Experiment 5: Stirling Cycle

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Abstract

The objective of this experiment was to study the Stirling cycle using a Stirling engine. To check its efficiency from its power output and compare it to a theoretical value by testing the unloaded engine and the engine with a resistor and a bulb as a load. The Efficiency for the unloaded and resistor were determined to be 1.1 $\% \pm 0.2\%$ *and 0.742* $\% \pm 0.1\%$*, respectively, with a theoretical efficiency of 32.7* $\%$.

1 Introduction

The Stirling cycle consists of two isotherms and two isochoric sections on a P-V diagram. Positive work is done in the clockwise direction and negative work done us done counter-clockwise. Comparing this cycle to the Carnot cycle, which has two adiabatic sections and two isotherms, we can see that both cycles use different routes to get to a different temperature. The Carnot cycle uses an adiabatic process and the Stirling cycle uses an isochoric process.

The Stirling engine used for this experiment has an alcohol burner as a heat source under the piston. The apparatus has a flywheel which could be turned to get the engine running. The experiment ran the engine for 20 seconds. The first 10 seconds was used to measure the pressure and volume across time without the presence of a load, and the last 10 seconds was used to measure the quantities with a load (a 100Ω resistor and a light bulb).

1.1 Equations

$$\eta = \frac{P_1}{P_2} \tag{1}$$

Equation 1 is a ratio of power output P_1 and power input P_2 of the Stirling engine used in the experiment to give us the efficiency.

$$\eta = 1 - \frac{T_1}{T_2} \tag{2}$$

Equation 2 is the theoretical efficiency where the two temperatures of the isotherms are T_1 and T_2 are in Kelvin and $T_2 > T_1$.

$$P_1 = f \times W \tag{3}$$

Equation 3 is the power output determined through the product of the work, W, acquired from the area of the loop in a P-V plot and the rotational frequency, f, (cycles/second) acquired from the period.

$$T = \frac{t_2 - t_1}{N} \tag{4}$$

Equation 4 shows how the period was acquired using two times at two different peaks and taking their difference $(t_2 - t_1)$ and dividing by the number of cycles, N.

$$P_{lost} = \frac{V^2}{R} \tag{5}$$

Equation 5 gives us the power lost through the resistor, where V is the voltage in volts and R is the resistance in Ohms.

1.2 Calculations

$$\begin{aligned} T_{unloaded} &= \frac{1.795s - 0.130s}{14} \\ &= 0.1189s \\ f_{unloaded} &= \frac{1}{T_{unloaded}} \\ &= \frac{1}{0.1189s} \\ &= 8.41Hz \end{aligned} \tag{6}$$

$$\begin{aligned} T_{resistor} &= \frac{19.910s - 18.115s}{11} \\ &= 0.1632s \\ f_{resistor} &= \frac{1}{T_{resistor}} \\ &= \frac{1}{0.1632s} \\ &= 6.128Hz \end{aligned} \tag{7}$$

The above calculations show the period and the frequency for the unloaded and the resistor loaded experiment.

$$\begin{aligned} P_{1,unloaded} &= f_{unloaded} \times W_{unloaded} \\ &= 8.41hz \times 0.1787J \\ &= 1.502W \end{aligned} \tag{8}$$

$$\begin{aligned} P_{1,resistor} &= f_{resistor} \times W_{resistor} \\ &= 6.128hz \times 0.1697J \\ &= 1.039W \end{aligned} \tag{9}$$

The above calculations show the power output for the unloaded and the resistor loaded experiment.

$$\begin{aligned} \eta_{unloaded} &= \frac{P_1}{P_2} \\ &= \frac{1.502W}{140W \pm 20W} \\ &= 0.011 \pm 0.002 \\ &= 1.1\% \pm 0.2\% \end{aligned} \tag{10}$$

$$\begin{aligned} \eta_{resistor} &= \frac{P_1}{P_2} \\ &= \frac{1.039W}{140W \pm 20W} \\ &= 0.00742 \pm 0.001 \\ &= 0.742\% \pm 0.1\% \end{aligned} \tag{11}$$

The above calculations show the experimental efficiency for the unloaded and the resistor loaded experiment.

$$\begin{aligned} \eta &= 1 - \frac{T_1}{T_2} \\ &= 1 - \frac{305.15K}{453.15K} \\ &= 0.3266 \\ &= 32.7\% \end{aligned} \tag{12}$$

The above calculation shows the theoretical efficiency from the two temperatures collected.

$$\begin{aligned} P_{lost} &= \frac{V^2}{R} \\ &= \frac{2.152V^2}{100\Omega} \\ &= 0.0463W \end{aligned} \tag{13}$$

$$\begin{aligned} P_{1,unloaded} - P_{1,resistor} &= 1.502W - 1.039W \\ &= 0.463W \end{aligned} \tag{14}$$

