Tutorial Sheet No. 08

Course: B.Tech. (CSE, IT, ECE, EEE, ME, CE, FT)

Year & Semester: I / II

Subject & Code: Mathematics – II (BAS – 203)

Unit & Topic: III / Fourier Series

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1. If $f(x) = x + x^2$ for $-\pi < x < \pi$, find the Fourier series of $f(x)$.

[Ans.: $f(x) = \frac{\pi^3}{3} + 4\left(-\cos x + \frac{1}{4}\cos 2x - \frac{1}{9}\cos 3x + \cdots\right) + 2\left(\sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \cdots\right)$]

2. Find the Fourier series of $f(x) = \pi - x$ for $0 < x < 2\pi$.

[Ans.: $f(x) = 2\left(\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \cdots\right)$]

3. Find the Fourier series of $f(x) = |x|$ for $-2 < x < 2$.

[Ans.: $f(x) = 1 - \frac{8}{\pi^2}\left(\frac{1}{1^2}\cos\frac{\pi x}{2} + \frac{1}{3^2}\cos\frac{3\pi x}{2} - \frac{1}{9}\cos 3x + \cdots\right)$

4. Find the Fourier series of $f(x) = \begin{cases} -1; & -\pi < x < -\frac{\pi}{2} \\ 0: & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 1; & \frac{\pi}{2} < x < \pi \end{cases}$.

[Ans.: $f(x) = \frac{1}{\pi}\left(2\sin x - 2\sin 2x + \frac{2}{3}\sin 3x + \cdots\right)$]

5. Find the sine and cosine series of $f(x) = \begin{cases} 0 & ; 0 < x < \frac{\pi}{2} \\ \pi - x; & \frac{\pi}{2} < x < \pi \end{cases}$.

[Ans.: (i) $f(x) = \frac{4}{\pi}\left(\sin x - \frac{1}{3^2}\sin 3x + \frac{1}{5^2}\sin 3x - \cdots\right)$

(ii) $f(x) = \frac{\pi}{4} - \frac{2}{\pi}\left(\frac{1}{1^2}\cos 2x + \frac{1}{3^2}\cos 6x + \cdots\right)$]