Lecture 5: Evidence from the CAPM and the APT (and related models)

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Tests of the CAPM

- To test the validity of a model, we need to test its predictions
- CAPM predicts
  1. A linear relationship between expected excess returns and beta
  2. No other variable has marginal explanatory power/alpha is zero
  3. The risk premium for Beta is positive and equal to the market risk premium
- These predictions can be tested in various ways

Time series tests

- Jensen's alpha: from the CAPM regression,

$$r_{i,t} - r_f = \alpha_i + \beta_i \left(r_{m,t} - r_f \right) + \epsilon_{i,t}$$

- Prediction: α_i should be jointly zero, i.e. $\alpha_1 = 0, \alpha_2 = 0, \ldots$
- Historical tests have not typically been favorable towards the CAPM
  - Reject that alphas are jointly zero (p=0.02) (Campbell, Lo, Mackinlay 1997)

Time series tests

- Importance of time-series tests, however, is in their interpretation
- Can show joint tests of alpha are equivalent to the following:

1. Find the Sharpe ratio of the market portfolio and compare it to the portfolio with the highest realized Sharpe ratio over a given period

2. Time series tests provide statistical comparison of the predicted MVE (the market) and the actual MVE portfolio

Time series tests

- Said otherwise, non-zero alphas suggest the market portfolio lags the realized maximum Sharpe ratio portfolios by more than the CAPM would suggest
- Generally, it is useful to think about CAPM tests/factor model tests as figuring out if the market/factor portfolios are “efficient”
    - Big question: which stocks should we overweight in the market portfolio to make it more efficient?

Cross-sectional tests

- Cross-sectional tests have been somewhat more favorable
- Rather than regressing returns on returns, we now regress returns on betas
  - Hope to find that betas and returns line up as predicted by security market line
- Fama-Macbeth (1973) provides the standard framework for these tests

Fama-Macbeth Cookbook

Two-step procedure:

1. Run time-series regressions to estimate beta for all stocks

$$r_{i,t} - r_f = \alpha_i + \beta_i(r_{m,t} - r_f) + \epsilon_{i,t}$$

2. Run cross-section regression of average excess returns $r_i - r_f$ on estimated betas

$$r_i - r_f = \lambda_0 + \lambda_1 \hat{\beta}_i + u_i$$

Prediction: $\lambda_0 = 0$ and $\lambda_1 = E(r_m - r_f)$

Fama-Macbeth Cookbook

- However, note that beta estimates are noisy
  - Regressing any variable on a noisy proxy will flatten the slope coefficient
- Why?
  - Imagine noise is added to x so that you observe x+e
  - True OLS coefficient is $\lambda_1 = \frac{Cov(x,y)}{Var(x)}$
  - Estimated OLS coefficient is $\lambda_1 = \frac{Cov(x+\epsilon,y)}{Var(x+\epsilon)} = \frac{Cov(x,y)}{Var(x)+Var(\epsilon)}$
- So, we expect any regression estimate of the security market line (SML) to be too flat.

