Developing mass appraisal models with fuzzy systems

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Abstract

The use of regression techniques in mass appraisal is a common practice. Real data have several sources of error or imprecision, such as the lack of correct specification of model format, multiple simultaneous relationships among the explanatory variables, and not clear transitions between submarkets, generating difficulties to construct mass appraisal models. An alternative to develop more flexible and comprehensive models is to use fuzzy systems. However, fuzzy systems may not learn market characteristics alone and generally fuzzy systems are developed jointly with other techniques, such as artificial neural networks (ANNs) and genetic algorithms (GAs), performing hybrid systems. This chapter is about fuzzy systems, developed with neural network or genetic algorithms.

1. Introduction

Studies developed to research or property valuation must consider several aspects of the housing market, such as heterogeneity, durability, high cost and fixed spatial positioning of the properties. One of most important of them is the diversity among the units, in terms of physical features of the property and location attributes. Properties have large differences on size, age, type and quality of construction, neighbourhood quality, accessibility and other characteristics, which are reflected in variations in their market prices. Surveys of empirical studies indicate significative differences among the presented models. Moreover, they have indicated that the relevance of each attribute can change even in adjacent neighbours or for similar property types. Therefore, decision about what attributes to include and which model format to adopt is an open question, which is empirically solved until now (Balchin and Kieve, 1986; Ball, 1973; Boyle and Kiel, 2001; Lavender, 1990; Robinson, 1979; Sheppard, 1999; Smith et al., 1988). Besides of that, in many cases submarkets are not clearly divided in crisp and homogeneous parts. In another words, a property can partially belong to two or more segments of market simultaneously. For example, in the case of location may be there interpenetration between contiguous market regions and delineate the boundaries will be a difficult task. Excessive segmentation or splitting of data in partial samples causes the need of obtains too much data to develop significant citywide models, which not always is available. An additional reason for a different approach is that an *a priori* segmentation is not convenient to mass appraisal models, which need be replicated periodically (Bourassa and Hoesli, 1999; Bourassa et al., 1999; Kauko, 1997; Kauko, 2000; Goodman and Thibodeau, 2003).

The common approach to develop mass appraisal models is based on multiple regression analysis (MRA), which has some problems with real estate market. In fact, conventional statistical techniques such as clustering or regression analysis have difficulties in consider imprecision in data.

A more automated (or less subjective) approach can improve the quality of the models and reduce the time of compilation, using the artificial intelligence or machine learning paradigms, for example. In the last years, several papers presented artificial neural networks (ANNs) as an alternative to MRA, but ANN models have problems in the explanation of the results, because of their "black box" nature. By another hand, an interesting alternative is the fuzzy rule-based systems (FRBSs), which are able to generate flexible systems and may be useful in considering vagueness or imprecision presents in real estate market. However, the FRBSs are not capable of learning alone the rules. In general, the fuzzy systems are constructed in hybrid approaches, using neural networks or genetic algorithms in the training or learning phase (Cordón et al., 2001). Some characteristics of these methods are exposed in this text.

2. Fuzzy rule-based systems

There are several studies about fuzzy logic in other domains but very little on real estate. Byrne (1995) proposed to apply fuzzy logic considering the risk and uncertainty presents on real estate appraisal. Bagnoli and Smith (1998) used fuzzy logic to handle vagueness and imprecision in the subjective measures of property attributes. Bonissone et al. (1998) used fuzzy logic in real estate appraisal, in two sub-systems.

One of them uses fuzzy logic to determinate similarity to select cases in Case-Based Reasoning. The second part uses a neuro-fuzzy approach, with fuzzy rules tuned with neural networks. They has used fuzzy components in the neural network (converting crisp inputs to fuzzy labels) and generated a fuzzy rule set using a special architecture to neural network (E-ANFIS). Siniak (2002) has suggested the use of fuzzy numbers to adjust the three conventional approaches to valuation, weighting cost, income and market values in a final, improved assessment.

A fuzzy rule-based system (FRBS) is an extension of the classical rule-based systems, using fuzzy rules instead of classical logic rules. In fuzzy sets the membership can assume values in the continuous range [0,1], not just {0,1} as in classic set theory. Fuzzy rules can be obtained from data using genetic algorithms or neural networks. They have been successfully applied to a wide range of problems (Cordón and Herrera, 1999; Cordón et al., 2001; Kosko, 1992).

