Exercise 1

a)

In [2]:
```
from PIL import Image
from IPython.display import display
img = Image.open('EX01_a.jpeg')
display(img,)
```

$E = -\mu B_0$ (1) $\quad \mu = I\gamma$ (2) $\quad r = \frac{n\downarrow}{n\uparrow}$ (3) $\quad I = \pm\frac{1}{2}\hbar$ (4)

from (1) & (2) $\Rightarrow E = -I\gamma B_0$ (5)

from (4) & (5) $\rightarrow I = +\frac{1}{2}\hbar \rightarrow E_\uparrow = -\frac{1}{2}\hbar\gamma B_0$
$\hookrightarrow I = -\frac{1}{2}\hbar \rightarrow E_\downarrow = +\frac{1}{2}\hbar\gamma B_0$ (6)

given $\gamma > 0$ for hydrogen, the low-energy (parallel) state becomes the $E = -\frac{1}{2}\hbar\gamma B_0$ and this is how the signs $\uparrow\downarrow$ were assigned

from $n = n\uparrow + n\downarrow$ and (3):

$n_\downarrow = r\,n\uparrow \rightarrow n = n\uparrow + r\,n\uparrow \rightarrow n = n\uparrow(1+r) \rightarrow n\uparrow = \frac{n}{1+r}$

$\rightarrow n\downarrow = \frac{rn}{1+r}$

Now we count the number of uncancelled number of spins:

$n\uparrow - n\downarrow = \frac{n}{1+r} - \frac{rn}{1+r} = \frac{(1-r)n}{1+r}$

each uncancelled spin contributes $\mu = \frac{1}{2}\hbar\gamma$ so $M_0 = \frac{(1-r)n}{(1+r)} \cdot \frac{\hbar\gamma}{2}$

substituting values:

$r = e^{-\frac{\Delta E}{K_B T}}$
$\rightarrow T = 310\ K$
$\rightarrow K_B = 1{,}381e^{-23}\ J/K$
$\hookrightarrow \Delta E = \frac{1}{2}\hbar\gamma B_0 + \frac{1}{2}\hbar\gamma B_0 = \hbar\gamma B_0$
$\rightarrow \hbar = 1{,}054e^{-34}\ Js$
$\rightarrow B_0 = 1{,}5\ T$
$\hookrightarrow \gamma = 267{,}513e^{6}\ rad/s.T$

$\Delta E = 1{,}054e^{-34} \cdot 1{,}5 \cdot 267{,}513e^{6} \simeq 4{,}23e^{-26}\ J$

$\frac{-\Delta E}{K_B T} = \frac{4{,}23e^{-26}}{1{,}381e^{-23} \cdot 310} \simeq 9{,}89e^{-6}$

