# UNIVERSITY OF RUHUNA

## Faculty of Engineering

End-Semester 6 Examination in Engineering: October 2024

**Module Number: EE6302** **Module Name: Control System Design (C-18)**

**[Three Hours]**

[Answer **all questions**, each question carries 12 marks]

Note: Formulas you may require are given in page 5. A table of Laplace transforms is attached in page 6.

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Q1 a) i) Using necessary block diagrams, explain the terms "open-loop control" and "closed-loop control" in control engineering.

ii) Describe the main advantage and the main disadvantage of the closed-loop control systems.

[2.5 Marks]

b) i) Drawing a suitable time response, explain the terms; rise time, settling time, maximum overshoot and peak time, associated with a control system.

ii) An underdamped second order system is shown in Figure Q1(b). Assume that k>0. It is required to design the system so that it gets 5% maximum overshoot and 2 s peak time. Determine whether both specifications can be met simultaneously by selecting a value for k.

[4.5 Marks]

c) i) Consider the system shown in Figure Q1(c1). Show that a non-zero steady-state error exists in the system for a unit-ramp input.

ii) In order to eliminate the steady-state error for a unit-ramp input, an input filter is added to the system as shown in Figure Q1(c2). Determine the input filter transfer function H(s).

[5 Marks]

Q2 a) i) In terms of the characteristic equation of a system, what is the necessary condition to be fulfilled to have a stable system?

ii) State the Routh's necessary and sufficient condition to have a stable system.

[1.5 Marks]

b) The characteristic equation of a system is given by

$$s^4 + 2s^3 + (4 + k)s^2 + 9s + 25 = 0$$

Using Routh's stability criterion, determine the range of k so that the system becomes stable.

[3 Marks]

c) Explain a method to check the stability, when the transfer function of the system is not known.

[1.5 Marks]

d) i) Write the general form of matrix equations so that a system is represented in state-variable form. Name the matrices in your matrix equations.

ii) Consider the RLC circuit shown in Figure Q2. The input voltage is $V_i$ and the output voltage is $V_o$, the voltage across the capacitor. Writing differential equations for the RLC circuit, obtain the state-variable form of the system. Take the state vector $x$ as

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, where , $x_1 = V_o$ and $x_2 = \dot{V}_o$

Input and output of the system is $V_i$ and $V_o$, respectively.

iii) Using the state-variable form obtained in part ii), obtain the poles of the system. Hence, derive the condition to be fulfilled so that the system becomes an underdamped system.

[6 Marks]

Q3 a) Figure Q3(a) shows the root locus of a system whose plant transfer function is G(s).

i) What is a root locus?

ii) Is the open-loop system stable for any gain K? Briefly explain the reasons for your answer.

iii) Is the closed-loop system stable for any gain K? Briefly explain the reasons for your answer.

iii) Derive the transfer function of the plant, G(s).

[4.0 Marks]

b) Consider the closed-loop system given in Figure Q3(b), where

$$G(s) = \frac{(s+2)(s+5)}{(s^2 - 5s + 11)}$$

i) Sketch the root locus for the system after finding the following.
- Break-away/ Break-in points if exists
- Imaginary axis crossings
- Departure angles at open-loop complex poles/Arrival angles at open-loop complex zeros

ii) Find the range of gain K where the closed-loop system stable.

iii) Determine whether the point -1.8+2.65j is on the root locus.

iv) Find the closed-loop poles when the gain is 10.

v) Find the gain corresponding to closed-loop pole -2.8+1.51j.

[8.0 Marks]

Q4 a) i) What are the functions of a controller in a closed-loop system.

ii) State four types of controllers that can be used in closed-loop systems and briefly explain their functions.

[3.0 Marks]

b) Using your knowledge of root locus design technique, answer this question. Note that it is NOT necessary to sketch the root locus of the system.

Figure Q4 shows a closed-loop control system where the plant's transfer function is given by:

$$G(s) = \frac{1}{(s+15)(s^2+2s+4)}$$

Design a PID compensator for this system to achieve the following specifications for a unit step input:

Percent overshoot, $M_p\% \approx 10\%$,

Peak time, $t_p \approx 0.3$ seconds,

Steady state error = 0.

State and justify any assumptions made.

[9.0 Marks]

Q5 a) i) Draw the Bode diagrams for the following system using asymptotic approximations.

$$G(s) = \frac{25(s+30)}{s(s+1)(s+5)}$$

ii) Draw the frequency responses of a lag compensator and a lead compensator.

[4.0 Marks]

b) Using your knowledge of frequency response design technique, answer this question. Note that it is NOT necessary to draw the frequency response of the system.

Figure Q4 shows a closed-loop control system where the plant's transfer function is given by:

$$G(s) = \frac{1}{s^2+7s+15}$$

Design a Lag compensator for this system to achieve the following specifications for a unit step input:

Percent overshoot, $M_p\% \approx 12\%$,

Steady state error = 0.1%.