Fuzzy rules are composed by a precedent (fuzzy) part and a consequent part, which is a function of the input variables. Majority of the FRBS systems uses Mamdani or TSK (Takagi-Sugeno-Kang) rules. The main difference between them is that the TSK rules have a function in the consequent part and Mamdani rules have a linguistic output such as “small, medium, large”. In general a TSK rule has this format (Equation 1):

$$\text{IF } x_1 \text{ is } A_1 \text{ and ... and } x_k \text{ is } A_k \text{ THEN } y_j = p_1.x_1 + ... + p_k.x_k + p_0, \tag{1}$$

Where x_i are input variables, A_i are fuzzy sets specifying their meanings, p_i are the coefficients of equation and y_j is the output variable. The output of a fuzzy system is computed as a weighted average of the individual rule outputs using the matching degree among inputs and the antecedent part of each rule. Several rules may be applied to a single input vector and the number of rules is dependent of application complexity. Therefore, TSK rules are more convenient to real estate appraisal (Cios et al., 1998; Cordón et al., 2001). Genetic algorithms and neural networks may be viewed as techniques to knowledge acquisition phases in the work to generate FRBS, performing then genetic fuzzy systems (GFS) or neuro-fuzzy systems (NFS).

3. Neuro-fuzzy systems

Neuro-fuzzy systems (NFS) are systems that use neural networks and fuzzy logic. In recent years artificial neural networks (ANN) were studied in models to housing price valuation like an alternative tool to regression. There are several works such as, for example, Borst (1991), Kathman (1993), Tay and Ho (1994), Evans *et al.* (1995), Worzala *et al.* (1995), Lenk *et al.* (1997), McCluskey (1996), Kauko (1997), Lewis *et al.* (1997), McCluskey and Borst (1997), Rossini (1997), Bonissone *et al.* (1998), Connelan and James (1998), McGreal *et al.* (1998), Cechin *et al.* (2000), Panayoutou *et al.* (2000), and Nguyen and Cripps (2001). However these valuation models based on neural networks are not complete substitutes to hedonic models based on regression because they do not present the implicit (shadow) prices.

Artificial neural networks have been proposed as an alternative tool to valuation because of their ability in domains with non-linear relationships or initially unknown models. Nevertheless the major drawback of neural networks is the explanation of its predictions. ANN generally are called of “black boxes” because of the lack of a simple and explicit model mapping the relationships among inputs and outputs, which is important in some applications, such as taxation or insurance.

Neural networks are systems of massively distributed parallel processing. Initially inspired in the human brain, they use learning mechanisms to knowledge acquisition and save this knowledge in weighted connections (Rumelhart and McClelland, 1986; Haykin, 1999). An artificial neural network is composed of a set of neurons (nodes) and a number of weighted connections among them. The neurons have two parts: an initial sum of the weighted inputs and an activation function (generally non-linear) which give the neuron’ output (Figure 1).

Usually artificial neural networks are used in classification and regression tasks and are competitors of the regression models in several applications. Knowledge initially is on a form of training data and is acquired by training the network, that is, progressively adjusting the set of connection’ weights, became embodied in these weights.

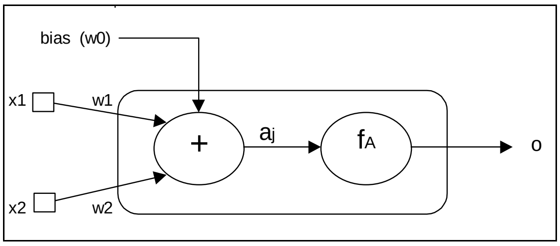

Figure 1 – The component parts of a neuron

The most widely used format of ANN in valuation is based on the feed forward multi-layer Perceptron with three layers (input, hidden and output) with a back propagation scheme of learning. This kind of network typically has an initial random set of weights, which is progressively adjusted by an error-correction algorithm. The system compute network' output in the first stage (propagation) determining the difference (error) to the expected output (actual output). This error is distributed among the connections' weights in the second stage (back propagation) in order to progressively reduce the error. These steps are repeated for hundreds or thousands of cycles in the training phase of the network, changing the case presented to the network (current example). When desired error level is obtained training phase ends and the network is then tested with unseen data to verify its generalization capability. The network has an aspect like in the Figure 2:

