Tutorial Sheet No. 01

Course: B.Tech. (CSE, IT, ECE, EEE, ME, CE, FT)

Year & Semester: I / II

Subject & Code: Mathematics – II (BAS – 203)

Unit & Topic: I / ODE's of Higher Order

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1. Solve the following differential equations:

(a) $4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 3y = e^{2x}$. [Ans.: $y = C_1 e^{\frac{1}{2}x} + C_2 e^{-\frac{3}{2}x} + \frac{1}{21}e^{2x}$]

(b) $y''' - 3y' + 2y = e^{-2x} + 2\sinh x$

[Ans.: $y = C_1 e^{-2x} + (C_2 + C_3 x)e^{x} + \frac{1}{9}xe^{-2x} + \frac{1}{6}x^2 e^{x} + \frac{1}{4}e^{-x}$]

(c) $y''' - 3y' + 3y' - y = e^{x} + 2$ [Ans.: $y = (C_1 + C_2 x + C_3 x^2)e^{x} + \frac{1}{6}x^3 e^{x} - 2$]

(d) $4y'' + 4y = \sin 3x + \cos 2x$ [Ans.: $y = C_1 \cos 2x + C_2 \sin 2x - \frac{1}{5}\sin 3x + \frac{1}{4}x \sin 2x$]

(e) $\frac{d^4y}{dx^4} - m^4 y = \cos mx$ [Ans.: $y = C_1 e^{mx} + C_2 e^{-mx} + C_3 \cos mx + C_4 \sin mx - \frac{x}{4m^3}\sin mx$]

(f) $y'' + 5y' + 4y = x^2 + 7x + 9$ [Ans.: $y = C_1 e^{-4x} + C_2 e^{-x} + \frac{1}{4}(x^2 + \frac{9}{2}x + \frac{23}{8})$]

(g) $(D-2)^2 y = 8(e^{2x} + \sin 2x + x^2)$

[Ans.: $y = (C_1 + C_2 x)e^{2x} + 4x^2 e^{2x} + \cos 2x + 2x^2 + 4x + 3$]

(h) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 12y = (x-1)e^{2x}$ [Ans.: $y = C_1 e^{2x} + C_2 e^{-6x} + e^{2x}\left(\frac{1}{16}x^2 - \frac{9}{64}x\right)$]

(i) $y'' + 3y' + 2y = e^{e^x}$ [Ans.: $y = C_1 e^{-x} + C_2 e^{-2x} + e^{-2x} e^{e^x}$

(j) $y'' + 9y = \sec 3x$ [Ans.: $y = C_1 \cos 3x + C_2 \sin 3x - \frac{1}{3}\left\{x \sin 3x + \frac{1}{3}\cos 3x \,.\log(\cos 3x)\right\}$]

2. Solve $y'' + 2y' + 10y + 37\sin x = 0$ and find the value of y at $x = \frac{\pi}{2}$ if $y = 3$ and $y' = 0$ at $x = 0$. [Ans.: $y = e^{-x}(C_1 \cos 3x + C_2 \sin 3x) + 6\cos 3x - \sin 3x$ and $y = 1$]

3. Solve $\frac{d^2y}{dx^2} + y = 0$; given that $y(0) = 2$ and $y\left(\frac{\pi}{2}\right) = -2$. [Ans.: $y = 2(\cos x - \sin x)$]