

Agenda :-

1. Probability :- All concepts.
2. App'g probability in python.

Probability :- Naive Bayes.

Probability terms: -

1. Experiment or Random Experiment
2. Outcome
3. Sample Space
4. Event.
5. Probability Rules
(Intersection, Union, Complement)

1. Experiment :-

Defined as Uncertain situations which has multiple outcomes.

eg:- 1. Tossing a coin.

2. Rolling a dice

3. Whether it will rain tomorrow?


4. Whether India will win or lose?

2. Outcome:- Result of an experiment

eg:- Toss a coin :- H, T.

2. Roll a dice:- 1, 2, 3, 4, 5, 6

(1)  4 ← outcome

(2)  2 ← outcome for 2nd time.

3. Sample Space :- Collection of all possible results of an experiment

eg:- Tossing a coin :- H, T \leftarrow T \leftarrow outcome

$$S(T) = \{H, T\}$$

Rolling ^{one} a dice :-

Sample Space :- $\{1, 2, 3, 4, 5, 6\}$

Q. Write the Sample Space for 2 coins tossed.

$$SS :- \{(HH), (TT), (HT), (TH)\}$$

Q. Write the Sample Space for 2 dice rolled.

$$SS = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), \\ \vdots \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

$$n(SS) = \underline{\underline{36}}$$

4. Event :- Subset of Sample Space.

eg: - ① A coin is tossed and head turns up.
is called an event.

$$S = (H, T) \Rightarrow H \leftarrow \underline{\underline{\text{outcome}}}$$

Subset

② India won the match with Aus.

$$S = (W, L)$$

I W \leftarrow
 \uparrow outcome

eg:- 1 Tossing a coin, head turns up
 $S = \{H, T\} \Rightarrow \underline{n(S)} = \underline{2}$

Event A:- Head turning up.

$$\underline{\underline{S(A)}} = \underline{\underline{\{H\}}} = \underline{\underline{n(S(A))}} = \underline{\underline{1}}$$

eg²:- Exp:- Rolling a dice

$$S(RD) = \{1, 2, 3, 4, 5, 6\}$$

Event:- When a dice rolled, even nos. turn up.

$$S(RD) = \{1, 2, 3, 4, 5, 6\}$$

$$n(S) = 6.$$

Event A:- Even nos. turning up.

$$S(A) = \{2, 4, 6\} \Rightarrow \underline{\underline{n(S(A)) = 3}}$$

eg:- Exp :- Toss 2 coins and atleast 1 head turns up.

$$S = \{(HH), (HT), (TH), (TT)\} = n(S) = 4$$

Event A :- Atleast one head turns up.

$$S = \{(HH), (HT), (TH)\} \Rightarrow n(A) = 3.$$

Event B :- No tail turns up

$$S = \{HH\} \Rightarrow n(B) = \underline{\underline{1}}$$

eg: 2] Exp:- Rolling 2 dice & the sum of two dice is 10.

$$S = \{(1, 1) \dots (6, 6)\}$$

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

$$n(S) = \underline{\underline{36}}$$

A :- the sum of two dice turns up as 10

$$S(A) = \{(5, 5), (6, 4), (4, 6)\} \Rightarrow \underline{\underline{n(A)}} = \underline{\underline{3}}$$

eg:- 3 Exp:- Rolling one dice & Tossing one coin at the same time.

$$S = \{ (H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), \\ (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6) \}$$

$$\boxed{n(S) = 12}$$

Event A :- Only Head turns up

$$S(A) = \{ (H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6) \}$$

$$n(S(A)) = 6$$

$$\boxed{P(A) = \frac{6}{12}} = \frac{1}{2}$$

Event B :- Get only even nos. on the dice

$$S(B) = \{ (H, 2), (H, 4), (H, 6), \\ (T, 2), (T, 4), (T, 6) \} \Rightarrow n(S(B)) = 6$$

$$\boxed{P(B) = \frac{6}{12}}$$

5. Probability :-
of an event A

$$\underline{Prob} = \frac{n(S(A))}{n(S)}$$

A :- is an event.

eg:- Exp:- Tossing a coin $S(H, T) \rightarrow \boxed{n(S) = 2}$

Event A :- When you toss a coin head turns up

$$S(H) = n(S(A)) = 1$$

$$\boxed{P(A) = \frac{1}{2}}$$

Probability Rules :-

1: Complement :-

eg:- Roll 1 dice $S = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 6$

Event A:- Getting all nos. divisible by 3.

