Experiment 18: The Gyroscope

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May 7, 2024

Abstract

The final results for precession period produced a ratio of $0.471 \pm 3.34 \times 10^{-4}$*, which is almost half when you double the mass. The moments of inertia about the spin and perpendicular axes were* $0.00479 \pm 3.4 \times 10^{-4} kgm^2$ *and* $0.004275 \pm 8.559 \times 10^{-4} kgm^2$*, with the theoretical value for the spin axis turning out to be* $0.00421 \pm 5.052 \times 10^{-6} kgm^2$*.* k/m *and* I_1/I_3 *ratios were* 0.85 ± 0.06 *and* 0.98 ± 0.20*, respectively.*

1 Introduction

The objective of this experiment is to study a gyroscope. A gyroscope has many interesting features that can be studied. Some such features include what this experiment will be studying, such as precession, spin around different axes, moments of inertia, and nutation. This gyroscope is a steel ball with an aluminum rod insert with a balancing weight on it. Precession is the rotation of aluminum rod about some axis, in this experiment it will primarily be the vertical axis. The steel ball can float slightly in place with the use of air flow underneath it, creating a friction-less environment. Using the spin axis in the horizontal plane creates precession about the vertical axis with angular frequency ω for spin and angular precession frequency Ω. These quantities can be acquired from the period which can be measured using a stop clock. The moment of inertia can be calculated for different rotational axes. Nutation is something that occurs when the gyroscope is introduced to a sudden torque on the rod while in precession. The motion of the rod is circular while it precesses. Nutation frequency can then be measured by determining the period. The nutation and the angular frequency are related to the moments of inertia about the spin and perpendicular axes via equation 5.

1.1 Equations

$$\Omega = \frac{mgl}{I\omega} \tag{1}$$

Ω is the angular precession frequency, I is the moment of inertia, ω is the angular frequency of the steel ball, and l is the distance from the surface to the mass at the end of the rod.

$$f = \frac{2\pi}{T} \tag{2}$$

Frequency f is related to the period T by this equation.

$$I_1 = \frac{mgl}{slope} \tag{3}$$

I_1 is the moment of inertia about the spin axis and the slope is obtained from the graph of $1/\Omega$ and ω.

$$T = 2\pi \frac{I_3}{mga} \tag{4}$$

I_3 is the moment of inertia about the perpendicular axis and a is the distance from the zero gravitational torque balance point of the balancing mass to the new location closer towards the steel ball. This is used for part three of the experiment.

$$\frac{k}{\omega} = \frac{I_1}{I_3} \tag{5}$$

k is the nutation frequency

2 Data and Calculations and Graphs

Data and Calculations

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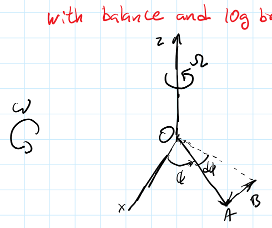
1) Precession

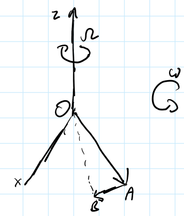
For 5g : $T = 28.945\,s \pm 1.447\,s$, For 10g : $13.647\,s \pm 0.682\,s$

$$\frac{13.647\,s \pm 0.682\,s}{28.945\,s \pm 1.447\,s} = 0.471 \pm 3.34\times10^{-4}$$

with balance and 10g brass mass



without balance and 10g brass mass



2)

Theoretical $I = \frac{2}{5}MR^2 = \frac{2}{5}(4.080\,kg \pm 0.005)(0.0508\,m)^2$
$= 0.00421\,kgm^2 \pm 5.052\times10^{-6}\,kgm^2$

$\tau = mgl = (0.001\,kg \pm 5\%)(9.8\,m/s^2)(0.2\,m) = 0.0196\,kg\frac{m^2}{s^2} \pm 9.8\times10^{-4}\,kgm^2/s^2$

k = nutation rate $= \frac{2\pi}{T_k}$, ω = spin rate $= \frac{2\pi}{T_\omega}$

	Ω_n	$1/\Omega_n$	ω_n
1	0.6849	1.460067	7.47
2	0.336	2.9762	14.87
3	0.2667	3.749531	16.5
4	0.2544	3.930818	17.8
5	0.216	4.62963	20.32
6	0.2114	4.730369	21.52

Graph (linear)
$y = 4.0842x + 1.7944$
$\omega_n = 4.0842\left(\frac{1}{\Omega_n}\right) + 1.7944$

$$I_1 = \frac{mgl}{slope} = \frac{0.0196\,kg\frac{m^2}{s^2} \pm 9.8\times10^{-4}\,kgm^2/s^2}{4.0842\,\frac{1}{s^2} \pm 0.20421\,1/s^2}$$
$= 0.00479\,kgm^2 \pm 3.4\times10^{-4}\,kgm^2$

$$T = 2\pi\sqrt{\frac{I_3}{mga}} \Rightarrow I_3 = \frac{T^2}{4\pi^2}\cdot mga$$

$T_1 = 8.969 \pm 5\%$, $a_1 = 2cm$ $\Rightarrow I_3 = 0.00399 \pm 3.99\times10^{-4}\,kgm^2$