[8.0 Marks]

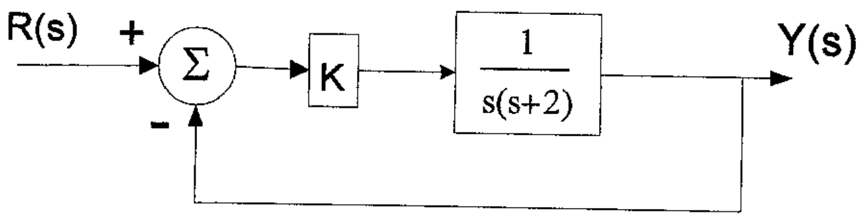


Figure Q1(b).

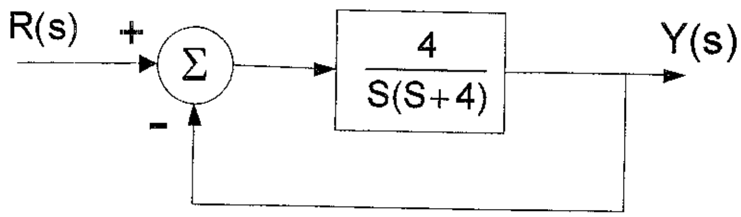


Figure Q1(c1).

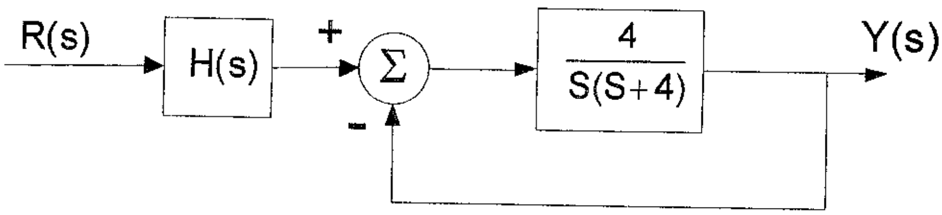


Figure Q1(c2).

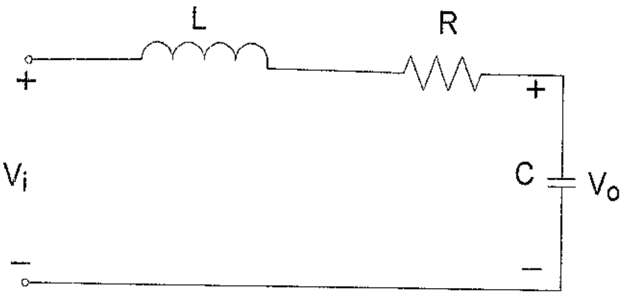


Figure Q2.

