UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 5 Examination in Engineering: October 2024

Module Number: EE5351 **Module Name: Control Systems Design**

[Three Hours]

[Answer **all questions**, each question carries 15.0 marks]

All the notations have their usual meanings unless it is explicitly specified.

Q1 a) i) If the damping ratio and undamped natural frequency of a system are denoted by ζ and ω_n, write the transfer function of general second order system.

ii) For the general second order system, write the corresponding range of ζ when the system response is underdamped, critically damped and overdamped, respectively.

iii) Obtain an expression for the system poles when the system response is underdamped.

iv) Show that the unit step response $[y(t)]$ of a general second order system is expressed as in Q1.1.

$$y(t) = u(t) - e^{-\zeta\omega_n t}\left(\cos\left(\omega_n\sqrt{1-\zeta^2}\right)t + \frac{\zeta\omega_n}{\sqrt{1-\zeta^2}}\sin\left(\omega_n\sqrt{1-\zeta^2}\right)t\right) \text{ - Q1.1}$$

[5.0 Marks]

b) The circuit diagram of a separately excited DC motor is shown in Figure Q1 (b). The electrical resistance (R_a), inductance (L_a), and the moment of inertia (J) of the DC motor are 3 Ω, 0.6 H and 0.8 kgm², respectively. The torque constant (k_m) and voltage constant (k_b) are 2.4 NmA⁻¹ and 0.8 V/rads⁻¹, respectively.

i) Consider the circuit shown in Figure Q1 (b). Find a suitable state vector to express the system in state-variable form.

ii) Obtain the state-space model of the circuit given in Figure Q1 (b).

iii) Hence, show that the transfer function of the DC motor $(G_M(s))$ is,

$$G_M(s) = \frac{\frac{k_m}{JL_a}}{s^2 + \frac{R_a}{L_a}s + \frac{k_m k_b}{JL_a}}$$

[4.0 Marks]

c) A closed loop speed control system for the DC motor considered in Part b) is depicted in Figure Q1 (c). Here, $k_p = 15$.

i) Obtain the overall transfer function of closed loop speed control system of the DC motor.

ii) If the speed of the motor must be kept at 1000 rpm, calculate the steady state value of the system response.

iii) Find the maximum speed of the DC motor considered in Part c) ii).

[6.0 Marks]

Q2 a) i) Write the transfer functions for controller type P, PI, PD and PID. Here, P, I and D denote proportional, integral, and derivative, respectively.

ii) What is the main objective of adding derivative control to a system?

iii) Clearly stating your assumptions, obtain an expression for the steady state error of negative feedback system with a unit step input. Assume that the open loop transfer function of the system is $G(s)$.

[4.5 Marks]

b) Show that the ***steady state error*** of the system shown in Figure Q2 (b) can be eliminated by adding a PI type controller.

[2.5 Marks]

c) You are required to design a simple speed control system for a DC motor as shown in Figure Q2 (c). The electrical resistance (R_a), inductance (L_a), and the moment of inertia of the DC motor are 1 Ω, 0.5 H and 0.2 kgm², respectively. The torque constant (k_m) and viscous friction constant (b) are 5 NmA⁻¹ and 1 Nms. The DC motor runs a load with 3 Nm load torque. Initially, a P controller is used as D(s).

i) Calculate the proportional constant (k_p) of D(s) when the motor speed due to only the load torque undergoes critically damped stage.

ii) Find the range of k_p to keep the steady state error due to the load torque less than 0.01 rads⁻¹.

iii) Find the k_p and k_D parameters of a PD controller used as D(s) which damp out the torque disturbance within 0.5 s in terms of 1% settling time. The peak overshoot of the motor speed due to the load torque is detected after 100 ms since the motor starts.

[8.0 Marks]

Q3 a) i) Define the root locus considering a negative feedback system.

ii) Explain the magnitude and the phase conditions to be satisfied at a point on the root locus.

iii) If s = -0.85 lies on the root locus of a system having $1 + kL(s)$, find the value of k using the magnitude condition. $L(s) = 1/[s(s+1)(s+3)]$.

