Lecture 18:

Putting it all together

Algebra, Systems design

Today's Lecture

1. Relational Algebra and Optimization

2. Systems design example

Logical Optimization

RDBMS Architecture

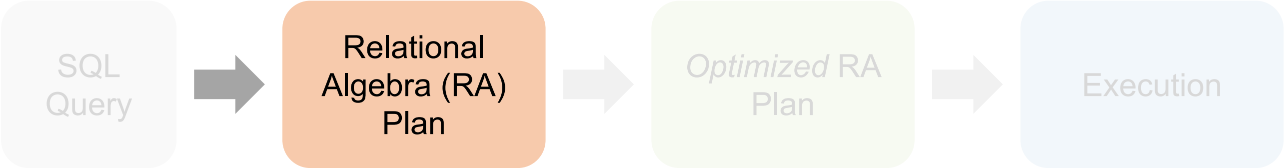
How does a SQL engine work ?

SQL Query → Relational Algebra (RA) Plan → *Optimized* RA Plan → Execution

SQL Query	Relational Algebra (RA) Plan	*Optimized* RA Plan	Execution
Declarative query (from user)	Translate to relational algebra expression	*Find logically equivalent- but more **cost-efficient**- RA expression*	Execute each operator of the optimized plan!

RDBMS Architecture

How does a SQL engine work ?



Relational Algebra allows us to translate declarative (SQL) queries into precise and optimizable expressions!

Relational Algebra (RA)

Five **basic** operators:

1. Selection: σ
2. Projection: Π
3. Cartesian Product: $\times$
4. Union: $\cup$
5. Difference: -

Derived or auxiliary operators:

- Intersection, complement
- Joins: $\bowtie$ (natural, equi-join, semi-join)
- Renaming: ρ

Converting SFW Query to RA

Students(sid,sname,gpa)
People(ssn,sname,address)

```
SELECT DISTINCT
  gpa,
  address
FROM Students S,
    People P
WHERE gpa > 3.5 AND
   sname = pname;
```

$$\Pi_{gpa,address}(\sigma_{gpa>3.5}(S \bowtie P))$$

How do we represent this query in RA?

Logical Equivalence of RA Plans

- Given relations R(A,B) and S(B,C):
  - Here, projection & selection commute:
    - $\sigma_{A=5}(\Pi_A(R)) = \Pi_A(\sigma_{A=5}(R))$
  - What about here?
    - $\boxed{\sigma_{A=5}(\Pi_B(R))} ?= \Pi_B(\sigma_{A=5}(R))$

losing access to A

RDBMS Architecture

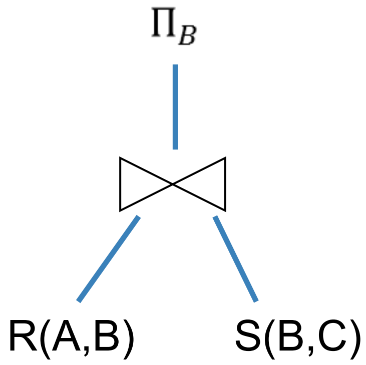
How does a SQL engine work ?

SQL Query → Relational Algebra (RA) Plan → *Optimized* RA Plan → Execution

We'll look at how to then optimize these plans now

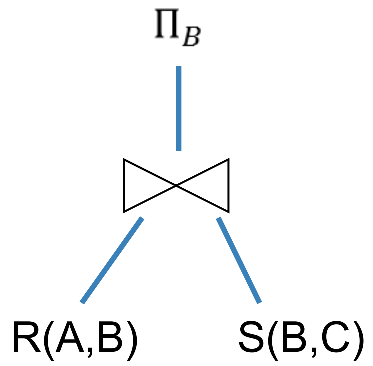
Visualize the plan as a tree

$$\Pi_B(R(A,B) \bowtie S(B,C))$$



Bottom-up tree traversal = order of operation execution!