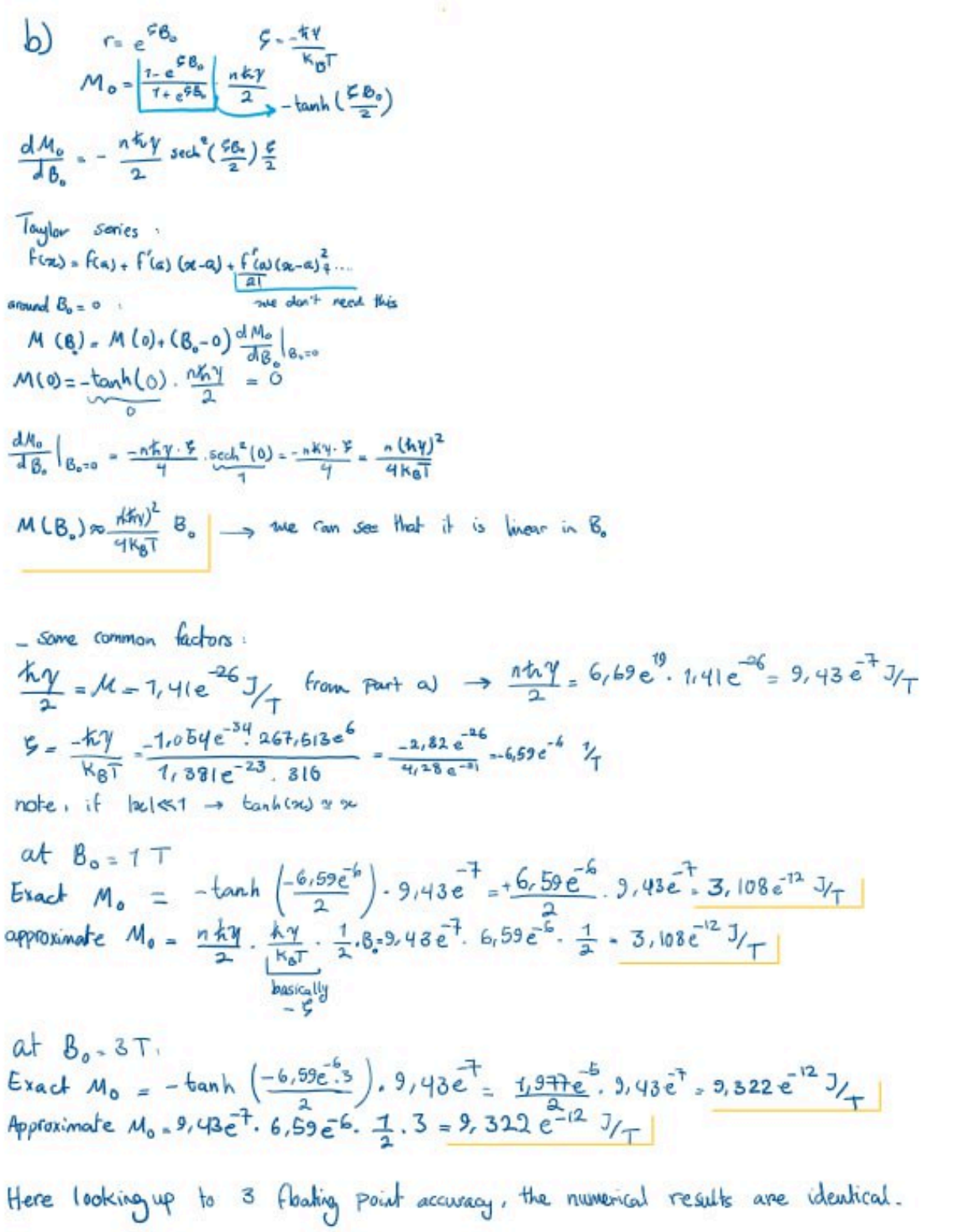
$r = e^{-9{,}89e^{-6}} \simeq 0{,}999 \rightarrow$ I avoid approximation to 1 to avoid multiplication by 0 in M_0.

$\mu = \frac{1}{2}\hbar\gamma = \frac{1}{2} \cdot 1{,}054e^{-34} \cdot 267{,}513e^{6} \simeq 1{,}41e^{-26}\ J/T$

$M_0 = \frac{(1-0{,}999)}{(1+0{,}999)} 6{,}69e^{19} \cdot 1{,}41e^{-26} \simeq 4{,}66e^{-12}\ J/T$