[4.0 Marks]

b) Consider the system shown in Figure Q3 (b)-1. The root locus of this system is illustrated in Figure Q3 (b)-2.

i) Obtain the open loop transfer function G(s).

ii) Write the general characteristic equation of the closed loop plant.

iii) With the help of root locus given or the answer in Part i), determine,

I. Number of asymptotes.

II. Asymptote(s) angle(s) and intersection point(s).

III. Break-away and/or Break-in points.

IV. Corresponding departure or arrival angle at each pole and zero of the open-loop plant.

iv) With the help of root locus given in Figure Q3 (b)-2, find only one set of system poles for each of the following.

I. The system response is marginally stable.

II. The system response is critically damped.

III. The system response is underdamped.

[7.0 Marks]

c) Consider the root locus given in Figure Q3 (b)-2. It is desired to to use a compensator [i.e. $D(s) = k(s+z)/(s+p)$] to increase the undamped natural frequency (ω_n) to 5 rads^{-1} while maintaining the damping ratio (ζ) at 0.5. The noise suppression requirements require that the lead pole to be at -1. Determine the transfer function of the compensator.

[4.0 Marks]

Q4 a) i) What is the frequency response of a system?

ii) Define amplitude ratio (M) and phase (ϕ) related to the frequency response of a system whose transfer function is $G(s)$.

iii) Define the terms; phase margin and gain margin, associated with the Bode plots.

iv) How do we examine the system stability using the stability margins?

[6.0 Marks]

b) It is required to control the angular displacement of a plant as a unity feedback system. The transfer function of the plant is,

$$G(s) = \frac{\theta_L(s)}{V_T(s)} = \frac{10}{s(s+1)(s+10)(s^2+s+1)}$$

i) Obtain the steady-state output of the plant, when it is subjected to the input $V_T = 2\,sin(4t - 30^0)$.

ii) Draw the approximate Bode plots for the system.

[6.0 Marks]

c) The Bode plots for the open-loop plant of a water level control system of a tank is shown in Figure Q4 (c).

i) Obtain the phase margin and gain margin of the system from Bode plots.

ii) Hence, discuss the stability of the system.

[3:0 Marks]

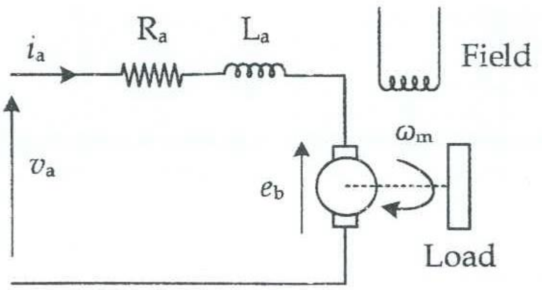

Figure Q1 (b)

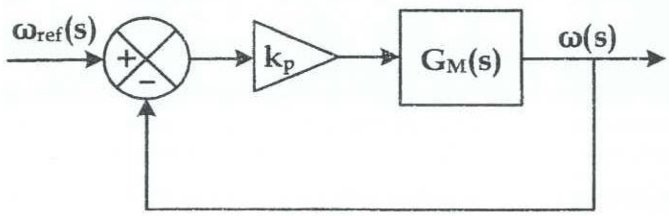

Figure Q1 (c)

