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ASSIGNMENT ON -

LINEAR PROGRAMMING

SUBMITTED TO-
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DEPT.- SCHOOL OF MANAGEMENT

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CERTIFICATE

This is to certify that Mr. PRAVEEN KUMAR [174CA049] MCA first year student of "National Institute of Technology Karnataka (Surathkal)" has done ~~on~~ Assignment on Linear Programming, Related to Managerial Economics under the guidance of Mr. Gopalakrishna B V during the year 2017-18 in the partial fulfillment of ~~Acco~~ Managerial Economics Assignment

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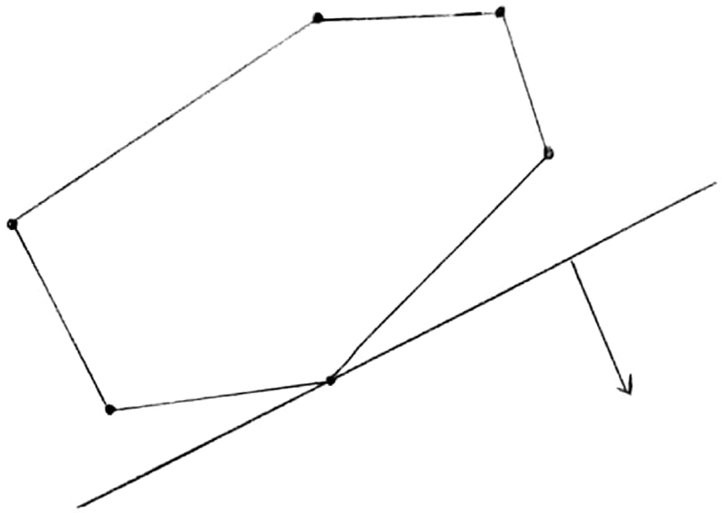
INTRODUCTION :-

Linear programming (LP) is also called linear optimization is a method to achieve the best outcome in a mathematical model whose requirements are represented by linear relationship. Linear programming is a special case of mathematical programming (mathematical optimization).

More formally, linear programming is a technique for the optimization of a linear objective function, subject to linear equality and linear inequality constraints. Its feasible region is a convex polytope, which is a set defined as the intersection of finitely many half space, each of which is defined by a linear inequality. Its objective function is a real valued affine (linear) function defined on different shape. A linear programming algorithm finds a point in the polyhedron where this function has the smallest (or largest) value if such a point exists



A pictorial representation of a two variables and six inequalities in simple linear program. The set of fesible solution is depicted in yellow and form a polygon, a 2-dimensional polytope. The linear cost function is represented by the red line and the arrow: The red line is a level set of the cost function, and the arrow indicates the direction in which we optimizing.

Chief Characterstics of a Linear Programming Problem :-

All problem where linear programming is applicable have the following chief characterstics in common:

1. Objective Function

There must be a certain clearly defined objective which can be stated quantitatively. In business problem, this is usually the maximization of profits or contribution, or the minimisation of costs. The objective saught after is known as the objective function'. To illustrate, suppose that a manufacturer produces Q_A, Q_B and Q_C quantities of three comodities A, B and C respectively. Let the profit per unit in the case of these commodities be P_A, P_B and P_C respectively. The producer wants to maximise profit

P from them. The objective function would, then, be expressed as under.

$$Q_A P_A + Q_B P_B + Q_C P_C = \text{Maximum}$$

2. Constraints :-

A constraints is simply an algebraic statement of the limits of a resource or input. Suppose that commodities A, B and C each require per unit of product x, y and z hours of the machine time and only m hours of machine time are available. The constraint on the production of A, B and C, then is as under:

$$xQ_A + yQ_B + zQ_C \leq m$$

The constraints, like an objective function, must be capable of arithmetic or algebraic expression. For example, a requirement that any solution shall not lower the quality or attractiveness of a product is not a constraint in the linear programming sense, as these characterstics can not expressed numerically.

