Experiment 16: Coupled Oscillation

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Abstract

The final results for the three and eight glider systems were all consistent with the predicted theoretical values. The results for the testing the relative frequencies of the fundamental mode and comparing it to 0.707 was inconsistent.

1 Introduction

The purpose of this experiment is to study the normal modes of oscillation of a system of gliders of same mass moving in 1D. The gliders are connected by springs of equivalent spring constant. Normal modes for this experiment are when the wavelength is an integral number times 'a' in the sine function in equation 2. A frequency f can be determined from this by equation 4 and the normal modes can be seen as standing waves. These can be observed in the motion of the three gliders in the first part of the experiment, as they would move in phase in mode one for one example. The second mode would have two end gliders moving out of phase with each other and the middle glider stationary. Similarly, if more gliders are added then normal modes can be seen at different frequencies. For the fundamental mode, the two opposite gliders should have the same amplitude of 0.707a and the middle should have the amplitude a, this will be tested and compared.

$$f = \frac{\omega_m}{2\pi} \tag{1}$$

The frequency f determined from the angular frequency ω_m

$$\omega_m = 2\sqrt{\frac{K}{M}}sin(\frac{\pi a}{\lambda_m}) \tag{2}$$

The angular frequency where K, M are the average spring constant value and the glider mass, respectively. The 'a' in the equation cancels when equation 3 is substituted into the equation.

$$\lambda_m = \frac{2(N+1)a}{m} \tag{3}$$

The normal mode wavelengths for mode m. The N in this case is the number of gliders.

$$A_n = Asin(\frac{2\pi na}{\lambda_m}) \tag{4}$$

The amplitude of the longitudinal oscillation nth mass.

2 Experimental Procedure and Design

The experiment apparatus is an air track that is straight and long enough to hold 9 gliders, which can be connected to springs. This setup is connected to a precision sine drive which is capable of oscillating the springs at various frequencies.

A point by point procedure can be outlined via:

1. The first task was to determine the mass of the gliders and springs, as well as spring constant k, for the springs. Since the masses and the springs are supposed to be the same, the average was taken to be used to later calculations.

2. The glider mass and spring mass was determined using a weight scale. Then the spring had a mass attached to one end and made to oscillate. The period of oscillation of this for 10 cycles was measured to get the average for one spring, and repeated for two more springs to get another average. The spring constant was calculated for each and averaged.

3. With the quantities determined, the next part of the experiment involved studying the normal modes for a three glider system connected with four springs.

4. First the theoretical frequency f, was determined from the angular frequency equation 2. Since there are three gliders, N=3, and m=1,2,3. The frequency f for the three modes are calculated from equation 1.

5. From the frequency, the next step is to see if this theoretical frequency matches the experimental one. This is done by turning the air track on to generate a friction-less environment for the gliders.

6. The sine drive was activated and set to the frequency that was theoretically predicted for mode 1. If the theoretical was correct, then the gliders would oscillate in sink with one another for this as it can be analogized to one standing wave. The frequency was unlikely to be exactly the theoretical so minor adjustments were made to determine the best possible experimental result.

7. After the first mode, the same procedure was repeated for modes 2 and 3. These two modes could be analogized to two and three standing waves, respectively and can be observed by the motion of the gliders.

8. The theoretical and experimental values were then compared.

9. For the fundamental mode, m=1, the amplitudes of each of the three gliders were measured using a ruler and using equation 4 the results were compared.

10. The final part of the experiment is similar to the first part, but instead of three gliders, this part uses eight gliders all connected by springs.

11. The normal modes for this part occur on modes m=1,2,3,6 since that's when the sine function in equation 2 has a wavelength that in an integral number time 'a'.

12. The same procedure from the three glider system was repeated and the theoretical and experimental results were compared.