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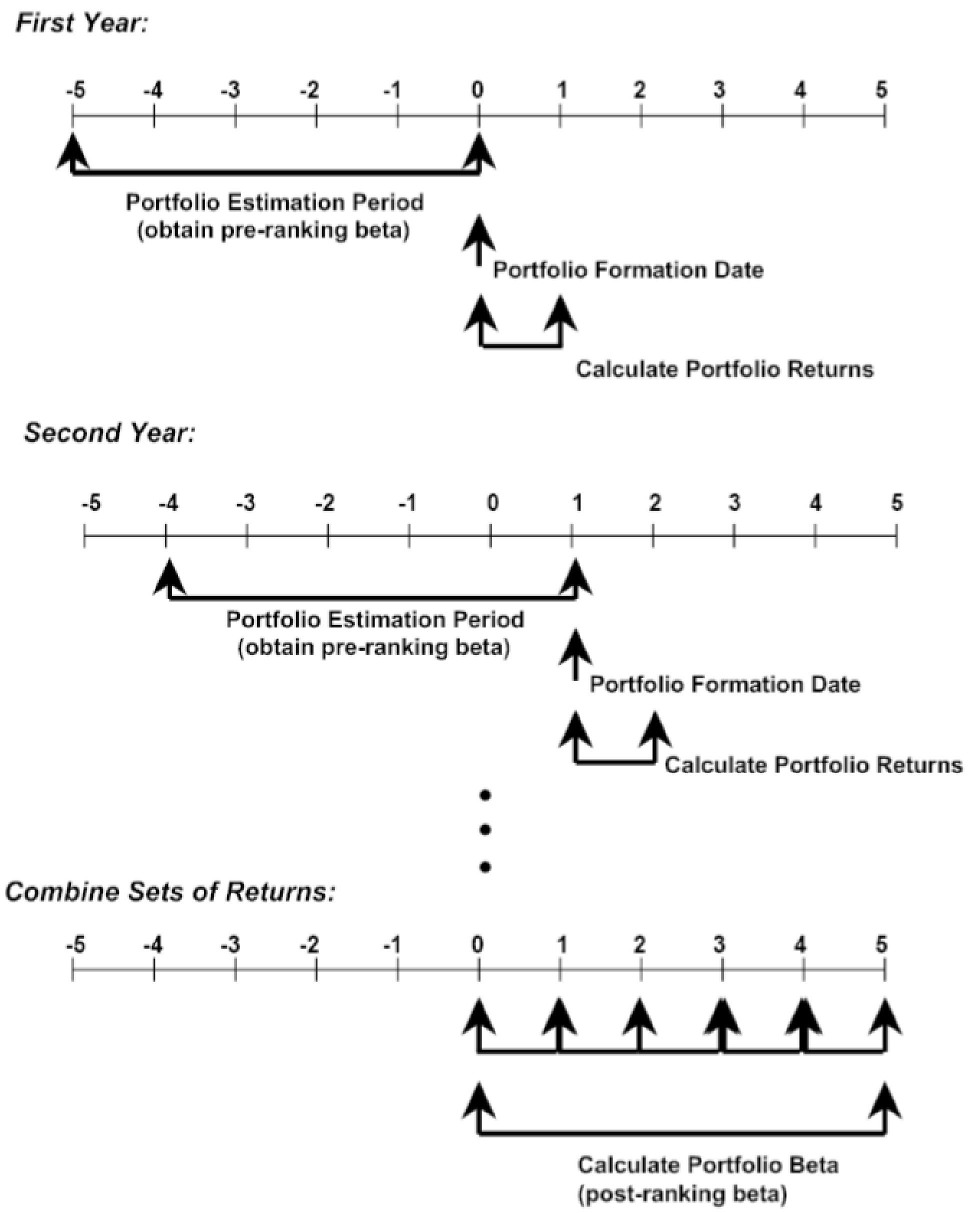
Fama-Macbeth Cookbook

