On a decomposition method for finding winning strategy in Hex game

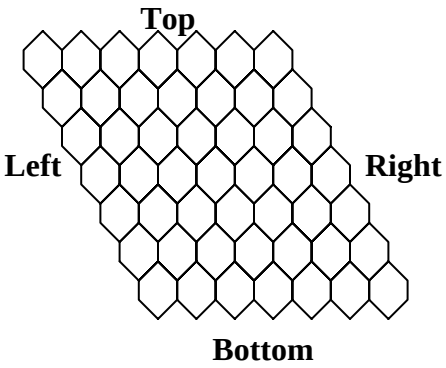
Jing Yang, Simon Liao, Mirek Pawlak

Abstract--In this paper, a decomposition method is described that is very powerful on finding winning strategies in Hex game. A detail description of our solution for Hex on 7 by 7 board is also given. Up to now, 7 by 7 is the largest Hex board for which a winning strategy is actually known.
Index Terms--AI, Hex, winning strategy, decomposition method

I. Introduction

Hex is a board game in which two players play in turn and try to build a connected chain of pieces across opposite sides of the board. The Hex board is a hexagonal tiling of n rows and n columns. Usually n =11 with 11 by 11 is the widely accepted standard board size. Figure 1 is an empty 7 by 7 Hex board.

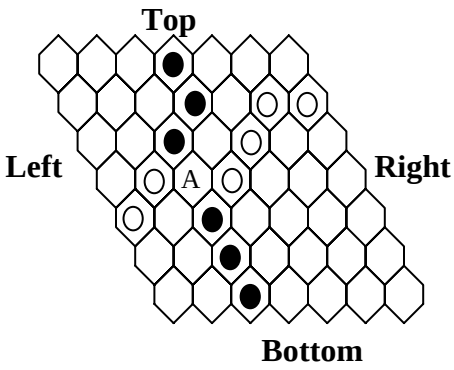
Figure 1: An empty Hex 7 by 7 board



The rules of Hex game are simple:
• One player plays Black and the other plays White. Black owns the Top and Bottom sides and White owns the Left and Right sides.
• Black player plays first.
• Players take turns placing a piece of their color on an unoccupied hexagon.
• A game is won when one player establishes an unbroken chain of their pieces connecting their sides of the board.

For example, Figure 2 is in the middle stage of a Hex game and the next move is Black's turn. If Black plays the next move at position "A", Black will win the game. However, if Black plays at any other position rather than "A", White will play at position "A" and win the game.

Figure 2: Take position "A" to win the game



Hex was invented by Piet Hein in 1942. In 1949, John Nash proved that there is no tie in Hex game so that the first player has a theoretical win for Hex in any size. Later, Nash won Nobel Prize in Economics for his work in game theory. However, John Nash's proof is non-constructive. He didn't indicate how to win a Hex game. Over half a century, Hex only belongs to an ultra-weak solved game and a solution (winning strategy) for Hex has continued to elude researchers.

II. The approach for finding Solution

Hex is a NP-complete problem. It has a huge number of possible games when the size of the board is larger than or equal to seven. Fortunately, two important characteristics of Hex make it possible to find a winning solution on smaller size game. One is that Hex has a relatively smaller move selection space in most of situations comparing with several other board games such as Go and Chess. This characteristic is more obvious on smaller size Hex. For example, in Figure 3, if White doesn't play on "A", "B", "C", "D", "E", "F", "G", "H", and "I", Black will go "C" to win the game because three black stones (in a vertical line)

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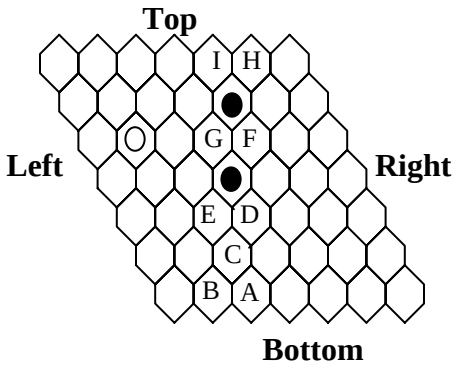
will certainly connect from Top to Bottom. Since our goal is to find a huge solution tree for Hex. This good characteristic means that, in this huge solution tree, many branches often terminate early. We call it the "sudden death" property.

Figure 3: The "sudden death" in Hex game



Another important characteristic of Hex is that a given Hex game sometimes can be viewed as the sum of several smaller "local games". We call those "local games" LocalPatterns. A "local game" is a part of a region in a given Hex game. "Local games" in a given Hex game do not overlap each other. They also must satisfy that the strategies for the given Hex game must able to be described by the sum of strategies on each of "local games". For example, considering the game in Figure 4, later we will know that the two black stones will certainly connect with Top side when Black plays with a correct strategy. And, the connection status will not be threaten unless some white stones have been played on △ hexagon region. Similarly, the two black stones will certainly connect with Bottom side and the connection status will not be threaten unless some white stones have been played on × local area. Region △ and region × are typical two "local games". They don't overlap each other and the strategies used on the given game in Figure 4 can be separated from the two local regions. Note that, we ignore those blank (empty) hexagons in Figure 4 because they don't have any effect to Black's connection whether they are Black or White.

Small "local games" can be further decomposed into smaller "local games". The game in Figure 5 developed from Figure 4 gives another example for "local game" concept. Those different marks denote different "local games". If we prove that those "local games" are locally connected, it is obvious that Black in Figure 5 has a connected group from Top to Bottom. If we decompose the Hex board from beginning, and figure out all possible decomposition ways by different white's defensive moves and the further decompositions for each "local game", that means we found the winning solution. An advantage of the "local game" idea is that many "local games" in a given game situation can be used in the other game situations when they are decomposed. In fact, we just use total 41 different "local games" to present the whole solution tree for Hex 7 by 7.

Figure 4:

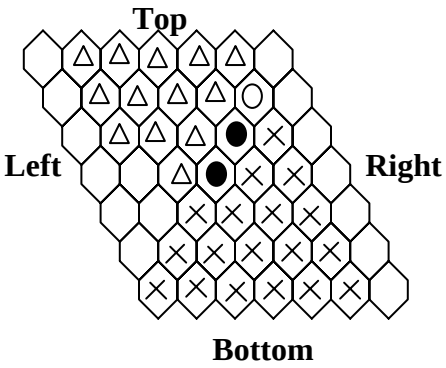
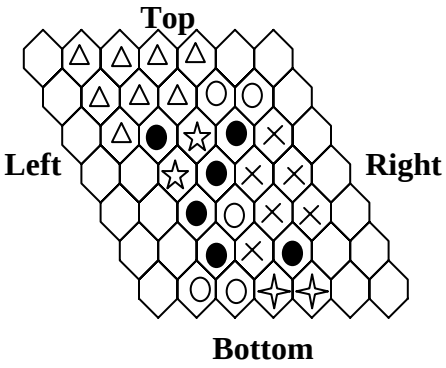


Figure 5: Local game further developed from Figure 4



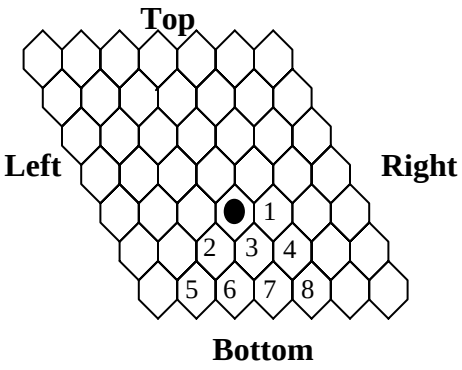
III. THE IMPLEMENTATION DETAIL

Based on the "local game" idea, a solution tree which covers all possible White defensive moves on 7 by 7 board has been found. The solution has been implemented by Java applet programming to make everyone be able to check the solution online. The web site is: http://www.ee.umanitoba.ca/~jingyang/jhex.html.

To make the solution tree concise, we use a kind of special logical language which is almost the same as C. The value of variable WhiteMove always shows the position White plays on the given LocalPattern (corresponding the position number shown on the localPattern). Similarly, the value of variable BlackMove shows Black's playing position. The variable SumOfLocalGames can be viewed as a list that stores all the "local games" information on current situation in the game. An addition operation "+" on SumOfLocalGames means insert a new local game item on the list, and a subtraction operation "-" on it means delete an old local game item from the list. LocalPatternK denotes the number K LocalPattern that associates with a lot of strategies and position information. The parameters in LocalPatternK, for example the 1,2,3,4,5,6,7,8 in LocalPatternK(1,2,3,4,5,6,7,8), are used to represent the positions in LocalPatternK correspondence with the given Pattern (the father of the LocalPatternK). Before the proof, let's use Pattern13 as an example to explain it in detail.

A. Detail Explanations for LocalPattern13

Figure 6: LocalPattern13




```
If (WhiteMove == 1 || 3 || 4 || 7 || 8) {
BlackMove = 2;
SumOfLocalGames = SumOfLocalGames - LocalPattern13
+ LocalPattern2(5,6); }
else if (WhiteMove == 2 || 5 || 6) {
BlackMove = 4;
SumOfLocalGames = SumOfLocalGames - LocalPattern13
+ LocalPattern2(1,3) + LocalPattern2(7,8);  }
```

The above LocalPattern13 and its associated playing strategies can be translated as follow: If the White move is on position 1, position 3, position 4, position 7, or position 8 in above LocalPattern13, Black will response on position 2. And then on the "local games" list (SumOfLocalGames), LocalPattern13 (position 1-8) will be deleted and a new LocalPattern2 will be added. The new LocalPattern2 will apply local game with strategies on LocalPattern2 and the new position 1 (in LocalPattern2) is the old position 5 (in LocalPattern13), the new position 2 (in LocalPattern2) is the old position 6 in (localPattern13). Figure 7 shows the updated situation in this case.

If the White move is on position 2, position 5, or position 6 in above LocalPattern13, Black will response on position 4. And then on the "local games" list (SumOfLocalGames), LocalPattern13 will be removed and two new LocalPattern2 will be added. In one of LocalPattern2, new position1 is on the old position 1 (LocalPattern13), and new position 2 is on the old position 3 (LocalPattern13). In the other LocalPattern2, new position 1 is on the old position 7 (LocalPattern13), and new position 2 is on the old position 8 (LocalPattern13). Figure 8 shows the updated situation in this case.

