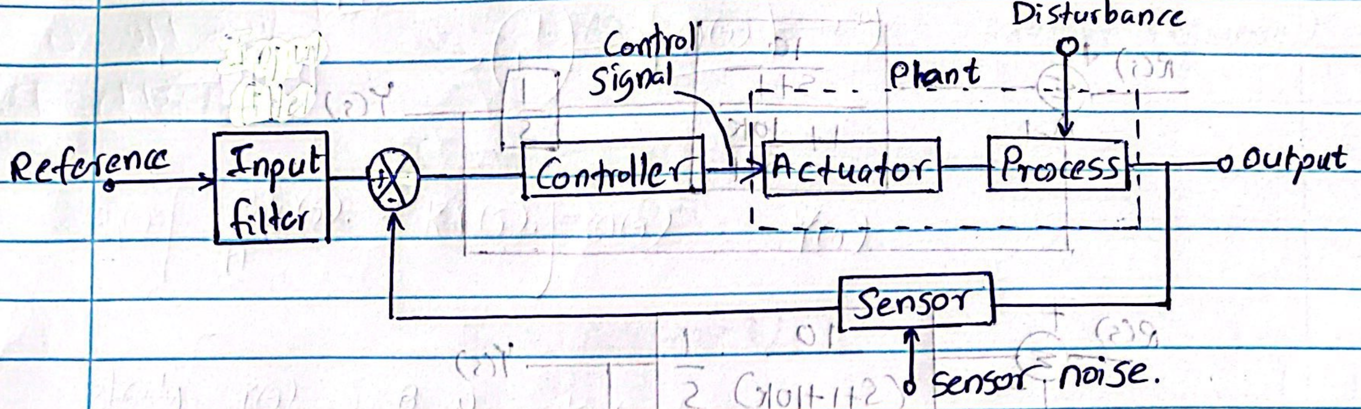
Q1, Q2

Date / / №. 1

Control Systems - 2020.

(Q1)

a) i) General block diagram for a closed-loop control system.



ii) Controller

The component that actually compute the desired control signal is the controller. It decides suitable control signal to the actuator.

Actuator

The device which gets the controlled signal from the control and converts into variable which influences the process output.

Sensor

The device which converts the physical variable into an electrical signal to be used in the controller.

iii) Disturbance is something interferes to the process output which means a signal that has some adverse effect on the value of output.

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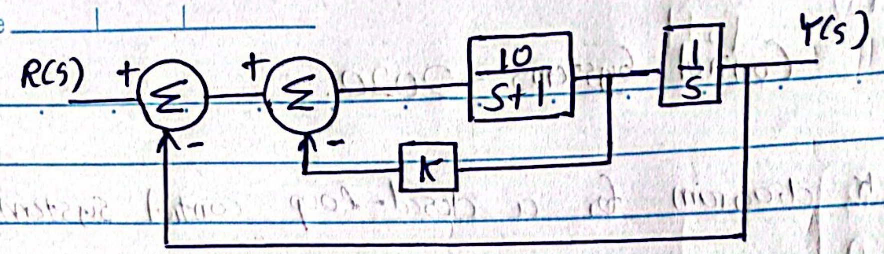
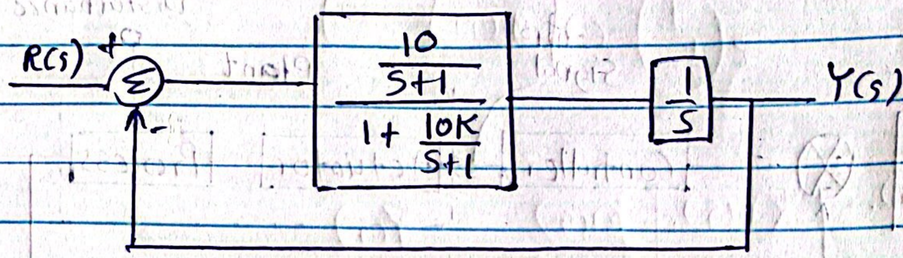
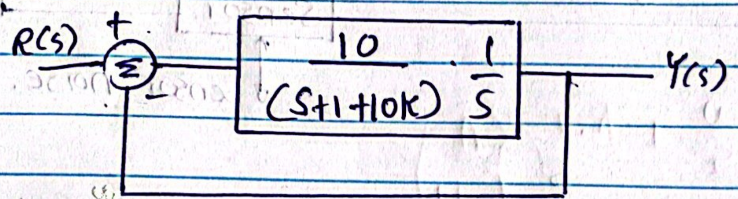
b)







$$G(s) = \frac{\frac{10}{s(s+1+10K)}}{1+\frac{10}{s(s+1+10K)}}$$

System transfer f^n,

$$G(s) = \frac{10}{s(s+1+10K)+10}$$

Charachteristic equation,

$$s(s+1+10K)+10=0$$

$$s^2+s(1+10K)+10=0 \quad \text{— (A)}$$

But standard from of second order system,

$$G(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$\therefore$ charachteristic eq^n,

$$s^2+2\xi\omega_n s+\omega_n^2 = 0 \quad \text{— (B)}$$

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Using/Comparing Ⓐ, Ⓑ eq^{n}s,

$$\omega_n^2 = 10$$
$$\omega_n = \sqrt{10}$$
$$\omega_n = 3.162 \quad \text{(undamped natural frequency)}$$

and,

$$2\xi\omega_n = 1+10K$$

for $\xi = 0.5$,

$$K = \frac{1}{10}(2\xi\omega_n - 1)$$
$$= \frac{1}{10}(2\times 0.5\times\sqrt{10} - 1)$$
$$\underline{\underline{K = 0.216}}$$

Method 2

Characteristic eq^n,

$$s^2 + (10K+1)s + 10 = 0$$

Poles of the system,

$$s = \frac{-(10K+1) \pm \sqrt{(10K+1)^2 - 4.1.10}}{2.1}$$
$$= \frac{-(10K+1)}{2} \pm \frac{\sqrt{(10K+1)^2 - 40}}{2}$$
$$= -\sigma \pm j\omega_d$$
$$= -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

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ii) Poles of the system,

$$s = -\sigma \pm j\omega_d$$

$$= -\xi\omega_n \pm j\,\omega_n\sqrt{1-\xi^2}$$

$\sigma = \xi\omega_n$
$= 0.5 \times \sqrt{10}$
$= 1.581$

$\omega_d = \omega_n\sqrt{1-\xi^2}$
$= \sqrt{10} \times \sqrt{1-0.5^2}$
$= 2.7386$

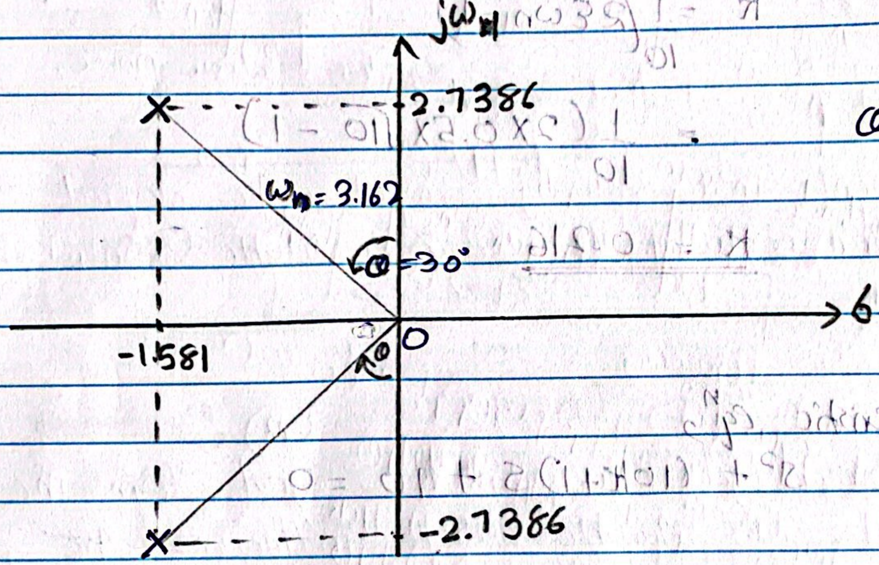
II Method

$s^2 + 2 \times 0.5 \times \sqrt{10}s + 10 = 0$

$s^2 + \sqrt{10}s + 10 = 0$

$s = -1.581 \pm j2.738$

$\therefore s = -1.581 \pm j2.7386$



$\theta = \sin^{-1}(\xi)$
$= \sin^{-1} 0.5$
$= 30°$

iii) for $K = 0.216$

$$G(s) = \frac{10}{s^2 + (1 + 10 \times 0.2162)s + 10}$$

$$G(s) = \frac{10}{s^2 + \sqrt{10}s + (\sqrt{10})^2}$$

Also $\frac{Y(s)}{R(s)} = G(s)$

But for a unit - step response,

$R(s) = 1/s$

$\therefore Y(s) = \frac{G(s)}{s}$

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$$\therefore Y(s) = \frac{10}{S(S^2+\sqrt{10}S+10)}$$

