ACHARYA NARENDRA DEV COLLEGE

B.Sc. (H) Computer Science
Semester – IIIrd
2022

GE 3 :Differential Equations Practical

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Problems

1. Plotting of First Order Solution family of Differential Equation

Example 1:

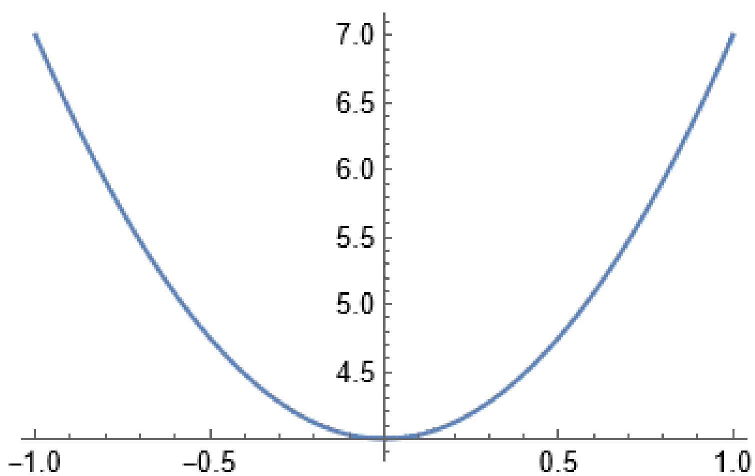
```
In[•]:= DSolve[{Y'[X] == X*6}, Y[X], X]
        Y[X] /. DSolve[{Y'[X] == X*6}, Y[X], X]
        sol = DSolve[{Y'[X] == X*6, Y[0] == 4}, Y[X], X]
        Plot[Y[X] /. sol, {X, -1, 1}]
```

Out[•]= $\{\{Y[X] \to 3X^2 + c_1\}\}$

Out[•]= $\{3X^2 + c_1\}$

Out[•]= $\{\{Y[X] \to 4 + 3X^2\}\}$

Out[•]=



Example 2:

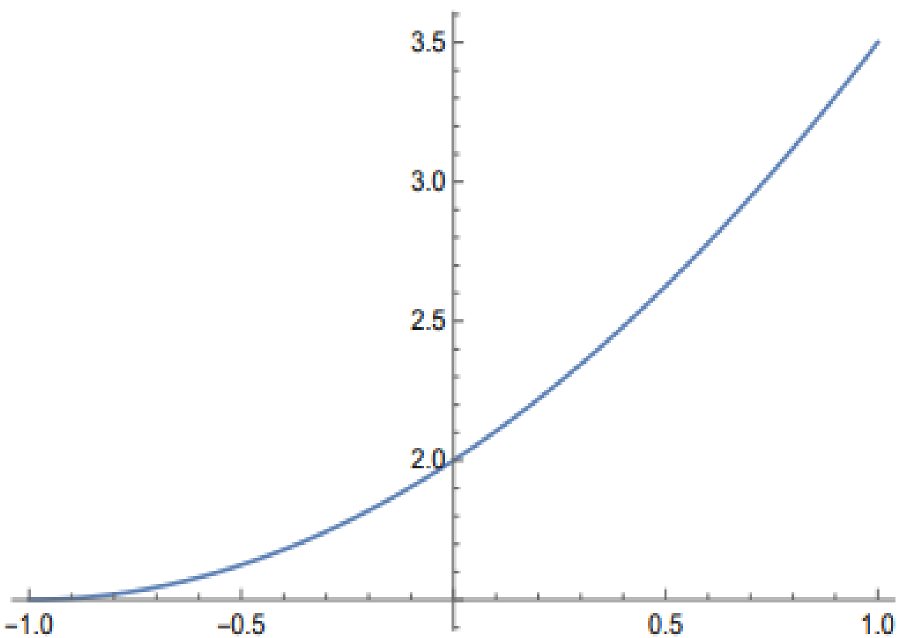
```
In[ ]:= DSolve[{Y'[X] == X + 1}, Y[X], X]
        Y[X] /. DSolve[{Y'[X] == X + 1}, Y[X], X]
        sol = DSolve[{Y'[X] == X + 1, Y[0] == 2}, Y[X], X]
        Plot[Y[X] /. sol, {X, -1, 1}]
```

Out[ ]= $\left\{\left\{Y[X] \to X + \frac{X^2}{2} + c_1\right\}\right\}$

Out[ ]= $\left\{X + \frac{X^2}{2} + c_1\right\}$

Out[ ]= $\left\{\left\{Y[X] \to \frac{1}{2}\left(4 + 2X + X^2\right)\right\}\right\}$

Out[ ]=

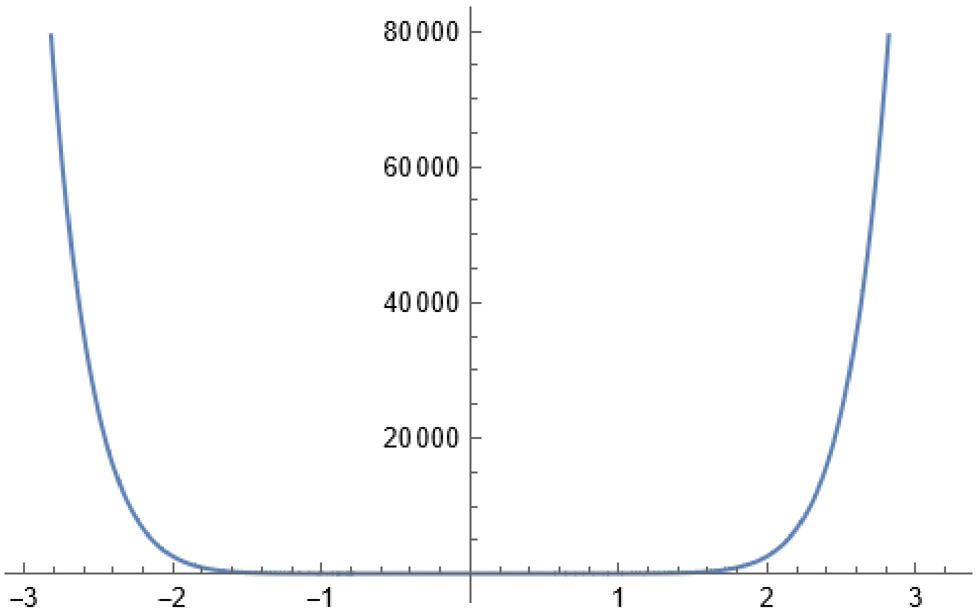


Example 3:

