Experiment 15: Conservation Principles

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Abstract

The final results of this experiment produced graphs which shows constant velocity for the motion of the system as seen in figure 1 and constant total angular momentum as seen in figure 4. Another method for obtaining the constant angular momentum value was not as accurate by plotting $1/\omega$ vs r^2, as will be discussed.

1 Introduction

The objective of this experiment is to use a two-mass connected by a spring system to test if this system moves with constant velocity and that its linear and angular momentum is constant when in motion. The experiment sets up an environment that is ideal such that the motion is friction-less and the spring is assumed to be of negligible weight. The friction-less motion is due to air produced from a table which lifts the system slightly off the table. The pucks also hold a magnet on their circumference to avoid collision.

$$I = \frac{1}{2}mr^2 + mR^2 \tag{1}$$

The equation for the moment of inertia of the system. Where m, r, R are the mass of each puck, the distance between the pucks, and the radius of the pucks, respectively. The two pucks are identical and the mass of the spring is neglected.

$$L = I\omega \tag{2}$$

This is the formula for the total angular momentum of the system used the moment of inertia and the angular frequency ω.

$$COM = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}) \tag{3}$$

The coordinates for the center of mass (x,y).

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \tag{4}$$

The distance between the pucks is calculated by from the position.

$$\theta_2 - \theta_1 = \Delta\theta = tan^{-1}(\frac{y_2 - y_1}{x_2 - x_1}) - tan^{-1}(\frac{y_2 - y_1}{x_2 - x_1}) = \omega_{12} \tag{5}$$

The difference in angle calculated using the inverse tan. The two pucks have an angle between them with respect to a reference axis. This is equal to the angular frequency ω_{12}.

$$\frac{1}{\omega} = \frac{mr^2}{2L} + \frac{mR^2}{L} \tag{6}$$

Rearranging equation 2 gives this formula which can be used to plot $1/\omega$ vs r^2.

2 Sample Calculations

Uncertainties were avoided due to the large size of the data. The average mass of the pucks (since they're identical) was 0.27225kg and the radius of the pucks was 4.1cm.

$$\begin{aligned} COM &= (\frac{x_1 + x + 2}{2}, \frac{y_1 + y_2}{2}) \\ &= (\frac{1.33cm + 1.45cm}{2}, \frac{1.2cm + 1.04cm}{2}) \\ &= (1.27cm, 1.12cm) \end{aligned} \tag{7}$$

Sample calculation for the center of mass.

$$\begin{aligned} r &= \sqrt{(1.45cm - 1.33cm)^2 + (1.2cm - 1.04cm)^2} \\ &= 0.2cm \end{aligned} \tag{8}$$

Sample calculation for the distance between the pucks. This stays more or less constant for the duration of the motion.

$$\begin{aligned} L &= (\frac{1}{2}mr^2 + mR^2)\omega \\ &= (\frac{1}{2}(0.27225kg)(0.002m)^2 + (0.27225kg)(0.041m)^2)(0.0024rads/s) \\ &= 1.642kgm^2/s \end{aligned} \tag{9}$$

Sample calculation of angular momentum for a given ω

$$\begin{aligned}\theta_2 - \theta_1 &= \Delta\theta = tan^{-1}(\frac{y_2 - y_1}{x_2 - x_1}) - tan^{-1}(\frac{y_2 - y_1}{x_2 - x_1}) = \omega_{12} \\ &= \Delta\theta = tan^{-1}(\frac{1.0042cm - 1.1803cm}{1.404cm - 1.33cm}) - tan^{-1}(\frac{0.996cm - 1.174cm}{1.387cm - 1.328cm}) = \omega_{12} \\ &= 0.0021rads/s\end{aligned} \tag{10}$$

Sample calculation of the difference in θ to get an angular momentum value between the two positions.

3 Experimental Procedure and Design

The basic setup of the experiment is a system of two pucks connected by a negligible spring on an air table. A camera was used to record the motion of the system which will be used as the data set that will be used to do the analysis for the experiment. Wmcap and LoggerPro software's were used to record and analyse the video, respectively. A Variac was also used to make the air table produce the air flow to make the surface friction-less. Excel was used to analyse the large data set.

A point by point procedure can be outlined via:

1. A program called Wmcap was started and set to record 30 frames per second.

2. The Variac was turned on such that there was air flow on the table to lift the system. The camera was then set to capture and the system was set in motion diagonally across the table.

3. The video was uploaded to LoggerPro which allows points to be set on the video and a scale to be set as the length of the table of 1 meter. Once the scale was set the motion of each puck was captured frame-by-frame by adding points from the beginning after being released from the experimenter until it hit the wall on the opposite side. Once the motion for one of the pucks was recorded. The same procedure was repeated for the joint puck.

4. The coordinates (x1,y1) and (x2,y2) from the two pucks produced a graph with respect to time.

5. The analysis was then started with the use of excel spreadsheets. All the time and position data was copied onto the spreadsheet.

6. From the position data and the center of mass (COM) formula 3, the COM was calculated for all the data and plotted against each other.

