Experiment 7: Speed of Light

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May 7, 2024

Abstract

This experiment determines the speed of light using two different methods. One using a rotating Michelson mirror and the other using using time of flight method, both to be explained in the introduction. The speed of light determined from the rotating mirror method was $300785098.9m/s \pm 17746320.84m/s$ *which was consistent with the theoretical value of 299792458 m/s. The time of flight method determined the speed of light for three different distances, the result being* $323773584.9m/s \pm 12303396.23m/s$, $307807692.3m/s \pm 11669243.9m/s$, $307085365.9m/s \pm 15047182.93m/s$, *for each distance. The first two distances being inconsistent with the theoretical value and the last being consistent.*

1 Introduction

The speed of light will be determined using two different methods:

1.1 Method 1: Michelson rotating mirror

The rotating mirror method apparatus here consists of a laser travelling through a beam splitter to a rapidly rotating mirror. The rotating mirror reflects the light through a lens towards a mirror M_1 which reflects it to mirror M_2. The light then follows this path back towards the beam splitter, which reflects the beam towards the eyepiece attached to a travelling microscope. We also have the original light sent through the beam splitter to the eyepiece to be observed. Now, we have two rays converging on the eyepiece, one from the laser to the eyepiece, and the other takes the path mentioned reflecting off of the rotating mirror all the way to M_2 and back to the eyepiece (see figures 3 and 4 for reference). This longer travelled path will be shifted to the right since the mirror is rotating clock-wise. This is because as the light travels the path with the rotating mirror to M_2 and back, the rotating mirror would have rotated by some angle θ and 2θ as the light returns. What is observed on from the eyepiece is the displacement, y, of the laser to the right since the mirror is rotating clock-wise. This displacement can be measured using the travelling microscope. This displacement increases depending on the rotation speed of the mirror. The faster the frequency at which the mirror rotates will farther displace the image observed in the eyepiece, since the angle is directly proportional to the displacement as shown in equation 1. The speed of light is simply (distance/time) where the time can be determined using equation 2 and the distance is the length the light travels from the rotating mirror to M_2 and back, which is 2a, where, a is the distance from the rotating mirror to M_2. A linear curve of the displacement as a function of frequency can be made by obtaining the displacement for various incremental values of the rotational frequency of the mirror. The slope of the line will be the quantity (y/f), which can be used in equation 3.

1.2 Time of Flight Method

The second method to determine the speed of light will the time of flight for light to go three different distances and return. A laser will be used to send light through a beam splitter mirror to a detector which can be considered a 'starting position' while the laser goes through the beam splitter to a top mirror, which reflects the beam to a distant mirror and comes back through a lens to be recorded on another detector (see figure 5).

1.3 Equations (Rotating Mirror Method)

$$2\theta = \frac{y}{b} \tag{1}$$

Equation 1 shows the angle θ as the mirror rotates as mentioned in the introduction and it's relation to the quantities y and b, which are the deviation of the image on the eyepiece measured by the travelling microscope and the distance between the eyepiece and the rotating mirror, respectively.

$$t = \frac{\theta}{2\pi f} = \frac{y}{4\pi b f} \tag{2}$$

Equation 2 is the equation for time, t, and its relation to θ and the frequency, f, which is half of the frequency obtained from the Oscilloscope when selecting different frequencies for the mirror rotation with the Variac (see procedure for details). The time, t, is also the time is takes for the light to travel from the rotating mirror to M_2 and back to the rotating mirror.

$$v = \frac{2a}{t} = \frac{8\pi a b f}{y} \tag{3}$$

Equation 3 is the experimental speed of light obtained from the Michelson rotating mirror method. The quantity, v, is the speed of light and, a, is the distance from the rotating mirror to the mirror M_2.

1.4 Equations (Time of flight method)

$$v = \frac{2d}{\Delta t} \tag{4}$$

The above equation is the basic distance over time equation used to measure the speed of light for a certain distance it travels in a certain amount of time from the starting position to the ending position. The different in time here is divided by 2 since we only need the time for which the light travels one way to the mirror, and not the return trip.

2 Experimental Procedure and Design

2.1 Michelson Rotating mirror method procedure

The Michelson rotating mirror apparatus was set up (refer to figures 3 and 4) such that the laser reflects and passes through the lenses and polarizer forming the image seen on the eyepiece in figure 6. Any adjustments that needed to be made to get the image, were made by using the appropriate adjustable settings as seen on figure 4.