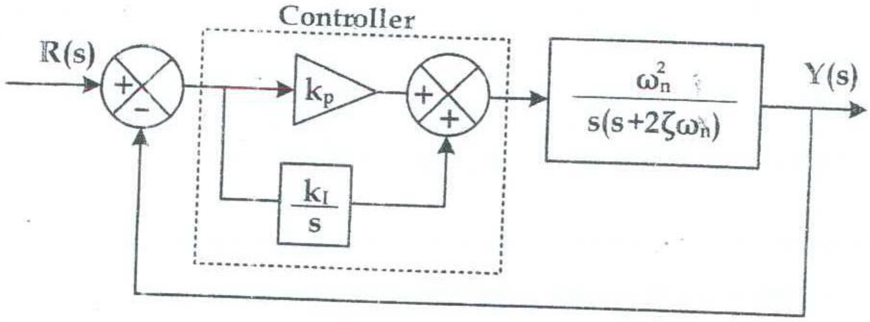

Figure Q2 (b)

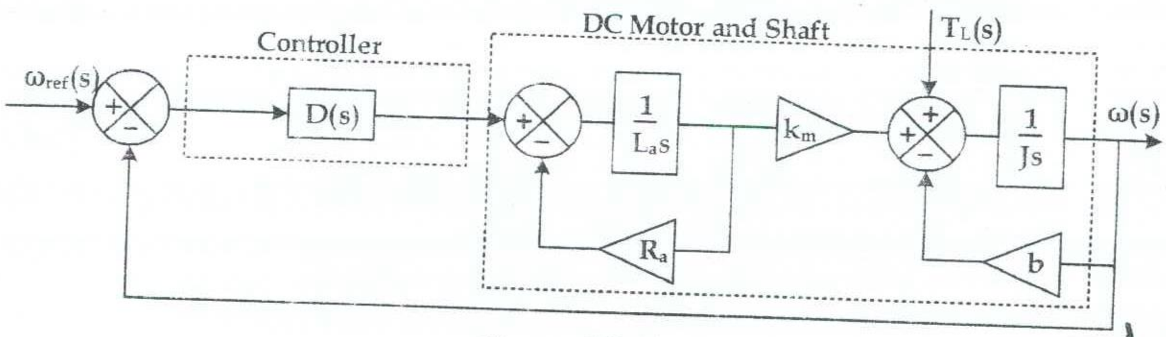

Figure Q2 (c)

