Experiment 13: Compound Pendulum

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Abstract

The final results for the values for gravity, radius of gyration, and moment of inertia for the bar were, $9.917 \pm 0.1462 m/s^2$, $0.288m \pm 1.2\%$, *and* $0.0350 kg/m^2 \pm 2.4\%$, *respectively and the results for gravity in comparison with another method used to determine its value of* $9.43 \pm 8.9\% m/s^2$ *was found to be inconsistent. The moment of inertia was found to be consistent with the theoretical value for both bar and disk. The results for the values for gravity, radius of gyration, and moment of inertia for the disk were* $9.823 m/s^2 \pm 8.3\%$, $0.1755m \pm 6\%$, *and* $0.0369 kg/m^2 \pm 12\%$.

1 Introduction

The objective of this experiment is to study the motion of compound pendulums and to determine the moment of inertia and radius of gyration of a rod and disk. A simple pendulum exhibits simple harmonic motion which assumes the a point mass at the end of the string without the string having a mass and the mass itself isn't point size. In the compound pendulum the rigid body oscillates about an axis that's not the centre of gravity of the body. This experiment has two compound pendulums that are studied, one is a bar and the other a disk. The bar and the disk have notches on them such that it can be oscillated on these notches. The bar has notches 1cm apart for up to 99cm with the centre of gravity being at about 49cm (in the middle of the rod as expected). The has only 7 notches on which it can be oscillated. Each compound pendulum has a mass and length measurement, the bar has a length of 99cm a mass of 421.7g, and the disk has a radius of 49.6cm and a mass of 1201g. Oscillating about the notches each takes time which means there's a measure of period. Taking the average period for each notch of the bar until the centre of mass is reached would produce an array of period data, and along with the length data can produce a linear line on a graph. The slope of which can be used to determine an experimental value for gravity, g. Similarly, this method can be used for the disk. Using the linear fit of the graph, the experimental value of the radius of gravitation can also be determined by where the line intercepts the x-axis (y=0). For the bar, another method of determining the value for gravity can be used, without the need for the radius of gyration. When taking the measurements, down the notches towards the center of gravity of the bar the periods will vary. However, there exist two notches where the periods are equivalent. These two points could be used to plot a different graph from which an experimental value for gravity could be determined. Another quantity that will be determined is the moment of inertia, which can be determined from the mass and radius of gyration for both pendulums.

1.1 Equations

$$g = \frac{4\pi^2}{T^2}(l_1 + l_2) \tag{1}$$

This equation helps determine the value for gravity using the method described in which there exist two notches where the periods are equivalent. Where g is the experimental value for gravity, T is the common period and, $l_1 and l_2$ are the distances away from the centre of mass.

$$slope = \frac{4\pi^2}{g} \tag{2}$$

This equation will be used for determining the value for gravity using the slope of the linear fitted graph.

$$lT^2 = \frac{4\pi^2}{g}(k_0^2 + l^2) \tag{3}$$

This equation shows the relationship of the radius of gyration k_0 to other quantities.

$$k_0^2 = -\frac{(y - intercept)}{slope} \tag{4}$$

The radius of gyration will be determined using the equation of a straight line acquired from the graph. Square-rooting this will give the experimental value for k_0.

$$I_G = mk_0^2 \tag{5}$$

The experimental value for the moment of inertia can be calculated using the mass and the radius of gyration.

$$I_{bar} = \frac{1}{12}ml^2 \tag{6}$$

This is the theoretical value for the moment of inertia for the bar.

$$I_{disk} = \frac{1}{2}mR^2 \tag{7}$$

This is the theoretical value for the moment of inertia for the disk, where R is the radius of the disk.

2 Calculations

Basic Values: $M_{bar} = 0.4217kg \pm 0.001kg$, $M_{disk} = 1.201kg \pm 0.001kg$, $R_{disk} = 0.248m \pm 0.001m$, $l_{bar} = 99cm$.

