Lecture Title: Steps and Practical Aspects of Hypothesis Testing

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Introduction to Hypothesis Testing

- **Definition**: Hypothesis testing is a statistical method used to make inferences about a population parameter based on sample data. It is a core tool in scientific research, business analytics, and decision-making processes.
- **Purpose**: The goal is to determine whether a claim (hypothesis) about a population is supported by sample data. For example, a pharmaceutical company might test whether a new drug is more effective than an existing one.
- **Key Idea**: Hypothesis testing involves comparing observed data with what we would expect under a null hypothesis (H_0). If the observed data is highly unlikely under H_0, we reject H_0 in favour of an alternative hypothesis (H_1).

Key Concepts in Hypothesis Testing

Population vs. Sample

- **Population**: The entire group of interest (e.g., all adults in a country).
- **Sample**: A subset of the population used for analysis (e.g., 1,000 adults surveyed).

Parameter vs. Statistic

- **Parameter**: A numerical characteristic of the population (e.g., population mean μ).
- **Statistic**: A numerical characteristic of the sample (e.g., sample mean $\bar{x}$).

Statistical Significance

- A result is statistically significant if the probability of observing a result under H_0 is very low (typically below $\alpha = 0.05$).
- **Example**: If p-value = 0.03, the result is statistically significant at $\alpha = 0.05$.

Key Components of Hypothesis Testing

Null Hypothesis (H_0)

- The default assumption that there is no effect or difference.
- **Example**: H_0: The mean height of men and women is the same.

Alternative Hypothesis (H_1 or Ha)

- The hypothesis that contradicts H_0 and represents the research question.
- **Example**: H_1: The mean height of men is greater than that of women.

Test Statistic

- A standardized value used to decide whether to reject H_0.
- **Example**: Z-score, t-score, or chi-square statistic.

Key Components of Hypothesis Testing

P-value

- The probability of obtaining results at least as extreme as the observed results, assuming H_0 is true.
- **Example**: p-value = 0.04 means there is a 4% chance of observing the data if H_0 is true.

Significance Level (α)

- The threshold (commonly 0.05) used to determine whether to reject H_0.
- **Example**: If $p \leq 0.05$, reject H_0.

Decision Rule

- If $p \leq \alpha$, reject H_0; otherwise, fail to reject H_0.

The 7-Step Process of Hypothesis Testing

Step 1: Formulate Hypotheses

- Clearly define H_0 and H_1 based on the research question.
- **Example**: Testing if a coin is fair.
  - H_0: The probability of heads = 0.5.
  - H_1: The probability of heads ≠ 0.5.

Step 2: Choose a Significance Level (α)

- Common choices: 0.01, 0.05, or 0.10.
- **Trade-off**: Lower α reduces Type I error but increases Type II error.

The 7-Step Process of Hypothesis Testing

Step 3: Select an Appropriate Test Statistic

- Depends on data type and hypothesis:
  - **Z-test**: Large sample sizes, known population variance.
  - **T-test**: Small samples, unknown population variance.
  - **Chi-square test**: Categorical data.
  - **ANOVA**: Comparing means of multiple groups.
  - **Non-parametric tests**: Used when assumptions of normality are violated.

The 7-Step Process of Hypothesis Testing

Step 4: Collect Data and Compute Test Statistic

- Gather the sample data and compute the required statistic.
- **Example**: For a Z-test,

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

where:

- $\bar{x}$ = sample mean,
- μ = population mean,
- σ = population standard deviation,
- n = sample size.

The 7-Step Process of Hypothesis Testing

Step 5: Determine the P-value and Compare with α

- The p-value indicates the likelihood of observing the data given H_0 is true.
- **Example**: If $p = 0.03$ and $\alpha = 0.05$, reject H_0.

Step 6: Make a Decision

- If rejecting H_0, conclude that there is enough evidence to support H_1.
- If failing to reject H_0, conclude that the sample data does not provide strong enough evidence to disprove H_0.

Step 7: Interpret and Report Results

- Provide a clear explanation of findings.
- Include confidence intervals when applicable.

Special Cases and Considerations

One-Tailed vs. Two-Tailed Tests

- **One-tailed test**: Tests for an effect in one direction (e.g., greater than or less than).
- **Two-tailed test**: Tests for an effect in both directions.
- **Example**:
  - One-tailed: H_1: $\mu > 50$.
  - Two-tailed: H_1: $\mu \neq 50$.

Bonferroni Correction

- Used when multiple comparisons are made to reduce Type I error.
- Adjust the significance level: $\alpha_{adjusted} = \alpha / m$, where *m* is the number of tests.
- **Example**: If $\alpha = 0.05$ and $m = 5$, $\alpha_{adjusted} = 0.01$

Practical Example

Problem: The historical mean age of college students in city X is 23 years. A random sample of 42 students has a mean age of 23.8. Assuming ages are normally distributed with a population standard deviation $\sigma=2.4$, test at $\alpha=0.05$ whether the population mean age has changed.

Solution:

Step 1: Formulate Hypotheses:

Null Hypothesis (H_0): $\mu=23$ (Mean age has not changed).

