Lecture 11: Hypothesis Testing Basics

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1 Introduction to Hypothesis Testing

Hypothesis testing is a statistical method used to make inferences or draw conclusions about a population based on sample data. It evaluates whether the observed data supports a specific claim or hypothesis.

1.1 Importance in Statistical Analysis

- Validates assumptions about population parameters.
- Provides a framework for decision-making under uncertainty.
- Helps determine the effectiveness of interventions or treatments.

Example: Testing whether a new drug is more effective than an existing one.

2 Key Components of Hypothesis Testing

2.1 Null Hypothesis (H_0)

The null hypothesis is a statement of no effect or no difference. It represents the default assumption.

- **Example:** H_0: The mean sales of two products are equal.

2.2 Alternative Hypothesis (H_1)

The alternative hypothesis is a statement that contradicts the null hypothesis. It represents the claim we aim to support.

- **Example:** H_1: The mean sales of two products are not equal.

2.3 Statistical Significance

Statistical significance measures whether the observed data is unlikely under the null hypothesis. It is determined by the **p-value** and a predefined significance level (α).

2.4 P-Value

Definition: The p-value is the probability of observing results as extreme as the current data, assuming the null hypothesis is true.

- **Low p-value:** Strong evidence against H_0.
- **High p-value:** Weak evidence against H_0.

2.5 Significance Thresholds

Common significance levels:

- $\alpha = 0.05$: 5% chance of rejecting H_0 when it is true.
- $\alpha = 0.01$: 1% chance of rejecting H_0 when it is true.

3 Types of Errors in Hypothesis Testing

3.1 Type I Error (False Positive)

Definition: Rejecting the null hypothesis when it is true.

- **Probability:** α
- **Example:** Concluding a new drug works when it does not.

3.2 Type II Error (False Negative)

Definition: Failing to reject the null hypothesis when it is false.

- **Probability:** β
- **Example:** Concluding a new drug does not work when it does.

3.3 Balancing Errors

- Lowering α reduces Type I errors but increases Type II errors.
- Power of a test: $1 - \beta$, representing the probability of correctly rejecting H_0.

4 Steps in Hypothesis Testing

1. **State the Hypotheses:** Define H_0 and H_1.
2. **Select the Significance Level (α):** Common values are 0.05 or 0.01.
3. **Choose the Test Statistic:** Depends on the data type and sample size (e.g., z-test, t-test).
4. **Calculate the Test Statistic and P-Value:** Use sample data to compute.
5. **Make a Decision:** Compare the p-value to α.

5 Examples

5.1 Example 1: Two-Sample t-Test

Scenario: Comparing mean test scores between two teaching methods.

- H_0: Mean scores are equal.
- H_1: Mean scores are different.
- Significance Level: $\alpha = 0.05$.

Outcome: If p-value < 0.05, reject H_0 and conclude a significant difference.

5.2 Example 2: A/B Testing

Scenario: Testing whether a new website design increases user engagement.

- H_0: The engagement rate is the same for both designs.
- H_1: The engagement rate is higher for the new design.

Outcome: Use a z-test for proportions to evaluate significance.

6 Conclusion

Hypothesis testing is a cornerstone of statistical analysis, providing a structured approach to decision-making. Understanding null and alternative hypotheses, p-values, significance thresholds, and error types ensures robust and reliable inferences in various applications.