UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 6 Examination in Engineering: September 2023

Module Number: EE6302 **Module Name: Control System Design (C/18)**

[Three Hours]

[Answer **all questions**, each question carries 12.0 marks]

Note: Formulas you may require are given in page 6. A table of Laplace transforms is attached in page 7.

Q1 a) i) Write expressions for ***rise time, overshoot,*** and ***settling time*** with their usual notations for a second order system.

ii) Sketch a suitable time response to illustrate the terms mentioned in part (i).

[2.0 Marks]

b) You are required to design a controller for the general second order system shown in Figure Q1 (b). All the notations have their usual meanings. Show that the damping of the system can be improved by adding a PD type controller as illustrated in Figure Q1 (b).

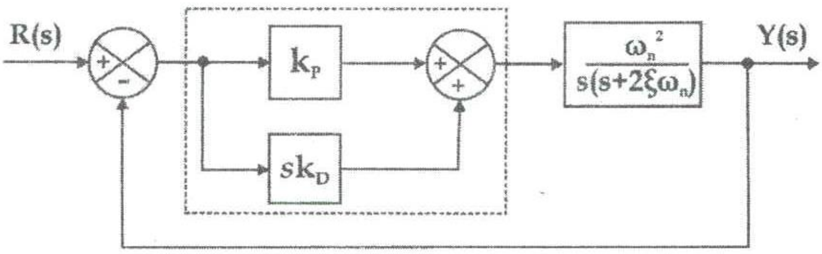

Figure Q1(b).

[2.5 Marks]

c) You are required to design a simple speed control system of a dc motor as shown in Figure Q1 (c). All the notations have their usual meanings.

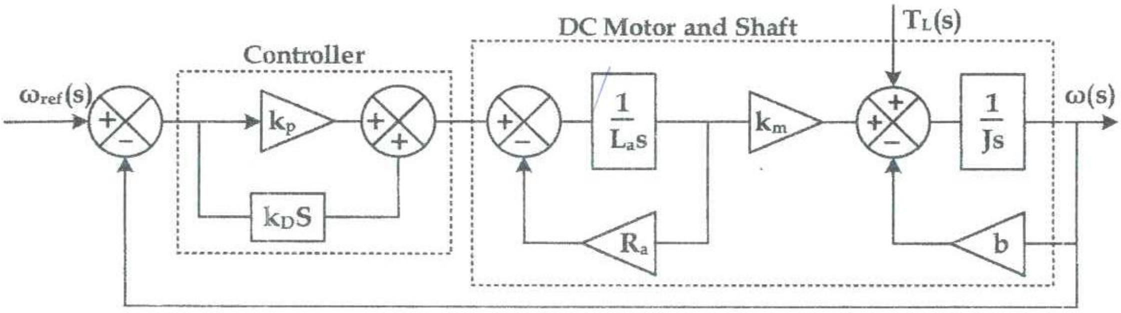

Figure Q1 (c).

i) Initially, the motor runs at no-load with negligible load torque. Show that the overall transfer function of the DC motor speed control system without a controller ($G_M(s)$) is

$$G_M(s) = \frac{k_m}{(Js + b)(L_a s + R_a) + k_m}$$

ii) The electrical resistance (R_a), inductance (L_a), and the moment of inertia of the DC motor are 1 Ω, 0.5 H and 0.2 kgm^2, respectively. The torque constant (k_m) and viscous friction constant (b) are 5 NmA^{-1} and 1 Nms. Calculate the ***overshoot*** of the system time response.

iii) Calculate the required (k_D) parameter of the PD type controller to damp out the present overshoot by 40%.

iv) If the DC motor runs a load with 3 Nm load torque, find the range of (k_p) to keep the steady state error due to the load torque less than 0.01 $rads^{-1}$.

[7.5 Marks]

Q2. a) Write the general form of matrix equations so that a system is represented in state space variable form. Name the matrices in your matrix equations.

[2.5 Marks]

b) Consider the plant of which the block diagram is shown in Figure Q2 (b).

