Forward Kinematics.

Lecture 11 - 26/11/2024

IA 3205 – Introduction to Robotics

U.V.Hiran Sameera
Lecturer (Probationary)
Department Of Instrumentation And Automation Technology
Faculty of Technology
University of Colombo

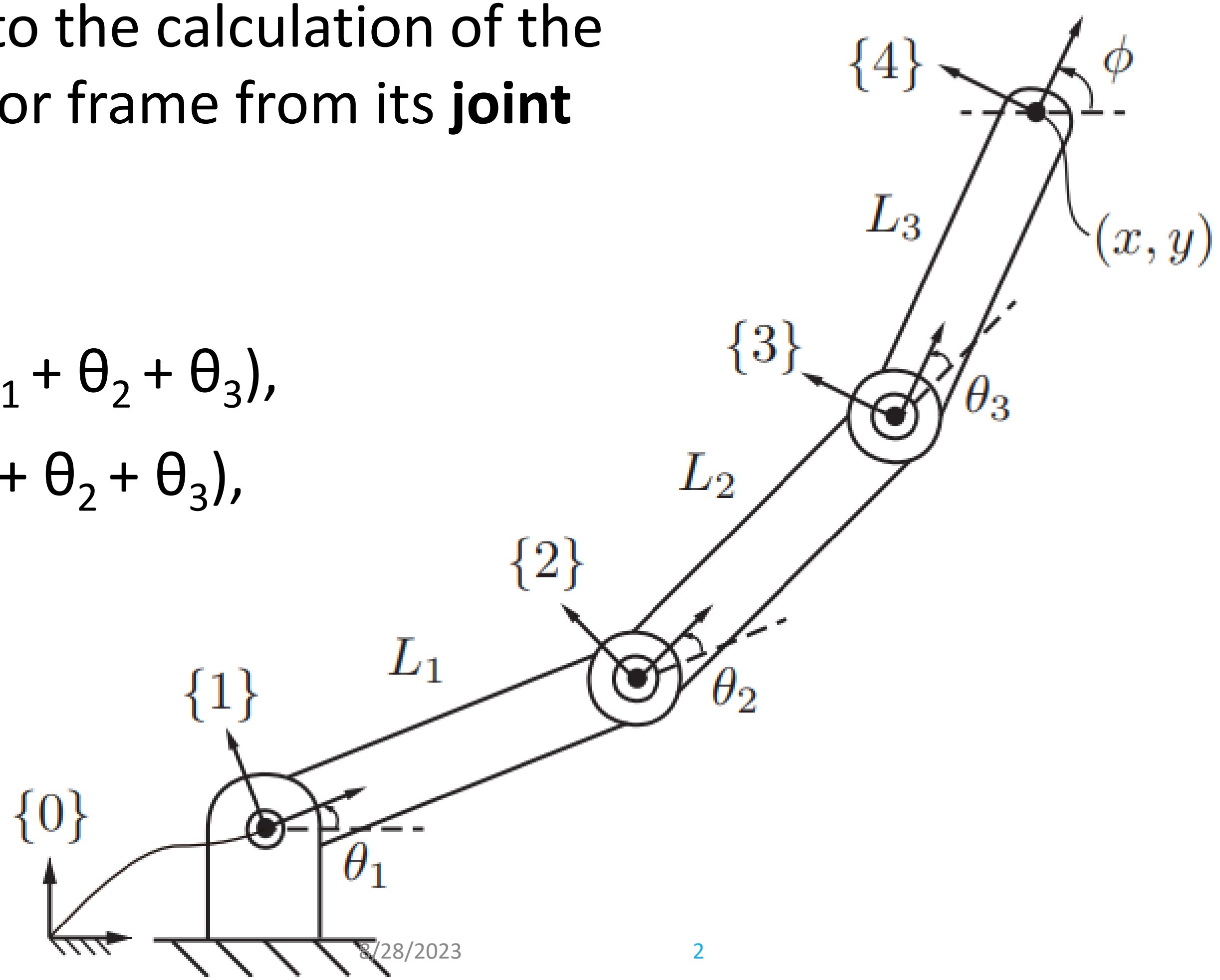
Forward Kinematics

- The forward kinematics of a robot refers to the calculation of the **position and orientation** of its end-effector frame from its **joint coordinates θ.**

$x = L_1 \cos \theta_1 + L_2 \cos(\theta_1 + \theta_2) + L_3 \cos(\theta_1 + \theta_2 + \theta_3),$

$y = L_1 \sin \theta_1 + L_2 \sin(\theta_1 + \theta_2) + L_3 \sin(\theta_1 + \theta_2 + \theta_3),$