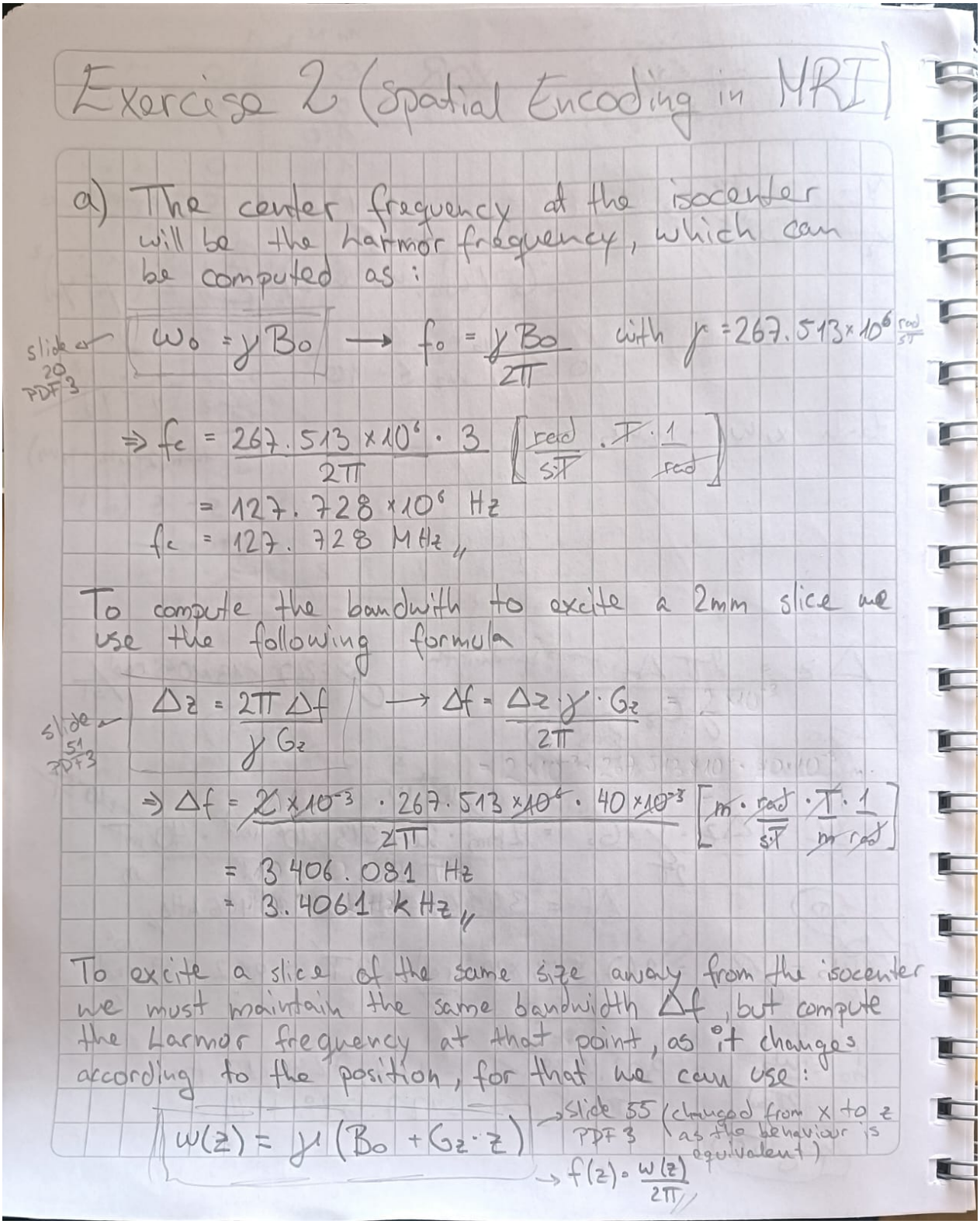
b)

In [5]:
```
from PIL import Image
from IPython.display import display
img = Image.open('EX01_b.jpeg')
display(img,)
```