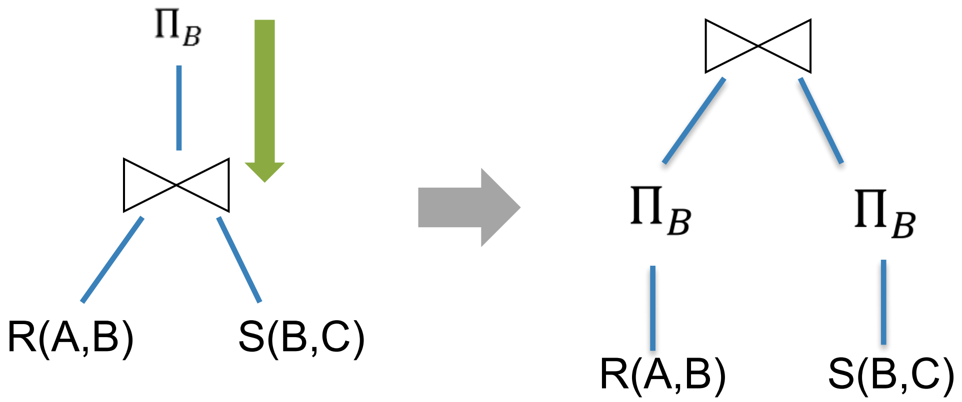
A simple plan



What SQL query does this correspond to?

Are there any logically equivalent RA expressions?

“Push down” projection



Why might we prefer this plan?

Logical Optimization

- Heuristically, we want selections and projections to occur as early as possible in the plan
  - Terminology: “push down **selections** and **projections”**

- **Intuition:** We will usually have fewer tuples in a plan.

  Exceptions

  - Could fail if the selection condition is very expensive (e.g, run image processing algorithm)
  - Projection could be a waste of effort, but more rarely