$\phi = \theta_1 + \theta_2 + \theta_3$



Forward Kinematics

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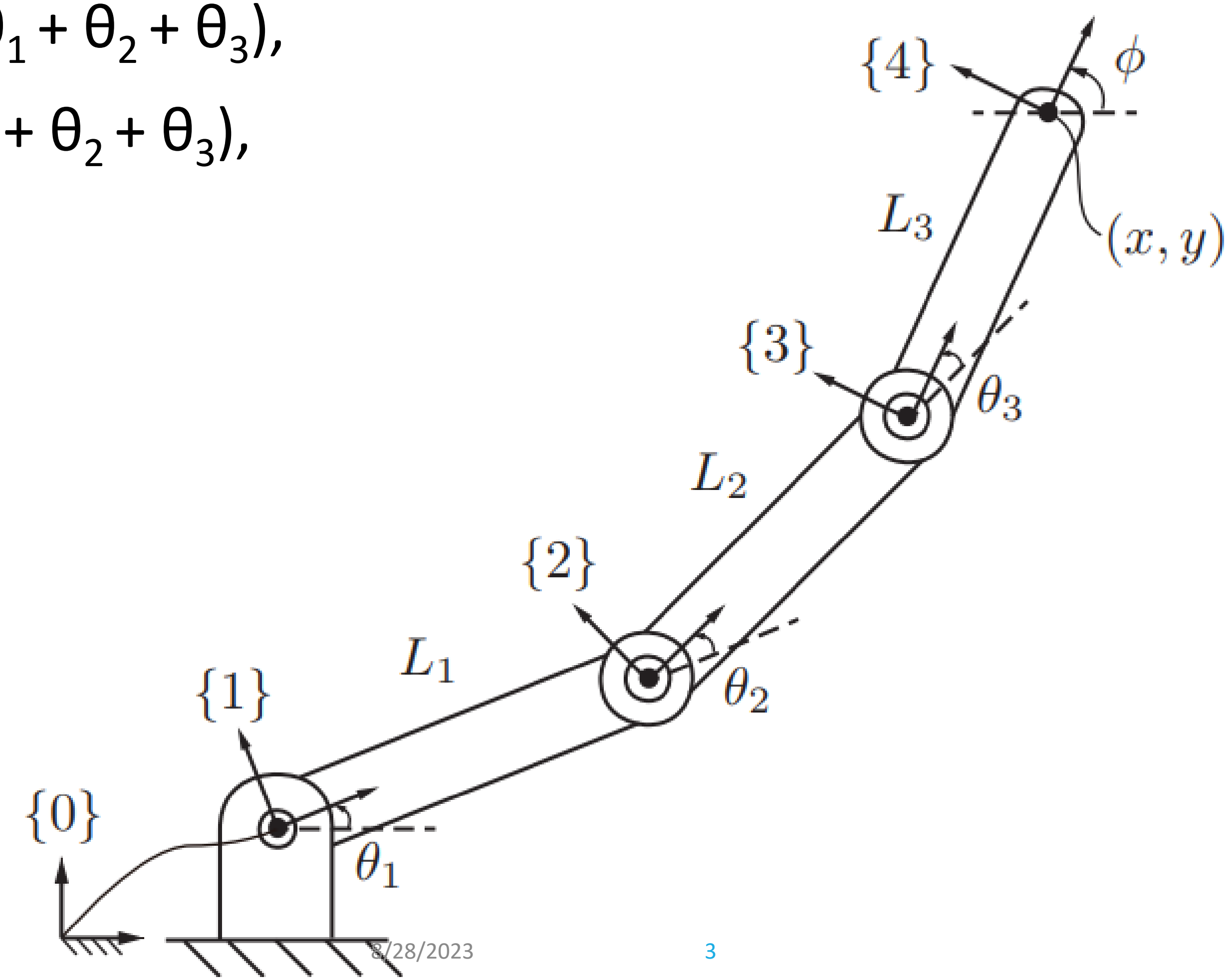
$$y = L_1 \sin \theta_1 + L_2 \sin(\theta_1 + \theta_2) + L_3 \sin(\theta_1 + \theta_2 + \theta_3),$$

$$\phi = \theta_1 + \theta_2 + \theta_3$$

- Derivation of forward kinematics can become complicated for 3D robot mechanisms.
- Therefore, a more convenient method involving transformation matrices is used.