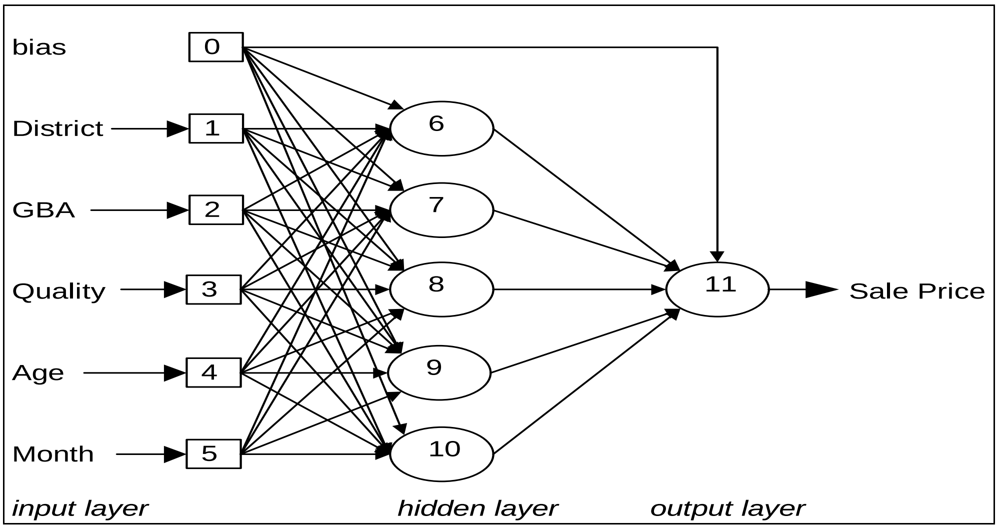

Figure 2 – An example of neural network applied to property valuation

There are not a reliable theory about definition of the network architecture and about the learning parameters, including number of hidden layers, number of hidden nodes, selection of learning algorithm (such as Back propagation, Back propagation with Momentum term, Resilient Propagation, Weight Decay, etc.), definition of algorithm' parameters (learning rate, number of cycles of training, etc.). These are questions solved by a trial-and-error strategy, commonly using some heuristics to define architecture and cross validation (with a holdout sample) to verify the results (Haykin, 1999; Kosko, 1992).

A major drawback on neural networks is that knowledge collected from data is stored only in the weights of the network and has not direct significance to the analyst, different from coefficients of a regression model, for example. Due to this, neural networks are generally called a "black box". There is a great effort of research looking for methods to explain neural networks and now there are several ways to interpret a network, using simulation on inputs to examine the behavior of the outputs or extracting fuzzy rules from a trained network. By these methods, the inner behavior of the network can be understood. In other sense, explanation of the network helps to improve the network, identifying relevance of the inputs or hidden neurons. Nowadays one of most complete and interesting approaches is the extraction of a fuzzy rule set from a trained network.

Previous studies showed that under some assumptions an artificial neural network could be approximated to any desired degree of accuracy for a fuzzy system, and vice-versa (Benítez et al., 1997). Neuro-fuzzy systems have been successfully applied to extract knowledge from data as fuzzy rules, exploring the best properties of neural networks and fuzzy systems. There are several methods to extract knowledge of neural networks as fuzzy rules, but apparently few can be applied directly in valuation task, due to the characteristics of real estate data. Constraints in some methods include binary outputs, discrete valued inputs and need of transformations or limitations of special formats to neural networks. In others methods continuous-valued inputs must be fuzzy variables, using a set of linguistic terms to convert a continuous input range in a set of binary variables, perhaps with the risk of biased results. Moreover, most of these methods are suitable only for classification purposes (Arbatli and Akin, 1997; Bonissone et al., 1998; Cordón et al., 2001; Huang and Xing, 2002; Ishikawa, 2000; Maire, 1999; Setiono, 1997 and 2000; Setiono et al., 1998).

Two methods that work directly with network weights' of a network trained by any algorithm and are suitable to regression purposes (with a continuous target variable) are the method proposed in Benítez et al., (1997) and extended in Castro et al., (2002), and FAGNIS (Fuzzy Automatically Generated Neural Inferred System), developed by Cechin (1998).

The first is applicable in property valuation but the generated system has complex precedents and the explanation to final user is not too much improved, in practice. Rules have a special logical connector introduced by the authors ("interactive-or"), which is an additional element to increase complexity (Benítez et al., 1997; Castro et al., 2002).