In[ ]:=
```
eq = Y'[X] - 25 X^9;
s = DSolve[{eq == 6}, Y[X], X];
s1 = s /. {C[1] → 12}
Plot[Y[X] /. s1, {X, -3, 3.25}]
```

Out[ ]= $\left\{\left\{Y[X] \to 12 + 6 X + \frac{5 X^{10}}{2}\right\}\right\}$

Out[ ]=



2. Plotting of Second Order Solution family of Differential Equation

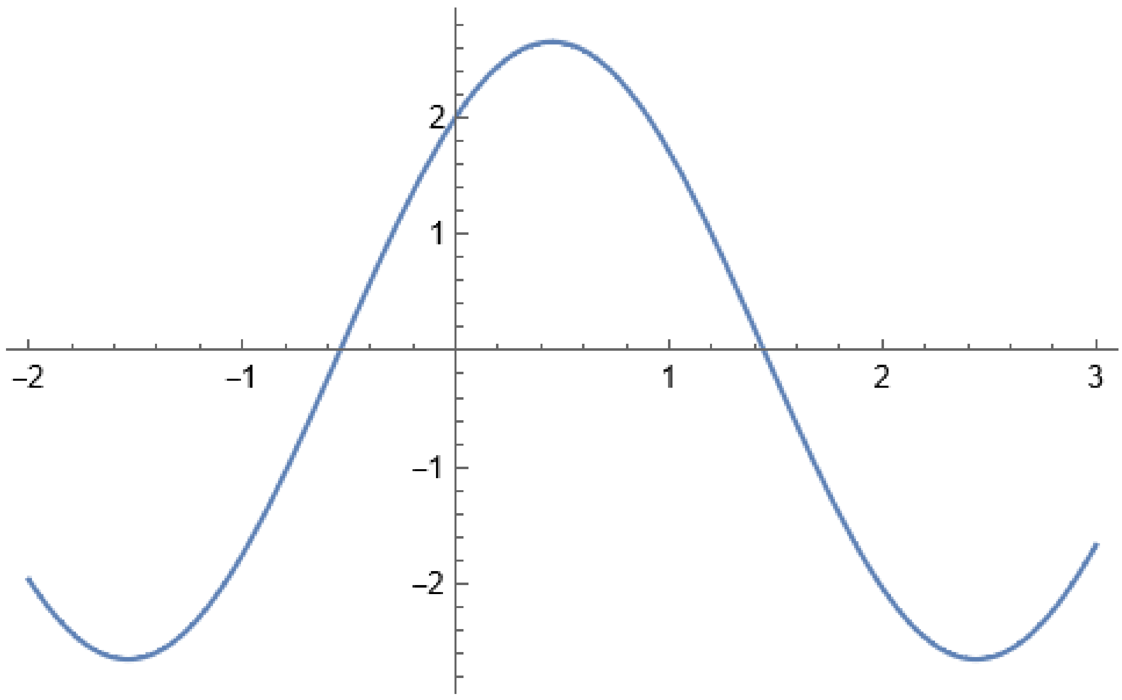
Example 4:

```
In[]:= eqn1 := 2*Y''[X] + 5*Y[X];
       ab = DSolve[eqn1 == 0, Y[X], X]
       a1 = ab /. {C[1] → 2, C[2] → √3}
       Plot[Y[X] /. a1, {X, -2, 3}]
```

Out[]= $\left\{\left\{Y[X] \to c_1 \mathrm{Cos}\left[\sqrt{\frac{5}{2}}\, X\right] + c_2 \mathrm{Sin}\left[\sqrt{\frac{5}{2}}\, X\right]\right\}\right\}$

Out[]= $\left\{\left\{Y[X] \to 2\,\mathrm{Cos}\left[\sqrt{\frac{5}{2}}\, X\right] + \sqrt{3}\,\mathrm{Sin}\left[\sqrt{\frac{5}{2}}\, X\right]\right\}\right\}$

Out[]=



Example 5:

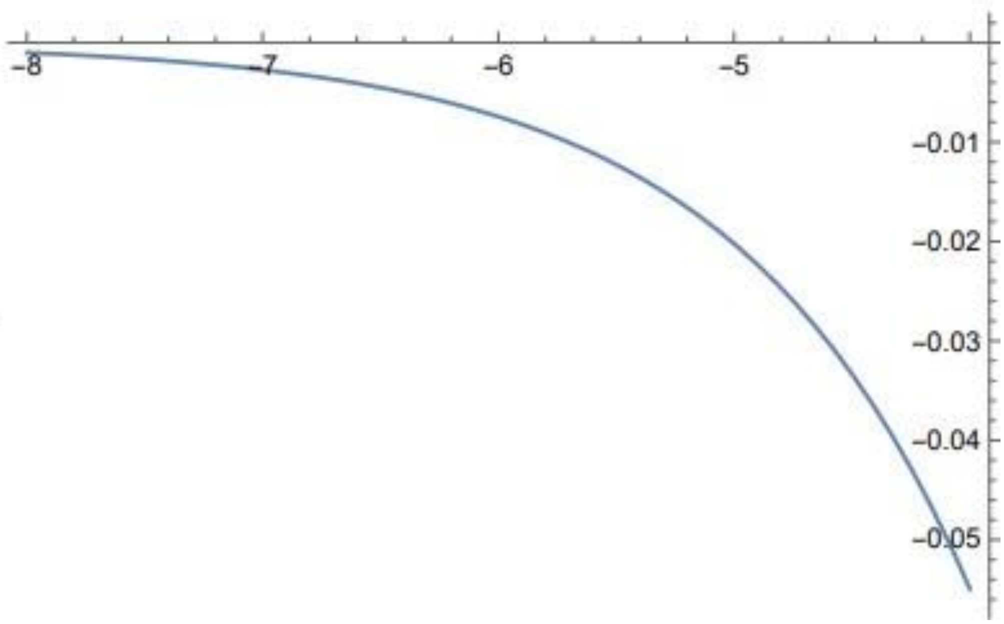
```
In[ ]:= eqn = Y''[X] - 5*Y'[X] + 4*Y[X];
        s = DSolve[eqn == 0, Y[X], X]
        Y[X] /. s
        s1 = s /. {C[1] → -3, C[2] → 7}
        Plot[Y[X] /. s1, {X, -4, -8}]
```

