Experiment 32: Charge Oscillations and Energy Transfers in an LRC Circuit

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Abstract

The purpose of this lab is to study the behaviour of an LRC circuit under three different damping conditions. The main interest will be the graphs of potentials and those of energies of the individual components. The potentials will be plotted using LoggerPro and a function generator connecting the LRC circuit. The results of the lab were curves that followed the expected pattern in any of the three damping conditions, and the total energy was shown to be conserved.

1 Introduction

This experiment studies the effects of three types of damping on an LRC circuit. The three damping effects include normal, critical, and over-damping. In an LRC circuit, the conditions for setting up each of these cases depends on the three component quantities of the circuit, which are the Resistance, inductance, and capacitance. The first case that will be studied is the normal damping case where the discharge of the capacitor is aperiodic, and decreases slowly in comparison to the other two cases. The second case is critical damping which is also aperiodic and decreases quicker than normal damping. The last case is over-damping which is periodic and has oscillatory behaviour. The LRC circuit in this experiment will change in reaction to an applied voltage. From the potential curves of the graphs plotted over time, it's possible to observe and compare the behaviour of these systems in these damping conditions. The energies of the capacitor and inductor can also be plotted along with the total energy and heat dissipated in the system to determine the the energy is conserved or is there some energy loss. The experiment will answer questions about what the differences in the potential curves, and what happens to the energy in the capacitor and inductor, and whether or not the energy is conserved.

1.1 Equations

$$(R/2L)^2 - 1/LC > 0 \tag{1}$$

Condition for normal damping case, where R,L,C is the Resistance, inductance, and capacitance, respectively.

$$(R/2L)^2 = 1/LC \tag{2}$$

Condition for critical damping case

$$(R/2L)^2 < 1/LC \tag{3}$$

Condition for over-damping case which creates oscillatory motion

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \tag{4}$$

The natural frequency of the oscillation of the circuit without any damping.

$$jw = \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} \tag{5}$$

The frequency $\omega fornon-vanishingcircuitresistance$.

$$E_L = \frac{1}{2}Li^2 \tag{6}$$

energy of inductor where i is $v_R/R_resistor$

$$E_L = \frac{1}{2}Cv_C^2 \tag{7}$$

energy of capacitor

$$E_H = Ri_{av}^2\Delta t \tag{8}$$

energy dissipated in resistance in a given Δt interval.

$$V_c = V_0(1 + \frac{2}{RC}t)e^{-\frac{R}{2L}t} \tag{9}$$

Theoretical equation for the potential across the capacitor.

$$V_R = -\frac{V_0}{L}te^{-\frac{R}{2L}t} \tag{10}$$

Theoretical equation for the potential across the resistor.

1.2 Sample Calculations

$$\begin{aligned} (R/2L)^2 &= 1/LC \\ R &= 2L\frac{1}{LC} \\ R &= 2(0.96H \pm 0.050H)\sqrt{\frac{1}{(0.96H \pm 0.050H)(10.15 \times 10^{-6}F \pm 2.04 \times 10^{-7}F)}} \\ &= 615.08\Omega \pm 16\Omega \end{aligned} \tag{11}$$

The critical Resistance for case two is calculated

$$\begin{aligned} R_{crit} - R_{fg} - R_L \\ &= 615.08 \pm 16\Omega - 50\Omega - 89.2 \pm 0.449\Omega \\ &= 475.88 \pm 16.18\Omega \end{aligned} \tag{12}$$

The total resistance used was critical resistance minus that of the function generator and inductor.

$$
\begin{aligned}
2R_{crit} - R_{fg} - R_L & \\
&= 2 * (615.08) \pm 16\Omega - 50\Omega - 89.2 \pm 0.449\Omega \\
&= 1090.96 \pm 31.64\Omega
\end{aligned} \tag{13}
$$

The resistance used for case one, which was twice the critical resistance minus that of the function generator and inductor.