$$\frac{\frac{1}{L_a s}}{1+\frac{R_a}{L_a s}}$$

$$\frac{\frac{1}{Js}}{1+\frac{b}{Js}}$$

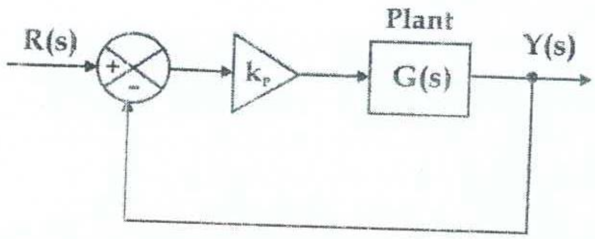

Figure Q3 (b)-1

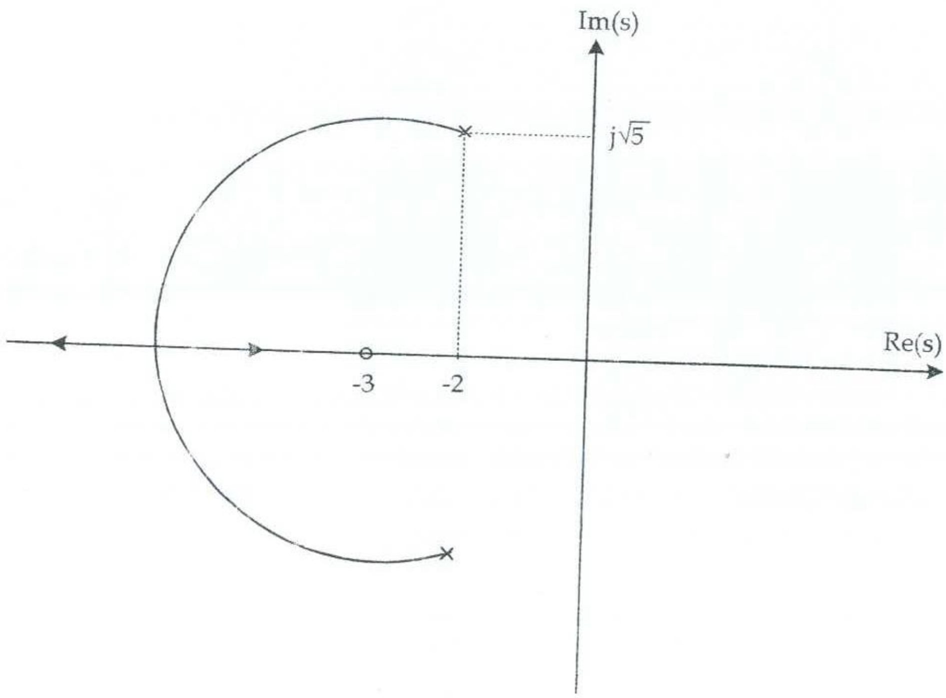

Figure Q3 (b)-2

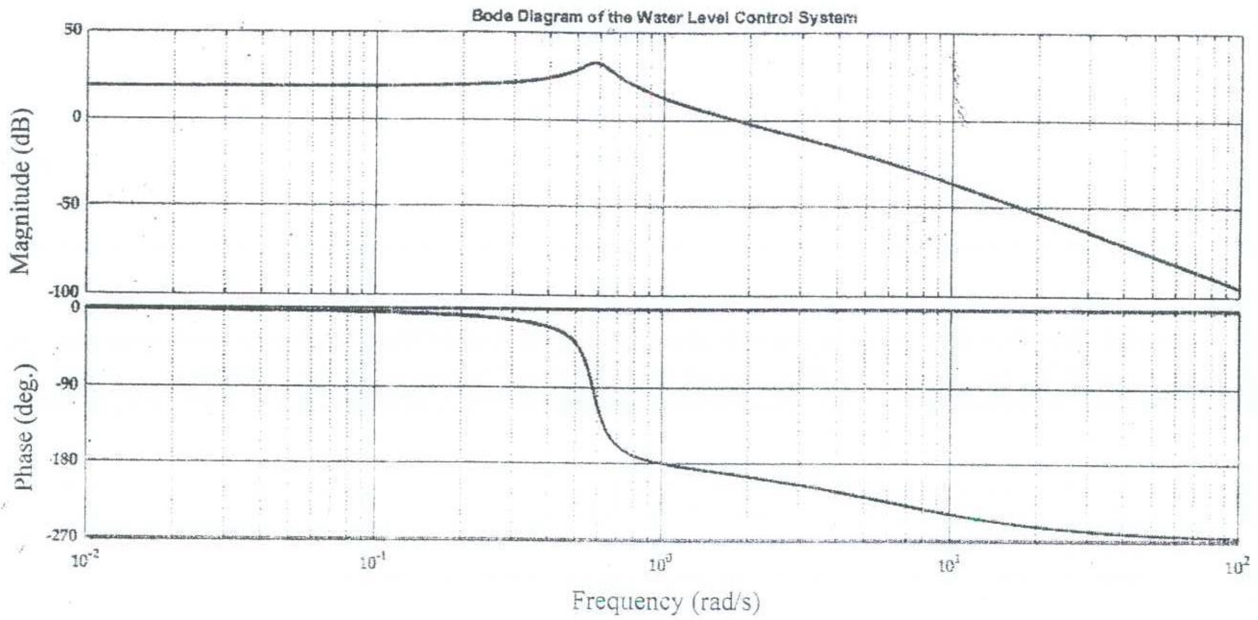

Figure Q4 (c)

Table 1: Laplace Transform Table

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$1/s$
e^{-at}	$\frac{1}{s+a}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\frac{df(t)}{dt}$	$sF(s)-f(0)$
$\int f(t)dt$	$\frac{F(s)}{s}+\frac{1}{s}\int(f(t)dt)\vert_{t=0}$