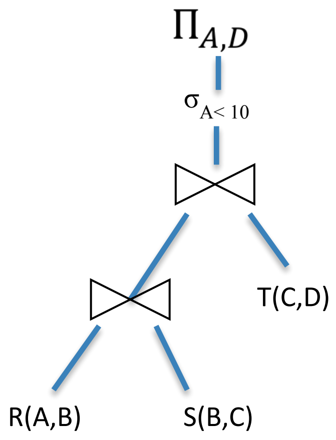
Optimizing the SFW RA Plan

Translating to RA

R(A,B) S(B,C) T(C,D)

```
SELECT R.A,S.D
FROM R,S,T
WHERE R.B = S.B
  AND S.C = T.C
  AND R.A < 10;
```

$$\Pi_{A,D}(\sigma_{A<10}(T \bowtie (R \bowtie S)))$$



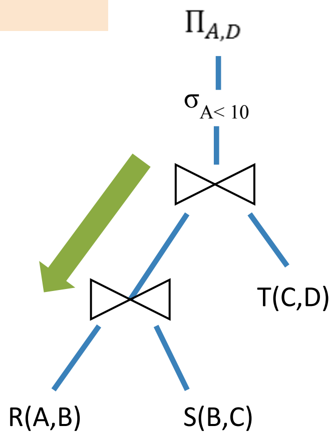
Optimizing RA Plan

R(A,B) S(B,C) T(C,D)

```
SELECT R.A,S.D
FROM R,S,T
WHERE R.B = S.B
  AND S.C = T.C
  AND R.A < 10;
```

$$\Pi_{A,D}(\sigma_{A<10}(T \bowtie (R \bowtie S)))$$

Push down selection on A so it occurs earlier



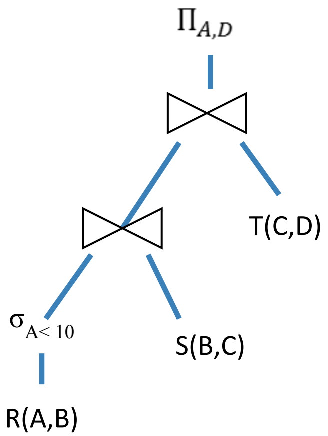
Optimizing RA Plan

R(A,B) S(B,C) T(C,D)

```
SELECT R.A,S.D
FROM R,S,T
WHERE R.B = S.B
  AND S.C = T.C
  AND R.A < 10;
```

$$\Pi_{A,D}\left(T \bowtie \left(\sigma_{A<10}(R) \bowtie S\right)\right)$$

Push down selection on A so it occurs earlier



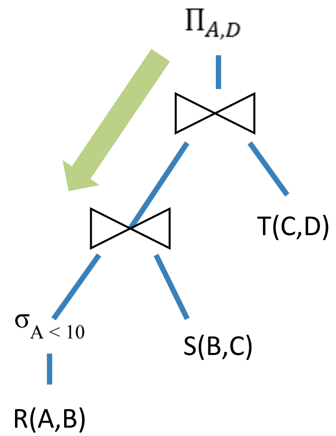
Optimizing RA Plan

R(A,B) S(B,C) T(C,D)

```
SELECT R.A,S.D
FROM R,S,T
WHERE R.B = S.B
  AND S.C = T.C
  AND R.A < 10;
```

$$\Pi_{A,D}\left(T \bowtie (\sigma_{A<10}(R) \bowtie S)\right)$$

Push down projection so it occurs earlier



Optimizing RA Plan

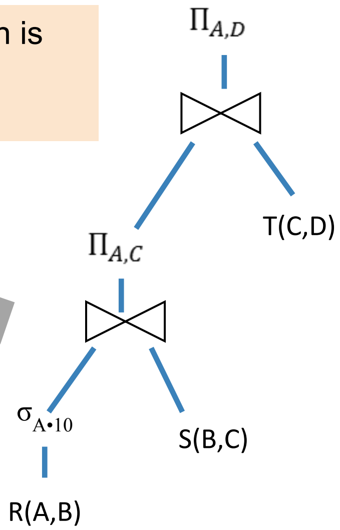
R(A,B) S(B,C) T(C,D)

```
SELECT R.A,S.D
FROM R,S,T
WHERE R.B = S.B
  AND S.C = T.C
  AND R.A < 10;
```

$$\Pi_{A,D}\left(T \bowtie \Pi_{A,c}(\sigma_{A<10}(R) \bowtie S)\right)$$

We eliminate B earlier!

In general, when is an attribute not needed…?



Basic RA commutators

- Push **projection** through **(1) selection**, **(2) join**
- Push **selection** through **(3) selection, (4) projection, (5) join**
- *Also:* Joins can be re-ordered!

⇒ Note that this is not an exhaustive set of operations

This covers *local re-writes; global re-writes possible but much harder*

This simple set of tools allows us to greatly improve the execution time of queries by optimizing RA plans!

Takeaways

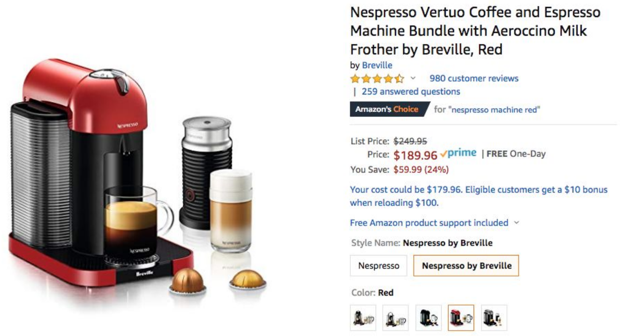
- This process is called logical optimization
- Many equivalent plans used to search for “good plans”
- Relational algebra is a simple and elegant abstraction

Data Systems Design

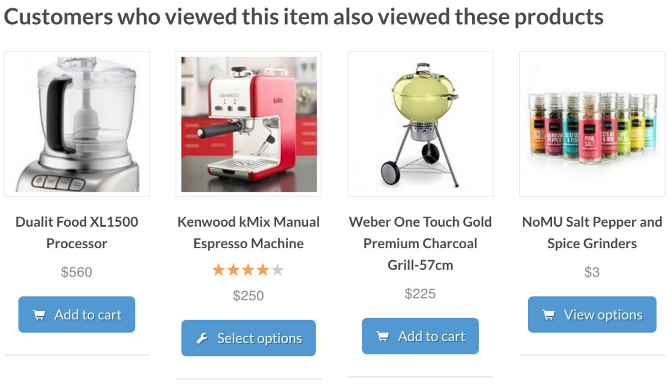
Systems Design Example:

Product CoOccur

Counting product views for billion products

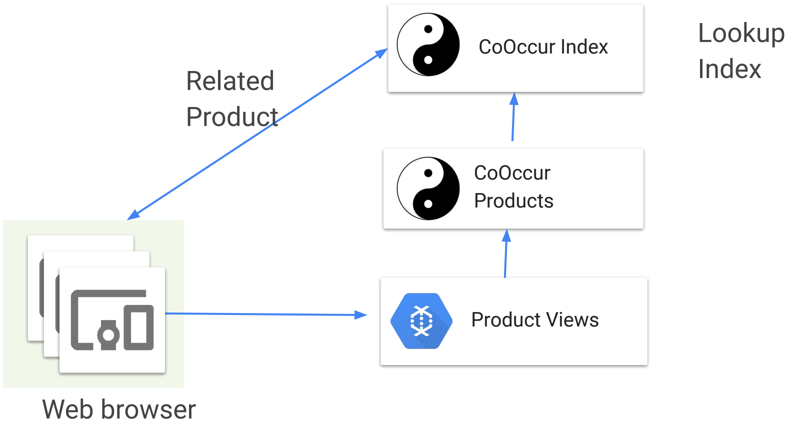


Counting popular product-pairs



Data Systems Design Example:

Product CoOccur



Popular Systems design pattern

1. Efficiently compute 'batch' of data **e.g. last week's data**

2. Build Lookup index on result

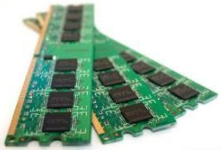
3. For 'streaming' data, update with 'micro batches'

Scale, scale, scale recap

Big Scaling (with Indexes)

Roadmap

Primary data structures/algorithms

	Hashing	Sorting	Counting
	HashTables ($hash_i(x)$)	BucketSort, QuickSort MergeSort	HashTable + Counter ($hash_i$(key) --> <count>)
	Hashes for disk location ($hash_i(x)$)	MergeSortedFiles SortFiles	
	Hashes for machines, shards ($hash_i(x)$)	MergeSortedFiles SortFiles	

Basic numbers

Cheatsheet

- Disk speeds: 10 msec/seek, 100 MBs/sec

- Typical machine assumptions (unless problem states otherwise):
  - 64 GB RAM, 64 KB (for disk block, RAM page size)
  - 4 byte integers, 8 byte long integers, 8 bytes pointers

- 2^10 = 1024, 2^20 ~= 1Million, 2^30 ~= 1 Billion (10^9), 2^40 ~= 1 Trillion (10^12)
  - To store records (4 bytes each): 1 Million records = 4MB, 1 Billion records = 4 GB

B+ tree Index recap

B+ Tree Index Search [recap]

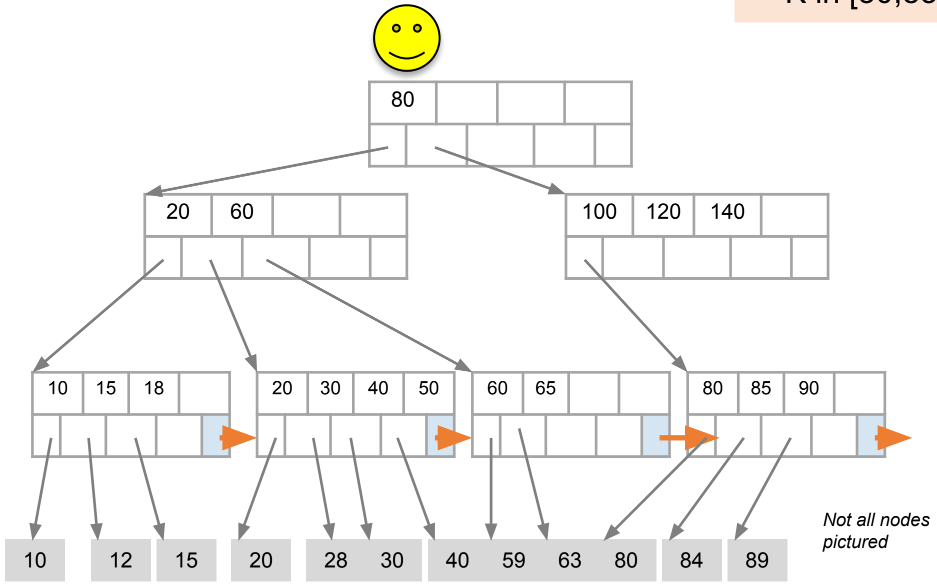
K in [30,85]?