$$
\begin{aligned}
R &= R_{res} + R_{fg} + R_L \\
&= 15\Omega + 50\Omega + 89.2 \pm 0.449\Omega \\
&= 154.2 \pm 0.447\Omega
\end{aligned} \tag{14}
$$

The resistance used for case three. A low value for R_{res} of 15Ω was used.

$$
\begin{aligned}
f &= \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \\
&= \frac{1}{2\pi}\sqrt{\frac{1}{(0.96H \pm 0.050H)(10.15 \times 10^{-6}F \pm 2.04 \times 10^{-7}F)}} \\
&= 50.99 \pm 1.33Hz
\end{aligned} \tag{15}
$$

The theoretical natural frequency is calculated, and the experimental frequency is obtained from the period of the E_C curve for case three, and is determined to be 94.83Hz.

$$
\begin{aligned}
jw &= \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} \\
&= \sqrt{\frac{(154.2 \pm 0.447\Omega)^2}{4(0.96 \pm 0.050H)^2} - \frac{1}{(0.96H \pm 0.050H)(10.15 \times 10^{-6}F \pm 2.04 \times 10^{-7}F))}} \\
&= 310.12 \pm 7.75Hz
\end{aligned} \tag{16}
$$

The frequency obtained using equation 5, using the case three resistance value, which will be compared to the determined value of 94.83Hz.

2 Experimental Procedure and Design

The apparatus had four main components that were connected together with wires. The four components are the function generator, decade resistor, inductor, and capacitor. Before starting the experiment, the resistor, inductor, and capacitor were connected and using the function generator set to produce a square wave and a minimum and maximum frequency of 0 and 4 volts, respectively. LoggerPro was also set up to be a continuous reading with a sampling rate of 100/s. The resistor, capacitor, and inductor were connected to a channel on the LabPro unit to measure the potential using LoggerPro. The resistance values for the inductor and function generator were measured. The inductance and capacitance of the inductor and capacitor was also measured.

A point by point procedure can be outlined via:

1. The first part involved the normal damping case where the condition for equation 1 was met. The objective was to measure the voltage over time from the resistor, capacitor, and inductor channels, and plot them on the same figure.

2. The resistance for the resistor was determined using equation 2, solving for R. This value is for critical damping, but the value for R used in normal damping will will twice this value calculated from critical damping minus the resistances from the function generator and inductor.

3. Setting R to this new value on the resistor, the potentials over time for all three channels were collected and plotted on the same figure.

4. The next part involved the critical damping case where equation 2 is relevant. The R value was set to the R value calculated using equation 2 minus the resistance of the function generator and inductor.

5. The plots of potential from all three channels were created using LoggerPro.

6. The final case involved over-damping, where equation 3 is relevant. The resistance of the decade resistor was set to 15 ohms and the total resistance was the sum of the resistor, inductor, and function generator resistance values.

7. The potential over time curves for this case were plotted.

3 Results

The results for this experiment are primarily shown in the plots created. From case 1, plot 1, was the only one created where the potential curves are plotted for the capacitor, inductor, and resistor as a function of time. The plot shows the inductor potential (red line) start very low and quickly approach zero. The resistor (blue line) decreases in potential into the negative and increase to approach zero. The capacitor (green line) simply decreases to zero exponentially. For case two, three graphs were created that can be seen in figures 2, 4, and 6. The first is another potential curves graph which shows all the potential lines for the three components. This graph is similar to the potential curves for case three, but the convergence to zero for all of the lines are much more rapid, which is expected in a critically damped system. The second curve contains the curves of $E_L, E_C, E_T, E_{HT}, E_H, andE_cons$. The curves for this plot are discussed more in the discussion section. The final graph for case two is the curves for theoretical and experimental potential curves plotted on the same graph for comparison purposes. The theoretical curves were determined using equations 9 and 10. These curves are also discussed more in the discussion section. For case three, there were two plots that were plotted as seen in figures 3, and 5. The first graph is of the potential curves. The potential curves for over-damping is different from the previous two cases. All three of the curves show oscillatory behaviour and converge to zero as time progresses. The inductor and capacitor curves seem to start at the opposite ends of potential values and stay symmetric to some degree. The resistor on the other hand oscillates, but stays close to zero. The final curves for case three is another plot of the $E_L, E_C, E_T, E_{HT}, E_H, and E_c ons$ curves. These will be discussed more in the discussion section.

