2022 - November.

සෑම ප්‍රශ්නයක් සඳහා ම පිළිතුරු අලුත් පිටුවකින් ආරම්භ කරන්න./
ஒவ்வொரு வினாவுக்குமான விடையைப் புதிய பக்கத்தில் ஆரம்பிக்க.

ප්‍රශ්න අංකය : / வினா எண் :

විභාග අංකය : / பரீட்சைச் சுட்டெண் :

පිටු අංකය : / பக்க எண் : 01

Q1

2016

i) for a second order system,

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

rise time, $t_r = \frac{1.8}{\omega_n}$

overshoot, $M_p = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}$

Setting time, (t_s)

Assuming 1% criteria,

$$t_s = \frac{4.6}{\xi\omega_n}$$

ii) P controller, $G_p(s) = K_p$

PI ", $G_{PI}(s) = \frac{K(s+Z_c)}{s}$

PD controller, $G_{PD}(s) = K(s+Z_c)$

PID ", $G_{PID}(s) = \frac{K(s+Z_{lag})(s+Z_{lead})}{s}$

iii) To eliminate the steady state error.

iv) The disturbance is something interfere to the process output.

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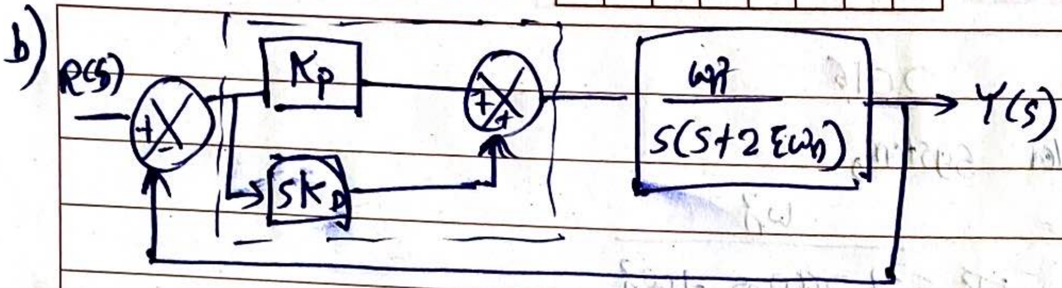
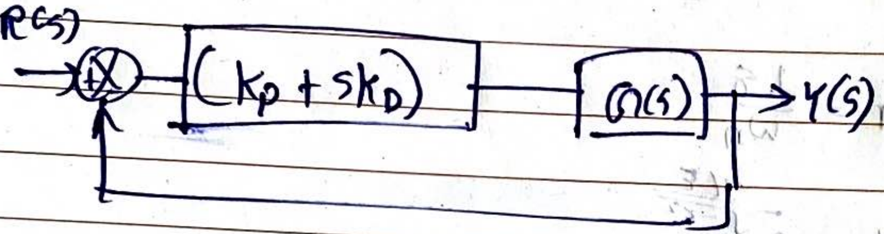
b)





$$\therefore G_{OL}(s) = \frac{(K_p + sK_D)\,\omega_n^2}{s(s+2\xi\omega_n)}$$

$$G_{CL}(s) = \frac{\omega_n^2(K_p + sK_D)}{s^2 + 2\xi\omega_n s + (K_p + sK_D)\omega_n^2}$$

$$= \frac{\omega_n^2 K_p + \omega_n^2 K_D s}{s^2 + (2\xi\omega_n + K_D\omega_n^2)s + K_p\omega_n^2}$$

$$= \frac{\omega_n^2 K_p}{s^2 + 2\omega_n\left(\frac{2\xi + K_D\omega_n}{2}\right)s + K_p\omega_n^2} + \frac{\omega_n^2 K_D s}{s^2 + 2\omega_n(2\xi + K_D\omega_n)s + K_p\omega_n^2}$$