A point by point procedure can be outlined via:

1. The Variac was set to 50Hz and the position on the travelling microscope was set to 0mm. Increasing the Variac frequency to 100Hz, the displacement of the image was measured and recorded.
2. The frequency was turned by increments of 50Hz and the displacement was recorded for a maximum frequency of 600Hz.
3. Steps 1 and 2 were repeated twice for a total of two data sets.
4. The graph of the displacement vs frequency, f (where f = (Variac frequency/2), was plotted for both data sets on the same graph in order to obtain the slope. The slope as stated is the quantity (y/f). See figure 1
5. Finally, the speed of light was determined using equation 3.

2.2 Time of Flight Method procedure

The time of flight method measures the time it takes light to travel three different distances. A laser was used to send pulses of light as shown in figure 5 such that an oscilloscope could be used to time the flight of the light. Figures 7 and 8 show how the oscilloscope is used to measure the time of flight for the first and second distances.

1. Starting with the first distance the light will travel, which is the shortest, the distance from the apparatus to the first door (where the first mirror was attached) was measured using a separate laser distance measuring device.
2. Once, the distance was measured, the laser was sent from the apparatus to the mirror and back and recorded on the oscilloscope. If the mirror wasn't aligned properly, then adjusting it was a simple task such that the laser bounces back to the apparatus.

3. The difference in time was measured and recorded and from the oscilloscope and equation 4 was used to determine the speed of light. It should be noted that this time measured is twice the time needed since the light travels to and from the mirror. Therefore, the difference in time needs to be divided by two.

4. The above steps were repeated for a second longer distance and a third longest distance to determine the speed of light from three different distances.

5. There were some corrections made to get a more accurate result. These corrections involved subtracting the distance that the laser travels from the laser to the first photo detector and adding the distance from the top mirror to the second photo detector. The mirrors were also attached a small distance away from the door at which the original distance was measured from the laser measuring device. So, a correction to the distance was made by subtracting this small distance from the original measured distance using the laser. Overall, this should improve the accuracy of the speed of light found.

3 Calculations

3.1 Rotating mirror calculations

$$\begin{aligned} v &= \frac{2a}{t} = \frac{8\pi abf}{y} \\ &= \frac{8\pi(15.46m \pm 0.01)(6.58m \pm 0.01m)}{8.5 \times 10^{-6}m/Hz \pm 5 \times 10^{-7}m/Hz} \\ &= 300785098.9m/s \pm 5.9\% \\ &= 300785098.9m/s \pm 17746320.84m/s \end{aligned} \tag{5}$$

$$\begin{aligned} \text{Consistency check} &= (experimental - theoretical) \leq 17746320.84m/s \\ &= (300785098.9m/s - 299792458m/s) \leq 17746320.84m/s \\ &= 992640.9m/s \leq 17746320.84m/s \\ \textbf{consistent} \end{aligned} \tag{6}$$

$$\begin{aligned} f &= \frac{\text{variac frequency}}{2} \\ &= \frac{100Hz}{2} \\ f &= 50Hz \end{aligned} \tag{7}$$

3.2 Time of Flight calculations

$$\begin{aligned} v &= \frac{2d}{\Delta t} \\ &= \frac{17.16m \pm 0.0005m}{53ns \pm 5\%} \\ &= 323773584.9m/s \pm 3.8\% \\ &= 323773584.9m/s \pm 12303396.23m/s \end{aligned} \tag{8}$$

$$\begin{aligned} \text{Consistency check} &= (experimental - theoretical) \leq 12303396.23m/s \\ &= (323773584.9m/s - 299792458m/s) \leq 12303396.23m/s \\ &= 23981126.9m/s \leq 12303396.23m/s \\ \textbf{inconsistent} \end{aligned} \tag{9}$$

$$\begin{aligned} v &= \frac{2d}{\Delta t} \\ &= \frac{24.009m \pm 0.0005m}{78ns \pm 5\%} \\ &= 307807692.3m/s \pm 3.8\% \\ &= 307807692.3m/s \pm 11669243.9m/s \end{aligned} \tag{10}$$