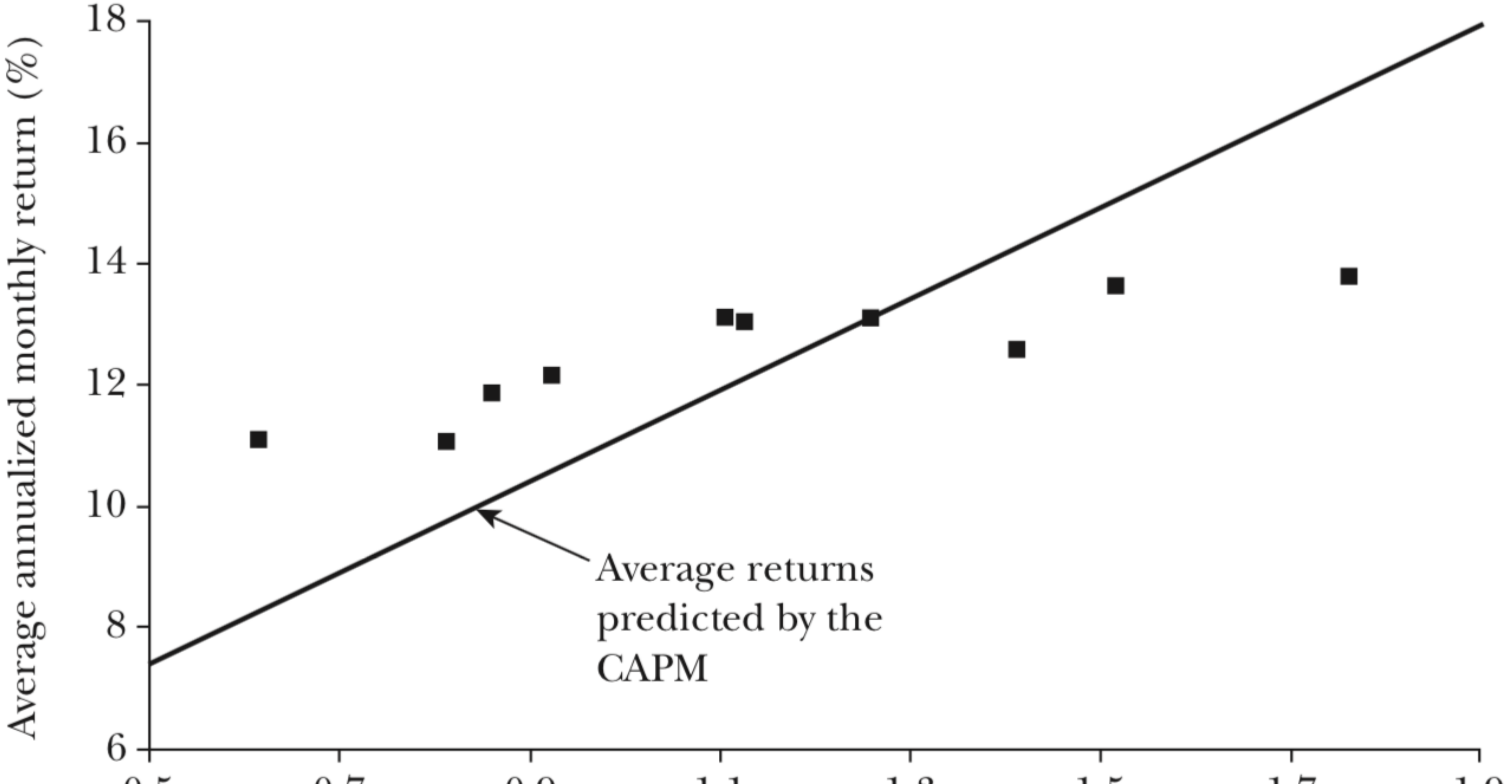
- In response, we form portfolios of stocks and hope that idiosyncratic noise in beta estimates disappears
  - Why might this help?
- Portfolios are formed based on firm betas
  - Why not random assignment?
- Actual procedure:
  1. Each year, calculate betas for all firms (past five years data – 60 months)
  2. Form 10 decile portfolios based on estimated betas
  3. Calculate realized portfolio returns and betas for 10 portfolios


First Year:
-5 -4 -3 -2 -1 0 1 2 3 4 5
Portfolio Estimation Period
(obtain pre-ranking beta)
Portfolio Formation Date
Calculate Portfolio Returns
Second Year:
-5 -4 -3 -2 -1 0 1 2 3 4 5
Portfolio Estimation Period
(obtain pre-ranking beta)
Portfolio Formation Date
Calculate Portfolio Returns
Combine Sets of Returns:
-5 -4 -3 -2 -1 0 1 2 3 4 5
Calculate Portfolio Beta
(post-ranking beta)

Results

- Evidence is suggestive, but not completely consistent with CAPM
- Positive relation between beta and portfolio returns, but fitted line too flat (Fama, French, JEP 2004)

Average Annualized Monthly Return versus Beta for Value Weight Portfolios Formed on Prior Beta, 1928–2003



Other predictions

- CAPM predicts no other measures of risk will predict cross-sectional returns
- In particular, CAPM says only covariance risk matters
  - What about idiosyncratic risk?
  - Fama-Macbeth (1973) control for idiosyncratic risk by including residual variances from firm time-series regressions in second-stage regression
  - Also beta squared

$$r_i - r_f = \lambda_0 + \lambda_1 \widehat{\beta}_i + \lambda_2 \widehat{\beta}_i^2 + \lambda_3 \widehat{\sigma}_e^2 + e_i$$

- They find that only β seems to matter – it's a linear relationship

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The search for anomalies begins

- However, we can go beyond beta-squared and residual variation to predict returns...
- For example:
  1. Earnings-to-price ratio $\longrightarrow$ high returns (Basu, 1977)
  2. Market cap $\longrightarrow$ low returns (Banz, 1981)
  3. Leverage $\longrightarrow$ high returns (Bhandari, 1988)
  4. Book-to-market $\longrightarrow$ high returns (Statman, 1980)
- All turn out to have predictive power over beta, in particular, size (market cap) and book-to-market
- So which wins in a race, beta or size?