Out[ ]= $\{\{Y[X] \to e^{X} c_1 + e^{4X} c_2\}\}$

Out[ ]= $\{e^{X} c_1 + e^{4X} c_2\}$

Out[ ]= $\{\{Y[X] \to -3 e^{X} + 7 e^{4X}\}\}$

Out[ ]=



Example 6:

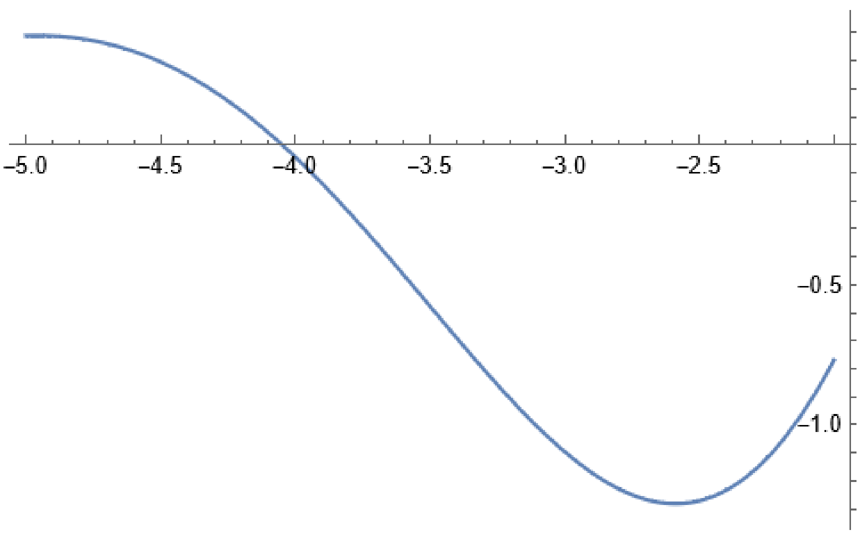
In[∘]:=
```
eqn = Y''[X] - Y'[X] + 2*Y[X];
s = DSolve[eqn == 0, Y[X], X]
Y[X] /. s
s1 = s /. {C[1] → -3, C[2] → 4}
Plot[Y[X] /. s1, {X, -2, -5}]
```

Out[∘]= $\left\{\left\{Y[X] \to e^{X/2} c_2 \operatorname{Cos}\left[\frac{\sqrt{7}\, X}{2}\right] + e^{X/2} c_1 \operatorname{Sin}\left[\frac{\sqrt{7}\, X}{2}\right]\right\}\right\}$

Out[∘]= $\left\{e^{X/2} c_2 \operatorname{Cos}\left[\frac{\sqrt{7}\, X}{2}\right] + e^{X/2} c_1 \operatorname{Sin}\left[\frac{\sqrt{7}\, X}{2}\right]\right\}$

Out[∘]= $\left\{\left\{Y[X] \to 4\, e^{X/2} \operatorname{Cos}\left[\frac{\sqrt{7}\, X}{2}\right] - 3\, e^{X/2} \operatorname{Sin}\left[\frac{\sqrt{7}\, X}{2}\right]\right\}\right\}$

Out[∘]=



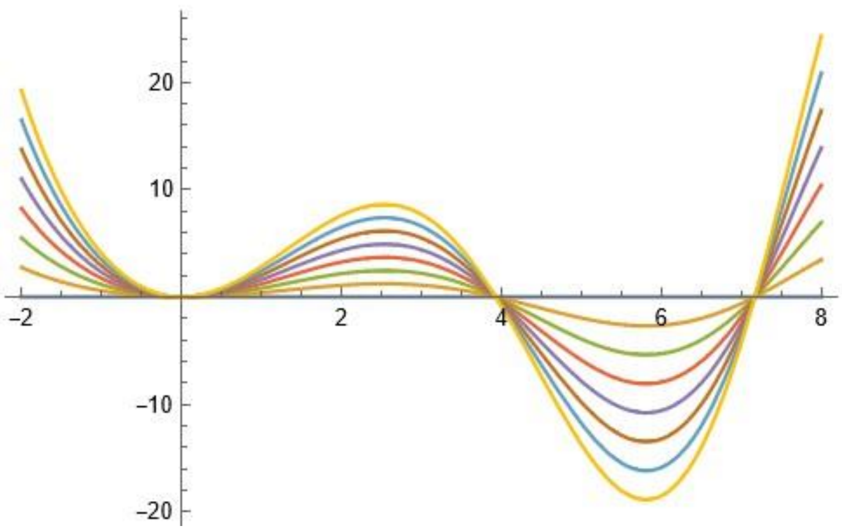
3. Plotting of Third Order Solution family of Differential Equation

Example 7:

In[ ]:=
```
e1 := 2 * Y'''[X] + Y''[X] + Y'[X] + 2 * Y[X];
S = DSolve[{e1 == 0, Y[0] == 0, Y'[0] == 0, Y''[0] == A}, Y[X], X]
Plot[Evaluate[Y[X] /. S /. A → Range[0, 7]], {X, -2, 8}]
```

Out[ ]=
$$\left\{\left\{Y[X] \to -\frac{2}{15} A e^{-X} \left(-3 + 3 e^{5X/4} \mathrm{Cos}\left[\frac{\sqrt{15}\, X}{4}\right] - \sqrt{15}\, e^{5X/4} \mathrm{Sin}\left[\frac{\sqrt{15}\, X}{4}\right]\right)\right\}\right\}$$