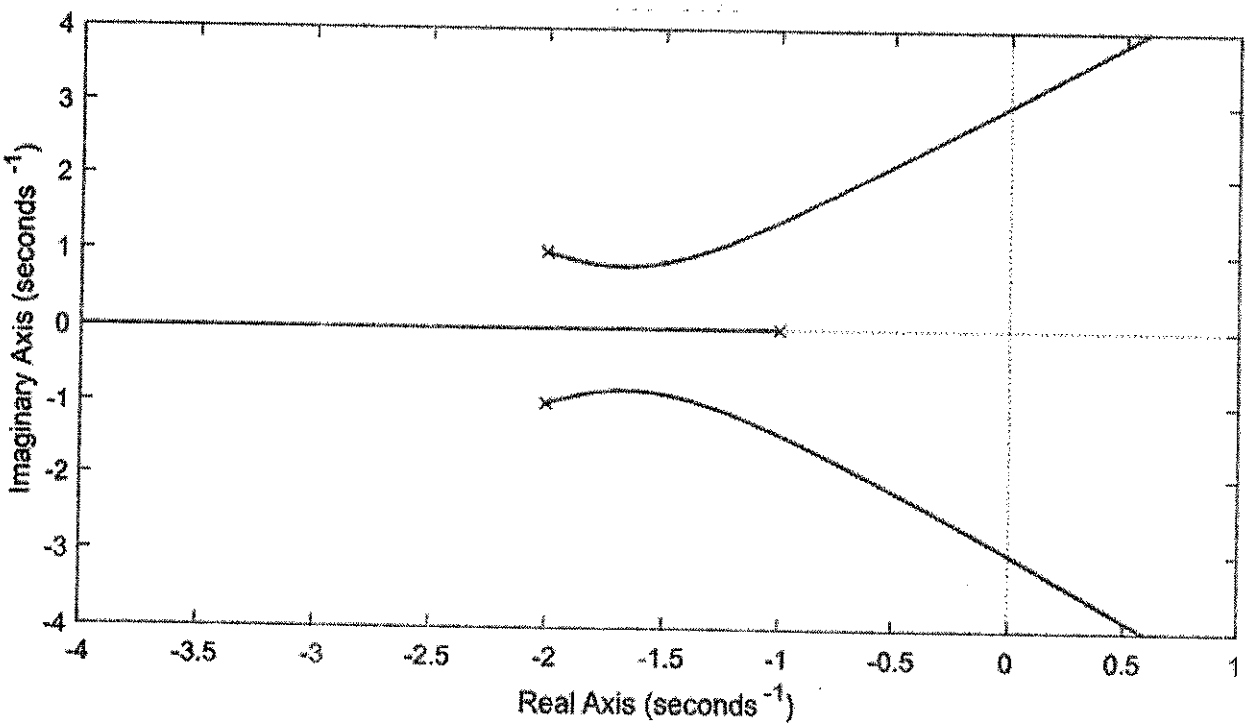


Figure Q3(a).

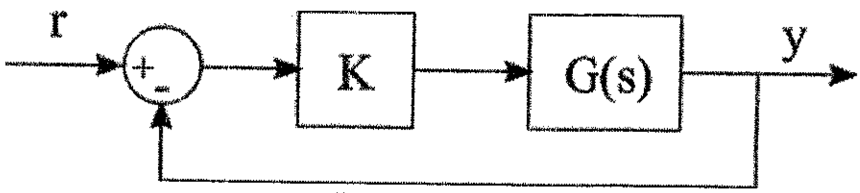


Figure Q3(b).

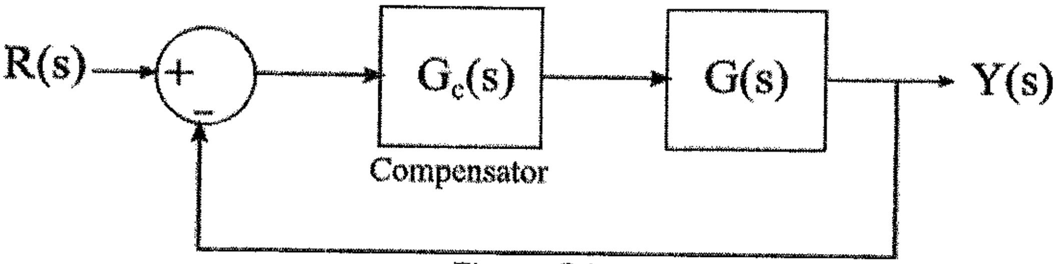


Figure Q4.

**Formulas you may require:**
(All notations have their usual meanings.)

For an underdamped second order system,

$$M_P = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$$
$$t_P = \frac{\pi}{\omega_d}$$
$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$\phi_{PM} = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}}$$

## Table of Laplace Transforms

| Number | $F(s)$ | $f(t), t \geq 0$ |
|---|---|---|
| 1 | $1$ | $\delta(t)$ |
| 2 | $\frac{1}{s}$ | $1(t)$ |
| 3 | $\frac{1}{s^2}$ | $t$ |
| 4 | $\frac{2!}{s^3}$ | $t^2$ |
| 5 | $\frac{3!}{s^4}$ | $t^3$ |
| 6 | $\frac{m!}{s^{m+1}}$ | $t^m$ |
| 7 | $\frac{1}{(s+a)}$ | $e^{-at}$ |
| 8 | $\frac{1}{(s+a)^2}$ | $te^{-at}$ |
| 9 | $\frac{1}{(s+a)^3}$ | $\frac{1}{2!}t^2e^{-at}$ |
| 10 | $\frac{1}{(s+a)^m}$ | $\frac{1}{(m-1)!}t^{m-1}e^{-at}$ |
| 11 | $\frac{a}{s(s+a)}$ | $1-e^{-at}$ |
| 12 | $\frac{a}{s^2(s+a)}$ | $\frac{1}{a}(at-1+e^{-at})$ |
| 13 | $\frac{b-a}{(s+a)(s+b)}$ | $e^{-at}-e^{-bt}$ |
| 14 | $\frac{s}{(s+a)^2}$ | $(1-at)e^{-at}$ |
| 15 | $\frac{a^2}{s(s+a)^2}$ | $1-e^{-at}(1+at)$ |
| 16 | $\frac{(b-a)s}{(s+a)(s+b)}$ | $be^{-bt}-ae^{-at}$ |
| 17 | $\frac{a}{(s^2+a^2)}$ | $\sin at$ |
| 18 | $\frac{s}{(s^2+a^2)}$ | $\cos at$ |
| 19 | $\frac{s+a}{(s+a)^2+b^2}$ | $e^{-at}\cos bt$ |
| 20 | $\frac{b}{(s+a)^2+b^2}$ | $e^{-at}\sin bt$ |
| 21 | $\frac{a^2+b^2}{s[(s+a)^2+b^2]}$ | $1-e^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$ |