$$\frac{2\xi + K_D\omega_n}{2} > \xi$$

$$\xi + K_D\omega_n > \xi$$

Overshoot,
$$M_p = e^{-\frac{\pi \xi}{\sqrt{1-\xi^2}}}$$

$$\ln M_p = \ln e^{-\frac{\pi \xi}{\sqrt{1-\xi^2}}}$$

$$\frac{-\pi \xi}{\sqrt{1-\xi^2}} = \ln(M_p)$$

$$\xi^2 = (\ln(M_p)/\pi)^2 (1-\xi^2)$$

$$\xi = \sqrt{\frac{(\ln(M_p)/\pi)^2}{1+(\ln(M_p)/\pi)^2}}$$

let $\xi = 0.5$

$$\therefore M_p = e^{-\frac{\pi \xi}{\sqrt{1-\xi^2}}}\Big|_{\xi=0.5}$$
$$= 0.163$$

when ~~improved~~ damping ratio is improved it's value increases,

$\therefore$ at $\xi = 0.8$,

$$M_p = 0.0152$$

$\therefore$ when ξ is improved from $\xi = 0.5$ to $\xi = 0.0152$

Overshoot <u>decreases</u>

Also, $t_s = \frac{4.6}{\xi \omega_n}$

$$\xi \propto \frac{1}{t_s}$$

$\therefore$ when $\xi \uparrow$ $t_s \downarrow$.

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(iii) $t_p = 0.6$

$M_p = \frac{5.815-5}{5} = 0.163 = 16.3\%$

$0.163 = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$

$\frac{-\pi\xi}{\sqrt{1-\xi^2}} = \ln(0.163)$

$\xi = 0.5$

$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$

$\omega_n = \frac{\pi}{0.6\sqrt{1-0.5^2}}$

$\omega_n = 6.046$

$G_{cL}(s) = \frac{\omega_n^2(K_p + sK_D)}{s^2 + (2\xi\omega_n + K_D\omega_n^2)s + K_p\omega_n^2}$

$= \frac{(K_p + sK_D)36.554}{s^2 + (6.046 + 36.554K_D)s + 36.554K_p}$

Routh's array

s^2	1	$36.554K_p$
s^1	$6.046 + 36.554K_D$	0
s^0	$36.554K_p$	

For system to be stable,

$6.046 + 36.554K_D > 0$ and $36.554K_p > 0$

$K_D > -0.1654$ $\therefore K_p > 0$

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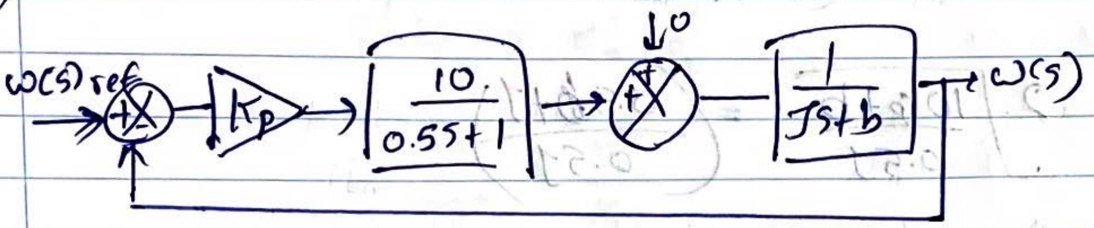
2017 - Control. $\boxed{Q1\ Q2}$

Q1) b)

i)



$$G_{OL}(s) = K_p G_1(s) G_2(s) \qquad \left(G_1(s) = \frac{10}{0.5s+1},\ G_2(s) = \frac{1}{Js+b} \right)$$

$$G_{CL}(s) = \frac{K_p G_1(s) G_2(s)}{1 + K_p G_1(s) G_2(s)}$$

$$= \frac{\frac{K_p(10)}{(0.5s+1)} \cdot \frac{1}{(Js+b)}}{1 + \frac{K_p(10)}{0.5s+1}\left(\frac{1}{Js+b}\right)}$$

$$G_{CL}(s) = \frac{10K_p}{(0.5s+1)(Js+b) + 10K_p}$$

$$= \frac{(10K_p+b)}{0.5Js^2 + (0.5b+J)s + (10K_p+b)} \left(\frac{10K_p}{10K_p+b}\right)$$

$$= \frac{(10K_p+b)/0.5J}{s^2 + \frac{(0.5b+J)}{0.5J}s + \left\{\frac{10K_p+b}{0.5J}\right\}} \left(\frac{10K_p}{10K_p+b}\right)$$

$$\omega_n^2 = \frac{10K_p+b}{0.5J} \Rightarrow \omega_n = \left(\frac{10K_p+b}{0.5J}\right)^{1/2} \quad -(1)$$

$$2\xi\omega_n = \left(\frac{0.5b+J}{0.5J}\right)$$

for critically damped, $\xi = 1$

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$$\therefore 2\omega_n = \left(\frac{0.5b+J}{0.5J}\right)$$