30 < 80

30 in [20,60)

30 in [30,40)

To the data!

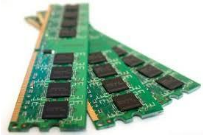


Simple Cost Model for Search [recap]

- Let:
  - ***f*** = fanout, which is **in [d+1, 2d+1]** ***(we'll assume it's constant for our cost model…)***
  - ***N*** = the total number of *pages* we need to index
  - ***F*** = fill-factor (usually ~= 2/3)
- Our B+ Tree needs to have room to index ***N / F*** pages!
  - We have the fill factor in order to leave some open slots for faster insertions
- What height (*h*) does our B+ Tree need to be?
  - h=1 → Just the root node- room to index f pages
  - h=2 → f leaf nodes- room to index f^2 pages
  - h=3 → f^2 leaf nodes- room to index f^3 pages
  - …
  - h → f^{h-1} leaf nodes- room to index f^h pages!

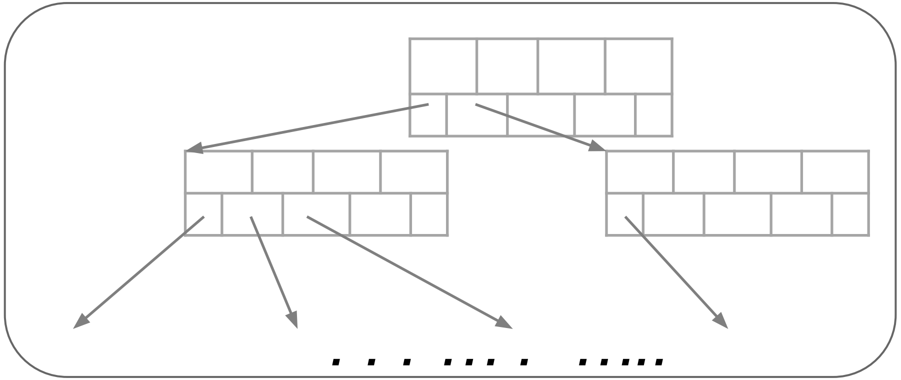
→ We need a B+ Tree of height h = $\left\lceil \log_f \frac{N}{F} \right\rceil$!

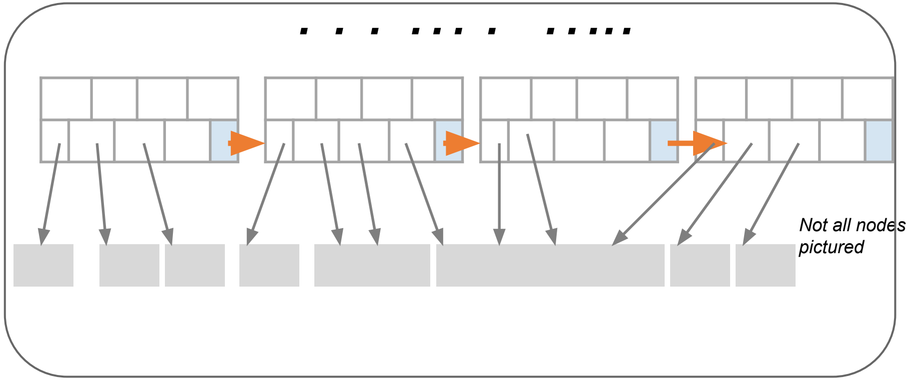
Search cost of B+ Tree [recap]



Read 1st levels
Into RAM buffer

Rest of index on
disk





$1 + f + f^2 + f^3 + \ldots <= B$

Keep 1st L_B levels in
RAM of size B

Algorithm: B+ Search
- Read 1 page per level
- Read 1 page for record

IO Cost: $\left\lceil \log_f \frac{N}{F} \right\rceil - L_B + 1$

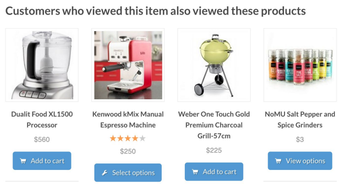
where $B \geq \sum_{l=0}^{L_B - 1} f^l$

Systems Design Example:

Product CoOccur

Counting popular product-pairs

(from primer doc)



1. Amazon/Walmart/Alibaba (AWA) need to compute 'related products' for all products so their users can explore and buy new products. AWA would like to use collaborative filtering (aka 'wisdom of crowds') from their website logs of user views of products. That is, each time a user views a set of products, those products are said to co-occur. By computing product pairs and their co-occurrence frequency (or **co-occur count)** across all users, AWA can compute related products for their catalog.