Figure 7:Developed from LocalPattern13 first rule

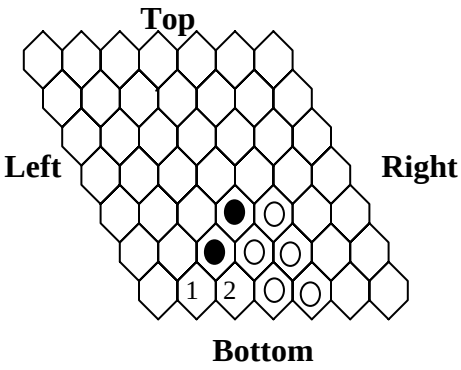
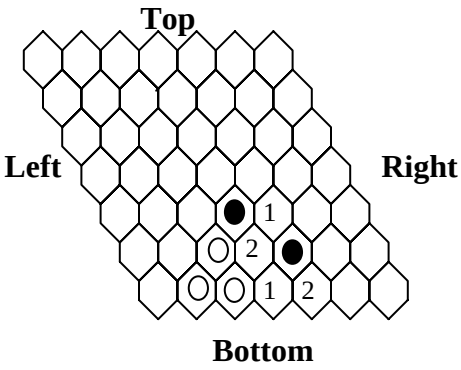


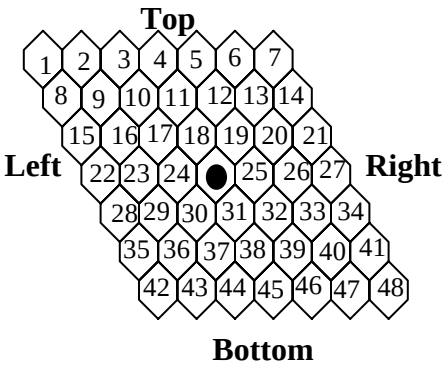
Figure 8: Developed from LocalPattern13 second rule



B. Whole solution tree description

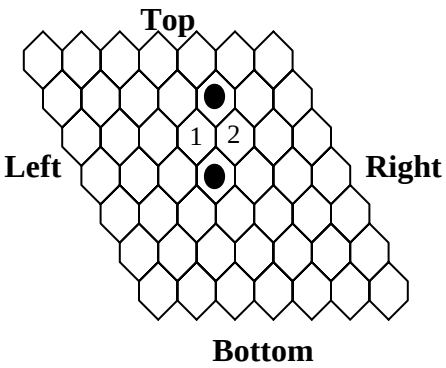
All other LocalPatterns can be explained in the same way as in LocalPattern13. Some LocalPatterns have different game patterns such as LocalPattern2(a) and LocalPattern2(b) but their associated playing strategies based on pattern position number are identical. We collect them into one LocalPattern distinguished by (a), (b) or more.

Local Pattern 1:

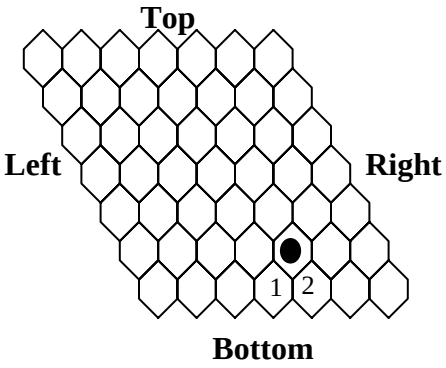


```
If (WhiteMove == 1 || 2 || 3 || 4 || 8 || 9 || 10 || 11 || 15 || 16 ||
17 || 22 || 23 || 24) {
BlackMove = 12;
SumOfLocalGames = SumOfLocalGames - LocalPattern1
+ LocalPattern2(5,6) + LocalPattern2(18,19) +
LocalPattern3(7,13,14,20,21,25,26,30,31,32,33,34,36,37,38
,39,40,41,42,43,44,45,46,47, 48);  }
else if (WhiteMove == 14 || 20 || 21) {
BlackMove = 37;
SumOfLocalGames = SumOfLocalGames - LocalPattern1
+ LocalPattern2(30,31)+ LocalPattern2(43,44) +
LocalPattern3(42,36,35,29,28,24,23,19,18,17,16,15,13,12,
11,10,9,8,7,6,5,4,3,2,1); }
else if (WhiteMove == 13 || 7) {
BlackMove =19;
SumOfLocalGames = SumOfLocalGames - LocalPattern1
+ LocalPattern4(24,18,17,16,12,11,10,9,7,6,5,4,3,2) +
LocalPattern5(20,25,26,30,31,32,33,36,37,38,39,40,42,43,
44,45,46,47); }
else if (WhiteMove == 12) {
BlackMove = 19;
SumOfLocalGames = SumOfLocalGames - LocalPattern1
+ LocalPattern6(24,18,17,16,13,11,10,9,7,6,5,4,3,2) +
LocalPattern5(20,25,26,30,31,32,33,36,37,38,39,40,42,43,
44,45,46,47); }
else if (WhiteMove == 6) {
BlackMove = 20;
SumOfLocalGames = SumOfLocalGames - LocalPattern1
+ LocalPattern2(19,25) +
LocalPattern7(24,18,17,16,14,13,11,10,9,7,6,5,4,3,2) +
LocalPattern8(26,30,31,32,33,36,37,38,39,40,42,43,44,45,
46,47); }
else if (WhiteMove == 5) {
BlackMove = 13;
SumOfLocalGames = SumOfLocalGames - LocalPattern1
+ LocalPattern2(6,7) +
LocalPattern9(19,18,17,12,11,10,4,3) +
LocalPattern10(20,25,26,30,31,32,33,36,37,38,39,40,42,43,
44,45,46,47); }
else if (WhiteMove == 18) {
BlackMove = 19;
SumOfLocalGames = SumOfLocalGames - LocalPattern1
+ LocalPattern11(3,4,5,6,7,10,11,12,13,14,16,17,20,23,24,
25,26,28,29,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,
45,46,47,48);  }
else if (WhiteMove == 19) {
BlackMove = 18;
SumOfLocalGames = SumOfLocalGames - LocalPattern1
+LocalPattern12(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,
20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,
39,40,41,42,43,44,45,46,47,48);
else (omit because of the symmetric characteristics)
```

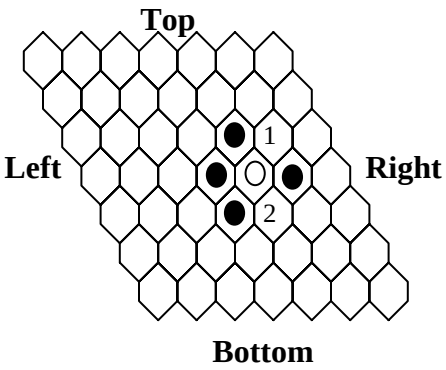
Local Pattern 2(a):



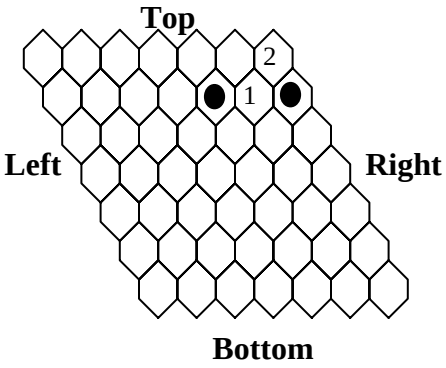
Local Pattern 2(b):



Local Pattern 2(c):



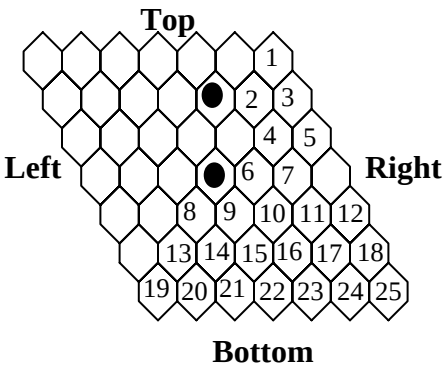
Local Pattern 2(d):




```
If (WhiteMove == 1) {
BlackMove = 2;
SumOfLocalGames = SumOfLocalGames - LocalPattern2;
}
```

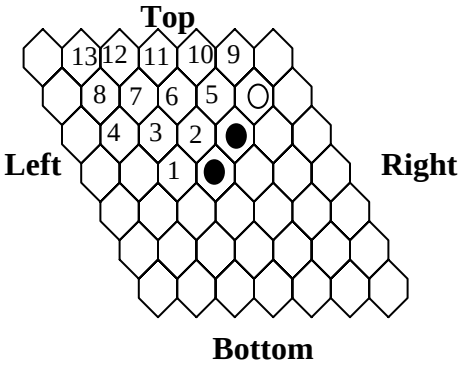
```
else if (WhiteMove == 2) {
BlackMove = 1;
SumOfLocalGames = SumOfLocalGames - LocalPattern2;
}
```

Local Pattern 3:



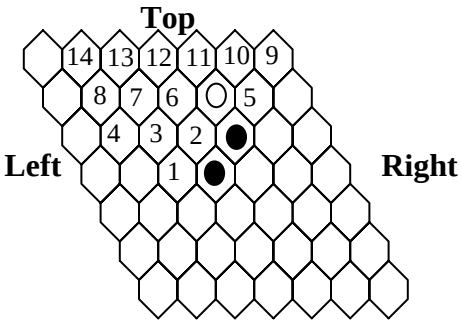

```
If (WhiteMove == 1 || 2 || 3 || 4 || 5 || 6 || 7 || 10 || 11 || 12 ||
13 || 15 || 16 || 17 || 18 || 19 || 22 || 23 || 24 || 25) {
BlackMove = 14;
SumOfLocalGames = SumOfLocalGames - LocalPattern3
+ LocalPattern2(8,9) + LocalPattern2(20,21); }
else if (WhiteMove == 8 || 14 || 20) {
BlackMove = 10;
SumOfLocalGames = SumOfLocalGames - LocalPattern3
+ LocalPattern2(6,9) +
LocalPattern13(11,15,16,17,21,22,23,24);  }
else if (WhiteMove == 21) {
BlackMove = 13;
SumOfLocalGames = SumOfLocalGames - LocalPattern3
+ LocalPattern2(19,20) +
LocalPattern9(8,9,10,14,15,16,22,23);  }
else if (WhiteMove == 9) {
BlackMove = 8;
SumOfLocalGames = SumOfLocalGames - LocalPattern3
+ LocalPattern14(1,2,3,4,5,6,7,10,11,13,14,15,16,19,20,21,
22,23); }
```

