Experiment 12: Harmonic Motion

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May 7, 2024

Abstract

The results for the spring constant was determined to be $0.674 kg/s^2 \pm 0.121 kg/s^2$. *The graphs for undriven motion in air and water show an exponential decrease in amplitude. The graphs for driven frequency and amplitude/phase angle showed that the amplitude increases and peaks at resonance and decreases again. The phase angle stays close to zero, until resonance, then 'jumps' to 180 degrees for driven motion. The* x_1/x_2 *values for undriven air and water was* $0.976 \pm 8.49 \times 10^{-4}$ *and* $0.644 \pm 2.19 \times 10^{-2}$, *respectively.*

1 Introduction

This experiment deal with harmonic oscillation and driven oscillations in air and in water so that the motion is more damped. The objective of the experiment will be to measure the variation of amplitude, period, and phase angle for a spring system with a hanging mass on the bottom. To study harmonic motion for forced and free oscillations in air and water. The spring can be driven from the top with some fixed amplitude that depends a driver that varies with frequency. The experiment measures the amplitude and for undriven and driving oscillations in air and water, respectively, for a fixed number of oscillations. The number of oscillations can be used to get the period using the natural frequency. The spring-mass system in air would oscillate more rapidly than it would in a medium such as water. The force is proportional to the spring constant K, and the displacement x as shown in equation 1. The spring constant K, can also be calculated using equation 2. This experiment uses the same spring for all the measurements, so the spring constant is an important quantity to determine. The quantity x_1/x_2 is the ratio of amplitudes between two successive oscillations. This will be determined for undriven motion for both air and water.

1.1 Equations

$$F = Kx \tag{1}$$

The force in proportional to the spring constant K, and displacement x.

$$K = 4\pi^2 f_0^2 M_T \tag{2}$$

The spring constant calculated using the natural frequency f_0 and total mass M_T.

$$M_T = \frac{1}{3}(M_s) + (M_m) \tag{3}$$

The total mass is calculated using one-third of the mass of the spring plus the mass of the weight on the bottom of the spring. $M_s = 2.76g \pm 0.1g$ and $M_m = 51.62g \pm 0.1g$

$$A(f) = \frac{\frac{F_0}{4\pi^2 M_T}}{\sqrt{(f_0^2 - f_d^2)^2 + (bf_d/2\pi M)^2}} \tag{4}$$

The amplitude for driven oscillations dependent on the natural frequency f_0 and driving frequency f_d.

$$tan\theta = \frac{bf_d/2\pi M}{f_0^2 - f_d^2} \tag{5}$$

The phase angle is dependent on the the natural and driving frequency, as well as mass can a constant of proportionality b.

$$A_0 e^{-bt/2M_T} \tag{6}$$

This is the exponential decrease of amplitude with time. M_T is the total mass, b is the constant of proportionality.

$$\frac{b}{2M_T}T = ln\frac{x_1}{x_2} \tag{7}$$

x_1 and x_2 are amplitudes of any two successive oscillations. x_1 is the amplitude at time t_1 and x_2 is the amplitude at time $t_1 + T$, where T is the period of oscillation.

2 Calculations

$$\begin{aligned} M_T &= \frac{1}{3}(M_s) + (M_m) \\ &= \frac{1}{3}(2.76g \pm 0.1g) + (51.62g \pm 0.1g) \\ &= 52.54g \pm 0.1g \\ &= 0.05254kg \pm 0.0001kg \end{aligned} \tag{8}$$

The total mass, calculated from one-third of the mass of the spring plus the mass of the weight.

$$
\begin{aligned}
K &= 4\pi^2 f_0^2 M_T \\
&= 4\pi^2 (0.57Hz \pm 0.05Hz)^2 (0.05254kg \pm 0.0001kg) \\
&= 0.674kg/s^2 \pm 0.121kg/s^2
\end{aligned} \tag{9}
$$

The spring constant K, calculated using equation 2, using the natural frequency and total mass.

$$
\begin{aligned}
\frac{b}{2M_T}T &= ln\frac{x_1}{x_2} \\
\frac{x_1}{x_2} &= e^{\frac{b}{2M_T}T} \\
\frac{x_1}{x_2} &= e^{-0.02436 \pm 8.750 \times 10^{-4}} \\
\frac{x_1}{x_2} &= 0.976 \pm 0.087\% \\
\frac{x_1}{x_2} &= 0.976 \pm 8.49 \times 10^{-4}
\end{aligned} \tag{10}
$$

