Tutorial Sheet No. 11

Course: B.Tech. (CSE, IT, ECE, EEE, ME, CE, FT)

Year & Semester: I / II

Subject & Code: Mathematics – II (BAS – 203)

Unit & Topic: V / Complex Integration (b)

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1. Expand the function $f(z) = \frac{z - \sin z}{z^2}$ in Maclaurian's series.

$$\left[\text{Ans.:} -\frac{1}{3!}z + \frac{1}{5!}z^3 - \cdots + 2z^{-1} - z^{-2}\right]$$

2. Expand the function $f(z) = \frac{1}{z}$ about $z = 2$. $\left[\text{Ans.:} \frac{1}{2} - \frac{1}{4}(z - 2) + \frac{1}{8}(z - 2)^2 + \frac{1}{16}(z - 2)^3 + \cdots\right]$

3. Find the Taylor's series of the function $f(z) = \frac{1}{(z-1)(z-2)}$ in the region $|z| = 1$.

$$\left[\text{Ans.:} \frac{1}{2} + \frac{3}{4}z + \frac{7}{8}z^2 + \frac{15}{16}z^3 + \cdots\right]$$

4. Expand the function $f(z) = \frac{z}{(z+1)(z+2)}$ about $z = 2$ in Laurent's series.

$$\left[\text{Ans.:} \left(\frac{1}{2} - \frac{1}{3}\right) - \left(\frac{1}{2^3} - \frac{1}{3^2}\right)(z - 2) + \left(\frac{1}{2^5} - \frac{1}{3^3}\right)(z - 2)^2 - \cdots\right]$$

5. Find the zeros and its order of the function $f(z) = \frac{z^2 + 6z + 9}{z^2 + 2z + 2}$.   [Ans.: $z = -3$ of order 2]

6. Find the zeros and singular points of the function $f(z) = \frac{z-2}{z^2}\sin\left(\frac{1}{z-1}\right)$.

$$\left[\text{Ans.: 2 and } \frac{1}{n\pi} + 1; n = \pm 1, \pm 2, \dots \text{ and } z = 0\right]$$

7. Find the singular points of a function $f(z) = \frac{1}{z(2-z)}$.   [Ans.: 0, 2]

8. If $f(z) = \frac{z^2}{(z-2)^4(z+3)^6}$ then find the poles and their orders. [Ans.: $z = 2$ of order 4 and $z - 3$ of order 6]

9. Find the sum of poles of $f(z) = \frac{1}{z^2 - 1}$.   [Ans.: 0]

10. Determine the pole and residue at the pole of the function $f(z) = \frac{z}{z-1}$.

11. If $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ then find Res.$f(-2)$.   [Ans: 4/9]

12. Sum of residues of $f(z) = \frac{1}{z^2 - 1}$ at its poles.   [Ans.: 0]

13. Evaluate the following integrals around unit circle:

(a) $\int_0^{2\pi} \frac{\cos 3\theta}{5 + 4\cos\theta} d\theta$ $\left[\text{Ans.:} -\frac{\pi}{12}\right]$ (b) $\int_0^\pi \frac{1}{17 - 8\cos\theta} d\theta$ $\left[\text{Ans.:} \frac{\pi}{15}\right]$

14. Evaluate the following integrals around semi – circle:

(a) $\int_{-\infty}^\infty \frac{1}{(1+x^2)^2} dx$ $\left[\text{Ans.:} \frac{\pi}{2}\right]$ (b) $\int_0^\infty \frac{x^2}{(x^2+9)(x^2+4)^2} dx$ $\left[\text{Ans.:} \frac{\pi}{200}\right]$