HAUSMAN INSTRUMENTS

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- Consider the (simplified) estimation problem from D&G (2020):

$$log(Q_{st}) = -\alpha \log(P_{st}) + \beta_s + \epsilon_{st}$$

- s : store, t : time (week), β_s includes a constant and f.e. for store
- Want to estimate α (demand elasticity w.r.t. price) but P is endogeneous

> *To address potential endogeneity of prices, we instrument $\log(P_{st})$ with the average of $\log(P_{st})$ across other stores in s's chain that are located outside s's DMA.* ***This is a version of the instrumenting approach introduced by Hausman (1996) and applied by Nevo (2001), where prices of a product in other markets serve as instruments.***

- Use $Z_{st} = N_{st}^{-1} \sum_{s' \neq s} P_{s't}$ as an instrument for P_{st}
- Today's talk: when does this work?
  - What does it assume? What are the implications?
  - How does the presence of more complex pricing models affect this?
    - E.g. uniform pricing or fixed costs

The identifying assumption [of the Hausman instrument] is that, controlling for brand-specific means and demographics, city-specific demand shocks are independent across cities. Given this assumption, a demand shock for a particular brand will be independent of prices of the same brand in other cities. Due to common regional marginal cost shocks, prices of a brand in different cities within a region will be correlated, and therefore can be used as valid instrumental variables.

[Hausman] writes [...] the reduced form equation for the price ... in city n at time t as

$$\log(p_{nt}) = \alpha_n + \delta log(c_t) + w_{nt}$$

Identification is to be found in the subscripts. [...] In economic language, any demand shock that affects prices is drawn independently in each city rather than being a nationwide effect. It is this assumed asymmetry of the geographical structure of cost vs. demand errors that drives identification.

> *Before we go on, let me point out that there is no controversy about what this would mean. If the error in prices that is correlated across cities is largely cost, then the estimates are fine and Hausman is right that the value of the new variety he studies is high. If, on the other hand, I am right and the nationwide component of the error contains a substantial demand component, then the direction of the bias is clear.*

- **Today:** Even with national cost shocks, particular assumption about market structure and conduct needed for this intuition

[T]he key assumption is that the price in market t is (mean) independent of [demand shocks in market t'] conditional on [controls]. This would fail if the demand shocks are correlated [across markets], for example, through seasonal variation in demand that is not captured by the observable product characteristics. More generally, to use any proxy for an exogenous change in firm costs as an instrument, the proxy error must also be exogenous.

- Recall the canonical Hausman instrument: $Z_{st} = N_{st}^{-1} \sum_{s' \neq s} P_{s't}$,
  - Averaging over markets that are not the focal market
  - Key feature: exogeneous structural shock can be estimated through inversion of price function
  - Can do additional restrictions (only certain locations) and controls (residualize for year fixed effects, using only short-run variation)

- Hasuman"-like": instruments that transform this proxy using additional measures
  - "weighted average cost advantage that chains in market m have for UPCs in product group j":

$$Z_{mt} = \frac{\sum_r w_r \Delta \log(p_{rt,-m})}{\sum_r w_r}$$

- In more structural IO settings, many of these features would be clear (see, for e.g., Berry Haile (2021) discussion).
    - Growth in *reduced form* work that employs Hausman-style instruments
- Hausman-style instruments used in lots of canonical settings:
    - Kroft et al. (2022), Atkin, Faber and Gonzalez-Navarro (2018), Alcott et al. (2019)
- Hausman instrument has become more popular in several dimensions:
    - Uniform pricing setting (DellaVigna and Gentzkow (2020)
    - Labor settings (Azar, Berry and Ionescu (2022), Hazell, Patterson and Sarsons (2021))

> *To address potential endogeneity of prices, we instrument [price] with the average of [price] across other stores ... outside the [store's] DMA. This is a version of the instrumenting approach introduced by Hausman (1996) and applied by Nevo (2001), ... The existence of uniform pricing within chains combined with frequent sales makes this approach particularly convincing.*

- Literature studying settings with uniform prices across locations
- View that this uniform pricing *helps* with identification.
- We augment the simple model to capture spillover behavior across locations