By another hand FAGNIS reach both of two targets: precision on fuzzy inference system and simplicity of the rules. This is the method used here. A brief explanation about FAGNIS is provided and further explanation and details would be obtained in Cechin (1998). Because of peculiarities of data generally the activation functions (f_A) works on a small input range. FAGNIS is based in a simple idea: to substitute these activation functions for a set of linear segments. The value of f_A can be approximated by a set of linear segments, using a relationship as demonstrated in Equation 2:

$$f_A(a_j) \sim \Sigma_i \, [F_i(a_j)*(p_i*a_j+q_i)], \qquad (2)$$

Where $f_A(a_j)$ is the original non-linear function, a_j is the activation signal (weighted sum of the input vector), $F_i(a_j)$ is a function that links each value of a_j to correspondent linear segment(s), and $p_i*a_j+q_i$ are linear segments. To improve the precision of the approximation, $F_i(a_j)$ must be a fuzzy number (Cechin, 1998).

The activation function of each non-linear neuron is substituted by a Fuzzy Inference System (FIS) composed by a TSK rule in which the membership function $F_i(a_j)$ is the rule precedent and the linear segment $p_i*a_j+q_i$ is the consequent. The analyst choose the number of segments in attempt to desired error level. The fuzzy sets may be constants (defined equally to all rules and neurons) or variables (different among them). For example, if the neuron has a sigmoid activation function (such as $sigm(a_j)= 2*(1+exp^{-aj})^{-1}–1$) and analyst wish to use three constant fuzzy sets (three fixed segments), a possible FIS is the presented in the Figure 3 (Cechin, 1998, p.59).

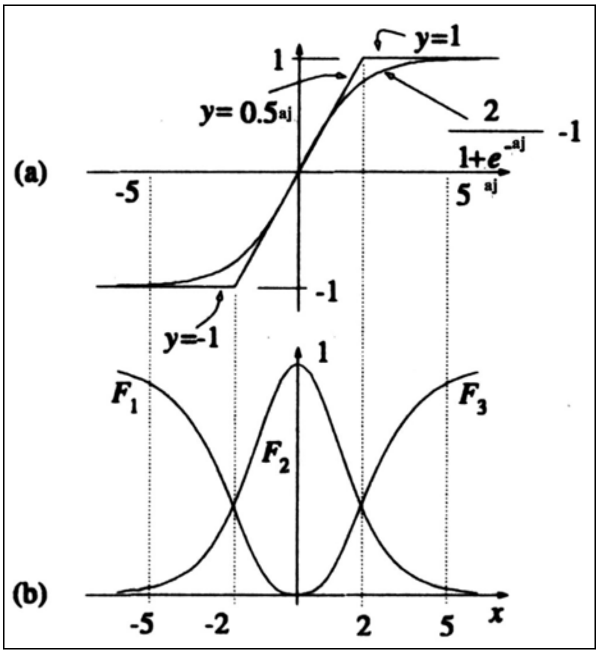

Figure 3 –Membership functions (adapted from Cechin, 1998, p.59)

A further constraint used to define the membership functions is that only two functions can have membership value different from zero simultaneously. So, for the system presented on Figure 3, the membership functions have the following expressions (Equations 3-5 – see Fig. 3):

$$F_1(a_j)= –sigm(a_j) + sigm'(a_j)*a_j, \text{ if } a_j<0 \text{ ; } F_1(a_j)=0, \text{ else} \quad (3)$$

$$F_2(a_j)= 2*sigm'(a_j) \quad (4)$$

$$F_3(a_j)= sigm(a_j) – sigm'(a_j)*a_j, \text{ if } a_j>0 \text{ ; } F_3(a_j)=0, \text{ else} \quad (5)$$

The linear segments in consequent part of FIS are calculated by means of the activation function and by it derivative on the desired point: $p_i= f_A'(a_x)$ and $q_i= f_A(a_x) – p_i.a_x$, where a_x is chosen to be the interval centre of non-linear actual range in each neuron. The membership functions associated to each linear segment are related with the activation function: $F_i(a_j)= f_A(a_j)/(p_i*a_j+q_i)$, that means, membership value is the approximation degree between non-linear and linear functions (Cechin, 1998).

