



## UNIVERSITY OF RUHUNA

### Faculty of Engineering

End-Semester 6 Examination in Engineering: November 2016

Module Number: EE6302

Module Name: Control System Design

[Three Hours]

[Answer all questions, each question carries 15 marks]

- Q1 a) i) Give expressions for *rise time*, *overshoot* and *settling time* with their usual notations for a second order system.  
ii) Write the transfer functions for the controller type P, PI, PD and PID.  
iii) What is the main objective of adding integral control to a system?  
iv) What is meant by disturbance rejection in control system design? [6.0 Marks]
- b) You are required to design a controller for the system shown in Figure Q1 (b1) to improve the damping of the system.  
i) Show that the design requirement can be achieved by adding a PD type controller as illustrated in Figure Q1 (b1).  
ii) Hence, show that the *overshoot* and *settling time* can be reduced.  
iii) If the time response of the plant  $G(s)$  is shown in Figure Q1 (b2), find the range of controller gains so that the overall unit feedback system is stable. [5.5 Marks]
- c) It is required to analyze the current controller module of a dc motor drive shown in Figure Q1 (c). The back EMF voltage ( $E_a$ ) of the dc motor can be considered as a disturbance input to the system. The system should keep the armature current at 600A constant level. Determine whether the system can track this current reference value, when the motor is rotating at a constant speed of 800 rpm? [3.5 Marks]

- Q2 a) i) Write the general form of matrix equations so that a system is represented in state-variable form. Name the matrices in your matrix equations.  
ii) Define the root locus considering a negative feedback system.  
iii) Explain the magnitude and the phase conditions to be satisfied at a point on the root locus.  
iv) Explain the usages of the root locus in control system design. [6.0 Marks]
- b) Consider the plant whose block diagram is shown in Figure Q2 (a).  
i) If the state vector of the plant can be denoted as  $[x_1 \ x_2 \ \dots \ x_n]$  and the block diagram has 6, 10, -12 and 8 for  $k_1, k_2, k_3$  and  $k_4$  respectively, formulate the state space model of the plant shown in Figure Q2 (a).  
ii) Hence, show that the transfer function of the plant is,  
$$G(s) = \frac{s^2 - 4s + 20}{s^2 + 6s + 8}$$
 [4.5 Marks]

- c) You are required to design a proportional controller for the system shown in Figure Q2 (c) using the root locus design method.
- Following necessary rules for drawing the root locus, sketch the root locus of the system shown in Figure Q2 (c).
  - Determine the range of the controller gain ( $k_p$ ) so that the system is stable.
  - For a stable system, it is required to have the damping factor  $\xi = 0.6$ . Determine the required controller gain ( $k_p$ ).

[4.5 Marks]

- Q3 a) i) Define poles and zeroes of a transfer function.  
ii) If the damping ratio and the undamped natural frequency of a system are denoted by  $\xi$  and  $\omega_n$ , locate a pair of complex conjugate poles in the s-plane and express the rectangular coordinates of the poles in terms of  $\xi$  and  $\omega_n$ .  
iii) Explain the basic function of a compensator associated with a control system in improving the system speed.  
iv) Explain why it is not recommended to add only poles or only zeroes for compensation.

[6.5 Marks]

- b) Consider the root locus given in Figure Q3 (b). It is desired to increase the system speed to  $\omega_n = 3\text{rads}^{-1}$  while maintaining the damping ratio ( $\xi$ ) at 0.5. Using the root locus of the closed loop system given in Figure Q3 (b), show that the design requirement cannot be achieved.

[2.0 Marks]

- c) In order to achieve the design requirements in Q3 b), it is decided to use a lead compensator [i.e.  $D(s) = k(s+z)/(s+p)$ ]. The noise suppression requirements require that the lead pole to be at -1.
- Using necessary geometrical constructions in the s-plane, determine the values for the zero ( $z$ ) and the gain ( $k$ ) of the lead compensator.
  - Roughly sketch the root locus of the compensated system and hence, discuss the system stability.

[6.5 Marks]

- Q4 a) i) What is frequency response of a system?  
ii) Define amplitude ratio M and phase  $\phi$  related to the frequency response of a system whose transfer function is  $G(s)$ .  
iii) Explain the Bode diagrams associated with the frequency response of a system.  
iv) Define the terms phase margin and gain margin associated with the Bode plots.