- Consider a linear demand model, with two (potential) augmented features:
    1. quadratic costs in the aggregate cost of producing across markets **[cost spillovers]**
    2. CEO recommending a price to maximize aggregate profits **[uniform pricing]**

- Model proceeds in two steps for uniform pricing
    1. CEO recommends a uniform price to maximize the overall profit, knowing that store managers may deviate [this is not microfounded (Adams and Williams (2019))]
    2. In the second stage, store managers choose their prices by maximizing their store profits

- Demand given by $Q_{mt} = \xi_{mt} - \alpha P_{mt}$ (single product and ignore location heterogeneity)
- Solve backwards :
  1. Store manager in m solves:

$$\max_{P_{mt}} P_{mt}Q_{mt} - \left(c_{mt}Q_{mt} + \underbrace{\frac{Q_{mt}}{Q_t}\omega Q_t^2}_{\text{cost spillovers}} \right) - \gamma \underbrace{\left(\overline{P}_t^{ceo} - P_{mt}\right)^2}_{\text{uniform pricing}}$$

  2. CEO then solves the uniform price setting problem as a function of the store-level prices:

$$\max_{\overline{P}_t^{ceo}} \sum_m \left[\xi_{mt} - \alpha P_{mt}(\overline{P}_t^{ceo})\right]\left[P_{mt}(\overline{P}_t^{ceo}) - c_{mt}\right] - \underbrace{\omega\left(\sum_m \left[\xi_{mt} - \alpha P_{mt}(\overline{P}_t^{ceo})\right]\right)^2}_{\text{cost spillovers}} \tag{1}$$

The equilibrium pricing condition from this model can be written as

$$P^*_{mt} = \pi_0 \left[P^*_{\text{flexible},mt}(\xi_{mt}, c_{mt}) + \kappa P^*_{\text{cost},t}(\bar{\xi}_t, \bar{c}_t) + (1-\kappa) 2\gamma P^*_{\text{uniform},t}(\bar{\xi}_t, \bar{c}_t) \right]$$

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$$P^*_{\text{flexible},mt}(\xi_{mt}, c_{mt}) = (1 + \alpha\omega)\xi_{mt} + \alpha c_{mt}$$

$$P^*_{\text{cost spillovers}}(\bar{\xi}_t, \bar{c}_t) = \frac{\alpha + 2\gamma}{\alpha}\bar{\xi}_t - \alpha \bar{c}_t$$

$$P^*_{\text{uniform}}(\bar{\xi}_t, \bar{c}_t) = \beta_\xi \bar{\xi}_t + \beta_c \bar{c}_t$$

$$\kappa = \frac{\alpha^2 \omega M}{2(\alpha + \gamma) + \alpha^2 \omega (1 + M)}, \qquad \pi_0 = \frac{1}{2(\alpha + \gamma) + \alpha^2 \omega}$$

- No Cost Spillovers or Uniform Pricing (Canoncial Hausman):

$$P^*_{mt} = \frac{1}{2\alpha}(\xi_{mt} + \alpha c_{mt})$$

- Only Uniform Pricing:

$$P^*_{mt} = \frac{1}{2(\alpha+\gamma)}\left[\xi_{mt} + \alpha c_{mt} + 2\gamma\left(\beta_\xi \overline{\xi}_t + \beta_c \overline{c}_t\right)\right]$$

- Only Cost Spillovers:

$$P^*_{mt} = \frac{1}{2(\alpha+\alpha^2\omega)}\left[(1+\alpha\omega)\xi_{mt} + \alpha c_{mt} + \kappa\left(\overline{\xi}_t - \alpha\overline{c}_t\right)\right]$$

N.B. Uniform pricing and cost spillover effects are similar functions of ξ and c

- Define setup using model:

$$
\begin{aligned}
Q_{mt} &= -\alpha P_{mt} + \underbrace{u_{mt}}_{X_{mt}\beta+\xi_{mt}} && \text{(Demand model)} \\
P_{mt} &= a_1 c_{mt} + b_1 \xi_{mt} + a_2 \bar{c}_t + b_2 \bar{\xi}_t && \text{(Pricing model)} \\
Z_{mt} &= (M-1)^{-1} \sum_{n \neq m} P_{nt} \\
&= a_1 (M-1)^{-1} \sum_{n \neq m} c_{nt} + b_1 (M-1)^{-1} \sum_{n \neq m} \xi_{nt} + a_2 \bar{c}_t + b_2 \bar{\xi}_t
\end{aligned}
$$

