UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 6 Examination in Engineering: July 2025

Module Number: EE6302 **Module Name: Control System Design (C-18)**

[Three Hours]

[Answer **all questions**, each question carries 12 marks]
Note: Formulas you may require are given in page 5. A table of Laplace transforms is attached in page 6.

Q1 a) i) Showing all main components, draw the general block diagram of a closed-loop control system. Explain why the block diagram you have drawn is called a closed-loop control system.

ii) What is the main disadvantage of the open-loop control system?

[2.5 Marks]

b) An underdamped second order system is shown in Figure Q1(b). Assume that $k_1>0$ and $k_2>0$. It is required to design the system to satisfy the following specifications:

1. Maximum overshoot (M_p) $\leq 5\%$
2. Settling time (t_s) ≤ 3 s (1% criteria).
3. Steady-state error (e_{ss}) $\leq 10\%$ to a unit ramp input.

i) Obtain the open-loop transfer function of the system.

ii) Obtain the closed-loop transfer function of the system.

iii) Determine the region in the complex plane where the closed-loop poles must lie to meet the given transient response specifications of the system. Then, compute the specific pole locations that satisfy the minimum transient response requirements.

iv) Due to a unit ramp input, find the steady-state error of the system in terms of k_1 and k_2.

v) If $k_1=25$, will it be possible to determine the k_2 so that all specifications can be met? Explain your answer using necessary calculations.

[9.5 Marks]

Q2 a) i) Why is stability an important consideration in control system design?

ii) State the Routh's necessary and sufficient condition to have a stable system.

[1.5 Marks]

b) The characteristic equation of a system is given by

$$s^4 + ks^3 + s^2 + s + 1 = 0$$

Using Routh's stability criterion, analyze the stability of the system in terms of the parameter k.

[4 Marks]

c) Explain a method to check the stability when the transfer function of the system is not known.

[1 Mark]

d) i) Write the general form of matrix equations so that a system is represented in state-variable form. Name all the matrices in your matrix equations.

ii) A system is described by the third-order differential equation $\dddot{y} + 3\ddot{y} + 2\dot{y} = u$, where y and u are the output and the input respectively. Taking the state vector x as

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \text{ where }, x_1 = y, x_2 = \dot{y} \text{ and } x_3 = \ddot{y},$$

obtain the system in state-variable form.

iii) Using the state-variable form obtained in part ii),

(I) write a matrix equation to derive the transfer function of the system.

(II) write a matrix equation to obtain the poles of the system.

[5.5 Marks]

Q3 a) i) Write the transfer functions for the controller types P, PI, PD and PID.

ii) What is the main objective of introducing integral control to a system?

iii) What is the main objective of adding derivative control to a system?

iv) What is meant by disturbance rejection in control system design?

[3 Marks]

b) For the system shown in Figure Q3(b), it is required to design the controller C(s). By sketching the rough shape of the root loci of the system, explain why by selecting a PD type controller for C(s) can achieve better performance than by selecting a P type controller.

[3 Marks]

c) Consider the system shown in Figure Q3(c). R(s) is reference input to the system and W(s) is disturbance input to the system. K is a real constant. It is required to design the controller D(s) for the system.

i) Writing a suitable expression for steady-state error, decide what type of controller, i.e. P, PI or PID, suits to stabilize and to reject a unit step disturbance in the system.

ii) What are the conditions to be fulfilled for the parameters of the selected controller?

iii) With your selected controller, what would be the steady-state error if there was a unit ramp disturbance in the system?

[6.0 Marks]

Q4 a) Consider the system shown in Figure Q4. Using root-locus design technique, design a lead compensator $G_c(s)$ such that the damping ratio ζ and the undamped natural frequency ω_n of the dominant closed-loop poles are 0.5 and 2 rad/sec, respectively. Choose the zero of the lead compensator at s = -1.

[9 Marks]

b) Write the required equations to draw the root-locus of the compensated system obtained in section a). Hence, sketch the rough shape of the root-locus of the compensated system. (Note: You are not required to solve all the equations you have written to sketch the rough shape of the root-locus)

[3 Marks]

Q5 a) i) Give definition for the frequency response of a system.

ii) Consider the negative unity-feedback system shown in Figure Q5(a). Obtain the steady state output of the system when it is subjected to the input $r(t) = sin(t + 30°)$.

[3 Marks]

b) i) Define the terms phase-margin and gain-margin associated with the Bode plots.

ii) Sketch the asymptotes of the Bode plot magnitude and approximate phase for the open-loop transfer function of the system shown in Figure Q5(b).

iii) By using the Bode plot diagrams sketched in part (ii), explain the stability of the system.

iv) Using necessary equations, briefly explain how to draw the Nyquist plot to examine the stability of this system.

