Tutorial Sheet No. 05

Course: B.Tech. (CSE, IT, ECE, EEE, ME, CE, FT)

Year & Semester: I / II

Subject & Code: Mathematics – II (BAS – 203)

Unit & Topic: II / Beta and Gamma Functions

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1. Evaluate the following functions:

(a) $\Gamma(3.5)$ [Ans.: $\frac{15}{8}\sqrt{\pi}$] (b) $\Gamma(4.5)$ [Ans.: $\frac{105}{16}\sqrt{\pi}$

(c) $\beta(1.5, 2.5)$ [Ans.: $\frac{\pi}{16}$] (d) $\frac{\Gamma(5/2)}{\Gamma(1/2)}$ [Ans.: $\frac{3}{4}$]

(e) $\frac{\Gamma(6)}{2\Gamma(3)}$ [Ans.:30] (f) $\Gamma(1/4).\Gamma(3/4)$ [Ans.: $\frac{\pi}{\sqrt{2}}$]

(g) $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta$. [Ans.: $\frac{2\sqrt{2}}{\pi}$]

2. Evaluate the following definite integrals:

(i) $\int_0^1 x^5(1-x^3)^{10}dx$ [Ans.: $\frac{1}{396}$] (ii) $\int_0^1 x^3\left(1-\sqrt{x}\right)^5 dx$ [Ans.: $\frac{1}{5148}$]

(iii) $\int_0^\infty \frac{x^3}{(1+x)^9}dx$ [Ans.: $\frac{1}{280}$] (iv) $\int_0^\infty \frac{x^c}{c^x}dx$ [Ans.: $\frac{\Gamma(c+1)}{(\log c)^{c+1}}$]

(v) $\int_0^\infty \frac{e^{-st}}{\sqrt{t}}dt$ [Ans.: $\sqrt{\frac{\pi}{s}}, s>0$] (vi) $\int_0^\infty 4x^4e^{-x^4}dx$ [Ans.: $\Gamma(\frac{5}{4})$]

(vii) $\int_0^\infty x^3e^{-x}dx$ [Ans.:6] (viii) $\int_0^6 x^6e^{-2x}dx$ [Ans.: $\frac{45}{8}$]

(ix) $\int_0^\infty e^{-x^2}dx$ [Ans.: $\frac{\sqrt{\pi}}{2}$] (x) $\int_0^\infty \sqrt{x}e^{-x^3}dx$ [Ans.: $\frac{\sqrt{\pi}}{3}$]

3. Show that

(i) $\int_0^1 x^{n-1}\left(\log\frac{1}{x}\right)^{m-1}dx = \frac{\Gamma(m)}{n^m}; m,n>0.$

(ii) $\int_0^2 \frac{x^2}{\sqrt{1-x^3}}dx = \frac{2}{3}$

(iii) $\int_0^\infty \frac{x^2}{1+x^4}dx = \frac{\pi}{2\sqrt{2}}$

(iv) $\beta(p,q) = \beta(p+1,q) + \beta(p,q+1)$

(v) $\frac{\beta(m+1,n)}{m} = \frac{\beta(m,n+1)}{n} = \frac{\beta(m,n)}{m+n}$