Local Pattern 4:



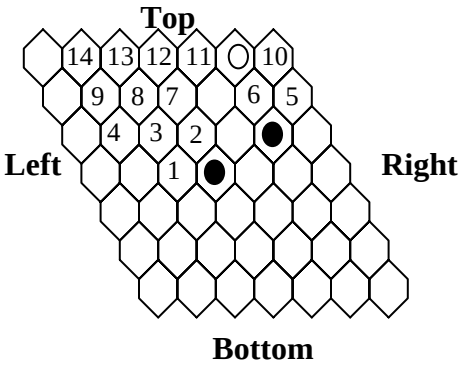

```
If (WhiteMove == 1 || 2 || 3 || 4 || 6 || 7 || 8 || 11 || 12 || 13) {
BlackMove = 5;
SumOfLocalGames = SumOfLocalGames - LocalPattern4
+ LocalPattern2(10,9);  }
else if (WhiteMove == 5 || 9) {
BlackMove = 3;
SumOfLocalGames = SumOfLocalGames - LocalPattern4
+ LocalPattern2(1,2) +
LocalPattern13(4,6,7,8,10,11,12,13);  }
else if (WhiteMove == 10) {
BlackMove = 5;
SumOfLocalGames = SumOfLocalGames - LocalPattern4
+ LocalPattern15(1,2,3,6,7,9,11,12);  }
```

Local Pattern 5:



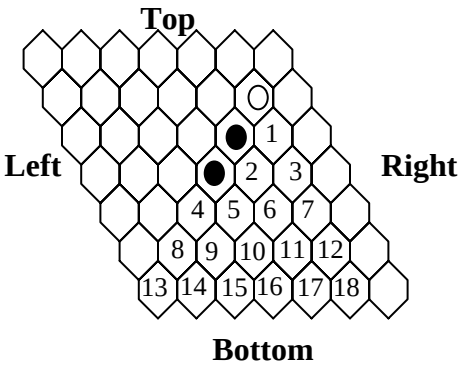

```
If (WhiteMove == (1 || 2 || 3 || 4 || 6 || 7 || 8 || 11 || 12 || 13 ||
14) {
BlackMove = 5;
SumOfLocalGames = SumOfLocalGames - LocalPattern6
+ LocalPattern2(9,10); }
else if (WhiteMove == 10 || 9 || 5) {
BlackMove = 3;
SumOfLocalGames = SumOfLocalGames - LocalPattern6
+ LocalPattern2(1,2) +
LocalPattern13(4,6,7,8,11,12,13,14);  }
```

Local Pattern 6:



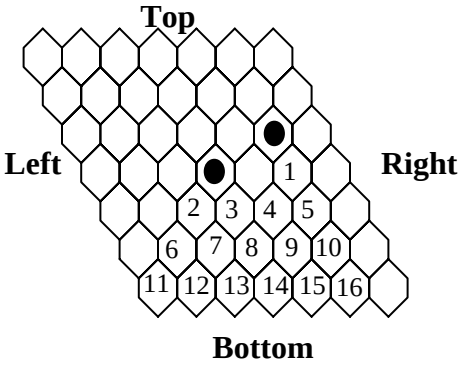

```
If (WhiteMove == (1 || 2 || 3 || 4 || 7 || 8 || 9 || 11 || 12 || 13 ||
14) {
BlackMove = 10;
SumOfLocalGames = SumOfLocalGames - LocalPattern7
+ LocalPattern2(5,6);  }
else if (WhiteMove == 10 || 6 || 5) {
BlackMove = 3;
SumOfLocalGames = SumOfLocalGames - LocalPattern7
+ LocalPattern2(1,2) +
LocalPattern13(4,7,8,9,11,12,13,14);  }
```

Local Pattern 7:



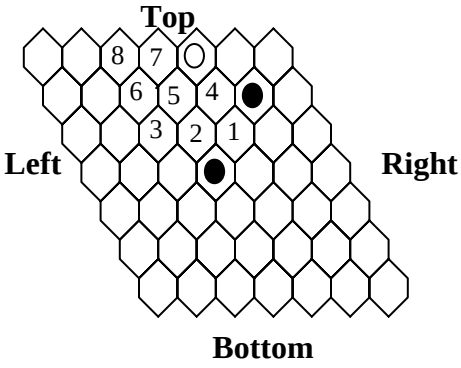

```
If (WhiteMove == (1 || 2 || 3 || 6 || 7 || 10 || 11 || 12 || 16 || 17
|| 18) {
BlackMove = 9;
SumOfLocalGames = SumOfLocalGames - LocalPattern5
+ LocalPattern2(4,5) + LocalPattern2(14,15);  }
else if (WhiteMove == 4 || 8 || 9 || 13 || 14) {
BlackMove = 6;
SumOfLocalGames = SumOfLocalGames - LocalPattern5
+ LocalPattern2(2,5) +
Localpattern13(7,10,11,12,15,16,17,18);  }
else if (WhiteMove == 15) {
BlackMove = 8;
SumOfLocalGames = SumOfLocalGames - LocalPattern5
+ LocalPattern2(13,14) +
LocalPattern9(4,5,6,9,10,11,16,17);  }
else if (WhiteMove == 5) {
BlackMove = 4;
SumOfLocalGames = SumOfLocalGames - LocalPattern5
+ LocalPattern16(1,2,3,6,7,8,9,10,11,13,14,15,16,17);  }
```

Local Pattern 8:



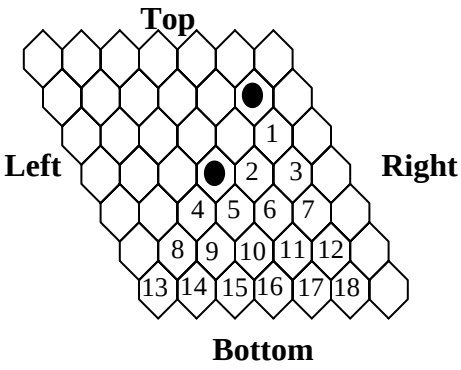

```
If (WhiteMove == 1 || 4 || 5 || 6 || 8 || 9 || 10 || 11 || 14 || 15 ||
16) {
BlackMove = 7;
SumOfLocalGames = SumOfLocalGames - LocalPattern8
+ LocalPattern2(2,3) + LocalPattern2(12,13);  }
else if (WhiteMove == 2 || 7 || 12) {
BlackMove = 4;
SumOfLocalGames = SumOfLocalGames - LocalPattern8
+ LocalPattern2(1,3) +
LocalPattern13(5,8,9,10,13,14,15,16); }
else if (WhiteMove == 13) {
BlackMove = 6;
SumOfLocalGames = SumOfLocalGames - LocalPattern8
+ LocalPattern2(11,12) + LocalPattern9(2,3,4,7,8,9,14,15);
}
else if (WhiteMove == 3) {
BlackMove = 2;
SumOfLocalGames = SumOfLocalGames - LocalPattern8
+ LocalPattern17(1,4,5,6,7,8,9,11,12,13,14,15); }
```

Local Pattern 9:




```
If (WhiteMove == 2 || 3 || 4 || 5 || 6 || 7 || 8) {
BlackMove = 1;
SumOfLocalGames = SumOfLocalGames - LocalPattern9;
}
else if (WhiteMove == 1) {
BlackMove = 2;
SumOfLocalGames = SumOfLocalGames - LocalPattern9
+ LocalPattern18(3,4,5,6,7,8); }
```

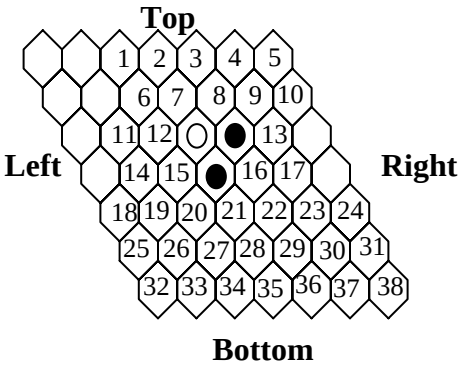
Local Pattern 10:




```
If (WhiteMove == 1 || 2 || 3 || 6 || 7 || 8 || 10 || 11 || 12 || 13 ||
16 || 17 || 18) {
BlackMove = 9;
SumOfLocalGames = SumOfLocalGames - LocalPattern10
+ LocalPattern2(4,5) + LocalPattern2(14,15);  }
else if (WhiteMove == 4 || 9 || 14) {
BlackMove = 6;
SumOfLocalGames = SumOfLocalGames - LocalPattern10
+ LocalPattern2(2,5) +
LocalPattern13(7,10,11,12,15,16,17,18);  }
else if (WhiteMove == 15) {
BlackMove = 8;
SumOfLocalGames = SumOfLocalGames - LocalPattern10
+ LocalPattern2(13,14) +
```

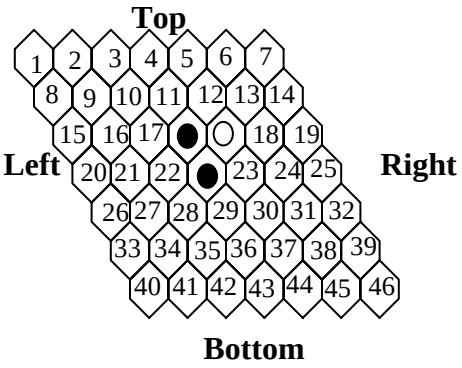
LocalPattern9(4,5,6,9,10,11,16,17); }
else if (WhiteMove ==5) {
BlackMove = 4;
SumOfLocalGames = SumOfLocalGames - LocalPattern10
+ LocalPattern19(1,2,3,6,7,8,9,10,11,13,14,15,16,17); }

Local Pattern 11:



If (WhiteMove == 1 || 2 || 3 || 5 || 6 || 7 || 10 || 11 || 12 || 14 ||
15 || 18 || 19 || 25) {
BlackMove = 4;
SumOfLocalGames = SumOfLocalGames - LocalPattern11
+ LocalPattern2(8,9) +
LocalPattern5(13,16,17,20,21,22,23,26,27,28,29,30,32,33,
34,35,36,37); }
else if (WhiteMove == 8) {
BlackMove = 9;
SumOfLocalGames = SumOfLocalGames - LocalPattern11
+ LocalPattern2(4,5) + LocalPattern5(13,16,17,20,21,22,23,
26,27,28,29,30,32,33,34,35,36,37); }
else if (WhiteMove ==9) {
BlackMove = 8;
SumOfLocalGames = SumOfLocalGames - LocalPattern11
+ LocalPattern2(3,4) + LocalPattern5(13,16,17,20,21,22,23,
26,27,28,29,30,32,33,34,35,36,37); }
else if (WhiteMove == 13 || 16 || 17 || 22 || 23 || 24 || 28 || 29
|| 30 || 31 || 35 || 36 || 37 || 38) {
BlackMove = 27;
SumOfLocalGames = SumOfLocalGames - LocalPattern11
+ LocalPattern2(20,21) + LocalPattern2(33,34) +
LocalPattern14(32,26,25,19,18,15,14,12,11,9,8,7,6,5,4,3,2,
1); }
else if (WhiteMove == 26 || 32) {
BlackMove = 20;
SumOfLocalGames = SumOfLocalGames - LocalPattern11
+ LocalPattern4(16,21,22,23,27,28,29,30,33,34,35,36,37) +
LocalPattern16(19,15,14,12,11,9,8,7,6,5,4,3,2,1); }
else if (WhiteMove == 33) {
BlackMove =19;
SumOfLocalGames = SumOfLocalGames - LocalPattern11
+ LocalPattern2(15,20) +
LocalPattern17(14,12,11,9,8,7,6,5,4,3,2,1) +
LocalPattern7(16,21,22,23,27,28,29,30,32,34,35,36,37); }
else if (WhiteMove == 34) {
BlackMove = 26;
SumOfLocalGames = SumOfLocalGames - LocalPattern11
+ LocalPattern2(32,33) +
LocalPattern9(20,21,22,27,28,29,35,36) +
LocalPattern17(14,12,11,9,8,7,6,5,4,3,2,1); }
else if (WhiteMove == 27) {
BlackMove = 20;
SumOfLocalGames = SumOfLocalGames - LocalPattern11
+
LocalPattern6(16,21,22,23,26,28,29,30,32,33,34,35,36,37)
+ LocalPattern16(19,15,14,12,11,9,8,7,6,5,4,3,2,1); }
else if (WhiteMove == 21) {
BlackMove = 20;
SumOfLocalGames = SumOfLocalGames - LocalPattern11
+
LocalPattern16(13,16,17,22,23,26,27,28,29,32,33,34,35,36)
+ LocalPattern16(19,15,14,12,11,9,8,7,6,5,4,3,2,1); }
else if (WhiteMove == 20) {
BlackMove = 21;
SumOfLocalGames = SumOfLocalGames - LocalPattern11
+ LocalPattern20(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,
18,19,22,23,24,25,26,27,28,29,30,31,32,33,34,35,
36,37,38); }
else if (WhiteMove == 4) {
BlackMove = 8;
SumOfLocalGames = SumOfLocalGames - LocalPattern11
+ LocalPattern35(1,2,3,6,7,11,12,13,14,15,16,17,18,19,20,
21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38); }

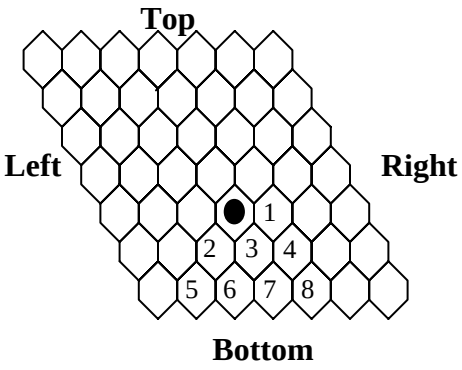
Local Pattern 12:



If (WhiteMove == 1 || 2 || 7 || 8 || 9 || 13 || 14 || 15 || 16 || 18 ||
19 || 20 || 21 || 22 || 23 || 24 || 25 || 26 || 27 || 30 || 31 || 32 || 33
|| 34 || 36 || 37 || 38 || 39 || 40 || 43 || 44 || 45 || 46) {
BlackMove = 35;
SumOfLocalGames = SumOfLocalGames - LocalPattern12
+ LocalPattern2(28,29) + LocalPattern2(41,42) +
LocalPattern13(17,12,11,10,6,5,4,3); }
else if (WhiteMove == 28 || 35 || 41) {
BlackMove = 30;
SumOfLocalGames = SumOfLocalGames - LocalPattern12
+ LocalPattern2(23,29) +
LocalPattern13(31,36,37,38,42,43,44,45) +
LocalPattern13(17,12,11,10,6,5,4,3); }
else if (WhiteMove == 3 || 4 || 10 || 11 || 17) {
BlackMove = 12;
SumOfLocalGames = SumOfLocalGames - LocalPattern12
+ LocalPattern2(5,6) +
LocalPattern3(7,13,14,18,19,23,24,28,29,30,31,32,34,35,36
,37,38,39,40,41,42,43,44,45, 46); }
else if (WhiteMove == 42) {
BlackMove = 34;

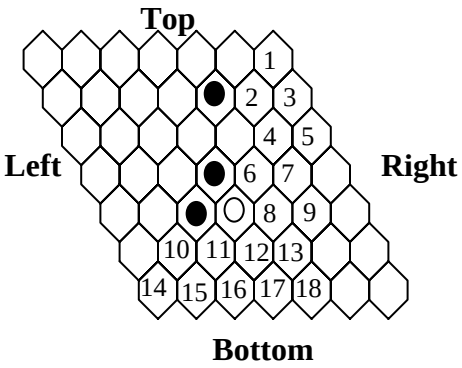
SumOfLocalGames = SumOfLocalGames - LocalPattern12 + LocalPattern2(40,41) + LocalPattern9(28,29,30,35,36,37,43,44) + LocalPattern13(17,12,11,10,6,5,4,3); }
else if (WhiteMove == 29) {
BlackMove = 28;
SumOfLocalGames = SumOfLocalGames - LocalPattern12 + LocalPattern20(44,43,42,41,40,37,36,35,34,33,31,30,27,24,23,22,21,19,18,17,16,15,14,13,12,11,10,9,8,7,6,5,4,3,2,1); }
else if (WhiteMove == 6 || 12) {
BlackMove = 28;
SumOfLocalGames = SumOfLocalGames - LocalPattern12 + LocalPattern13(29,34,35,36,40,41,42,43) + LocalPattern22(27,22,21,17,16,15,11,10,9,8,5,4,3,2,1); }
else if (WhiteMove == 5) {
BlackMove = 12;
SumOfLocalGames = SumOfLocalGames - LocalPattern12 + LocalPattern23(17,11,10,6,4,3) + LocalPattern3(7,13,14,18,19,23,24,28,29,30,31,32,34,35,36,37,38,39,40,41,42,43,44,45, 46); }

Local Pattern 13:



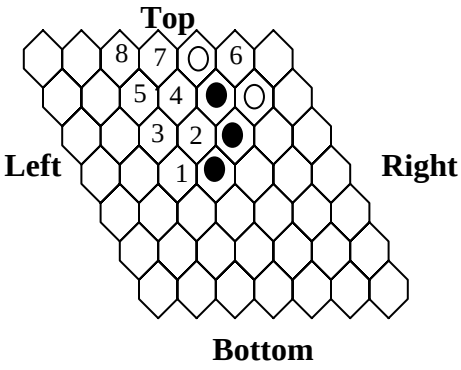
If (WhiteMove == 1 || 3 || 4 || 7 || 8) {
BlackMove = 2;
SumOfLocalGames = SumOfLocalGames - LocalPattern13 + LocalPattern2(5,6); }
else if (WhiteMove == 2 || 5 || 6) {
BlackMove = 4;
SumOfLocalGames = SumOfLocalGames - LocalPattern13 + LocalPattern2(1,3) + LocalPattern2(7,8); }

Local Pattern 14:



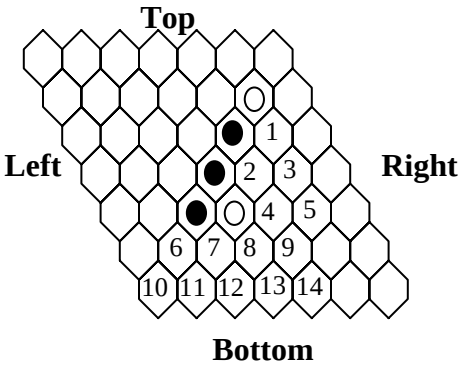
If (WhiteMove == 1 || 2 || 3 || 4 || 5 || 6 || 7 || 8 || 9 || 12 || 13 || 14 || 16 || 17 || 18) {
BlackMove = 15;
SumOfLocalGames = SumOfLocalGames - LocalPattern14 + LocalPattern2(10,11); }
else if (WhiteMove == 10) {
BlackMove = 11;
SumOfLocalGames = SumOfLocalGames - LocalPattern14 + LocalPattern2(15,16); }
else if (WhiteMove == 11) {
BlackMove = 10;
SumOfLocalGames = SumOfLocalGames - LocalPattern14 + LocalPattern2(14,15); }
else if (WhiteMove == 15) {
BlackMove = 11;
SumOfLocalGames = SumOfLocalGames - LocalPattern14 + LocalPattern24(1,2,3,4,5,6,7,8,9,12,13,16,17,18); }

Local Pattern 15:



If (WhiteMove == 1 || 2 || 3 || 4 || 5 || 7 || 8) {
BlackMove = 6;
SumOfLocalGames = SumOfLocalGames - LocalPattern15; }
else if (WhiteMove == 6) {
BlackMove = 5;
SumOfLocalGames = SumOfLocalGames - LocalPattern15 + LocalPattern2(7,8) + LocalPattern25(1,2,3,4); }

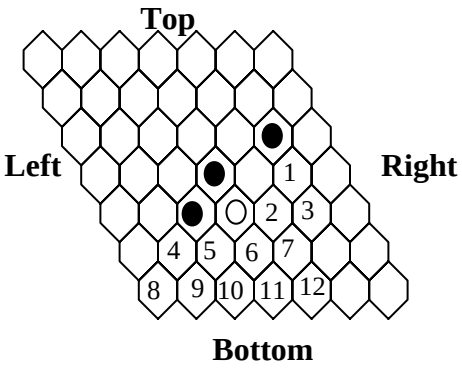
Local Pattern 16:



If (WhiteMove == 1 || 2 || 3 || 4 || 5 || 8 || 9 || 10 || 12 || 13 || 14) {
BlackMove = 11;
SumOfLocalGames = SumOfLocalGames - LocalPattern16 + LocalPattern2(6,7); }
else if (WhiteMove == 6) {

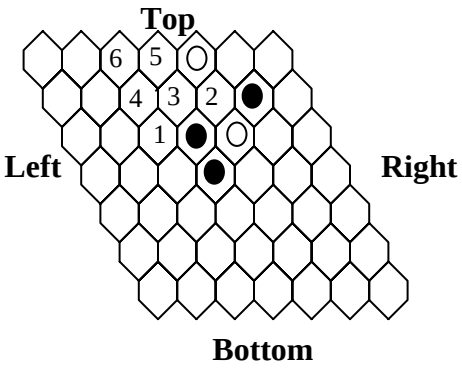
```
BlackMove = 7;
SumOfLocalGames = SumOfLocalGames - LocalPattern16
+ LocalPattern2(11,12);  }
else if (WhiteMove == 7) {
BlackMove = 6;
SumOfLocalGames = SumOfLocalGames - LocalPattern16
+ LocalPattern2(10,11); }
else if (WhiteMove == 11) {
BlackMove = 7;
SumOfLocalGames = SumOfLocalGames - LocalPattern16
+ LocalPattern26(1,2,3,4,5,8,9,12,13,14);  }
```

Local Pattern 17:



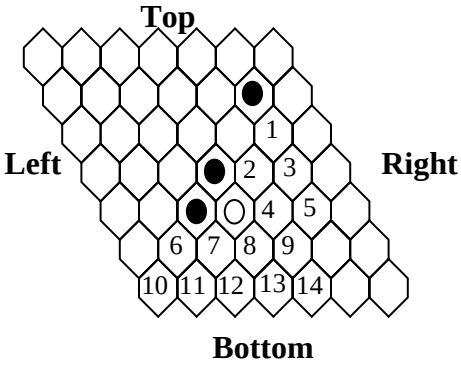

```
If (WhiteMove == 1 || 2 || 3 || 6 || 7 || 8 || 10 || 11 || 12) {
BlackMove =9;
SumOfLocalGames = SumOfLocalGames - LocalPattern17
+ LocalPattern2(4,5);  }
else if (WhiteMove == 4) {
BlackMove =5;
SumOfLocalGames = SumOfLocalGames - LocalPattern17
+ LocalPattern2(9,10):  }
else if (WhiteMove == 5) {
BlackMove = 4;
SumOfLocalGames = SumOfLocalGames - LocalPattern17
+ LocalPattern2(8,9);  }
else if (WhiteMove == 9) {
BlackMove = 5;
SumOfLocalGames = SumOfLocalGames - LocalPattern17
+ LocalPattern27(1,2,3,6,7,10,11,12);  }
```

Local Pattern 18:



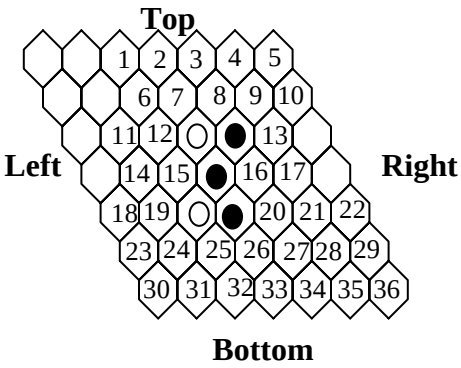

```
If (WhiteMove == 1 || 3 || 4 || 5 || 6) {
BlackMove = 2;
SumOfLocalGames = SumOfLocalGames -
LocalPattern18;  }
else if (WhiteMove == 2) {
BlackMove = 4;
SumOfLocalGames = SumOfLocalGames -
LocalPattern18+ LocalPattern2(1,3) + LocalPattern2(5,6);
```

Local Pattern 19:




```
If (WhiteMove == 1 || 2 || 3 || 4 || 5 || 8 || 9 || 10 || 12 || 13 ||
14) {
BlackMove = 11;
SumOfLocalGames = SumOfLocalGames - LocalPattern19
+ LocalPattern2(6,7);  }
else if (WhiteMove == 6) {
BlackMove = 7;
SumOfLocalGames = SumOfLocalGames - LocalPattern19
+ LocalPattern2(11,12);  }
else if (WhiteMove == 7) {
BlackMove = 6;
SumOfLocalGames = SumOfLocalGames - LocalPattern19
+ LocalPattern2(10,11);  }
else if (WhiteMove == 11) {
BlackMove = 7;
SumOfLocalGames = SumOfLocalGames - LocalPattern19
+ LocalPattern28(1,2,3,4,5,8,9,12,13,14);  }
```

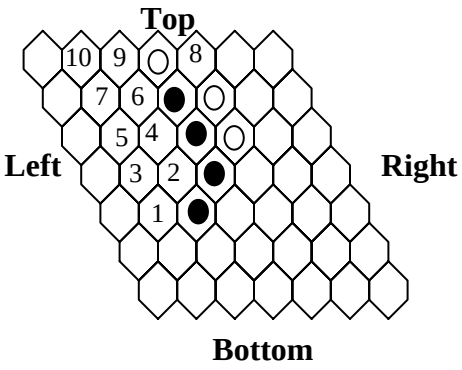
Local Pattern 20:




```
If (WhiteMove == 1 || 2 || 3 || 5 || 6 || 7 || 10 || 11 || 12 || 13 ||
14 || 15 || 16 || 17 || 18 || 19 || 21 || 22 || 23 || 24 || 28 || 29 || 30
|| 35 || 36) {
BlackMove = 4;
SumOfLocalGames = SumOfLocalGames - LocalPattern20
+ LocalPattern2(8,9) +
LocalPattern13(20,25,26,27,31,32,33,34);  }
else if (WhiteMove == 8) {
BlackMove = 9;
SumOfLocalGames = SumOfLocalGames - LocalPattern20
```

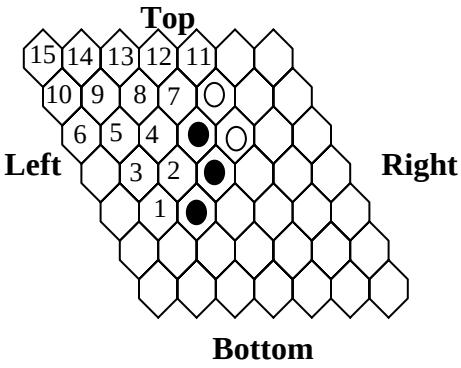
+ LocalPattern2(4,5) +
LocalPattern13(20,25,26,27,31,32,33,34); }
else if (WhiteMove == 9) {
BlackMove = 8;
SumOfLocalGames = SumOfLocalGames - LocalPattern20
+ LocalPattern2(3,4) +
LocalPattern13(20,25,26,27,31,32,33,34); }
else if (WhiteMove == 20 || 26 || 27 || 33 || 34) {
BlackMove = 25;
SumOfLocalGames = SumOfLocalGames - LocalPattern20
+ LocalPattern2(31,32) + LocalPattern14(30,24,23,19,18,
15,14,12,11,9,8,7,6,5,4,3,2,1); }
else if (WhiteMove == 25 || 31) {
BlackMove = 4;
SumOfLocalGames = SumOfLocalGames - LocalPattern20
+ LocalPattern2(8,9) + LocalPattern22(13,16,17,20,21,22,
26,27,28,29,32,33,34,35,36); }
else if (WhiteMove == 32) {
BlackMove = 25;
SumOfLocalGames = SumOfLocalGames - LocalPattern20
+ LocalPattern23(20,26,27,31,33,34) + LocalPattern14(30,
24,23,19,18,15,14,12,11,9,8,7,6,5,4,3,2,1); }
else if (WhiteMove == 4) {
BlackMove = 8;
SumOfLocalGames = SumOfLocalGames - LocalPattern20
+ LocalPattern29(1,2,3,6,7,11,12,13,14,15,16,17,18,19,20,
21,22,23,24,25,26,27,28,29,30, 31,32,33,34,35,36); }

Local Pattern 21:



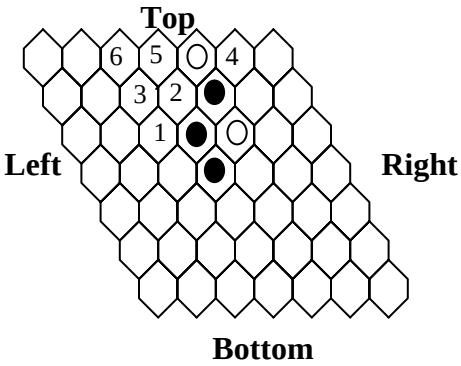
If (WhiteMove == 1 || 2 || 3 || 4 || 5 || 6 || 7 || 9 || 10) {
BlackMove = 8;
SumOfLocalGames = SumOfLocalGames -
LocalPattern21; }
else if (WhiteMove == 8) {
BlackMove = 7;
SumOfLocalGames = SumOfLocalGames - LocalPattern21
+ LocalPattern2(9,10) + LcoalPattern30(1,2,3,4,5,6); }

Local Pattern 22:



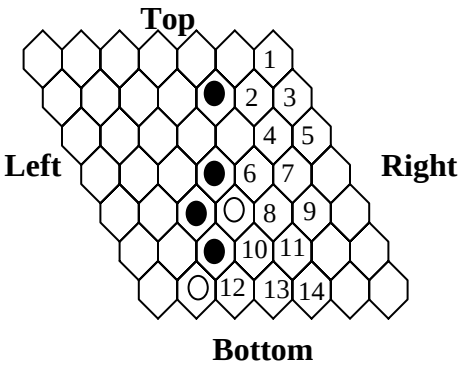
If (WhiteMove == 1 || 2 || 3 || 4 || 5 || 6 || 8 || 9 || 10 || 13 || 14
|| 15) {
BlackMove = 7;
SumOfLocalGames = SumOfLocalGames - LocalPattern22
+ LocalPattern2(11,12); }
else if (WhiteMove == 7 || 11) {
BlackMove = 5;
SumOfLocalGames = SumOfLocalGames - LocalPattern22
+ LocalPattern13(6,8,9,10,12,13,14,15) +
LocalPattern25(1,2,3,4); }
else if (WhiteMove == 12) {
BlackMove = 7;
SumOfLocalGames = SumOfLocalGames - LocalPattern22
+ LocalPattern21(1,2,3,4,5,8,9,11,13,14); }