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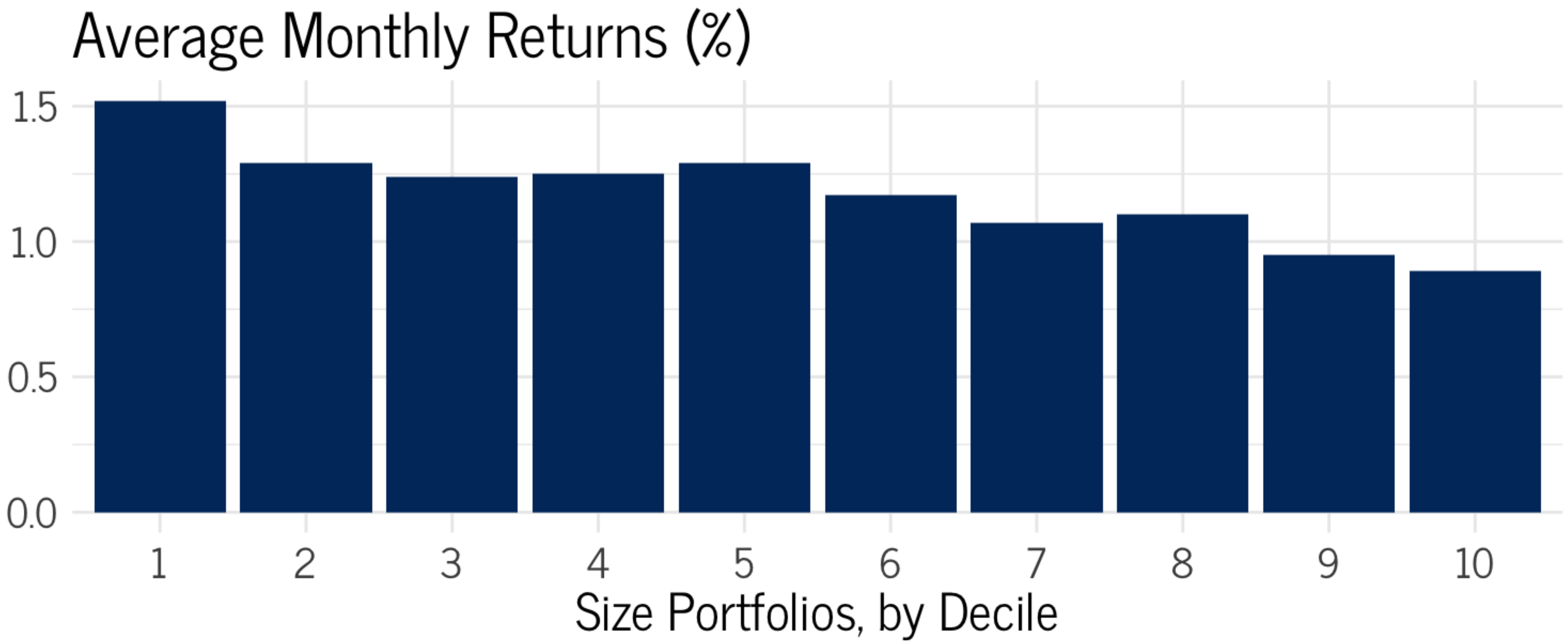
The search for anomalies begins

- Fama-French (1992) test this by creating double sorted portfolios
  - First, sort firms on size; then, sort on beta (cov with market)
  - Can do the same with book-to-market
- Set up a horse-race between beta and the two other factors
- If beta is a good predictor, it should predict even with a bin of similarly sized firms

Average Returns, Post-Ranking Betas and Average Size on Portfolios (Fama-French 1992)