7. The distance r using equation 4 was found for all position data.

8. The difference in angle θ could be found by using equation 5, which is the angular frequency between positions 1 and 2. This equation would be adjusted depending on where the pucks are on the coordinate plane.

9. The graph of $1/\omega$ vs r^2 was plotted using the data.

4 Results

One of the objectives of the experiment was to show that the velocity of the motion of this system was constant and the results of the experiment produced a linear line graph, figure 1, for the center of mass over time, which shows that the velocity is constant. The other objective was to show that the total angular momentum is constant, which can be seen in figure 4, with its near constant horizontal line. This line would ideally be horizontal if it wasn't for potential errors.

5 Discussion

5.1 Questions

1) Newton's first law does apply to the motion of the center of mass of this system. This can be seen in the linear line on graph 1. This happens because in free friction-less motion such as this experiment, there are no external forces acting on the system to provide any acceleration so the velocity of the system will be constant.

2) Yes, this can be seen in figure 4. This figure shows the total angular momentum over time and the trend is very close to being constant. The discrepancy would be due to any potential sources of errors.

3) From equation 6, the diver can lower r value to increase their angular momentum. This can be done by crunching the body, which would increase the rotational speed.

4) From figure 5, the equation $1 \times 10^{-6} + 0.032$. The two values (slope and intercept) aren't the same and this would be likely due to the improper adjustments discussed later.

5.2 More Discussion

The results for most of the experiment were similar to what was expected, but the final graph of the experiment as seen in figure 5, did not turn out to be a straight line as predicted. The graph is supposed to be created from the the ω values obtained from equation 5. The values were adjusted depending on the quadrant of the plane the pucks were supposed to be in with respect to the reference axis. Despite adjustments being made to the to the values, this graph seemed to be mostly vertically straight, with a few outlier points.

6 Conclusion

The experiment fulfilled its objectives and showed that the velocity of the motion of this system is constant and the total angular momentum is conserved.

7 Figures

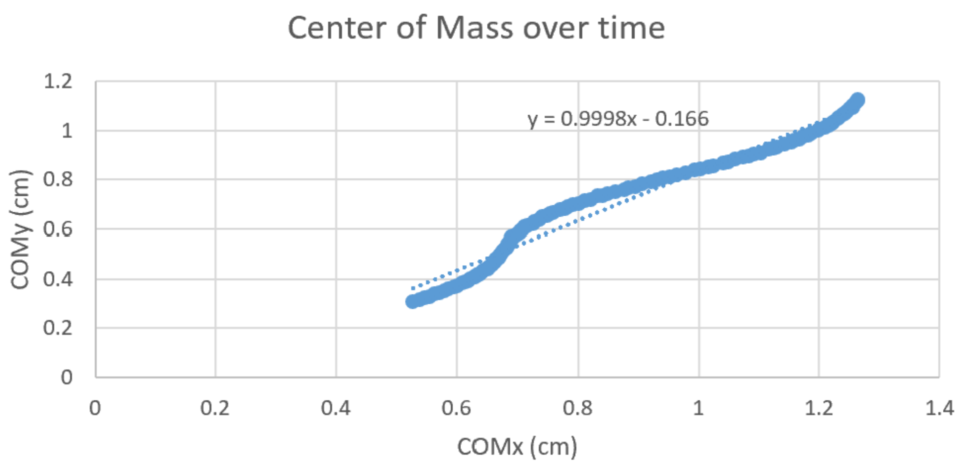

Figure 1: The graph shows coordinates of the center of mass (x,y) plotted against each other.

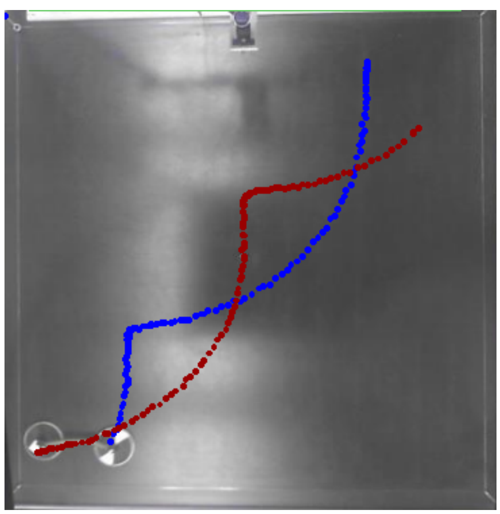

Figure 2: The figure shows the physical system on the air table along with the trail that the two pucks follow as they rotate.