$$= \frac{10}{S(S+1.58-j2.738)(S+1.58+j2.738)}$$

$$= \frac{A}{S} + \frac{B}{S+1.58-j2.738} + \frac{C}{S+1.58+j2.738}$$

$$A = 1 \;;\; B = \frac{10}{(-1.58+j2.738)(j5.476)} = -0.5+j0.2887$$

$$C = \frac{10}{(-1.58-j2.738)(-j5.476)} = -0.5-j0.2887$$

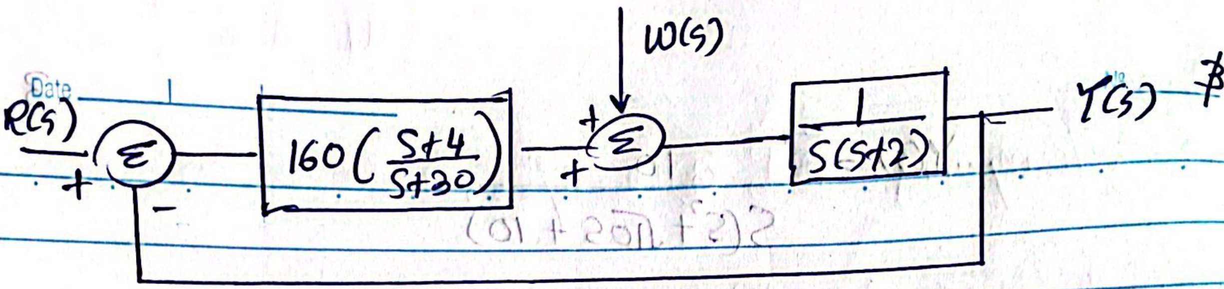
$$\therefore Y(s) = \frac{1}{S} + \frac{-0.5+j0.2887}{(S+1.58-j2.738)} - \frac{0.5+j0.2887}{(S+1.58+j2.738)}$$

Applying Inverse laplace transform,

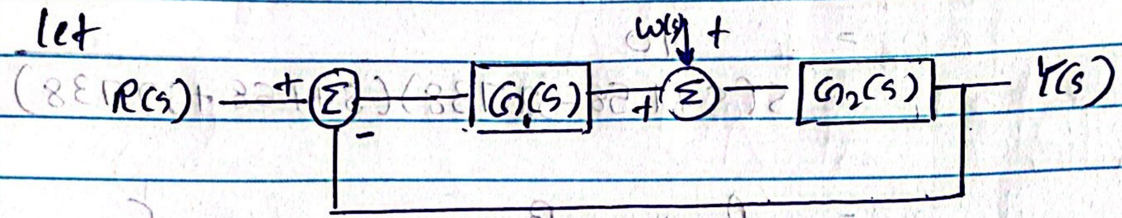
$$y(t) = \mathcal{L}^{-1}[Y(s)]$$

time function

$$y(t) = u(t) - (-0.5+j0.2887)e^{-(1.58-j2.736)t} - (0.5+j0.2887)e^{-(1.58+j2.738)t}$$

c) 

let

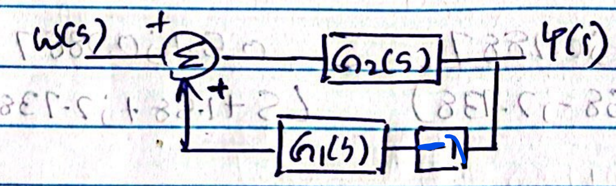


In the absense of disturbance,

$$Y_1(s) = R(s)G(s)$$

$$Y_1 = \frac{G_1G_2}{1+G_1G_2}R$$

In the absense of response,



Now, $Y_2(s) = G_2 W(s)G(s)$

$$Y_2 = \frac{G_2}{1+G_1G_2}W$$

∴ Total output,

$$Y(s) = Y_1(s) + Y_2(s)$$

$$= \frac{G_1(s)G_2(s)}{1+G_1(s)G_2(s)} \cdot R(s) + \frac{G_2(s)}{1+G_1(s)G_2(s)} W(s) \quad \text{— (A)}$$

i) In the absense of disturbance,

$$W(s) = 0$$

$$\therefore Y(s) = \frac{G_1G_2}{1+G_1G_2}R$$

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$$Y(s) = \frac{160\left(\frac{s+4}{s+30}\right)\frac{1}{s(s+2)}}{1+\frac{160(s+4)}{s(s+30)(s+2)}} \times R(s)$$

$$= \frac{160(s+4)}{s(s+30)(s+2)+160(s+4)} R(s)$$

for unit step response/reference, $R(s) = \frac{1}{s}$

$$Y(s) = \frac{160(s+4)}{s[s(s+30)(s+2)+160\,s(s+4)]}$$

But error, $E(s) = R(s) - Y(s)$

$$= \frac{1}{s} - \frac{160(s+4)}{s[s(s+30)(s+2)+160(s+4)]}$$

$$= \left(\frac{1}{s}\right)\frac{[s(s^2+32s+60)+160(s+4)-160(s+4)]}{s(s+30)(s+2)+160(s+4)}$$

$$= \frac{s(s^2+32s+60)}{s[s(s+30)(s+2)+160(s+4)]}$$

$$E(s) = \frac{s^2+32s+60}{s(s+30)(s+2)+160(s+4)} \quad \text{—(1)}$$

Steady state error,

$$e_{ss} = \lim_{t\to\infty} y(t)$$

By using final value theorem,

$$e_{ss} = \lim_{t\to\infty} y(t) = \lim_{s\to 0} sE(s)$$

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$$\therefore e_{ss} = \lim_{s\to 0} \frac{s(s^2+32s+60)}{s(s+30)(s+2)+160(s+4)}$$

$$e_{ss} = 0$$

therefore the system can track a unit step reference

ii) for unit step disturbance,

$W = \frac{1}{s}$ and $R(s) = 0$

(A) $\Rightarrow$ $$Y(s) = \frac{\frac{1}{s(s+2)}}{1+\frac{160(s+4)}{s(s+30)(s+2)}} \times \frac{1}{s}$$

$$Y(s) = \frac{1}{s[s(s+30)(s+2)+160(s+4)]}$$

error, $E(s) = R(s) - Y(s)$

$$= 0 - \frac{1}{s[s(s+30)(s+2)+160(s+4)]}$$

Steady state error,

$$y_{ss} = \lim_{s\to 0} sE(s)$$

$$= \lim_{s\to 0} \frac{-s}{s[s(s+30)(s+2)+160(s+4)]}$$

$$= \lim_{s\to 0} \frac{-1}{s(s+30)(s+2)+160(s+4)}$$

$$= -\frac{1}{640}$$

$$= -0.0015625$$

RATHNA

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Q2

a) i) All the ~~roots~~ coefficients of the charachteristic polynomial should be positive and cannot be zero or negative.

that is for $s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_n = 0$

$$a_1, a_2, a_3, \dots, a_n > 0$$

ii) A system is stable if and only if all the elements of the Routh array are positive.

b) Charachteristic eq^n,

$$F(s) = s^4 + 2s^3 + (4+k)s^2 + 9s + 25 = 0$$

Routh's array,

s^4	1	$4+k$	25
s^3	2	9	0
s^2	$\frac{2(4+k)-9}{2}$	25	0
s^1	$\frac{9(4+k-\frac{9}{2})-50}{(4+k-\frac{9}{2})}$	0	
s^0	25		

for the system to be stable all the elements of 1st column should have same sign (here +)