This is the ratio $\frac{x_1}{x_2}$ for the undriven air damping. The quantity $\frac{b}{2M_T}T$ comes from the slope of graph 2.

$$
\begin{aligned}
\frac{x_1}{x_2} &= e^{\frac{b}{2M_T}T} \\
\frac{x_1}{x_2} &= e^{-0.4401 \pm 3.384 \times 10^{-2}} \\
\frac{x_1}{x_2} &= 0.644 \pm 3.4\% \\
\frac{x_1}{x_2} &= 0.644 \pm 2.19 \times 10^{-2}
\end{aligned} \tag{11}
$$

This is the ratio $\frac{x_1}{x_2}$ for the undriven water damping. The quantity $\frac{b}{2M_T}T$ comes from the slope of graph 3.

3 Experimental Procedure and Design

The apparatus involves a spring with a mass on the bottom which has the ability to oscillate freely or with a driving force that's attached to the top of the spring-mass system. This experiment is conducted in two mediums, one of them is in air and the other is in water. The first set of measurements were taken in air for undriven and driven motion and the second were taken with the system in water. The driven motion is accomplished by a motor that varies with an adjustable frequency.

A point by point procedure can be outlined via:

3.1 For undriven motion in air and water

1. For undriven motion, the spring-mass system was made to oscillate by raising and releasing the spring. After releasing, the amplitude was recorded for every tenth oscillation. This was done until the hundredth oscillation.
2. For the system in water, the same step was taken as above, but with amplitude was recorded with every oscillation instead of every tenth oscillation. For the measurements, the oscillations were tracked by the observer, and were recorded by a camera to improve accuracy.

3.2 For driven motion in air and water

1. For driven motion, in air, the motor was turned on and set to a low frequency of 0.1Hz and slowly increased to almost 3Hz.
2. As the range of frequency was increased from (0.1Hz to 3Hz), the amplitude and phase angle was recorded. The increase in frequency was in intervals of 0.2Hz, while avoiding the natural frequency of 1.75Hz.
3. The same step as above was taken with the system in water.

3.3 Analysis

1. The analysis produced six graphs plotting the quantities that were measured.
2. The two graphs that were plotted first were the time vs the natural log of amplitude for undriven motion in air and water. This produces a linear line.
3. The next two graphs were plots of the driven frequency and phase angle of the the system in air and water.
4. The final two graphs were plots of the driven frequency and amplitude of the system in air and water.

4 Results

The results of the graphs are as follows: For the graphs of time vs natural log of amplitude of the system in air and water for undriven motion resulted in decreasing linear slopes as seen in figures 2 and 3. For the graphs of the driving frequency vs phase angle the system in air and water for driven motion resulted in graphs that seem constant in phase angle for most points except when the driving frequency is close to the natural frequency (resonance) at about 1.75Hz, where the phase angle 'jumps' from being at the bottom to the top. The phase angle ranges from 0-180 degrees. See figures 4 and 5. For the graphs of the driving frequency vs amplitude the system in air and water for driven motion resulted in graphs that start at a small amplitude then rise in amplitude exponentially and peaking at the resonant frequency, then decreasing exponentially again. See figures 6 and 7. The value for the spring constant was $0.674 kg/s^2 \pm 0.121 kg/s^2$. The results for the ratio of amplitudes for air and water in undriven motion was $0.976 \pm 8.49 \times 10^{-4}$ and $0.644 \pm 2.19 \times 10^{-2}$.