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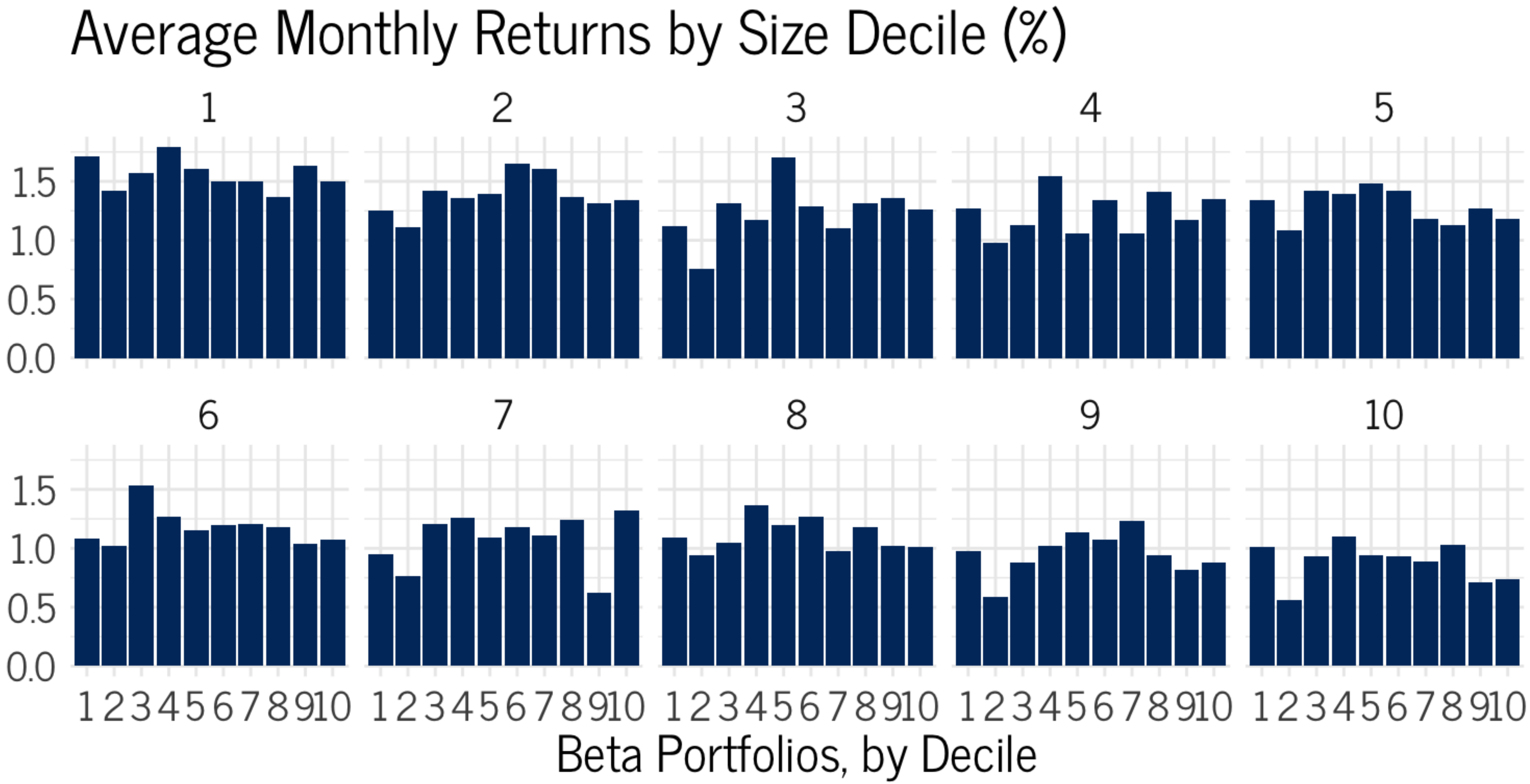