The above calculations show the power that's theoretically supposed to dissipated through the resistor, and the power that's actually lost via the difference in the power output from the unloaded and the loaded engine. This shows that 10 times as much power was lost in the actual experiment than theoretically supposed to be lost.

$$\begin{aligned} 1pa \times 1m^3 &= \frac{kg}{m \times s^2} \times m^3 \\ &= \frac{kg \times m^2}{s^2} \\ &= 1J \end{aligned} \tag{15}$$

The above calculation shows through dimensional analysis how the area within the P-V diagram gives us the work in Joules. Since units of pressure times units of volume will equal units of work.

2 Experimental Procedure and Design

This experiment will be conducted using a Stirling engine to collect temperature and pressure data. The apparatus has a pressure sensor, two temperature sensors, and a potentiometer to measure the displacement of the working piston of the engine. The LabPro unit is used to measure the piston displacement and the pressure to provide instantaneous data for both with respect to time. The data can also be used to plot a P-V diagram, which can be used to get a value for the work done. LoggerPro software was used to track the data acquired from the LabPro unit.

A point by point procedure can be outlined via:

1. To start, the LabPro input needed to be calibrated as well as some setup for LoggerPro. The experiment was set to collect 200 samples/sec for 20 seconds. The

LabPro recorded the potential across a slide-wire to get the displacement, but for the experiment required a reading of volume. This was easily calibrated on LoggerPro by setting the max and min values for the volume measured from the max and min positions of the slide-wire. The lowest volume was $32cm^3$ and the max volume was set to $44cm^3$.

2. The heat source was then lit under the piston and the temperature began to rise. The flywheel was then rotated clockwise slowly to get the engine going after the temperature reading on for the heat source exceeded 140 degrees Celsius.

3. The hot and cold temperatures were recorded when the temperature at the hot end reached 180 degrees Celsius and the LoggerPro started to record the data.

4. The first 10 seconds were used to record the data without any load. After it was done, the last 10 seconds was used to record the data with a 100Ω resistor as the load. When data for both were finished recording, a light bulb was set to be the load for observational purposes.

5. The P-V diagrams could be plotted by taking one cycle from the unloaded and the loaded parts of the experiment and taking that data to plot the two diagrams.

6. The period was obtained by using the pressure vs time graph, which consisted of lots of cycles, and measured the difference in time from two respective peaks and dividing by the number of cycles. See equation 4. From the period, the frequency was calculated.

3 Results

The final results for the unloaded and the loaded (resistor) efficiency were $1.1\% \pm 0.2\%$ and $0.742\% \pm 0.1\%$, respectively. A theoretical efficiency was calculated to be 32.7%. The results for other values determined in the experiment are shown in the table below. The power lost in the resistor was also determined to be 10 times what's expected to be lost.

Table 1: Data and results

Quantity	Value
$f_{unloaded}$	8.41Hz
$f_{resistor}$	6.128Hz
$P_{1,unloaded}$	1.502W
$P_{1,resistor}$	1.039W
$\eta_{unloaded}$	$1.1\% \pm 0.2\%$
$\eta_{resistor}$	$0.742\% \pm 0.1\%$
Steady state voltage	2.152V
Resistance	100Ω
P_2	$140W \pm 20W$
T_1	305.15 K
T_2	453.15 K
$\frac{V^2}{R}$	0.0463W
Power lost	0.463W

4 Discussion

The Stirling engine here was used to test the efficiency of the Stirling cycle by comparing the experimentally determined efficiency with the theoretical efficiency possible from the Carnot cycle. The results as stated previously show that the efficiency for the unloaded and the resistor loaded engine to be $1.1\% \pm 0.2\%$ and $0.742\% \pm 0.1\%$, respectively. This compared to the theoretical of 32.7% shows that there is a large difference in the efficiency of this engine and what's its theoretical case. This is to be expected as the engine used for the experiment was not utilizing its heat source (alcohol burner) to its full capability. The flame from the source was only partly being used to work the piston, while the rest of the energy from it was dissipated in the air. The power lost in the resistor was 0.463W, which is 10 times higher than was is expected from equation 5 because of the engine's inefficiency. In observing the engine while it powered the light bulb, it was noticed that the frequency dropped slightly more than it did when it dropped going from the unloaded to the resistor loaded case. The light bulb used more power than the resistor, and it's difficult to determine how much more power the light bulb used. This is because unlike the resistor case where the resistance is known, we don't know the resistance that's in the light bulb, which we need to calculate the power lost.

5 Conclusion

The final results for the experiment were as expected for the Stirling engine and the inefficiency of the engine explains why results acquired.