$$\begin{aligned} \text{Consistency check} &= (experimental - theoretical) \leq 11669243.9m/s \\ &= (307807692.3m/s - 299792458m/s) \leq 11669243.9m/s \\ &= 8015224.3m/s \leq 11669243.9m/s \\ \textbf{inconsistent} \end{aligned} \tag{11}$$

$$\begin{aligned} v &= \frac{2d}{\Delta t} \\ &= \frac{50.362m \pm 0.0005m}{164ns \pm 5\%} \\ &= 307085365.9m/s \pm 4.9\% \\ &= 307085365.9m/s \pm 15047182.93m/s \end{aligned} \tag{12}$$

$$
\begin{aligned}
\text{Consistency check} &= (experimental - theoretical) \leq 15047182.93m/s \\
&= (307085365.9m/s - 299792458m/s) \leq 15047182.93m/s \\
&= 7292907.9m/s \leq 15047182.93m/s \\
\textbf{consistent} &
\end{aligned} \tag{13}
$$

4 Figures and Graphs and Tables

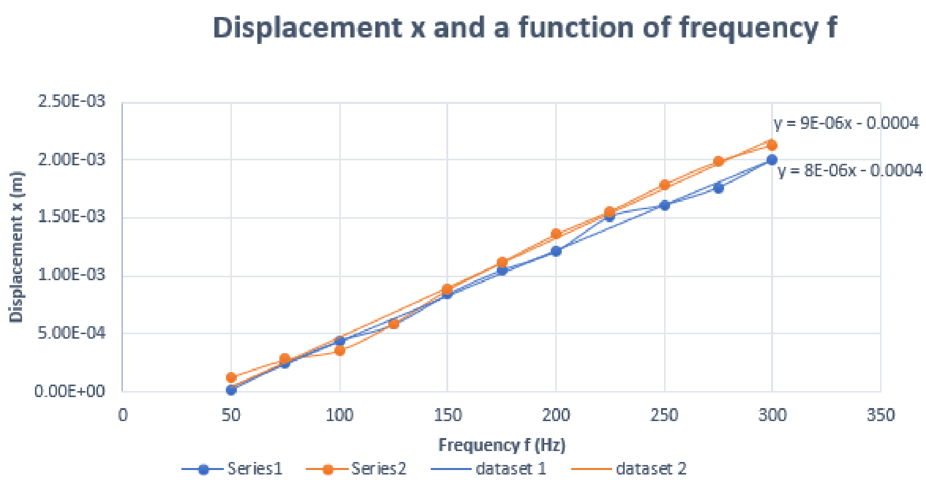

Figure 1: The graph shows the displacement as a function of the frequency from the Michelson rotating mirror part of the experiment. The two linear curves are plotted here from the two displacement data sets acquired from the experiment. The slopes for both of the graphs turn out to be $8 \times 10^{-6}m/Hz$ from trial 1 data, and $9 \times 10^{-6}m/Hz$ form trial 2 data.

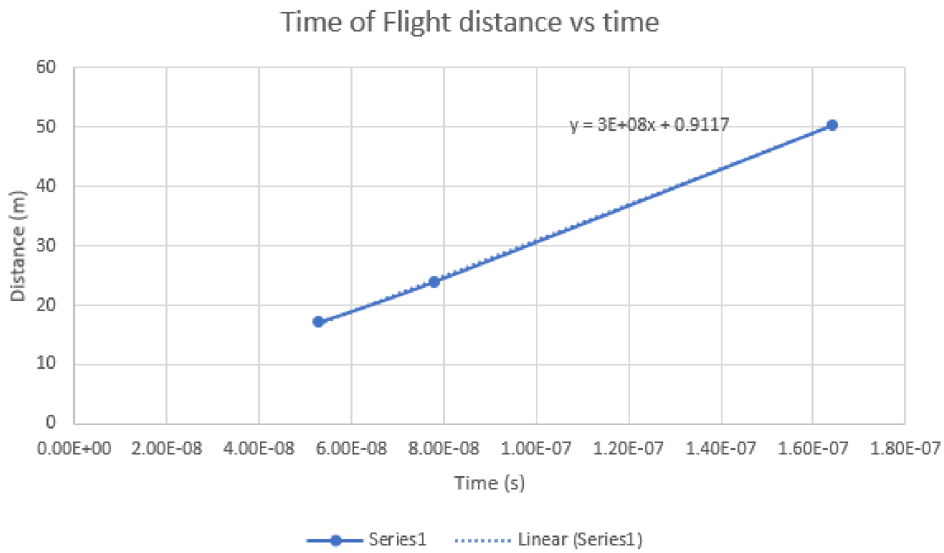

Figure 2: This graph plots the linear curve that is produced by plotting the distance as a function of time. The slope of the graph shows the speed of light.