2.1 For Bar

$$y = (3.981 \pm 0.05869s^2/m)x + (0.3301 \pm 0.006463ms^2) \tag{8}$$

The equation of the line for the l^2 vs lT^2 graph acquired using LoggerPro linear fit.

$$\begin{aligned} slope &= \frac{4\pi^2}{g} \\ g &= \frac{4\pi^2}{slope} \\ &= \frac{4\pi^2}{3.981 \pm 0.05869s^2/m} \\ &= 9.917 \pm 0.1462m/s^2 \end{aligned} \tag{9}$$

Gravity g, determined from the graph for the bar using equation 2.

$$\begin{aligned} k_0^2 &= -\frac{(y - intercept)}{slope} \\ &= -\frac{(0.3301 \pm 0.006463ms^2)}{3.981 \pm 0.05869s^2/m} \\ &= |-0.083m^2| \pm 2.5\% \\ k_0 &= 0.288m \pm 1.2\% \end{aligned} \tag{10}$$

Radius of gyration determined from the graph of the bar and the equation of the line above.

$$\begin{aligned} I_{bar} &= \frac{1}{12}ml^2 \\ &= \frac{1}{12}(0.4217kg \pm 0.001kg)(0.99m \pm 0.001)^2 \\ &= 0.0334kg/m^2 \pm 0.31\% \end{aligned} \tag{11}$$

Theoretical value for the moment of inertia of the bar.

$$\begin{aligned} I_G &= mk_0^2 \\ &= (0.4217kg \pm 0.001kg)(0.288m \pm 1.2\%)^2 \\ &= 0.0350kg/m^2 \pm 2.4\% \end{aligned} \tag{12}$$

The experimental value for the moment of inertia of the bar determined from equation 5.

$$\begin{aligned} g &= \frac{4\pi^2}{T^2}(l_1 + l_2) \\ &= \frac{4\pi^2}{1.5582s^2 \pm 0.2s}((0.21m + 0.37m) \pm 0.001m) \\ &= 9.43 \pm 8.9\% m/s^2 \end{aligned} \tag{13}$$

gravity determined by using two notches with the same period.

$$\begin{aligned} |0.0334 - 0.0350| &<= 0.0001 + 0.00084 \\ 0.0016 &<= 0.00094 \end{aligned} \tag{14}$$

Consistency check between the theoretical and experimental moment of inertia's. The result shows it to be inconsistent.

$$\begin{aligned} |9.43 - 9.917| &<= 0.83927 + 0.1462 \\ 0.487 &<= 0.98547 \end{aligned} \tag{15}$$

Consistency check between the theoretical and experimental gravity, g. The result shows it to be consistent.

2.2 For Disk

$$y = (4.019 \pm 0.3325s^2/m)x + (0.1236 \pm 0.011ms^2) \tag{16}$$

The equation of the line for the l^2 vs lT^2 graph acquired using LoggerPro linear fit.

$$\begin{aligned} slope &= \frac{4\pi^2}{g} \\ g &= \frac{4\pi^2}{slope} \\ &= \frac{4\pi^2}{4.019 \pm 0.3325 s^2/m} \\ &= 9.823 m/s^2 \pm 8.3\% \end{aligned} \tag{17}$$

Gravity g, determined from the graph for the disk using equation 2.

$$\begin{aligned} k_0^2 &= -\frac{(y - intercept)}{slope} \\ &= -\frac{(0.1236 \pm 0.011 ms^2)}{4.019 \pm 0.3325 s^2/m} \\ &= |-0.0308 m^2| \pm 12\% \\ k_0 &= 0.1755 m \pm 6\% \end{aligned} \tag{18}$$

Radius of gyration determined from the graph of the disk and the equation of the line above.

$$\begin{aligned} I_{disk} &= \frac{1}{2} mR^2 \\ &= \frac{1}{2}(1.201 kg \pm 0.001 kg)(.248 m \pm 0.001 m)^2 \\ &= 0.0369 kg/m^2 \pm 0.81\% \end{aligned} \tag{19}$$

Theoretical value for the moment of inertia of the disk.

$$\begin{aligned} I_G &= mk_0^2 \\ &= (1.201 kg \pm 0.001 kg)(0.1755 m 6\%)^2 \\ &= 0.0369 kg/m^2 \pm 12\% \end{aligned} \tag{20}$$

The experimental value for the moment of inertia of the disk determined from equation 5.

$$\begin{aligned} |0.0369 - 0.0369| &<= 0.004428 + 0.0003 \\ 0 &<= 0.004728 \end{aligned} \tag{21}$$

Consistency check between the theoretical and experimental moment of inertia's. The result shows it to be consistent.

3 Experimental Procedure and Design

The experiment measures the periods of a bar and a disk on notches away from the center of mass. The masses and lengths/radius were recorded for both compound pendulums. The apparatus used aside from the bar and disk were sliding pivot clamp which could be used to hang the bar and disk on a chosen notch giving the ability to choose an axis of rotation. A stop clock was used to collect data for the period in units of seconds. A point by point procedure can be outlined via:

1. Starting off with the bar, the objective was to get an average period from five swings of the pendulum using the stop clock. The pendulum started an a small angle and was released to swing and the clock was started at the same time.

