Tutorial Sheet No. 07

Course: B.Tech. (CSE, IT, ECE, EEE, ME, CE, FT)

Year & Semester: I / II

Subject & Code: Mathematics – II (BAS – 203)

Unit & Topic: III / Sequence and Series

Prepared By: Dr. Manoj Kumar Gupta, Assistant Professor

1. Test the convergence of the sequence $\langle u_n \rangle$ if

(i)
$$u_n = \frac{n}{n^2 + 1}$$

[Ans.: convergent] (ii)
$$u_n = [n + (-1)^n]^{-1}$$

[Ans.: convergent]

(iii)
$$u_n = \sin n$$

[Ans.: oscillatory] (iv) $u_n = 2n + 1$

(iv)
$$u_n = 2n + 1$$

[Ans.: divergent]

2. 1. Test the convergence of the series:

(i)
$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$$

[Ans.: convergent]

(ii)
$$-1 - 2 - 3 - 4 - \cdots$$

[Ans.: divergent]

(iii)
$$1 - 2 + 3 - 4 + 5 - \cdots$$

[Ans.: oscillatory]

(iv)
$$1 + 4 + 9 + 16 + 25 + \cdots$$

[Ans.: divergent]

(v)
$$1 + 8 + 27 + 64 + \cdots$$

[Ans.: divergent]

(vi)
$$\frac{1}{3} + \frac{4}{3^2} + \frac{9}{3^3} + \cdots$$

[Ans.: convergent]

3. Test the convergence of the series $\sum u_n$ where

(i)
$$u_n = \frac{2^n}{n^3}$$

[Ans.: divergent]

(ii)
$$u_n = \frac{\sqrt{n}}{\sqrt{n^2 + 1}} x^n$$
; $x > 0$

[Ans.: convergent for x < 1 and divergent for $x \ge 1$]

4. Test the convergence of the series $\frac{x}{1.2} + \frac{x^2}{3.4} + \frac{x^3}{5.6} + \cdots$

[Ans.: convergent for $x \le 1$ and divergent for x > 1]

5. Test the convergence of the series $\sum \frac{1}{\sqrt{n+1}-1}$ [Ans.: divergent]