In the extreme case can exist even one rule for case on training base, but often the rule base had only one or two rules, probably due to data ranges. In networks with several non-linear nodes an unique linear function to the network may be generated making the composition between the neurons (in parallel or in series) by the sum-product rule. After this rule, the membership functions are multiplied ($G_r(a_j)=F_1(a_j)*F_2(a_j)*...*F_n(a_j)$) and the linear segments are summed ($y_r = (p_1+p_2+...+p_n).a_j + (q_1+q_2+...+q_n)$).

In some cases, the coefficient of bias term (constant of equation) need an adjustment to reduce the model error. Finally, if there are more than one rule in the system, the applicable rules must be weighted by matching degree (membership value of the current inputs in each rule), using $y=(\Sigma_r G_r(X).y_r)/\Sigma_r G_r$, where y is the system' output, G_r is the membership function for the Rule r, X is the input vector ($X=\{x_1,x_2,...x_k\}$), and y_r is the output for the Rule r.

4. Genetic Fuzzy Rule-Based Systems

One interesting technique used to generate the fuzzy rules is genetic algorithm (GA). The GAs are optimization or search procedures, inspired by the "survival of the fittest" natural rule of biological systems. They employ a random search for locating the globally optimal solution. It has been proven that GAs are able to find near-optimal solutions to complex problems. A GA begins by randomly generating an initial population of chromosomes, which are potential solutions. The chromosomes are encoded in binary or real strings. The algorithm evolves these solutions in successive generations through selection and reproduction, running until that desired error or when the limit of time is reached. Some fittest individual can be selected directly (elitist selection). Offspring chromosomes are created by merging two parent chromosomes (crossover operator) or randomly modifying a chromosome (mutation operator). The chromosomes are evaluated in each generation on their performances with respect to the fitness function and fitter chromosomes have higher survival probabilities (Cordón et al., 2001; Goldberg, 1989).

There are two basic approaches for the training. In the Pittsburgh approach, all the rules of the systems are extracted from data simultaneously. In the Michigan approach, each rule is obtained individually, constructing the system progressively. In general, the first training scheme goes to systems that are more consistent. The advantages of the second alternative are the greater flexibility of the system (updated easily) and less computations' effort, training one rule each time (Cordón et al., 2001). In another view, according to variable-type, training may be developed by a one-dimensional variable or bi-dimensional form, like is developed in sequence.

4.1. Fuzzy rules based in one-dimensional characteristics

This model consists on a fuzzy rule-based system based in one-dimensional characteristics of market or properties. These systems uses n rules with the same format: **R_i: IF X is A_i THEN y_i= (model$_i$)**, where X is an important characteristic of property, A_i are the membership functions, y_i is the partial appraisal of the properties and model$_i$ are the equations extracted from database by a GA, with i=[1,n]. The set of membership functions may have an aspect like in the Figure 4, where the functions are assigned to the labels "very small" (vS), "small" (S), "small-medium" (SM), "medium" (M), "medium-large" (ML), "large" (L) and "very large" (vL). Different formats or configurations of the membership functions are obtained setting different limits to variable X. The membership functions A_i were defined in triangular shape.

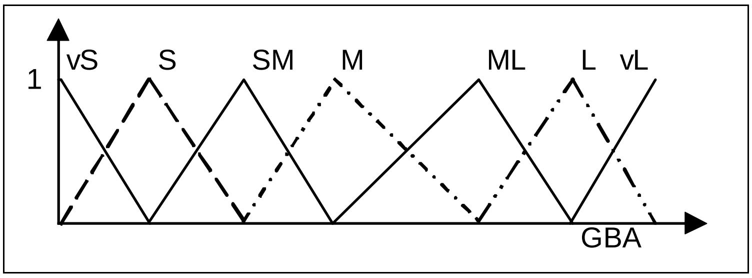

Figure 4: Distribution of possible fuzzy sets for variable X

Each membership function is defined using user-defined limits for X. The format of the three functions is $A(X)=1-[|X-c|/(b-a)]$, where c is the "center" of the function, b is the greater and a is the smaller X considered. The values calculated by A are in the range [0,1]. For a X outside of interval [a,b], A(X)=0, and for X=c, A(c)=1. The estimated value of SP for a property j is calculated by $SP^h_j=\Sigma_i(y^h_{i,j}*A_{i,j})$, where $\Sigma_i(A_{i,j})=1$.