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- Let $\boldsymbol{\xi}_t = (\xi_{1t}, \ldots, \xi_{Mt})$ and $\boldsymbol{c}_t = (c_{1t}, \ldots, c_{Mt})$ denote $1 \times M$ vectors for each time period. Let $\mathbf{X}_t = (X_{1t}, \ldots, X_{Mt})$ be a $K \times M$ matrix of controls.
- $(\boldsymbol{\xi}_t, \boldsymbol{c}_t, \mathbf{X}_t)$ i.i.d.
- It is possible to consider letting M grow, or T grow, or both. This will have implications in our results under uniform pricing.
  - Let n denote the sample subscript ($n = MT$)

- Consider two estimators (post FWL residualization):

$$\alpha_n^{OLS} - \alpha = \frac{\sum_t \sum_m P_{mt}\xi_{mt}}{\sum_t \sum_m P_{mt}^2}$$
$$\alpha_n^{IV} - \alpha = \frac{\sum_t \sum_m Z_{mt}\xi_{mt}}{\sum_t \sum_m Z_{mt}P_{mt}}.$$

- We now examine the properties of these estimators under different assumptions

- Consider a simple case where $c_{mt} = c_t + \tilde{c}_{mt}$, $\xi_{mt} = \xi_t + \tilde{\xi}_{mt}$, with $\tilde{c}_{mt}, \tilde{\xi}_{mt}$ i.i.d. across markets
    - Hence: $\bar{c}_t = c_t + M^{-1} \sum_m \tilde{c}_{mt}$, $\bar{\xi}_t = \xi_t + M^{-1} \sum_m \tilde{\xi}_{mt}$.

- Is it possible to invert P_{mt} to construct a proxy that is uncorrelated with some $\xi_{m't}$?

$$P_{mt} = a_1 c_{mt} + b_1 \xi_{mt} + a_2 c_t + a_2 M^{-1} \sum_m \tilde{c}_{mt} + b_2 \xi_t + b_2 M^{-1} \sum_m \tilde{\xi}_{mt} \tag{2}$$

- Intuitively, yes under functional form and statistical restrictions, but
    - Uniform pricing *works against Hausman instrument*

1. Assume correlated cost shocks across markets

$$T^{-1}\sum_{t} M^{-1}\sum_{m}(M-1)^{-1}\sum_{n\neq m} c_{mt}c_{nt} = C_{\text{cost},n}, \operatorname{plim}_{n\to\infty}|C_{\text{cost}}| > 0$$

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$$T^{-1}\sum_t M^{-1}\sum_m (M-1)^{-1}\sum_{n\neq m} c_{mt}c_{nt} = C_{\text{cost},n},\ \text{plim}_{n\to\infty}|C_{\text{cost}}| > 0$$

2. Assume no cross-market demand and cost correlation

$$T^{-1}\sum_t M^{-1}\sum_m (M-1)^{-1}\sum_{n\neq m} \xi_{mt}c_{nt} = C_{\text{cost,demand},n},\ \text{plim}_{n\to\infty} C_{\text{cost,demand},n} = 0$$

3. No cross-market demand correlation

$$T^{-1}\sum_t M^{-1}\sum_m (M-1)^{-1}\sum_{n\neq m}\xi_{mt}\xi_{nt} = C_{\text{demand},n},\ \text{plim}_{n\to\infty} C_{\text{demand},n} = 0$$

