Experiment 49: Electron Diffraction

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Abstract

The results for this experiment produced many wavelengths corresponding to the miller indices using Bragg's law and two de Broglie wavelengths for 14kV and 16kV, which were $1.035 \times 10^{-11} m$ *and* $1.369 \times 10^{-11} m$*. The Bragg's wavelengths were closer in value to the de Broglie wavelengths, for both 14kV and 16kV, for lower miller index values. The lattice constant was 5.43nm, which is not equal to the given value of 0.408nm.*

1 Introduction

The purpose of this experiment is to verify electrons have wave characteristics by measuring and observing diffraction patterns. Electrons are accelerated and hit a crystalline material, in this case gold. Two wavelengths will be determined from de Broglie wavelength equation 4, for 14kV and 16kV. A wavelength for each miller index will also be determined from Bragg's law and compared to the de Broglie wavelength. The electrons will be fired inside the apparatus onto gold and travel a distance to the screen to show a diffraction pattern. Comparison's will be done between the de Broglie and Bragg's wavelengths for different miller indices and the lattice constant will be determined.

2 Equations

$$\frac{1}{2}mv^2 = eV \tag{1}$$

The kinetic energy increases as the electrons travel in this potential.

$$2dsin(\theta) = n\lambda^* \tag{2}$$

Bragg's Law, where d is the perpendicular distance between successive crystal lattice planes, θ is the angle between the incident beam and reflecting plane. n is the order of the ring, in this case it's set to one. λ^* is the wavelength of the electrons.

$$d = \frac{na}{\sqrt{h^2 + k^2 + l^2}} \tag{3}$$

the distance d, used in Bragg's law. h, k, l are the miller indices defining the reflection planes. 'a' is the lattice constant, with a value of 0.408nm

$$\lambda = \frac{h}{\sqrt{2meV}} \tag{4}$$

de Broglie wavelength, where m,e, and V are the electron mass, charge, and voltage, respectively.

$$2\theta = arctan(\frac{r}{D}) \tag{5}$$

Formula used to calculate theta, where D is $257mm \pm 3mm$ and r is the radius. D is the distance between the gold material used and the screen.

$$a = \frac{M}{s^2} N_0 \tag{6}$$

The lattice constant a is calculated with the M (molecular weight), s (thickness of sheet 1000 atoms), and Avogadro's number (6.022×10^{-23}). M/s$^2 is the density.$

3 Sample Calculations

$$\begin{aligned} \lambda &= \frac{6.626 \times 10^{-34} Js}{\sqrt{2(9.109 \times 10^{-31} kg)(1.607 \times 10^{-19} c)(14000V)}} \\ &= 1.035 \times 10^{-11} m \end{aligned} \tag{7}$$

The calculation for the de Broglie wavelength for 14kV.

$$\begin{aligned} \lambda &= \frac{6.626 \times 10^{-34} Js}{\sqrt{2(9.109 \times 10^{-31} kg)(1.607 \times 10^{-19} c)(16000V)}} \\ &= 1.369 \times 10^{-11} m \end{aligned} \tag{8}$$

The calculation for the de Broglie wavelength for 16kV.

$$
\begin{aligned}
2\theta &= arctan(\frac{1.075\times10^{-2}}{257\times10^{-3}}) \\
\theta &= \frac{1}{2}arctan(\frac{1.075\times10^{-2}}{257\times10^{-3}}) \\
&= 0.0209 rads
\end{aligned}
\tag{9}
$$

The sample calculation for the angle theta that will be used in equation 2. This will be repeated for all ring radii, the final results for all of the radii are shown in table 1.

$$
\begin{aligned}
d &= \frac{0.408nm}{1^2+1^2+1^2} \\
&= \frac{0.408nm}{\sqrt{3}} \\
&= 2.356\times10^{-10}m
\end{aligned}
\tag{10}
$$

A sample calculation of determined the distance d for the miller index of (h,k,l)=(1,1,1).

$$
\begin{aligned}
a &= \frac{M}{s^2}N_0 \\
&= \frac{3.27\times10^{-25}}{1000^2}6.022\times10^{-23} \\
&= 5.43\times10^{9}m
\end{aligned}
\tag{11}
$$

lattice constant is determined here.

4 Experimental Procedure and Design

The procedure involved using a ruler to measure the diameter of the circular diffraction patterns produced by the electrons being fired at gold crystalline material. Four rings diameters for two different voltage values were measured. One for 14kV and the other is 16kV.

1. The first set of measurements were for the voltage set to 14kV. A metal tool to measure length was used to measure the diameter since if a ruler is placed on the screen of the device, it changes the diameter of the rings.

2. Starting from the innermost ring, the diameter was recorded, and multiple measurements were made at different angles to see if the resulting diameter differed. If it did then the average of the measurements was used as the final data point.