Table 1: The table shows the displacement values for a given Variac frequency for two trials. Note, that the Variac frequency is twice the frequency of the rotating mirror since the mirror is double sided. Therefore, the linear plot of will be of displacement as a function of frequency, f, and not the Variac frequency.

2f = Hz (Variac)	Displacement Trial 1	Displacement Trial 2
100	0.02mm	0.13mm
150	0.25mm	0.28mm
200	0.44mm	0.36mm
250	0.58mm	0.59mm
300	0.84mm	0.88mm
350	1.05mm	1.12mm
400	1.21mm	1.36mm
450	1.51mm	1.59mm
500	1.61mm	1.79mm
550	1.76mm	1.99mm
600	2.00mm	2.13mm

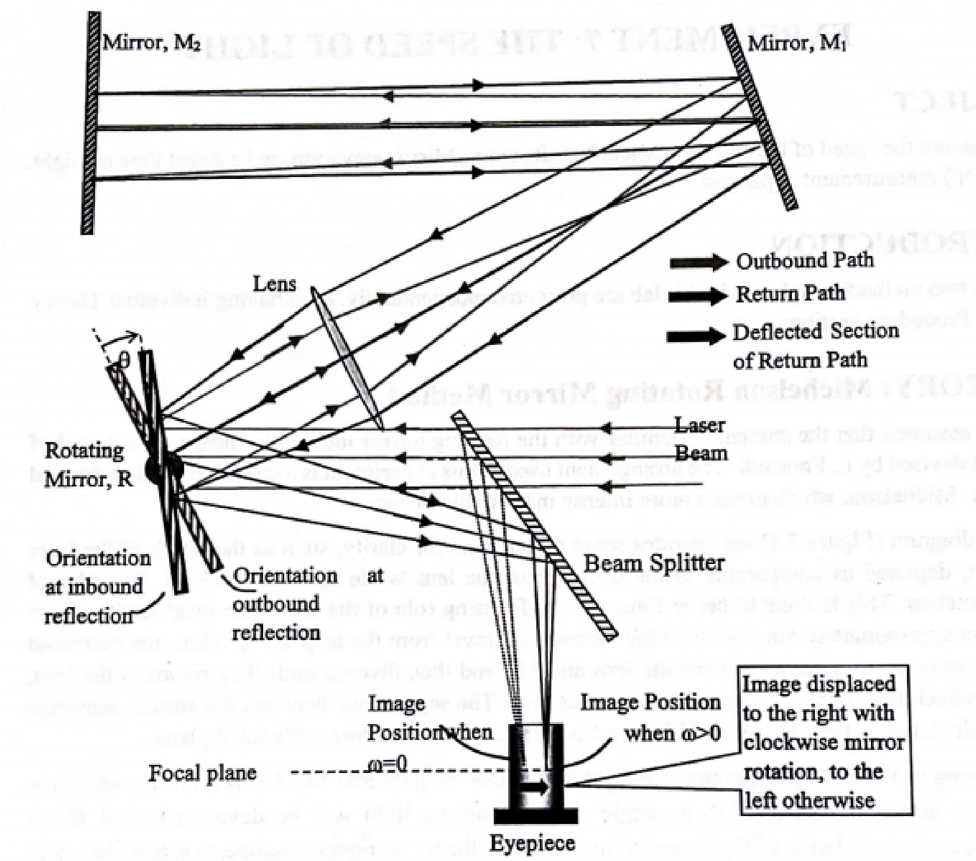

Figure 3: The diagram shows the pathway the laser beam travels from the laser to splitting at the beam splitter, creating two images at the eyepiece. One goes straight to the eyepiece, while the other path travels to the rotating mirror to M_2 and back to the eyepiece.