4. Own-market correlations

$$T^{-1}\sum_t M^{-1}\sum_m \xi_{mt}c_{mt} = \rho_{\xi,c,n},\ \text{plim}_{n\to\infty}\rho_{\xi,c,n} = \rho_{\xi,c},\ |\rho_{\xi,c}| < \infty$$
$$T^{-1}\sum_t M^{-1}\sum_m \xi_{mt}^2 = \rho_{\xi,n},\ \text{plim}_{n\to\infty}\rho_{\xi,n} = \rho_{\xi},\ |\rho_{\xi}| < \infty$$
$$T^{-1}\sum_t M^{-1}\sum_m c_{mt}^2 = \rho_{c,n},\ \text{plim}_{n\to\infty}\rho_{c,n} = \rho_{c},\ |\rho_{c}| < \infty$$

$$\alpha_n^{OLS} = \frac{a_1 \rho_{c,\xi,n} + b_1 \rho_{\xi,n} + a_n^{spillover}}{\Sigma_{P,n}}$$

$$\alpha_n^{2SLS} = \frac{a_1 C_{\text{cost,demand},n} + b_1 C_{\text{demand},n} + a_n^{spillover}}{\Sigma_{ZP,n}}$$

$$a_n^{spillover} = a_2 (C_{\text{cost,demand},n} + M^{-1} \rho_{c,\xi,n}) + b_2 (C_{\text{demand},n} + M^{-1} \rho_{\xi,n})$$

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1. Without spillovers, 2SLS numerator goes to zero only if $C_{\text{cost,demand},n}, C_{\text{demand},n} \to 0$.
2. With uniform pricing, both estimators are affected identically in the numerator
3. With spillovers, if $C_{\text{cost,demand},n}, C_{\text{demand},n} \to 0$, α_n^{2SLS} is consistent only if $M \to \infty$.

REWRITTEN ESTIMATORS (DENOMINATORS)

$$
\begin{aligned}
\Sigma_{P,n} &= a_1^2 \rho_{c,n} + b_1^2 \rho_{\xi,n} + 2a_1 b_1 \rho_{c,\xi,n} \\
&\quad + \Sigma_{\text{spillover}} \\
\Sigma_{ZP,n} &= a_1^2 C_{\text{cost},n} + b_1^2 C_{\text{demand},n} + 2a_1 b_1 C_{\text{cost,demand},n} \\
&\quad + \Sigma_{\text{spillover}} \\
\Sigma_{\text{spillover}} &= a_2^2 (C_{\text{cost},n} + M^{-1} \rho_{c,n}) + 2a_2 b_2 (C_{\text{cost,demand},n} + M^{-1} \rho_{c,\xi,n}) + b_2^2 (C_{\text{demand},n} + M^{-1} \rho_{\xi,n}) \\
&\quad + 2a_1 a_2 (C_{\text{cost},n} + M^{-1} \rho_{c,n}) + 2(a_1 b_2 + 2b_1 a_2)(C_{\text{cost,demand},n} + M^{-1} \rho_{c,\xi,n}) + 2b_1 b_2 (C_{\text{demand},n} +
\end{aligned}
$$

$$
\begin{aligned}
\Sigma_{P,n} &= a_1^2\rho_{c,n} + b_1^2\rho_{\xi,n} + 2a_1b_1\rho_{c,\xi,n} \\
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\Sigma_{ZP,n} &= a_1^2C_{\text{cost},n} + b_1^2C_{\text{demand},n} + 2a_1b_1C_{\text{cost,demand},n} \\
&\quad + \Sigma_{\text{spillover}} \\
\Sigma_{\text{spillover}} &= a_2^2(C_{\text{cost},n} + M^{-1}\rho_{c,n}) + 2a_2b_2(C_{\text{cost,demand},n} + M^{-1}\rho_{c,\xi,n}) + b_2^2(C_{\text{demand},n} + M^{-1}\rho_{\xi,n}) \\
&\quad + 2a_1a_2(C_{\text{cost},n} + M^{-1}\rho_{c,n}) + 2(a_1b_2 + 2b_1a_2)(C_{\text{cost,demand},n} + M^{-1}\rho_{c,\xi,n}) + 2b_1b_2(C_{\text{demand},n} +
\end{aligned}
$$

1. Uniform pricing (nearly) identically in denominator
   - Uniform pricing $\rightarrow$ time average of price $\rightarrow$ Hausman $\approx$ time average of price
2. If $C_{\text{cost,demand},n}, C_{\text{demand},n} \rightarrow 0$, and no uniform pricing, then need $|C_{\text{cost},n}| > 0$