Forward Kinematics

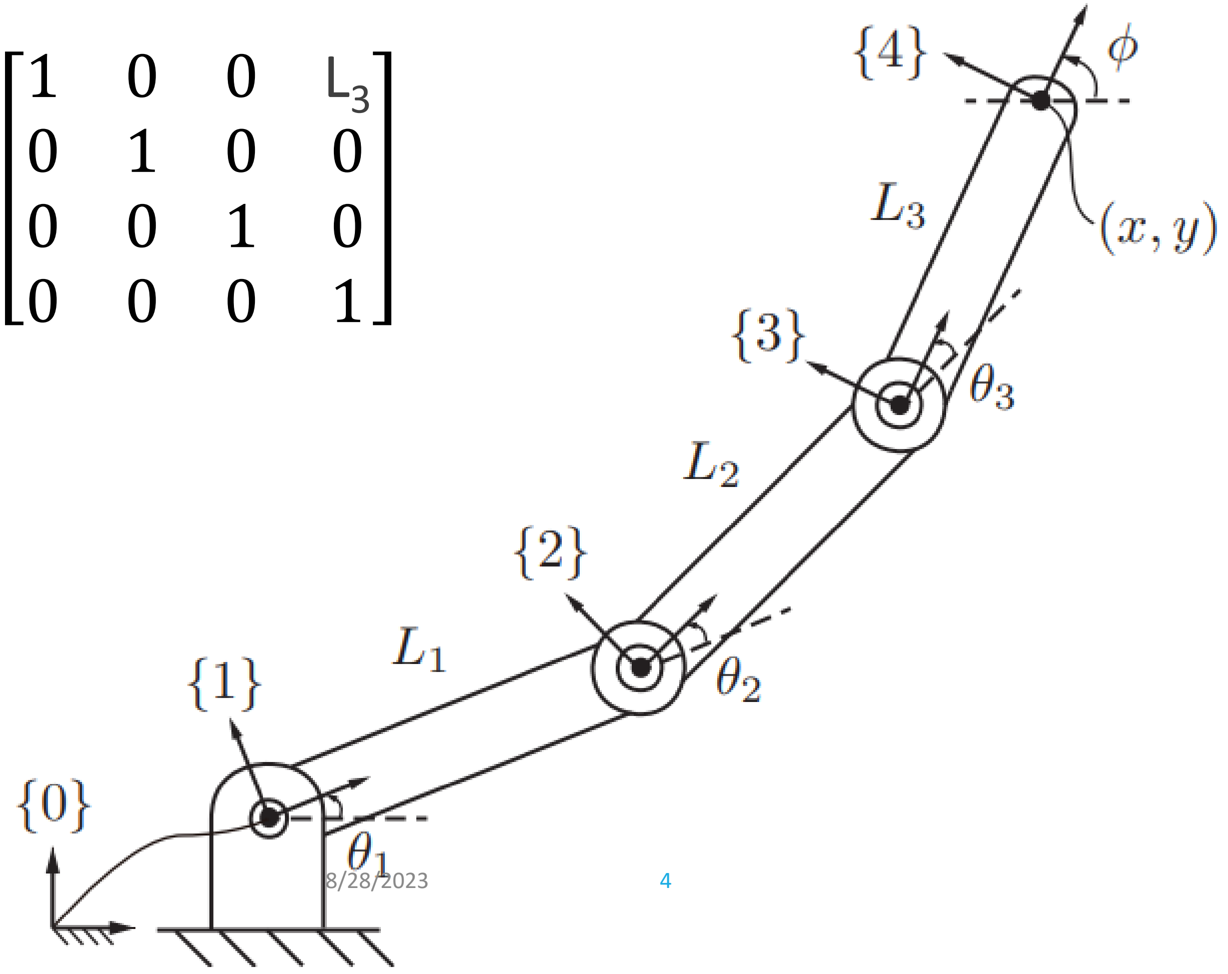
- Forward kinematics using transformation matrices.

$$[{}^{1}T_{0}] = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[{}^{4}T_{3}] = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[{}^{2}T_{1}] = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & L1 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

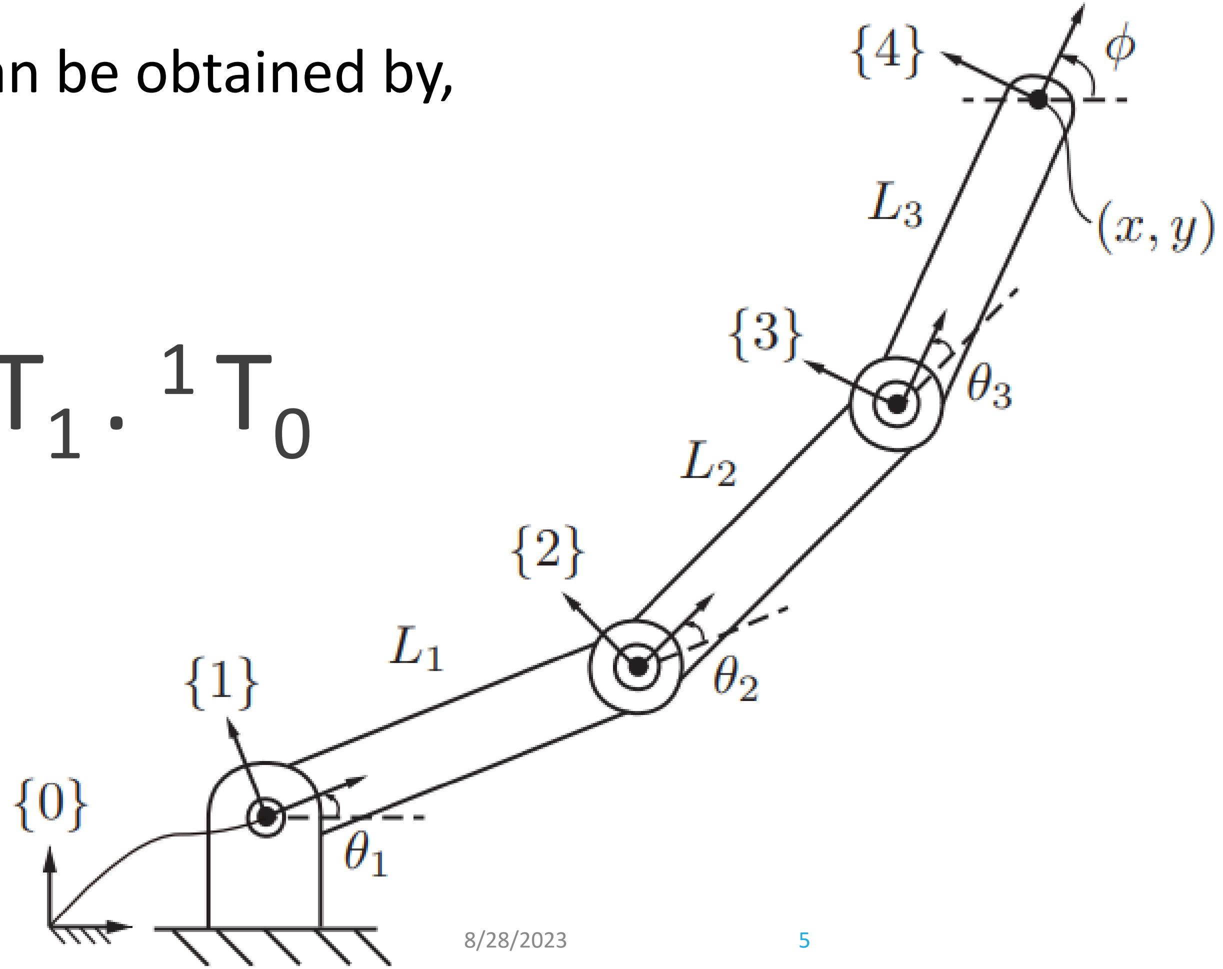
$$[{}^{3}T_{2}] = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & L2 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Forward Kinematics

