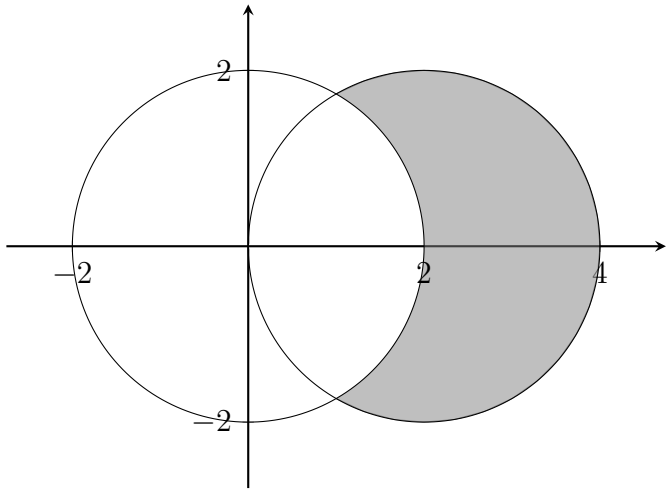
MATH 242: Calculus 3
TEST 1A

Appropriate calculators ARE permitted—show decimals to three places unless otherwise stated.
Please box your answers. Good luck!

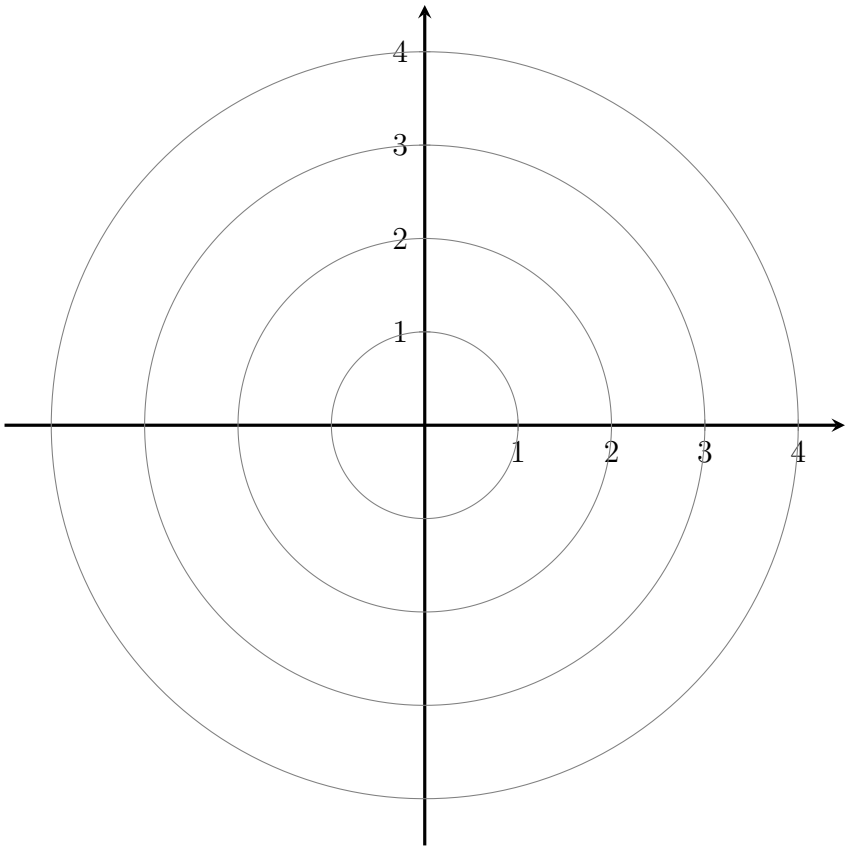
Page	Points	Score
1	8	
2	10	
3	11	
4	8	
5	9	
6	4	
Total:	50	

Name:_____

1. [4 pts] Set up an integral equal to the area inside $r = 4\cos\theta$ and outside $r = 2$, which is shown below.