- Bresnahan critique holds: without uniform pricing or spillovers, the Hausman instrument is valid if $C_{\text{cost,demand},n}, C_{\text{demand},n} \to 0$ and $C_{\text{cost},n} \neq 0$.
    - Aggregate cost shock, no national demand shocks, no cov. of costs and demand across markets

- With uniform pricing or spillovers, the Hausman instrument is valid if $C_{\text{cost},n} \neq 0$, $C_{\text{cost,demand},n}, C_{\text{demand},n} \to 0$ and $M \to \infty$.
    - Individual markets need to be very small to not contaminate the instrument
    - However, this is almost identical to running the time series version of OLS (what does that imply about the estimates?)

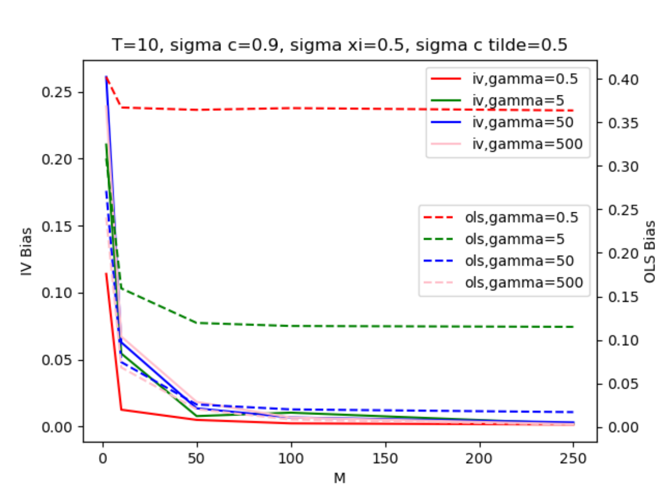
- What if $C_{\text{cost},n} = 0$? (No aggregate cost shocks?)

$$\alpha_n^{2SLS} = \frac{a_2 \rho_{\xi,c,n} + b_2 \rho_{\xi,n}}{(a_2^2 + 2a_1 a_2)\rho_{c,n} + (b_2^2 + 2b_1 b_2)\rho_{\xi,n} + (2a_1 b_2 + 2b_1 a_2 + 2a_2 b_2)\rho_{c,\xi,n}}$$

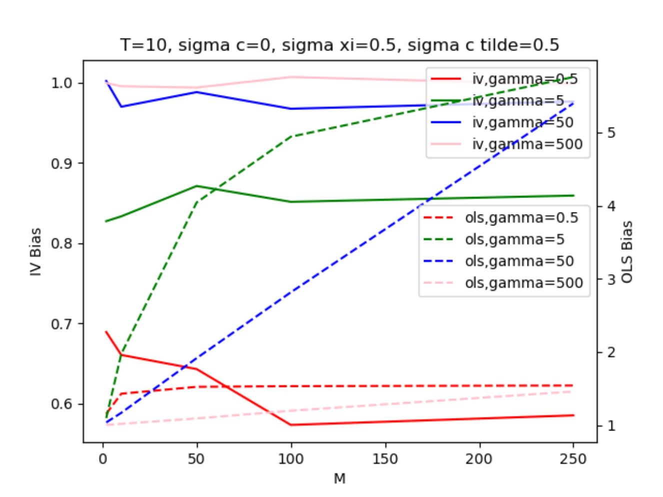
- Without an aggregate shock in costs, the 2SLS is no longer consistent
- The bias will shrink as idiosyncratic cost shocks become a larger share of local prices
    - True with and without uniform pricing / cost spillovers

- Focusing on own-market vs. all other markets is a special case
    - General problem: which structural shocks and markets are uncorrelated?
- Thought experiment: we have uniform pricing, and there are *some* locations that have large idiosyncratic shocks
    - Shock needs to be large enough to move average price (spillover mechanism)
    - In progress to operatioanlize, but likely requires heterogeneity in sizes
- 2SLS with Hausman is (nearly) identical to jackknife IV with time fixed effects
    - Implications for inference and estimation?