$\therefore \frac{2(4+k)-9}{2} > 0$ and $\frac{9(4+k-\frac{9}{2})-50}{4+k-\frac{9}{2}} > 0$

$k > \frac{9}{2} - 4$

$k > 0.5$

$9 - \frac{50}{4+k-\frac{9}{2}} > 0$

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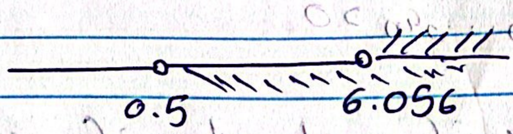
$$9 > \frac{50}{4+K-\frac{9}{2}}$$

$$9 < \frac{4+K-\frac{9}{2}}{50}$$

$$9\left(4+K-\frac{9}{2}\right) - 50 > 0$$

$$4+K-\frac{9}{2} > \frac{50}{9}$$

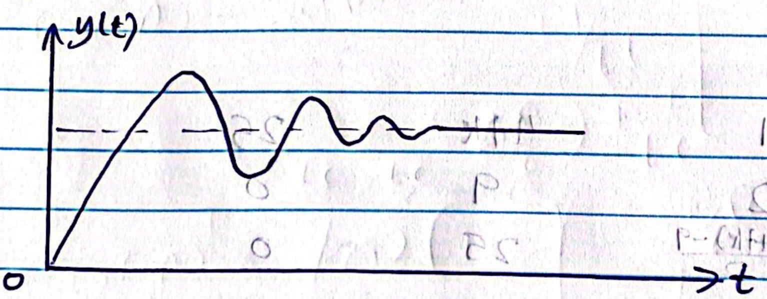
$$K > 6.056$$



$\therefore K > 6.056$

$\therefore$ for the system to be stable $\underline{\underline{K > 6.056}}$

c) By using time domain response.



Observe system's time-domain response to diff^n input signals. If the o/p remains bounded for different inputs it indicates stability. Key features to examine are overshoot, rise time (t_r), settling time (t_s) and peak time (t_p).

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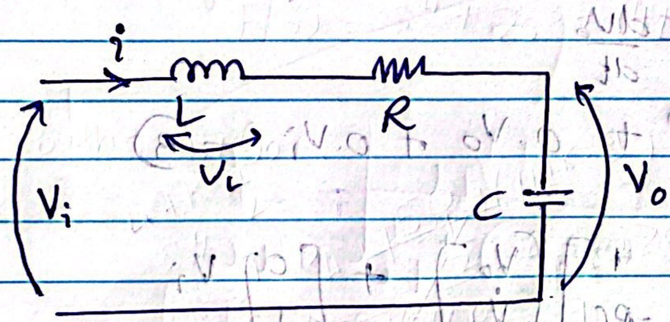
d)

i)

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

x - Column vector n (state vector)
$\dot{x}$ - Derivatives of the states of the system
y - output of the system.
u - input to the system
A - System matrix
B - input matrix
C - output matrix
D - Direct transmission matrix.

ii)



~~Converting~~ Applying KVL,

$$V_i = L\frac{di}{dt} + iR + V_o \quad \text{—} ①$$

~~Considering~~ capacitor,

$$i_c = C\frac{dV_o}{dt} = C\dot{V} \quad \text{—} ②$$

output $y = V_o$ — ③

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} V_o \\ \dot{V}_o \end{bmatrix} \qquad \therefore \dot{x} = \begin{bmatrix} \dot{V}_o \\ \ddot{V}_o \end{bmatrix}$$

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D_1 (2) ⇒ $$V_i = L\frac{d}{dt}\left(C\frac{dV_o}{dt}\right) + iR + V_o$$

$$V_i = LC\frac{d^2V_o}{dt^2} + iR + V_o$$

$$LC\frac{d^2V_o}{dt^2} = -V_o - iR + V_i$$

$$= -V_o - \left(C\frac{dV_o}{dt}\right)R + V_i$$

$$\ddot{V}_o = \frac{1}{LC}\left(-V_o - RC\dot{V}_o + V_i\right) \quad \text{— (A)}$$

(2) ⇒ $$\dot{V} = \frac{i_c}{C}$$

$$= \frac{1}{C}\cdot C\frac{dV_o}{dt}$$

$$\dot{V}_o = \dot{V}_o + 0.V_o + 0.V_i \quad \text{— (B)}$$

(A)(B) ⇒ $$\begin{bmatrix}\dot{V}_o \\ \ddot{V}_o\end{bmatrix} = \begin{bmatrix}0 & 1 \\ \frac{-1}{LC} & \frac{-R}{L}\end{bmatrix}\begin{bmatrix}V_o \\ \dot{V}_o\end{bmatrix} + \begin{bmatrix}0 \\ \frac{1}{LC}\end{bmatrix}V_i$$

(3) ⇒ $$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}V_o \\ \dot{V}_o\end{bmatrix} + \begin{bmatrix}0\end{bmatrix}V_i$$

Now $A = \begin{bmatrix}0 & 1 \\ \frac{-1}{LC} & \frac{-R}{L}\end{bmatrix}$ $B = \begin{bmatrix}0 \\ \frac{1}{LC}\end{bmatrix}$

$C = \begin{bmatrix}1 & 0\end{bmatrix}$ $D = \begin{bmatrix}0\end{bmatrix}$

Transfer f^n of the sys is given by,

$$G(s) = C(SI-A)^{-1}B + D$$

Date

Q1

$$SI-A = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & \frac{R}{L} \end{bmatrix}$$

$$SI-A = \begin{bmatrix} S & -1 \\ \frac{1}{LC} & S-\frac{R}{L} \end{bmatrix}$$

$$C(SI-A) = [1\ 0]\begin{bmatrix} S & -1 \\ \frac{1}{LC} & S-\frac{R}{L} \end{bmatrix}$$

$$C(SI-A)B = \begin{bmatrix} S & -1 \end{bmatrix}\begin{bmatrix} 0 \\ \frac{1}{LC} \end{bmatrix}$$

$$= -\frac{1}{LC}$$

$$(SI-A)^{-1} = \frac{1}{S(S-\frac{R}{L})+\frac{1}{LC}}\begin{bmatrix} S-\frac{R}{L} & 1 \\ -\frac{1}{LC} & S \end{bmatrix}$$

$$C(SI-A)^{-1} = \phi(s)\,[1\ 0]\begin{bmatrix} S-\frac{R}{L} & 1 \\ -\frac{1}{LC} & S \end{bmatrix}$$

$$C(SI-A)^{-1}B = \frac{LC}{(S(LCS-RC)+1)}\begin{bmatrix} S-\frac{R}{L} & 1 \end{bmatrix}\begin{bmatrix} 0 \\ \frac{1}{LC} \end{bmatrix}$$

$$C(SI-A)^{-1}B+D = \frac{LC}{S(LCS-RC)+1}\cdot\frac{1}{LC}$$

Transfer fn

$$G(s) = \frac{1}{LCS^2-RCS+1}$$

Root Locus - 2020

Q3

a) i) *Greater accuracy than open-loop systems.

*Less sensitive to noise, disturbances and changes in the environment

*Transient response and steady state error can be controlled more conveniently with greater flexibility.

ii)
$$G(s) = \frac{1}{(s+z_1)(s+z_2)}$$

In a proportional controller o/p is directly proportional to error signal,

$$\therefore y_c(t) = K_p e(t)$$

TF,

$$G_c(s) = K_p$$

For closed loop system

Closed loop gain,

$$G_{cl}(s) = \frac{KG(s)}{1+KG(s)}$$

Error,

$$E(s) = R(s) - Y(s)$$
$$= R(s) - G_{cl}(s)R(s)$$
$$= \left[1 - \frac{KG(s)}{1+KG(s)}\right]R(s)$$

$$E(s) = \frac{1}{1+KG(s)} R(s)$$

මෙම කඩදාසි විභාග ශාලාවෙන් පිටතට ගෙනයාම හෝ අවසර නොමැතිව පෞද්ගලික ස්ථානයක තබා ගැනීම හෝ වෙනයම් කටයුත්තක් සඳහා යොදා ගැනීම හෝ දඬුවම් ලැබිය හැකි වරදකි. විභාග කොමසාරිස් ජනරාල්, ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව.