- Then, final forward kinematics can be obtained by,

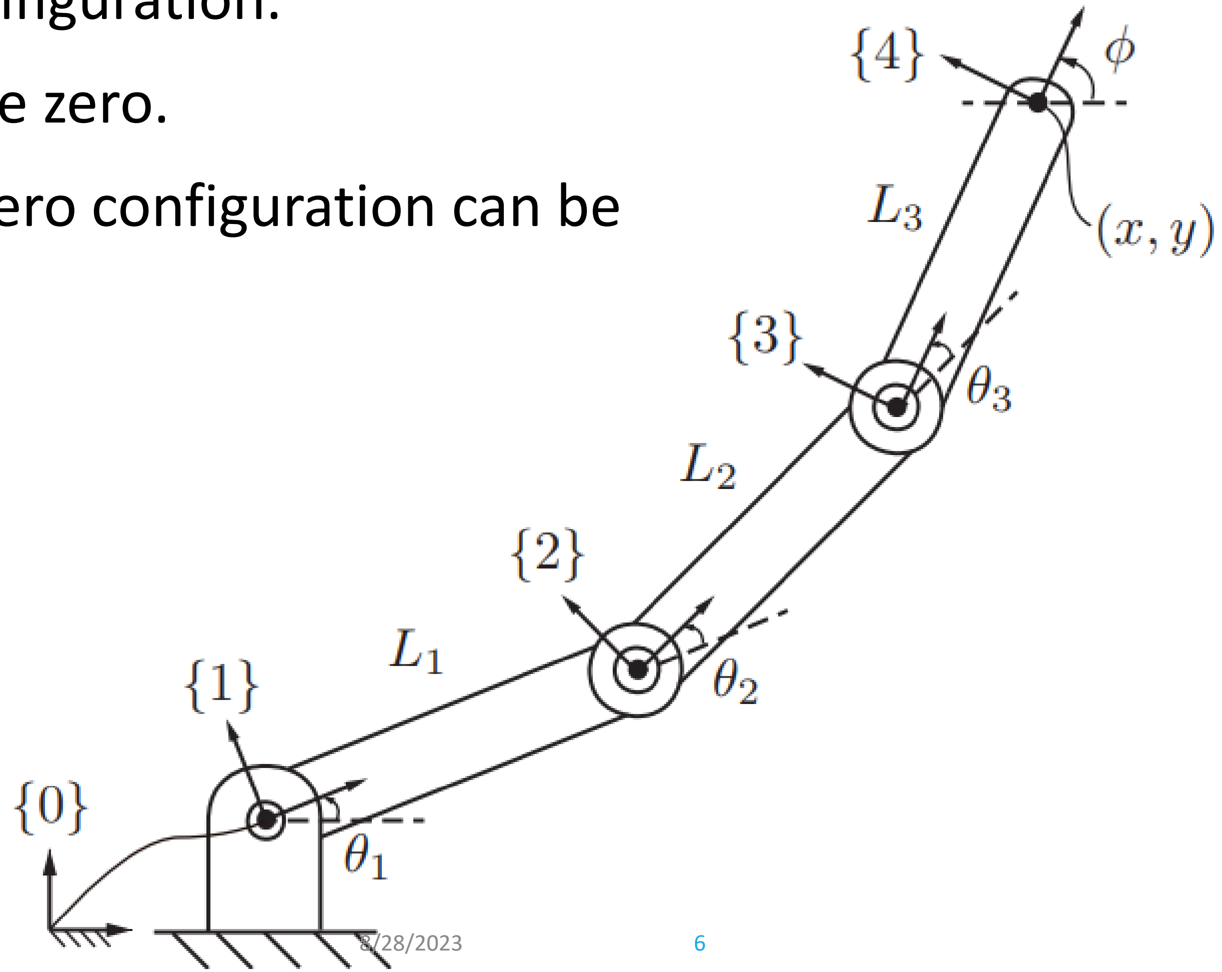
$$^{4}T_{0} = {}^{4}T_{3} \cdot {}^{3}T_{2} \cdot {}^{2}T_{1} \cdot {}^{1}T_{0}$$