Local Pattern 23:



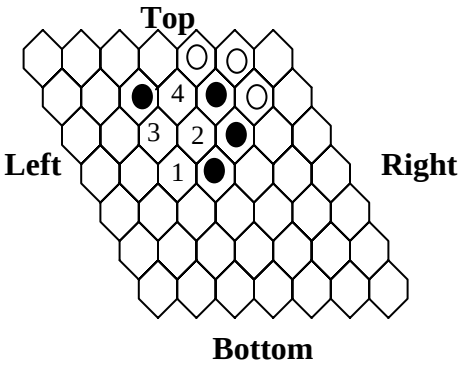
If (WhiteMove == 1 || 2 || 3 || 5 || 6) {
BlackMove = 4;
SumOfLocalGames = SumOfLocalGames -
LocalPattern23; }
else if (WhiteMove == 4) {
BlackMove = 3;
SumOfLocalGames = SumOfLocalGames - LocalPattern23
+ LocalPattern2(1,2) + LocalPattern2(5,6); }

Local Pattern 24:

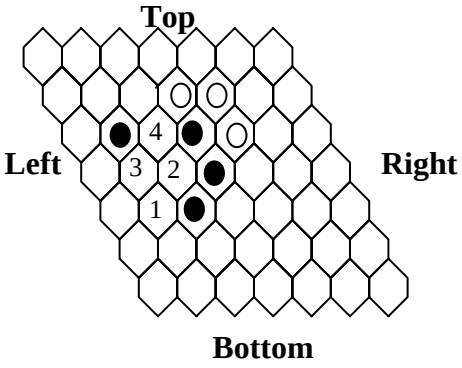



```
If (WhiteMove == 1 || 2 || 3 || 4 || 5 || 6 || 7 || 8 || 9 || 10 || 11 ||
13 || 14) {
BlackMove = 12;
SumOfLocalGames = SumOfLocalGames -
LocalPattern24;  }
else if (WhiteMove == 12) {
BlackMove = 11;
SumOfLocalGames = SumOfLocalGames - LocalPattern24
+ LocalPattern2(13,14) +
LocalPattern31(1,2,3,4,5,6,7,8,9,10);   }
```

Local Pattern 25(a):

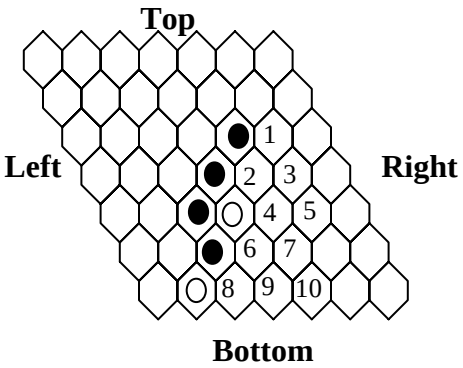


Local Pattern 25(b):



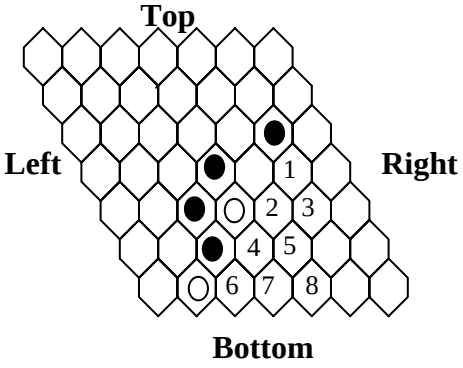

```
If (WhiteMove == 1 || 2 || 3) {
BlackMove = 4;
SumOfLocalGames = SumOfLocalGames -
LocalPattern25;  }
else if (WhiteMove == 4) {
BlackMove = 3;
SumOfLocalGames = SumOfLocalGames - LocalPattern25
+ LocalPattern2(1,2);  }
```

Local Pattern 26:



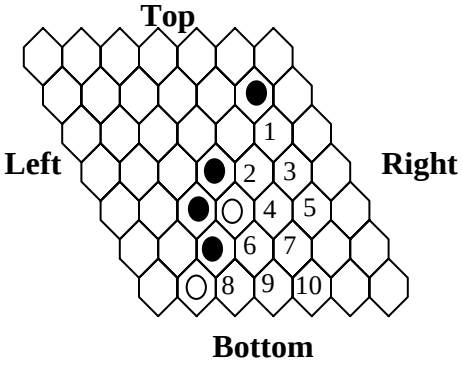

```
If (WhiteMove == 1 || 2 || 3 || 4 || 5 || 6 || 7 || 9 || 10) {
BlackMove = 8;
SumOfLocalGames = SumOfLocalGames -
LocalPattern26;  }
else if (WhiteMove == 8) {
BlackMove = 7;
SumOfLocalGames = SumOfLocalGames - LocalPattern26
+ LocalPattern2(9,10) + LocalPattern32(1,2,3,4,5,6);  }
```

Local Pattern 27:



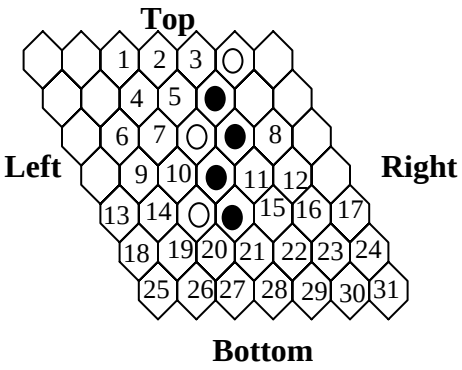

```
If (WhiteMove == 1 || 2 || 3 || 4 || 5 || 7 || 8) {
BlackMove = 6;
SumOfLocalGames = SumOfLocalGames -
LocalPattern27;  }
else if (WhiteMove == 6) {
BlackMove = 5;
SumOfLocalGames = SumOfLocalGames - LocalPattern27
+ LocalPattern2(7,8) + LocalPattern33(1,2,3,4);  }
```

Local Pattern 28:



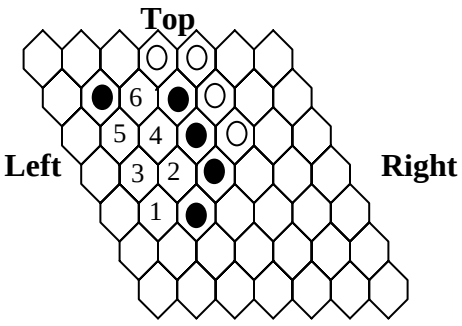
If (WhiteMove == 1 || 2 || 3 || 4 || 5 || 6 || 7 || 9 || 10) {
BlackMove = 8;
SumOfLocalGames = SumOfLocalGames -
LocalPattern28; }
else if (WhiteMove == 8) {
BlackMove = 7;
SumOfLocalGames = SumOfLocalGames - LocalPattern28
+ LocalPattern2(9,10) + LocalPattern34(1,2,3,4,5,6); }

Local Pattern 29:



If (WhiteMove == 1 || 2 || 4 || 5 || 6 || 7 || 8 || 9 || 10 || 11 || 12
|| 13 || 14 || 16 || 17 || 18 || 19 || 23 || 24 || 25 || 30 || 31) {
BlackMove = 3;
SumOfLocalGames = SumOfLocalGames - LocalPattern29
+ LocalPattern13(15,20,21,22,26,27,28,29); }
else if (WhiteMove == 15 || 21 || 22 || 28 || 29) {
BlackMove = 20;
SumOfLocalGames = SumOfLocalGames - LocalPattern29
+ LocalPattern2(26,27) +
LocalPattern24(25,19,18,14,13,10,9,7,6,5,4,3,2,1); }
else if (WhiteMove == 20 || 26) {
BlackMove = 3;
SumOfLocalGames = SumOfLocalGames - LocalPattern29
+ LocalPattern22(8,11,12,15,16,17,21,22,23,24,27,28,29,
30,31); }
else if (WhiteMove == 27) {
BlackMove = 20;
SumOfLocalGames = SumOfLocalGames - LocalPattern29
+ LocalPattern23(15,21,22,26,28,29) +
LocalPattern24(25,19,18,14,13,10,9,7,6,5,4,3,2,1); }
else if (WhiteMove == 3) {
BlackMove = 4;
SumOfLocalGames = SumOfLocalGames - LocalPattern29
+ LocalPattern2(1,2) +LocalPattern36(5,6,7,8,9,10,11,12,13
,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31); }

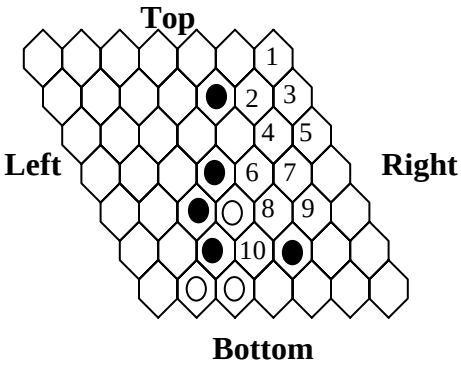
Local Pattern 30:



Bottom

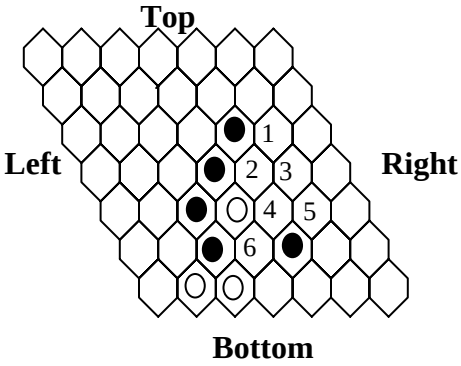
If (WhiteMove == 1 || 2 || 3 || 4 || 5) {
BlackMove = 6;
SumOfLocalGames = SumOfLocalGames -
LocalPattern30; }
else if (WhiteMove == 6) {
BlackMove = 5;
SumOfLocalGames = SumOfLocalGames - LocalPattern30
+ LocalPattern25(1,2,3,4); }

Local Pattern 31:

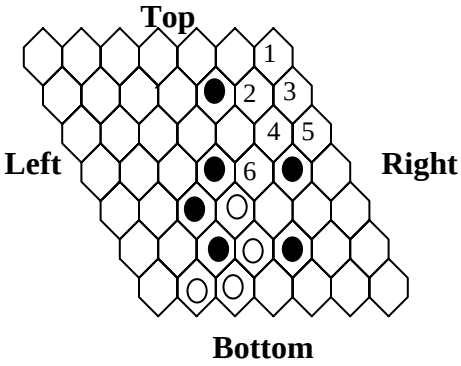


If (WhiteMove == 1 || 2 || 3 || 4 || 5 || 6 || 7 || 8 || 9) {
BlackMove = 10;
SumOfLocalGames = SumOfLocalGames -
LocalPattern31; }
else if (WhiteMove == 10) {
BlackMove = 7;
SumOfLocalGames = SumOfLocalGames - LocalPattern31
+ LocalPattern2(8,9) + LocalPattern32(1,2,3,4,5,6); }

Local Pattern 32(a):

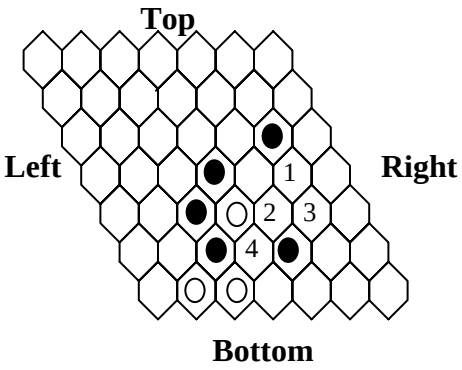


Local Pattern 32(b):



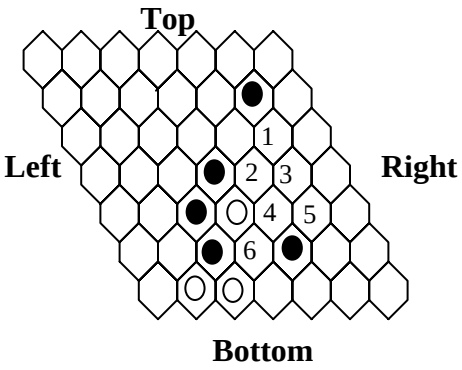
```
If (WhiteMove == 1 || 2 || 3 || 4 || 5) {
BlackMove = 6;
SumOfLocalGames = SumOfLocalGames -
LocalPattern32;  }
else if (WhiteMove == 6) {
BlackMove = 3;
SumOfLocalGames = SumOfLocalGames - LocalPattern32
+ LocalPattern2(1,2) + LocalPattern2(4,5);  }
```

Local Pattern 33:



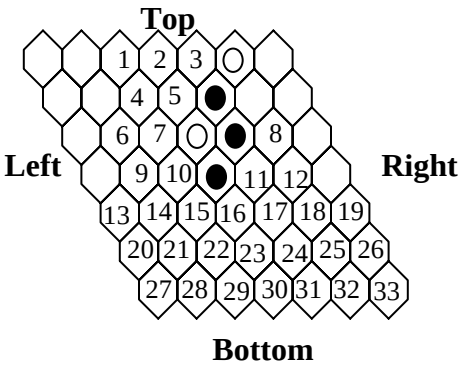

```
If (WhiteMove == 1 || 2 || 3) {
BlackMove = 4;
SumOfLocalGames = SumOfLocalGames -
LocalPattern33;  }
else if (WhiteMove == 4) {
BlackMove = 1;
SumOfLocalGames = SumOfLocalGames - LocalPattern33
+ LocalPattern2(2,3);  }
```

Local Pattern 34:




```
If (WhiteMove == 1 || 2 || 3 || 4 ||5) {
BlackMove = 6;
SumOfLocalGames = SumOfLocalGames -
LocalPattern34;  }
else if (WhiteMove == 6) {
BlackMove = 3;
SumOfLocalGames = SumOfLocalGames - LocalPattern34
+ LocalPattern2(1,2) + LocalPattern2(4,5);  }
```

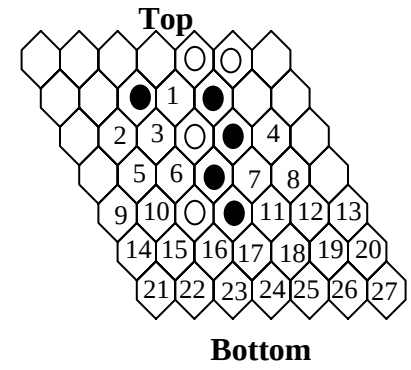
Local Pattern 35:




```
If (WhiteMove == 1 || 2 || 4 || 5 || 6 || 7 || 9 || 10 || 13 || 14 ||
20) {
BlackMove = 3;
SumOfLocalGames = SumOfLocalGames - LocalPattern35
+ LocalPattern5(8,11,12,15,16,17,18,21,22,23,24,25,27,28,
29,30,31,32);  }
else if (WhiteMove == 8 || 11 || 12 || 17 || 18 || 19 || 23 || 24 ||
25 || 26 || 30 || 31 || 32 || 33) {
BlackMove = 22;
SumOfLocalGames = SumOfLocalGames - LocalPattern35
+ LocalPattern2(15,16) + LocalPattern2(28,29) +
LocalPattern24(27,21,20,14,13,10,9,7,6,5,4,3,2,1);  }
else if (WhiteMove == 21 || 27) {
BlackMove = 15;
SumOfLocalGames = SumOfLocalGames - LocalPattern35
+ LocalPattern26(14,10,9,7,6,5,4,3,2,1) +
LocalPattern4(11,16,17,18,22,23,24,25,28,29,30,31,32);  }
else if (WhiteMove == 28) {
BlackMove = 14;
SumOfLocalGames = SumOfLocalGames - LocalPattern35
+ LocalPattern2(10,15) +
LocalPattern7(11,16,17,18,20,21,23,24,25,27,29,30,31,32)
+ LocalPattern27(9,7,6,5,4,3,2,1);  }
else if (WhiteMove == 29) {
BlackMove = 21;
SumOfLocalGames = SumOfLocalGames - LocalPattern35
+ LocalPattern2(27,28) +
LocalPattern9(15,16,17,22,23,24,30,31) +
LocalPattern28(14,10,9,7,6,5,4,3,2,1);  }
else if (WhiteMove == 22) {
BlackMove = 15;
SumOfLocalGames = SumOfLocalGames - LocalPattern35
+ LocalPattern26(14,10,9,7,6,5,4,3,2,1) +
LocalPattern6(11,16,17,18,21,23,24,25,27,28,29,30,31,32);
}
else if (WhiteMove == 15) {
BlackMove = 16;
SumOfLocalGames = SumOfLocalGames - LocalPattern35
+ LocalPattern29(1,2,3,4,5,6,7,8,9,10,11,12,13,14,17,18,19,
20,21,22,23,24,25,26,27,28,29,30,31,32,33);  }
else if (WhiteMove == 16) {
BlackMove = 15;
SumOfLocalGames = SumOfLocalGames - LocalPattern35
+ LocalPattern26(14,10,9,7,6,5,4,3,2,1) +
LocalPattern16(8,11,12,17,18,21,22,23,24,27,28,29,30,31);
}
```

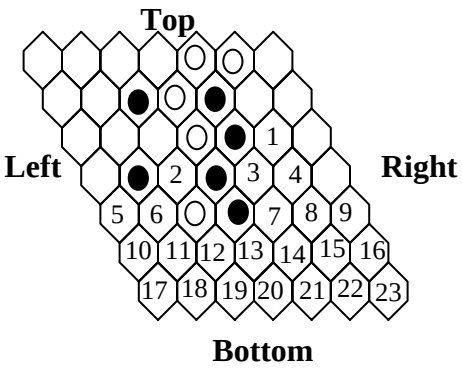
else if (WhiteMove == 3) {
BlackMove = 4;
SumOfLocalGames = SumOfLocalGames - LocalPattern35 + LocalPattern2(1,2) + LocalPattern40(5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20, 21,22,23,24,25,26,27,28,29, 30,31,32,33); }

Local Pattern 36:



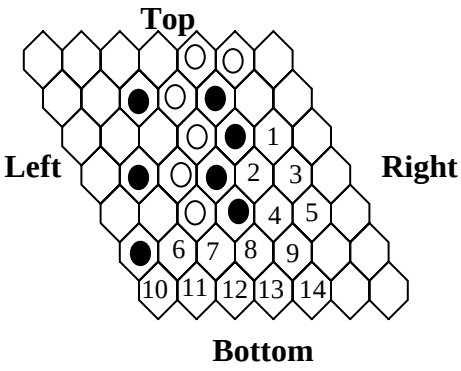
If (WhiteMove == 2 || 3 || 4 || 5 || 6 || 7 || 8 || 9 || 10 || 12 || 13 || 14 || 15 || 19 || 20 || 21 || 26 || 27) {
BlackMove = 1;
SumOfLocalGames = SumOfLocalGames - LocalPattern36 + LocalPattern13(11,16,17,18,22,23,24,25); }
else if (WhiteMove == 11 || 17 || 18 || 24 || 25) {
BlackMove = 16;
SumOfLocalGames = SumOfLocalGames - LocalPattern36 + LocalPattern2(22,23) + LocalPattern31(21,15,14,10,9,6,5,3,2,1); }
else if (WhiteMove == 16 || 22) {
BlackMove = 1;
SumOfLocalGames = SumOfLocalGames - LocalPattern36 + LocalPattern22(4,7,8,11,12,13,17,18,19,20,23,24,25,26, 27); }
else if (WhiteMove == 23) {
BlackMove = 16;
SumOfLocalGames = SumOfLocalGames - LocalPattern36 + LocalPattern23(11,17,18,22,24,25) + LocalPattern31(21,15,14,10,9,6,5,3,2,1); }
else if (WhiteMove == 1) {
BlackMove = 5;
SumOfLocalGames = SumOfLocalGames - LocalPattern36 + LocalPattern2(2,3) + LocalPattern37(4,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20, 21,22,23,24,25,26,27); }

Local Pattern 37:



