# Building a neural network FROM SCRATCH

#### **Important**

The content and code in this document are based on a YouTube tutorial by the original creator (credited in the online reference). These are my personal notes summarizing and implementing the concepts demonstrated in the video.

https://petite-salmon-b93.notion.site/Building-a-neural-network-FROM-SCRATCH-1fe136d1791480079f04ca81cd562ab9?pvs=73

# **OVERVIEW**

In this, we will be building a NEURAL NETWORK from scratch.

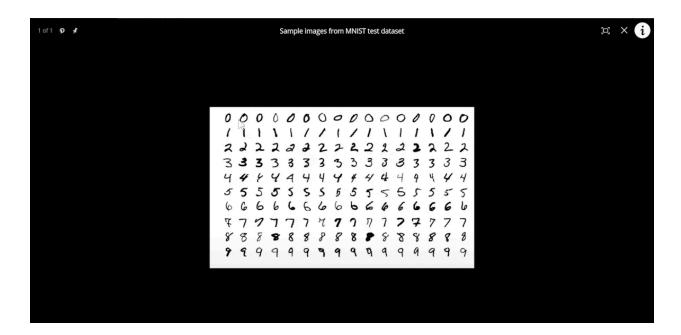
Using only:-

- 1. NUMPY
- 2. LINEAR ALGEBRA AND EQUATIONS

For this tutorial, we will have 3 layers (1  $\rightarrow$  Input Layer, 2  $\rightarrow$  Hidden Layer and 3  $\rightarrow$  Output Node/Layer)

# PROBLEM STATEMENT

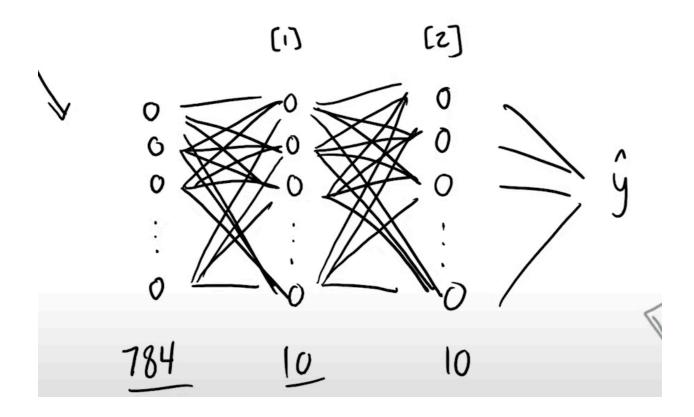
For the example, we are using images of hand written digits (0 to 9).



Neural Network which will classify any image of hand written Digit to the actual digit.

# **WORKING**

# Graph





Here the reason 784, 10,10 is there its because

Also the dimensions of the layers are as follows:

Input layer: 784 features (pixels)

Hidden layer: 10 neurons (can vary, chosen arbitrarily for simplicity)

Output layer: 10 neurons (one for each digit)

### Math

The training images are  $28 \times 28 = 784$  pixels (IN TOTAL)

Each pixel value has a range 0 to 255 (255 = Black and 0 = white)...

In laymen term, we can convert the each image into 25 rows and 25 columns. Each data having a value from 0 to 255.

Total dataset = m

Total Pixels = 784m (NOT IMP IG... Just for info)

**HENCE** 

$$\chi = \left[egin{array}{c} x^{(1)} \ x^{(2)} \ dots \ x^{(m)} \end{array}
ight]^{\mathsf{T}} = \left[egin{array}{cccc} x^{(1)} & x^{(2)} & \cdots & x^{(m)} \end{array}
ight]$$

Each x is an image, and hence we have m rows...

Each row will have 784 values... Correct??? Or we can say each row will have 784 columns

And after we transpose it, IT BECOMES VICE VERSA...

Remember the first layer will be the INPUT LAYER or 0th layer and will have 784 nodes (for each pixel).

### STEP 1 → FOR (Forward Propagation) PROPOGATION

#### **BASIC PRINCIPLE:-**



Neural Network Node Principle

Each neuron's output is computed by taking a weighted sum of its inputs, adding a bias, and passing the result through an activation function:

$$a = g(w op x + b)a = g(w op x + b)a = g(w op x + b)$$



#### Why do we use Activation Function?

We add them because if there is no activation function, the value from the output layer will be a sum of linear expression of all the hidden nodes.

Eg:- input layer. the first Layer will be a linear expression of input layer. Second Hidden layer will be another linear expression of the prev one (which is already a linear expression of the input layer) and there is no point of having hidden layer. And the output is just a Linear expression rather than being a complex algo.

