VIT Vellore Institute of Technology

Reg. No. : [redacted]

Final Assessment Test (FAT) – November/December 2022

Programme	B.Tech.	Semester	Fall Semester 2022-23
Course Title	COMPLEX VARIABLES AND LINEAR ALGEBRA	Course Code	BMAT201L
Faculty Name	Prof. Dr. SUSHMITHA	Slot	A2+TA2+TAA2
		Class Nbr	CH2022231001195
Time	3 Hours	Max. Marks	100

Part A (10 X 10 Marks)
Answer <u>any 10</u> questions

1. If $u = x^2 - y^2$, $v = -\dfrac{y}{(x^2+y^2)}$, then show that both u and v satisfy Laplace's equation, but $u + iv$ is not an analytic function of z. [10]

2. a) Find the bilinear transformation that maps the points $z = -1, 0, 1$ in z-plane onto the points $w = 0, i, 3i$ in w-plane. [10]
   b) Find the fixed points and the image of upper half of the z-plane under the transformation $w = \dfrac{1}{1-z}$. **[5 +5 Marks]**

3. a) Find the image of the line segment from 1 to i under the complex mapping $w = -i\bar{z}$, where $\bar{z}$ represents the conjugate of z. [10]
   b) Find all the points where the mapping $f(z) = \sin(z)$ is conformal. **[7+3 Marks]**

4. a) Expand $f(z) = \dfrac{1}{(z-1)^2(z-3)}$ in a Laurent's series valid for $0 < |z-1| < 2$. [10]
   b) Define singularity of a complex function and determine the type of singularity of

$$f(z) = z e^{\frac{z-1}{z^3-1-3z^2+3z}} - \frac{e^{\frac{1}{z-1}}}{e^{\frac{1}{1-z}}}.$$ **[5+5 Marks]**

5. Evaluate $\int \dfrac{z\cosh(z)}{z^5 + 2iz^4} dz$ over a closed curve C, where C is a unit circle, using Cauchy's integral formula. [10]

6. Find a basis of row space, column space and null space for the given matrix A and hence, verify the rank-nullity theorem [10]

$$A = \begin{pmatrix} 1 & 2 & 0 & 2 & 5 \\ -2 & -5 & 1 & -1 & -8 \\ 0 & -3 & 3 & 4 & 1 \\ 3 & 6 & 0 & -7 & 2 \end{pmatrix}$$

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7. Check whether the following sets are linearly independent or not. [10]
   a) $S_1 = \{(1, 2, -1, 4), (1, 0, 1, 0), (1, -1, 0, 1)\} \subset \mathbb{R}^4$.
   b) $S_2 = \{t-1, t^2-1, -t^2+2t-1\} \subset \mathcal{P}_2(\mathbb{R})$, where $\mathcal{P}_2(\mathbb{R})$ represents the vector space of polynomials of degree at most 2. **[5+5 Marks]**

8. Consider the following matrix [10]

$$A = \begin{bmatrix} -8 & 5 \\ -41 & 24 \\ -3 & 2 \end{bmatrix}$$

a) Find the unique linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ so that A is the associated matrix of T with respect to bases, $\alpha = \{(1,3), (2,5)\}$ and $\beta = \{(1,0,0), (1,1,0), (1,1,1)\}$.

b) Find $T(2,3)$.

9. Let $F : \mathbb{R}^4 \to \mathbb{R}^3$ be linear mapping defined by $F(x, y, s, t) = (x - y + s + t, x + 2s - t, x + y + 3s - 3t)$. Find basis and dimension of image of F. [10]

10. Find an orthonormal basis for the solution space of the homogeneous system of linear equations $x + y - 2z + w = 0;\ y + 2z + w = 0.$ [10]

11. Solve the system of linear equations $x + 2y - z = -1, 3x + 8y + 2z = 28, 4x + 9y - z = 14$ using the Gauss-Jordan method. [10]

12. Find the eigenvalues and eigenvectors of the given matrix [10]

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

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4r = 3r

LXF 4λ

$x_1 = 2\lambda$

$2x_1 = -\lambda$