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ஒவ்வொரு வினாவுக்குமான விடையைப் புதிய பக்கத்தில் ஆரம்பிக்க.

ප්‍රශ්න අංකය : / வினா எண் :
විභාග අංකය : / பரீட்சைச் சுட்டெண் :
පිටු අංකය : / பக்க எண் :

Assuming a unit step input,

Steady state error,

$$e_{ss} = \lim_{s \to 0} sE(s)$$

$$= \lim_{s \to 0} s \cdot \frac{1}{1+KG(s)} \cdot \frac{1}{s}$$

$$= \frac{1}{1+KG(0)} \qquad G(s) = \frac{1}{(s+z_1)(s+z_2)}$$

$$e_{ss,CL} = \frac{1}{1+\frac{K}{z_1 z_2}}$$

$$\boxed{t_s = \frac{4}{\sigma} \quad t_p = \frac{\pi}{\omega}}$$

For open loop system

$$R(s) \to \boxed{K} \to \boxed{G(s)} \to Y(s)$$

$$E(s) = R(s) - Y(s)$$

$$= [1 - KG(s)]R(s)$$

Steady state error,

$$\lim_{s \to 0} sE(s) = \lim_{s \to 0} s[1 - KG(s)] \cdot \frac{1}{s}$$

$$= \cancel{\lim} \; 1 - KG(0)$$

$$= 1 - \frac{K}{z_1 z_2}$$

පරීක්ෂකවරයාගේ ප්‍රයෝජනය සඳහා පමණි.
பரீட்சகரின் உபயோகத்திற்கு மாத்திரம்

ஒவ்வொரு வினாவுக்குமான விடையைப் புதிய பக்கத்தில் ஆரம்பிக்க.

ප්‍රශ්න අංකය : வினா எண் : | විභාග අංකය : பரீட்சைச் சுட்டெண் : | පිටු අංකය : பக்க எண் : 01

Root Locus - 2020

b)

i)

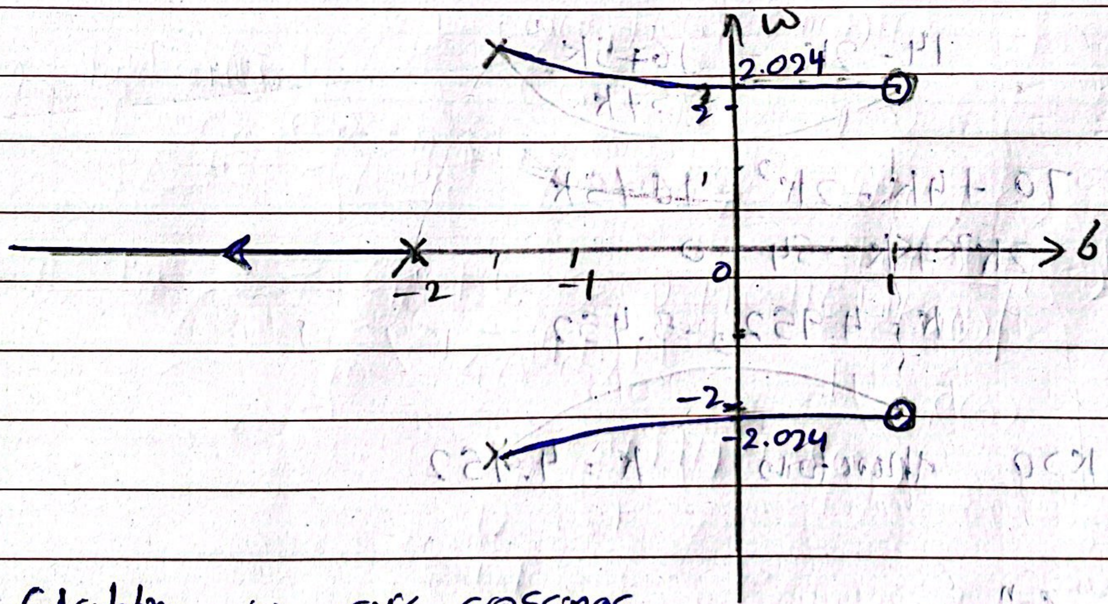
$$G(s) = \frac{(s^2-2s+5)}{(s+2)(s^2+3s+8)}$$

$$G(s) = \frac{(s-1-2j)(s-1+2j)}{(s+2)(s+1.5+j2.398)(s+1.5-j2.398)}$$

Poles $\Rightarrow$ -2, $-1.5+j2.398$, $-1.5-j2.398$

zeros $\Rightarrow$ $1+2j$, $1-2j$

\# Asymptotes = 3 - 2 = 1



ii) Calculating $j\omega$ axis crossings.

Eq^n of the charachteristic eq^n,

$$1 + KG(s) = 0$$

$$1 + \frac{K(s^2-2s+5)}{(s+2)(s^2+3s+8)} = 0$$

පරීක්ෂකවරයාගේ ප්‍රයෝජනය සඳහා පමණි.

பரீட்சகரின் உபயோகத்திற்கு மாத்திரம்

මෙම කඩදාසි විභාග ශාලාවෙන් පිටතට ගෙනයාම හෝ අවසර නොමැතිව පෞද්ගලික ස්ථානයක තබා ගැනීම හෝ වෙනයම් කටයුත්තක් සඳහා යොදා ගැනීම හෝ දඬුවම් ලැබිය හැකි වරදකි. විභාග කොමසාරිස් ජනරාල්, ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව.

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සෑම ප්‍රශ්නයක් සඳහා ම පිළිතුරු අලුත් පිටුවකින් ආරම්භ කරන්න.
ஒவ்வொரு வினாவுக்குமான விடையைப் புதிய பக்கத்தில் ஆரம்பிக்க.

ප්‍රශ්න අංකය : / வினா எண் : 1 | විභාග අංකය : / பரீட்சைச் சுட்டெண் : | පිටු අංකය : / பக்க எண் :

$$s^3+5s^2+14s+16+K(s^2-2s+5)=0$$
$$s^3+(5+K)s^2+(14-2K)s+(16+5K)=0$$

Routh's array,

$$\begin{array}{lll} s^3 & 1 & 14-2K \\ s^2 & 5+K & 16+5K \\ s^1 & \frac{(5+K)(14-2K)-(16+5K)}{5+K} & \\ s^0 & 16+5K & \end{array}$$

to obtain auxiliary eq^n and to calculate K value,

$$\frac{(14-2K)(5+K)-(16+5K)}{5+K}=0$$

$$14-2K=\frac{16+5K}{5+K}$$

$$70+4K-2K^2=16+5K$$
$$2K^2+K-54=0$$
$$K=4.952,\ -5.452$$

$K>0$ therefore $K=4.952$

$\therefore$ Auxiliary eq^n,

$$(5+K)s^2+(16+5K)=0$$
$$(5+4.952)s^2=-(16+5\times4.952)$$
$$s=\pm j\,2.024$$

$\therefore j\omega$ axis crossings $=\pm j\,2.024$

iii) For stability

$5+K>0 \Rightarrow K>-5$

$(14-2K)-\dfrac{(16+5K)}{5+K}>0$ and

$16+5K>0 \Rightarrow K>-3.2$

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$(K-4.952)(K+5.452) > 0$

$\therefore$ $K-4.952 > 0$ and $K+5.452 > 0$

or

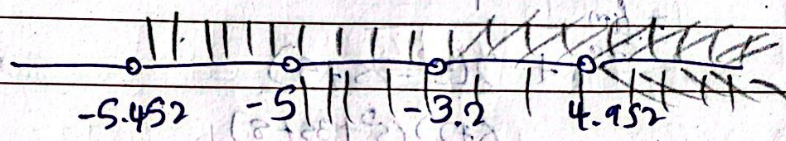
$K-4.952 < 0$ and $K+5.452 < 0$

$\therefore$

$K > 4.952$ and $K > -5.452$

or

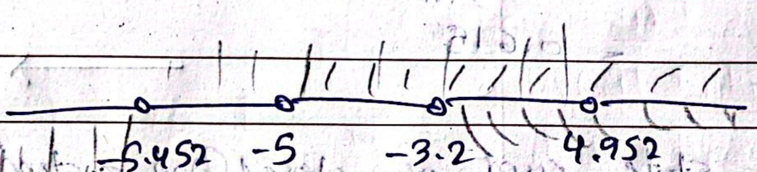
$K \cancel{>} 4.952$ and $K < -5.452$



$\therefore$ for stability

$K > 4.952$

or



this is not valid.