Forward Kinematics

- For every robot, there is a **zero/home** configuration.
- For this robot it is when all joint angles are zero.
- End effector position and orientation at zero configuration can be expressed using a transformation matrix.

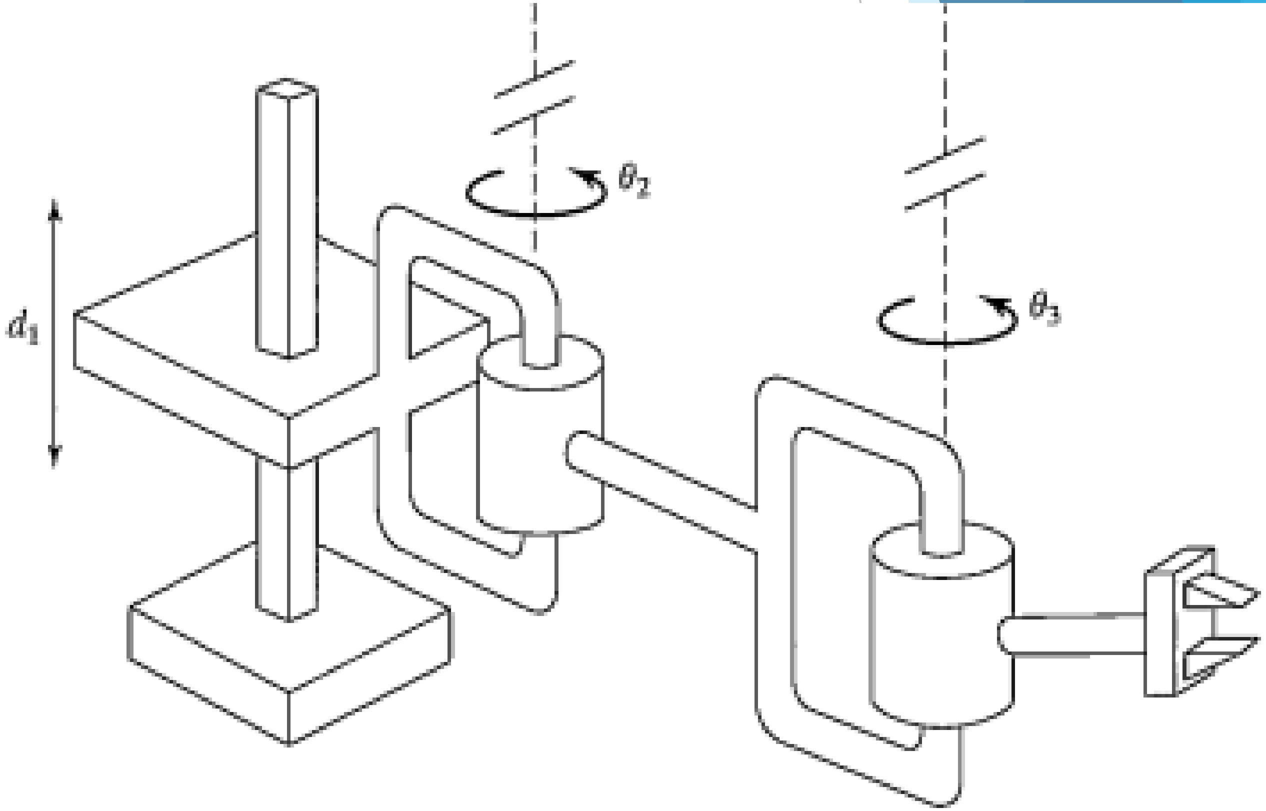
$$[H] = \begin{bmatrix} 1 & 0 & 0 & L_1 + L_2 + L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Forward VS Inverse Kinematics