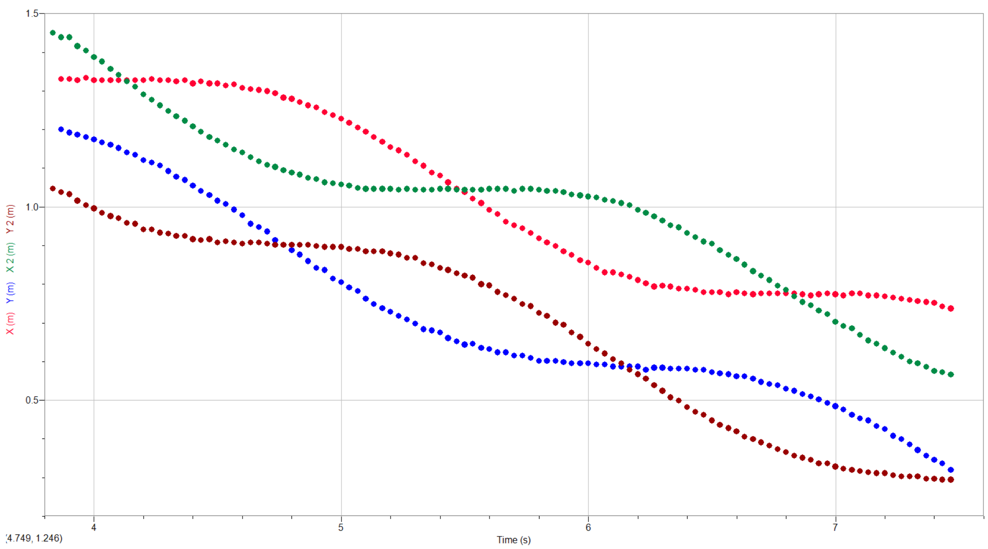

Figure 3: Graph produced by LoggerPro as the coordinates of the two pucks change over time.

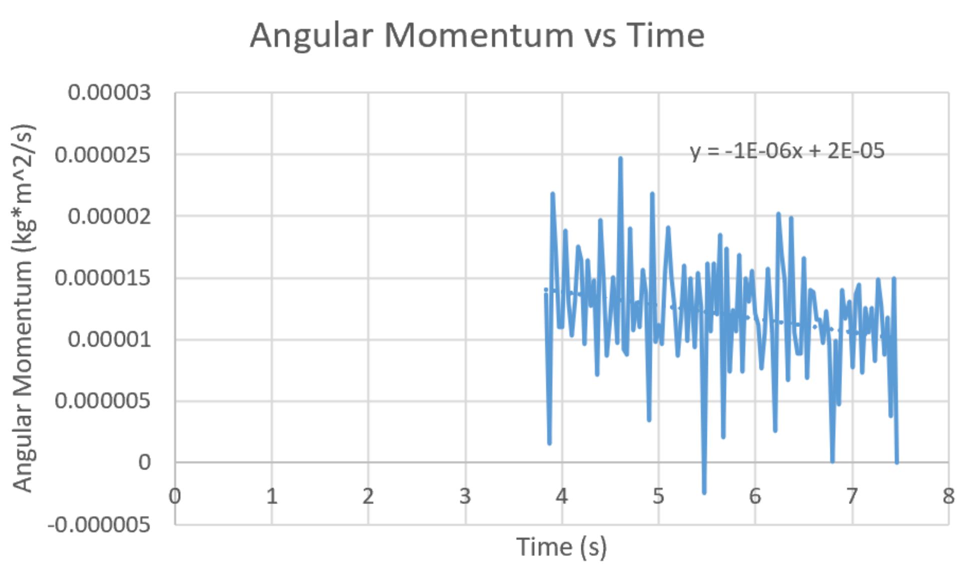

Figure 4: The plot of the total angular momentum vs time graphed using excel and equation 2. This figure also shows that the total angular momentum is constant, this is difficult to see because of the graph is noisy, but the trend is very close to a horizontal line.

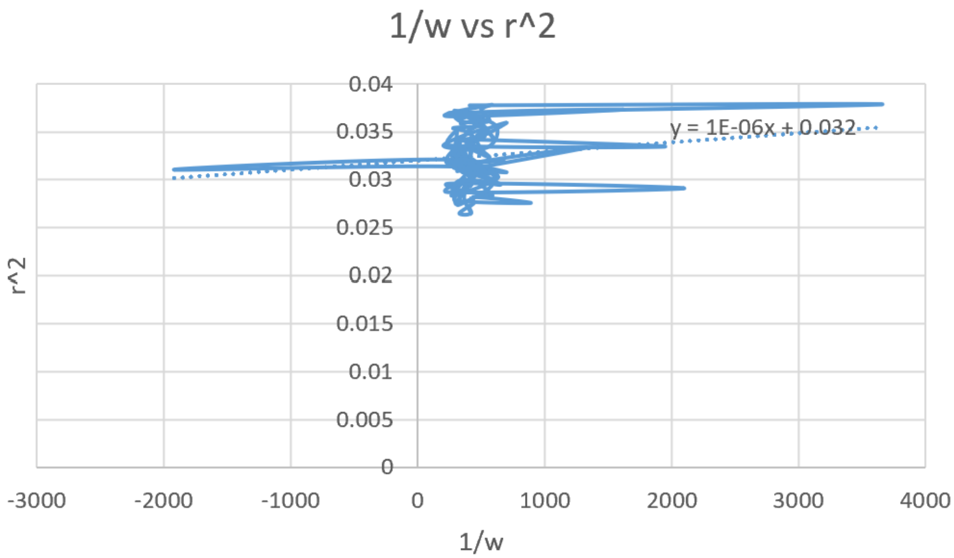

Figure 5: This figure shows the graph of $1/\omega$ vs r^2.