$\therefore$ only possible range of gain K,

$$\underline{\underline{K > 4.952}}$$

iv) $G_{OL}(s) = KG(s)$

$$G_{CL}(s) = \frac{KG(s)}{1+KG(s)}$$

$E(s) = R(s) - Y(s)$

$= \left[1 - G_{CL}(s)\right]R(s)$

$$= \frac{1}{1+KG(s)} R(s)$$

ඔබ ලියන්නේ කුමන ප්‍රශ්නයද? පිළිතුරු ආරම්භ කරන්න
ஒவ்வொரு வினாவுக்குமான விடையைப் புதிய பக்கத்தில் ஆரம்பிக்கவும்

ප්‍රශ්න අංකය : வினா எண் :
විභාග අංකය : பரீட்சைச் சுட்டெண் :
පිටු අංකය : பக்க எண் :

Steady state error,

$$e_{ss} = \lim_{s \to 0} sE(s)$$

$$= \lim_{s \to 0} \frac{sR(s)}{1+KG(s)}$$

for unit step inputs

$$R(s) = 1/s$$

$$\therefore e_{ss} = \lim_{s \to 0} \frac{s \cdot 1/s}{1+KG(s)}$$

$$= \lim_{s \to 0} \frac{1}{1 + \frac{2(s^2-2s+5)}{(s+2)(s^2+3s+8)}}$$

$$= \frac{1}{1+\frac{2.5}{2.8}}$$

$$= 0.615$$

$\therefore$ steady state error of closed loop system's o/p y is 0.615

v) for compensated system,

$$e_{ss,comp} = \frac{e_{ss,uncomp}}{10}$$

$$= \frac{0.615}{10}$$

$$= 0.0615$$

Static error constant of uncompensated system,

K=2,

$$K_{vo} = \lim_{s \to 0} sG(s) = \frac{2(0-0+5)}{(0+2)(0+8)}$$

$$K_{vo} = 0.625$$

for compensated system,

$$e_{ss} = 0.0615 = \frac{1}{1+K_{vn}}$$

$$K_{vn} = 15.26$$

But $K_{vn} = K_{vo} \cdot \frac{Z_c}{P_c}$

$$\therefore 15.26 = 0.625 \frac{Z_c}{P_c}$$

$$\frac{Z_c}{P_c} = 24.416$$

Taking

$$P_c = 0.01$$

$$Z_c = 24.416 \times 0.01$$

$$= 0.2442$$

(c) suitable lag compensator to the system,

$$G_{lag}(s) = \frac{2(s+0.2442)}{(s+0.01)}$$

පරීක්ෂකවරයාගේ ප්‍රයෝජනය සඳහා පමණි.
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සෑම ප්‍රශ්නයක් සඳහා ම පිළිතුරු අලුත් පිටුවකින් ආරම්භ කරන්න.
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ප්‍රශ්න අංකය : வினா எண் : | විභාග අංකය : பரீட்சைச் சுட்டெண் : | පිටු අංකය : பக்க எண் :

(04) a)

i) I - Lag compensator.

II - Lead "

III - Lead "

IV - ~~PID controller~~ Lag-Lead compensator.

ii) G(s) can be assumed as,

$$G(s) = \frac{(s+z_1)}{(s+p_1)(s+p_2)}$$

~~Both~~ PI compensator,

$$G_{PI}(s) = \frac{K(s+z_c)}{s}$$

∴ open loop transfer function,

$$G_{OL}(s) = K\frac{(s+z_1)}{(s+p_1)(s+p_2)} \cdot \frac{(s+z_c)}{s}$$

$$E(s) = \frac{1}{1+KG(s)} R(s)$$

$$e_{ss} = \lim_{s\to 0} sE(s)$$

$$= \lim_{s\to 0} \frac{s \cdot 1/s}{1 + \frac{K(s+z_1)(s+z_c)}{s(s+p_1)(s+p_2)}} \quad (R(s) = 1/s \text{ unit step})$$

$$= \lim_{s\to 0} \frac{s(s+p_1)(s+p_2)}{s(s+p_1)(s+p_2) + K(s+z_1)(s+z_c)}$$

$$e_{ss} = 0$$

මෙම කඩදාසි විභාග ශාලාවෙන් පිටතට ගෙනයාම හෝ අවසර නොමැතිව පෞද්ගලික ස්ථානයක තබා ගැනීම හෝ වෙනයම් කටයුත්තක් සඳහා යොදා ගැනීම හෝ දඬුවම් ලැබිය හැකි වරදකි. විභාග කොමසාරිස් ජනරාල්, ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව.

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ප්‍රශ්න අංකය : / வினா எண் :
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පිටු අංකය : / பக்க எண் :

National Evaluation and Testing Service NETS

For the uncompensated system.

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{1/s}{1+\frac{K(s+z_1)}{(s+P_1)(s+P_2)}}$$

$$= \frac{1}{1+\frac{Kz_1}{P_1P_2}} \neq 0$$

Therefore, steady state error of the system can be driven to zero when PI compensator is added.

b) $$G(s) = \frac{K}{(s+10)(s^2+4s+2)}$$

Assuming 2nd order approximation,

i) Overshoot, $20\% = 0.2 = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$

$$\ln(0.2) = \ln e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$$

$$\frac{-\pi\xi}{\sqrt{1-\xi^2}} = -1.609$$

$$\xi^2 = 0.262(1-\xi^2)$$

$$\xi = 0.456$$

$$\sin\theta = 0.456 = \xi$$

$$\theta = 27.126^\circ$$

$$G(s) = \frac{K}{(s+10)(s+3.414)(s+0.586)}$$

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සෑම ප්‍රශ්නයක් සඳහා ම පිළිතුරු අලුත් පිටුවකින් ආරම්භ කරන්න.
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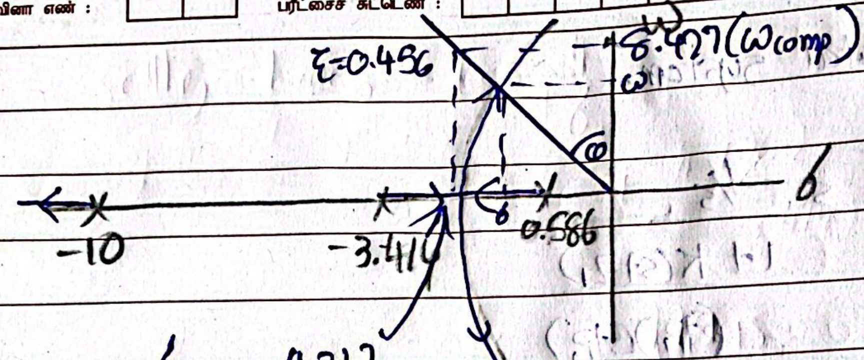
ප්‍රශ්න අංකය : வினா எண் :   විභාග අංකය : பரீட்சைச் சுட்டெண் :   පිටු අංකය : பக்க எண் :



$\sigma_{comp} = 4.317$

$$\tan\theta = \sigma/\omega$$

$$\tan(27.126^\circ) = \sigma/\omega$$

$$\sigma = 0.512\omega$$

Crossing point,

$$s = -\sigma + j\omega$$
$$= -0.512\omega + j\omega$$
$$= \omega(-0.512+j) \quad \text{— (1)}$$

This point is a root of the root locus (eqⁿ),

$$1+KG(s)=0$$

$$1 + \frac{K}{(s+10)(s^2+4s+2)} = 0$$

$$s^3 + 14s^2 + 42s + 20 + K = 0$$

$$(1) \Rightarrow \omega^3(-0.512+j)^3 + 14\omega^2(-0.512+j)^2 + 42\omega(-0.512+j) + 20 + K = 0$$

$$\omega^3(1.402 - j0.214) + \omega^2(-10.329 - j14.336) + \omega(-21.504 + j42) + 20 + K = 0$$

$$1.402\omega^3 - 10.329\omega^2 - 21.504\omega + 20 + K = 0 \quad \text{— (2)}$$

$$-0.214\omega^3 - 14.336\omega^2 + 42\omega = 0 \quad \text{— (3)}$$

මෙම කඩදාසි විභාග ශාලාවෙන් පිටතට ගෙනයාම හෝ අවසර නොමැතිව පෞද්ගලික ස්ථානයක තබා ගැනීම හෝ ...