The rules R_i are estimated using a genetic algorithm, in the Pittsburgh approach. Each individual in GA is a complete rule system (with n rules). The populations of potential rules are generated using MRA model or another estimate as initial values and adding random variations of ±50% in each coefficient, for example. The reason is that this initial solution may accelerate the search.

A further detail need to be considered. The properties have very different characteristics and prices. A competition based on RMSE may generate a preference to models adjusted to small properties, which have smaller residuals, in absolute figures. In this view, the evolution of the rules produce a system adjusted only to small and less valued properties, with crescent error to medium and large properties. To avoid this problem, fitness function was based on mean absolute percentual error, with the format in Equation 6.

$$F_i=1/(1+MAPE_i), \qquad \text{with} \quad MAPE_i=\Sigma_{i,j}\,(|Y_j-Y^h_{i,j}|\,/\,Y_j * 100\,) \qquad (6)$$

Where F_i is the fitness to the rule system i, $MAPE_i$ is the mean absolute percentual error calculated for the system i with training data, Y_j is SP_j and $Y^h_{i,j}$ is the estimated price by system i to case j (the SP^h for the j^{th} property).

4.2. Fuzzy system based on bi-dimensional characteristics

Other alternative is based on bi-dimensional variables, like location of the properties. The main differences from the one-dimensional variable-based fuzzy system are the procedure used to construct the system, which follow a Michigan approach in this case.

In the case of location, each rule is specialized in a determined region of the city, in spite of contributes to all estimates. The first step is to define centroids for the rules, using clustering in the co-ordinates of the properties. Using this scheme, were defined 10 sets of (X, Y) co-ordinates, corresponding to 10 rules. The adjustment of the system is similar to the other system, except for adjusting a single rule for step, and for using a different membership function, based on location (it is a “spatial membership function”). The format used is an exponential one (Equation 7).

$$B_{i,j} = 1 / [1 + ((X_i - X_j)^2 + (Y_i - Y_j)^2)^{0,5}] \qquad (7)$$

Where $B_{i,j}$ is membership to case j to the rule i, (X_i,Y_i) is the centroid of the rule i, and (X_j,Y_j) are the co-ordinates of each property j. These functions define membership values in the urban space, obtained by rotation of the curve $B_{i,j}$ in 360^o. Because of this format, the sum of membership values is not normalized (the sum differs from 1).

The fitness function also is different, using the membership value $B_{i,j}$ as a penalty to force “spatialized” adjustment. It uses the relationship presented in Equation 8, where F_i is the measure of adjustment of the rule i, $mMAPE_i$ is the modified mean absolute percentual error, and the other parameters are equal to descript above, except that the Y_j and $Y^h_{i,j}$ are concerned to a single rule.

$$F_i = 1/(1+mMAPE_i), \qquad \text{with} \quad mMAPE_i = \Sigma_{i,j} (|Y_j - Y^h_{i,j}| / Y_j * 100 / B_{i,j}) \qquad (8)$$

This modified fitness function increase the error measure proportionally to the distance from case j to rule i. In the phase of adjustment (generation of rule base), each rule must be individually adjusted using GA or another procedure.

5. References

ARBATLI, A.Duygu; AKIN, H. Levent. Rule extraction from trained neural networks using genetic algorithms. **Nonlinear Analysis, Theory, Methods and Applications**, v.30, n.3, p.1639-1648, 1997.

BAGNOLI, Carlo; SMITH, Halbert C. The theory of fuzzy logic and its application to real estate valuation. **Journal of Real Estate Research**, 1998.

BALCHIN, Paul N.; KIEVE, J.L. **Urban land economics**, 3ed. London: MacMillan, 1986.

BALL, Michael J. Recent empirical work on the determinants of relative house prices. **Urban Studies**, v.10, p.213-233, 1973.

BENÍTEZ, José M.; CASTRO, Juan L.; REQUEÑA, I. Are artificial neural networks black boxes? **IEEE Transactions on Neural Networks**, v.8, n.5, p.1156-1164, Sept. 1997.

BONISSONE, Piero. P.; CHEETAM, William; GOLIBERSUCH, David. C.; KHEDKAR, Pratap. Automated residential property valuation: An accurate and reliable approach based on soft computing. In: RIBEIRO, R.; ZIMMERMANN,H.; YAGER, R.R.; KACPRZYK, J. (eds.). **Soft Computing in Financial Engineering**. Heidelberg: Physica-Verlag, 1998.