$T_2 = 7.524 \pm 5\%$, $a_2 = 3cm$ $\Rightarrow I_3 = 0.00421 \pm 4.21\times10^{-4}\,kgm^2$

$T_3 = 6.741 \pm 5\%$, $a_3 = 4cm$ $\Rightarrow I_3 = 0.00451 \pm 4.51\times10^{-4}\,kgm^2$

$T_4 = 5.930 \pm 5\%$, $a_4 = 5cm$ $\Rightarrow I_3 = 0.00439 \pm 4.39\times10^{-4}\,kgm^2$

Avg $= 0.004275\,kgm^2 \pm 8.559\times10^{-4}\,kgm^2$

4) $K = \frac{I_1\omega}{I_3} \Rightarrow \frac{K}{\omega} = \frac{I_1}{I_3}$

Sample Calculation

$T_{k_1} = 0.3437$ seconds
$T_{\omega_1} = 0.2899$ seconds

$k_1 = \frac{2\pi}{T_{k_1}} = \frac{2\pi}{0.3437s \pm 5\%} = 18.28\,1/s \pm 0.92\,1/s$

$\omega_1 = \frac{2\pi}{} = \frac{2\pi}{} = 21.67\,1/s \pm 1.08\,1/s$

$$\left.\right\} \frac{k_1}{\omega_1} = \frac{18.28 \pm 0.92}{21.67 \pm 1.08}\,1/s = 0.84 \pm 0.06$$

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$$\frac{K_1}{\omega_1} = \frac{18.28 \pm 0.92}{21.67 \pm 1.08}\ 1/s = 0.84 \pm 0.06$$

$K_2 = 2.1437 \pm 0.11\ 1/s$ $\quad \omega_2 = 2.45 \pm 0.12\ 1/s$ $\quad \Rightarrow \frac{K_2}{\omega_2} = 0.87 \pm 0.06$

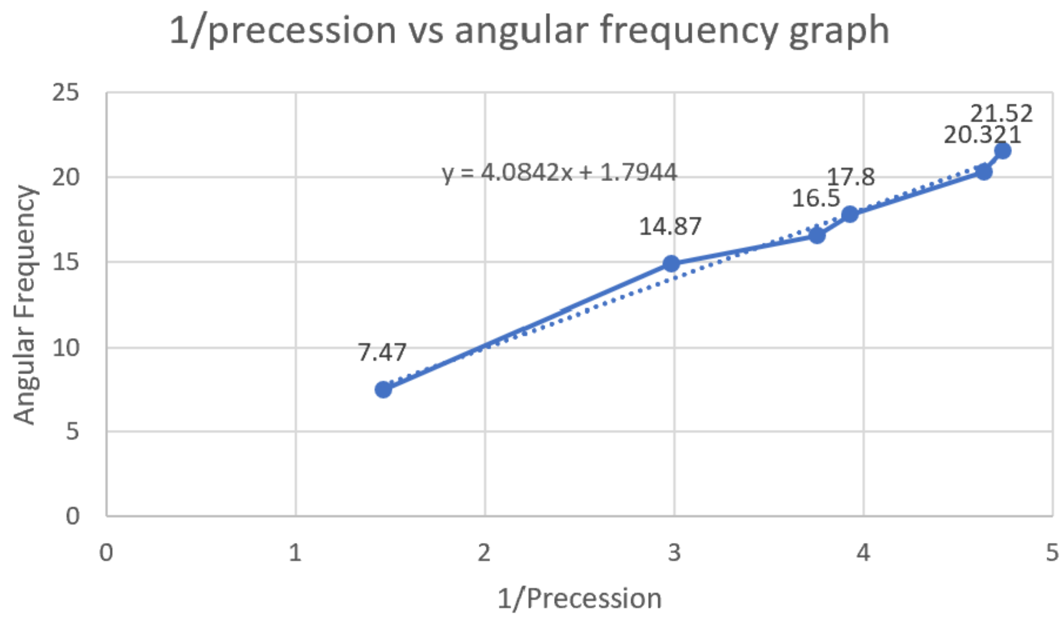
$K_3 = 1.99 \pm 0.1\ 1/s$ $\quad \omega_3 = 2.41 \pm 0.12\ 1/s$ $\quad \Rightarrow \frac{K_3}{\omega_3} = 0.83 \pm 0.06$

avg: 0.85 ± 0.06

Theoretical (not experimental) → I_1

experimental → I_3

$$\frac{I_1}{I_3} = \frac{0.00421\ kgm^2 \pm 5.052 \times 10^{-6}\ kgm^2}{0.004275\ kgm^2 \pm 8.559 \times 10^{-4}\ kgm^2} = 0.98 \pm 0.20$$



3 Experimental Procedure and Design

The Experiment equipment is a solid steel ball with an aluminum insert with a weight on it to make it balanced. This system in placed on a level apparatus. The ball can spin in place in a friction-less environment because of a light air flow underneath the ball. The ball can spin either the horizontal or vertical plane. The procedure had four parts. The first part has to do with precession about the vertical axis. The second and third parts have to do with moments of inertia about the spin and perpendicular axis, respectively. The last part has to do with nutation. Measurements can be taken of the angular frequency ω and angular precession Ω, by using a stop clock to measure the period and from that calculate the frequencies.