2. Data input: AWA's product catalog is 1 billion items. Each product record is ~1MB with a product description and image. AWA has 10 billion product views each week, from 1 billion users. Each log record stores <userID, productID, viewID, viewtime>.

3. Your mission is to design an efficient system to compute co-occur counts on *Sundays* from weekly logs and produce a CoOccurCount table <productID, productID, count>
   - AWA's data quality magicians recommend (a) retaining only the top billion popular pairs, and (b) dropping product pairs with co-occur counts less than million. Also, assume users view ten products on average (**UserSession** assumption).
   - For simplicity, LogOfViews is stored sorted by <userID, productID>. You can sequentially scan the log and produce co-occurring product pairs for each user. In other words, (p_i, p_j) if a user viewed p_i and p_j. This "stream" of tuples (TempCoOccur) may then be (a) stored on disk or (b) discarded after updating any data structures.

Systems Design Example:

Product CoOccur

Pre-design

	Size	Why?
ProductId		
UserID		
LogOfViewsID		
Product		
Users		
LogOfViews		
CoOccur		
TempCoOccur		
TempCoOccur (with UserSession assumption, of ~10 views/user)		

Systems Design Example:

Product CoOccur

Pre-design

	Size	Why?
ProductId	4 bytes	1 Billion products ⇒ Need at least 30 bits (2^30 ~= 1 Billion) to represent each product uniquely. So use 4 bytes.
UserID	4 bytes	"
LogOfViewsID	8 bytes	10 Billion product views.
Product	1 PB	1 Billion products of 1 MB each
Users	Unknown	
LogOfViews	240 GB	Each record is <userID, productID, viewID, viewtime>. Assume: we use 8 bytes for viewTime. So that's 24 bytes per record. 10 Billion*24 bytes = 240 GBs.
CoOccur	12 GB	The output should be <productID, productID, count> for the co-occur counts. That is, 12 bytes per record (4 + 4 + 4 for the two productIDs and 4 bytes for count). To keep top billion product pairs (as recommended by AWA data quality), you need 1 billion * 12 bytes = 12 GBs.
TempCoOccur	10^9 * 12 GB	To count all product pairs as we scan input, we may need 1 billion*1 billion (10^18) counters (10^9 * 12 GB of storage).
TempCoOccur (with UserSession assumption, of ~10 views/user)	800 GB	# product pairs produced: 1 billion users * 10^2 = 100 billion Size @8 bytes/record = 800 GBs.

Systems Design Example: Product CoOccur

Managing RAM/Disk

	Keep table in RAM? Size?	Sequentially scan from disk? If so, how many disk pages?
Products		-
Users		-
LogOfViews		
CoOccur		
TempCoOccur		
TempCoOccur (with UserSession assumption)		

Systems Design Example:

Product CoOccur

Managing RAM/Disk

	Keep table in RAM? Size?	Sequentially scan from disk? If so, how many disk pages?
Products	Not needed for problem	-
Users	Not needed for problem	-
LogOfViews	No, 240 GB You need to only scan the per-user records once. No need for random access.	
CoOccur	Yes, 12 GB. Prefer random access.	
TempCoOccur	No. Worst-case: 1 billion * 1 billion counters. Size = 10^18*12 bytes	
TempCoOccur (with UserSession assumption)	No. 800 GBs	

Systems Design Example:

Product CoOccur

Managing RAM/Disk

	Keep table in RAM? Size?	Sequentially scan from disk? If so, how many disk pages?
Products	Not needed for problem	-
Users	Not needed for problem	-
LogOfViews	No, 240 GB You need to only scan the per-user records once. No need for random access.	Yes ~4 million disk blocks (recall: 64KB per page, disk block)
CoOccur	Yes, 12 GB. Prefer random access.	No, Keep in RAM. (Flush later to disk, if necessary.)
TempCoOccur	No. Worst-case: 1 billion * 1 billion counters. Size = 10^18*12 bytes	Yes, must leave on disk. Worst case: 12 * 10^18/64KB pages (= 18.75 * 10^13 pages)
TempCoOccur (with UserSession assumption)	No. 800 GBs	Yes, must leave on disk. # pages: 800 GBs/64 KB ~= 12.5 million

Systems Design Example:

Product CoOccur

Design #2

Design #2: With 1 machine, Analyze with UserSession assumption.