2. After five full swings, the clock was stopped and the time recorded for that specific notch.

3. The previous steps were repeated for all notched on the bar until the notch right before the center of mass was reached, at about 49cm from the top of either side of the bar.

4. Similarly, for the disk, the previous steps were followed to acquire the period data for seven notches on along the diameter of the disk.

5. From the data for period and length acquired, two graphs of $l^2 vs l T^2$ were plotted. These plots produce a linear line, the slope of which could be used to determine the value for gravity and the x-intercept to determine the value for radius of gyration using equations 2 and 4, respectively.

6. Another method was used for the bar in which, two notches were located where the period for each notch was nearly equivalent. Using equation 1, the value for gravity could also be obtained.

4 Results

4.1 BAR

The final results for the bar were $(3.981 \pm 0.05869 s^2/m)x + (0.3301 \pm 0.006463 ms^2)$, which in the equation of the line plotted in figure 3. From this, the result for gravity and k_0 was determined, which was $9.917 \pm 0.1462 m/s^2$ and $0.288 m \pm 1.2\%$, respectively. The experimental and theoretical values for the moment of inertia were determined to be $0.0350 kg/m^2 \pm 2.4\%$ and $0.0334 kg/m^2 \pm 0.31\%$, respectively. The comparison of gravity values between the two different methods used showed that the values were consistent with each other. The moment of inertia, theoretical and experimental values were not consistent.

4.2 DISK

The final results for the disk were $y = (4.019 \pm 0.3325 s^2/m)x + (0.1236 \pm 0.011 ms^2)$, which in the equation of the line plotted in figure 4. From this, the result for gravity and k_0 was determined, which was $9.823 m/s^2 \pm 8.3\%$ and $0.1755 m \pm 6\%$, respectively. The experimental and theoretical values for the moment of inertia were determined to be $0.0369 kg/m^2 \pm 12\%$ and $0.0369 kg/m^2 \pm 0.81\%$, respectively. The theoretical and experimental values for the moment of inertia of the disk were consistent.

5 Discussion

The objective of this lab was to study compound pendulum's in the form of a bar and disk with notches to swing the pendulum's at different axes of rotation. The results of the lab demonstrate that the experiment was a success due to the consistency in the moment of inertia's of both bar and disk, as well as, the values for gravity were close to their expected value despite the inconsistency in the two values for the bar. Potential sources of error that contribute to the difference in values could be random error due to the measurement of the periods using the stop clock. One way to make sure the results are more accurate may be to average over more periods than the five used for this experiment. The measurements of the radius and length were only taken once, and not checked twice or more times for an average value, so more measurements of the basic parameters would also increase accuracy in the results.

6 Conclusion

Compound pendulums are useful to study since they can be used to determine the force of gravity with relative ease. The results of this experiment were consistent for the moment of inertia's for the bar and the disk. The result for the gravity, g was not consistent with the value acquired using another method for the bar, but the values were close enough to what was expected without being consistent.

References

[1] Physics 321A, Laboratory Manual. University of Victoria, 2022.