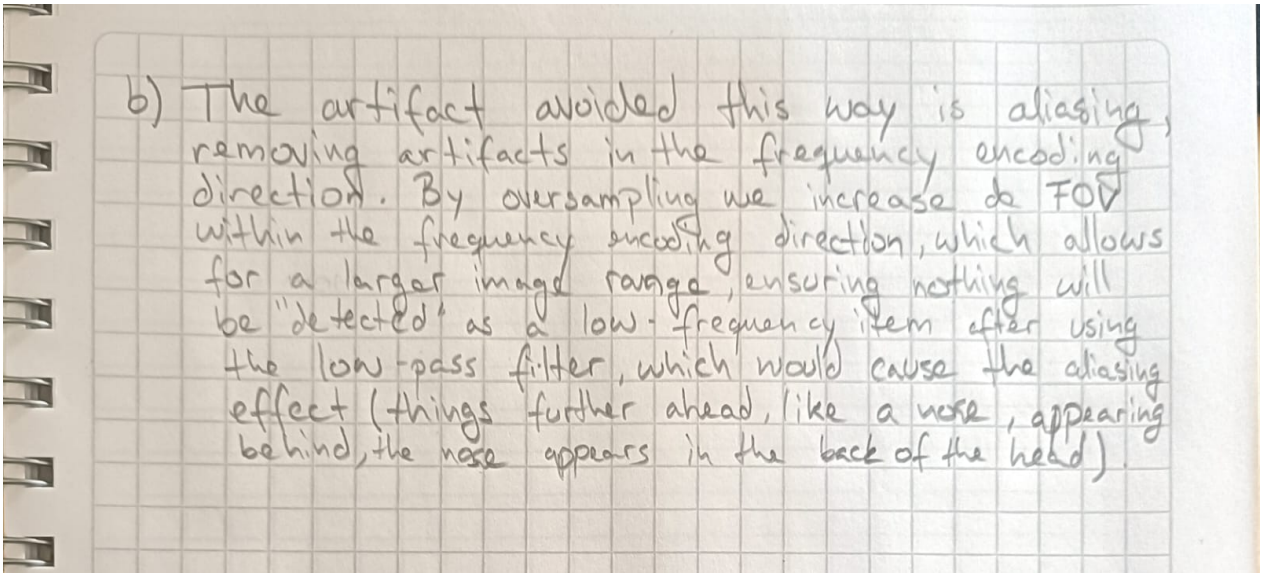


Exercise 2

In [6]:
```
from PIL import Image
from IPython.display import display
img = Image.open('IAAN_A2_Ex2a.jpeg')
display(img,)
```



In [7]:
```
img = Image.open('IAAN_A2_Ex2b.jpeg')
display(img)
```

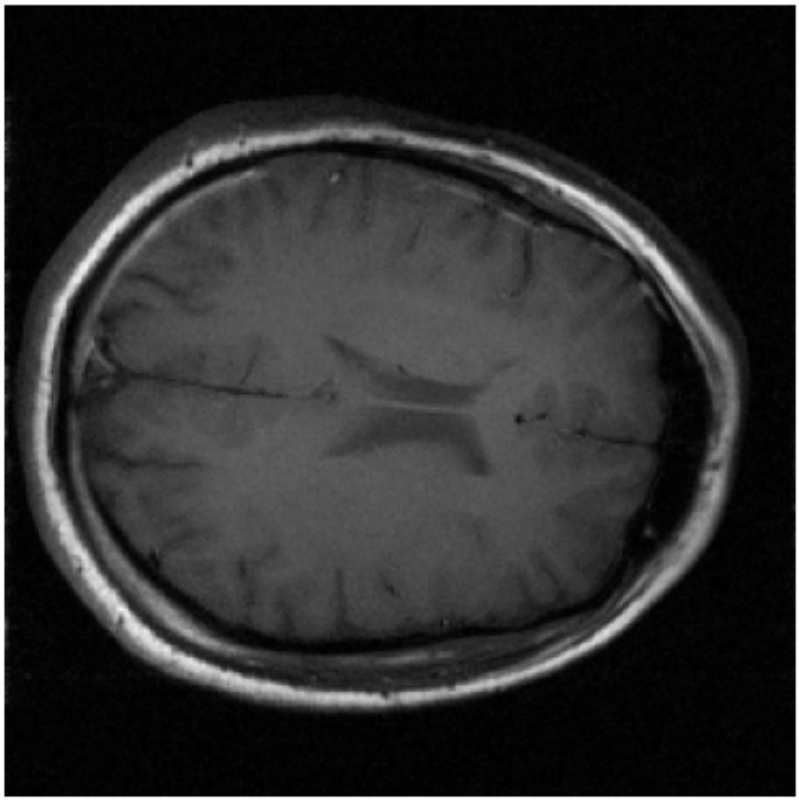


Exercise 3

a)

In [13]:
```python
import numpy as np
import matplotlib.pyplot as plt

# --- 1. Load and Reformat k-space Data ---

# Define the file name
filename = 'mrimage1d.txt'


# Read the text file, flatten it, and 'view' pairs of floats as complex n
kspace_data = np.loadtxt(filename).ravel().view(complex).reshape((256, 25

# --- 2. Reconstruct the Image ---

# The canonical MR reconstruction pipeline:
# 1. np.fft.ifftshift: Undo the "centered" k-space. Moves the center
#    (low frequencies) from (128,128) to the corner (0,0) for the FFT alg
# 2. np.fft.ifft2: Perform the 2D Inverse Fourier Transform.
# 3. np.fft.fftshift: Center the resulting image. Moves the image's
#    center (which is at the corner (0,0) after ifft2) back to (128,128).

img_complex = np.fft.fftshift(np.fft.ifft2(np.fft.ifftshift(kspace_data))

# --- 3. Visualize the Magnitude Image ---

# Get the magnitude (absolute value) of the complex image
img_magnitude = np.abs(img_complex)

# Display the image
plt.imshow(img_magnitude, cmap='gray')
plt.title('Reconstructed Magnitude Image')
plt.axis('off') # Hide the x and y axes
plt.show()
```

