1. Solution of First Order of Differential equation

Example 1 :-

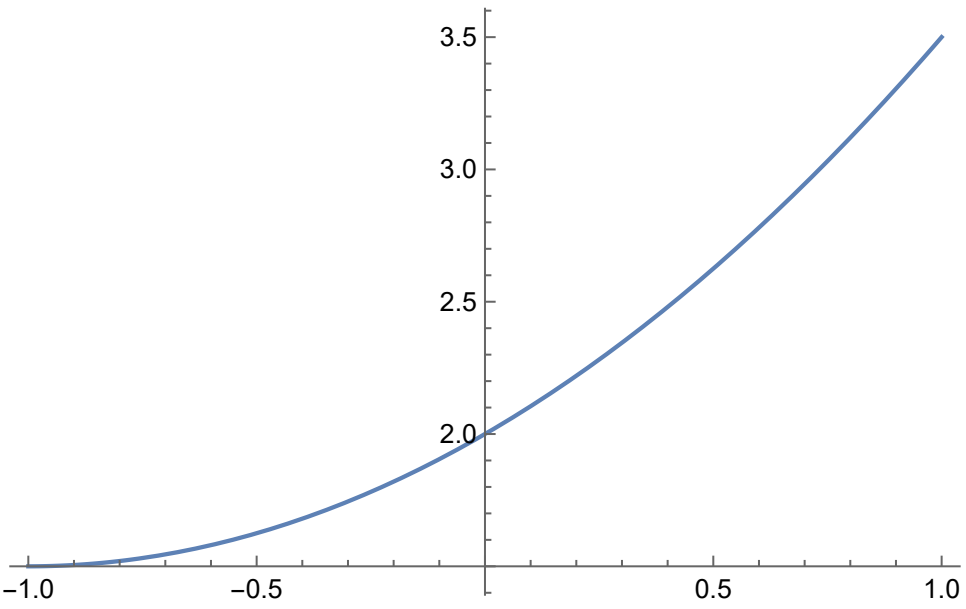
In[ ]:=
```
DSolve[{Y'[X] == X + 1}, Y[X], X]
Y[X] /. DSolve[{Y'[X] == X + 1}, Y[X], X]
sol = DSolve[{Y'[X] == X + 1, Y[0] == 2}, Y[X], X]
Plot[Y[X] /. sol, {X, -1, 1}]
```

Out[ ]= $\left\{\left\{Y[X] \to X + \frac{X^2}{2} + C_1\right\}\right\}$

Out[ ]= $\left\{X + \frac{X^2}{2} + C_1\right\}$

Out[ ]= $\left\{\left\{Y[X] \to \frac{1}{2}\left(4 + 2X + X^2\right)\right\}\right\}$

Out[ ]=

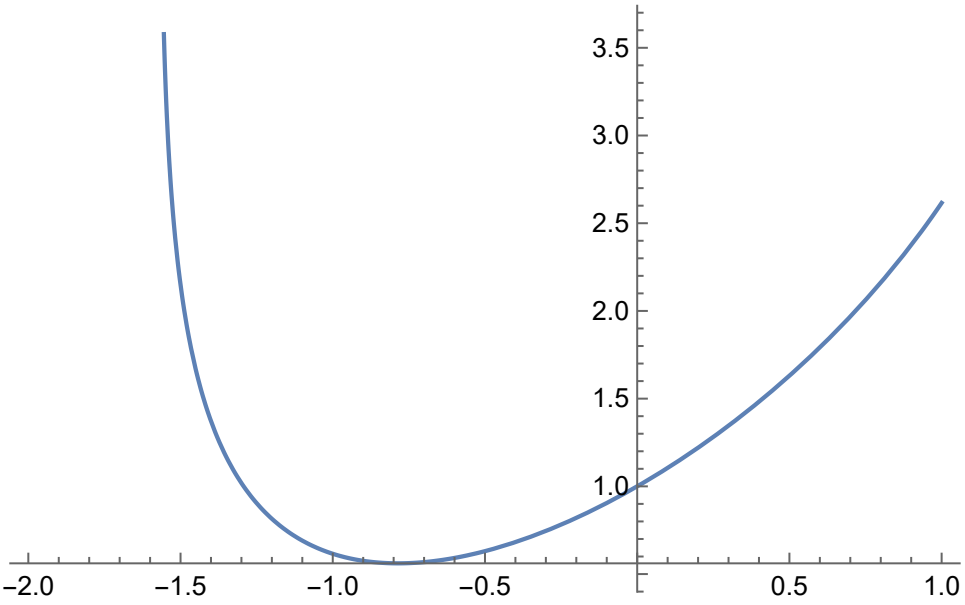


Example 2 :-

In[ ]:=
```
eqn := Y'[X] - Tan[X] - 1;
s = DSolve[{eqn == 0}, Y[X], X];
s1 = s /. {C[1] → 1}
Plot[Y[X] /. s1, {X, -2, 1}]
```