3. After the innermost ring, the ring outside of it was measured, until four total rings were measured.

4. Then the steps above were repeated for a voltage of 16kV.

5. Using excel, the wavelength of electrons at each permitted reflection (h,k,l) was calculated using equation 2. The permitted reflections indices are (111, 200, 220, 311, 222, 331, 333, 420, 422, 440, 511).

5 Results

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Volt(14K)	111	200	220	311	222	331	333	420	422	440	511	diameter	radius	theta
2	Ring 1	1.97E-11	1.71E-11	1.21E-11	1.03E-11	9.84E-12	7.82E-12	6.56E-12	7.63E-12	6.96E-12	6.03E-12	6.56E-12	2.15	1.075	0.041804
3	Ring 2	3.11E-11	2.69E-11	1.90E-11	1.62E-11	1.55E-11	1.24E-11	1.04E-11	1.20E-11	1.10E-11	9.52E-12	1.04E-11	3.4	1.7	0.066052
4	Ring 3	3.61E-11	3.13E-11	2.21E-11	1.89E-11	1.80E-11	1.43E-11	1.20E-11	1.40E-11	1.28E-11	1.11E-11	1.20E-11	3.95	1.975	0.076698
5	Ring 4	4.70E-11	4.07E-11	2.88E-11	2.45E-11	2.35E-11	1.87E-11	1.57E-11	1.82E-11	1.66E-11	1.44E-11	1.57E-11	5.15	2.575	0.099861
6															
7															
8															
9															
10	4.08E-10	2.36E-10	2.04E-10	1.44E-10	1.23E-10	1.18E-10	9.36E-11	7.85E-11	9.12E-11	8.33E-11	7.21E-11	7.85E-11	These are d values		
11															
12	Volt(16K)	111	200	220	311	222	331	333	420	422	440	511	diameter	radius	theta
13	Ring 1	1.83E-11	1.59E-11	1.12E-11	9.57E-12	9.16E-12	7.28E-12	6.11E-12	7.09E-12	6.48E-12	5.61E-12	6.11E-12	2	1	0.038891
14	Ring 2	2.88E-11	2.50E-11	1.76E-11	1.50E-11	1.44E-11	1.15E-11	9.61E-12	1.12E-11	1.02E-11	8.82E-12	9.61E-12	3.15	1.575	0.061207
15	Ring 3	3.38E-11	2.93E-11	2.07E-11	1.77E-11	1.69E-11	1.34E-11	1.13E-11	1.31E-11	1.20E-11	1.04E-11	1.13E-11	3.7	1.85	0.07186
16	Ring 4	4.43E-11	3.83E-11	2.71E-11	2.31E-11	2.21E-11	1.76E-11	1.48E-11	1.71E-11	1.56E-11	1.36E-11	1.48E-11	4.85	2.425	0.094079

Figure 1: The data and calculations are shown in the excel table for both 14kV and 16kV. The diameter, radius an theta values are shown on the right end. The distance d values are shown in the middle. The calculations for λ^* for a given miller index is shown for each ring for each voltage. Note: Uncertainties were avoided because of the large set of calculations.

The results for the de Broglie wavelength determined for 14kV and 16kV from equation 4 were $1.035 \times 10^{-11} m$ and $1.369 \times 10^{-11} m$, respectively.

6 Discussion

The results for the wavelength's using Bragg's law produced the table shown with the λ^* values for each index. Comparing this to the results determined from de Broglie, the wavelengths λ^* and λ were within the same order of magnitude for lesser indices and larger ring diameters. The closest match between the the de Broglie wavelength and the Bragg's wavelength was for index (2,2,0) on ring 3 for 14kV, and index (2,2,0) on ring 4 for 16kV. There is more deviation from the de Broglie wavelength as the the miller indices increase in value, but all of the wavelengths are on the magnitude 10^{-11} or 10^{-12}. When the voltage is increased the ring diameters decrease. This may be because as the voltage increases, the wavelength decreases. The decrease in wavelength means there's an increase in frequency, and the electrons would be travelling at a higher velocity. A higher velocity means the electrons have more kinetic energy, so there's less spread in the diameter of the rings observed on the screen. Potential sources of error could have been in the measurement of the ring diameters. This is because the ring diameter changes slightly if measured by a ruler up close. Precautions were taken and a metal measurement device was used to measure the length and a ruler to then give that length a value. The uncertainties for this experiment were avoid due to the large set of calculations needed. The lattice constant was calculated to be $5.43nm$, which is not equal to the value given of 0.408nm. The difference in magnitude of this is surely due to incorrect equation used to calculate the constant.

7 Conclusion

The purpose of this lab was to verify that electrons have wave characteristics using their diffraction patterns, and the results of the wavelengths verify the observations from the rings.