5 Discussion

5.1 undriven motion graph discussion

The decreasing linear lines of figures 2 and 3 show that the amplitude decreases exponentially as time increases. The plots of driving frequency vs phase angle show the phase angle close to zero degrees before the resonance frequency and close to 180 degrees after the resonance frequency. The plots of driving frequency and amplitude show the amplitude spiking at resonance. The degree of damping affects the shape of the amplitude for both driven and non-driven motion. For non-driven motion the amplitude reaches zero about a hundred times for water than for air. For driven motion, The amplitude graphs both look similar, in that they both peak at resonance. The difference is that the amplitude at resonance is almost twice as large for the system in air than for the system in water. The maximum amplitude would occur when the driving frequency matches the natural frequency. This can be seen from analysing equation 4, and can be seen in plots 6 and 7. The experimental resonant frequency was at about $1.75 Hz \pm 0.05 Hz$ as see in the amplitude graphs. At very low and very high frequencies, produce limiting amplitudes $F_0/4\pi^2 M_T f_0^2$ and some amplitude close to zero, respectively from equation 4. The experimental results show this as the graphs for both the mediums confirm this. The limiting phase angles predicted from equation 5 are something close to zero for low frequencies and something close to 180 degrees for a driving frequency that's very high. The experimental results show that this is accurate. There does seem to be a problem with figure 5, where the phase angle decreases at some of the very high frequencies, but this might be due to errors occurring during the experiment. Probably some systematic error that caused the data for high frequencies to deviate from the expected result of somewhere close to 180 degrees. This can probably be fixed by resetting this part of the experiment and retaking the data for phase angle.

6 Conclusion

This experiment was successful in that it accomplished its original objective of studying forced harmonic motion under different dampening conditions to see how the quantities such as amplitude, period, and phase angles are affected.

References

[1] Physics 321A, Laboratory Manual. University of Victoria, 2022.

A Graphs and Data

Data Set

A (mm)	Y	LnA	T air	A water	LnA water	Water os	twater	Df_air (Hz)	phase_air (deg)	drive freq_water (Hz)	p_water (deg)	D_amp_A (mm)	D_amp_W (mm)
37	10	3.611	5.700	54	3.989	1	0.570	0.1	5	0.2	4	2	10
30	20	3.401	11.400	54	3.989	2	1.140	0.3	6	0.3	36	11	10
27	30	3.296	17.100	36	3.584	3	1.710	0.5	3	0.5	33	11	11
23	40	3.135	22.800	24	3.178	4	2.280	0.7	4	0.7	33	12	12
20	50	2.996	28.500	19	2.944	5	2.850	0.9	4	0.9	31	14	14
17	60	2.833	34.200	14	2.639	6	3.420	1.1	3	1.1	25	17	17
17	70	2.833	39.900	13	2.565	7	3.990	1.3	2	1.3	22	22	23
13	80	2.565	45.600	9	2.197	8	4.560	1.5	2	1.5	33	39	41
12	90	2.485	51.300	9	2.197	9	5.130	1.6	2	1.7	93	65	62
10	100	2.303	57.000	6	1.792	10	5.700	1.65	3	1.9	163	108	35
								2	170	2.1	173	40	19
								2.2	176	2.3	155	20	12
								2.4	176	2.5	137	10	9
								2.6	170	2.7	110	9	7
								2.9	170	2.9	180	5	6

Figure 1: The data set containing all the quantities measured in the experiment.
A-amplitude for undriven motion in air
Y-number of oscillations for undriven motion in air
LnA-natural log of the amplitude (undriven in air)
T air- time calculated by (of oscillations air * 0.57Hz)
A water- amplitude of motion in water (undriven)
LnA water - natural log of amplitude of motion in water
water os - number of oscillations in water
twater - time calculated by (of oscillations water * 0.57Hz)
Df air (hz) - driving frequency used to get phase angle and amplitude in air
phase air(deg) - phase angle for a given driven frequency in air
Drive freq water (hz) - driving frequency used to get phase angle and amplitude in water
p-water - phase angle for a given frequency in water
D-amp-A (mm) - Amplitude for a given driving frequency in air
D-amp-W (mm) - Amplitude for a given driving frequency in water

