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Image Acquisition and Analysis in Neuroscience

Assignment Sheet 1

Solution has to be uploaded by October 27, 2025, 10:00 a.m., via eCampus

If you have questions concerning the exercises, please use the forum on eCampus.

- Please work on this exercise in **small groups** of 3 students. Submit each solution only once, but clearly indicate who contributed to it by forming a team in eCampus. Remember that all team members have to be able to explain all answers.
- Please submit your answers in PDF format, and your scripts as *.py/*.ipynb files. If you are using Jupyter notebook, please also export your scripts and results as PDF, e.g., using your browser's "Print to PDF" feature.
- Admission to the exam requires **at least 75 points** (50% of the overall points from all six sheets) and presenting an answer in class **at least once.**

We will use Python for the practical part of the exercises. You will need the following open source software, which is available for Linux, Windows, and Mac:

- A Python 3 interpreter, and Python packages NumPy, SciPy, matplotlib, and scikit-image.
- Feel free to use other packages you might know and find helpful (e.g., seaborn)
- We recommend using Jupyter notebook, or a suitable integrated development environment.

In case you should require an introduction to Python, please refer to `https://docs.python.org/3/tutorial/`. A useful reference for NumPy is at `https://numpy.org/doc/stable/user/index.html`.

Exercise 1 (Complex numbers and their geometric intuition, *4 Points*)

Perform the following two operations in the polar representation $z = |z|\mathrm{e}^{i\phi}$:

a) Multiplication of two complex numbers: $(1 - i) \cdot (0 + 2i)$
b) Division of two complex numbers: $(0 + i)/(1 - \sqrt{3}i)$

Illustrate the geometric intuition behind these operations by plotting the complex operands and the result of each operation in the complex plane.

Exercise 2 (Fourier transform of two-sided decaying exponentials, *6 Points*)

Please determine the Fourier transform $F(k)$ of the two-sided decaying exponentials function:

$$f(t) = \begin{cases} e^{-t} & \text{for } t \geq 0 \\ e^{t} & \text{for } t < 0 \end{cases} \tag{1}$$

You may use all theorems and results provided in the lecture.

Exercise 3 (Understanding the Discrete Fourier Transform (DFT), *15 Points*)

Similar to the continuous Fourier transform that was treated in the lecture, a Fourier transform for discrete signals $x \in \mathbb{C}^N$ is commonly defined as

$$X_k = \sum_{n=0}^{N-1} e^{-2\pi i k(n/N)} x_n \text{ for } k = 0, \ldots, N-1 \tag{2}$$

with the corresponding inverse DFT

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} e^{2\pi i k(n/N)} X_k. \tag{3}$$

a) Convince yourself that the real and imaginary parts of the DFT basis functions can be interpreted as cosine and sine functions with increasing frequencies by plotting the real and imaginary parts of $e^{2\pi i k t}$ for $k \in \{0, 1, \ldots, N-1\}, N = 5$, and $t \in [0, 1]$. Highlight the points on the graph that correspond to $t = n/N, n \in \{0, 1, \ldots, N-1\}$. (3P)

b) For integer n, derive the equality

$$e^{2\pi i (N-k)\frac{n}{N}} = e^{-2\pi i k \frac{n}{N}} \tag{4}$$

which allows us to re-interpret high positive frequencies $N-k, k < N/2$, as lower negative frequencies $-k$. A negative frequency can be interpreted in terms of time running backwards, i.e., the graph of the resulting function will be mirrored across the y axis compared to the corresponding positive frequency. Verify that intuition by plotting the basis functions for $k \in \{-2, -1, 0, 1, 2\}$. When comparing the highlighted points on the resulting graph with the one from a), you should be able to see that, for all $n \in \{0, 1, \ldots, N-1\}$, we still obtain the same values. (3P)

c) In the DFT as defined by Equation (2), the zero frequency corresponds to the first coefficient X_0, which will be shown on the left side when plotting the spectrum. Many people find it more intuitive to place the zero frequency at the center of the plot, corresponding to the above-described interpretation of the DFT in terms of positive and negative frequencies. Numerical packages provide a function called `fftshift` that re-arranges the X_k accordingly.[1] Try this out by generating a real discrete signal of length $N = 5$, using `scipy` to apply the FFT, and plotting the resulting real and imaginary components with and without the `fftshift`. In each case, label the frequency axis in a meaningful way. You should be able to see the symmetry for real signals that was discussed in Chapter 2, Slide 26. Also find out and state how `fftshift` deals with cases in which N is even. (4P)

d) Write a function that, for any sufficiently large integer N, sets $X_k \in \mathbb{C}^N$ to suitable Fourier coefficients and applies the inverse FFT to obtain a discrete signal that samples one fundamental period of the sine. Reason about the scaling that is required to obtain unit amplitude, about how to achieve a real-valued signal (note that very small imaginary components might still arise due to round-off in the inverse FFT), and ensure the correct phase. Consider if and at what point(s) you need to use `fftshift` and/or its inverse `ifftshift`. Verify your result by plotting the real and imaginary parts of the resulting signal for different values of N, both odd and even. Finally, provide a similar function that generates two periods of the cosine. (5P)

[1] FFT refers to the Fast Fourier Transform, a fast algorithm for computing the DFT.