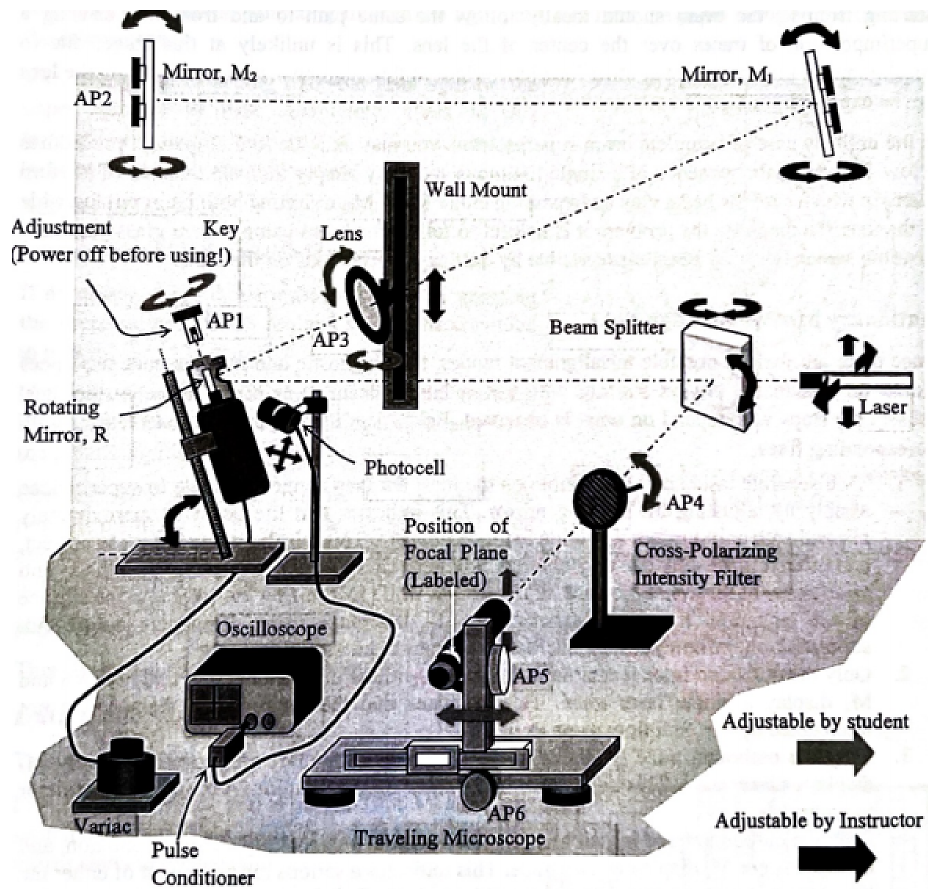

Figure 4: Shows a more detailed diagram of the Michelson apparatus with the equipment used and basic layout of the system as well as how the apparatus can be adjusted.