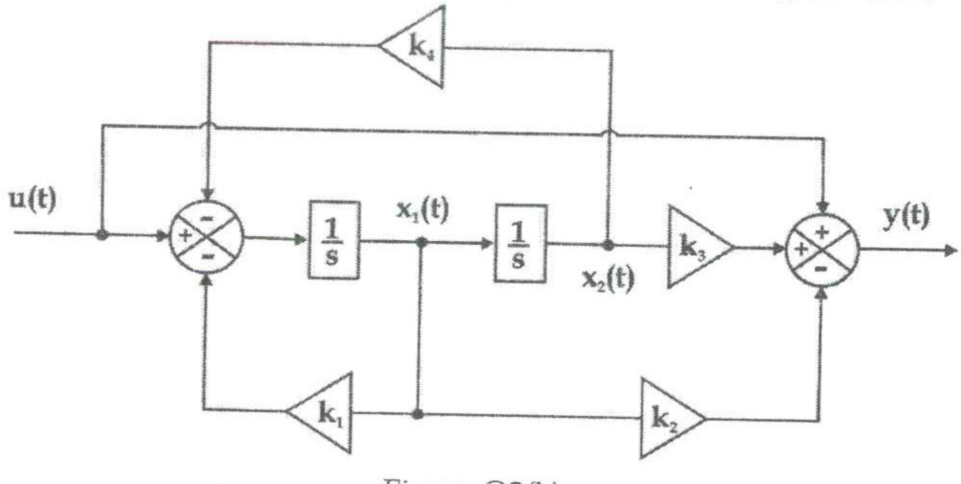

Figure Q2(b).

i) If the state vector of the plant can be denoted as $[x_1 \quad x_2 \quad \cdots \quad x_n]$ and the block diagram has 6, 2, 1 and 8 for k_1, k_2, k_3 and k_4 respectively, formulate the state space model of the plant shown in Figure Q2 (b).

ii) Hence, show that the transfer function of the plant is,

$$G(s) = \frac{s^2 + 4s + 9}{s^2 + 6s + 8}$$

[4.5 Marks]

c) Consider the transfer function obtained in part (b). The input and output parameters of the plant are angular displacement of a load and voltage.

i) Find the variation of angular displacement of load with time to the applied voltage $V_T(t) = 2e^{-3t}U(t)\ V$.

ii) Hence, obtain the steady state value of the system time response.

iii) Discuss the stability of the control system.

[5.0 Marks]

Q3. a) i) State the definition of the root locus.

ii) Consider the closed-loop system shown in Figure Q3.a).i) where;

$$G(s) = \frac{\prod_{i=1}^{m}(s + z_i)}{\prod_{i=1}^{n}(s + p_i)}$$

Derive the mathematical expression of the root-locus for this system.

iii) Derive the mathematical expressions for the two main properties of the root locus for the system shown in Figure Q3.a).i).

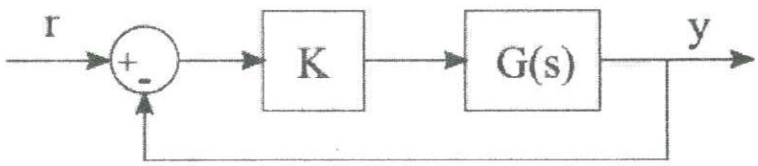

Figure Q3.a).i). Closed-loop control system.

iv) Figure Q3.a).ii) depicts the root locus of a second-order closed-loop system. Can the percent overshoot of the closed-loop unit step response of this system be kept below 5% by adjusting the gain alone? Provide an explanation for your answer.