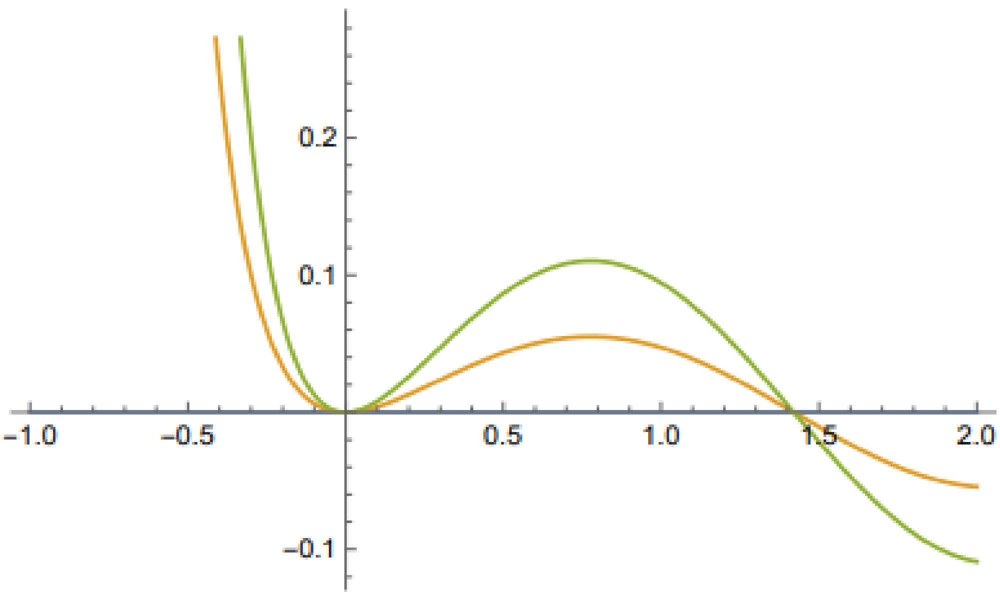
Out[ ]=



Example 8:

In[ ]:=
```
eqn := Y'''[X] + 7*Y''[X] + 6*Y'[X] + 42*Y[X];
s = DSolve[{eqn == 0, Y[0] == 0, Y'[0] == 0, Y''[0] == A},
  Y[X], X]
Plot[Evaluate[Y[X] /. s /. A → Range[0, 2]], {X, -1, 2}]
```

Out[ ]= $\left\{\left\{Y[X] \to -\frac{1}{330} A e^{-7X} \left(-6 + 6 e^{7X} \text{Cos}\left[\sqrt{6}\, X\right] - 7\sqrt{6}\, e^{7X} \text{Sin}\left[\sqrt{6}\, X\right]\right)\right\}\right\}$

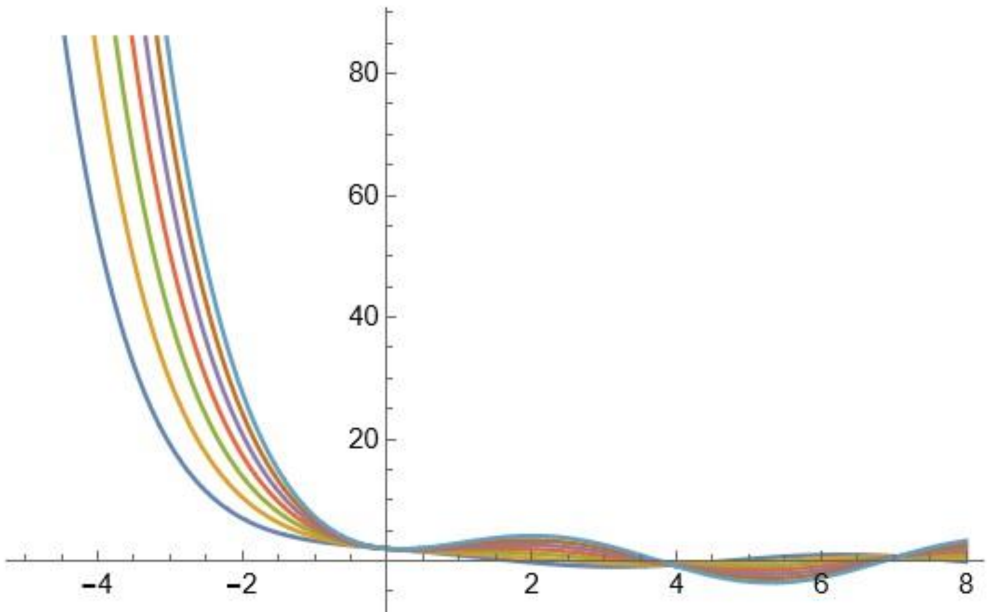
Out[ ]=


Example 9:

```
In[•]:= eqn := Y'''[X] + Y''[X] + Y'[X] + Y[X];
        s = DSolve[{eqn == 0, Y[0] == 2, Y'[0] == -1, Y''[0] == A}, Y[X], X]
        Plot[Evaluate[Y[X] /. s /. A → Range[0, 6]], {X, -5, 8}]
```

Out[•]= $\left\{\left\{Y[X] \to -\frac{1}{2} e^{-X} \left(-2 - A - 2 e^{X} \text{Cos}[X] + A e^{X} \text{Cos}[X] - A e^{X} \text{Sin}[X]\right)\right\}\right\}$

Out[•]=



4. Solution of Differential Equation by Variation of Parameter Method

Example 10:

```
In[ ]:= homsol = DSolve[{y''[x] + 3*y'[x] + 2*y[x] - ℯ^(2*x)*30 == 0},
          y[x], x]
        y1[x_] = ℯ^-x;
        y2[x_] = ℯ^(-2*x);
        caps = {y1[x], y2[x]};
        ws = Simplify[Det[{caps, ∂x caps}]]
        f[x_] = 30*ℯ^(2*x);
        u1prime = -y2[x]*f[x] / ws;
        u2prime = y1[x]*f[x] / ws;
        u1[x_] = ∫u1prime dx;
        u2[x_] = ∫u2prime dx;
        yp[x_] = y1[x]*u1[x] + y2[x]*u2[x] // Simplify
```

Out[ ]= $\{\{y[x] \to e^{-2x} c_1 + e^{-x} c_2 + \left(15\, e^{-2x} \left(-e^{x\,(2+2\,\mathrm{Log}[e])} + 2\, e^{x+x\,(1+2\,\mathrm{Log}[e])} - 2\, e^{x\,(2+2\,\mathrm{Log}[e])}\,\mathrm{Log}[e] + 2\, e^{x+x\,(1+2\,\mathrm{Log}[e])}\,\mathrm{Log}[e]\right)\right) / \left((1+\mathrm{Log}[e])\,(1+2\,\mathrm{Log}[e])\right)\}\}$

Out[ ]= $-e^{-3x}\,\mathrm{Log}[e]$

Out[ ]= $\dfrac{5\, e^{2x}}{2\,\mathrm{Log}[e]^2}$

Example 11:

```
In[•]:= sol = DSolve[Y''[X] - 2*Y'[X] + Y[X] == ⅇ^X*Sin[X], Y[X], X]
        Y1[X_] = ⅇ^X;
        Y2[X_] = X*ⅇ^X;
        CA = {Y1[X], Y2[X]};
        WS = Simplify[Det[{CA, ∂_X CA}]]
        F[X_] = ⅇ^X Sin[X];
        U1P = -Y2[X]*F[X]/WS
        U2P = Y1[X]*F[X]/WS
        U1[X_] = ∫U1P ⅆX;
        U2[X_] = ∫U2P ⅆX; YP[X_] = Y1[X]*U1[X] + Y2[X]*U2[X] // Simplify
```