3 Results

The final results for the three glider system for this experiment produced theoretical frequencies of $0.3889Hz \pm 0.0113Hz$, $0.719Hz \pm 0.021Hz$, and $0.9388Hz \pm 0.0.0272Hz$ for m=1,2,3 modes, respectively. The corresponding experimental values were determined to be $0.372Hz \pm 0.01Hz$, $0.718Hz \pm 0.01Hz$, $0.937Hz \pm 0.01Hz$. All experimental results were consistent with the theoretical values predicted from equation 2. For the eight glider system, the theoretical frequencies $0.177Hz \pm 0.005Hz$, $0.348Hz \pm 0.01Hz$, $0.508Hz \pm 0.015Hz$, and $0.880Hz \pm 0.026Hz$ for modes m=1,2,3,6. The corresponding experimental frequencies were $0.166Hz \pm 0.01Hz$, $0.348Hz \pm 0.01Hz$, $0.501Hz \pm 0.01Hz$, and $0.861Hz \pm 0.01Hz$. All experimental results were also consistent for the eight glider system. The results for the relative amplitudes and their comparison to the expected results from equation 4 were 0.512 ± 0.002 and 0.566 ± 0.002 and both results were inconsistent with the theoretical of 0.707.

4 Discussion

The experiment overall produced good results that were consistent for the glider frequency measurements. The inconsistency came from the testing of the relative amplitudes of a three glider system in the fundamental frequency. The inconsistency might be a result of measurement error with the ruler. The results for testing the harmonics of the gliders were very good, and could potentially be improved with more measurements of the springs and gliders. Measuring all the gliders and springs would also improve the result instead of taking the average of three because it's assumed to be the same weight.

5 Conclusion

Overall, the experiment produced good results and met the objective of the lab.

6 Data and Calculations

Exp 16

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Exp. 16

$T = 2\pi\sqrt{\frac{m_{eff}}{k}}$ $m_{eff} = \frac{1}{3}m_s + m_w$ m_s: mass spring, m_w: mass weight

$= \frac{1}{3}(7.7g) + 50.5g = 53.07g \pm 0.1g$

Spring #	1	2	3
m_s =	7.7g	7.7g	7.7g

$m_w = 50.5g \pm 0.1g$

Period T for 10 oscillations:

$T_1 = \frac{11.135s}{10}$, $T_2 = \frac{11.204s}{10}$, $T_3 = \frac{11.203s}{10}$

$= 1.1135s$ $= 1.1204s$ $= 1.1203s$ $\pm 5\%$

$\boxed{k = \frac{4m\pi^2}{T^2}}$

$k_1 = \frac{4(53.07\times10^{-3}kg \pm 0.0001kg)\pi^2}{(1.1135s \pm 5\%)^2} = 1.689\ kg/s^2 \pm 0.1689\ kg/s^2$

$= 1.689\ kg/s^2 \pm 10\%$ $k_2 = 1.669\ kg/s^2 \pm 10\%$ $k_3 = 1.669\ kg/s^2 \pm 10\%$

Avg. $k = 1.676\ kg/s^2 \pm \underbrace{5.8\%}_{\pm 0.0972\ kg/s^2}$

Glider mass

glider 6: $m_6 = 164.2g \pm 0.1g$

glider 7: $m_7 = 164.6g \pm 0.1g$

glider 8: $m_8 = 164.3g \pm 0.1g$

Avg mass $= 0.1644kg \pm 0.0002kg = M_g$

$\omega_m = 2\sqrt{\frac{k}{M_g}}\sin\left(\frac{\pi a}{\lambda_m}\right)$ $\lambda_m = \frac{2(N+1)a}{m}$

$\omega_m = 2\sqrt{\frac{k}{M_g}}\sin\left(\frac{\pi m}{2(N+1)}\right)$

$N = 3$, $m = 1, 2, 3$

$f = \frac{\omega_1}{2\pi} = 0.3889$ $\omega = 2\pi f \rightarrow f = \frac{\omega}{2\pi}$

$m=1$: $2\sqrt{\frac{1.676\ kg/s^2 \pm 0.0972\ kg/s^2}{0.1644kg \pm 0.0002kg}}\sin\left(\frac{\pi(1)}{2(4)}\right) = 2.444Hz \pm 0.0709Hz = \omega_1 \rightarrow f_1 = \frac{2.444Hz}{2\pi} = 0.3889Hz \pm 0.0113Hz$

exp.: $f = 0.372Hz \pm 0.01Hz$

$m=2$: $2\sqrt{\frac{1.676\ kg/s^2}{0.1644kg}}\sin\left(\frac{2\pi}{8}\right) = 4.513Hz \pm 0.1309Hz = \omega_2 \rightarrow f_2 = \frac{\omega_2}{2\pi} = 0.719Hz \pm 0.021Hz$ $f = 0.718Hz \pm 0.01Hz$