#### FOR LAYER 0 TO 1:-

To calculate the value at each layer we use these functions:-

$$A^{(0)} = X \quad (784 imes m) \ W^{(1)} : 10 imes 784 \ Z^{(1)} = W^{(1)}A^{(0)} + b^{(1)} \quad A^{(0)} : 784 imes m \ b^{(1)} : 10 imes 1 \Rightarrow 10 imes m \ A^{(1)} = g(Z^{(1)}) = ext{ReLU}(Z^{(1)})$$

There are many Types of Activation Function which will be covered in another PAGE

In our case, we will be using ReLu Activation Function

$$ext{ReLU}(x) = egin{array}{ll} x & ext{if } x > 0 \ 0 & ext{if } x \leq 0 \end{array}$$

#### FOR LAYER 1 TO 2:-

The process for this layer is same as the above one but the KEY Difference is ACTIVATION FUNCTION.

Rather than Using the ReLu Activation Function, we are using Softmax Activation Function

$$Z^{(2)} = W^{(2)}A^{(1)} + b^{(2)} egin{array}{c} W^{(2)} : 10 imes 10 \ A^{(1)} : 10 imes m \ b^{(2)} : 10 imes 1 \Rightarrow 10 imes m \ A^{(2)} = \mathrm{softmax}(Z^{(2)}) \end{array}$$

**SOFTMAX EQUATION** 

$$\operatorname{softmax}(z_i) = rac{e^{z_i}}{\sum_{j=1}^K e^{z_j}}$$

So it will look like this

Input vector: 
$$\mathbf{z} = \begin{bmatrix} 1.3 \\ 5.1 \\ 2.2 \\ 0.7 \\ 1.1 \end{bmatrix}$$

Softmax formula: softmax
$$(z_i) = \frac{e^{z_i}}{\sum_{j=1}^K e^{z_j}}$$

$$e^{1.3}pprox 3.669$$

$$e^{5.1}pprox 164.022$$

**Exponentials:** 
$$e^{2.2} \approx 9.025$$

$$e^{0.7}pprox 2.014$$

$$e^{1.1} pprox 3.004$$

**Denominator (sum):** 
$$\sum_{j=1}^{5}e^{z_{j}}=3.669+164.022+9.025+$$
  $2.014+3.004$ 

$$= 181.734$$

$$ext{softmax}(z_1) = rac{3.669}{181.734} pprox 0.02$$

$$ext{softmax}(z_2) = rac{164.022}{181.734} pprox 0.90$$

Softmax outputs: 
$$\operatorname{softmax}(z_3) = \frac{9.025}{181.734} \approx 0.05$$

$$ext{softmax}(z_4) = rac{2.014}{181.734} pprox 0.01$$

$$ext{softmax}(z_4) = rac{2.014}{181.734} pprox 0.01 \ ext{softmax}(z_5) = rac{3.004}{181.734} pprox 0.02$$

Final probabilities: 
$$softmax(\mathbf{z}) = \begin{bmatrix} 0.02\\0.90\\0.05\\0.01\\0.02 \end{bmatrix}$$



#### WHY USE SOFTMAX???

The Reason is simple that each node's value will lie in the range of 0 to 1. Here 1 is Absolute certainty and 0 is the opposite.

#### STEP 2 → BACK (Backward Propagation) PROPOGATION

#### **BASIC PRINCIPLE:-**



#### **Principle**

As the name suggests, backward propagation works from the last layer to the first, computing gradients and adjusting the weights and biases accordingly. This allows the network to learn which parameters contribute most to the error. To optimize the weights and biases, this step is repeated iteratively rather than performed just once. By running multiple iterations, the network progressively adjusts its parameters to minimize the loss and improve its predictions.

In Laymen Term's we can say:-

We Start with the prediction and we find out how much the prediction deviated with the actual input. This will give us the **ERROR**.

Then we find how much each of the **Bias and Weights** contributed to that error. Then, we adjust them accordingly.