[6.0 Marks]

- b) A gear train system shown in Figure Q4 (b1) consists of a torque generator whose locations of the poles and zeroes in the s-plane are illustrated in Figure Q4(b2) and a gear unit. The dynamics of the gear unit are described in (1), (2) and (3).

$$J_{eq} = J_2 + J_1 \left( \frac{N_2}{N_1} \right)^2 \quad (1), \quad D_{eq} = D_2 + D_1 \left( \frac{N_2}{N_1} \right) \quad (2), \quad T_G \left( \frac{N_2}{N_1} \right) = J_{eq} \frac{d^2 \theta_L}{dt^2} + D_{eq} \frac{d\theta_L}{dt} \quad (3)$$

- i) The transfer function gain of the torque generator is 10. In Figure Q4 (b1),  $J_1 = 0.1 \text{ kgm}^2$ ,  $J_2 = 2.4 \text{ kgm}^2$ ,  $D_1 = 9 \text{ Nm/rads}^{-1}$ ,  $D_2 = 4 \text{ Nm/rads}^{-1}$ ,  $N_1 = 3$  and  $N_2 = 12$ . Show that the overall transfer function of the gear train system is,

$$G(s) = \frac{\theta_L(s)}{V_T(s)} = \frac{10}{s(s+1)(s+10)}$$

- ii) Obtain the steady-state output of the system mentioned in part b) (i), when it is subjected to the input  $V_T = 2\sin(2t - 60^\circ)$

[4.0 Marks]

- c) It is required to control the angular displacement of the load as a unity feedback system. Drawing approximate Bode plots and giving reasons, analyze the stability of the system.

[5.0 Marks]

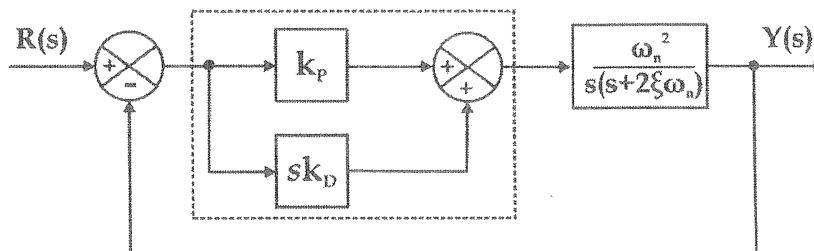


Figure Q1(b1)

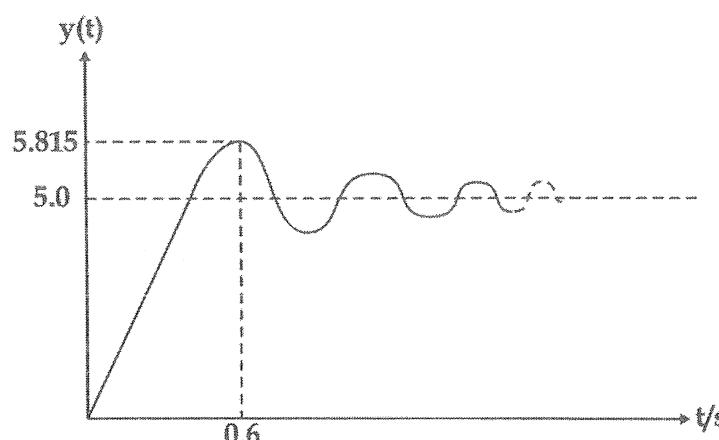


Figure Q1(b2)

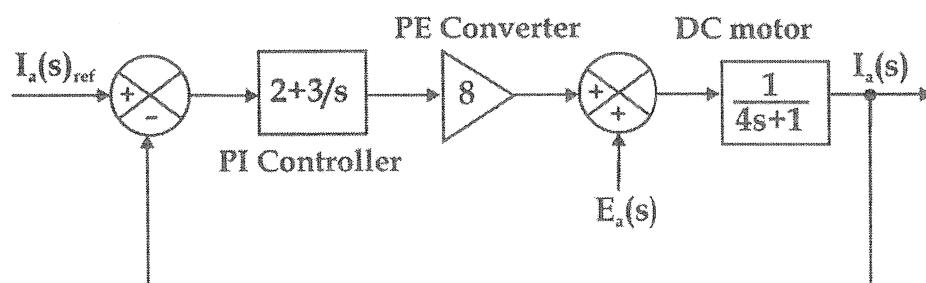


Figure Q1(c)