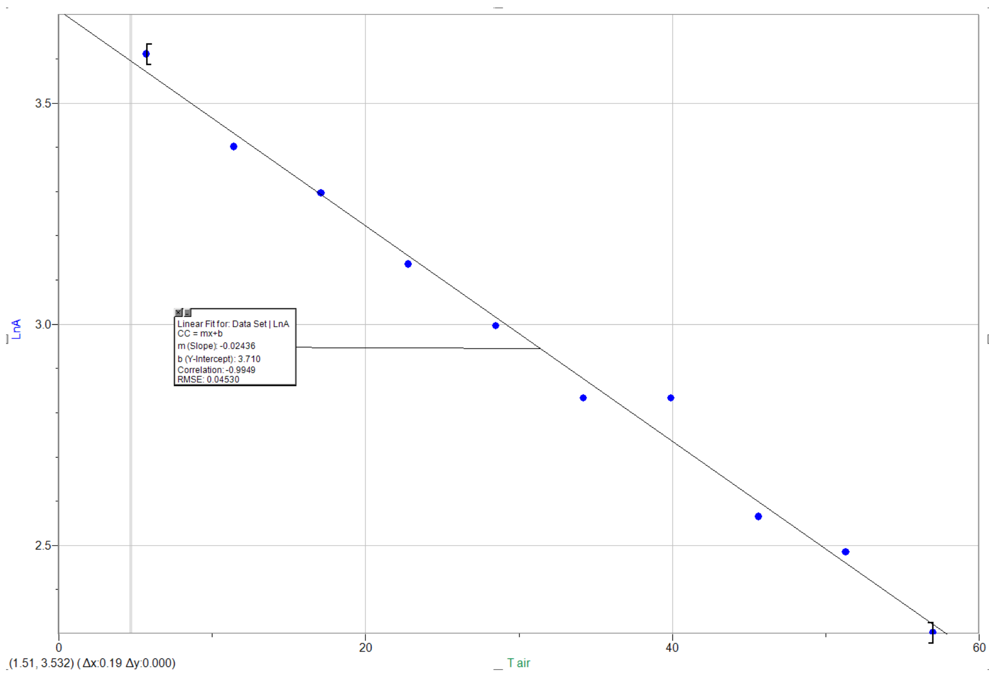

Figure 2: The graph of time vs the natural log of the amplitude (mm) for undriven motion in air.

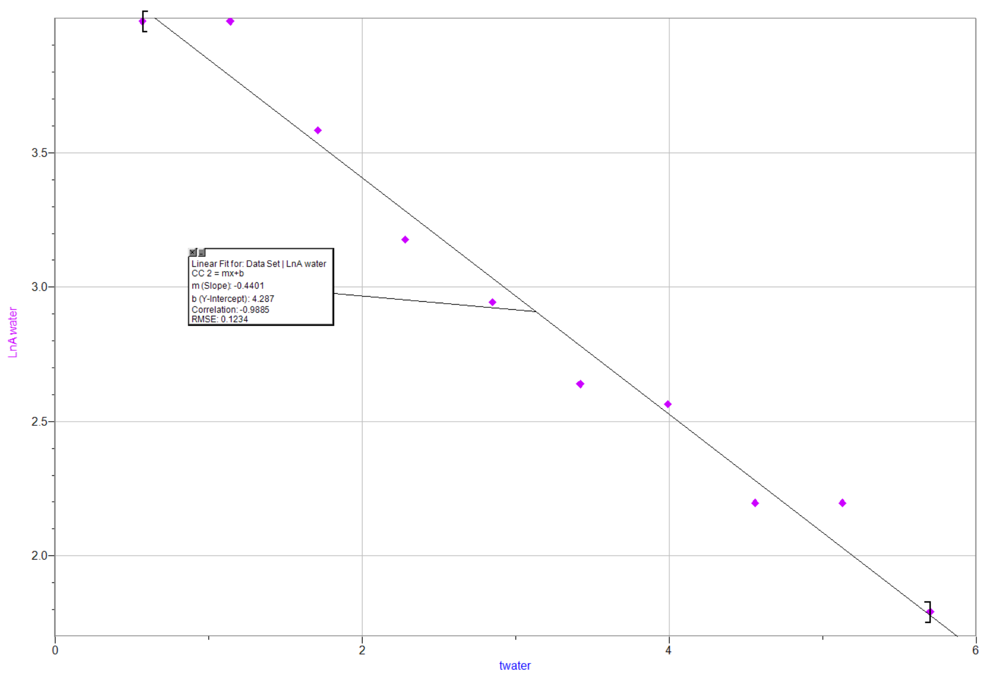

Figure 3: The graph of time vs the natural log of the amplitude (mm) for undriven motion in water.

