

Tutorial Sheet No. 06

Course: B.Tech. (CSE, IT, ECE, EEE, ME, CE, FT)

Year & Semester: I / II

Subject & Code: Mathematics – II (BAS – 203)

Unit & Topic: II / Dirichlet's Integral & Volume and Surface Area of Revolution

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1. Find the mass of an octant of the ellipsoid $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$, the density at any point being $\rho = kxyz$.
[Ans.: $\frac{ka^2b^2c^2}{48}$]

2. Evaluate $\iint x^{2l-1}y^{2m-1}dxdy$ for all positive values of x and y such that $x^2 + y^2 \leq c^2$.

3. Find the volume of the solid generated by the portion of the arc $y = \sqrt{1+x^2}$ lying between $x = 0$ and $x = 4$, as it revolves about the x – axis. [Ans.: $\frac{76}{3}$]

4. Show that the volume of the solid obtained by revolving the region enclosed between the curves $y = \sin x$ and $y = \cos x$ about the x – axis is $\frac{\pi(\pi-2)}{4}$.

5. The part of the parabola $y^2 = 4ax$ bounded by the latus rectum revolves about the tangent at the vertex. Find the area of the curved surface of the reel thus generated.

[Ans.: $a^2\pi\{3\sqrt{2} - \log(\sqrt{2} + 1)\}$]

6. Show that the surface of the solid obtained by revolving the arc of the curve $y = \sin x$ from $x = 0$ to $x = \pi$ about the x – axis is $\pi^2\{\sqrt{2} + \log(1 + \sqrt{2})\}$.

7. Find the volume and surface area of a sphere of radius a by the method of revolution.

[Ans.: $\frac{4}{3}\pi a^3$ and $4\pi a^2$]

8. Find the volume and surface area of a right circular cone formed by the revolution of a right – angled triangle about a side containing the right angle. [Ans.: $\frac{1}{3}\pi r^2h$ and $\pi r\sqrt{r^2 + h^2}$]

9. Find the area of the surface formed by the revolution of $y^2 = 4ax$ about its axis, by the arc from the vertex to one end of the latus rectum. [Ans.: $\frac{8\pi a^2}{3}(2\sqrt{2} - 1)$]