Out[•]= $\{\{Y[X] \to e^X c_1 + e^X X c_2 - e^X \mathrm{Sin}[X]\}\}$

Out[•]= e^{2X}

Out[•]= $-X\,\mathrm{Sin}[X]$

Out[•]= $\mathrm{Sin}[X]$

Out[•]= $-e^X\,\mathrm{Sin}[X]$

Example 12:

```
In[*]:= new = DSolve[X^2 * Y''[X] - X * Y'[X] + 5 * Y[X] == X, Y[X], X]
        Y1[X_] = X * Cos[2 Log[X]];
        Y2[X_] = X * Sin[2 Log[X]];
        CA = {Y1[X], Y2[X]};
        WS = Simplify[Det[{CA, ∂X CA}]]
        F[X_] = 1 / X;
        U1P = -Y2[X] * F[X] / WS
        U2P = Y1[X] * F[X] / WS
        U1[X_] = ∫U1P dX;
        U2[X_] = ∫U2P dX;
        YP[X_] = Y1[X] * U1[X] + Y2[X] * U2[X] // Simplify
```

Out[*]= $\left\{\left\{Y[X] \to X\, c_2 \operatorname{Cos}[2 \operatorname{Log}[X]] + X\, c_1 \operatorname{Sin}[2 \operatorname{Log}[X]] + \frac{1}{4}\left(2\, X \operatorname{Cos}[\operatorname{Log}[X]]^2 \operatorname{Cos}[2 \operatorname{Log}[X]] + X \operatorname{Sin}[2 \operatorname{Log}[X]]^2\right)\right\}\right\}$

Out[*]= $2\, X$

Out[*]= $-\frac{\operatorname{Sin}[2 \operatorname{Log}[X]]}{2\, X}$

Out[*]= $\frac{\operatorname{Cos}[2 \operatorname{Log}[X]]}{2\, X}$

Out[*]= $\frac{1}{2} X \operatorname{Cos}[\operatorname{Log}[X]]^2$

5. Solution of system of Ordinary Differential Equation

Example 13:

```
In[ ]:= DSolve[{2*x'[t] == Sin[t] + Tan[t], 4*x'[t] == -6*y[t]}, {x, y}, t]
```

Out[ ]= $\left\{\left\{x \to \text{Function}\left[\{t\}, c_1 - \frac{\text{Cos}[t]}{2} - \frac{1}{2} \text{Log}[\text{Cos}[t]]\right], y \to \text{Function}\left[\{t\}, \frac{1}{3} (-\text{Sin}[t] - \text{Tan}[t])\right]\right\}\right\}$

Example 14 :

```
In[ ]:= DSolve[{y[x] == z'[x], z[x] == -3*y'[x]}, {y, z}, x]
```

Out[ ]= $\left\{\left\{y \to \text{Function}\left[\{x\}, c_1 \text{Cos}\left[\frac{x}{\sqrt{3}}\right] - \frac{c_2 \text{Sin}\left[\frac{x}{\sqrt{3}}\right]}{\sqrt{3}}\right], z \to \text{Function}\left[\{x\}, c_2 \text{Cos}\left[\frac{x}{\sqrt{3}}\right] + \sqrt{3}\, c_1 \text{Sin}\left[\frac{x}{\sqrt{3}}\right]\right]\right\}\right\}$

Example 15:

```
In[ ]:= DSolve[{Y[X] == 4*Z'[X], Z[X] == -Y'[X]}, {Y, Z}, X]
```

Out[ ]= $\left\{\left\{Y \to \text{Function}\left[\{X\}, c_1 \text{Cos}\left[\frac{X}{2}\right] - 2 c_2 \text{Sin}\left[\frac{X}{2}\right]\right], Z \to \text{Function}\left[\{X\}, c_2 \text{Cos}\left[\frac{X}{2}\right] + \frac{1}{2} c_1 \text{Sin}\left[\frac{X}{2}\right]\right]\right\}\right\}$

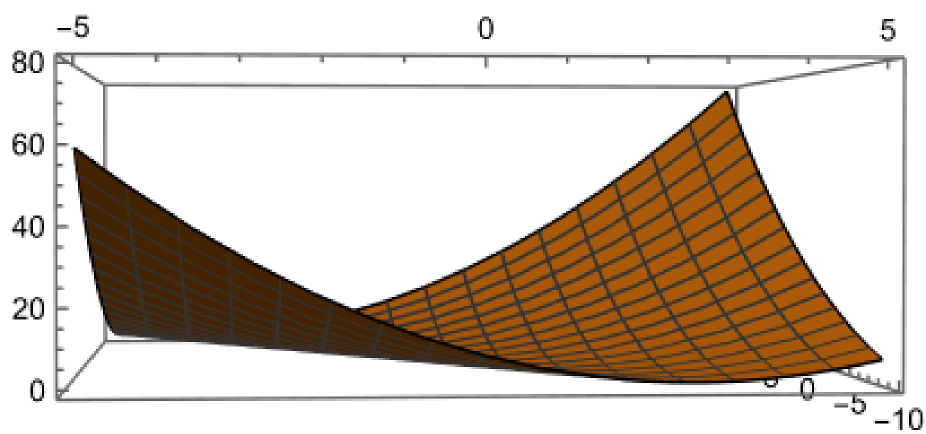
6. Solution of Cauchy problem for First Order Partial differential equation

Example 16:

```
In[•]:= eq4 = D[u[x, y], x] - 3 D[u[x, y], y] == 1 / 6;
        sol = DSolve[{eq4, u[x, -2] == x^2},
          u[x, y], {x, y}]
        Plot3D[sol[[1, 1, 2]], {x, -5, 5}, {y, -10, 10}]
```

Out[•]= $\left\{\left\{u[x, y] \to \frac{1}{18}\left(6 + 24\,x + 18\,x^2 + 7\,y + 12\,x\,y + 2\,y^2\right)\right\}\right\}$