Design 2

1. Scan LogOfviews. For each user, append <p_i, p_j> to a log TempCoOccurLog if the user has viewed product p_i and p_j. (i.e., produce per-user co-occur product pair)
2. Externally sort TempCoOccurLog on disk, so identical product pairs are adjacent to each other in the sorted file
3. Scan sorted TempCoOccurLog. This With a single pass, you can count co-occur pairs. Drop co-occur pairs with < 1 million.

Systems Design Example:

Product CoOccur

Design #2

Design #2: With 1 machine, Analyze with UserSession assumption.

Steps	Cost (time)	Why?
Scan LogOfViews Append <p_i, p_j> to TempCoOccurLog		
Externally sort TempCoOccurLog on disk (Assume sort cost is ~2N, where N is number of pages for table and B is number of buffers, and B ~~ N)		
Scan TempCoOccurLog (sorted) and keep counts in CoOccur		

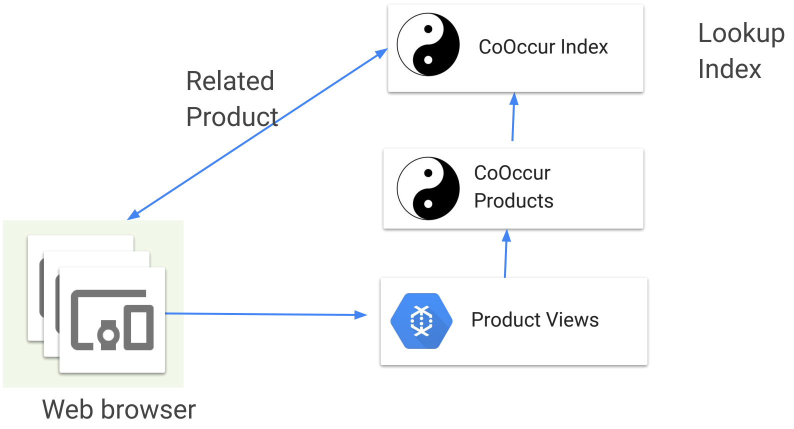
Systems Design Example: Product CoOccur Design #2

Design #2: With 1 machine, Analyze with UserSession assumption.

Steps	Cost (time)	Why?
Scan LogOfViews Append <p_i, p_j> to TempCoOccurLog	~2400 secs ~8000 secs	240GB @100 MB/sec 800 GB @100 MB/sec
Externally sort TempCoOccurLog on disk (Assume sort cost is ~2N, where N is number of pages for table and B is number of buffers, and B ~~ N)	~16,000 secs	IO cost is (appx) 2 * (1 seek + scan cost for 12.5 million pages* 64 KB/per page) = 2* scan cost of 800 GBs. That is, 16000 secs (2*800 GB @100 MB/sec). Assume TempCoOccurLog (and runs) are stored sequentially.
Scan TempCoOccurLog (sorted) and keep counts in CoOccur	~8000 secs	800 GB @100 MB/sec

Data Systems Design Example:

Product CoOccur



Systems Design Example:

Product CoOccur

B+ tree index

Evaluate the cost of lookups in a clustered B+ tree, clustered on productId we look up. How many IO lookups can we expect if we had 1 GB of RAM for the index?

Recall: Let ‘d’ be the degree of the B+ tree nodes, and ‘F’ be fill-factor. Leaf nodes have between d and 2d keys.