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$\omega = 2.812\,,\ -69.802\,,\ 0$

$k = 90.970\,,\ 525.627\times10^3\,,\ -20$

$\sigma = 1.439\,,\ -35.739\,,\ 0$

Finding the third closed loop poles,

$$s^3 + 14s^2 + 47s + (20+K) = 0$$

$K = 90.970$,

$s = -11.121\,,\ -1.439 \pm j2.812$

$K = 525.627\times10^3$,

$s = -85.467\,,\ 35.733 \pm j69.809$

Closed loop dominent poles	Third pole	Gain
~~$-2.812 \pm j1.439$~~		
① $-1.439 \pm j2.812$	-11.121	90.970
② $-35.739 \pm j69.802$	-85.467	525×10^3

In ①, Third closed loop pole is 5 times more farther than dominent closed loop poles. Therefore, it can be neglected. $(P_c = 11.121)$

∴ System can be approximated to a 2^{nd} order sys.

∴ Gain K required, $\underline{\underline{K = 90.97}}$

ii) Gain K was calculated assuming this is a second order system. And above I have shown that the approximation is valid.

පරීක්ෂකවරයාගේ ප්‍රයෝජනය සඳහා පමණි.
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settling time, $t_s = \frac{4}{\sigma}$

$= \frac{4}{1.439}$

$\underline{t_s = 2.779\ s}$

Peak time, $t_p = \frac{\pi}{\omega}$

$= \frac{\pi}{2.812}$

$\underline{t_p = 1.117\ s}$

iv) Compensated system,

$ts_{comp} = \frac{ts_{uncomp}}{3}$

$= \frac{2.779}{3}$

$= 0.927$

$0.927 = \frac{4}{\sigma_{comp}}$

$\sigma_{comp} = 4.317$

$\omega_{comp} = \frac{4.317}{\tan(27.176)}$

$= 8.427$

∴ Dominent second order poles of the compensated sys.

$S = -4.317 \pm j8.427$

පරීක්ෂකවරයාගේ ප්‍රයෝජනය සඳහා පමණි.

பரீட்சகரின் உபயோகத்திற்கு மாத்திரம்

මෙම කඩදාසි විභාග ශාලාවෙන් පිටතට ගෙනයාම හෝ අවසර නොමැතිව පෞද්ගලික ස්ථානයක තබා ගැනීම හෝ වෙනයම් කටයුත්තක් සඳහා සොදා ගැනීම හෝ දඬුවම් ලැබිය හැකි වරදකි. විභාග කොමසාරිස් ජනරාල්, ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව.

I Method

Designing a PD controller,

$$G_{PD}(s) = K(s+Z_c)$$

∴ Root locus of the compensated sys,

$$1 + KG(s) = 0$$

$$1 + \frac{K(s+Z_c)}{(s+10)(s^2+4s+2)} = 0$$

$$\sum \theta_z - \sum \theta_p = 180(2n+1) \quad n = 0,1,2,\ldots$$

$$\angle s+Z_c - [\angle s+10 + \angle s+3.414 + \angle s+0.586] = 180°$$

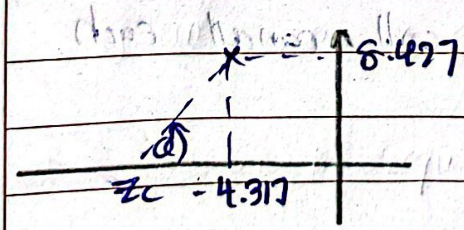
Now $s = -4.317 + j8.427$ is a point of this,

$$\angle -4.317+j8.427+Z_c - [\angle -4.317+j8.427+10 + \angle -4.317+j\ldots] = 180$$

$$\theta_{Z_c} - [56.005 + 96.116 + 113.881] = 180°$$

$$\theta_{Z_c} = 446.002$$

$$= 86.002$$



$$\tan\theta = \omega/\sigma$$

$$\tan 86.002 = \frac{8.427}{Z_c - 4.317}$$

$$Z_c = 4.906$$

∴ Suitable PD compensator,

$$G_{PD}(s) = K(s+4.906)$$

පරීක්ෂකවරුන්ගේ ප්‍රයෝජනය සඳහා පමණි.
பரீட்சகரின் உபயோகத்திற்கு மாத்திரம்
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සෑම ප්‍රශ්නයක් සඳහා ම පිළිතුරු අලුත් පිටුවකින් ආරම්භ කරන්න./
ஒவ்வொரு வினாவுக்குமான விடையைப் புதிய பக்கத்தில் ஆரம்பிக்க.

ප්‍රශ්න අංකය : / வினா எண் :

විභාග අංකය : / பரீட்சைச் சுட்டெண் :

පිටු අංකය : / பக்க எண் :

Compensated System, Root locus

$$\frac{K(s+4.906)}{(s+10)(s+3.414)(s+0.586)} + 1 = 0$$

$s = -4.317 + j8.427$ is a root of this,

$$K = \frac{-(s+10)(s+3.414)(s+0.586)}{s+4.906}$$

$$= 93.981 + j4.505\times10^{-4}$$

$\therefore K = 93.981$

$\therefore$ compensated system,

$$1 + \frac{93.981(s+4.906)}{(s+10)(s+3.414)(s+0.586)} = 0$$

$$s^3 + 14s^2 + 42s + 20 + 93.981s + 461.071 = 0$$

$$s = -5.366,\ -4.317 \pm j8.427$$

Third closed loop pole is closed to the closed loop zero of the sys. $\therefore$ They will cancell each other

පරීක්ෂකවරයාගේ ප්‍රයෝජනය සඳහා පමණි.

பரீட்சகரின் உபயோகத்திற்கு மாத்திரம்

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இத்தாளைப் பரீட்சை மண்டபத்திலிருந்து வெளியே எடுத்துச் செல்லல், அல்லது அனுமதியின்றித் தனிப்பட்டோர் வைத்திருத்தல், அல்லது
தேவைகளுக்குப் பயன்படுத்தல் என்பன தண்டனைக்குரிய

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සෑම ප්‍රශ්නයක් සඳහා ම පිළිතුරු අලුත් පිටුවකින් ආරම්භ කරන්න./
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ප්‍රශ්න අංකය : / வினா எண் : | විභාග අංකය : / பரீட்சைச் சுட்டெண் : | පිටු අංකය : / பக்க எண் :

II Method

Designing a lead compensator,

$$G(s) = \frac{K(S+Z_c)}{(S+P_c)}$$

Assuming $Z_c = 3$.

Compensated system,

$$1 + KG(s) = 0$$

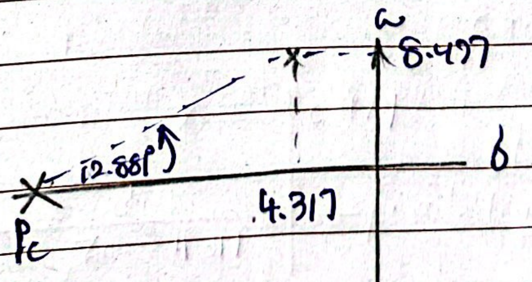
$$1 + \frac{K(S+3)}{(S+P_c)} \cdot \frac{1}{(S+10)(S^2+4S+2)} = 0$$

$$\Sigma\theta_z - \Sigma\theta_p = 180(2h+1); \quad h = 0,1,2,\ldots$$

$$(\angle S+3) - [\angle S+P_c + \angle S+10 + \angle S+3.414 + \angle S+0.586] = 180$$

$$98.883 - (\theta_{P_c} + 56.005 + 96.116 + 113.881) = 180$$

$$\theta_{P_c} = -347.119^\circ$$
$$= 360 - 347.119^\circ$$
$$= 12.881$$



$$\tan 12.881^\circ = \frac{8.427}{P_c - 4.317}$$

$$P_c = 41.167$$

$\therefore$ lead comp,

$$G_{Lead}(s) = \frac{K(S+41.167)}{(S+3)}$$

පරීක්ෂකවරයාගේ ප්‍රයෝජනය සඳහා පමණි.
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Q5

a) i)

