\frac{n(n+1)}{}} \frac{n^2+n}{n^3}$

$\frac{1}{2} \lim \left[\frac{1}{n} + \frac{1}{n^2}\right)$

$\frac{1}{2} \lim [0+0]$

$= 0$

d.) $n(n+1)/2 \in \Omega(n)$

$\lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{n}$

$\frac{1}{2} \lim_{n\to\infty} \frac{n(n+1)}{n}$

$\frac{1}{2} \lim_{n\to\infty} \frac{n^2+n}{n}$

$\frac{1}{2} \lim_{n\to\infty} (n+1)$

$\frac{1}{2} \cancel{\lim_{n\to\infty}} (\infty+1)$

$\frac{1}{2}$ //