A Figures

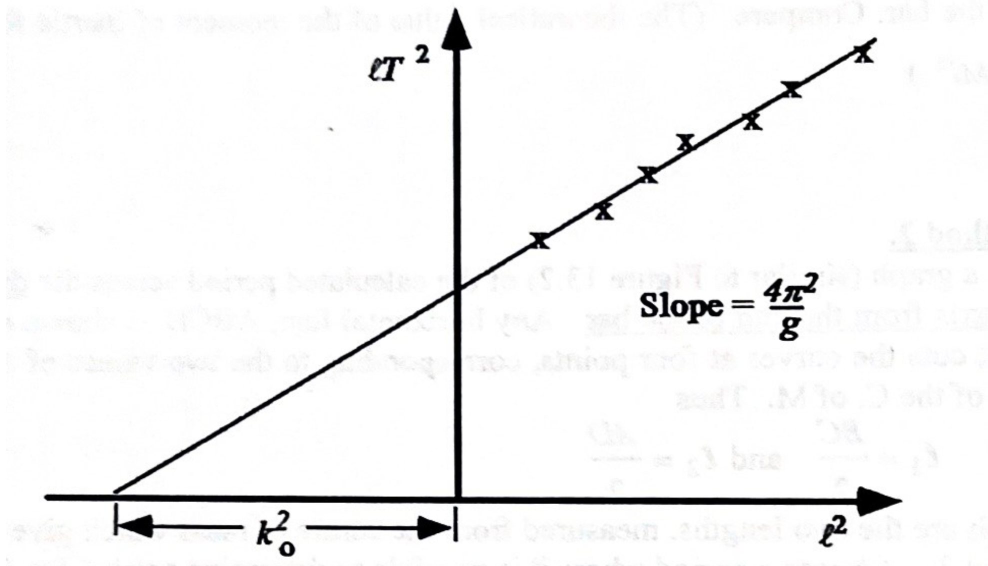

Fig. 13.3 Graph of ℓT^2 versus ℓ^2

Figure 1: This is an example of the graph that should be produced by plotting $l^2 vs l T^2$ for the bar and the disk. How the experimental values for the determination of gravity and the radius of gyration are shown.

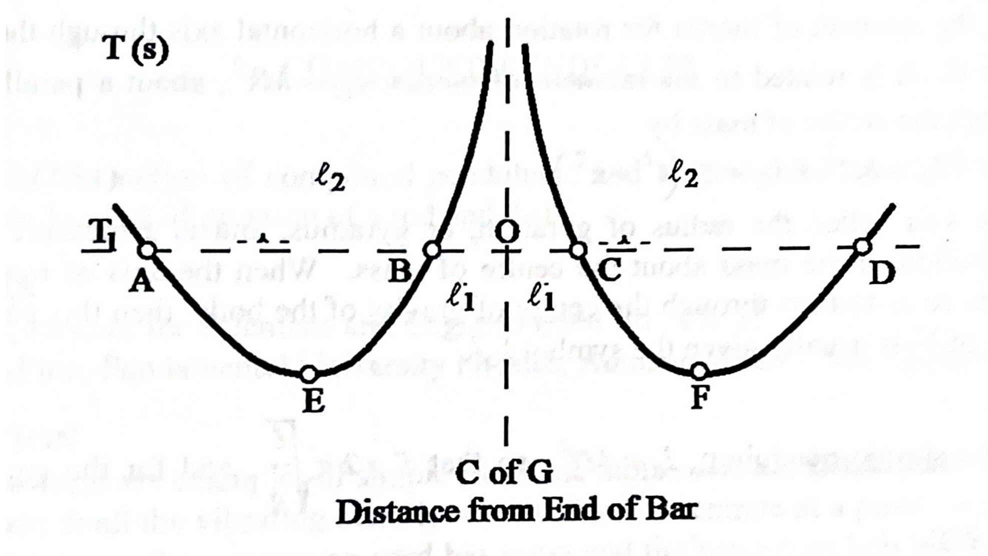

Figure 2: This graph is an example of the graph that should be produced by plotting the distance away from the center of mass and as shown, there exist two notches some distance away where the period T_1 is equivalent. This experiment will consider only the right-hand side of this graph since the two sides are identical.

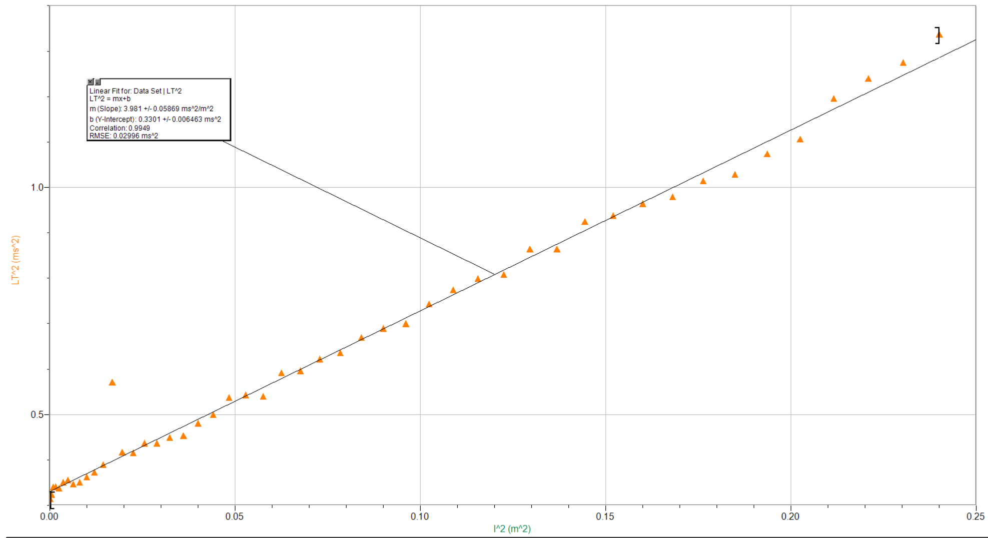

Figure 3: The linear graph produced by plotting $l^2 vs lT^2$ for the bar. The slope and y-intercept are shown giving the equation for the line.