Reconstructed Magnitude Image

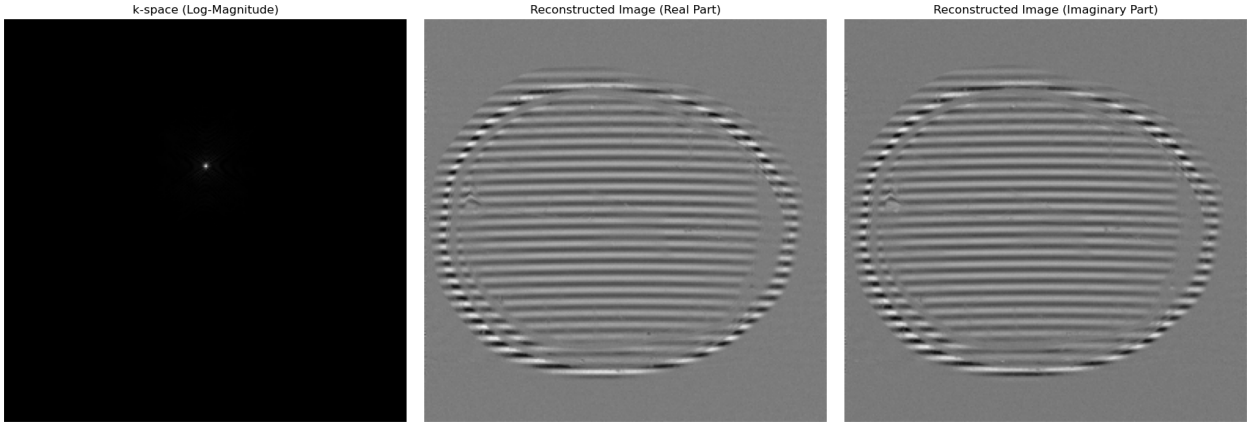


b)

In [14]:

```python
import numpy as np
import matplotlib.pyplot as plt

#2. Prepare Data for Visualization

# k-space: Log-magnitude.
# We use np.log1p(x) which calculates log(1+x)
# This handles x=0 gracefully (log(1)=0) and scales the view.
kspace_log_mag = np.log1p(np.abs(kspace_data))

#Image: Separate real and imaginary parts
img_real = np.real(img_complex)
img_imag = np.imag(img_complex)

#3. Visualize All Three

fig, (ax1, ax2, ax3) = plt.subplots(1, 3, figsize=(18, 6))

# Plot 1: Log-Magnitude k-space
ax1.imshow(kspace_log_mag, cmap='gray')
ax1.set_title('k-space (Log-Magnitude)')
ax1.axis('off')

# Plot 2: Real part of the image
ax2.imshow(img_real, cmap='gray')
ax2.set_title('Reconstructed Image (Real Part)')
ax2.axis('off')

# Plot 3: Imaginary part of the image
```

```
ax3.imshow(img_imag, cmap='gray')
ax3.set_title('Reconstructed Image (Imaginary Part)')
ax3.axis('off')

plt.tight_layout()
plt.savefig('my_reconstruction.png')
plt.show()
```



k-space Visualization (Frequency Domain) Yes, the k-space spectrum is centered as expected. The first plot shows that the highest-magnitude (brightest) values, which represent the low-frequency components of the image, are located in the center of the 256x256 matrix. The magnitudes decay as you move outward toward the high-frequency edges.

Image Visualization (Spatial Domain) Yes, the reconstructed image is real-valued.

Spatial-Frequency Connection This observation is a direct consequence of a fundamental property of the Fourier Transform: A real-valued signal in the spatial domain (the image) must correspond to a frequency-domain signal (k-space) that has conjugate symmetry.