3. NON-NEGATIVE CONDITIONS :- The values for the decision variables must be zero or positive and not negative. Thus, if the firm makes three products A, B and C, the non-negativity conditions would be.

$$Q_A \geq 0, \; Q_B \geq 0, \; Q_C \geq 0$$

This means that production of A, B and C should be positive or zero and not negative. Negative production would mean that products are indeed deproduced or dismantled, and obviously this is non-sensical.

4. LINEAR RELATIONSHIP :- The various relationship to be expressed either in the form of equations or inequalities must be linear. The adjective 'linear' is used to describe a proportional relationship between or among two or more variables. i.e. the exponents of all variables must be one. This would mean that raw materials or number of hour required will be proportional to the number of units produced. To take an example, linearity would imply that a 15 percent change in number of productive hours used in a certain process will lead to a 15 per cent change in output.

METHODS OF LINEAR PROGRAMMING

A linear programming problem can be solved either by the graphical method or by the simplex method.

1. LINEAR PROGRAMMING - GRAPHICAL METHOD

To explain the use of the graphical method to solve a linear programming problem ~~to~~ let us take a simple example. Suppose the objective of a firm is to maximise profits by producing product A and /or Product B, both of which have to be processed on two Machines 1 and 2. Product A required 2 hours on both the machine 1 and 2 while

Product B requires 3 hours on Machine 1 but only 1 hour on Machine 2. ~~and 2~~ There are only 12 and 8 hours available on Machine 1 and 2 respectively. The profit per unit is estimated at Rs 6. and Rs. 7 in case of A and B, respectively. Here we have dependent variable 'Profit' which is to be maximised and which is a function of two independent variables A and B, the production of which is restricted by the machine time available.

Now, if

P = Profit

Rs $6A$ = Total profit from sale of Product A
Rs $7B$ = Total profit from sale of Product B

$$P = \text{Rs } 6A + \text{Rs } 7B$$

Again, time taken in making the two products must not certainly exceed the total time available on each of the machines. It may be less or equal to the time available on the machines. Mathematically. We can write these constraints as under:

$$2A + 3B \leq 12, \quad 2A + B \leq 8$$

The first inequality above states that the hours required to produce one unit of A (2 hours) multiplied by the number of units A produced plus the hours required to produce one unit of B (3 hours) multiplied by the number of units of B produced must be equal to or less then 12 hours available on Machine 1. A Similar explanation holds for the second inequality. Both these inequalities represent capacity restrictions on output and hence on profit.

Finally, in order to obtain meaningful answers, the values calculated for A and B must be positive. In order to producing negative quantities of A and B does not convey any meaning. Thus Solution for A and B must be either zero or greater than zero ($A \geq 0$; $B \geq 0$)

Summing up. the problem would be as under:

Maximize $P = 6A + 7B$

Subject to constraints

$2A + 3B \leq 12$, $2A + 3B \leq 8$, $A \geq 0$, $A \geq 0$

Second ~~step~~ Method is "Simplex Method"

SIMPLEX METHOD :-

Simplex method is an iterative produce that allows to improve the solution at each step. This procedure is finished when isn't possible to improve the solution.

Starting from a random vertex value of the objective functions, simplex method tries to find repeatedly another vertex value that improves the one you have before. The search is done through the side of the polygon (or the edges of the polyhedron, if the number of variables is higher). As the number of vertices (and edges) is finite, it will always be able to find the result.

Simplex method is based on the following property: if objective function, F, doesn't take the max value in the A vertex, then there is an edge starting at A, along which the value of the function grows.

you should take care about simplex method only works "≤" type inequality and independent coefficient higher or equal to zero, and you will have to standrize the restrictions for the algorithm. Case after this ~~prod~~ procedure ">" o "=" type restrictions appear (or not modified) you should try other

NUMERICAL PROBLEMS

Problem Based on "GRAPHICAL METHOD"

The graphical method of solving a linear programming problem is used when there are only two decision variables. If the problem has 3 or more variables the graphical Method is not suitable then we have to go for the next Method i.e. called "Simplex Method"

STEPS FOR GRAPHICAL METHOD :-

STEP-1 :- Formulate the linear Programming Problem.