Out[•]=

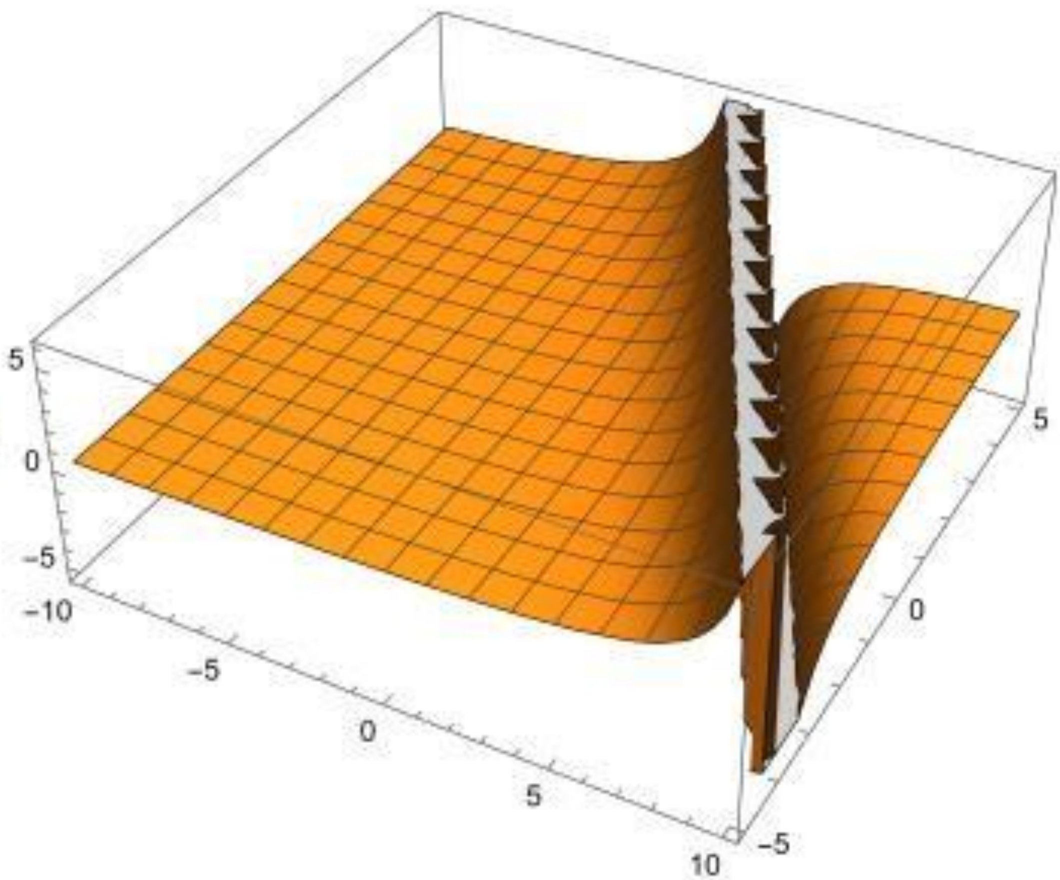


Example 17:

```
In[4]:= eq1 = D[2 * u[x, y], x] + D[4 * u[x, y], y] == u[x, y] * u[x, y];
Sol1 = DSolve[{eq1, u[x, -x] == 1}, u[x, y], {x, y}]
Plot3D[Sol1[ [1, 1, 2]], {x, -10, 10}, {y, -5, 5}]
```

Out[5]= $\left\{\left\{u[x, y] \rightarrow -\frac{6}{-6+x+y}\right\}\right\}$

Out[6]=



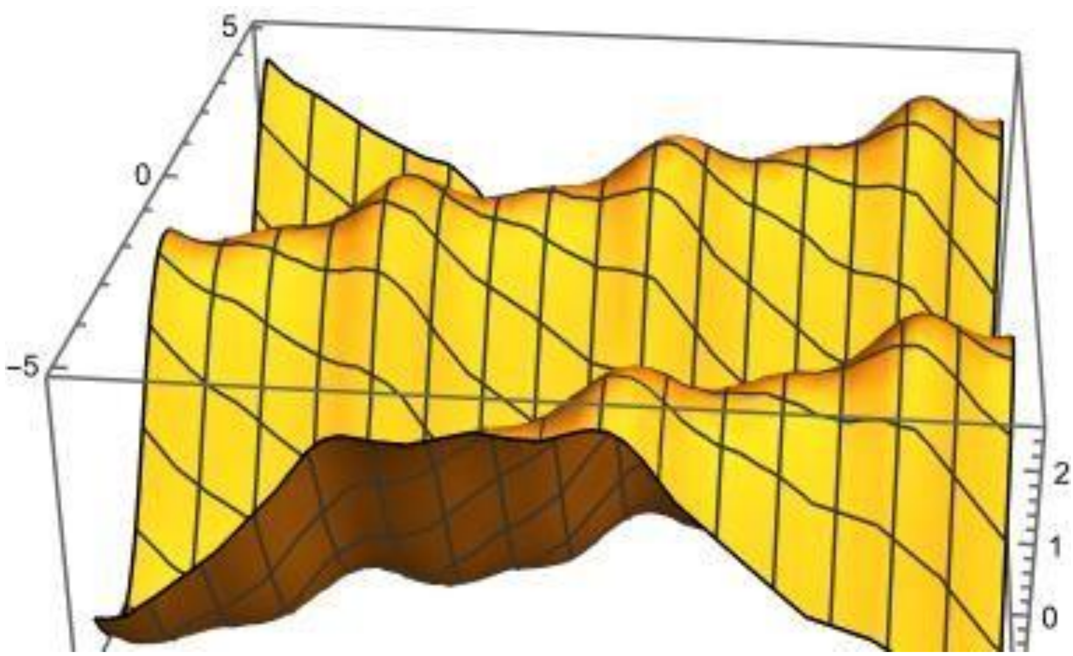
Example 18:

```
In[ ]:= sol1 = DSolve[{D[y[x, t], t] + 4 D[y[x, t], x] == Sin[2 x] + Cos[x], y[0, t] == 2 Sin[t]}, y[x, t], {x, t}]
        sol2 = sol1[1, 1, 2]
        sol2 /. {t → 1, x → 2}
        Plot3D[sol1[[1, 1, 2]], {x, -10, 10}, {t, -5, 5}]
```

Out[ ]= $\left\{\left\{y[x,t] \to \frac{1}{8}\left(1 - \mathrm{Cos}[2x] + 16\,\mathrm{Sin}\left[t - \frac{x}{4}\right] + 2\,\mathrm{Sin}[x]\right)\right\}\right\}$