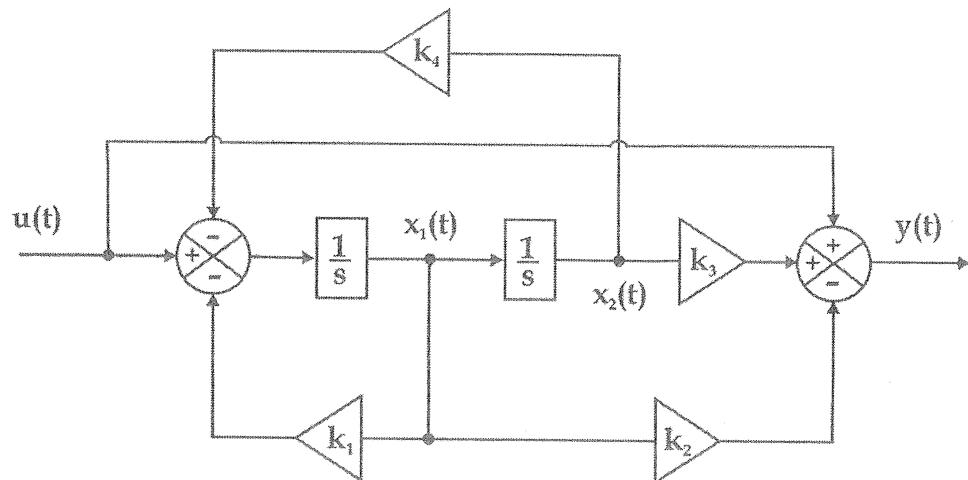


Figure Q2(a)

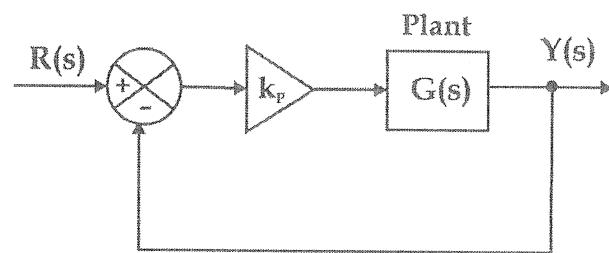


Figure Q2(c)

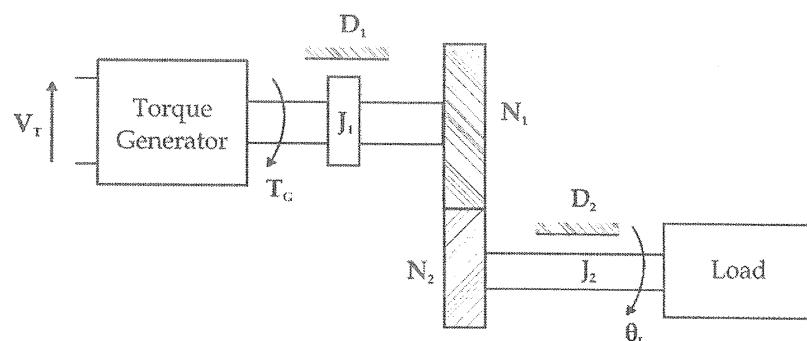


Figure Q4(b1)

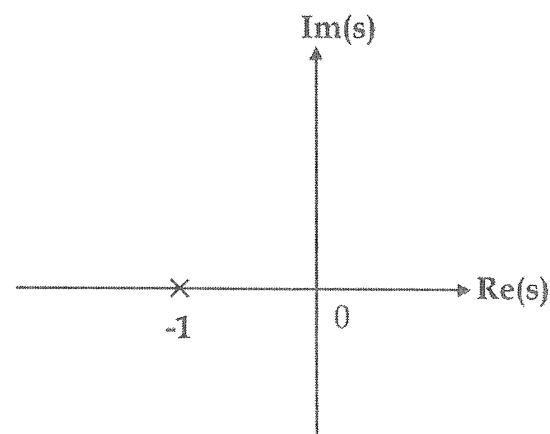


Figure Q4(b2)