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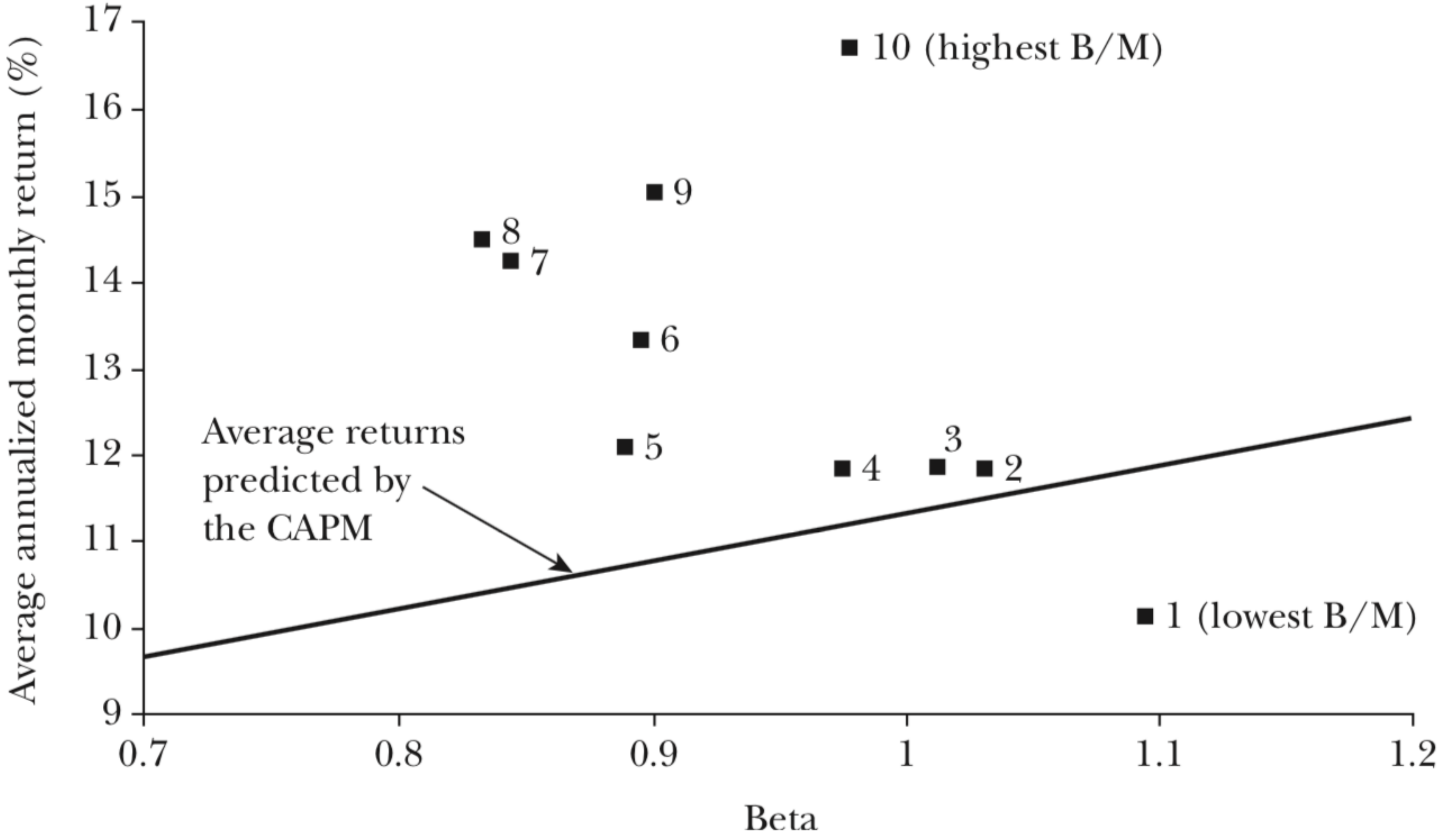
Average Annualized Monthly Return versus Beta for Value Weight Portfolios Formed on B/M, 1963–2003



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Is Beta is dead?

- 30 years after its birth, hard to say that CAPM isn't dead
- In reality, however, hard to say if CAPM or tests of the CAPM are flawed
- Roll's critique:
  - Tests of the CAPM are infeasible because the market portfolio is unobservable
  - Tests of CAPM are only tests of the efficiency of the market proxy used

Factor Models

- In spite of being largely credited with the temporary demise of the CAPM, Fama-French argue we need more flexible market proxies
- They advocate multiple factor models that capture the spirit of the CAPM
  - i.e. expected returns dictated by exposure to non- diversifiable risk
  - Size and book-to-market are not “characteristics” but proxies for economic risk factors

Fama-French 3 factor model

- Create factor mimicking portfolios
  - HML (returns from high B/M stocks less returns from low B/M stocks)
  - SMB (returns from high market cap less returns from low market cap stocks)

  - Data available here:
    http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
- Sort firms into portfolios based on size and value
  - Estimate the following regression for different portfolios

$$r_i - r_f = \alpha + b_i \times (r_m - r_f) + s_i \times SMB + h_i \times HML + \epsilon_i$$