$m=3$: $2\sqrt{\frac{1.676\ kg/s^2}{0.1644kg}}\sin\left(\frac{3\pi}{8}\right) = 5.899Hz \pm 0.1711Hz = \omega_3 \rightarrow f_3 = \frac{\omega_3}{2\pi} = 0.9388Hz \pm 0.0272Hz$ $f = 0.937Hz \pm 0.01Hz$

$A_n = A\sin\left(\frac{2\pi n a}{\lambda_m}\right) \rightarrow A_1 = A\sin\left(\frac{2\pi a m}{2(N+1)a}\right) \rightarrow A_1 = A\sin\left(\frac{m\pi}{4}\right)$

Middle Glider 7: Oscillation Amp. $2a = 172.7cm \pm 0.05cm - 131.5cm \pm 0.05$
$= 41.2cm \pm 0.07cm$
$a = (41.2cm \pm 0.07cm)/2 = (20.6 \pm 0.04)cm$

left Glider 6: $a = (137.7cm - 116.6cm) \pm 0.05cm/2$
$= (10.55 \pm 0.04)cm$

Right Glider 8: $a = (190.5cm - 167.2cm) \pm 0.05cm/2$
$= (11.65 \pm 0.03)cm$

$\omega_m = 2\sqrt{\frac{k}{M_g}}\sin\left(\frac{\pi m}{2(N+1)}\right)$ $\lambda_m = \frac{2(N+1)a}{m}$

$N = 8$, $m = 1, 2, \ldots, 8$

$m=1$: $2\sqrt{1.676\ kg/s^2 \pm 0.0972\ kg/s^2}$ ...

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$$\omega_m = 2\sqrt{\frac{k}{m_g}}\sin\left(\frac{\pi m}{2(N+1)}\right) \qquad \lambda_m = \frac{2(N+1)a}{m}$$

$N=8$, $m = 1, 2, \ldots, 8$

$$m=1:\ 2\sqrt{\frac{1.676\ \text{kg/s}^2 \pm 0.0972\ \text{kg/s}^2}{0.1644\ \text{kg} \pm 0.0002\ \text{kg}}}\ \sin\left(\frac{\pi(1)}{2(9)}\right) = 1.111\ \text{Hz} \pm 0.032\ \text{Hz} = \omega_1$$

$$f_1 = \frac{\omega_1}{2\pi} = \frac{1.111\ \text{Hz} \pm 0.032\ \text{Hz}}{2\pi} = 0.177\ \text{Hz} \pm 0.005\ \text{Hz}$$ exp. $f_1 = 0.166\ \text{Hz} \pm 0.01\ \text{Hz}$

$$m=2:\ 2\sqrt{\frac{1.676\ \text{kg/s}^2 \pm 0.0972\ \text{kg/s}^2}{0.1644\ \text{kg} \pm 0.0002\ \text{kg}}}\ \sin\left(\frac{\pi(2)}{2(9)}\right) = (2.184 \pm 0.063)\ \text{Hz}$$

$$f_2 = \frac{\omega_2}{2\pi} = \frac{2.184\ \text{Hz} \pm 0.063\ \text{Hz}}{2\pi} = (0.348 \pm 0.01)\ \text{Hz}$$ exp $f_2 = 0.348\ \text{Hz} \pm 0.01\ \text{Hz}$

$$m=3:\ 2\sqrt{\frac{1.676\ \text{kg/s}^2 \pm 0.0972\ \text{kg/s}^2}{0.1644\ \text{kg} \pm 0.0002\ \text{kg}}}\ \sin\left(\frac{\pi(3)}{2(9)}\right) = (3.193 \pm 0.093)\ \text{Hz}$$

$$f_3 = \frac{\omega_3}{2\pi} = \frac{3.193\ \text{Hz} \pm 0.093\ \text{Hz}}{2\pi} = (0.508 \pm 0.015)\ \text{Hz}$$ exp $f_3 = 0.501\ \text{Hz} \pm 0.01\ \text{Hz}$