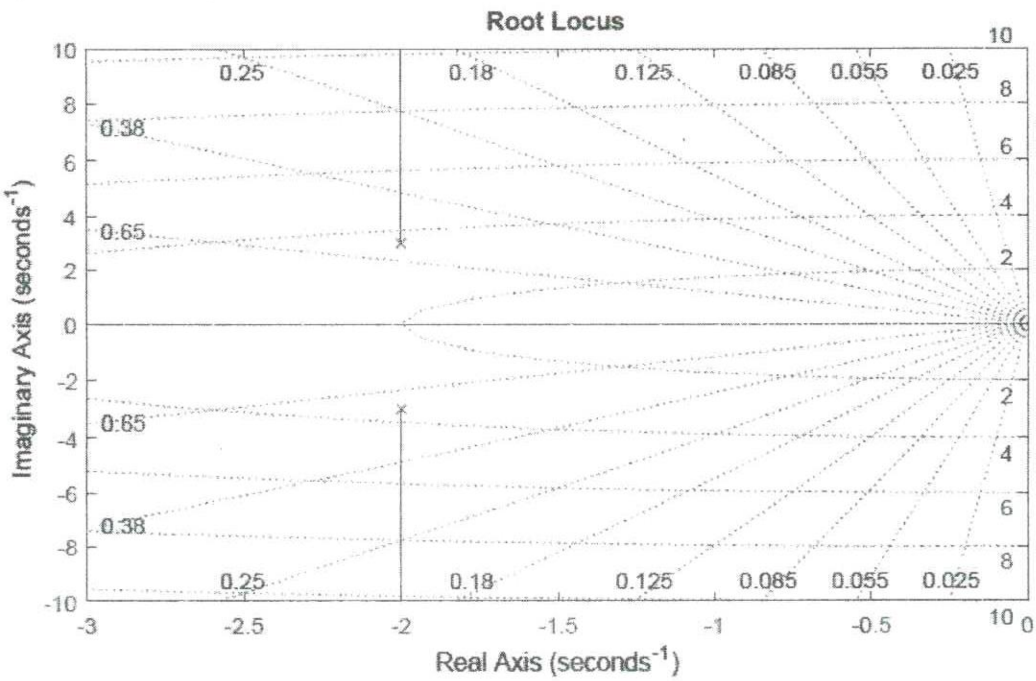

Figure Q3. a) ii).

[5.0 Marks]

b) Consider the closed-loop control system shown in Q3.a).i). where;

$$G(s) = \frac{s+10}{(s+3)(s^2+3s+7)}$$

Answer the following questions with respect to the root locus of this system.

i) Calculate the imaginary axis crossings of the root locus.

ii) Find the asymptotes.

iii) Find the departure angles at open-loop complex poles.

iv) Sketch the root locus of the system.

v) Determine whether the point s = -1+3.8j is a point on the root locus.

vi) Calculate the poles of the closed-loop system when the gain is 15. Is the closed-loop system stable at this gain?

vii) Find the range of gain K where the closed-loop system is stable.

[7.0 Marks]

Q4. a) Figure Q4.a) illustrates the response of a closed-loop system to a unit step input.
Note: The same figure is given on page 8, use that figure to answer the followings and attach that page with your answers.

i) Sketch the anticipated response of the closed-loop system to a unit step input as the open-loop gain is increased.

ii) Sketch the anticipated response of the closed-loop system to a unit step input when a PI controller is added.

iii) How would you enhance the response in part ii), if you want to achieve a better settling time?