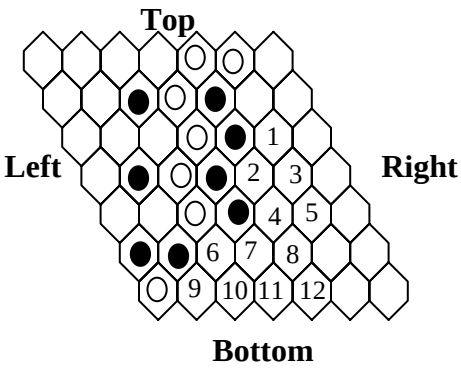
If (WhiteMove == 1 || 3 || 4 || 5 || 6 || 8 || 9 || 10 || 11 || 15 || 16 || 17 || 22 || 23) {
BlackMove = 2;
SumOfLocalGames = SumOfLocalGames - LocalPattern37 + LocalPattern13(7,12,13,14,18,19,20,21); }
else if (WhiteMove == 7 || 13 || 14 || 20 || 21) {
BlackMove = 12;
SumOfLocalGames = SumOfLocalGames - LocalPattern37 + LocalPattern2(18,19) + LocalPattern32(17,11,10,6,5,2); }
else if (WhiteMove == 12 || 18) {
BlackMove = 2;
SumOfLocalGames = SumOfLocalGames - LocalPattern37 + LocalPattern22(1,3,4,7,8,9,13,14,15,16,19,20,21,22,23); }
else if (WhiteMove == 19) {
BlackMove = 12;
SumOfLocalGames = SumOfLocalGames - LocalPattern37 + LocalPattern32(17,11,10,6,5,2) + LocalPattern23(7,13,14,18,20,21); }
else if (WhiteMove == 2) {
BlackMove =10;
SumOfLocalGames = SumOfLocalGames - LocalPattern37 + LocalPattern2(5,6) + LocalPattern38(1,3,4,7,8,11,12,13,14,17,18,19,20,21); }

Local Pattern 38:



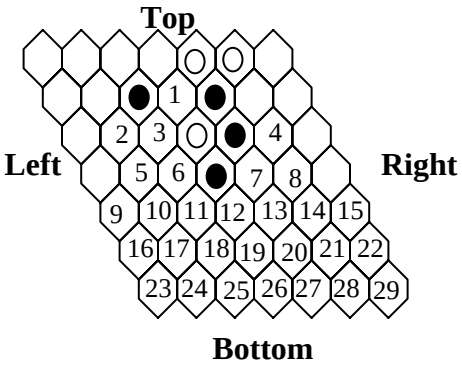
If (WhiteMove == 1 || 2 || 3 || 4 || 5 || 6 || 7 || 8 || 9 || 11 || 12 || 13 || 14) {
BlackMove = 10;
SumOfLocalGames = SumOfLocalGames - LocalPattern38; }
else if (WhiteMove == 10) {
BlackMove = 6;
SumOfLocalGames = SumOfLocalGames - LocalPattern38 + LocalPattern39(1,2,3,4,5,7,8,9,11,12,13,14); }

Local Pattern 39:



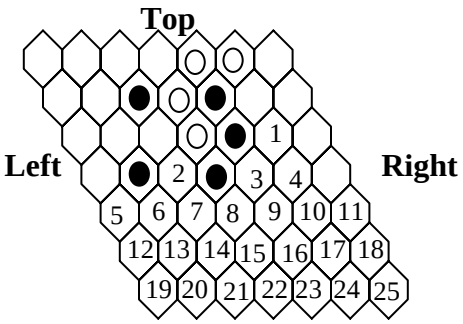
```
If (WhiteMove == 1 || 2 || 3 || 4 || 5 || 6 || 7 || 8 || 10 || 11 || 12)
{
BlackMove = 9;
SumOfLocalGames = SumOfLocalGames -
LocalPattern39;  }
else if (WhiteMove == 9) {
BlackMove = 6;
SumOfLocalGames = SumOfLocalGames - LocalPattern39
+ LocalPattern26(1,2,3,4,5,7,8,10,11,12);  }
```

Local Pattern 40:




```
If (WhiteMove == 2 || 3 || 5 || 6 || 9 || 10 || 16) {
BlackMove = 1;
SumOfLocalGames = SumOfLocalGames - LocalPattern40
+ LocalPattern5(4,7,8,11,12,13,14,17,18,19,20,21,23,24,25,
26,27,28);  }
else if (WhiteMove == 4 || 7 || 8 || 13 || 14 || 15 || 19 || 20 || 21
|| 22 || 26 || 27 || 28 || 29) {
BlackMove = 18;
SumOfLocalGames = SumOfLocalGames - LocalPattern40
+ LocalPattern2(11,12) + LocalPattern2(24,25) +
LocalPattern31(23,17,16,10,9,6,5,3,2,1);  }
else if (WhiteMove == 17 || 23) {
BlackMove = 11;
SumOfLocalGames = SumOfLocalGames - LocalPattern40
+ LocalPattern32(10,6,5,3,2,1) +
LocalPattern4(7,12,13,14,18,19,20,21,24,25,26,27,28);  }
else if (WhiteMove == 24) {
BlackMove = 10;
SumOfLocalGames = SumOfLocalGames - LocalPattern40
+ LocalPattern2(6,11) +
LocalPattern7(7,12,13,14,16,17,19,20,21,23,25,26,27,28) +
LocalPattern33(5,3,2,1);  }
else if (WhiteMove == 25) {
BlackMove = 17;
SumOfLocalGames = SumOfLocalGames - LocalPattern40
+ LocalPattern2(23,24) +
LocalPattern9(11,12,13,18,19,20,26,27) +
LocalPattern34(10,6,5,3,2,1);  }
else if (WhiteMove == 18) {
BlackMove = 11;
SumOfLocalGames = SumOfLocalGames - LocalPattern40
+ LocalPattern32(10,6,5,3,2,1) +
LocalPattern6(7,12,13,14,17,19,20,21,23,24,25,26,27,28); }
else if (WhiteMove == 11) {
BlackMove = 12;
SumOfLocalGames = SumOfLocalGames - LocalPattern40
+ LocalPattern36(1,2,3,4,5,6,7,8,9,10,13,14,15,16,17,18,19,
20,21,22,23,24,25,26,27,28,29);  }
else if (WhiteMove == 12) {
BlackMove = 11;
SumOfLocalGames = SumOfLocalGames - LocalPattern40
+ LocalPattern32(10,6,5,3,2,1) +
LocalPattern16(4,7,8,13,14,17,18,19,20,23,24,25,26,27);  }
else if (WhiteMove == 1) {
BlackMove = 5;
SumOfLocalGames = SumOfLocalGames - LocalPattern40
+ LocalPattern2(2,3),
LocalPattern41(4,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,
21,22,23,24,25,26,27,28,29);  }
```

Local Pattern 41:




```
If (WhiteMove == 5 || 6 || 12) {
BlackMove = 2;
SumOfLocalGames = SumOfLocalGames - LocalPattern41
+ LocalPattern5(1,3,4,7,8,9,10,13,14,15,16,17,19,20,21,22,
23,24);  }
else if (WhiteMove == 1 || 3 || 4 || 9 || 10 || 11 || 15 || 16 || 17
|| 18 || 22 || 23 || 24 || 25) {
BlackMove = 14;
SumOfLocalGames = SumOfLocalGames - LocalPattern41
+ LocalPattern2(7,8) + LocalPattern2(20,21) +
LocalPattern32(19,13,12,6,5,2);  }
else if (WhiteMove == 13 || 19) {
BlackMove = 7;
SumOfLocalGames = SumOfLocalGames - LocalPattern41
+ LocalPattern2(6,2) +
LocalPattern4(3,8,9,10,14,15,16,17,20,21,22,23,24);  }
else if (WhiteMove == 20) {
BlackMove = 6;
SumOfLocalGames = SumOfLocalGames - LocalPattern41
+ LocalPattern2(2,7) +
LocalPattern7(3,8,9,10,12,13,15,16,17,19,21,22,23,24);  }
else if (WhiteMove == 21) {
BlackMove = 13;
SumOfLocalGames = SumOfLocalGames - LocalPattern41
+ LocalPattern2(19,20) + LocalPattern2(2,6) +
LocalPattern9(7,8,9,14,15,16,22,23);  }
else if (WhiteMove == 14) {
BlackMove = 7;
SumOfLocalGames = SumOfLocalGames - LocalPattern41
+ LocalPattern2(2,6) +
LocalPattern6(3,8,9,10,13,15,16,17,19,20,21,22,23,24);  }
else if (WhiteMove == 7) {
```

```
BlackMove = 8;
SumOfLocalGames = SumOfLocalGames - LocalPattern41
+ LocalPattern37(1,2,3,4,5,6,9,10,11,12,13,14,15,16,17,18,
19,20,21,22,23,24,25);  }
else if (WhiteMove == 8) {
BlackMove = 7;
SumOfLocalGames = SumOfLocalGames - LocalPattern41
+ LocalPattern2(2,6) +
LocalPattern16(1,3,4,9,10,13,14,15,16,19,20,21,22,23);  }
else if (WhiteMove == 2) {
BlackMove = 12;
SumOfLocalGames = SumOfLocalGames - LocalPattern41
+ LocalPattern2(5,6) + LocalPattern2(7,8) +
LocalPattern38(1,3,4,9,10,13,14,15,16,19,20,21,22,23); }
```

IV. Conclusion

In this paper, we use heuristics to decompose the whole board into a sum of disjunctive small “local” games because moves made in one part of the positions do not often affect that other parts. Based on this method, we found the winning strategy on 7 by 7 Hex board. According to Ian Stewart’s “Hex Marks the Spot” in “Scientific American” (September 2000), this is the largest Hex board for which a winning strategy is actually known. In fact, we have found the winning strategy on 8 by 8 Hex board by the same idea. The online solution for Hex 8 by 8 is just on a debugging stage. The web site is: http://www.ee.umanitoba.ca/~jingyang/hex88.html. We believe the decomposition method can be further improved to solve other combination explosion problems in computer game.

V. References

[1] Ian Stewart, "Hex Marks the Spot", Scientific American, Sept. 2000.

[2] Cameron Browne, "Hex Strategy: Making the Right Connections", 2000

[3] Martin Muller, "Computer Go as a Sum of Local Games: An Application of Combinatorial Game Theory", (Ph.D. Dissertation 1995)

[4] L. Victor Allis, "Searching for Solutions in Games and Artificial Intelligence", 1995

[5] J. H. Conway, "On Numbers and Games", (Book 1976)