- Verify results in Monte Carlo simulations
    - True demand elasticity of $\alpha = 0.9$
    - $c_{mt} = c_t + \tilde{c}_{mt}$.
    - $\tilde{c}_{mt} \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma^2_{\tilde{c}})$ market-specific cost shocks
    - $c_t \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma^2_c)$ are national cost shocks affecting all markets.
    - $\xi_{mt} \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma^2_\xi)$ market-specific demand shocks
- Consider various experiments:
    - $\gamma \in \{0.5,\ 5,\ 50,\ 500\}$
    - $\sigma_\xi \in \{0.5,\ 1,\ 1.5,\ 5\}$
    - $\sigma_c \in \{0.9, 0\}$
    - M and T selected from the set $\{2, 10, 50, 100, 250\}$

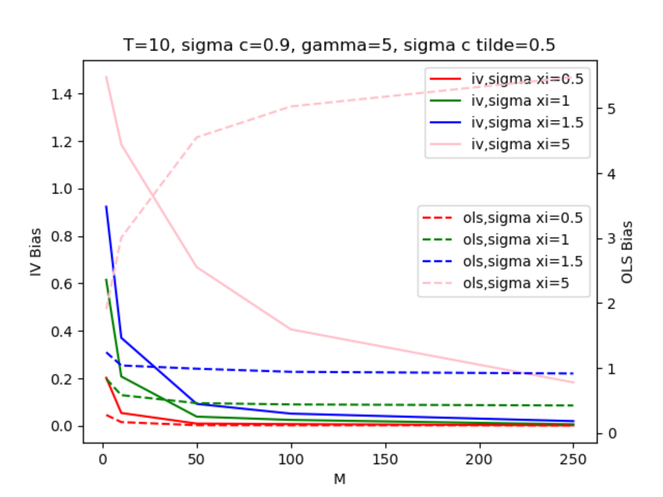


- As M grows, 2SLS bias to zero if aggregate shock
- As γ grows, OLS bias is closer to zero



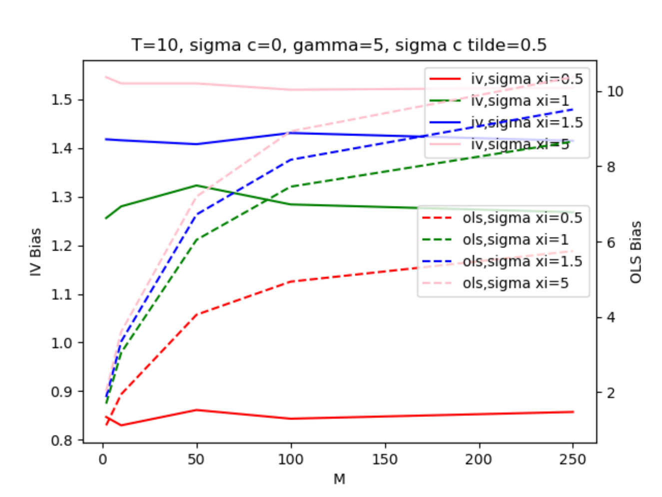
- As M grows, 2SLS bias to zero if aggregate shock
- As γ grows, OLS bias is closer to zero
- Both estimators biased without aggregate shock

SPILLOVERS, INCREASING DEMAND SHOCK BIAS σ_ξ, WITH AND WITHOUT AGGREGATE SHOCK



- M needs to be much larger when σ_ξ is large
- Relative size of cost shock vs. bias terms