Out[ ]= $\left\{\left\{y[x,t] \to \frac{1}{8}\left(1 - \mathrm{Cos}[2x] + 16\,\mathrm{Sin}\left[t - \frac{x}{4}\right] + 2\,\mathrm{Sin}[x]\right)\right\}\right\}[1, 1, 2]$

Out[ ]= $\left\{\left\{y[2,1] \to \frac{1}{8}\left(1 - \mathrm{Cos}[4] + 16\,\mathrm{Sin}\left[\frac{1}{2}\right] + 2\,\mathrm{Sin}[2]\right)\right\}\right\}[1, 1, 2]$

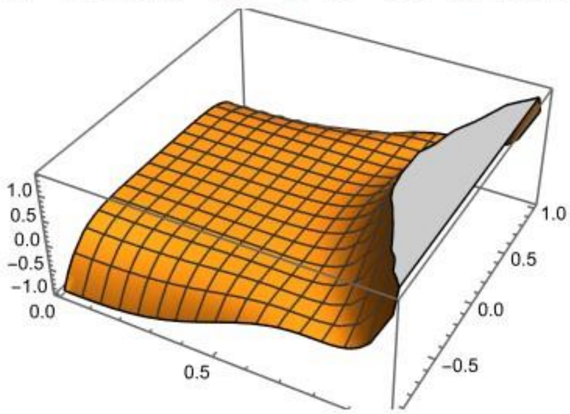


7. Plotting the Characteristics of the First Order Partial Differential Equations

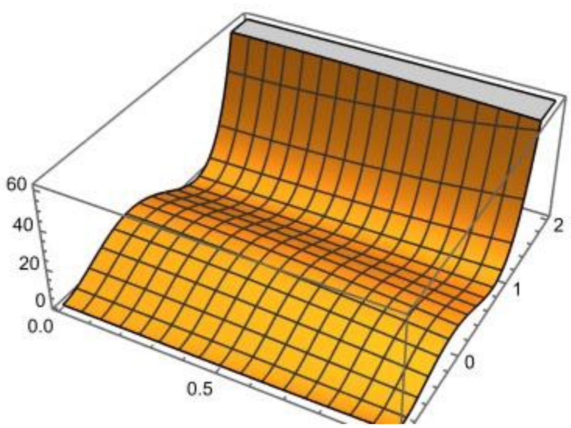
Example 19:

```
In[ ]:= p0 = Plot3D[(2 x^3 - y^2)^(3), {x, 0, 1}, {y, -1, 1}, PlotPoints → 10]
        p1 = Plot3D[(4 - x^2 - 4 y^2)^(2), {x, 0, 1}, {y, 2, -1}, PlotPoints → 55]
```

Out[ ]=



Out[ ]=

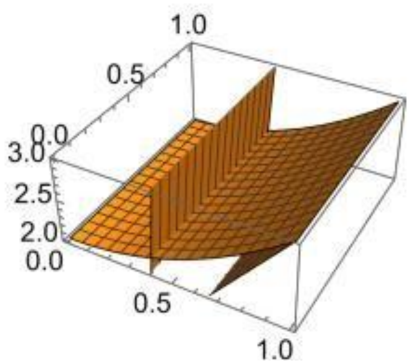


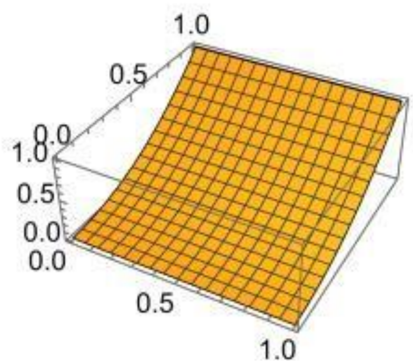
Example 20:

```
In[ ]:= f0 = Plot3D[2 + x^2, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
f1 = Plot3D[x * 3, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
f2 = Plot3D[15 - 5 * x^(-1), {x, 0, 1}, {y, 0, 1},
   PlotPoints → 10];
g1 = Show[f0, f1, f2];
h0 = Plot3D[ y^(2), {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
h1 = Plot3D[2 - y^5, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
h2 = Plot3D[10 - y^4, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
g2 = Show[h0, h1, h2];
Show[GraphicsArray[{g1, g2}]]
```

··· GraphicsArray: GraphicsArray is obsolete. Switching to GraphicsGrid.

Out[ ]=



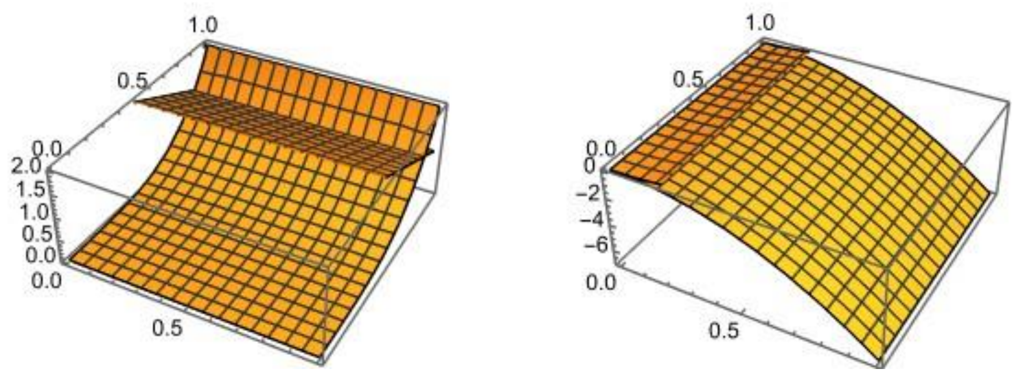


Example 21:

```
In[ ]:= f0 = Plot3D[2 y^4, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
        f1 = Plot3D[3 - 2 y, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
        f2 = Plot3D[6 - 4 y^(1/2), {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
        g1 = Show[f0, f1, f2];
        h0 = Plot3D[-7 x^2, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
        h1 = Plot3D[5 * x^2, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
        h2 = Plot3D[4 - x^6, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
        g2 = Show[h0, h1, h2];
```

··· GraphicsArray: GraphicsArray is obsolete. Switching to GraphicsGrid.

