Tutorial Sheet No. 09

Course: B.Tech. (CSE, IT, ECE, EEE, ME, CE, FT)

Year & Semester: I / II

Subject & Code: Mathematics – II (BAS – 203)

Unit & Topic: IV / Complex Differentiation

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1. If $z = r(\cos\theta + i\sin\theta)$ then find $|z|^3$. [Ans.: r^3]

2. If $x + iy = \sqrt{2} + 3i$ then find $(x^2 + y)$. [Ans.: 5]

3. Test the existence of $\lim_{z\to 0} \frac{z}{|z|}$. [Ans.: does not exist]

4. Evaluate $\lim_{z\to 0} \frac{z^2+6z+3}{z^2+2z+2}$. [Ans.: $\frac{3}{2}$]

5. Show that the following limits do not exist:

(i) $\lim_{z\to 0} \frac{Im(z^3)}{Re(z^3)}$ (ii) $\lim_{z\to 0} \frac{z}{(\bar{z})^2}$

6. Show that the function $f(z) = \begin{cases} \frac{Re(z)}{z}; & z \neq 0 \\ 0; & z = 0 \end{cases}$ is discontinuous at $z = 0$.

7. Define an analytic function and singular point with examples. Write the necessary and sufficient condition for a complex function to be analytic in both Cartesian and polar coordinates.

8. Determine whether the following functions are analytic or not:

(i) $f(z) = z^2 + 3$ [Ans.: analytic] (ii) $f(z) = (z-1)^2$ [Ans.: analytic]

(iii) $f(z) = z^2 + z + 1$ [Ans.: analytic] (iv) $f(z) = \sinh z$ [Ans.: analytic]

9. Test the analyticity of the function $f(z) = \frac{1}{z}$. [Ans.: analytic everywhere except at $z = 0$]

10. Show that function $f(z) = z.|z|$ is not analytic anywhere. [Ans.: is not analytic anywhere except at $z = 0$]

11. Find the values of a and b such that the function $f(z) = x^2 + ay^2 - 2xy + i(bx^2 - y^2 + 2xy)$ is analytic. Also find $f'(z)$. [Ans.: $a = -1$ and $b = 1$; $f'(z) = 2(1+i)z$]

12. Find p such that the function $f(z)$ expressed in polar coordinates as $f(z) = r^2\cos 2\theta + ir^2 \sin p\theta$ is analytic. [Ans.: $p = 2$]

13. Prove that the following functions are harmonic:

(i) $u(x,y) = x^2 - y^2$ (ii) $u = x^2 - y^2 - 2xy - 2x + 3y$

(iii) $u = \frac{1}{2}\log_e(x^2 + y^2)$ (iv) (i) $u(x,y) = x^4 - 6x^2y^2 + y^4$

14. If $2x - x^2 + ay^2$ is harmonic then find the value of a. [Ans.: $a = 1$]

15. If $u(x,y) = 2x(1-y)$ then find the conjugate function $v(x,y)$ so that $f(z) = u(x,y) + iv(x,y)$ is analytic, by Milne – Thomson's method. [Ans.: $v(x,y) = x^2 - y^2 + 2y$]