UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 5 Examination in Engineering: January 2024

Module Number: IS5311 **Module Name: Discrete Mathematics**

[Three hours]

[Answer **all questions,** each question carries twelve marks]

Q1 a) i) Briefly explain what it means by 'a tautology' and 'a contradiction'

ii) Use truth table to determine whether $p \vee \neg(p \wedge q)$ and $\neg p \vee (p \wedge \neg q)$ are tautologies.

[3 Marks]

b) Suppose that during the most recent financial year, the annual revenue of Nitro Technologies was 145 billion dollars and its net profit was 15 billion dollars, the annual revenue of Zennon Computers was 234 billion dollars and its net profit was 14 billion dollars, the annual revenue of Hexa Designs was 211 billion dollars and its net profit was 19 billion dollars and the annual revenue of Renex Software was 186 billion dollars and its net profit was 8 billion dollars. Determine the truth value of each of the following propositions for the most recent financial year.

i) Nitro Technologies or Hexa Designs had the largest annual revenue.

ii) Zennon Computers had the lowest net profit and Renex Software had the largest annual revenue.

iii) Not both Nitro Technologies and Renex Software had the lowest net profit.

iv) If Hexa Designs had the lowest annual revenue or Zennon Computers had the largest net profit then Nitro Technologies had the second largest net profit.

v) Renex Software had the lowest net profit and Nitro Technologies had the largest annual revenue if and only if Hexa Designs had the largest net profit or Zennon Computers had the lowest net annual revenue.

[4 Marks]

c) i) For the given input p, q and r, build the logic circuit to obtain the output $s: (p \vee \neg r) \wedge (\neg q \wedge (\neg p \vee r))$

ii) Compute $(0101011 \vee 1101001) \oplus (1101101 \wedge 0110011)$

[3 Marks]

d) Let $p(x)$ be a propositional function defined on $\mathbb{N}$. then find the truth set of

i) $P(x): 0 < x - 2 < 4$

ii) $P(x): 0 < 3x + 8 < 10$

iii) Write down the negation of $p(x): \forall x,\ x \in A \rightarrow x \in B$.

[2 Marks]

Q2 a) Let A be a given set and R be a binary relation defined on the set A. Explain by giving an example for each case, when R is

i) Reflexive

ii) Symmetric

iii) Anti-symmetric

iv) Transitive

[4 Marks]

b) i) What is an equivalence relation?

ii) Consider the relation R on the set $A = \{a, b, c, d\}$

$$R = \{(a,a), (a,b), (a,d), (b,a), (b,b), (b,c), (c,b), (c,c), (d,a), (d,d)\}.$$

Determine whether R is reflexive, Symmetric, Anti-symmetric and transitive. Justify your answer.

iii) Is R an equivalence relation? Explain your answer.

iv) Construct the matrix for relation R.

[4 Marks]

c) State whether the following statements are True or False. Justify your answer.

i) $P = [\{1,3,5,7\}, \{2,6\}, \{4,5,8\}, [\{9\}]]$ is a partition of $S = \{1,2,3,4,5,6,7,8,9\}$.

ii) $(A \cap B)^c = A^c \cap B^c$.

iii) The relation $\subseteq$ of set inclusion is a partial ordering relation.

[4 Marks]

Q3 a) i) State and prove the division theorem of numbers.

ii) Show that, if $b|a$ and $c|b$ then $c|a$.

iii) Show that there are infinitely many prime numbers.

iv) Use Euclidian algorithm to show that 1960 and 1377 are relatively prime.

[6 Marks]

b) i) How many different 5 letter words can be made with the letters E, N, G, I, N, E, E, R, I, N, G.

ii) For positive integers n, r $(n \geq r)$, show that

$$^{n+1}C_r = {}^{n}C_{r-1} + {}^{n}C_r$$

iii) Write down the coefficients of x^6y^8 and x^7y^7 in the expansion of $(x+y)^{14}$.
Determine the coefficient of x^7 in the expansion of $(2-x)^{15}$, by using the relationship in ii) above

[6 Marks]

Q4 a) Briefly explain the following terms of a graph $G(V, E)$ by giving a non-trivial example for each.

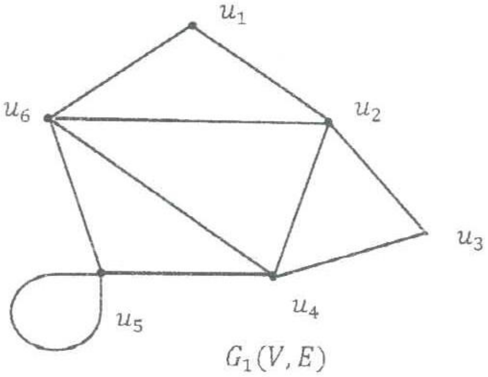
i) Order and Size.

ii) Degree sequence.

iii) Adjacency and incidence matrices.