1) $$2\sqrt{\frac{10k_p+b}{0.5J}} = \left(\frac{0.5b+J}{0.5J}\right)$$

$$\frac{10k_p+b}{0.5J} = \frac{1}{4}\cdot\frac{(0.5b+J)^2}{(0.5J)^2}$$

$$K_p \; \cancel{10k_p+b} = \frac{(0.5b+J)^2}{10\times0.5J\times4} - \frac{b}{10}$$

$$K_p = \frac{(0.5b+J)^2}{20J} - \frac{b}{10}$$

ii) $b \to 0$

$$M_p = 85\% \to 35\%$$

85%,

$$0.85 = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$$

$$\ln(0.85) = \ln e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$$

$$\frac{-\pi\xi}{\sqrt{1-\xi^2}} = -0.1625$$

$$\xi_1 = 0.052$$

35%,

$$0.35 = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$$

$$\xi_2 = 0.317$$

b is negligible,

$$\therefore \omega_n = \left(\frac{10K_p}{0.5J}\right)^{1/2}$$

$$\omega_n = \sqrt{\frac{20K_p}{J}}$$

$$2\xi\omega_n = \left(\frac{0.5b+J}{0.5J}\right) = \frac{J}{0.5J} = 2 \qquad (b\to0)$$

$$\xi = \frac{1}{\omega_n} = \sqrt{\frac{J}{20K_p}}$$

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No (2)

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$$\xi \propto \frac{1}{\sqrt{K_p}}$$

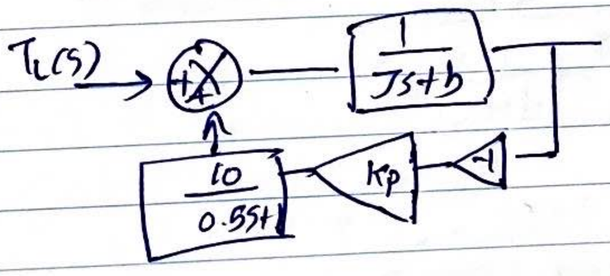
$$\xi_1^2 K_{p_1} = \xi_2^2 K_{p_2}$$

$$\frac{0.052^2}{0.317^2} = \frac{K_{p_2}}{K_{p_1}}$$

$$\frac{K_{p_2}}{K_{p_1}} = 0.027$$

$\therefore$ factor that should be multiplied is $\underline{0.027}$

c)
1) Assuming $\omega(s)_{ref} = 0$



$$G_{cL}(s) = \frac{\frac{1}{Js+b}}{1+\left(\frac{1}{Js+b}\right)\left(\frac{10K_p}{0.5s+1}\right)}$$

$$= \frac{0.5s+1}{(Js+b)(0.5s+1)+10K_p}$$

$$G_{cL}(s) = \frac{0.5s+1}{0.5Js^2+(0.5b+J)s+(10K_p+b)}$$

$R(s) = 2/s$ $\qquad E(s) = R(s) - R(s)G(s)$

$$e_{ss} = \lim_{s\to 0} sE(s)$$

$$= \lim_{s\to 0} s\left(\frac{2}{s}\right)\frac{0.5Js^2+(0.5b+J-0.5)s+(10K_p+b-1)}{0.5Js^2+(0.5b+J)s+(10K_p+b)}$$