[9 Marks]

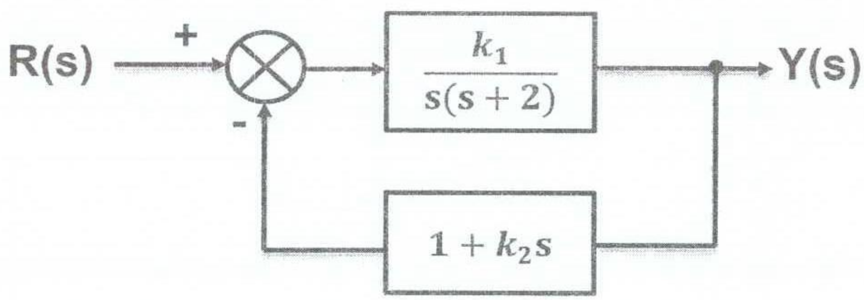

Figure Q1(b).

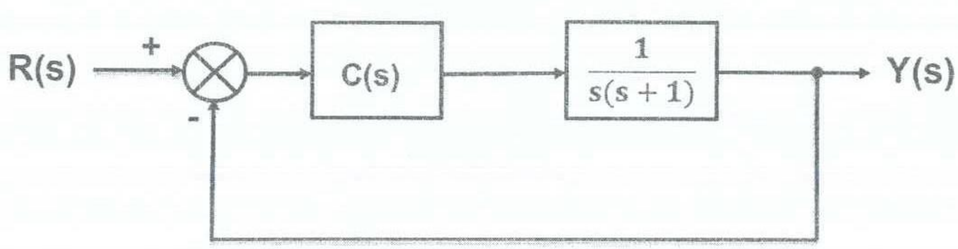

Figure Q3(b).

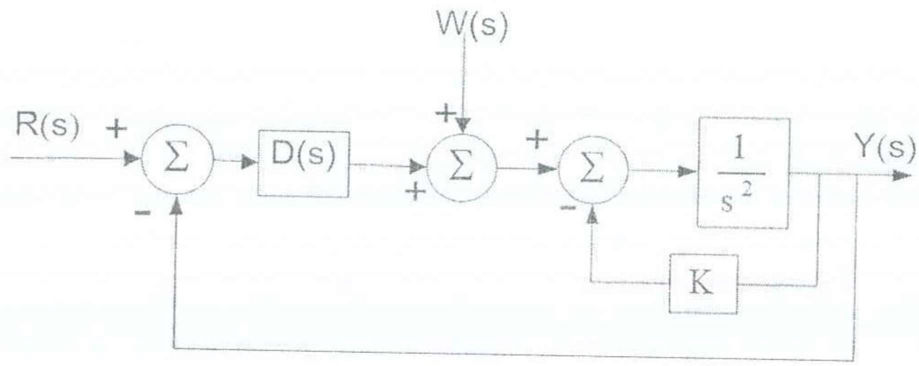

Figure Q3(c).

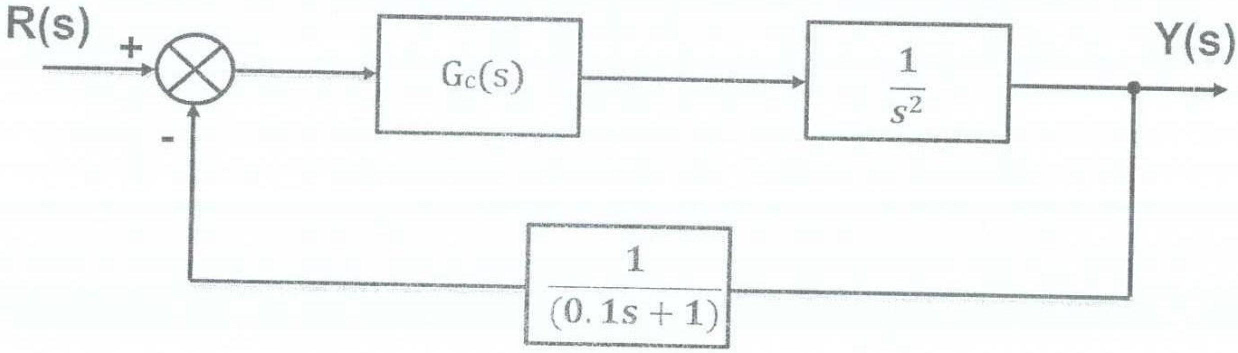

Figure Q4.

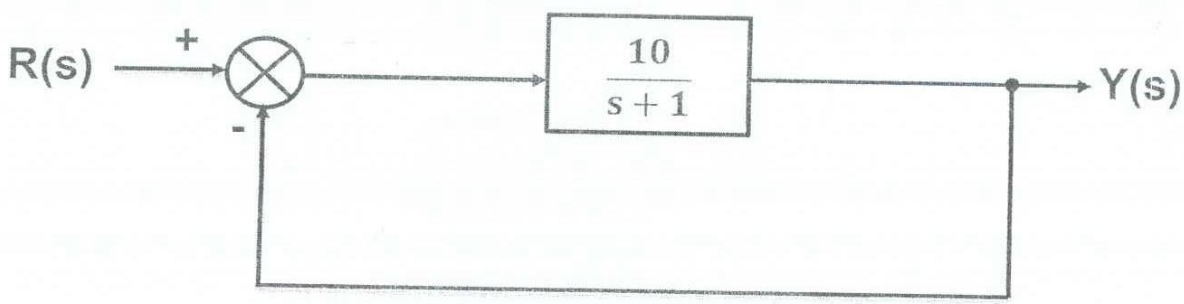

Figure Q5(a).