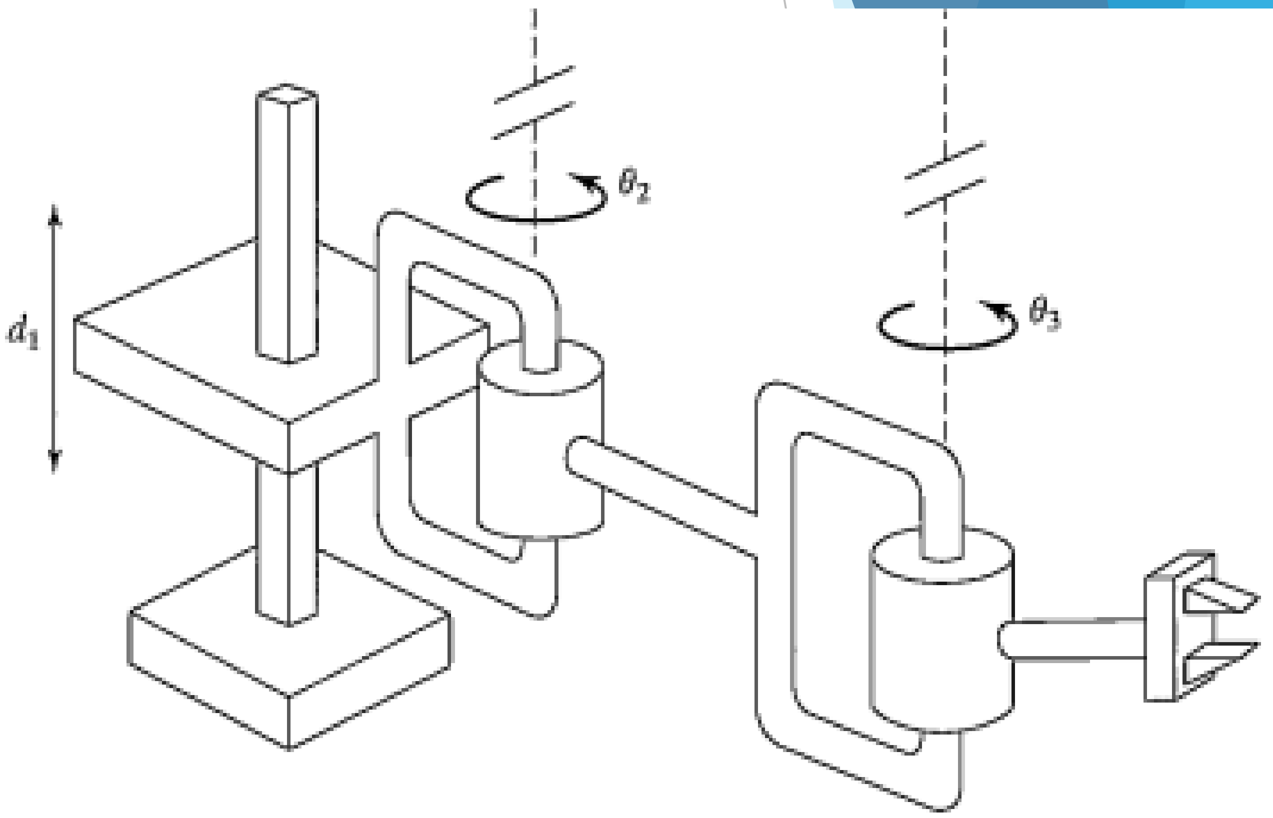
Assigning frames to a robot manipulator

- General Purpose robot should have 6 DOFs.
  - 6 < Redundant
  - 6 > Deficient
- Consider the following robot arm.



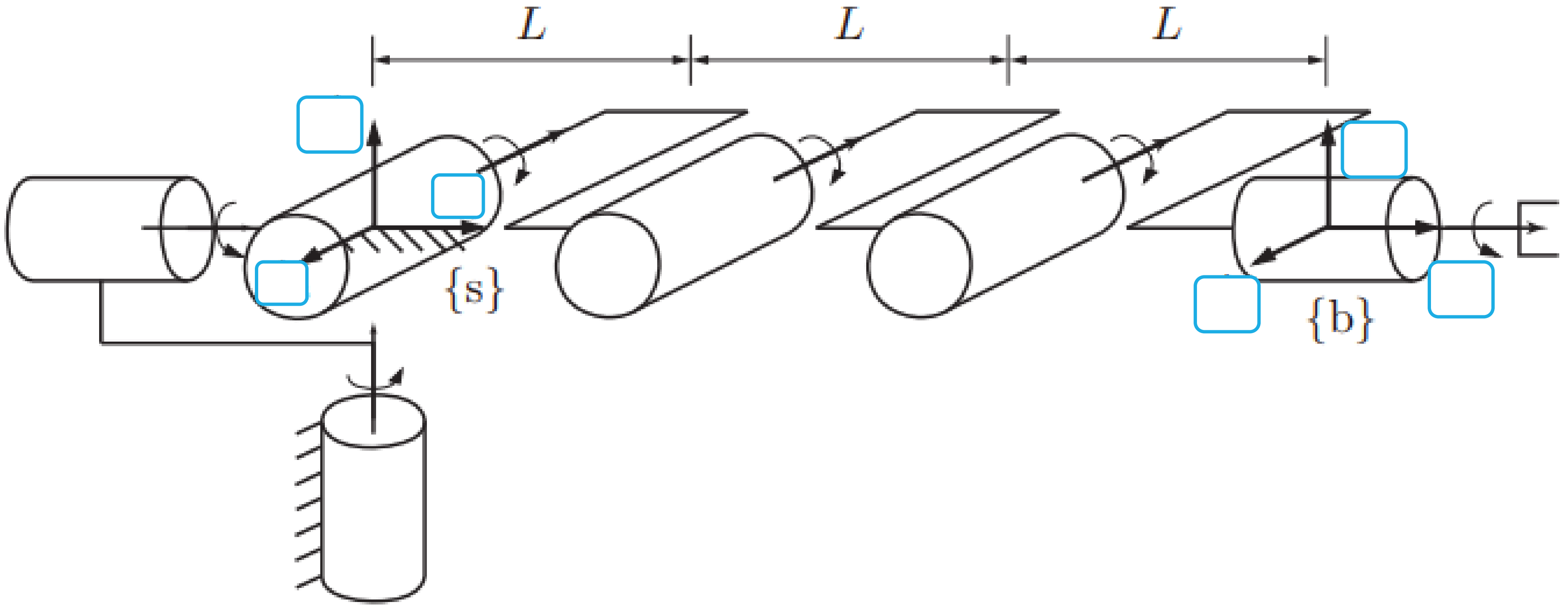
Conventions for assigning frames.