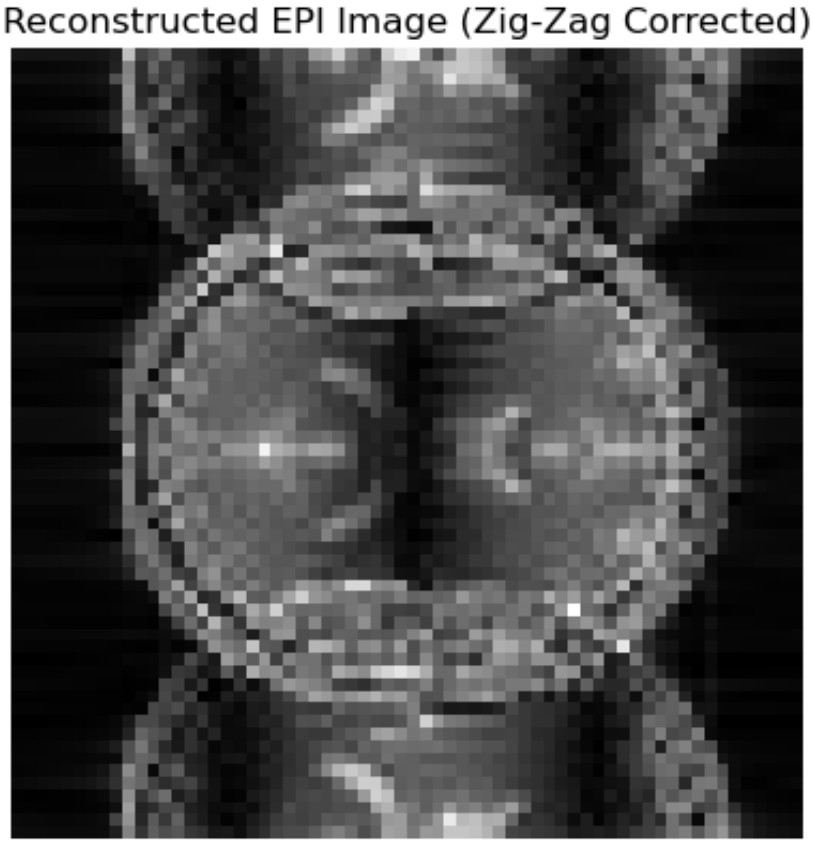
Our k-space log-magnitude plot visually confirms this centrosymmetry (it would look the same if rotated 180 degrees). Because the measured k-space data has this property, the inverse Fourier transform (our reconstruction) results in an image that is real-valued, with all the meaningful information captured in the real component.

C)

In [15]:
```
import numpy as np
import matplotlib.pyplot as plt

filename = 'sepi.npy'

# 1. Load the data
raw_data = np.load(filename)

# 2. Reshape the data
# Reshape the 1D vector of 4096 points into a 2D (64, 64) k-space matrix
kspace = raw_data.reshape((64, 64))
```

```
# 3. Correct for EPI zig-zag
# We apply the correction to a copy to be safe
kspace_corrected = kspace.copy()

# Select every other row (1, 3, 5, ...)
# Reverse the order of elements in that row (::-1)
# Take the complex conjugate
kspace_corrected[1::2] = np.conjugate(kspace_corrected[1::2, ::-1])

# 4. Reconstruct the image
# Standard MR reconstruction pipeline:
# 1. Move k-space center to corner for FFT
# 2. 2D Inverse FFT
# 3. Move image center from corner back to center
img_complex = np.fft.fftshift(np.fft.ifft2(np.fft.ifftshift(kspace_correc

# 5. Visualize the magnitude
img_magnitude = np.abs(img_complex)

plt.imshow(img_magnitude, cmap='gray')
plt.title('Reconstructed EPI Image (Zig-Zag Corrected)')
plt.axis('off')

# Save the image
output_filename = 'epi_reconstruction_zigzag.png'
plt.savefig(output_filename)
print(f"Reconstructed image saved as {output_filename}")
```

```
Reconstructed image saved as epi_reconstruction_zigzag.png
```