2. [4 pts] Graph and label the polar points $(2, \frac{3\pi}{2})$, $(-1, \frac{7\pi}{6})$, $(-\sqrt{2}, \frac{-\pi}{4})$, and $(3, 3)$ on the axes provided.



3. [2 pts] Find a formula for the general term a_n of the sequence $\left\{\frac{1}{5}, \frac{1}{25}, \frac{1}{125}, \ldots\right\}$.

4. [5 pts] Determine if each of the following sequences is convergent or divergent and state the limit of each, if it exists.

   (a) $a_n = 1 - (.7)^n$

   (b) $a_n = \dfrac{2^n}{n!}$

   (c) $\{1, -1, 1, -1, 1, \ldots\}$

5. [3 pts] Mark the following as true (T) or false (F):

   (a) _____ If a sequence is convergent, then it must be bounded.

   (b) _____ If a sequence is bounded, then it must be convergent.

   (c) _____ If a sequence is convergent, then it must be monotonic.

6. [3 pts] Eliminate the parameter from the following parametric curve to obtain a Cartesian equation.

$x = e^{4t}$
$y = \sqrt[3]{t+1}$

7. [2 pts] Rewrite the Cartesian equation $y = x^2 - 3$ as a polar equation.

8. [3 pts] Find the Cartesian coordinates of the polar point $\left(3, \frac{13\pi}{12}\right)$.

9. [3 pts] Find the polar coordinates of the Cartesian point $(-4, 3)$.

10. Consider the parametric curve given by $x = \frac{1}{3}t^3 + t^2 - 3t, \quad y = t^3 - 3t + 1.$

(a) [3 pts] Find $\frac{dy}{dx}$ for the given curve.

(b) [5 pts] Find the ordered pair(s) at which the curve has a horizontal tangent.

(c) [2 bonus] Determine the value of $\lim_{t \to 1} \frac{dy}{dx}$ and explain what it represents.

11. Set up (but do not evaluate) integrals equal to the arc lengths of the given curves.

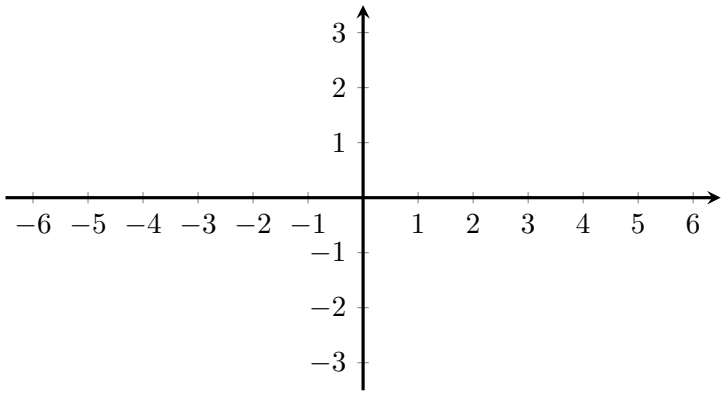
(a) [3 pts] $x = t^3, \quad y = e^{2t}, \quad -1 \leq t \leq 5$

(b) [2 pts] $r = \sin^2(\theta), \quad 0 \leq \theta \leq \frac{\pi}{2}$

12. [4 pts] Calculate the slope of the polar curve $r = 2\cos\theta$ at $\theta = \dfrac{\pi}{6}$.

13. [2 pts] Say that specific parametric equations yield the following table of values. Graph the function on the axes provided.

t	x	y
-2	-4	2
-1	0	0
0	1	-2
1	2	-2
2	6	0



14. [1 pt] Give an example of a parametric function whose graph fails the vertical line test.

15. [1 pt] The graph of the parametric equations $x = t^2 + t - 3$, $y = t^2 - t - 3$ is shown below. Use an arrow to label the direction of increase.