

Tutorial Sheet No. 10

Course: B.Tech. (CSE, IT, ECE, EEE, ME, CE, FT)

Year & Semester: I / II

Subject & Code: Mathematics – II (BAS – 203)

Unit & Topic: V / Complex Integration (a)

Prepared By: Dr. Manoj Kumar Gupta, Assistant Professor

1. Evaluate:

(i) $\int_0^{1+i} (x - y + ix^2) dz$ along the line from $z = 0$ to $z = 1 + i$. [Ans.: $-\frac{1}{3} + \frac{1}{3}i$]

(ii) $\int_0^{1+i} (x^2 - iy) dz$ along the path $y = x$ [Ans.: $\frac{5}{6} - i\frac{1}{6}$]

(b) $\int_C |z| dz$ where C is the straight line from $z = -i$ to $z = i$. [Ans.: i]

(c) $\int_C [(x + y)dx + x^2y dy]$ where C is the path $y = x^2$ having $(0, 0)$ and $(3, 9)$ as end points.

[Ans: 513 / 2]

(d) $\int_0^{2+i} (|z|)^2 dz$ along the line $y = x/2$ is [Ans.: $\frac{5}{3}(2 - i)$]

2. Find the value of the integral $\int_C \frac{1}{z-a} dz$ where C is the circle $|z - a| = r$. [Ans.: $2\pi i$]

3. Evaluate the following integrals:

(i) $\int_C \frac{z^2 - z + 1}{z - 1} dz$ where C is the circle $|z| = 1$ [Ans.: $2\pi i$]

(ii) $\int_C \frac{\sin^2 z}{(z - \frac{\pi}{6})^3} dz$ where C is the circle $|z| = 1$ [Ans.: πi]

(iii) $\int_C \frac{z+1}{z^3 - 2z^2} dz$ where C is a circle $|z| = 1$ [Ans.: $-\frac{3}{2}\pi i$]

(iv) $\int_C \frac{z^2 + 1}{(z+1)(z+2)} dz$ where C is a circle $|z| = \frac{3}{2}$ is [Ans.: $4\pi i$]

(v) $\int_C \frac{4z^2 + z + 5}{z + 4} dz$ where C is a circle $9x^2 + 4y^2 = 36$ [Ans.: 0]

(v) $\int_C \frac{1}{z^2 - 1} dz$ If C is a circle with $|z| = 2$ [Ans.: 0]

4. Evaluate the integral $\int_C \frac{\cos z}{z} dz$; where C is the ellipse $4x^2 + 9y^2 = 36$ by Cauchy's integral formula. [Ans.: $2\pi i$]

5. Evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$ where C is the circle $|z| = \frac{3}{2}$. [Ans.: $2\pi i$]

6. Evaluate $\oint_C \frac{3z^2 + z}{z^2 - 1} dz$ where C is the circle $|z - 1| = 1$. [Ans.: $4\pi i$]