SPILLOVERS, INCREASING DEMAND SHOCK BIAS σ_ξ, WITH AND WITHOUT AGGREGATE SHOCK



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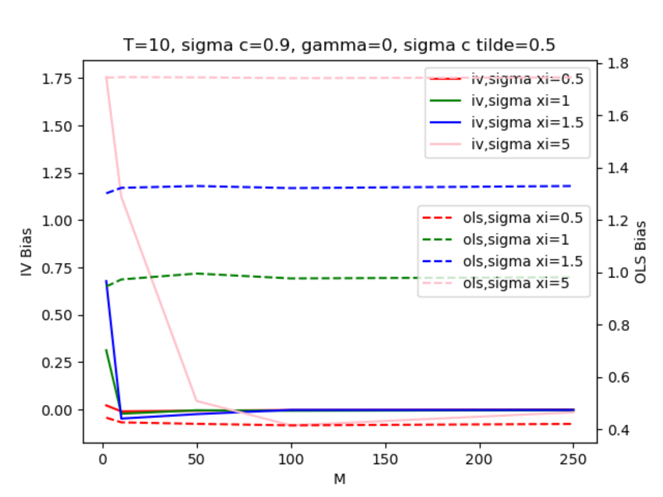
NO SPILLOVERS, INCREASING DEMAND SHOCK BIAS σ_ξ, WITH AND WITHOUT AGGREGATE SHOCK



- Without spillovers, IV works well with an aggregate cost shock.
- Without an aggregate cost shock, close to OLS

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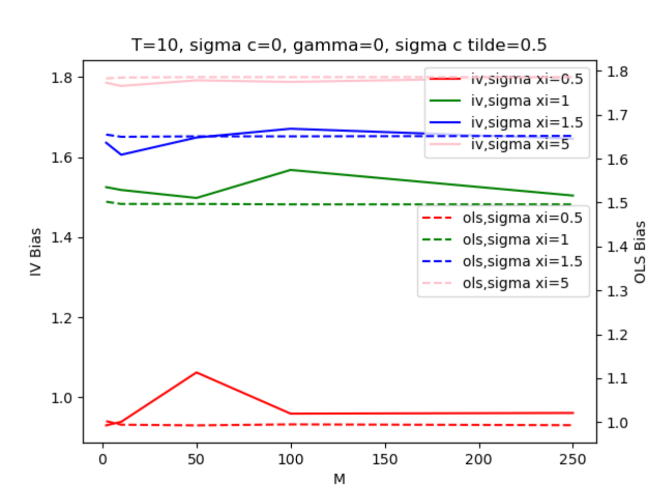

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- Formalizing some folk wisdom about Hausman instruments
    - Aggregate cost shocks in the absence of any other aggregate demand shocks
- Uniform pricing creates important wrinkle. Not necessarily a mechanism for IV
    - Hausman may still "work" in these settings, but aggregate cost shock main driver
- Key point: identification based on two features:
    1. the equilibrium quantity and price are a function of the competitive environment (e.g. monopolistic competition vs. oligopoly), the cost structure, *and* the demand system parameterization.
    2. the statistical properties of the unobserved components in these models will play an important role in the identification of the demand elasticity.

Researchers using Hausman instruments need to understand underlying economic assumption when using averages as proxy for unobserved cost shocks

- What about in general settings (e.g. not linear models)?
- How does this affect inference? Aggregate cost shocks are source of random variation – how to think about standard errors?
    - What about JIVE analogy?
- How does heterogeneity in demand elasticities affect estimation?
- Others?

Thank you! paulgp@gmail.com

A monopolist chooses the prices in the markets to maximize its overall profit:

$$\max_{P_1,\ldots,P_M} \quad \sum_m P_m Q_m(P_m) - C(Q_1, \ldots, Q_M)$$

which leads to FOC

$$P_m^* = C_m'(Q_1, \ldots, Q_M) - \frac{Q_m(P_m^*)}{Q_m'(P_m^*)},$$

where C_m' denotes the first-order derivative with respect to the price in market m.

- In general, if the marginal cost function depends on the sales in all markets, we expect the optimal price in market m to depend on *all* unobserved demand shocks across markets, ξ_m.

- With a separable cost function, the monopolist chooses an optimal price in each market

$$\max_{P_m} \quad P_m Q_m(P_m) - C_m(Q_m(P_m))$$

which leads to FOC $P_m^* = C_m'(Q_m(P_m^*)) - \frac{Q_m(P_m^*)}{Q_m'(P_m^*)}$.

  - Note that the optimal price depends on only the focal market.

- Often assume that $C_m(Q_m) = C(Q_m, c_m)$, where c_m represents unobserved cost shifters.
- Hausman instruments are valid if c_m are correlated and ξ_m are independent. In fact, researchers most often assume $C_m(Q_m) = c + c_m$.