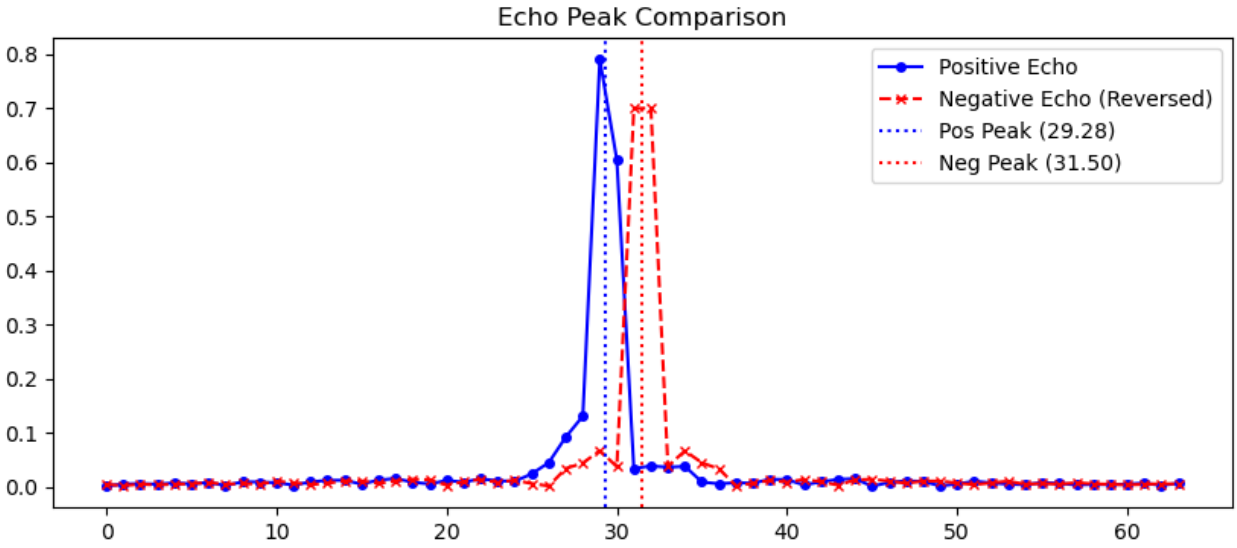
d)