[4 Marks]

b) Use the graph G_1 given below to answer i) – iv)



$G_1(V, E)$

i) Write down the degree sequence of G_1.

ii) Find a circuit of G_1, that is not a cycle.

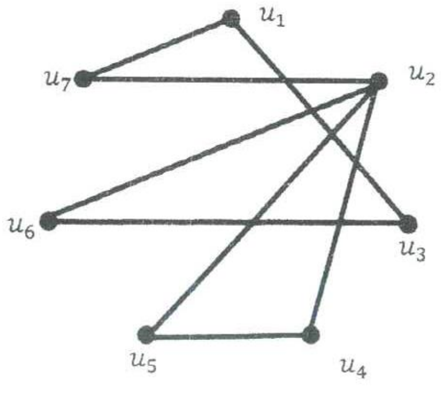
iii) Is G_1 an Eulerian? Explain your answer.

iv) Find a Hamiltonian cycle of length 6.

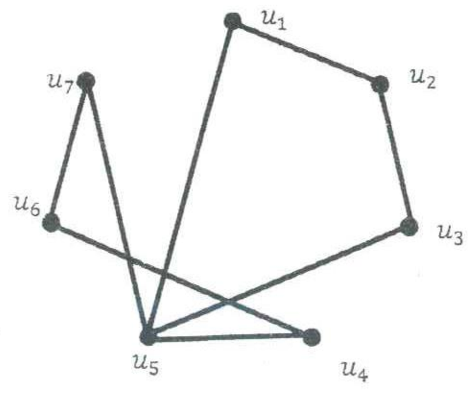
v) Give an example of a graph, which is both Hamiltonian and Eulerian.

[4 Marks]

c) i) Determine whether the following graphs G_2 and G_3 are bipartite. If bipartite, explicitly list the partition.

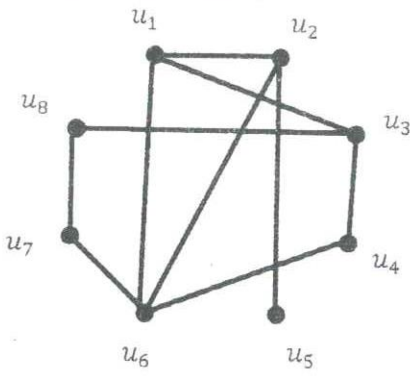


$G_2(V, E)$

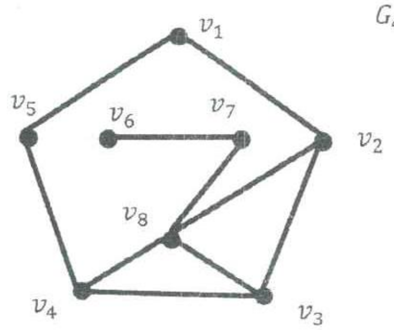


$G_3(V, E)$

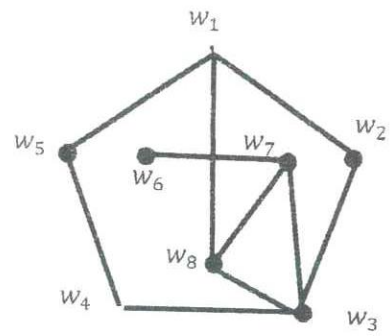
ii) Are the following graphs G_5 and G_6 isomorphic with the graph G_4. Produce the appropriate mapping of isomorphism.



$G_4(V,E)$



$G_5(V,E)$



$G_6(V,E)$

[4 Marks]

Q5 a) Briefly explain what are

i) Stochastic Processes, and

ii) Markov property.

[2 Marks]

b) Define the State space and the Parameter space of the stochastic processes given below.

i) $X(n)$ be the four types of transactions a person submits to an on-line data-base service, and time n corresponds to the number of transactions submitted.

ii) $Y(t)$ be the number of cars parked in a parking garage at a shopping mall, and time t corresponds to hours.

[2 Marks]

c) Consider a random walk on the finite states $\{-2,-1,0,1,2\}$. If the process is in state i; where $i=-1,0,1$ at time n, then it moves to either $i-1$ or $i+1$ at time $n+1$ with equal probability. If the process is in state -2 or 2 at time n, then it moves to state -1, 0 or 1 at time $n+1$ with equal probability.

i) Draw the state transition diagram for this random walk.

ii) Write the transition probability matrix P for this random walk.

iii) Calculate 2- step transition probability matrix.

[8 Marks]