1. Identify the joints and axes and imagine (draw) a line along its axis of movement.
2. Assign the Zi axis pointing along the joint axis. (using right hand thumb rule)
3. Set the origin of the frame of the ith axis, at the
   a) Point of intersection between joint axes (I and i+1)
   b) Start of the common perpendicular between the joint axes (I and i+1)
4. Assign Xi (X-axis)
   a) Along the common perpendicular.
   b) If the axes intersect, Xi would be normal to the plane containing the two axes.
5. Assign the Yi axis to complete a left-handed coordinate system. (this is not important when we use DH parameters).
6. Assign frame {0} to match frame{1} when the first joint variable is Zero.



Practice Problem

- Obtain the transformation matrix for forward kinematics



Practice Problem

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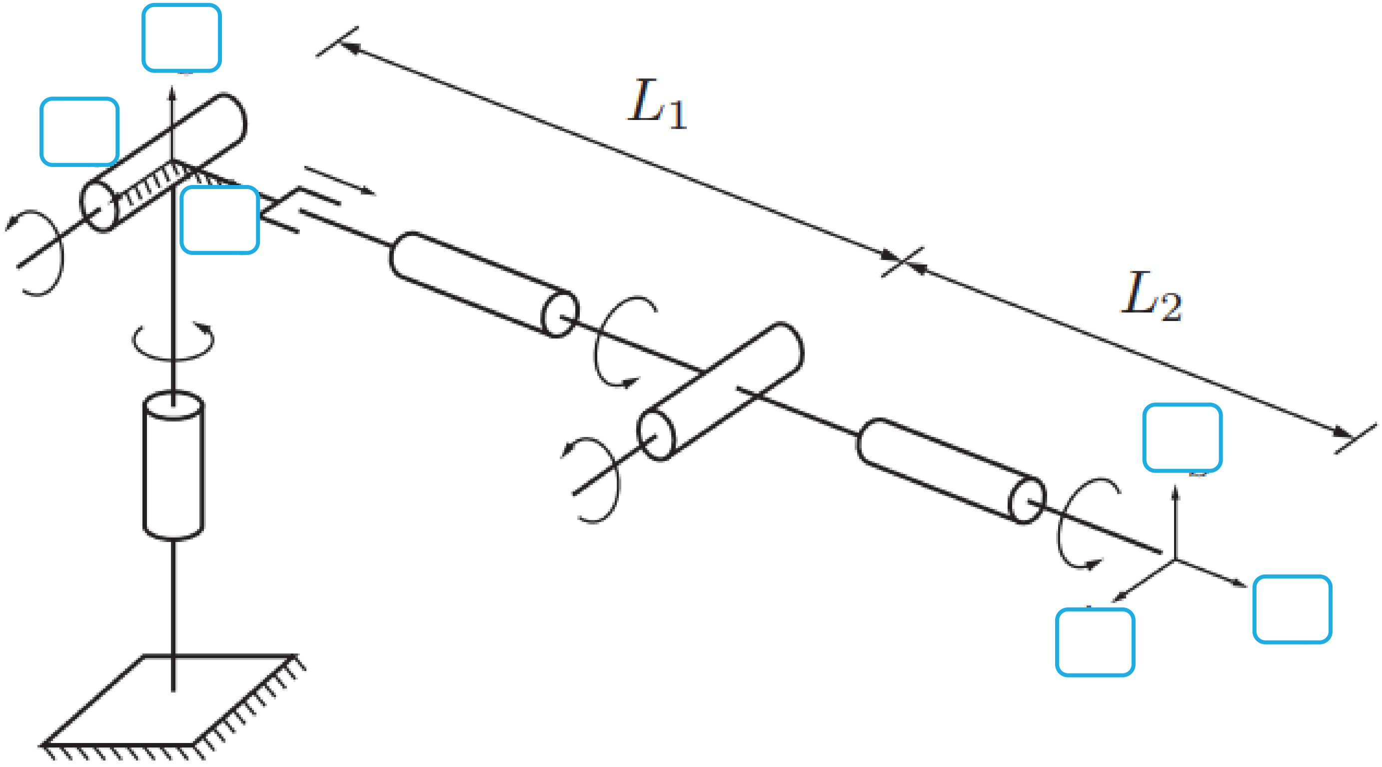
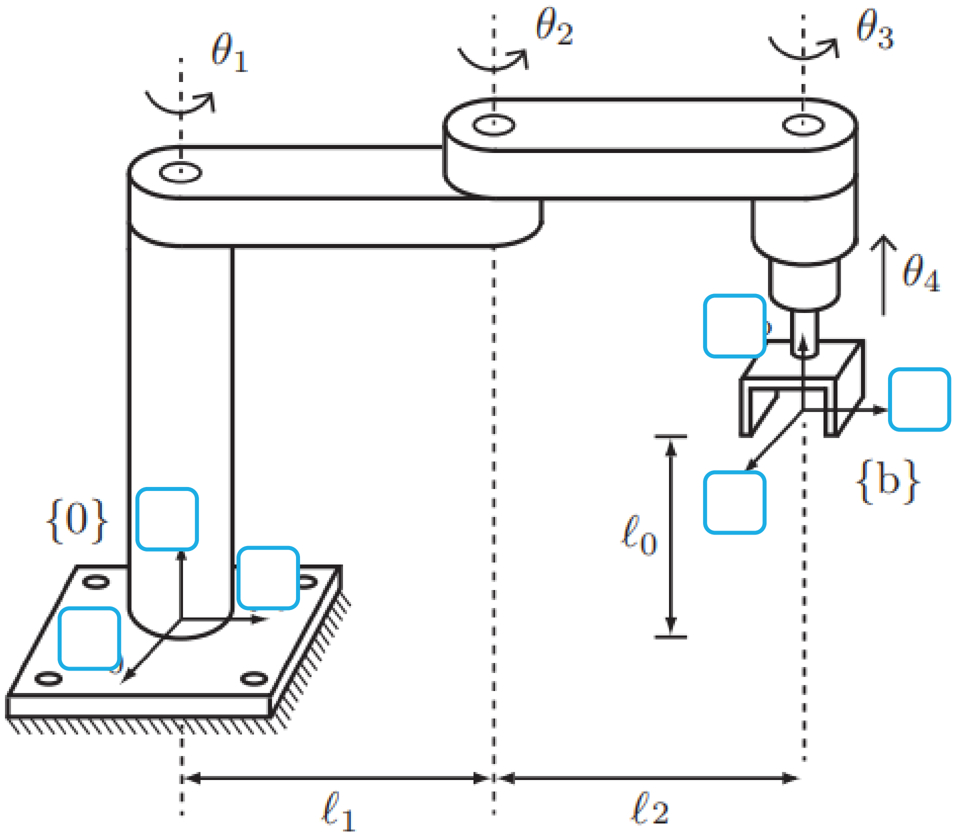
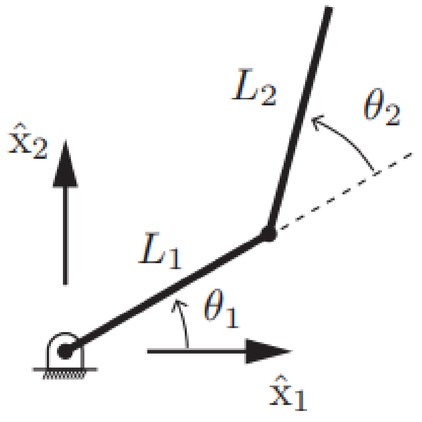


Practice Problem

- Obtain the transformation matrix for forward kinematics



Velocity Kinematics