BORST, R. A. Artificial neural networks: The next modeling/calibration technology for the assessment community? **Property Tax Journal**, v.10, n.1, p.69-94, Mar. 1991.

BOURASSA, S. C.; HAMELIN, F.; HOESLI, M.; MACGREGOR, B. D. Defining housing submarkets. **Journal of Housing Economics**, v.8, n.2, p. 160-183, June, 1999.

BOURASSA, S. C.; HOESLI, M. The structure of housing submarkets in a metropolitan region. **Paper 99.15** - Ecole des Hautes Etudes Commerciales. Genève: Université de Genève, 1999.

BOYLE, Melissa A.; KIEL, Katherine A. A survey of house price hedonic studies of the impact of environmental externalities. **Journal of Real Estate Literature**, v.9, n.2, p.117-144, 2001.

BYRNE, P. Fuzzy analysis: A vague way of dealing with uncertainty in real estate analysis? **Journal of Property Valuation and Investment**, v.13, n.3, p.22-41, 1995.

CASTRO, Juan L.; MANTAS, Carlos J.; BENÍTEZ, José M. Interpretation of artificial neural networks by means of fuzzy rules. **IEEE Transactions on Neural Networks**, v.13, n.1, p.101-116, Jan. 2002.

CECHIN, Adelmo Luis, SOUTO, Antonio, GONZÁLEZ, Marco Aurélio Stumpf. Real estate value at Porto Alegre city using artificial neural networks. **Sixth Brazilian Symposium on Neural Networks (SBRN'2000)**. Rio de Janeiro: SBRN, Nov. 2000.

CECHIN, Adelmo Luis. **The extraction of fuzzy rules from neural networks**. Aachen: Shaker Verlag,1998. (PhD Thesis).

CIOS, Krzysztof J.; PEDRYCZ , Wiltold; SWINIARSKI, Roman W. **Data mining**: Methods for knowledge discovery. Boston: Kluwer, 1998.

CONNELLAN, O.; JAMES, H. Estimate realisation price (ERP) by neural networks: forecasting commercial property values. **Journal of Property Valuation and Investment**, v.16, n.1, p.71-86, 1998.

CORDÓN, Oscar; HERRERA, Francisco. A two-stage evolucionary process of designing TSK fuzzy rule-based systems**. IEEE Transactions on Systems, Man, and Cybernetics** - Part B: Cybernetics, v.29, n.6, 1999.

CORDÓN, Oscar; HERRERA, Francisco; HOFFMANN, Frank; MAGDALENA, Luis. **Genetic fuzzy systems:** Evolutionary tuning and learning of fuzzy knowledge bases. World Scientific: Singapore, 2001.

EVANS, Alec; JAMES, Howard; COLLINS, Alan. Artificial neural networks: An application to residential valuation in UK. **Journal of Property Tax Assessment & Administration**, v.1, n.3, p.78-92, May 1995.

GOLDBERG, D. E. **Genetic algorithms in search, optimization, and machine learning**. Reading: Addison-Wesley, 1989.

GOODMAN, A.C.; THIBODEAU, T.G. Housing market segmentation and hedonic prediction accuracy. **Journal of Housing Economics**, v.12, p.181-201, 2003.

HAYKIN, Simon. **Neural networks** - A comprehensive foundation. Prentice-Hall, 1999.

HUANG, Samuel H.; XING, Hao. Extract intelligible and concise fuzzy rules from neural networks. **Fuzzy Sets and Systems**, 2002 (to appear).

ISHIKAWA, M. Rule extraction by sucessive regularization. **Neural Networks**, v.13, p.1171-1183, 2000.

KATHMAN, R. M. Neural networks for the mass appraisal of real estate. **Computer Environment and Urban Systems**, v.17, p.373-384, 1993.

KAUKO, T. Can housing market segmentation be captured with a neural network modeling approach? In: Conference of the European Network for Housing Research (ENHR 2000), 2000, Gävle (Sweden). **Proceedings...** Uppsala (Sweden): Uppsala Universitet, 2000.

KAUKO, Tom. Exploring the prices of residential apartments and locality features within an artificial neural network approach – Evidence from Finland. **AREUEA International Conference**. UCLA: Berkeley, June 1997.