$$S(A) = \{3, 6\} \Rightarrow n(S(A)) = 2.$$

$$P(A) = 2/6$$

Event B:- Getting all nos. not divisible by 3.

$$S(B) = \{1, 2, 4, 5\} \Rightarrow n(S(B)) = 4$$

$$P(B) = 4/6$$

$$P(A) = \frac{2}{6}$$

$$P(A) = 1 - \frac{2}{6} \Rightarrow \frac{6-2}{6} \Rightarrow \left(\frac{4}{6}\right) \leftarrow P(B)$$

$$P(A) = 1 - P(A^c)$$

eg:- Event C:- Getting all nos. multiples of 2.

$$S(C) = \{2, 4, 6\} \Rightarrow n(S(C)) = 3$$

$$P(C) = 3/6$$

$$P(C^c) = 1 - P(C) \Rightarrow 1 - \frac{3}{6} = \underline{3}$$

$$\boxed{P(C^c) = 1 - P(C)} \Rightarrow 1 - \frac{3}{6} = \frac{3}{6}$$

Exmp C^c :- Getting all nos. non multiple of 2.

$$S(C^c) = \{1, 3, 5\} \Rightarrow n(S(C^c)) = 3$$

$$\boxed{P(C^c) = 3/6}$$

II Rule:- Roll a dice :-

$$S = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 6$$

A :- Get all nos. > 2 & < 6 .

$$S(A) = (3, 4, 5) \Rightarrow n(S(A)) = 3$$

$$B :- S(B) = \{1, 5, 6\} \Rightarrow n(S(B)) = 3.$$

$$S(A \cap B) = \{5\}.$$

$$P(A \cap B) = 1/6$$

\cap = Inter
Section

$\cap \rightarrow$ Intersection :- Common elements in Both the events.

III :- Union :- $S :- \{1, 2, 3, 4, 5, 6\}$
 (U)

$$S(A) = \{2, 4, 6\}$$

$$S(B) = \{1, 2, 5\}$$

$$S(A \cup B) = \{1, 2, 4, 5, 6\}$$

$$P(A \cup B) = 5/6$$

eg:- Roll one dice & toss one coin simultaneously
 $|S| = 12$

A:- Getting all odds when it's fail.

$$S(A) = 3$$

$$|P(A)| = 3/12$$

*Law of Probability :-

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

eg:- Dice :- $S = \{1, 2, 3, 4, 5, 6\}$

$$S(A) = \{2, 4, 6\} \Rightarrow P(A) = 3/6$$

$$S(B) = \{1, 2\}$$

$$P(B) = 2/6$$

$$S(A \cap B) = \{2\}$$

$$P(A \cap B) = 1/6$$

$$P(A \cup B) = \frac{3}{6} + \frac{2}{6} - \frac{1}{6} = \frac{3+2-1}{6} = \frac{4}{6}$$

$$\overline{P(A \cap B)} \times \overline{P(B)} \times \overline{P(A \cup B)} = \underline{\underline{P(A)}}$$

Q. What is the range of probability?

R:- $[0, 1] \Rightarrow 0 \& 1$ both are inclusive

eg:- Event A:- Sun rising in west.

$$P(A) = 0$$

Event B:- Sun rising in east

$$P(B) = 1$$



Note:- Probability value will never be greater than 1 or less than 0.

eg:-

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$S(A) = \{2, 4, 6\}$$

$$S(B) = \{1, 3, 5\}$$

$$S(C) = \{\}$$

$$\Rightarrow \boxed{P(C) \Rightarrow 0/6}$$

↑ empty set or a null set

For 2 Events, Event A & Event C.

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$S(A) = \{2, 4, 6\}$$

$$S(C) = \{1, 3, 5\}$$

$$S(\underline{A \cap C}) = \{\} \leftarrow \text{Event A \& Event C are mutually exclusive events or disjoint events}$$

eg:- Tossing a coin.

$$S(A) = \{\} \leftarrow \text{mutually exclusive?} \Rightarrow \underline{\text{No}}$$

Recap :-

1. Experiment.
2. Outcome.
3. Sample Space
4. Event.

Rules :-

5. Probability (Event)
6. Complement \Rightarrow $\underline{P(A) = 1 - P(A^c)}$
7. Intersection

8. Union.

9. Range $\Rightarrow [0, 1]$

Law of
prob

10. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

11. Empty Set.

12. Mutually Exclusive Event
(Disjoint Event)