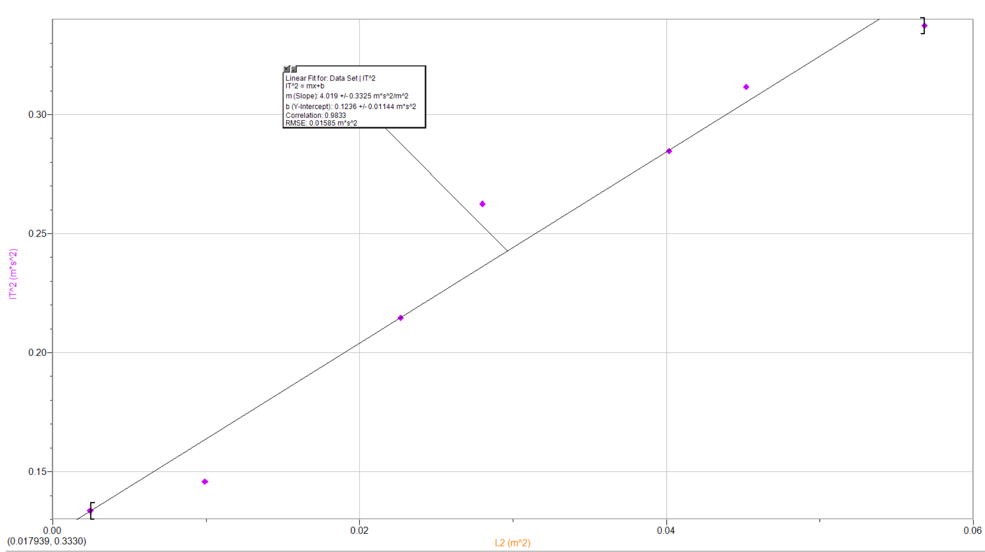

Figure 4: The linear graph produced by plotting $l^2 vs lT^2$ for the disk. The slope and y-intercept are shown giving the equation for the line.

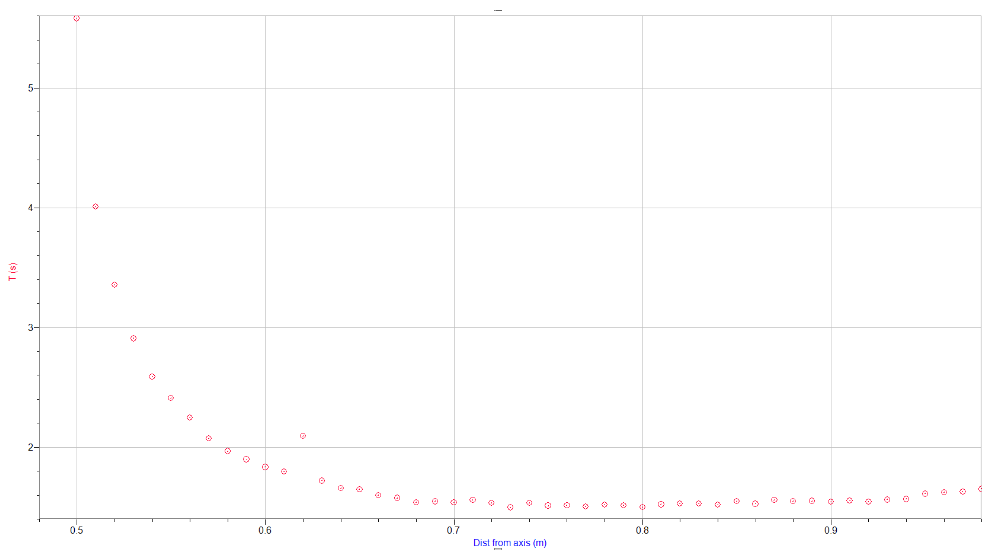

Figure 5: The graph is used to determine the gravity g, for the bar using two points on this graph with the nearly equivalent periods.