#### FOR LAYER 2 TO 1:-

To calculate the value at each layer we use these functions:-

$$dz^{[2]} = A^{[2]} - Y (1)$$

$$dW^{[2]} = rac{1}{m} dz^{[2]} A^{[1]T}$$
 (2)

$$db^{[2]} = rac{1}{m} \sum dz^{[2]}$$
 (3)

Here dz<sup>2</sup> is the error of the 2<sup>2</sup> Layer

#### FOR LAYER 1 TO 0:-

To calculate the value at each layer we use these functions:-

$$egin{align} dz^{[1]} &= W^{[2]T} \cdot dz^{[2]} \circ g'(z^{[1]}) \ dW^{[1]} &= rac{1}{m} dz^{[1]} \cdot X^T \ db^{[2]} &= rac{1}{m} \sum dz^{[1]} \ \end{array}$$

Here  $dz^2$  is the error of the  $2^2$  Layer g'(z) is the Derivative of the Activation Function

# STEP 3 → Updating the Values

In this step, we update 2 values:-

- 1. Weights
- 2. Biases

$$egin{aligned} W^{[1]} &:= W^{[1]} - lpha \, dW^{[1]} \ b^{[1]} &:= b^{[1]} - lpha \, db^{[1]} \ W^{[2]} &:= W^{[2]} - lpha \, dW^{[2]} \ b^{[2]} &:= b^{[2]} - lpha \, db^{[2]} \end{aligned}$$



 $\alpha$  is the the hyper-paramater. It is not trained by the Model. When you runt eht eabove cycle aka Gradient Descent, is set by us.

#### CODE

Below is the link to the code and in the comments are the notes:-

https://github.com/AviralTanwar/Nerual\_Network\_with\_Numpy

# **OUTPUT**

Since I have written this in python file (i.e. .py file) and not in jupiter or .ipnyb file, the code ran on terminal and many of the output was lost.