$e_{ss} \le 0.01$

$$\frac{2(10K_p+b-1)}{10K_p+b} \le 0.01$$

$$\frac{2(10K_p+1-1)}{10K_p+1} \le 0.01$$

$20K_p \le 0.1K_p + 0.01$

$19.9K_p \le 0.01$

$$\underline{\underline{K_p \le 5.025\times10^{-4}}}$$

ii) $t_p = 100ms = 0.1s$

t_s 1% $\Rightarrow 0.5$

$e^{-\sigma t} = 0.01$

$-\sigma t = 4.605$

$\xi\omega_n t_s = 4.605$

$$t_s = \frac{4.605}{\sigma}$$

$$\therefore 0.5 = \frac{4.605}{\sigma}$$

$\sigma = 9.21 = \xi\omega_n$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

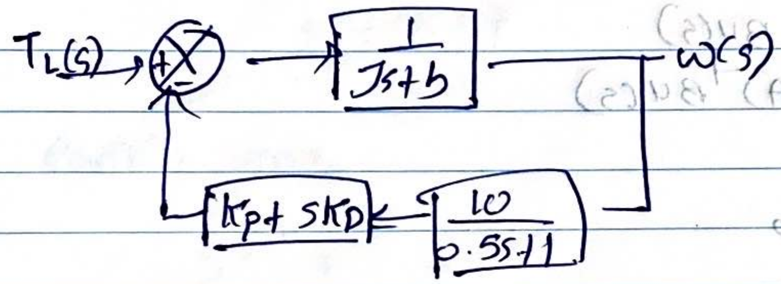
$$0.1 = \frac{\pi}{\sqrt{\omega_n^2 - \xi^2\omega_n^2}} = \frac{\pi}{\sqrt{\omega_n^2 - 9.21^2}}$$

$$\omega_n^2 - 9.21^2 = \left(\frac{\pi}{0.1}\right)^2$$

$$\omega_n = 32.738$$

$$\xi = \frac{9.21}{32.738}$$

$$\xi = 0.281$$



$$G_d(s) = \frac{\frac{1}{Js+b}}{1+\left(\frac{1}{Js+b}\right)\cdot(K_p+sK_D)\left(\frac{10}{0.5s+1}\right)}$$

$$= \frac{0.5s+1}{(0.5s+1)(Js+b)+10(K_p+sK_D)}$$

$$= \frac{0.5s+1}{0.5Js^2+(0.5b+J+10K_D)s+b+10K_p}$$

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(Q3a)

i) ✓

ii) $\dot{x} = Ax + Bu$ — (1)

$y = Cx + Du$ — (2)

Laplace T.F, of (1)

$$sX(s) = AX(s) + BU(s)$$
$$X(s)[s-A] = BU(s)$$
$$X(s) = (sI-A)^{-1}BU(s)$$

Laplace TF eq^n (2),

$$Y(s) = CX(s) + DU(s)$$
$$Y(s) = C(sI-A)^{-1}BU(s) + DU(s)$$

Transfer function,

$$G(s) = \frac{Y(s)}{U(s)} = C(sI-A)^{-1}B + D$$

iii) Controllability

If an i/p to the system can be found that forces every state variable from a desired ~~initial~~ final state, it's controllable.

observability

iv) for a closed loop sys,

$$G(s) = \frac{N(s)}{D(s)}$$

characteristic. eq^n = $D(s)$.

A system is stable if & only if, all the elements in the 1st column of the routh's array are positive.

b) Same as excercise.

c)

(i) ~~Bo~~ $G(s) = \dfrac{4}{(s+1)(s+4)}$

$$= \frac{4}{s^2+5s+4}$$

Routh's array,

s^2	1	4
s^1	5	0
s^0	4	

for stability, all the elements in the first column of the routh's array should be positive

In above,

$1>0$, $5>0$ and $4>0$

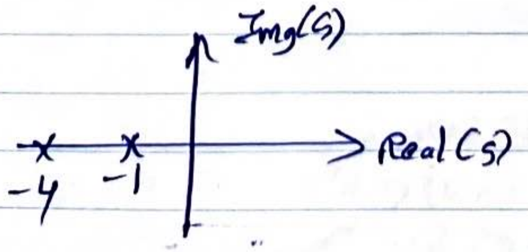
$\therefore$ all values/elements in 1^{st} column are positive.

$\therefore$ The system is <u>stable</u>.

<u>Verifying</u>.

here the poles of the system,

$P = -1, -4$



all poles of the closed loop system is on LHS of the s-plane. $\therefore$ The system is <u>stable</u>