4 Discussion

4.1 Questions

Q1) What is the relationship between E_L and E_C? The relationship between these two can be seen in figure 4. The capacitor seems to discharge rapidly, while the inductor energy seems to increase slightly then decrease again. Both reaching zero at about the same time. As the capacitor is discharging the inductor responds since its energy increases. Q2) How do the E_T and E_{HT} curves relate to one another? Comment on implications of $E_c ons$. As the total energy decreases to zero, the heat dissipated increases until some total amount is reached. This makes sense since the energy total is converted to heat energy because energy must be conserved. The curve of $E_c ons$ shows the sum of the two curves and is always at the total value of energy. For case 2 concerning critical damping, the plots of the experimental and theoretical curves for the potentials across the capacitor and resistor were plotted as seen in figure 6. The V_0 value used in equations 9 and 10 was 4 volts, thus effecting the curves slightly. However, the curves for both capacitor and resistor potentials match their theoretical equivalent, with a higher amplitude perhaps. The general shape does seem to match. Compared to the curves for case 1 potentials of capacitor and resistor, the general shape of both curves match. See figures 1 and 6 for the comparisons. Q3) What is the relationship between E_L and E_C as well as E_T and E_{HT} in case three? The energy for the capacitor rapidly decreases to zero and rises and drops again for a total of two times before staying at zero, in other words it displays oscillatory behaviour as it's discharging. The inductor energy quickly rises and plateau's for a very small amount of time before decreasing to zero and it also displays oscillatory behaviour. The curves for the total energy and energy dissipated through heat are decreasing and increasing, respectively. This would be expected as the energy must be conserved. As E_{HT} is increasing there are two intervals where the energy plateau's slightly, similarly for E_T. The reason for this might have to do with the nature of over-damping, where the oscillatory behaviour causes E_T and E_{HT} to have small plateaus. The $E_c ons$ curve seems to match with the E_{HT} curve as time increases. Given this, there seems to be no energy loss that can be seen from the graph. See figure 5. The experimental natural frequency for case three was determined to be 94.83Hz from the period of E_C curve. Equations 4 and 5 were used to obtain theoretical frequencies to compare to this value. The values obtained from these equations were $50.99 \pm 1.3Hz$ and $310.12 \pm 7.75Hz$, respectively. From these values, the first result is more consistent with the observed frequency. From this, it can be concluded that the resistance for case three is very small and the frequency of oscillation is close to that of a circuit that's absent in damping.

5 Conclusion

The purpose of this experiment was to study the behaviour of an LRC circuit in three different damping scenarios. The potential curves behaved as would be expected in their respective damping conditions. The energy curves also behaved as expected since the total energy of the system was conserved.

