Wednesday 18th November, 2020
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Exercise Sheet 2

Homework submission deadline: Tuesday 24th November, 2020, 14:00.

Groupwork

Exercise 1.
Let $f : \mathbb{N} \to \mathbb{R}$ and $g : \mathbb{N} \to \mathbb{R}$ be two functions defined as

$$f(n) = 8n^3 + n^2 + 76n \quad \text{and} \quad g(n) = n^3.$$

Please prove or disprove:

1. $f(n) \in \mathcal{O}(g(n))$
2. $f(n) \in \Omega(g(n))$
3. $f(n) \in \Theta(g(n))$

Exercise 2.
Please calculate the running times of the following procedures as a function of n. An elementary arithmetic operation (e.g. addition or comparison) has a constant running time c. Furthermore, determine asymptotic upper bounds.

```
def for_loop(n):
    m = 1
    for i in range(n):
        m += 1
    return m

def nested_for_loops(n):
    m = 0
    for i in range(n):
        for j in range(n):
            m += 1
    return m

def nested_for_loops_2(n):
```

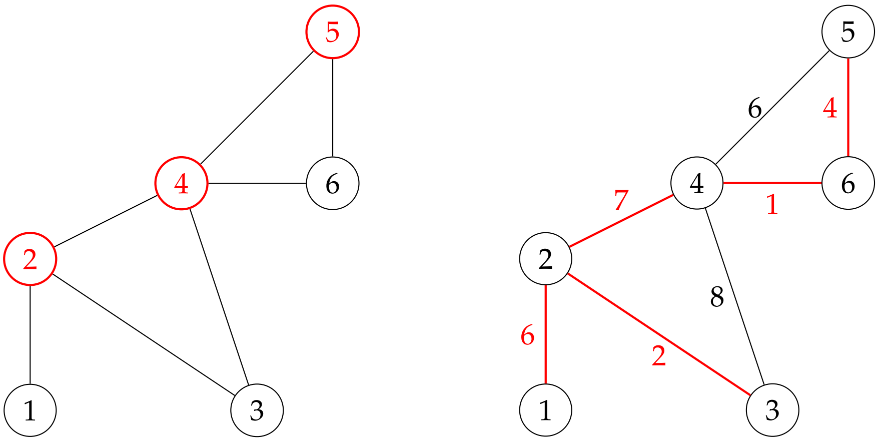
```
    m = 0
    for i in range(n):
        for j in range(i):
            m += 1
    return m

def if_statements(n):
    m = 0
    if n <= 100:
        m = m + 100
    else:
        m = m - 1
    return m

def sequencing_of_statements(n):
    x = 20
    m = 0
    for i in range(n):
        m = m + 10
    for i in range(n):
        for j in range(n):
            m = m + 1
            x = x + 1
    return m, x
```

Exercise 3.

a) Give an $\mathcal{NP}$-algorithm for each of the following decision problems. For the second and third question you can find an example in Figure 1. An $\mathcal{NP}$-algorithm is an algorithm that first uses some black box function that returns a possible solution and then checks if this solution is valid.

- Given a sequence of numbers. Is there a sub-sequence of length k that is sorted in descending order?
- Given a graph $G = (V, E)$. Can you find a vertex cover of size $|vc(G)| \leq k$? A vertex cover $vc(G)$ is a subset of the vertices V of a graph G such that at least one endpoint for each edge $e \in E$ is contained in $vc(G)$.
- Given a graph $G = (V, E)$. Is there a spanning tree T with $weight(T) \leq k$? A spanning tree is a connected subgraph $T = (V, E') \subseteq G = (V, E)$ that spans G, i.e. it contains all vertices, and is minimal in the number of edges. [1] The weight $weight(T)$ is defined as the sum of all edge weights in E'.

[1] There are a lot of definitions for (spanning) trees. Another possibility is for example to consider a spanning tree as an acyclic subset of the edges with maximum size. Furthermore, a tree must always contain $|V| - 1$ edges, where $|V|$ is the number of vertices.



(a) Vertex cover with size $|vc(G)| = 3$. (b) Spanning tree T with $weight(T) = 20$.

Figure 1: Example for vertex covers and spanning trees.

b) For which of these problems do you know that they are in $\mathcal{P}$?

Homework

Exercise 4. 10 P.

Please sort the sequence `ERLANGEN` lexicographically with

a) Merge Sort

b) Quicksort (with rightmost element as pivot)

Homework - Programming

Exercise 5. 10 P.

Complete the code snippets in StudOn to implement

a) Merge Sort,

b) Bubble Sort.

Feel free to create additional function if needed. Bubble Sort [2] is a sorting algorithm that compares elements pairwise from left to right. In each step, two adjacent elements are compared and they are swapped if the left one is greater than the right one. This goes on repetitively until the list is sorted.

[2] A slightly different visualization can be found here: https://www.youtube.com/watch?v=lyZQPjUT5B4