DS 288 HW6
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Stability analysis of the equation $\frac{dy}{dt} = -3y(t)$.

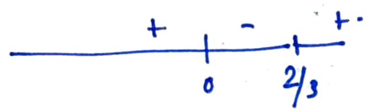
- Midpoint RK-2

$$
\begin{aligned}
w_{i+1} &= w_i + h f\left(w_i + \frac{h}{2} f_i,\ t_i + \frac{h}{2}\right) \\
&= w_i + h\left[-3\left(w_i + \frac{h}{2} f_i\right)\right] \\
&= w_i\left[1 - 3h\right] - \frac{3h^2}{2} f_i \\
&= w_i\left[1 - 3h\right] - \frac{3h^2}{2}(-3w_i) \\
&= w_i\left[1 - 3h + \frac{9h^2}{2}\right]
\end{aligned}
$$

We want $\frac{9h^2}{2} - 3h + 1 < 1$

or $\frac{9h^2}{2} - 3h < 0$

or $h\left(\frac{9h}{2} - 3\right) < 0$



$\therefore$ ~~$h < 0$ or $h > \frac{2}{3}$~~

$\therefore$ $h \in \left(0, \frac{2}{3}\right)$

Adam Bashforth 2 step method

$$w_{i+1} = w_i + \frac{h}{2}\left[3f_i - f_{i-1}\right] \qquad f(t) = -3y(t).$$

$$= w_i + \frac{h}{2}\left[3(-3w_i) - (-3w_{i-1})\right]$$

$$w_{i+1} = w_i\left(1 - \frac{9h}{2}\right) + w_{i-1}\left(\frac{3h}{2}\right)$$

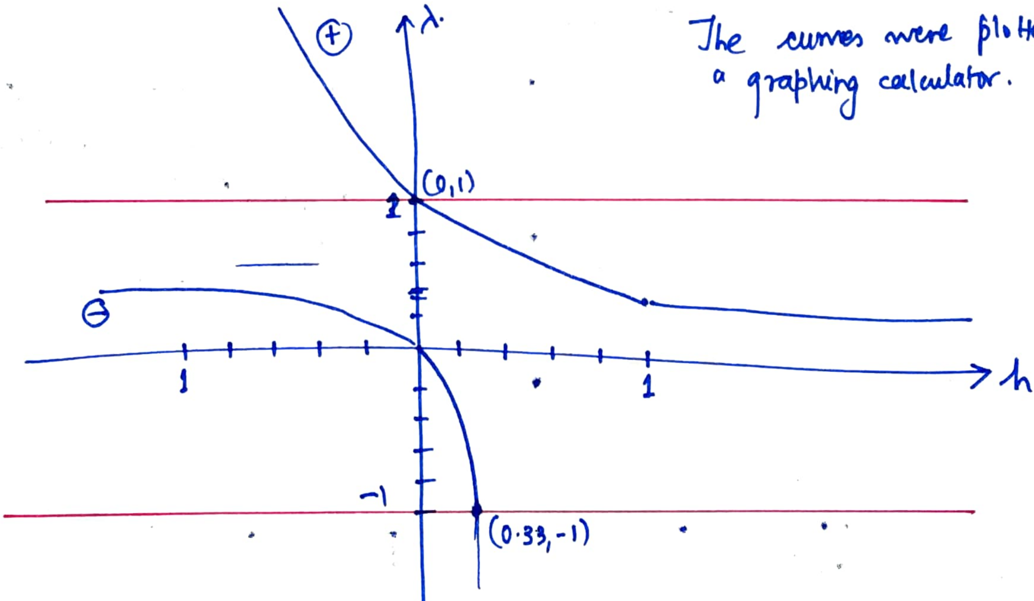
Assume $w_i = k\lambda^i$

$$\therefore \cancel{k}\lambda^{i+1} = \cancel{k}\lambda^i\left(1 - \frac{9h}{2}\right) + \cancel{k}\lambda^{i-1}\left(\frac{3h}{2}\right).$$

or $2\lambda^2 - \lambda(2-9h) - 3h = 0.$

$$\lambda = \frac{(2-9h) \pm \sqrt{81h^2 + 4 - 36h + 24h}}{4}$$

The curves were plotted with a graphing calculator.



- Basically we want $|\lambda| < 1$ and $h > 0$

$\therefore$ from the graph we see that the ~~correct~~ step size interval for which the method is stable is ~~(0, 1/3)~~ $(0, 1/3)$

Adam's Moulton 2 Step Method

$$y_{i+1} = y_i + \frac{h}{12}\left[5f_{i+1} + 8f_i - f_{i-1}\right]$$

$$w_{i+1} = w_i + \frac{h}{12}\left[5(-3w_{i+1}) + 8(-3w_i) - (-3w_{i-1})\right]$$

$$\therefore w_{i+1}\left(1 + \frac{\cancel{15}^{5} h}{\cancel{12}_{4}}\right) + w_i\left(\frac{\cancel{24}^{8} h}{\cancel{12}_{4}} - 1\right) - \frac{\cancel{3}h}{\cancel{12}_{4}} w_{i-1} = 0.$$

