UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 5 Examination in Engineering: November 2024

Module Number: IS5311   Module Name: Discrete Mathematics

[Three hours]

[Answer **all questions,** each question carries twelve marks]

Q1 a) i) Define the terms 'Negation' and 'Conditional Statement' in propositional logic.

  ii) Use truth table to determine whether $(p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p)$ and $(p \vee q) \rightarrow (q \wedge p)$ are tautologies.

[3 Marks]

  b) In a recent comparison of three smartphones, the following specifications were noted: Smartphone X has 128 GB of internal storage, 6 GB of RAM, and a camera resolution of 12 MP; Smartphone Y has 256 GB of internal storage, 8 GB of RAM, and a camera resolution of 10 MP; and Smartphone Z has 64 GB of internal storage, 4 GB of RAM, and a camera resolution of 8 MP. Based on these specifications, construct truth tables to determine the truth value for each of the following propositions.

  i) Smartphone Y has the most RAM out of these three smartphones.

  ii) Smartphone Z has more internal storage or a higher camera resolution than Smartphone Y.

  iii) Smartphone X has more RAM, more internal storage, and a higher camera resolution than Smartphone Z.

  iv) If Smartphone Y has more RAM and more internal storage than Smartphone X, then it also has a higher camera resolution than X.

  v) Smartphone X has more internal storage than Smartphone Z if and only if Smartphone Z has more internal storage than Smartphone X.

[4 Marks]

  c) i) For the given input p, q and r, build the logic circuit to obtain the output $s: (p \vee \neg r) \wedge (\neg p \vee (q \vee \neg r))$

  ii) Compute $(101011 \oplus 110011) \wedge (011010 \vee 100110)$

[3 Marks]

  d) Let $P(x)$ and $Q(x)$ be propositional functions defined on the set of real numbers $\mathbb{R}$.

  i) Find the truth set of $P(x): -2 \leq x - 3 \leq 4$

  ii) Find the truth set of $Q(x): x - 1 > 2$

  iii) Write the negation of the statement:
  "Every student in your class has taken a course in calculus."

[2 Marks]

Q2 a) Define the following terms related to binary relations on a set, and provide an example for each.

    i)   Reflexive Relation

    ii)   Symmetric Relation

    iii)   Anti-symmetric Relation

    iv)   Transitive Relation

[4 Marks]

  b) i) Define a partial ordering relation.

    ii)   Consider the relation R on the set $A = \{2, 4, 6, 8\}$

$$R = \{(2,2), (2,4), (2,6), (4,4), (6,6), (4,6), (6,8), (8,8)\}.$$

Determine whether R is Reflexive, Symmetric, Anti-symmetric, and Transitive. Justify your answer.

    iii) Is R an equivalence relation? Explain your answer.

    iv) Construct the matrix for relation R.

[4 Marks]

  c) Let $A = \{1,2,3\}$ and $B = \{a, b\}$. Consider the relation $R \subseteq A \times B$ defined as

$$R = \{(1, a), (2, b), (3, a)\}$$

    i)   Find the cartesian product $A \times B$.

    ii)   Find the inverse relation R^{-1}.

[2 Marks]

  d) State whether the following statements are True or False. Justify your answer.

    i)   The relation of congruence is an equivalence relation.

    ii) $(A \cup B)^c = A^c \cup B^c$

[2 Marks]

Q3 a) i) State the division algorithm of numbers.

    ii)   Let a, b, c are integers, where $a \neq 0$. Show that, if $a|b$ and $a|c$ then $a|mb + nc$, whenever m and n are integers.

    iii) State the Fundamental Theorem of Arithmetic.

    iv) Use Euclidian algorithm to show that 4620 and 101 are relatively prime.

[5 Marks]

  b) i) Show that if $a \equiv b(mod\ m)$ and $c \equiv d(mod\ m)$, then $a + c \equiv b + d(mod\ m)$ and $ac \equiv bd(mod\ m)$.

    ii)   Find an inverse of 5 modulo 11.

    iii) Find the solutions of the linear congruence $5x \equiv 3(mod\ 11)$.

[4 Marks]

c) In a computer engineering course, students must complete a series of coding assignments every 5 days. The first assignment is due on the 2nd day of the month.

    i) Express the due dates for the assignments as a congruence relation.

    ii) If the month has 30 days, list all the days in the month when assignments are due.

[3 Marks]

Q4 a) Briefly explain the following of a graph $G(V, E)$.

    i) Multigraph

    ii) Loops

    iii) In-degree and the Out-degree

    iv) Degree Sequence

[2 Marks]

b) i) State the Handshaking Theorem.

    ii) A graph has 12 edges and each vertex has a degree of 3. How many vertices does this graph have?

[2 Marks]

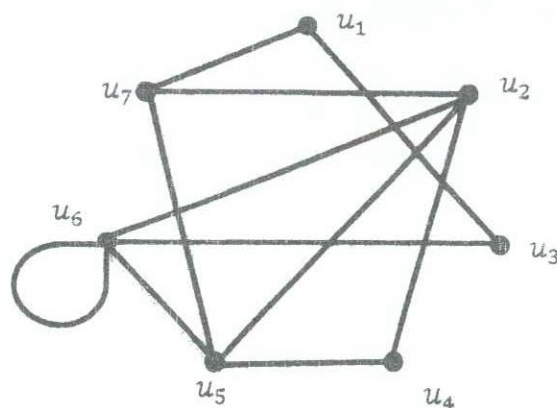c) Use the graph G_1 given in the Figure Q4(c) below to answer i) – iv)

Figure Q4(c): Graph $G_1(V, E)$

    i) Write down the degree sequence of G_1.

    ii) Find a circuit of G_1, that is not a cycle.

    iii) Is G_1 an Eulerian? Explain your answer.

[2 Marks]

d) i) Find the number of different 3-letter words that can be constructed by the letters D, I, S, C, R, E, T, E.

    ii) How many 7-digit numbers starting with numbers 450 are there, if the remaining digits consist of numbers greater than or equal to 2 and contain at least one digit that repeats itself?

    iii) Write down the coefficients of $x^4 y^5$ in the expansion of $(2x + 3y)^9$.

[6 Marks]

Q5 a) Briefly explain the following terms.

    i) Stochastic Process

    ii) Queuing Theory

[2 Marks]

b) Given a Transition Probability Matrix of a Markov Chain

$$P^1 = P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

with initial probability of $\pi(0) = p^{(0)} = (0.7 \quad 0.2 \quad 0.1)$, find the following probabilities.

    i) $P(X_2 = 3)$

    ii) $P(X_2 = 3|X_1 = 2)$

    iii) $P(X_2 = 3|X_0 = 1)$

[4 Marks]

c) i) Define the position of the random walk X_n at the nth step so that walks start from the origin.

    ii) Find the Mean and Variance of X_n.

    iii) Find the probability that the position of the walk at the 100th step with $p = 0.7$ lies on or between positions 35 and 45.

[6 Marks]