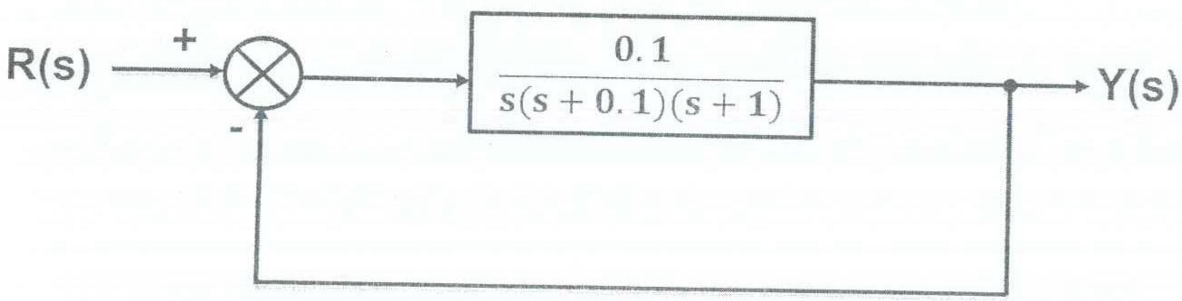

Figure Q5(b).

Formulas you may require:
(All notations have their usual meaning)

For an underdamped second order system

$$M_P = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$$

$$t_P = \frac{\pi}{\omega_d}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$t_s = \frac{4.6}{\sigma} \quad \text{(for 1\% criteria)}$$

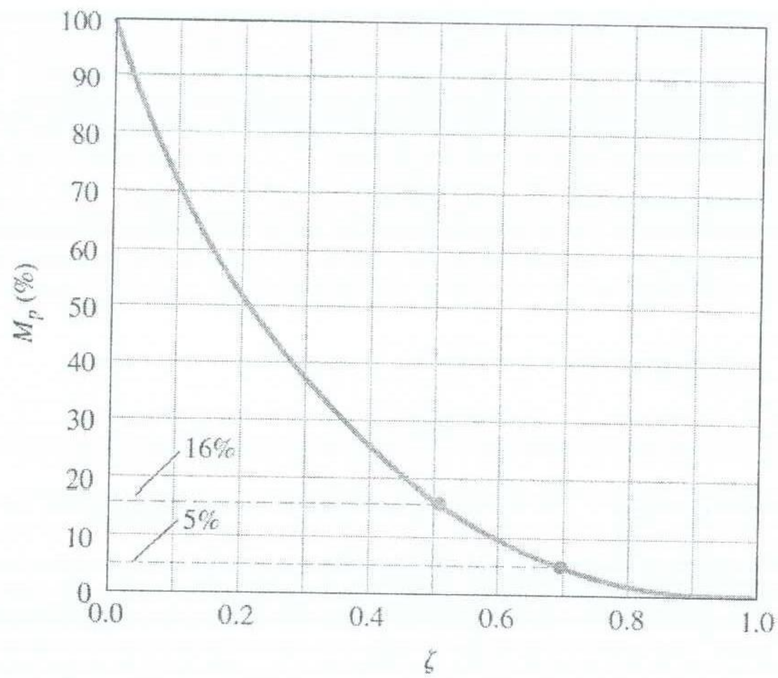

Figure 1: Maximum overshoot (as a percentage) versus damping ratio for the underdamped second order system.

Table of Laplace Transforms

Number	$F(s)$	$f(t), t \geq 0$
1	1	$\delta(t)$
2	$\frac{1}{s}$	$1(t)$
3	$\frac{1}{s^2}$	t
4	$\frac{2!}{s^3}$	t^2
5	$\frac{3!}{s^4}$	t^3
6	$\frac{m!}{s^{m+1}}$	t^m
7	$\frac{1}{(s+a)}$	e^{-at}
8	$\frac{1}{(s+a)^2}$	te^{-at}
9	$\frac{1}{(s+a)^3}$	$\frac{1}{2!}t^2e^{-at}$
10	$\frac{1}{(s+a)^m}$	$\frac{1}{(m-1)!}t^{m-1}e^{-at}$
11	$\frac{a}{s(s+a)}$	$1-e^{-at}$
12	$\frac{a}{s^2(s+a)}$	$\frac{1}{a}(at-1+e^{-at})$
13	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$
14	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$
15	$\frac{a^2}{s(s+a)^2}$	$1-e^{-at}(1+at)$
16	$\frac{(b-a)s}{(s+a)(s+b)}$	$be^{-at}-ae^{-at}$
17	$\frac{a}{(s^2+a^2)}$	$\sin at$
18	$\frac{s}{(s^2+a^2)}$	$\cos at$
19	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos bt$
20	$\frac{b}{(s+a)^2+b^2}$	$e^{-at}\sin bt$
21	$\frac{a^2+b^2}{s\left[(s+a)^2+b^2\right]}$	$1-e^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$