$\cancel{w_{i+1}(12+15h) + w_i}$

$$w_{i+1}(4+5h) + w_i(8h-4) - h w_{i-1} = 0.$$

$$w_i = k\lambda^i$$

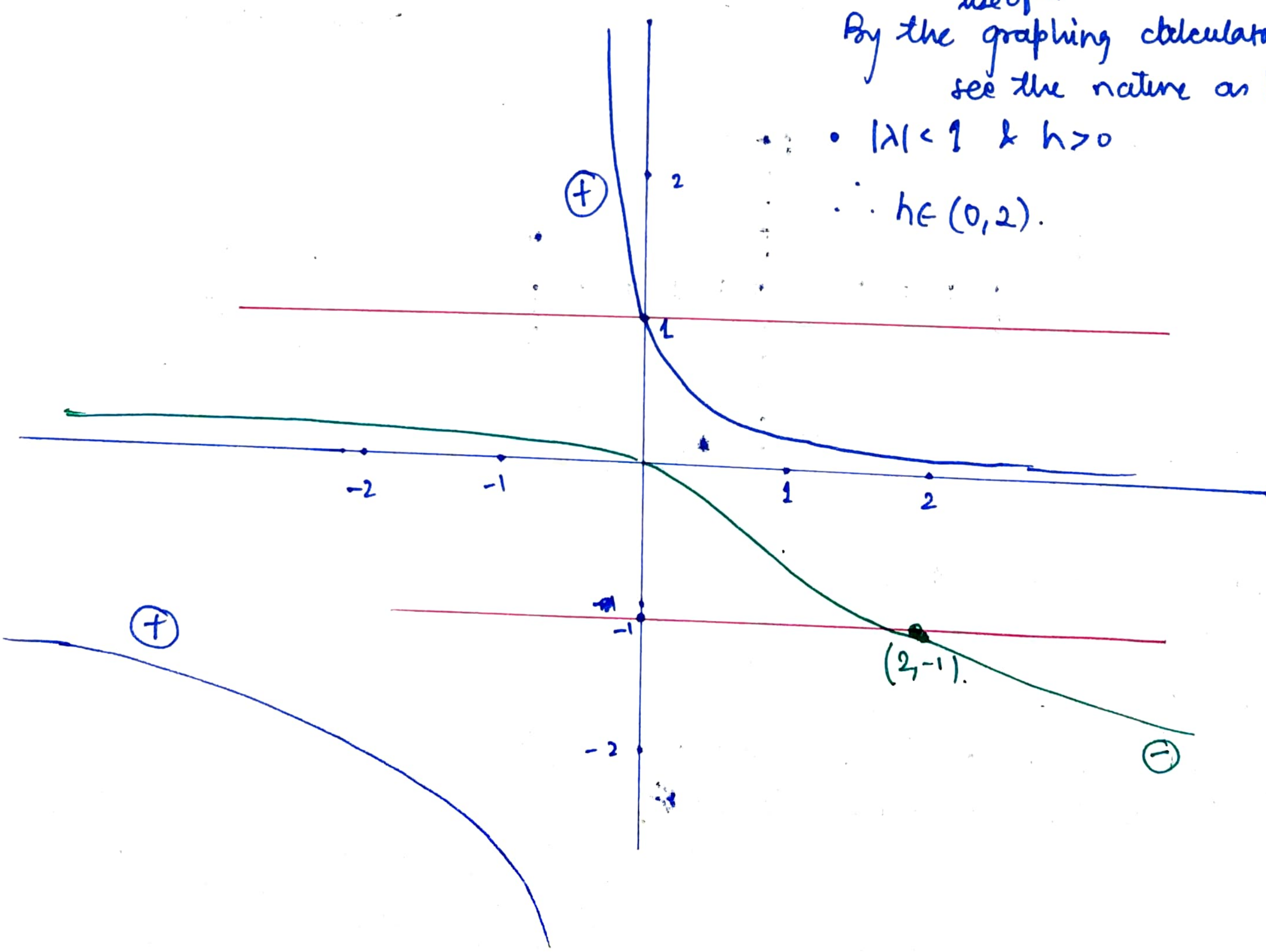
$$\lambda^2(4+5h) + \lambda(8h-4) - h = 0$$

$$\lambda = \frac{(4-8h) \pm \sqrt{64h^2 + 16 - 64h + 16h + 20h^2}}{2(4+5h)}$$

By the use of a graphing calculator, we see the nature as below.

- $|\lambda| < 1$ & $h > 0$

$\therefore h \in (0, 2)$.



Adam Bashforth Predictor Corrector Order 2

It is composed of 2 steps: for prediction & correction.
Prediction is done with Adams Bashforth. & correction is done with Adams-Molton.

The maximum allowable step sizes for the two are 1/3 & 2 respectively. Hence for P/C method of order 2, the maximum allowable stepsize will be 1/3. The same can be corroborated by the code by varying the step sizes.