$$m=6:\ 2\sqrt{\frac{1.676\ \text{kg/s}^2 \pm 0.0972\ \text{kg/s}^2}{0.1644\ \text{kg} \pm 0.0002\ \text{kg}}}\ \sin\left(\frac{\pi(6)}{2(9)}\right) = (5.530 \pm 0.160)\ \text{Hz}$$

$$f_6 = \frac{\omega_6}{2\pi} = \frac{5.530\ \text{Hz} \pm 0.160\ \text{Hz}}{2\pi} = (0.880 \pm 0.026)\ \text{Hz}$$ exp $f_6 = 0.861\ \text{Hz} \pm 0.01\ \text{Hz}$

Consistency Checks:

3 Glider system:

$m=1$ $\quad |0.3889\ \text{Hz} - 0.372\ \text{Hz}| \le 0.0113\ \text{Hz} + 0.01\ \text{Hz}$

$0.0169\ \text{Hz} \le 0.0213\ \text{Hz}$ consistent

$m=2$ $\quad |0.719\ \text{Hz} - 0.718\ \text{Hz}| \le 0.021\ \text{Hz} + 0.01\ \text{Hz}$

$0.001\ \text{Hz} \le 0.031\ \text{Hz}$ Consistent

$m=3$ $\quad |0.9388\ \text{Hz} - 0.937\ \text{Hz}| \le 0.0272\ \text{Hz} + 0.01\ \text{Hz}$ Consistent

$0.0018\ \text{Hz} \le 0.0372\ \text{Hz}$

8 Glider system: Consistent

$m=1$ $|0.177\,Hz - 0.166\,Hz| \le 0.005\,Hz + 0.01\,Hz$

$0.011\,Hz \le 0.015\,Hz$ consistent

$m=2$ $|0.348\,Hz - 0.348\,Hz| \le 0.01\,Hz + 0.01\,Hz$

$0 \le 0.02\,Hz$ Consistent

$m=3$ $|0.508\,Hz - 0.501\,Hz| \le 0.015\,Hz + 0.01\,Hz$

$0.007\,Hz \le 0.025\,Hz$ Consistent

$m=6$ $|0.880\,Hz - 0.861\,Hz| \le 0.026\,Hz + 0.01\,Hz$

$0.019\,Hz \le 0.036\,Hz$ Consistent

$$A_n = A\sin\left(\frac{2\pi n a}{\lambda_m}\right) \rightarrow A_1 = A\sin\left(\frac{2\pi a m}{2(N+1)a}\right) \rightarrow A_1 = A\sin\left(\frac{m\pi}{4}\right)$$

Middle Glider (7) : Oscillation Amp. $2a = 172.7\,\text{cm} \pm 0.05\,\text{cm} - 131.5\,\text{cm} \pm 0.05$

$= 41.2\,\text{cm} \pm 0.07\,\text{cm}$

$a = (41.2\,\text{cm} \pm 0.07\,\text{cm})/2 = (20.6 \pm 0.04)\,\text{cm}$

left Glider (6) : $a = (137.7\,\text{cm} - 116.6\,\text{cm}) \pm 0.05\,\text{cm}/2$

$= (10.55 \pm 0.04)\,\text{cm}$

Right Glider (8) : $a = (190.5\,\text{cm} - 167.2\,\text{cm}) \pm 0.05\,\text{cm}/2$

$= (11.65 \pm 0.03)\,\text{cm}$

$$\frac{(6)}{(7)} = \frac{(10.55 \pm 0.04)\,\text{cm}}{(20.6 \pm 0.04)\,\text{cm}} = 0.512 \pm 0.002$$

$$\frac{(8)}{(7)} = \frac{(11.65 \pm 0.03)\,\text{cm}}{(20.6 \pm 0.04)\,\text{cm}} = 0.566 \pm 0.002$$

Consistency Check

theoretical

$|0.512 - 0.707| \leq 0.002$

$0.195 \leq 0.002$ Inconsistent

$|0.566 - 0.707| \leq 0.002$

$0.141 \leq 0.002$ Inconsistent