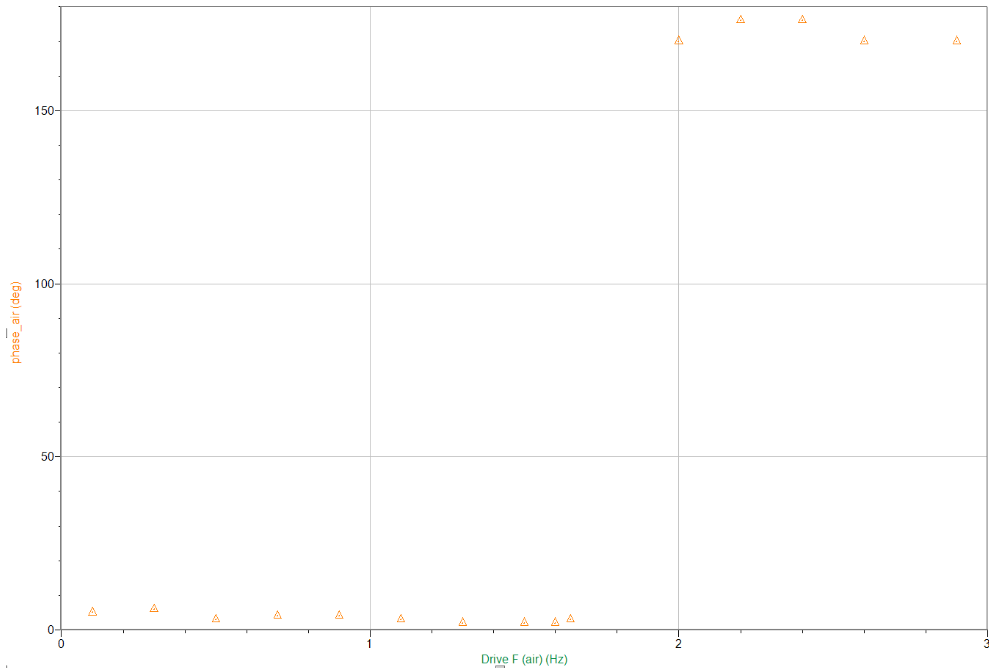

Figure 4: The graph of the driving frequency (Hz) vs the phase angle in degrees in air.

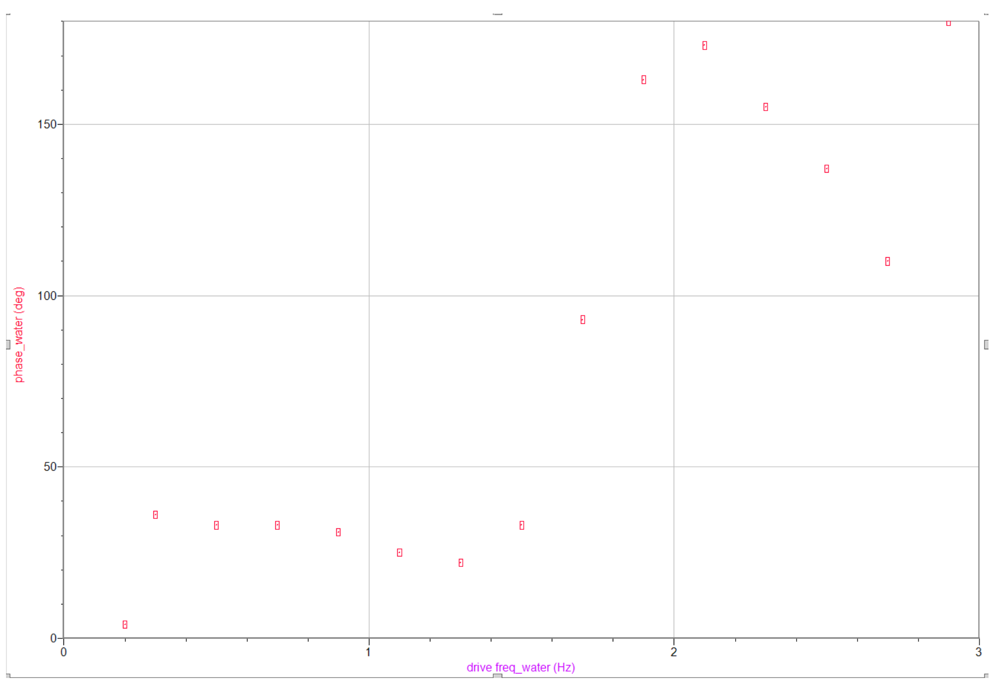

Figure 5: The graph of the driving frequency (Hz) vs the phase angle in degrees in water.