KAUKO, Tom. Exploring the prices of residential apartments and locality features within an artificial neural network approach – Evidence from Finland. **AREUEA International Conference**. UCLA: Berkeley, June 1997.

KOSKO, Bart. **Neural networks and fuzzy systems**: A dynamical systems approach to machine intelligence. Englewood Cliffs: Prentice-Hall, 1992

LAVENDER, Stephen D. **Economics for builders and surveyors**. Essex (UK): Longman, 1990.

LENK, Margarita M.; WORZALA, Elaine M.; SILVA, Ana. High-tech valuation: Should artificial neural networks bypass the human valuer? **Journal of Property Valuation and Investment**, v.15, n.1, p.8-26, 1997.

LEWIS, O. M.; WARE, J. A.; JENKINS, D. A novel neural network technique for the valuation of residential property. **Neural Computing and Applications**, v.5, n.4, p.224-229, 1997.

MAIRE, F. Rule-extraction by backpropagation of polyedra. **Neural Networks**, v.12, p.717-725, 1999.

McCLUSKEY, W. J. Predictive accuracy of machine learning models for mass appraisal of residential property. **New Zealand Valuer's Journal**, p.41-47, July, 1996.

MCCLUSKEY, W.; BORST, R. An evaluation of MRA, comparable sale analysis, and ANNs for the mass appraisal of residential properties in North Ireland. **Assessment Journal**, v.4, n.1, p.47-55, Jan/Feb., 1997

McGREAL, S.; ADAIR, A.; MCBURNEY, D.; PATTERSON, D. Neural networks: The prediction of residential values. **Journal of Property Valuation and Investment**, v.16, n.1, p.57-70, 1998.

NGUYEN, Nghiep; CRIPPS, Al. Predicting housing value: A comparison of multiple regression analysis and artificial neural networks. **Journal of Real Estate Research**, v.22, n.3, 2001.

PANAYIOTOU, P. A.; PATTICHIS, C.; JENKINS, D.; PLIMMER, F.; ECONOMIDES, C; PATTICHIS, C. S; MALIOTIS, G; PAPACONSTANTINOU, M; MICHAELIDES, N; SCHIZAS, C. N. A modular artificial neural network valuation system. In: 10th Mediterranean Electrotechnical Conference (MeleCon2000). **Proceedings...** Piscataway (NJ): IEEE Press, 2000, v.2, p.457-460.

ROBINSON, Ray. **Housing economics and public policy**. London: MacMillan, 1979.

ROSSINI, Peter. Application of artificial neural networks to the valuation of residential property. **Third Annual Pacific-Rim Real Estate Society Conference**. Palmerston North, New Zealand, Jan. 1997.

RUMELHART, David. E.; MACCLELLAND, James. L**. Parallel distributed processing**: Explorations in the microstructure of cognition. Cambridge: MIT, 1995

SETIONO, Rudy. Extracting M-of-N rules from trained neural networks. **IEEE Transactions on Neural Networks**, v.11, n.2, p.512-519, Mar. 2000.

SETIONO, Rudy. Extracting rules from neural networks by pruning and hidden-unit splitting. **Neural Computation**, n.9, p.205-225, 1997.

SETIONO, Rudy; THONG, James Y.L.; YAP, Chee-Sing. Symbolic rule extraction from neural networks. An application to identifying organizations adopting IT. **Information and Management**, v.14, p.91-101, 1998.

SHEPPARD, Stephen. Hedonic analysis of housing markets. In: CHESHIRE, P.C.; MILLS, E.S. (Eds.) **Handbook of applied urban economics**, v.3, chap.8. New York: Elsevier, 1999.

SINIAK, N. Fuzzy numbers for thre real estate valuation. In: 9th European Real Estate Society Conference (ERES 2002), June, 2002, Glasgow. **Proceedings...** Glasgow: ERES, 2002.

SMITH, Lawrence B.; ROSEN, Kenneth T.; FALLIS, George. Recent development in economic models of housing markets. **Journal of Economic Literature**, v.26, p.29-64, Mar, 1988.

TAY, Danny. P. H.; HO, David K. H. Intelligent mass appraisal. **Journal of Property Tax Assessment and Administration**, v.1, n.1, p.5-25, Sept. 1994.

WORZALA, Elaine; LENK, Margarita; SILVA, Ana. An exploration of neural networks and its application to real estate valuation. **The Journal of Real Estate Research**, v.10, n.2, p.185-201, 1995.