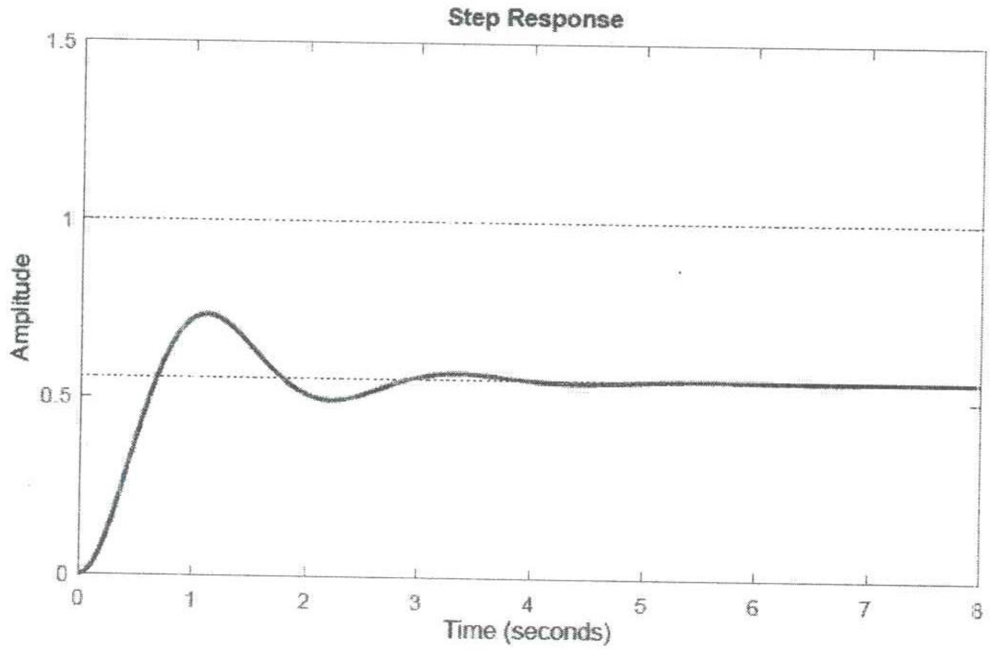

Figure Q4. a).

[3.0 Marks]

b) Answer this question using your knowledge on root locus design technique. However, it is NOT necessary to sketch the root locus of the given system.

Figure Q4.b) shows a closed-loop control system. The transfer function of the plant is

$$G(s) = \frac{1}{(s+1)(s+2)(s+5)}$$

Design a PID compensator for the above system to meet the following specifications when subjected to a unit step input.

Percent overshoot, $M_p\% \approx 12\ \%$

Peak time, $t_p \approx 1$ seconds

Steady state error = 0.

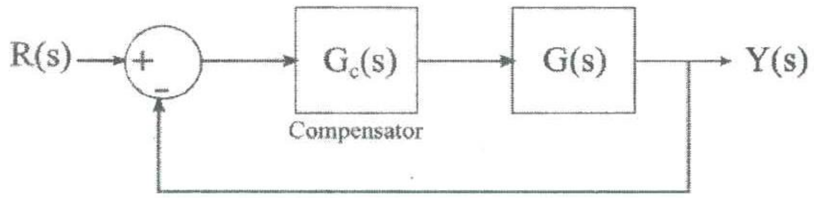

Figure Q4.b).

[9.0 Marks]

Q5. a) i) Draw the bode diagrams for the following system using asymptotic approximations.

$$G(s) = \frac{s(s+5)}{(s+1)(s+15)}$$

ii) Briefly explain the relationship between the open-loop frequency response and the stability of the closed-loop system.

[4.0 Marks]

b) Answer this question using your knowledge on frequency response design technique. However, it is NOT necessary to draw the frequency response of the given system.

Figure Q4.b) shows a closed-loop control system. The transfer function of the plant is

$$G(s) = \frac{1}{(s+4)(s+11)}$$

Design a lead compensator for the above system to meet the following specifications when subjected to a unit step input.

Percent overshoot, $M_p\% \approx 10\ \%$

Steady state error = 1%

[8.0 Marks]

Formulas you may require:

(All notations have their usual meaning)

$$M_P = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$$

$$t_P = \frac{\pi}{\omega_d}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$\phi_{PM} = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}}$$

For the lead compensator

$$G_{lead}(s) = \frac{1}{\beta}\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}.$$