Systems Design Example:

Product CoOccur

B+ tree index

CoOccur # of data records	1 Billion	Given
Index page size	64KB	Given
Number of pages you can fit with 1 GB RAM		
How large is d?		
Avg number of index records per page, with F=3/4		
Number of index pages to index (N)		
Root node (level 0)		
# Pages at level 1, 2		
# of levels in B+ tree to index 1 billion data records		
Number of IOs to get data record for query=urlID		

Systems Design Example:

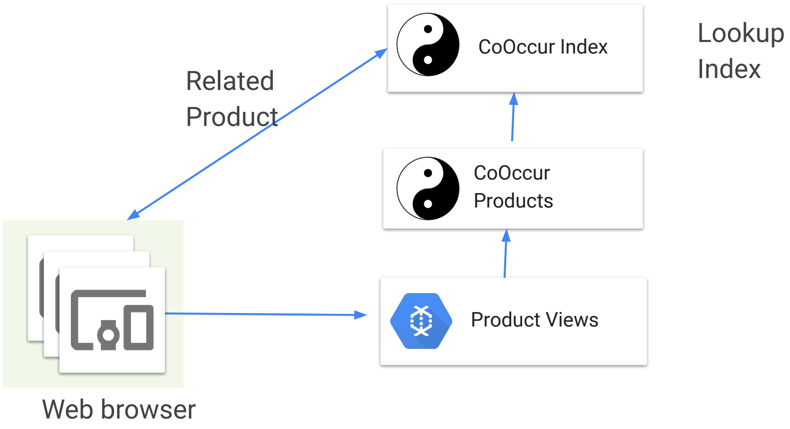
Product CoOccur

B+ tree index

CoOccur # of data records	1 Billion	Given
Index page size	64KB	Given
Number of pages you can fit with 1 GB RAM	~16k pages	1 GB/64KB
How large is d?	~2730	4 bytes for productId + 8 bytes for pointers @ 64KB/page 2d * 4 + (2d+1)*8 <= 64k ⇒ d ~= 2730
Avg number of index records per page, with F=3/4	4000	¾ of 2d ~= 4000
Number of index pages to index (N)	250,000	1 billion records/4000
Root node (level 0)	1 page	Has ~4000 pointers
# Pages at level 1, 2	4000, 4000^2	At level n, has 4000^n pages
# of levels in B+ tree to index 1 billion data records	Root + 2	4000 <= 250,000 4000 + 4000^2 >= 250,000
Number of IOs to get data record for query=urlID	2	Assume root & level1 can be in RAM (Level2 needs 4000^2 pages >> 16k pages in RAM) # IOs: 1 for Level 2, 1 for data record

Data Systems Design Example:

Product CoOccur



Data Systems Design Example:

Bigger Product CoOccur

Problem so far

- AWA's product catalog is 1 billion items. AWA has 10 billion product views each week, from 1 billion users. Each log record stores <userID, productID, viewID, viewtime>

Consider 1000x Bigger problem!

- Product catalog is 1 trillion items. AWA has 10 billion product views. Rest stays same

⇒ What changes?

Systems Design Example:

Product CoOccur

Pre-design

	Size	Why?
ProductId	8 bytes	1 trillion products ⇒ Need at least 40 bits (2^40 ~= 1 Billion) to represent each product uniquely. So use 8 bytes.
UserID	4 bytes	"
LogOfViewsID	8 bytes	10 Billion product views.
Product	1000 PB	1 Trillion products of 1 MB each
Users	Unknown	
LogOfViews	280 GB	Each record is <userID, productID, viewID, viewtime>. Assume: we use 8 bytes for viewTime. So that's 28 bytes per record. 10 Billion*28 bytes = 280 GBs.
CoOccur	20 GBs	The output should be <productID, productID, count> for the co-occur counts. That is, 20 bytes per record (8 + 8 + 4 for the two productIDs and 4 bytes for count). To keep top billion product pairs (as recommended by AWA data quality), you need 1 billion * 20 bytes = 20 GBs.
TempCoOccur	10^24 counters	To count all product pairs as we scan input, we may need 1 trillion*1 trillion (10^24) counters .
TempCoOccur (with UserSession assumption, of ~10 views/user)	1600 GB	# product pairs produced: 1 billion users * 10^2 = 100 billion Size @16 bytes/record = 1600 GBs.

Data Systems Design

Popular Systems design pattern

1. Efficiently compute 'batch' of data (sort, hash, count)
2. Build Lookup index on result (b+ tree, hash table)
3. For 'streaming' data, update with 'micro batches'

Popular problems

1. Related videos (youtube), people (Facebook), pages (web)
2. Security threats, malware (security), correlation analysis

THANK YOU!