Lecture Title: Hypothesis Testing Basics

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Hypothesis Testing in Statistical Analysis

- Hypothesis testing is a cornerstone of statistical analysis, enabling researchers and analysts to make informed decisions based on sample data. It is widely used across various fields, including:
- **Research**: To validate theories or models. For example, A researcher wants to determine if a new teaching method improves students' math scores compared to the traditional method..
- **Quality Control**: To ensure products meet specifications. For instance, a manufacturer might test if the average diameter of bolts produced is within acceptable limits.
- **Medicine**: To evaluate the efficacy of treatments. For example, a pharmaceutical company might test if a new drug lowers cholesterol levels more effectively than an existing drug.
- **Anomaly Detection**: To identify unusual patterns in data. For example, a financial institution might use hypothesis testing to detect fraudulent transactions.

Null Hypothesis (H_0) and Alternative Hypothesis (H_1)

- **Null Hypothesis (H_0)**: This is the default assumption that there is no effect or difference. It represents the status quo or a statement of "no change."
  - **Example**: "A new drug has no effect on blood pressure."
- **Alternative Hypothesis (H_1)**: This is the statement we seek evidence for. It represents a new theory or a statement of "change."
  - **Example**: "A new drug lowers blood pressure."
- **Example**
- Suppose a company claims that its machine produces items with a mean weight of 500g. To test this claim, we set up the following hypotheses:
- **H_0**: The mean weight of items produced by the machine is 500g.
- **H_1**: The mean weight of items produced by the machine is not 500g.
- We collect a sample of items, measure their weights, and use statistical methods to determine whether to reject H_0 in favor of H_1.

Statistical Significance

- **Understanding p-values**
- The p-value is a crucial concept in hypothesis testing. It represents the probability of observing results as extreme as the test statistic, assuming that the null hypothesis (H_o) is true.
- **Low p-value**: Suggests strong evidence against H_o, leading us to reject it.
  - **Example**: A p-value of 0.03 means there is a 3% chance that the observed result is due to random variation under H_o. If our significance level (α) is 0.05, we would reject H_o.
- **High p-value**: Suggests weak evidence against H_o, leading us to fail to reject it.
  - **Example**: A p-value of 0.20 means there is a 20% chance that the observed result is due to random variation under H_o. If our significance level (α) is 0.05, we would fail to reject H_o.

Statistical Significance

- **Thresholds for Significance (0.05, 0.01, 0.001)**
- Common significance levels (α) used in hypothesis testing are:
- **α = 0.05**: Moderate evidence against H_o.
  - **Example**: In clinical trials, a p-value less than 0.05 might be used to conclude that a new treatment is effective.
- **α = 0.01**: Strong evidence against H_o.
  - **Example**: In high-stakes research, such as drug safety studies, a more stringent α of 0.01 might be used to minimize the risk of false positives.
- **α = 0.001**: Very strong evidence against H_o.
  - **Example**: In particle physics, where the stakes of a false discovery are extremely high, a p-value threshold of 0.001 might be used.

Type I and Type II Errors

Type I Error (False Positive):

- **Definition**: Occurs when we reject H_0 when it is actually true.
- **Example**: A medical test wrongly indicates that a patient has a disease when they do not.
- **Probability**: The probability of a Type I error is equal to the significance level (α).
- **Trade-off**: Lowering α reduces the risk of Type I errors but increases the risk of Type II errors.

Type I and Type II Errors

Type II Error (False Negative)

- **Definition**: Occurs when we fail to reject H_o when it is false.
- **Example**: A medical test fails to detect a disease in a patient who actually has it.
- **Probability**: The probability of a Type II error is denoted by β.
- **Power of a Test**: The power of a test is 1 - β, representing the ability to detect an effect when it exists.

Example of Type I & II Errors

Reality	Decision	Error Type
H_o True	Reject H_o	Type I Error
H_o False	Fail to Reject H_o	Type II Error