Out[ ]=

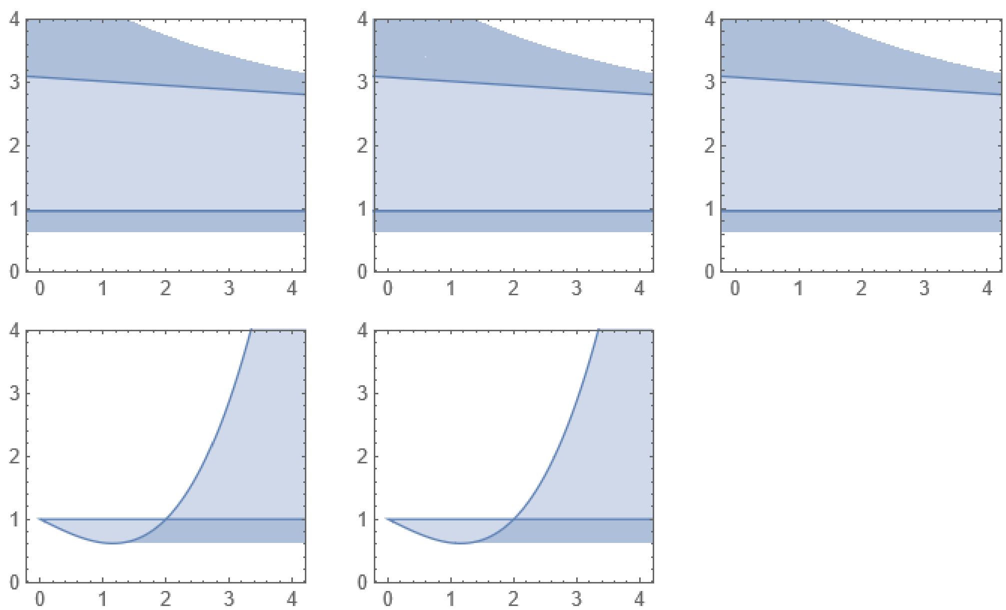


8. Plot the integral surfaces of First Order Partial Differential Equations with Initial Data

Example 22:

In[ ]:=
```
u[s_] := s^3 - s + 1;
x[s_, t_] := 4 t / s^3 - 3 * s^t + 4 * t;
y[s_, t_] := s * 2 + 6 t;
h0 = ParametricPlot[{x[s, t], u[s]}, {s, 0, 2}, {t, 0, 5}, PlotRange → {0, 4}];
h1 = ParametricPlot[{x[s, t], u[s]}, {s, 0, 2}, {t, 0, 5}, PlotRange → {0, 4}];
h2 = ParametricPlot[{x[s, t], u[s]}, {s, 0, 2}, {t, 0, 5}, PlotRange → {0, 4}];
h3 = ParametricPlot[{y[s, t], u[s]}, {s, 0, 2}, {t, 0, 3}, PlotRange → {0, 4}];
h4 = ParametricPlot[{y[s, t], u[s]}, {s, 0, 2}, {t, 0, 3}, PlotRange → {0, 4}];
Show[GraphicsArray[{{h0, h1, h2}, {h3, h4}}], FrameTicks → None, Frame → False]
```

Out[ ]=

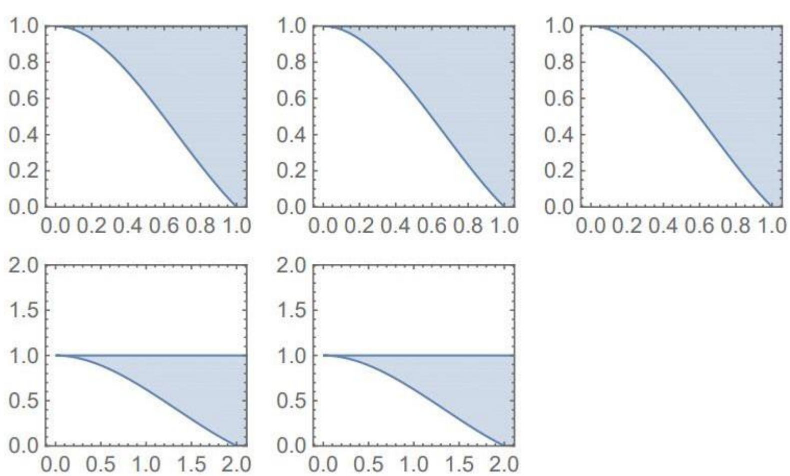


Example 23:

```
In[ ]:= u[s_] := s^3 - 2 * s^2 + 1;
x[s_, t_] := s + t * s^3 - 3 * s^2 * t + 4 * t;
y[s_, t_] := 2 * s + 3 * t^4;
h0 = ParametricPlot[{x[s, t], u[s]}, {s, 0, 2},
   {t, 0, 5}, PlotRange → {0, 1}];
h1 = ParametricPlot[{x[s, t], u[s]}, {s, 0, 2}, {t, 0, 5},
   PlotRange → {0, 1}];
h2 = ParametricPlot[{x[s, t], u[s]}, {s, 0, 2}, {t, 0, 5},
   PlotRange → {0, 1}];
h3 = ParametricPlot[{y[s, t], u[s]}, {s, 0, 1}, {t, 0, 4},
   PlotRange → {0, 2}];
h4 = ParametricPlot[{y[s, t], u[s]}, {s, 0, 1}, {t, 0, 4},
   PlotRange → {0, 2}];
Show[GraphicsArray[{{h0, h1, h2}, {h3, h4}}],
 FrameTicks → None, Frame → False]
```

GraphicsArray: GraphicsArray is obsolete. Switching to GraphicsGrid.

Out[ ]=



Example 24:

In[ ]:=
```
u[s_] := s^2 + 4 * s^3 + 1;
x[s_, t_] := 4 s + t * s^3 - 3 * s^2 * t + 4 * t;
y[s_, t_] := 4 s^2 / (5 t);
h0 = ParametricPlot[{x[s, t], u[s]}, {s, 0, 2}, {t, 0, 5}, PlotRange → {0, 4}];
h1 = ParametricPlot[{x[s, t], u[s]}, {s, 0, 2}, {t, 0, 5}, PlotRange → {0, 4}];
h2 = ParametricPlot[{x[s, t], u[s]}, {s, 0, 2}, {t, 0, 5}, PlotRange → {0, 4}];
h3 = ParametricPlot[{y[s, t], u[s]}, {s, 0, 2}, {t, 0, 3}, PlotRange → {0, 4}];
h4 = ParametricPlot[{y[s, t], u[s]}, {s, 0, 2}, {t, 0, 3}, PlotRange → {0, 4}];
Show[GraphicsArray[{{h0, h1, h2}, {h3, h4}}], FrameTicks → None, Frame → False]
```

Out[ ]=