Frequency Response - 2020

$$G(s) = \frac{12(s+10)}{(s+2)(s+3)}$$

$$= \frac{12s+120}{s^2+5s+6}$$

$$G(j\omega) = \frac{j12\omega+120}{-\omega^2+j5\omega+6} = \frac{12(10+j\omega)}{2}$$

$$= \frac{120(1+j\omega/10)}{6(1+j\omega/2)(1+j\omega/3)}$$

$$= \frac{20(1+j\omega/10)}{(1+j\omega/2)(1+j\omega/3)}$$

Magnitude plot,

$$|G(j\omega)|_{dB} = 20\log(G(j\omega))$$

$$= 20\log 20 + 20\log(1+j\omega/10) + 20\log\left(\frac{1}{1+j\omega/2}\right) + 20\log\left(\frac{1}{1+j\omega/3}\right)$$

$$= 26.021 + 20\log(1+j\omega/10) + 20\log\left(\frac{1}{1+j\omega/2}\right) + 20\log\left(\frac{1}{1+j\omega/3}\right)$$

Phase plot/response,

$$\angle G(j\omega) = \angle(j\omega/10+1) + \angle\frac{1}{1+j\omega/2} + \angle\frac{1}{1+j\omega/3}$$

Breaking frequencies ⇒ 10, 2, 3

පරීක්ෂකවරයාගේ ප්‍රයෝජනය සඳහා පමණි.
பரீட்சகரின் உபயோகத்திற்கு மாத்திரம்

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இத்தாளைப் பரீட்சை மண்டபத்திலிருந்து வெளியே எடுத்துச் செல்லல், அல்லது அனுமதியின்றித் தனிப்பட்டோர் வைத்திருத்தல், அல்லது வேறு தேவைகளுக்குப் பயன்படுத்தல் என்பன தண்டனைக்குரிய குற்றமாகும். பரீட்சை ஆணையாளர் நாயகம், இலங்கைப் பரீட்சைத் திணைக்களம்.
SPC 2018

සෑම ප්‍රශ්නයක් සඳහා ම පිළිතුරු අලුත් පිටුවකින් ආරම්භ කරන්න./
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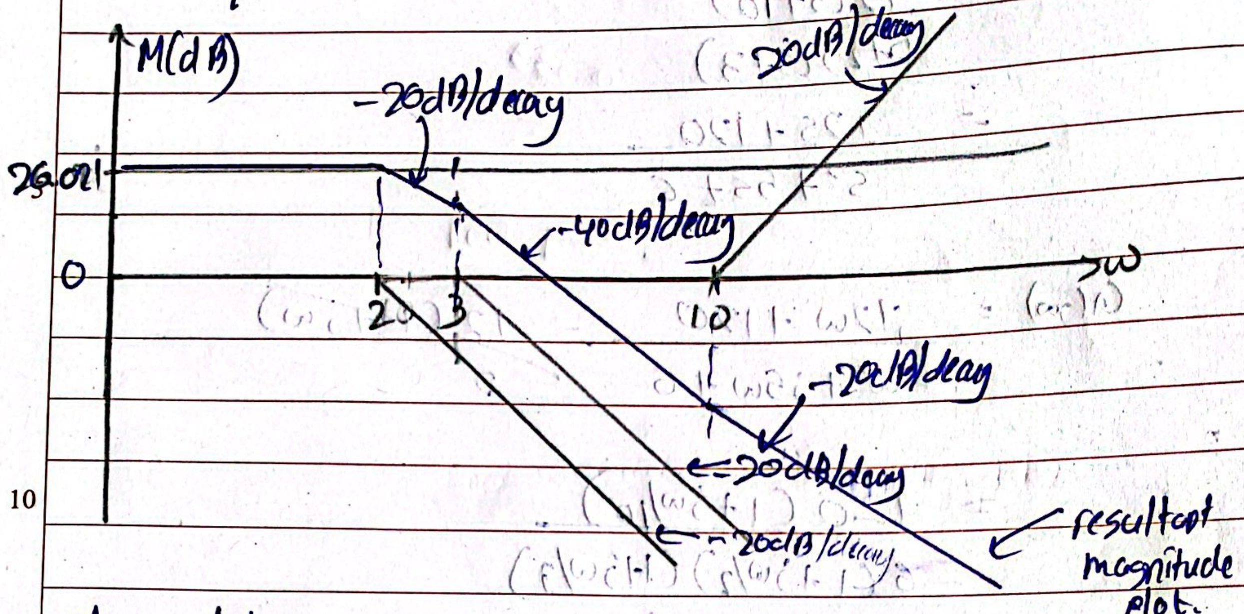
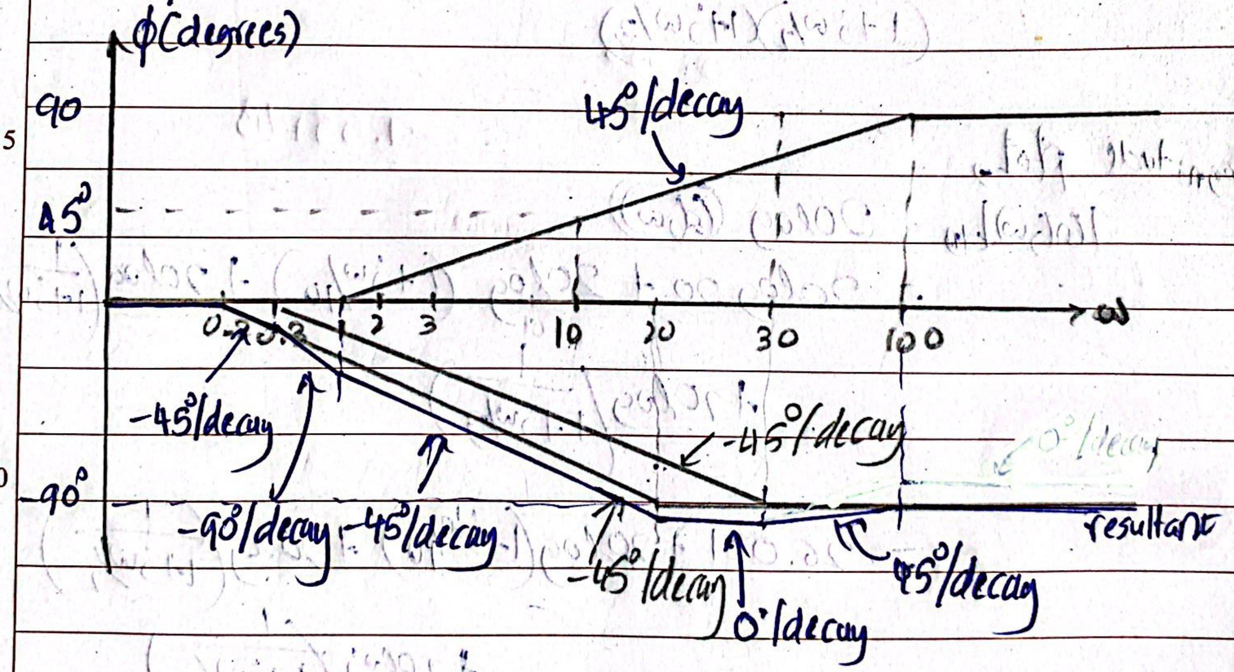
ප්‍රශ්න අංකය : / வினா எண் : 1 විභාග අංකය : / பரீட்சைச் சுட்டெண் : පිටු අංකය : / பக்க எண் :

Magnitude plot.



Phase plot.



ii) An open loop stable system is stable in closed loop, if the open loop magnitude response has gain less than 0dB at the frequency where the phase frequency response is -180° (degrees).

