Tutorial Sheet No. 03

Course: B.Tech. (CSE, IT, ECE, EEE, ME, CE, FT)

Year & Semester: I / II

Subject & Code: Mathematics – II (BAS – 203)

Unit & Topic: I / 2nd Order ODEs with Variable Coefficients

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1. Solve the following differential equations by the method of changing the independent variable:

(a) $\frac{d^2y}{dx^2} + \cot x \frac{dy}{dx} + 4y \operatorname{cosec}^2 x = 0.$ $\left[\text{Ans.}: y = C_1 \cos\left\{2 \log_e\left(\tan\frac{x}{2}\right)\right\} + C_2 \sin\left\{2 \log_e\left(\tan\frac{x}{2}\right)\right\}\right]$

(b) $\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} + 4x^2y = x^4.$ [Ans.: $y = C_1 \cos(x^2) + C_2 \sin(x^2) + \frac{x^2}{4}$]

2. Solve the following differential equations by the method of normal form (Removal of first derivative):

(a) $\frac{d}{dx}\left[(\cos^2 x)\frac{dy}{dx}\right] + y \cos^2 x = 0$ $\left[\text{Ans.}: y = \left(\cos\sqrt{2}x + C_2 \sin\sqrt{2}x\right).\sec x\right]$

(b) $x^2\frac{d^2y}{dx^2} - 2(x^2 + x)\frac{dy}{dx} + 2(x^2 + 2x + 2)y = 0.$ [Ans.: $y = x(C_1x + C_2).xe^x$]

3. Solve the following differential equations by the method of reduction of order:

(a) $x^2\frac{d^2y}{dx^2} - (2x - 1)\frac{dy}{dx} + (x - 1)y = e^x.$ [Ans.: $y = (C_1 \log_e x + C_2)e^x$]

(b) $x^2\frac{d^2y}{dx^2} - 2x(1 + x)\frac{dy}{dx} + 2(1 + x)y = x^3.$ [Ans.: $y = x\left(-\frac{x}{2} + \frac{C_1}{2}e^{2x} + C_2\right)$]

(c) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0.$ [Ans.: $y = \frac{A}{x} + C_2\left(x + \frac{1}{x}\right)$]

4. Find the series solution of the following equations by Frobenius method:

(a) $2x^2y'' + xy' - (x + 1)y = 0.$ $\left[\text{Ans.}: Ax\left(1 + \frac{1}{5}x + \frac{1}{70}x^2 + \cdots\right) + Bx^{-1/2}\left(1 - x - \frac{1}{2}x^2 + \cdots\right)\right]$

(b) $x^2y'' + x(x - 1)y' + (1 - x)y = 0.$ [Ans.: $y = Ax + B(x \log_e x - x + x^2 - \cdots)$]