In [16]:

```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.ndimage import shift

def find_subpixel_peak_shift(spc_file):
    print("--- Part 1: Finding Sub-pixel Shift ---")

    # --- 1. Load and process echo data ---
    spc_data = np.load(spc_file)
    # First 64 points are the positive echo
    echo_pos = spc_data[:64]
    # Second 64 points are the negative echo, which is time-reversed
    echo_neg_rev = spc_data[64:][::-1]

    # --- 2. Find sub-pixel peak locations ---
    def get_peak(data):
        """Finds sub-pixel peak by fitting a parabola to the max point."""
        peak_idx = np.argmax(np.abs(data))
        # Get 3 points around the peak for the fit
        y_points = np.abs(data[peak_idx-1 : peak_idx+2])
        # Fit y = ax^2 + bx + c to x = [-1, 0, 1]
        poly = np.polyfit([-1, 0, 1], y_points, 2)
        # The vertex (max) of the parabola is at x = -b / (2a)
        offset = -poly[1] / (2 * poly[0])
        return peak_idx + offset

    peak_pos = get_peak(echo_pos)
    peak_neg = get_peak(echo_neg_rev)

    # The shift to apply to the negative echo to align it with the positi
    pixel_shift = peak_pos - peak_neg

    print(f"Positive echo peak: {peak_pos:.4f}")
    print(f"Negative echo peak: {peak_neg:.4f}")
    print(f"Calculated shift:   {pixel_shift:.4f} samples")

    # --- 3. Plot the echoes ---
    plt.figure(figsize=(10, 4))
    plt.title('Echo Peak Comparison')
    plt.plot(np.abs(echo_pos), 'b-o', label='Positive Echo', markersize=4
    plt.plot(np.abs(echo_neg_rev), 'r--x', label='Negative Echo (Reversed
    # Plot vertical lines at calculated peaks
    plt.axvline(peak_pos, color='b', linestyle=':', label=f'Pos Peak ({pe
    plt.axvline(peak_neg, color='r', linestyle=':', label=f'Neg Peak ({pe
    plt.legend()
    plt.show()

    return pixel_shift

def correct_epi_image(sepi_file, pixel_shift):
    print("\n Part 2: Applying Correction to Image")

    #1. Load and apply initial zig-zag correction
    kspace = np.load(sepi_file).reshape((64, 64))

    #Create a copy for the uncorrected "before" image
    kspace_zigzag_corr = kspace.copy()
```

```
    kspace_zigzag_corr[1::2] = np.conjugate(kspace_zigzag_corr[1::2, ::-1

    #Create a second copy for the "after" image
    kspace_shifted = kspace_zigzag_corr.copy()

    #2. Apply sub-pixel shift correction
    # We apply the calculated shift to the odd-numbered lines (the negati
    # using cubic interpolation (order=3).
    print(f"Applying {pixel_shift:.4f} sample shift to odd k-space lines.
    for i in range(1, 64, 2):
        # We shift the complex data. scipy's shift handles this.
        kspace_shifted[i, :] = shift(kspace_shifted[i, :],
                                     shift=pixel_shift,
                                     order=3,
                                     mode='constant',
                                     cval=0.0)

    #3. Apply phase correction
    # We force the k-space center (DC component) to be real.
    # This removes a global, arbitrary phase offset.
    # The center of our 64x64 *centered* k-space is at (32, 32).
    center_phase = np.angle(kspace_shifted[32, 32])
    print(f"Applying global phase correction of {-center_phase:.4f} radia
    kspace_final = kspace_shifted * np.exp(-1j * center_phase)

    #4. Reconstruct both images for comparison
    # "Before" image (only zig-zag)
    img_before = np.abs(np.fft.fftshift(np.fft.ifft2(np.fft.ifftshift(ksp

    # "After" image (full correction)
    img_after = np.abs(np.fft.fftshift(np.fft.ifft2(np.fft.ifftshift(kspa

    #5. Display results
    fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(12, 6))

    ax1.imshow(img_before, cmap='gray')
    ax1.set_title('Figure 3 (Left) - Zig-Zag Correction Only')
    ax1.axis('off')

    ax2.imshow(img_after, cmap='gray')
    ax2.set_title('Figure 3 (Right) - Full Correction')
    ax2.axis('off')

    plt.tight_layout()
    plt.show()

#Main execution
# Part 1: Find the shift
shift_to_apply = find_subpixel_peak_shift('spc.npy')

# Part 2: Apply the shift
correct_epi_image('sepi.npy', shift_to_apply)
```

```
--- Part 1: Finding Sub-pixel Shift ---
Positive echo peak: 29.2804
Negative echo peak: 31.5014
Calculated shift:   -2.2210 samples
```




```
 Part 2: Applying Correction to Image
Applying -2.2210 sample shift to odd k-space lines...
Applying global phase correction of 2.8420 radians...
```

Figure 3 (Left) - Zig-Zag Correction Only

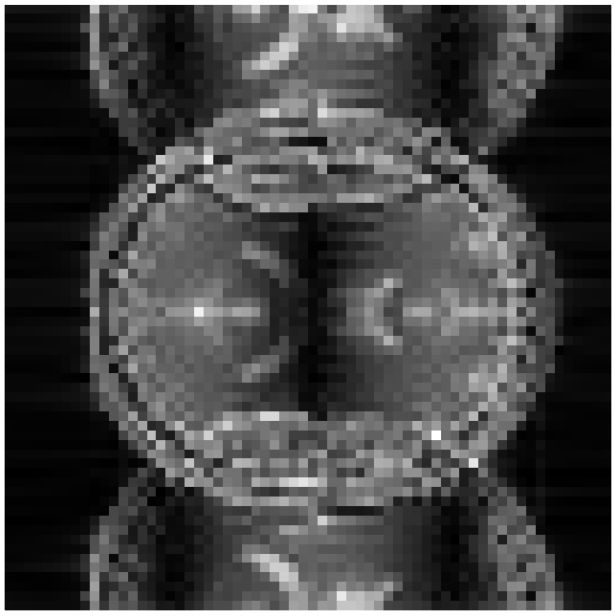


Figure 3 (Right) - Full Correction