පරීක්ෂකවරුන්ගේ ප්‍රයෝජනය සඳහා පමණි.
பரீட்சகரின் உபயோகத்திற்கு மாத்திரம்

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i) Calculating the steady state error of the uncompensated system, $e_{ss,uncomp}$.

$$e_{ss,uncomp} = \lim_{s \to 0} sE(s)$$

$$= \lim_{s \to 0} s[1 - G_{cL}(s)]R(s)$$

$$= \lim_{s \to 0} \frac{sR(s)}{1 + KG_{OL}(s)}$$

for unit step input,

$$R(s) = 1/s$$

$$e_{ss,uncomp} = \lim_{s \to 0} \frac{s \cdot 1/s}{1 + \frac{0.986(36.7)}{(3)(5)}}$$

$$= \cancel{0.23}\ 0.293$$

$\therefore$ Steady state error for output y without compensator $G_c(s)$ is

$$\underline{e_{ss,uncomp} = 0.293}$$

ii) Steady state error of the compensated system,

$$e_{ss,comp} = \frac{e_{ss,uncomp}}{5}$$

$$= \frac{0.293}{5}$$

$$= 0.0586$$

පරීක්ෂකවරයාගේ ප්‍රයෝජනය සඳහා පමණි.
பரீட்சகரின் உபயோகத்திற்கு மாத்திரம்

මෙම කඩදාසි විභාග ශාලාවෙන් පිටතට ගෙනයාම හෝ අවසර නොමැතිව පෞද්ගලික ස්ථානයක තබා ගැනීම හෝ වෙනයම් කටයුත්තක් සඳහා යොදා ගැනීම හෝ දඬුවම් ලැබිය හැකි වරදකි. විභාග කොමසාරිස් ජනරාල්, ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව.
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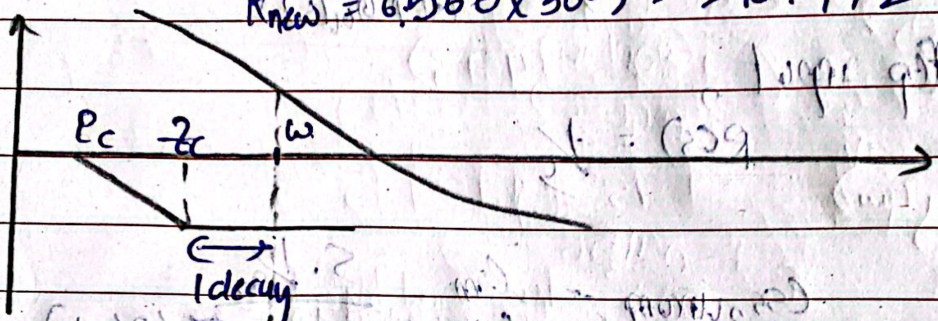
Calculating the new gain,

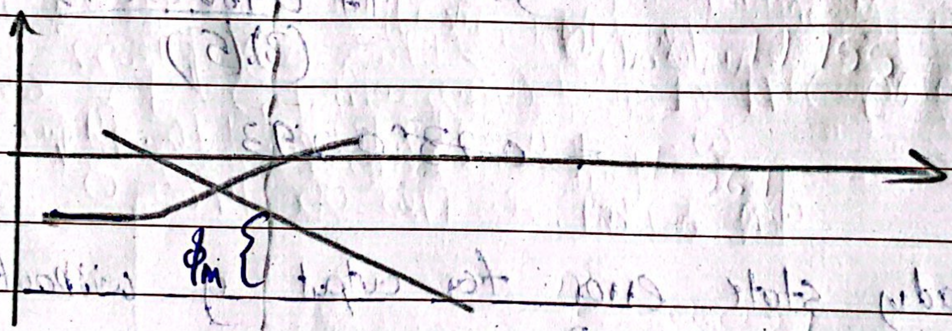
$$e_{ss,comp} = \frac{1}{1+KG_{OL}(0)}$$

$$0.0586 = \frac{1}{1+K\frac{(36.7)}{(3)(5)}}$$

$$K = 6.566$$

$$K_{new} = 6.566 \times 36.7 = 240.972$$





Required phase margin,

$$\phi_m = 48.148^\circ$$

$$\phi_M = \cancel{\phi_q}\ \phi_{m,req} + 10^\circ$$

$$= 48.148^\circ + 10^\circ$$

$$= 58.148^\circ$$

பரீட்சகரின் உபயோகத்திற்கு மாத்திரம்

සෑම ප්‍රශ්නයක් සඳහා ම පිළිතුරු අලුත් පිටුවකින් ආරම්භ කරන්න./
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National Evaluation and Testing Service — NETS

ප්‍රශ්න අංකය : / வினா எண் :
විභාග අංකය : / பரீட்சைச் சுட்டெண் :
පිටු අංකය : / பக்க எண் : 04

Find the phase margin freq where is $\phi_M = 58.146^\circ$

$$\therefore \tan^{-1}\left(\frac{-(17\omega-\omega^3)}{-7\omega^2+15}\right) = -180 + 58.146^\circ$$
$$= -121.852$$

$$-17\omega + \omega^3 = -11.267\omega^2 + 24.144$$
$$\omega^3 + 11.267\omega^2 - 17\omega - 24.144 = 0$$
$$\omega = \cancel{-12.449},\ -12.475,\ 2.12,\ -0.913$$

$$\omega > 0 \quad \therefore \omega = 2.12$$

$$\therefore z_c = \frac{\omega_{PM}}{10}$$
$$= \frac{2.12}{10}$$
$$z_c = 0.212$$

At $\omega = 2.12$ compensated system's magnitude must be 0dB.

$$|G(j\omega)| = \left.\frac{36.7 \times 6.566}{(s+3)(s^2+4s+5)}\right|_{s=j2.12}$$
$$= \cancel{1.176\angle -121.838}$$
$$7.722\angle -121.835$$

$$|G(j\omega)|_{dB} = 20\log \cancel{1.176}\ 7.722$$
$$= \cancel{1.408}\ 17.755$$

පරීක්ෂකවරයාගේ ප්‍රයෝජනය සඳහා පමණි.
பரீட்சகரின் உபயோகத்திற்கு மாத்திரம்

මෙම කඩදාසි විභාග ශාලාවෙන් පිටතට ගෙනයාම හෝ අවසර නොමැතිව පෞද්ගලික ස්ථානයක තබා ගැනීම හෝ වෙනයම් කටයුත්තක් සඳහා යොදා ගැනීම හෝ දඬුවම් ලැබිය හැකි වරදකි. විභාග කොමසාරිස් ජනරාල්, ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව.
இத்தாளைப் பரீட்சை மண்டபத்திலிருந்து வெளியே எடுத்துச் செல்லல், அல்லது அனுமதியின்றித் தனிப்பட்டோர் வைத்திருத்தல், அல்லது வேறு ... ஆணையாளர் நாயகம், இலங்கைப் பரீட்சைத் திணைக்களம்.

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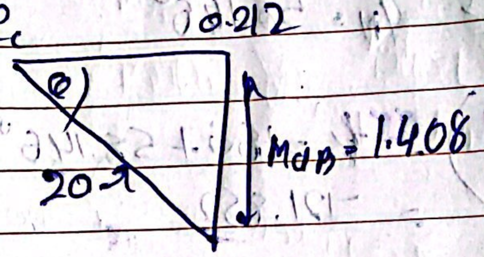

$\tan\alpha = 20 = \cancel{\frac{1.408}{P_c - 0.212}} = \frac{1.408}{20\log P_c - 20\log 0.212}$

$\log P_c = \frac{1.408}{20} + \log 0.212$

$\log P_c = -0.603$

$P_c = 10^{-0.603}$

$P_c = 0.249$

$\tan\alpha = 20 = \frac{\cancel{1.408}\ 17.755}{\log 0.212 - \log P_c}$

$\log P_c = \cancel{-0.744}\ -1.561$

$10^{-0.744} = P_c$

$\cancel{P_c = 0.18}\quad P_c = 0.027$

The lag compensator should have a dc gain of unity

$G_{lag}(s) = \frac{\gamma(s + 0.212)}{(s + \cancel{0.18}\ 0.027)}$

$|G_{lag}(s)|_{s=0} = \frac{\gamma(0.212)}{(\cancel{0.18}\ 0.027)} = 1$

$\gamma = \cancel{0.849}\ 0.129$

$\therefore G_{lag}(s) = \frac{0.129(s + 0.212)}{(s + 0.027)}$

පරීක්ෂකවරයාගේ ප්‍රයෝජනය සඳහා පමණී.
பரீட்சகரின் உபயோகத்திற்கு மாத்திரம்

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