Hypothesis Testing: Steps and Practical Aspects

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1 Introduction

Hypothesis testing is a statistical method used to determine whether there is enough evidence in a sample to infer conclusions about a population parameter. It is widely used in research, quality control, medicine, business, and various scientific fields.

2 Steps in Hypothesis Testing

2.1 Step 1: State the Null and Alternative Hypotheses

- **Null Hypothesis** (H_0): Represents the status quo or no effect/difference.
- **Alternative Hypothesis** (H_1 or H_a): Represents the claim to be tested.

Example: A factory claims its batteries last more than 400 hours on average.

$H_0 : \mu = 400$ (The average battery life is 400 hours)
$H_1 : \mu > 400$ (The average battery life is greater than 400 hours)

2.2 Step 2: Select the Significance Level (α)

The significance level (α) is the probability of rejecting the null hypothesis when it is true. Common values include:

- 0.05 (5%) - standard in most tests.
- 0.01 (1%) - more stringent.
- 0.10 (10%) - less stringent.

Example: We select $\alpha = 0.05$, meaning we allow a 5% risk of incorrectly rejecting H_0.

2.3 Step 3: Choose the Appropriate Test Statistic

The test selection depends on the data type and sample size:

- **Z-test**: For large samples ($n > 30$) with a known population variance.
- **T-test**: For small samples ($n < 30$) with an unknown population variance.
- **Chi-square test**: For categorical data.
- **ANOVA**: For comparing more than two groups.
- **Regression analysis**: For studying relationships between variables.

Example: Since battery life follows a normal distribution and the population variance is known, we use a **Z-test**.

2.4 Step 4: Calculate the Test Statistic

The test statistic for a Z-test is computed as:

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \tag{1}$$

where:

- $\bar{X}$ = Sample mean.
- μ = Population mean under H_0.
- σ = Population standard deviation.
- n = Sample size.

2.5 Step 5: Determine the Critical Value or P-value

- Find the critical Z or t-value using statistical tables.
- Compute the **p-value** (probability of obtaining a test statistic as extreme as the observed value).
- If $p \leq \alpha$, reject H_0.
- If $p > \alpha$, fail to reject H_0.

2.6 Step 6: Make a Decision

- If the test statistic falls in the rejection region or $p \leq \alpha$, reject H_0.
- Otherwise, do not reject H_0.

2.7 Step 7: Interpret the Results

- If H_0 is rejected, there is significant evidence supporting H_1.
- If H_0 is not rejected, there is not enough evidence to support H_1.

3 Solved Numerical Example

Problem: A company claims that the average life of its LED bulbs is 5,000 hours. A sample of 50 bulbs is tested, showing an average life of 5,150 hours with a standard deviation of 300 hours. Test the claim at $\alpha = 0.05$.

3.1 Step 1: State Hypotheses

$$H_0 : \mu = 5000 \quad \text{(No change in average life)}$$
$$H_1 : \mu > 5000 \quad \text{(Bulbs last longer)}$$

3.2 Step 2: Select $\alpha = 0.05$

3.3 Step 3: Compute the Z-score

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{5150 - 5000}{300/\sqrt{50}} \tag{2}$$

$$Z = \frac{150}{42.43} = 3.53 \tag{3}$$

3.4 Step 4: Find Critical Value

From the **Z-table**, the critical value at $\alpha = 0.05$ for a **one-tailed test** is 1.645.

3.5 Step 5: Decision

Since $Z = 3.53 > 1.645$, we reject H_0.

3.6 Step 6: Conclusion

The sample provides **strong evidence** that the average bulb life is **greater than 5,000 hours**.

4 Conclusion

Hypothesis testing is a powerful statistical tool for decision-making. By following the seven-step process, choosing the right test, and correctly interpreting results, one can draw meaningful conclusions with statistical confidence.