The main and important outputs are being shown below:-

```
Iteration: 1999
FINAL VA:UES OF THE WIEGHTS AND BIASES
W1: [[ 0.98411848 -1.29751216 0.28987067 ... 0.24964724 0.79800207
   0.80334867]
 [-0.4841924 1.10207873 0.65232149 ... -0.13034877 0.7621568 -1.46672642]
[-1.63936524 -1.11220145 -1.0146169 ... -0.43665716 0.22525685
  -0.39218341]
 [-1.14261946 \quad 2.81762964 \quad 0.70675579 \ \dots \quad 0.91739395 \quad 0.98839215
   -0.89004227]
 [ 1.9930897
              -0.54271756 0.08513634 ... 0.58301527 -0.81838441
   -0.73108455]
 b1: [[-0.04755573]
  1.70135233]
  -0.43539611
   0.94729015
   0.5198886
   1.60238648
   0.88289931
   -1.18815634]
   0.8980137
   -0.88235011]]
```

```
W2: [[-0.23175169 1.14114939 -1.00732695 0.04133117 0.73883826 -0.48001537 0.39659933 -0.03163318 -0.00712701 -1.18102818] [-0.09602214 -0.35500419 0.54141036 -0.05828282 -0.06987261 -0.52879247
 0.27734165 0.4865551 0.41932041 0.1889699 ]
[ 0.14770422 -0.31327452 0.28161426 0.23302932 0.43187098 -0.34471239
 -0.71052194 0.78744086 0.22631045 0.08882853]

[-0.13381231 -0.52021448 0.28067533 0.14746433 0.37300311 -0.27116849
     0.01092294 0.74995439 0.10791184 1.33202398]

    [-0.0437759
    -1.68103466
    0.1862439
    0.66870048
    0.53905865
    -0.17484516

    -1.32827969
    1.12289824
    0.37693665
    -2.62747926]

    [-0.25295205
    0.26312881
    0.29465887
    0.2619702
    0.59717791
    -0.16665529

 0.2451636 0.49931428 0.70404646 -0.95237423]
[-0.1852929 0.05381727 0.46139306 0.81658779 0.3762854 -0.88812929
     0.22803042 1.10201045 0.21774813 1.6513143 ]
-0.4255643 0.4725783 0.40265768 0.20762916 0.39963678 -0.12010362 -0.06032292 0.27629114 0.40924559 0.48366761]
[-0.10267804 0.03179111 0.35559466 0.78057489 0.34639001 -0.34101352 -0.45390375 1.13716717 0.36303738 0.4881365 ]]
b2: [[ 0.94043284]
     1.42841243]
     0.25842898
     -2.48785916
     0.60413411
     -0.41416557
     0.16786511
[-0.1697713 ]]
Predicted: [5]
Predicted: [2]
Predicted: [8]
Actual: 9
Predicted: [6]
```

```
Actual: 6
Predictions: [2067795899797942217375079499577939967
8810889150047257256314568912939370815
5039651906701576123217336739432001838
9103921081696293844177190532589868897
32312381781272921917060191193147002623
4122279945197969073985119606251079896
9394538756021390929822994220270934894
9705977260359295102941096829470772632
1279995396832516286960250871170371937
573955288621899381451105599792287597199
6618736023762659850551110558916940
088079667269978322206255125203039397
70620090889780661781559386760995391694
872086306972217977070367103998931694
872086306972217977070367103998931694
872086306972217977070367103998931694
872086306972217977070367103998931654
09109838207888291197110558817232089191
5991567657786078274573935052979720957
5975126521791616859599012990119645949
8637757249023178927002390519627802079
3793013854361850137051899975620060632
24505519130920886376441529055506488941
5038720060573221661758374905732254959
0259130145308921306792262954954099035
109291968813569237921926555913775179077
```

```
Y: [2 0 6 7 2 9 5 5 4 4 7 9 7 4 4 2 2 1 7 3 7 4 5 7 4 4 9 9 8 7 7 8 5 7 5 6 7
8810359750047257236314963412959370114
5 0 5 7 6 3 3 9 0 6 7 0 2 4 7 6 7 3 3 2 1 7 3 5 4 7 3 9 4 8 2 0 0 3 8 3 8
9 1 0 3 4 3 1 0 8 9 6 4 6 2 7 3 8 4 4 1 7 7 1 9 0 9 5 2 5 3 6 3 6 8 8 4 5
8 2 8 1 2 3 8 1 7 3 1 6 7 2 9 4 1 9 8 7 0 2 9 1 4 5 0 0 6 0 7 1 9 1 1 4 4
0703925892542287667731293721950018109
7607224118035572440980901195147602223