- to test

$$E(r_i - r_f) = b_i \times E(r_m - r_f) + s_i \times E(SMB) + h_i \times E(HML)$$

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Fama-French 3 factor model

$$E(r_i - r_f) = b_i \times E(r_m - r_f) + s_i \times E(SMB) + h_i \times E(HML)$$

	BE/ME	Size	Ex Ret	a	b	s	h	t(a)	t(b)	t(s)	t(h)	R^2
						7/29-6/97						
S/L	0.55	22.39	0.61	-0.42	1.06	1.39	0.09	-4.34	30.78	19.23	1.73	0.91
S/M	1.11	22.15	1.05	-0.01	0.97	1.16	0.37	-0.18	53.55	19.49	9.96	0.96
S/H	2.83	19.05	1.24	-0.03	1.03	1.12	0.77	-0.73	67.32	39.21	26.97	0.98
M/L	0.53	55.85	0.70	-0.06	1.04	0.59	-0.12	-1.29	55.83	18.01	-4.30	0.96
M/M	1.07	55.06	0.95	-0.01	1.05	0.47	0.34	-0.15	32.98	17.50	9.50	0.96
M/H	2.18	53.21	1.13	-0.04	1.08	0.53	0.73	-0.90	47.85	8.99	11.12	0.97
B/L	0.43	94.65	0.58	0.02	1.02	-0.10	-0.23	0.88	148.09	-6.88	-13.52	0.98
B/M	1.04	92.06	0.72	-0.09	1.01	-0.14	0.34	-1.76	61.61	-4.96	13.66	0.95
B/H	1.87	89.53	1.00	-0.09	1.06	-0.07	0.84	-1.40	52.12	-0.86	21.02	0.93

Fama-French 3 factor model

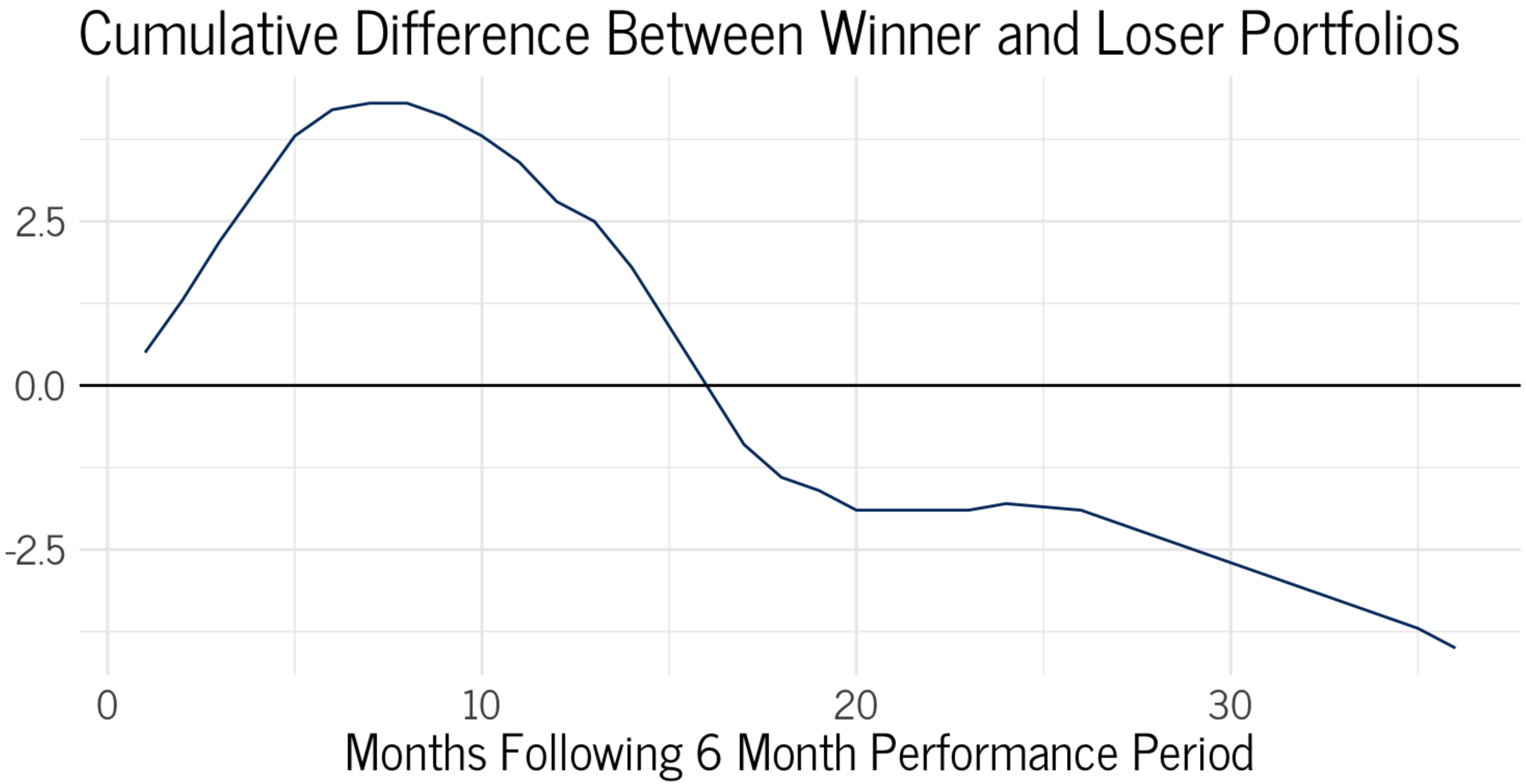
- Claim: Size and value premia reflect exposure to risk captured in SMB and HML
- High returns which are not associated with risk factors should be arbitraged away
- Alphas of size and book-to-market portfolios jointly zero, once we control for SMB and HML risk factors
- This ensures the model is closer to a CAPM/APT story, but is source of some debate