- Consider the 2-limb robot shown,
- Forward Kinematics

$$
\begin{aligned}
x_1 &= L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2) \\
x_2 &= L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2).
\end{aligned}
$$

- Forward velocity kinematics can be derived by taking time derivative

$$x(t) = f(\theta(t))$$

$$
\begin{aligned}
\dot{x} &= \frac{\partial f(\theta)}{\partial \theta}\frac{d\theta(t)}{dt} = \frac{\partial f(\theta)}{\partial \theta}\dot{\theta} \\
&= J(\theta)\dot{\theta},
\end{aligned}
$$

$$
\begin{aligned}
\dot{x}_1 &= -L_1\dot{\theta}_1 \sin\theta_1 - L_2(\dot{\theta}_1 + \dot{\theta}_2)\sin(\theta_1 + \theta_2) \\
\dot{x}_2 &= L_1\dot{\theta}_1 \cos\theta_1 + L_2(\dot{\theta}_1 + \dot{\theta}_2)\cos(\theta_1 + \theta_2),
\end{aligned}
$$

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -L_1 \sin\theta_1 - L_2 \sin(\theta_1 + \theta_2) & -L_2 \sin(\theta_1 + \theta_2) \\ L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2) & L_2 \cos(\theta_1 + \theta_2) \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}
$$