4 3 4 4 6 5 8 7 5 6 0 2 1 3 8 6 9 9 9 3 2 2 9 7 4 2 2 0 2 7 0 9 3 9 8 9 4
9 7 0 3 8 7 7 2 3 0 5 5 4 2 9 3 3 0 8 9 4 1 6 4 6 8 2 4 4 7 0 7 3 2 7 1 2
1279195846886316836960250871170279427
5 3 3 9 5 5 2 2 3 6 4 1 8 4 4 3 8 1 4 9 3 1 0 5 8 9 7 9 2 5 7 5 9 7 1 9 9
6 4 1 8 7 5 6 0 3 3 7 6 2 6 5 9 8 8 0 0 1 1 1 0 5 5 8 9 1 5 8 0 1 2 5 6 3
8 3 3 3 8 4 8 8 4 9 9 1 5 4 4 7 0 0 0 2 1 0 1 7 2 8 0 5 0 6 1 9 2 3 0 4 4
0 0 8 5 0 7 4 6 6 7 2 6 1 7 7 5 8 2 2 6 0 6 2 6 5 1 6 5 2 0 1 0 5 4 5 4 7
7062007039780611581559677790996391644
9 9 7 0 8 6 8 0 0 4 7 9 2 1 7 2 7 7 0 7 1 0 6 0 7 1 0 3 9 9 3 9 3 1 6 5 4
0 9 1 5 4 8 3 2 2 0 9 5 8 8 3 8 1 2 4 7 1 1 0 5 4 8 1 7 2 3 3 0 8 9 1 9 1
5 9 2 1 8 6 3 6 8 7 9 3 6 6 7 5 2 7 4 5 7 3 7 5 5 0 9 2 9 7 9 7 3 0 9 5 7
5 4 7 5 1 2 6 5 2 6 7 4 1 6 1 6 5 8 9 8 8 4 5 3 2 4 4 0 1 1 9 6 5 6 9 4 4
7 6 2 4 5 5 4 4 6 8 0 3 7 1 3 1 6 8 3 8 7 4 2 7 7 8 5 1 6 2 4 8 6 9 1 7 4
3 7 4 3 0 1 3 8 5 4 3 2 1 5 5 0 1 8 7 0 5 1 8 9 9 9 7 5 6 2 0 0 6 0 6 2 0
2 4 5 0 6 1 4 1 2 0 4 2 0 8 4 6 3 7 6 2 4 1 8 2 9 0 5 6 5 0 6 4 0 8 9 4 3
5 0 3 8 0 2 0 0 2 0 5 7 3 2 2 7 6 2 1 7 5 8 4 7 4 9 0 5 7 8 2 2 8 4 4 5 9
  5 3 9 1 3 0 1 4 5 8 0 9 9 2 3 3 0 3 7 9 8 2 4 2 4 5 4 4 9 4 0 1 7 5 3 8
  092419631386428792598633910775179077
```

```
Y: [2 0 6 7 2 9 5 5 4 4 7 9 7 4 4 2 2 1 7 3 7 4 5 7 4 4 9 9 8 7 7 8 5 7 5 6 7
 8 8 1 0 3 5 9 7 5 0 0 4 7 2 5 7 2 3 6 3 1 4 9 6 3 4 1 2 9 5 9 3 7 0 1 1 4
 5 0 5 7 6 3 3 9 0 6 7 0 2 4 7 6 7 3 3 2 1 7 3 5 4 7 3 9 4 8 2 0 0 3 8 3 8
 9 1 0 3 4 3 1 0 8 9 6 4 6 2 7 3 8 4 4 1 7 7 1 9 0 9 5 2 5 3 6 3 6 8 8 4 5
 8 2 8 1 2 3 8 1 7 3 1 6 7 2 9 4 1 9 8 7 0 2 9 1 4 5 0 0 6 0 7 1 9 1 1 4 4
 0703925892542287667731293721950018109
 7607224118035572440980901195147602223
 9 5 8 6 2 7 4 9 4 4 1 9 7 9 6 4 0 7 3 4 8 8 1 1 4 6 6 6 2 9 3 0 9 7
 4 3 4 4 6 5 8 7 5 6 0 2 1 3 8 6 9 9 9 3 2 2 9 7 4 2 2 0 2 7 0 9 3 9 8 9 4
 9 7 0 3 8 7 7 2 3 0 5 5 4 2 9 3 3 0 8 9 4 1 6 4 6 8 2 4 4 7 0 7 3 2 7 1 2
 1279195846886316836960250871170279427
 5 3 3 9 5 5 2 2 3 6 4 1 8 4 4 3 8 1 4 9 3 1 0 5 8 9 7 9 2 5 7 5 9 7 1 9 9
 6 4 1 8 7 5 6 0 3 3 7 6 2 6 5 9 8 8 0 0 1 1 1 0 5 5 8 9 1 5 8 0 1 2 5 6 3
 8 3 3 3 8 4 8 8 4 9 9 1 5 4 4 7 0 0 0 2 1 0 1 7 2 8 0 5 0 6 1 9 2 3 0 4 4
0 0 8 5 0 7 4 6 6 7 2 6 1 7 7 5 8 2 2 6 0 6 2 6 5 1 6 5 2 0 1 0 5 4 5 4 7
 7 0 6 2 0 0 7 0 3 9 7 8 0 6 1 1 5 8 1 5 5 9 6 7 7 7 9 0 9 9 6 3 9 1 6 4 4
 9 9 7 0 8 6 8 0 0 4 7 9 2 1 7 2 7 7 0 7 1 0 6 0 7 1 0 3 9 9 3 9 3 1 6 5 4
 0 9 1 5 4 8 3 2 2 0 9 5 8 8 3 8 1 2 4 7 1 1 0 5 4 8 1 7 2 3 3 0 8 9 1 9 1
 5 9 2 1 8 6 3 6 8 7 9 3 6 6 7 5 2 7 4 5 7 3 7 5 5 0 9 2 9 7 9 7 3 0 9 5 7
 5 4 7 5 1 2 6 5 2 6 7 4 1 6 1 6 5 8 9 8 8 4 5 3 2 4 4 0 1 1 9 6 5 6 9 4 4
 7624554468037131683874277851624869174
 3 4 5 7 7 8 7 2 4 9 0 3 3 1 9 8 9 2 7 0 0 2 5 9 0 3 1 7 6 2 7 2 0 2 0 7 9
 3 7 4 3 0 1 3 8 5 4 3 2 1 5 5 0 1 8 7 0 5 1 8 9 9 9 7 5 6 2 0 0 6 0 6 2 0
 2 4 5 0 6 1 4 1 2 0 4 2 0 8 4 6 3 7 6 2 4 1 8 2 9 0 5 6 5 0 6 4 0 8 9 4 3
 5 0 3 8 0 2 0 0 2 0 5 7 3 2 2 7 6 2 1 7 5 8 4 7 4 9 0 5 7 8 2 2 8 4 4 5 9
 0539130145809923303798242454494017538
 1 0 9 2 4 1 9 6 3 1 3 8 6 4 2 8 7 9 2 5 9 8 6 3 3 9 1 0 7 7 5 1 7 9 0 7 7
Development Set Accuracy: 0.696
```

# **ONLINE REFERENCES**

#### You tube

https://www.youtube.com/watch?v=w8yWXqWQYmU&ab\_channel=SamsonZhang

## **CODE on KAGGLE**

<u>https://www.kaggle.com/code/wwsalmon/simple-mnist-nn-from-scratch-numpy-no-tf-keras</u>

### **DATASET**

https://www.kaggle.com/datasets/ompanpatil12/simple-mnist-nn-from-scratch-numpy-no-tfkeras

# **Equations and Images**

Used ChatGPT... LOL