STEP 2 :- Graph the feasible region and find the corner points. The cordinates of the corner points can be the obtained by either inspection or by solving the two equations of the line intersecting at that point

STEP 3 :- Make a table listing the value of the objective function at each corner point.

STEP 4 :- Determine the optimal solution from the table in Step 3. If the problem is of maximization (minimization) type, the solution corresponding to the largest (smallest) value of the objective function is the optimal solution of the LPP.

Example 1 :- A furniture company produces inexpensive tables and chairs. the production process for each is similar in that both required a certain number of hours of Carpentry work and a certain number of labour hours in the painting department.

Each tables takes 4 hours of Carpentry and 2 hours in the painting department. Each chair requires 3 hours of Carpentry and 1 hour in painting department. During the current production period, 240 hours of Carpentry time are available and 100 hours in painting is available. Each table sold yield a profit of E7; each chair produced is sold yield a ~~profit~~ of for a E5 profit.

Find the best Combination of Tables of manufacture in order to reach the minimum profit.

SOLUTION :- We begin by summarizing the information needed to solve the problem in the form of table. This helps us understand the problem being faced

Department	Hours required to make 1 unit Tables	Chairs	Available hours
Carpentry	4	3	240
Painting	2	1	100
profit	7	5	

the objective is to maximize profit

the Constraints are

1. The hours of carpentry time used cannot exceed 240 hours per week.

2. the hours of painting time used can not exceed 100 hours per week.

3. The number of tables and chairs must be non-negative.

the decision variables that represent the actual decision to be made are defined as

x_1 = number of tables to be produced

x_2 = number of Chairs to be produced.

Now we can state the linear programming (LP) problem in term of x_1 and x_2 and profit (P).

maximize $\quad P = 7x_1 + 5x_2$ (objective function)

subject to

$4x_1 + 3x_2 \leq 240$ (hours of carpentry constraint)

$2x_1 + x_2 \leq 100$ (hours of painting constraint)

$x_1 \geq 0,\ x_2 \geq 0$ (Non-negative constraint)

To find the optimal solution to this LP using the graphical method we first indentify the region of feasible solutions and the corner points of the feasible region. The graph for this example is plotted in figure (2). In this example the corner points are (0,0), (50,0), (30,40) and (0,80). Testing these corner points on $P = 7x_1 + 5x_2$ gives

Corner Point	Profit
(0, 0)	0
(50, 0)	350
(30, 40)	410
(0, 80)	400

Because the point (30, 40) produces the highest profit we conclude the producing 30 tables and 40 chairs will yield a maximum profit of E410

SIMPLEX METHOD :-

Using simplex method to solve

maximize $z = x_1 + 3x_2$

Subject to: $x_1 + x_2 \leq 10$

$5x_1 + 2x_2 \leq 20$

$x_1 + 2x_2 \leq 36$

$x_1 \geq 0, \; x_2 \geq 0$

The slack variable are x_3, x_4 and x_5

$$x_1 + x_2 + x_3 = 10$$
$$5x_1 + 2x_2 + x_4 = 20$$
$$x_1 + 2x_2 + x_5 = 36$$

The objective function $z = x_1 + 3x_2$ need to be rewritten so that all the variables are on the left-hand side : $-x_1 - 3x_2 + z = 0$

The initial simplex tableau is

$$\begin{array}{cccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & z & \\ \hline 1 & 1 & 1 & 0 & 0 & 0 & 10 \\ 5 & 2 & 0 & 1 & 0 & 0 & 20 \\ 1 & 2 & 0 & 0 & 1 & 0 & 36 \\ \hline -1 & -3 & 0 & 0 & 0 & 1 & 0 \end{array}$$