More factors (?!): Momentum

- Often times a fourth factor – momentum – is added to the portfolio
- Based on results that suggest that a strategy of buying winners and selling losers can earn a significant premium over a buy-and-hold strategy

- Note: again, we have taken a firm characteristic (recent success), made a portfolio out of it, and called it a “risk-factor”
  - Is this reasonable?

Momentum Returns



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More factors – Liquidity

- Illiquid stocks tend to offer higher returns
  - Can be measured based on bid-ask spreads
  - CAPM assumes away transaction costs
- Alternatively, we can characterize liquidity as a risk factor
  - Illiquidity of stocks is correlated – "systemic" liqudity
  - Systemic liquidity varies over time
  - Stocks exposed to liuqidity risk need to compensate investors with additional risk premia

More factors—Liquidity

- Pastor and Stambaugh (2002) create a liquidity factor, LIQ
  - LIQ_t is low when order flows have a large impact on prices
- We can add this factor to our 3 factor model:

$$E(r_i - r_f) = \beta_i E(r_m - r_f) + s_i E(SMB) + h_i E(HML) + l_i E(RP_{liq})$$

- Hedge funds sell exposure to liquidity risk
  - As long as you don't need liquitiy when everyone else does, might as well get paid for it!

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Factors: risks or opportunities?

- Note the theme here
  - No shares are over/underpriced (almost)
  - Risk-premia paid on assets represent exposure to risk factors
  - Otherwise, "arbitrageurs" will quickly drive prices to equilibrium "correct" values
  - They need deep pockets!
- No free lunch → high returns = high risk exposure
- Different **kinds** of risk
  - Some institutions/investors prefer certain types of exposures

The Factor Zoo

- We discussed many potential factors
  - Fama French 3 Factors, Momentum, Liquidity
- Why stop at 5?
  - Can continue to capture as many risks as possible!
- Why such a small number of factors?

The Factor Zoo

- The estimation of systemic risk exposure relies on a limited amount of historical data
- Dumping in many historical risk factors that are correlated will give:
  - marginal gains
  - noisy estimates
- When you predict based on those estimates, you will get noisy output
  - Garbage in, garbage out!
- Many approaches to fix this

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Taming the Factor Zoo (Feng, Giglio and Xiu)

- A new approach: use machine learning to identify the “best” factors
- Feng Giglio and Xiu use “double-selection Lasso”, which will identify factors which capture the most important loadings in the cross-section

- They test 15 new contributed factors in the literature, and 4 out of 15 predictive beyond what was already studied in the literature.