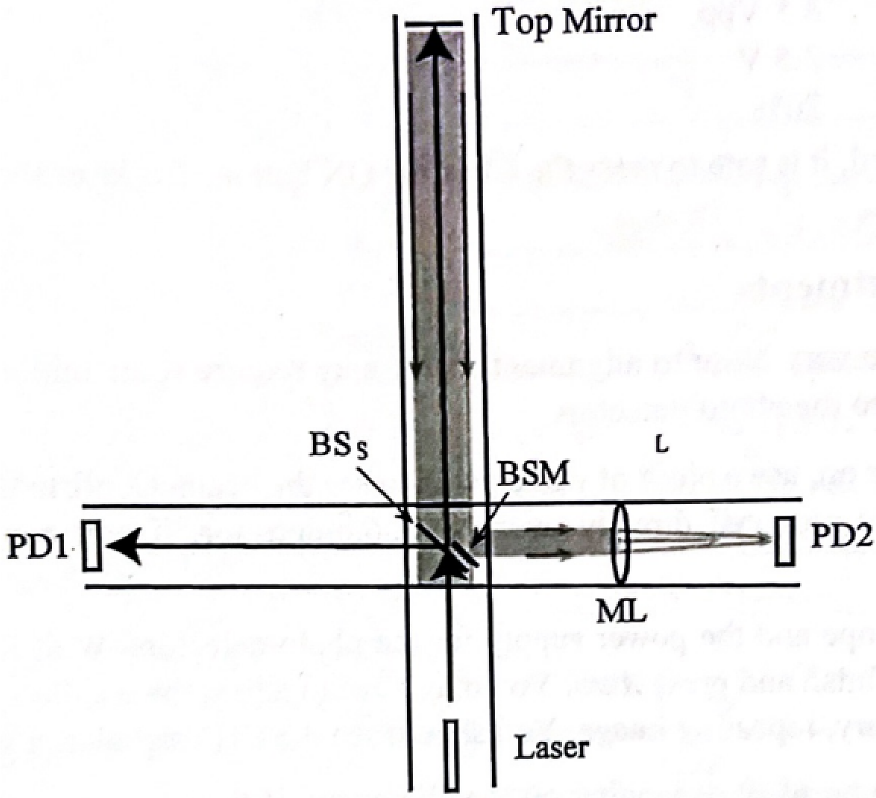

Figure 5: The figure shows the light from the laser travel and reflect off of the beam splitter mirror (BSM) to the first photo detector (PD1) and also a beam travelling to a top mirror which then would reflect light to a distance mirror, which would then return it to the top mirror and direct it to the BSM and through a movable lens (ML) to a second photo detector (PD2).

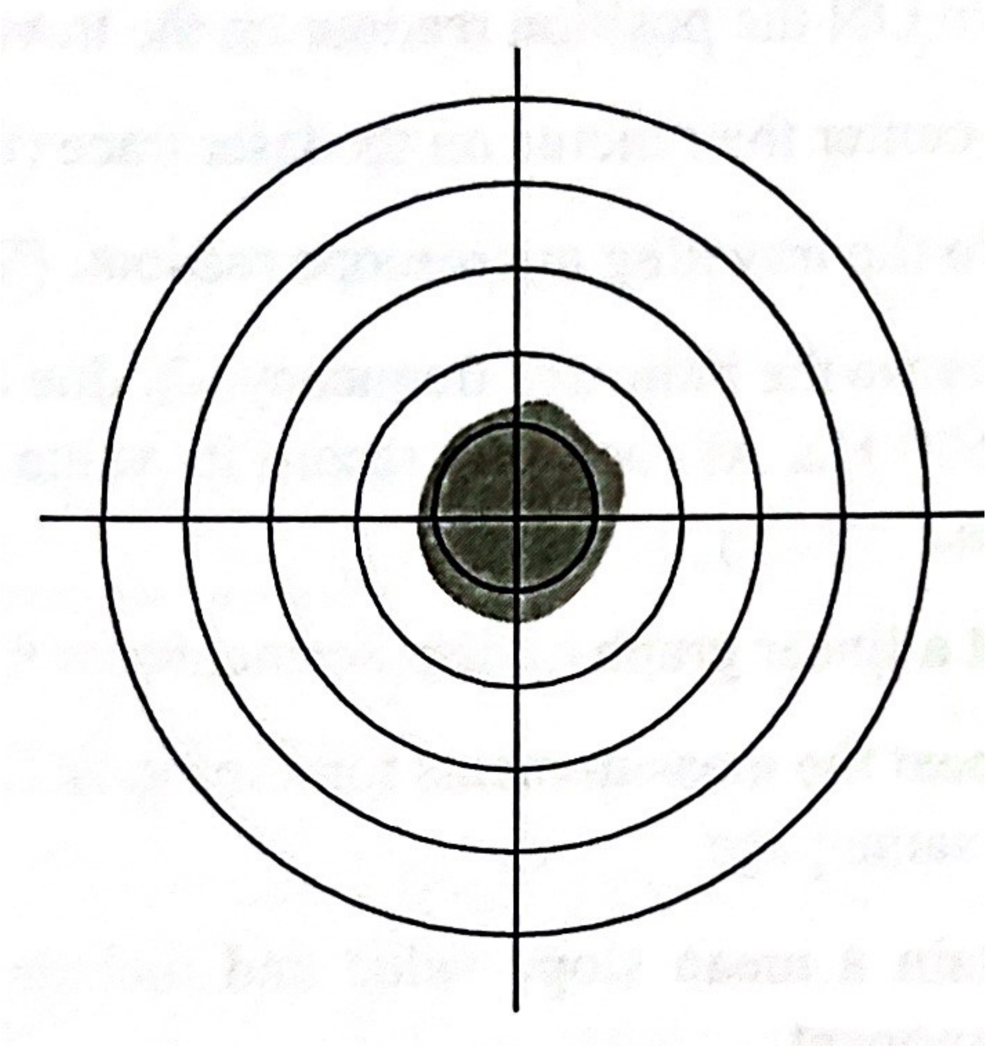

Figure 6: The image one would see looking into the eyepiece of the travelling microscope.