The second column is the pivot column, since -3 is the most negative indicator

Questions; $10/1 = 10$, $20/2 = 10$, $36/2 = 18$. Since there are two equally small quotients,

we can choose either entry in column 2 to the pivot

et choose the number 1 in column 2 to be

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pivot

$$\left[\begin{array}{cccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & z & \\ 1 & 1 & 1 & 0 & 0 & 0 & 10 \\ 5 & 2 & 0 & 1 & 0 & 0 & 20 \\ 1 & 2 & 0 & 0 & 1 & 0 & 36 \\ \hline -1 & -3 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \quad r_2 \to -2r_1 + r_2$$

$$\left[\begin{array}{cccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & z & \\ 1 & 1 & 1 & 0 & 0 & 0 & 10 \\ 3 & 0 & -2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 1 & 0 & 36 \\ \hline -1 & -3 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \quad r_3 \to -2r_1 + r_3$$

$$\begin{array}{rrrrrrrr} & -2 & -2 & -2 & 0 & 0 & 0 & -20 \\ + & 5 & 2 & 0 & 1 & 0 & 0 & 20 \\ \hline & 3 & 0 & -2 & 1 & 0 & 0 & 0 \end{array}$$

$$\begin{array}{rrrrrrrr} & -2 & -2 & -2 & 0 & 0 & 0 & -20 \\ + & 1 & 2 & 0 & 0 & 1 & 0 & 36 \\ \hline & -1 & 0 & -2 & 0 & 1 & 0 & 16 \end{array}$$

$$
\begin{array}{c}
\begin{array}{cccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & z & \end{array} \\
\left[\begin{array}{cccccc|c}
1 & \cancel{3}1 & 1 & 0 & 0 & 0 & 10 \\
3 & 0 & -2 & 1 & 0 & 0 & 0 \\
-1 & 0 & -2 & 0 & 1 & 0 & 16 \\
\hline
-1 & -3 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\end{array}
\quad r_4 \to 3r_1 + r_4
$$

$$
\begin{array}{c}
\begin{array}{cccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & z & \end{array} \\
\left[\begin{array}{cccccc|c}
1 & 1 & 1 & 0 & 0 & 0 & 10 \\
3 & 0 & -2 & 1 & 0 & 0 & 0 \\
-1 & 0 & -2 & 0 & 1 & 0 & 16 \\
\hline
2 & 0 & 3 & 0 & 0 & 1 & 30
\end{array}\right]
\end{array}
$$

$$
\begin{array}{rrrrrrrr}
 & 3 & 3 & 3 & 0 & 0 & 0 & 30 \\
+ & -1 & -1 & 0 & 0 & 0 & 1 & 0 \\
\hline
 & 2 & 0 & 3 & 0 & 0 & 1 & 3 \\
\hline
\end{array}
$$

Since there are no negative in the last row, We are ~~doing~~ Pivoting. The corresponding equation is,

$2x_1 + 3x_3 + z = 30$ or $z = 30 - 2x_1 - 3x_3$. The basic Variables are x_2, x_4, x_5 and z the non basic variables are x_1 and x_3. The solution is found by setting the nonbasic variables equal to zero. So $x_1 = 0$ and $x_3 = 0$, then from the last Tableau we have the following equations:

$$x_1 + x_2 + x_3 = 10$$

$$3x_1 - x_3 + x_4 = 0$$

$$-x_1 - 2x_3 + x_5 = 16$$

Substing $x_1 = 0$ and $x_3 = 0$ gives us $x_2 = 10$, $x_4 = 0$, and $x_5 = 16$ and the maximum value for the variable z is $z = 30$

Chief Characterstics of Linear Programming

1. objective function
2. Non-negative
3. Linearity.
4. ~~Fit~~ Finiteness
5. Maximization & Minimization of cost & profit.

Conclusion :-

Linear programming is an important branch of applied mathematics that solves a wide variety of optimization problems. It is widely used in production planning and scheduling problems. Many recent advances in the field have come from the airline industry where aircraft and crew scheduling have been great improved by the use of linear programming. It has also been used to solve a variety of assignment problems, such as the kary-otyping problem where 46 chromosomes are assigned to 24 classes. Although the revised simplex method is not theoretically satisfactory from a computational point of view, it is by far the most widely used method to solve linear programming problems and only rarely are its limitations encountered in practical application. The biggest advantages of linear programming as an optimization method is that it always achieves the optimal solution if one exists.

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