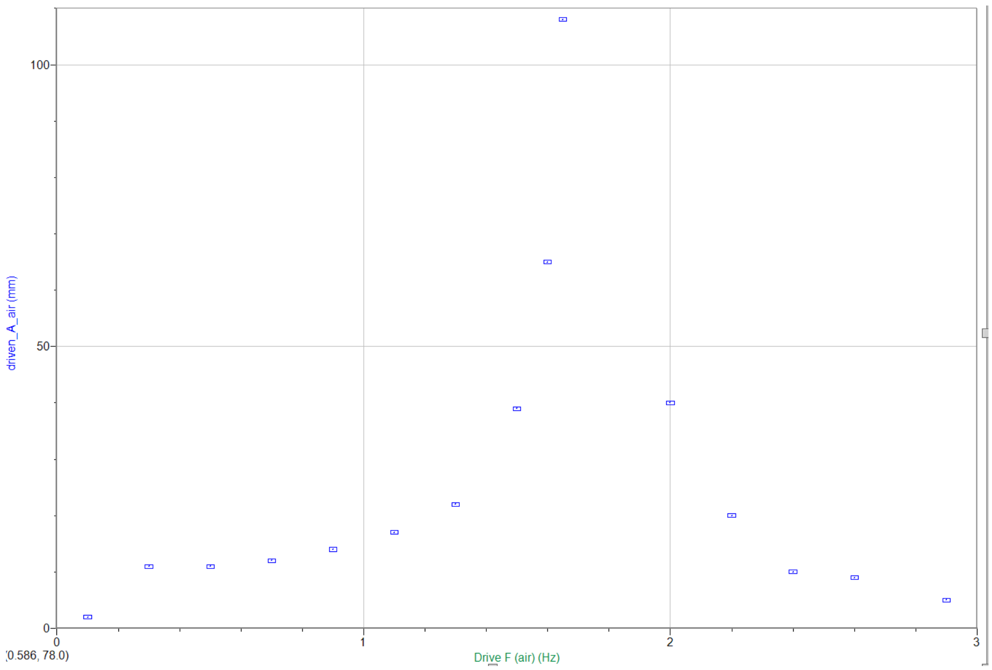

Figure 6: The graph of the driving frequency (Hz) vs the amplitude (mm) in air.

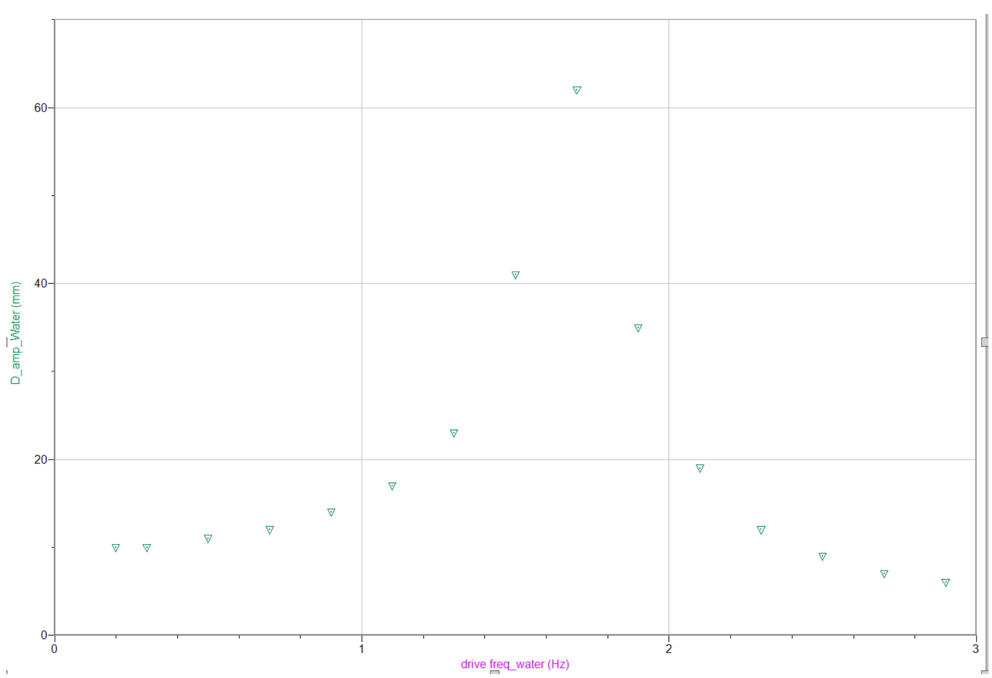

Figure 7: The graph of the driving frequency (Hz) vs the amplitude (mm) in water.