UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 6 Examination in Engineering: February 2020

Module Number: EE6302 **Module Name: Control System Design**

[Three Hours]

[Answer all questions, each question carries 12 marks]

Note: A table of Laplace transforms is attached.

---

1 a) i) Using a block diagram show the components of a closed-loop control system.

ii) Briefly explain the purpose of the controller, the actuator and the sensor in a closed-loop control system?

iii) What is meant by a disturbance to a control system?

[4 Marks]

b) Consider the system shown in Figure Q1(b).

i) Determine the value of k so that the damping ratio of the system is 0.5.

ii) In the s-plane, show the poles of the system indicating the damping ratio and the undamped natural frequency of the system.

iii) Calculate the time function of the unit-step response of the system when the damping ratio of the system is 0.5.

[4.5 Marks]

c) Consider the system shown in Figure Q1(c). W(s) is the disturbance to the system.

i) Determine whether the system can track a unit step reference in the absence of the disturbance.

ii) Can the system reject a unit step disturbance? Explain your answer using suitable calculations.

[3.5 Marks]

2 a) i) In terms of the characteristic equation of a system, what is the necessary condition to be fulfilled in order to have a stable system?

ii) State the Routh's necessary and sufficient condition to have a stable system.

[1.5 Marks]

b) The characteristic equation of a system is given by

$$s^4 + 2s^3 + (4 + k)s^2 + 9s + 25 = 0.$$

Using Routh's stability criterion, determine the range of k so that the system to become stable.

[2.5 Marks]

c) Explain a method to check the stability, when the transfer function of the system is not known.

[1.5 Marks]

d) i) Write the general form of matrix equations so that a system is represented in state-variable form. Name the matrices in your matrix equations.

ii) Consider the RLC circuit shown in Figure Q2. The input voltage is V_i and the output voltage is V_o, the voltage across the capacitor. Writing differential equations, represent the system in state-variable form. Take the state vector x as

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \text{ where}, x_1 = V_o \text{ and } x_2 = \dot{V}_o.$$

Input and output of the system is V_i and V_o, respectively.

iii) Using the state-variable form you obtained in part ii), derive the transfer function of the system.

[6.5 Marks]

Q3 a) i) State three advantages of the closed-loop control when it is compared to the open-loop control.

ii) Consider the closed-loop control system shown in Figure Q3 where

$$G(s) = \frac{1}{(s+\tau_1)(s+\tau_2)}$$

τ_1, τ_2 can be real or complex numbers.

The controller used here is a proportional controller with gain K. Explain how this closed loop control system can improve the transient response when compared with its open loop control system.

iii) State the definition of the root locus.

[5.0 Marks]

b) Consider the closed loop control system shown in Figure Q3 where

$$G(s) = \frac{(s^2 - 2s + 5)}{(s+2)(s^2 + 3s + 8)}$$

The reference r in this system is a unit step.

i) Plot the root locus of this system.

ii) Calculate the imaginary axis crossings of the root locus.

iii) Find the range of gain K where the closed-loop system is stable.

iv) Find the steady state error of the closed-loop system's output y for gain K = 2.

v) Design a lag compensator to improve the steady state error of the system in part b)iv) by a factor of 10.

[7.0 Marks]

a) i) Consider the closed-loop control system shown in Figure Q4. The reference, r in this system is a unit step. What types of compensators can be used for $G_c(s)$ for the following purposes?

I. Improve the steady state error of the output y.

II. Reduce the percent overshoot of the output y.

III. Decrease the settling time of the output y.

IV. Improve the steady state error and decrease the settling time of the output simultaneously.

ii) Consider the closed-loop control system shown in Figure Q4. The reference r in the system is a unit step. Prove that the steady state error of this system can be driven to zero when Proportional plus Integral (PI) compensator is used. Assume that G(s) does not contain a zero at the origin.

[4.0 Marks]

b) Consider the closed-loop control system shown in Figure Q4 where

$$G(s) = \frac{K}{(s+10)(s^2+4s+2)}$$

The reference r in this system is a unit step. You need to design the compensator $G_c(s)$ for this system to improve the settling time of the output y with the use of the root locus technique.

i) Calculate the gain K required to yield 20% overshoot of the output y.

ii) State the assumptions you made in calculating the gain K in part b) i). Justify your assumptions.

iii) Estimate the settling time and the peak time of the system response for the gain you found in part b) i).

iv) Design a suitable compensator to improve the settling time of the system response by a factor 3.