$$\phi_{lead,max} = \sin^{-1}\frac{1-\beta}{1+\beta}$$

$$|G_{lead}(j\omega_{max})| = \frac{1}{\sqrt{\beta}}$$

$$\omega_{max} = \frac{1}{T\sqrt{\beta}}$$

Table of Laplace Transforms

Number	$F(s)$	$f(t), t \geq 0$
1	1	$\delta(t)$
2	$\frac{1}{s}$	$1(t)$
3	$\frac{1}{s^2}$	t
4	$\frac{2!}{s^3}$	t^2
5	$\frac{3!}{s^4}$	t^3
6	$\frac{m!}{s^{m+1}}$	t^m
7	$\frac{1}{(s+a)}$	e^{-at}
8	$\frac{1}{(s+a)^2}$	te^{-at}
9	$\frac{1}{(s+a)^3}$	$\frac{1}{2!}t^2e^{-at}$
10	$\frac{1}{(s+a)^m}$	$\frac{1}{(m-1)!}t^{m-1}e^{-at}$
11	$\frac{a}{s(s+a)}$	$1-e^{-at}$
12	$\frac{a}{s^2(s+a)}$	$\frac{1}{a}(at-1+e^{-at})$
13	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$
14	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$
15	$\frac{a^2}{s(s+a)^2}$	$1-e^{-at}(1+at)$
16	$\frac{(b-a)s}{(s+a)(s+b)}$	$be^{-at}-ae^{-at}$
17	$\frac{a}{(s^2+a^2)}$	$\sin at$
18	$\frac{s}{(s^2+a^2)}$	$\cos at$
19	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos bt$
20	$\frac{b}{(s+a)^2+b^2}$	$e^{-at}\sin bt$
21	$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$1-e^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$

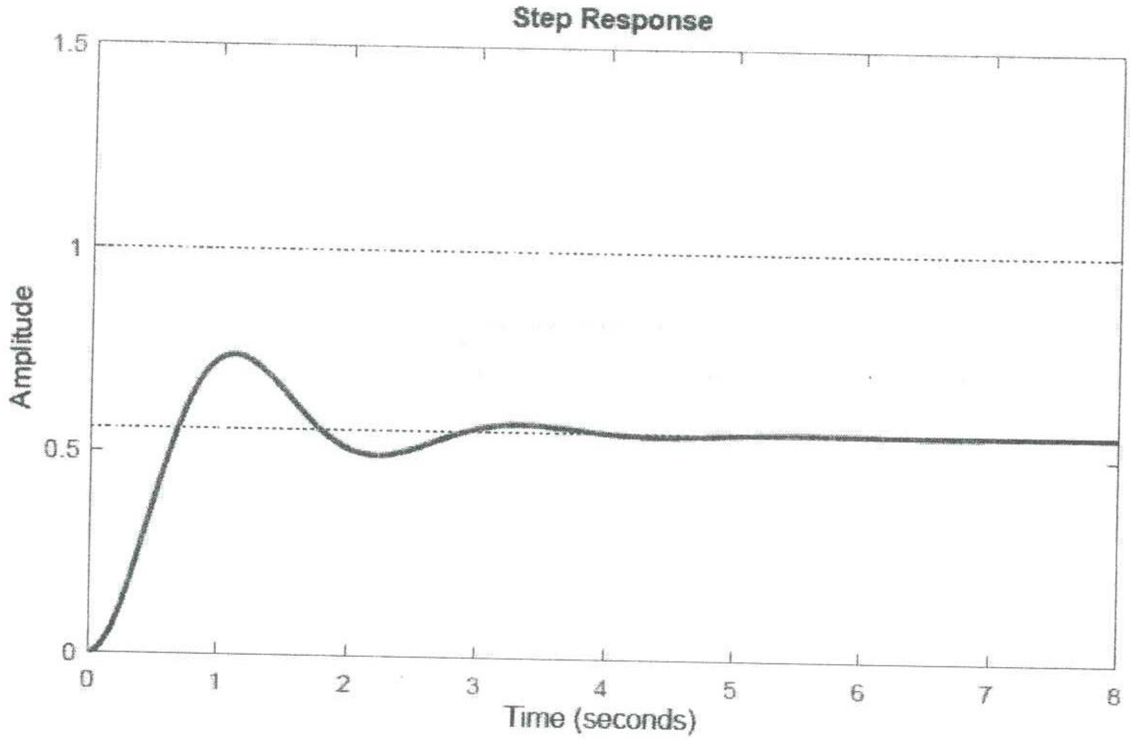

Figure Q4. a).