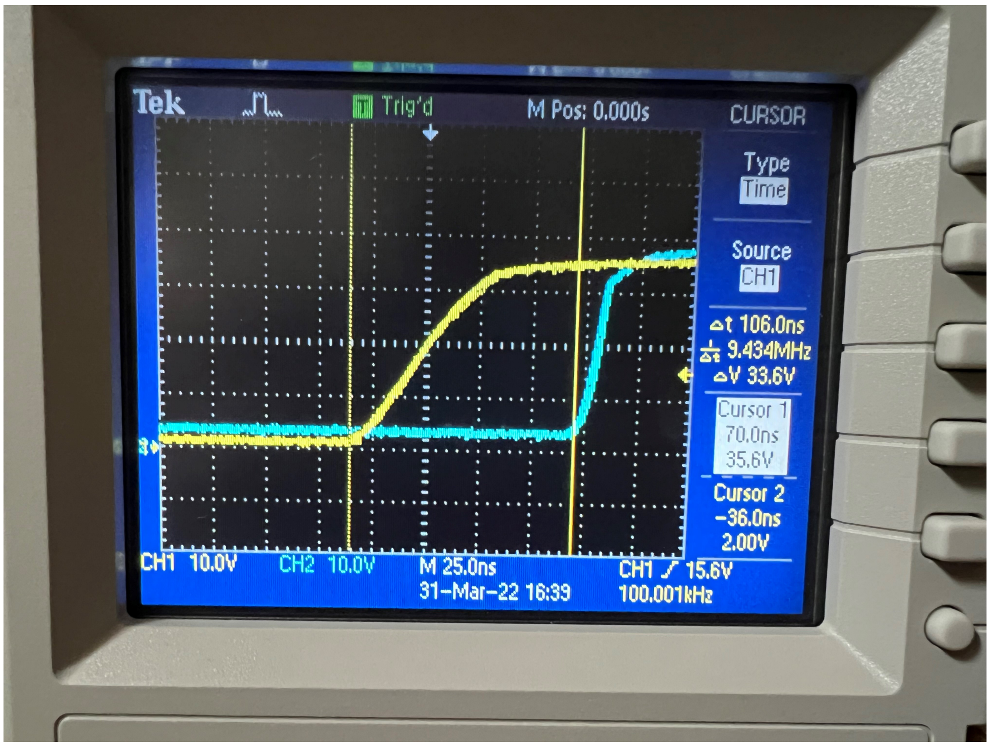

Figure 7: TOF measurement can be acquired from the difference in time between the first and second pulses, as measured here for the first (shortest) distance travelled by the light. The yellow line is the light recorded on the first photo detector, and the blue line is the return of the light recorded on the second photo detector. The vertical yellow lines are used to measure the time in nanoseconds of the light starting and ending position. The difference would give the total time.