A Figures and Graphs

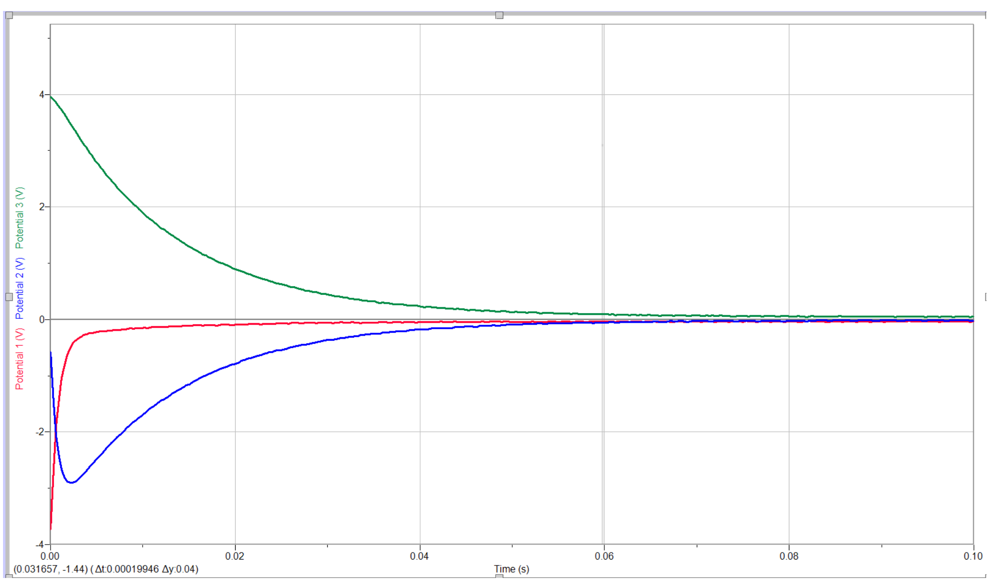

Figure 1: The potential curves for case 1 of normal damping, where the potentials 1, 2, 3 are for the inductor, decade resistor, and capacitor, respectively.

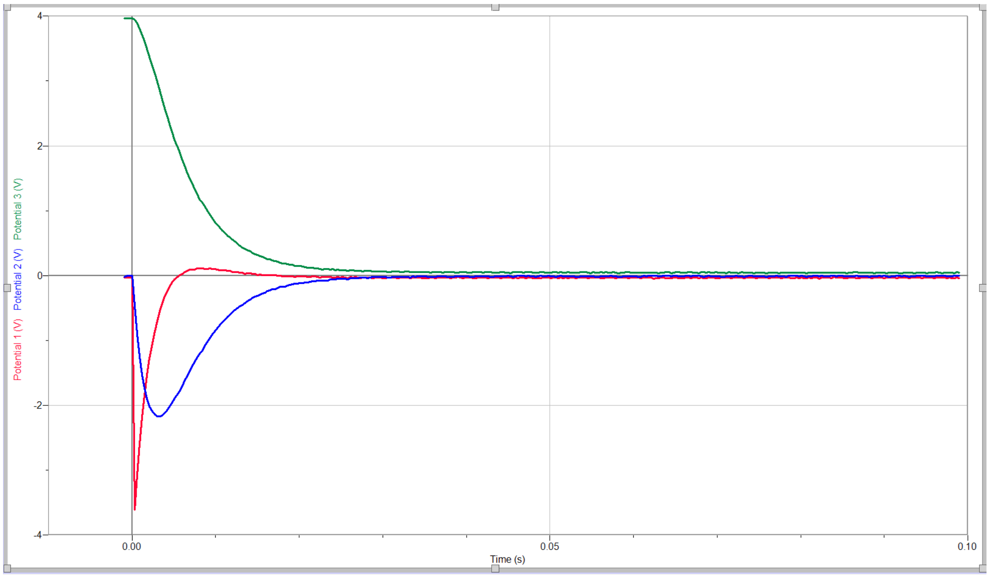

Figure 2: The potential curves for case 2 of critical damping, where the potentials 1, 2, 3 are for the inductor, decade resistor, and capacitor, respectively.