[8.0 Marks]

a) i) Draw the asymptotic approximations of the bode plots for the system with the transfer function

$$G(s) = \frac{12(s+10)}{(s+2)(s+3)}$$

ii) Explain how you determine whether an open loop system is stable in closed loop, with the use of frequency response of the open loop system.

[4.0 Marks]

b) Consider the closed-loop control system shown in Figure Q4 where

$$G(s) = \frac{36.7}{(s+3)(s^2+4s+5)}$$

The reference r in this system is a unit step. The percent overshoot of the system response is 20%. You need to design the compensator $G_c(s)$ for this system to improve the steady state error of the output y with the use of the frequency response technique. In order to answer the following questions you may require following formulas, where all the notations have usual meaning.

$$\emptyset_M = \tan^{-1}\left(\frac{2\xi}{\sqrt{-2\xi^2+\sqrt{1+4\xi^4}}}\right) \quad \text{and} \quad \omega_{BW} = \frac{4}{T_s\xi}\sqrt{(1-2\xi^2)+\sqrt{4\xi^4-4\xi^2+2}}$$

i) Calculate the steady state error of the output y without the compensator $G_c(s)$.

ii) Design a lag compensator for the system above to improve the steady state error by a factor of 5 without significantly changing the percent overshoot.

[8.0 Marks]

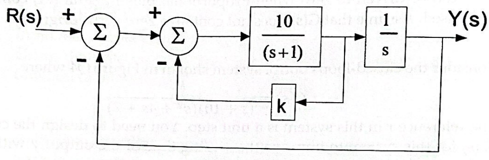

Figure Q1(b)

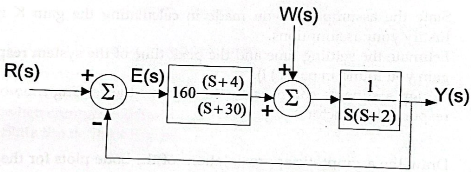

Figure Q1(c)

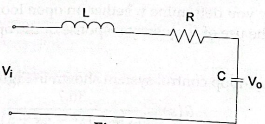

Figure Q2

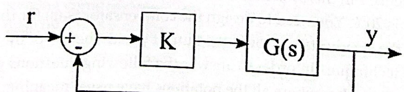

Figure Q3

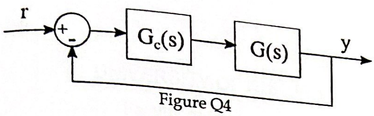

Figure Q4

Table of Laplace Transforms

Number	$F(s)$	$f(t), t \geq 0$
1	1	$\delta(t)$
2	$\frac{1}{s}$	$1(t)$
3	$\frac{1}{s^2}$	t
4	$\frac{2!}{s^3}$	t^2
5	$\frac{3!}{s^4}$	t^3
6	$\frac{m!}{s^{m+1}}$	t^m
7	$\frac{1}{(s+a)}$	e^{-at}
8	$\frac{1}{(s+a)^2}$	te^{-at}
9	$\frac{1}{(s+a)^3}$	$\frac{1}{2!}t^2e^{-at}$
10	$\frac{1}{(s+a)^m}$	$\frac{1}{(m-1)!}t^{m-1}e^{-at}$
11	$\frac{a}{s(s+a)}$	$1-e^{-at}$
12	$\frac{a}{s^2(s+a)}$	$\frac{1}{a}(at-1+e^{-at})$
13	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$
14	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$
15	$\frac{a^2}{s(s+a)^2}$	$1-e^{-at}(1+at)$
16	$\frac{(b-a)s}{(s+a)(s+b)}$	$be^{-at}-ae^{-at}$
17	$\frac{a}{(s^2+a^2)}$	$\sin at$
18	$\frac{s}{(s^2+a^2)}$	$\cos at$
19	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos bt$
20	$\frac{b}{(s+a)^2+b^2}$	$e^{-at}\sin bt$
21	$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$1-e^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$