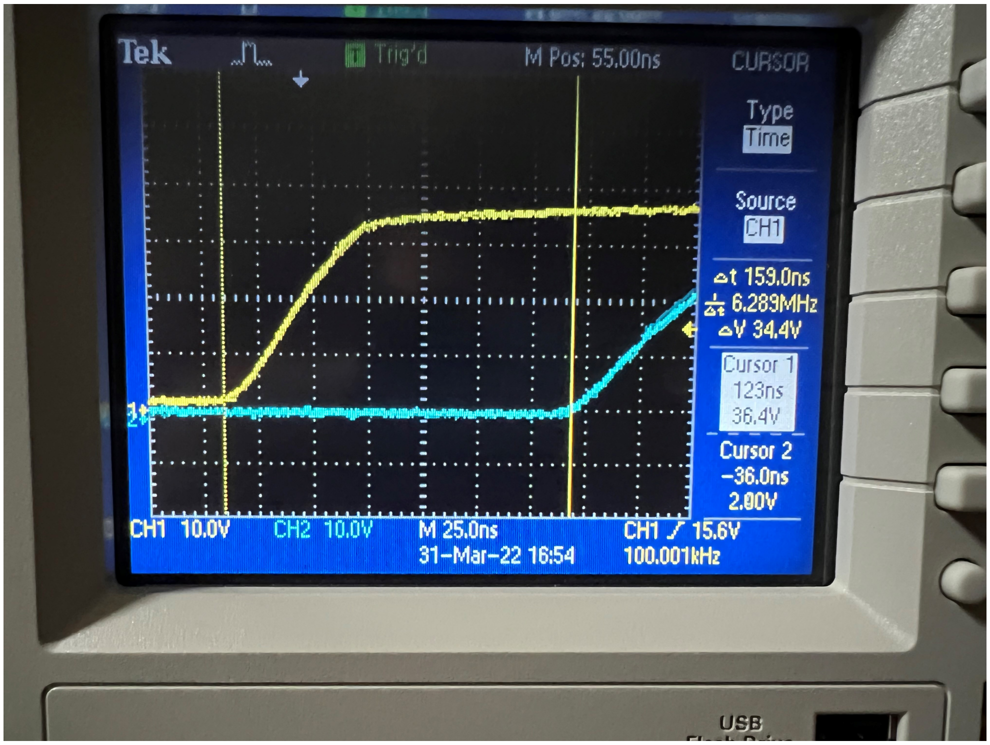

Figure 8: TOF measurement can be acquired from the difference in time between the first and second pulses, as measured here for the second distance travelled by the light. Notice that the difference in time, Δt, for this is larger than the first distance as this light beam travels a farther distance.

Table 2: The table shows the measured distances and time of travel of light for all three distances. The distances here have been corrected by adding and subtracting the small changes in length at the apparatus and the mirror. The laser starting position is at PD1 and the end position is at PD2, and there is a small distance between a given mirror and the door/wall to which it is attached. These are the distances that have been added or subtracted for from the original distances measured from the laser device. The start and stop times have been recorded, noting that the start time is always the same, while the stop times have increased as would be expected with a longer distance for the light to travel. The final time used to calculate the speed of light is the difference in time/2 since we only require the time that light travels for a one-way trip and not the return trip.

Distances	Start time (PD1)	stop time (Pd2)	$\Delta t/2$
17.16m	-36ns	70ns	53ns
24.009m	-36ns	120ns	78ns
50.362m	-36ns	206ns	164ns

Table 3: The table shows the distance from the rotating mirror to mirror M_2, (parameter a), and the distance from the rotating mirror to the eyepiece, (parameter b). There were two slopes produced in the graph

Rotating mirror $->$ M_2 (a)	$15.46m \pm 0.01m$
Rotating mirror $->$ focal plane/eyepiece (b)	$6.58m \pm 0.01m$
Slope measured from the graph for trial 1 (y/f)	$8 \times 10^{-6} m/Hz$
Slope measured from the graph for trial 2 (y/f)	$9 \times 10^{-6} m/Hz$
mean slope	$8.5 \times 10^{-6} m/Hz \pm 5 \times 10^{-7} m/Hz$

5 Results

The results for both Michelson and time of flight experiments are shown on table 4. Figures 1 and 2 aided in determining the values, and the time of flight graph slope resulted in the actual value for the speed of light.

Table 4: The table shows the speed of light determined from the Michelson rotating mirror method and the time of flight method for all three of the distances from the shortest to the longest and whether or not the result is consistent with the theoretical value as calculated in the calculations section.

Michelson speed	$300785098.9m/s \pm 17746320.84m/s$	consistent
TOF speed (shortest distance)	$323773584.9m/s \pm 12303396.23m/s$	inconsistent
TOF speed (mid)	$307807692.3m/s \pm 11669243.9m/s$	inconsistent
TOF speed (longest)	$307085365.9m/s \pm 15047182.93m/s$	consistent

6 Discussion

The speed of light for both experiments produced results close to the theoretical speed of light value, despite the inconsistency on two of the experimental values. Potential source of improvement include adding more trials in each method to improve accuracy, which would help reduce random errors. Adding more distances for the light to travel in the time of flight method, although this would require a lot of space to accomplish. From the results, it seems that the Michelson rotating method produced the most accurate result out of all the results. Therefore, this method seems to be more useful than the time of flight method, although the simplicity of the time of flight method in comparison to the rotating mirror makes this method quite useful for teaching purposes.

7 Conclusion

The experiment used two methods to determine the speed of light. The resulting speeds were accurate to the theoretical values despite inconsistencies.

References

[1] D. Rosa. Physics 325, Laboratory Manual. University of Victoria, 2021.