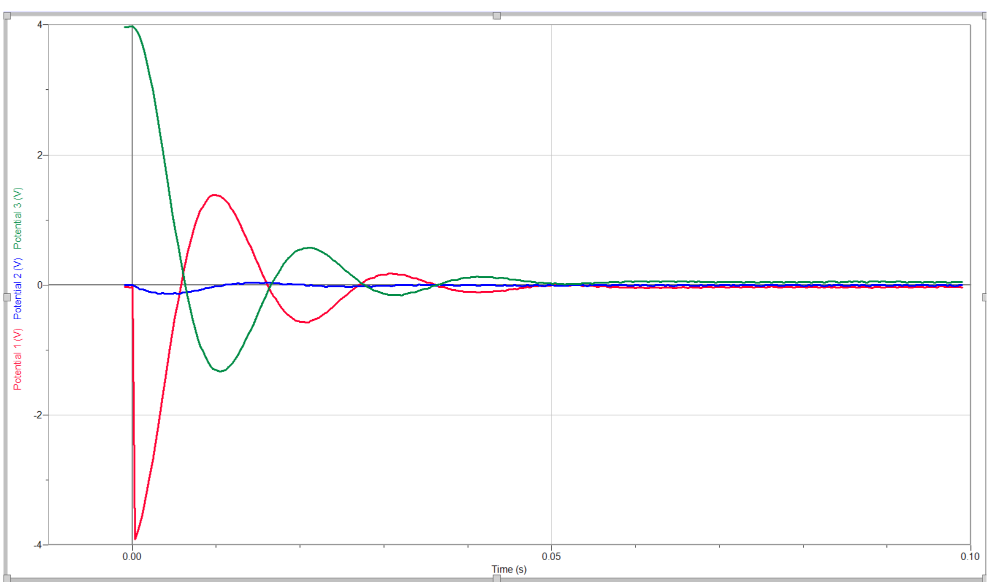

Figure 3: The potential curves for case 3 of over-damping, where the potentials 1, 2, 3 are for the inductor, decade resistor, and capacitor, respectively.

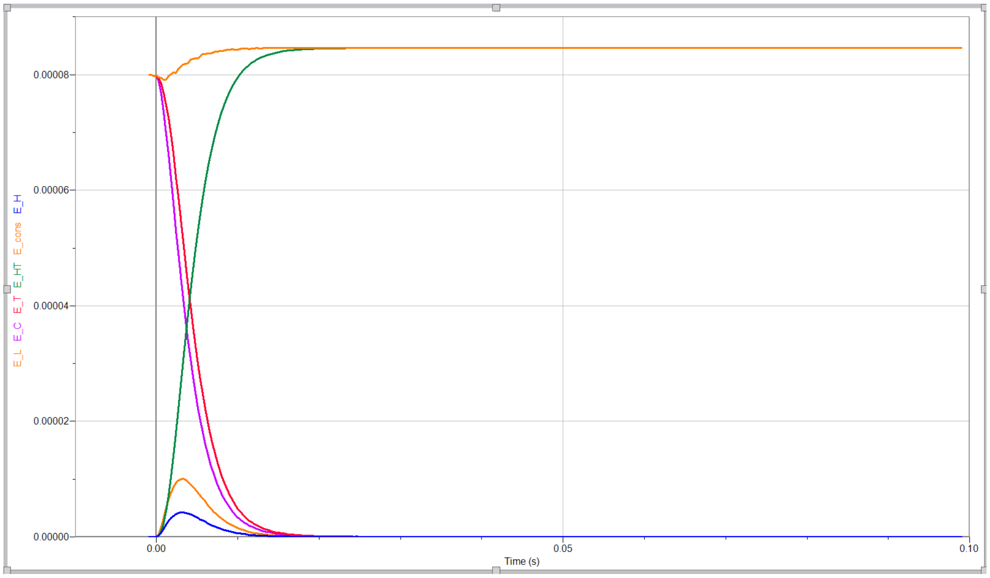

Figure 4: The curves of $E_L, E_C, E_T, E_H, E_{HT}, E_cons$, where the quantities are the energy levels for the inductor, capacitor, the sum of inductor and capacitor,heat dissipated over an interval, sum of the total heat dissipated, and the sum total energy and heat already dissipated, respectively.