Alternative Hypothesis (H_1): $\mu\neq23$

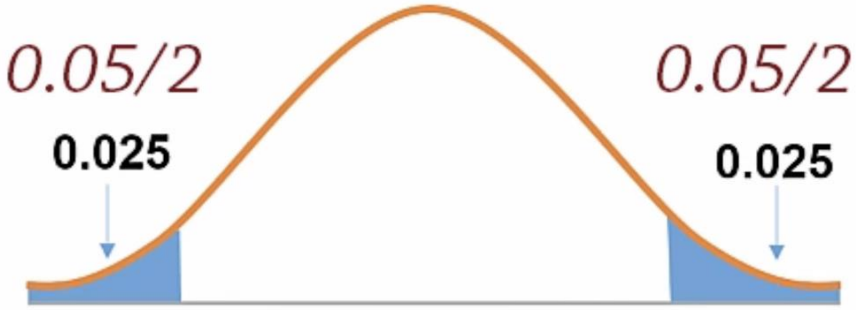
(Mean age has changed; two-tailed test)

$$H_0:\ \mu = 23$$
$$H_1:\ \mu \neq 23$$

Step 2: Choose Significance Level

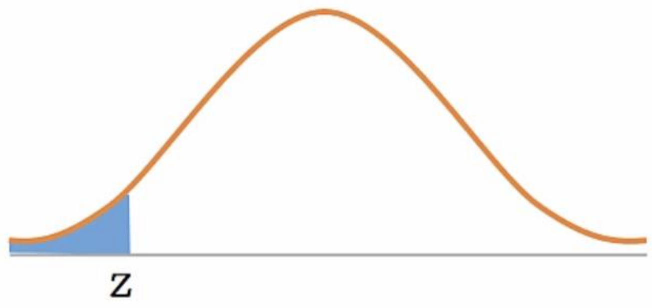
$\alpha=0.05$

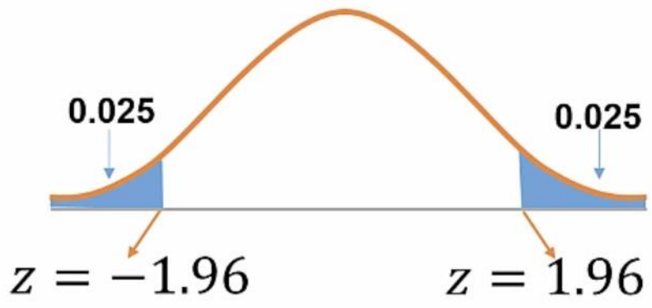
$$\alpha = 0.05$$



Step 2: Select Test Statistic

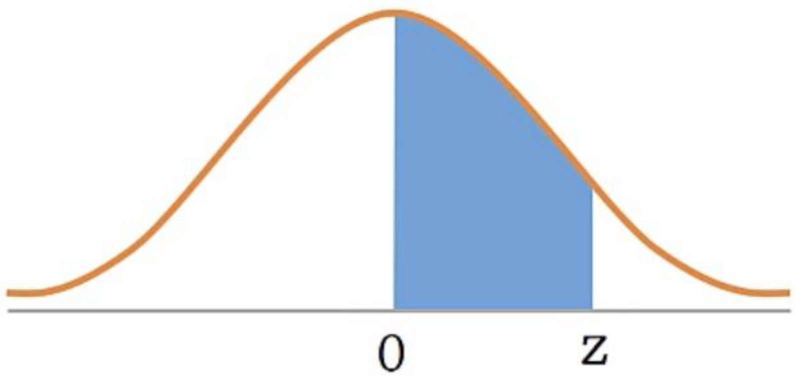
- Use the **Z-test** because:
  - Population standard deviation (σ) is known.
  - Sample size (n=42) is large (Central Limit Theorem applies i.e >30).
  - Find the critical value on the cumulative less than table

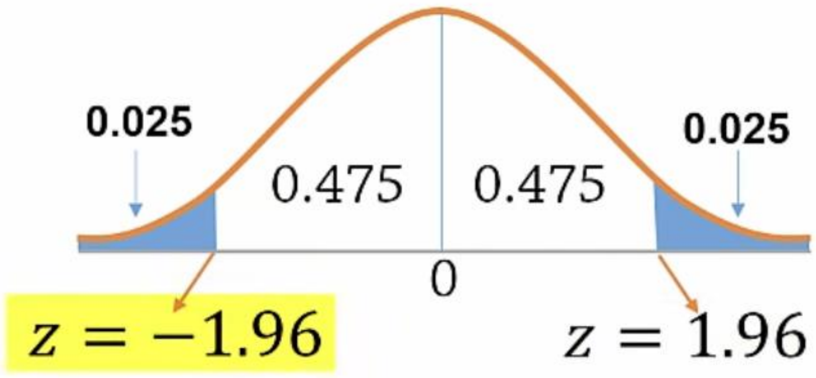




z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	$\boxed{0.0250}$	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559

Similarly, in the mean to z table:



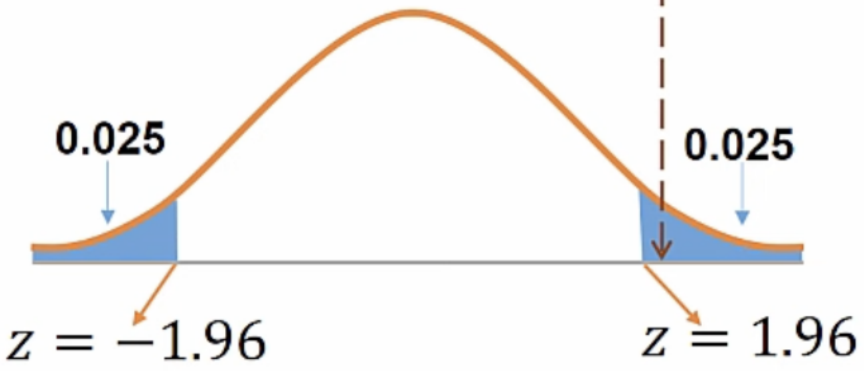


z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952

Step 4: Compute the Test Statistic

$$H_0:\ \mu = 23$$
$$H_1:\ \mu \neq 23$$

$$\alpha = 0.05$$



Reject H_0 if $z < -1.96$ or $z > 1.96$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{23.8 - 23}{2.4/\sqrt{42}} = 2.16$$

Since $z = 2.16 > 1.96$, reject H_0.