A point by point procedure can be outlined via:

3.1 Precession

1. The gyro was spun in the horizontal and it was noted that with a balanced gyro, there was no precession. Adding a 10 gram mass at the end of the axial rod and spinning again caused precession about the vertical axis.

2. Removing the weights made the rod unbalanced and the direction of precession changed to the opposite direction.

3. The balancing weight was added back to balance gravitational torque. The next thing that was done was to compare the precession rate between a 5 gram aluminum mass and a 10 gram brass mass.

3.2 Moments of Inertia about Spin Axis

4. Resetting the gyro by balancing it out, the next task was to find the moment of about the spin axis. This was done by first adding the 10 gram mass on the edge of the rod.

5. The ball was then spun in the horizontal plane. While it was spinning, a stop clock was used to measure the period of the precession as well as the period of the ball spinning. The average value of the period for the ball spinning was recorded since the ball was spinning too fast.

6. These two measurements were taken six times for varying spin rates of the ball to eventually calculate the angular frequency and precession (ω, Ω).

7. To obtain the moment of inertia, a graph was plotted of $1/\Omega$ and ω. The slope of which was used to get the moment of inertia via equation 3.

3.3 Moment of Inertia about Perpendicular Axis

8. Next, was to measure the moment of inertia about the perpendicular axis. To do this, the gyro was treated as a torsion pendulum.

9. The balancing weight was placed to remove the gravitational torque. From that position, the balancing weight could be shifted closer towards the sphere to create the pendulum swinging effect.

10. The weight was moved 2cm below the balance point and set to swing. The period was recorded. The weight was moved 2cm more and the same procedure was repeated three more times.

11. A value for the moment of inertia was recorded for each increment and from that the moment of inertia about the perpendicular axis was calculated using equation. The average of which was taken as the final result.

3.4 Nutation

12. Nutation k, has its angular frequency which was measured along with the angular frequency ω.

13. The balance mass was used to zero the gravitational torque and spun in the horizontal plane and set to precess. Flicking the rod causes the nutation.

14. The period for the nutation and the spring period was measured using the stop clock three times. The nutation and the angular frequency was calculated.

15. Using equation 5, and the moments of inertia obtained from parts two and three, the ratios of k/ω and I_1/I_3 were compared.

You can reference a figure like figure **??** just as you would with an equation.

4 Results

The result for the first part of the experiment dealing with precession can be seen in the vector diagrams that the direction of precession changes from counter-clockwise with the weights on to clockwise without the weights. If you double the mass from 5g to 10g and measure the periods of precession, the period is almost halved with an experimental ratio determined to be $0.471 \pm 3.34 \times 10^{-4}$. For the second part of the experiment, involving the determination of the moment of inertia about the spin axis had a final result of $0.00479 \pm 3.4 \times 10^{-4} kgm^2$ with the theoretical value being $0.00421 \pm 5.052 \times 10^{-6} kgm^2$. The experimental value was obtained using equation 3 and the slope obtained from the linear line in graph 2. The third part of the experiment involved the moment of inertia about the perpendicular axis. The final moment of inertia value obtained was $0.004275 \pm 8.559 \times 10^{-4} kgm^2$, obtained from equation 4. Lastly, the average ratio of nutation and angular frequency was calculated and compared with the results from parts 2 and 3. The result from part 2 was inaccurate, however for this comparison so the theoretical value was used in the ratio. The ratio of moments of inertia yielded a value of 0.98 ± 0.20 while the average k/ω value was 0.85 ± 0.06. A consistency check yields the result $|0.13| < 0.26$ so the result is consistent.

5 Discussion

The experiment overall was successful in meeting its objective of studying a gyroscope and its various qualities. The results as stated above were reasonable as they met what's expected. The only part where the result didn't work was the moment of inertia determined from part two when it was used in the ratio in part 4. The experimental value for part two is close to the theoretical value, but using the experimental value in the ratio for part 4 yields a number greater than 1 which shouldn't be the case. This is why the theoretical value was used. The reason for this might be due to error in measurement of the period, which would propagate to produce a linear graph with a slope that is slightly too small.

6 Conclusion

The experimental results were as expected for most of the experiment with slight error.