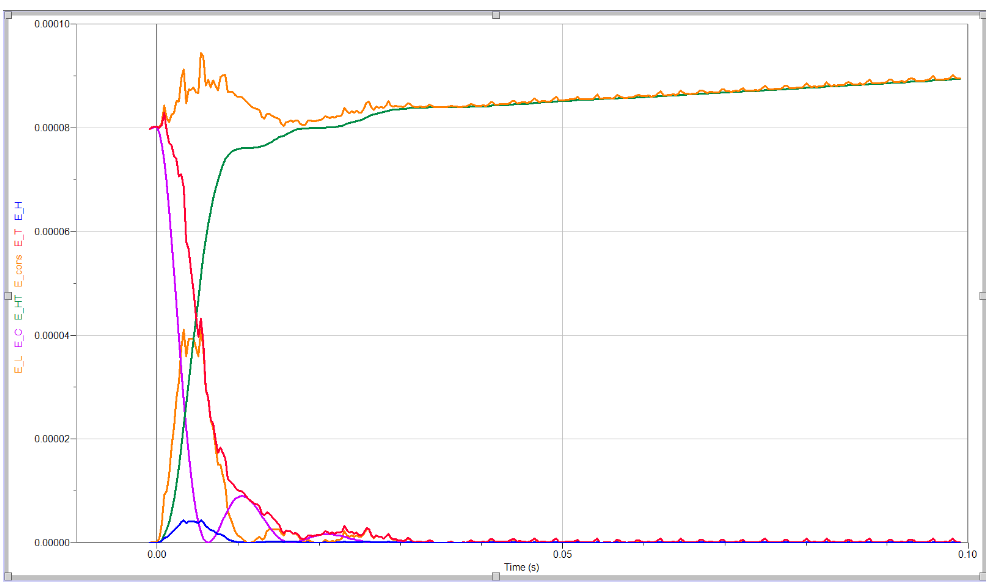

Figure 5: The curves of $E_L, E_C, E_T, E_H, E_{HT}, E_cons$, where the quantities are the energy levels for the inductor, capacitor, the sum of inductor and capacitor,heat dissipated over an interval, sum of the total heat dissipated, and the sum total energy and heat already dissipated, respectively.

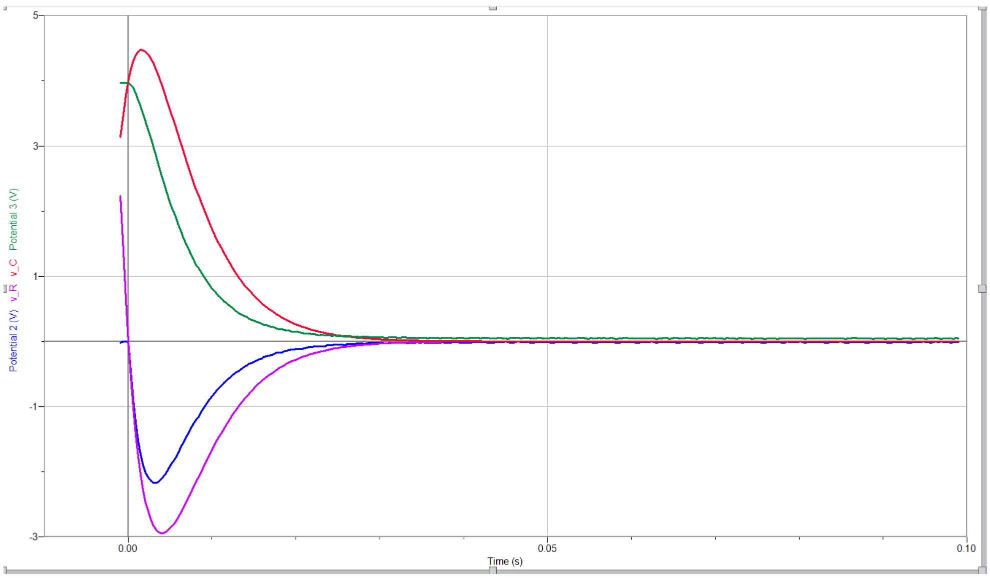

Figure 6: The theoretical and experimental curves for case 2 v_C and v_R are plotted from equation 9 and 10, respectively to see the difference in the curves.

A.1 Data Table

Table 1: Data collected from the experiment

$R_L[Ohm]$	L inductance [H]	R$_{fg}$	Capacitance [F]
$89.2 \pm 0.449\Omega$	$0.96H \pm 0.050H$	50Ω	$10.15 \times 10^{-6}F \pm 2.04 \times 10^{-7}F$