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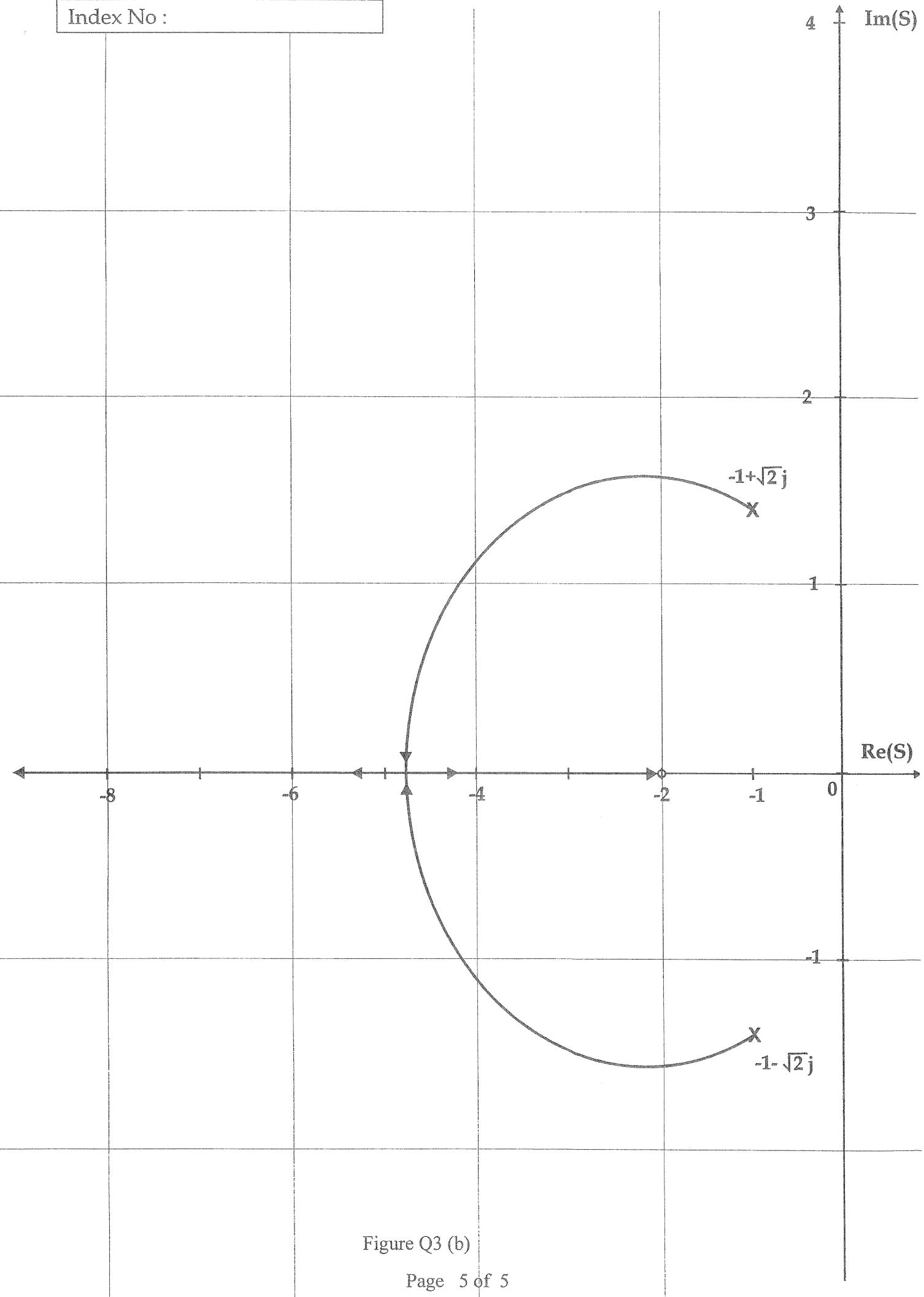


Figure Q3 (b)