Out[ ]=
```
{{Y[X] → 1 + X - Log[Cos[X]]}}
```

Out[ ]=

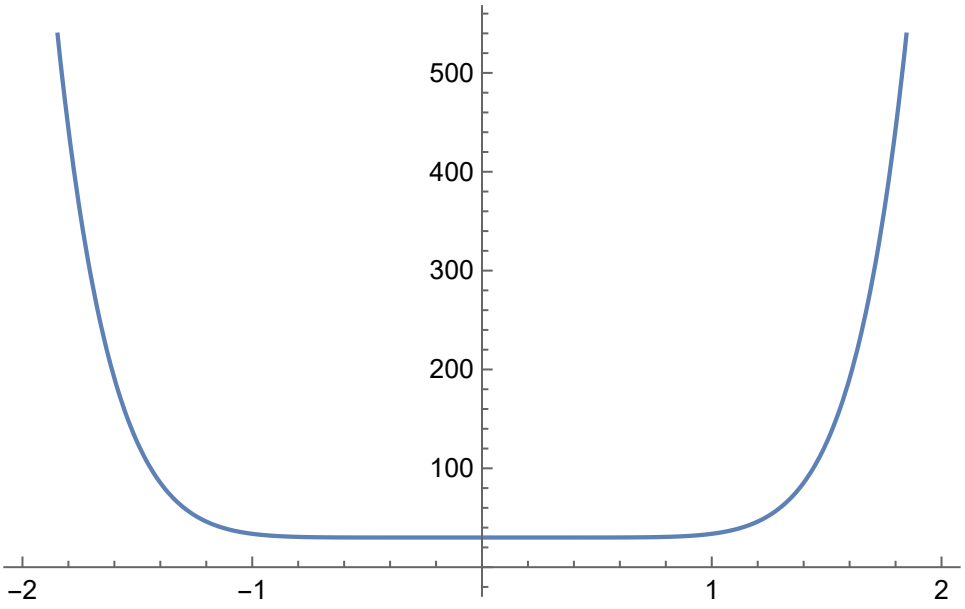


Example 3 :-

In[ ]:=
```
eqn := Y'[X] - 30 X^7;
s = DSolve[{eqn == 0}, Y[X], X];
s1 = s /. {C[1] → 30}
Plot[Y[X] /. s1, {X, -2, 2}]
```

Out[ ]= $\left\{\left\{Y[X] \to 30 + \frac{15\,X^8}{4}\right\}\right\}$

Out[ ]=



2. Plotting of Second Order Solution family of Differential Equation

Example 4 :-

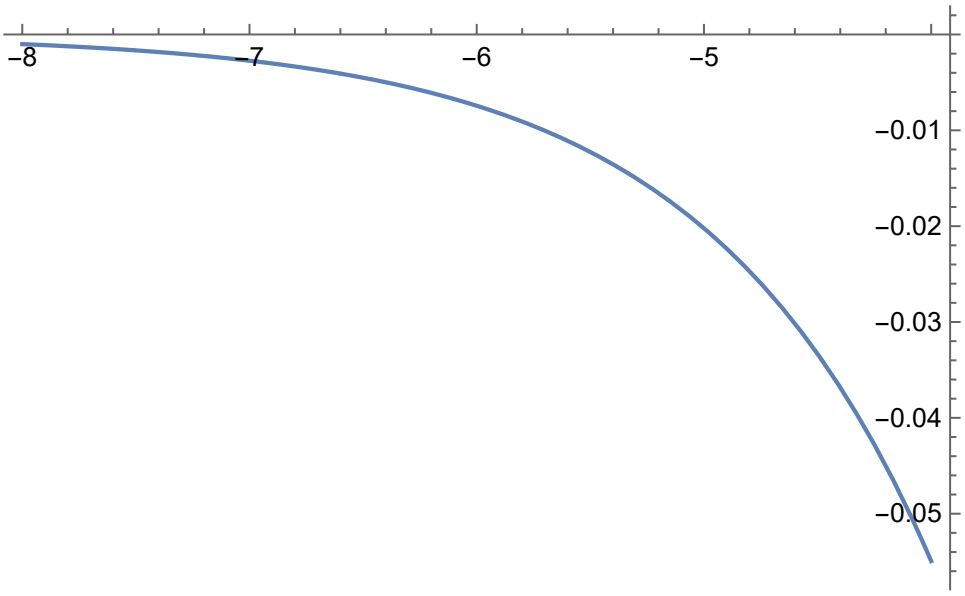
In[ ]:=
```
eqn = Y''[X] - 5 * Y'[X] + 4 * Y[X];
s = DSolve[eqn == 0, Y[X], X]
Y[X] /. s
s1 = s /. {C[1] → -3, C[2] → 7}
Plot[Y[X] /. s1, {X, -4, -8}]
```

Out[ ]= $\left\{\left\{Y[X] \to e^{X} C_1 + e^{4X} C_2\right\}\right\}$

Out[ ]= $\left\{e^{X} C_1 + e^{4X} C_2\right\}$

Out[ ]= $\left\{\left\{Y[X] \to -3\, e^{X} + 7\, e^{4X}\right\}\right\}$

Out[ ]=

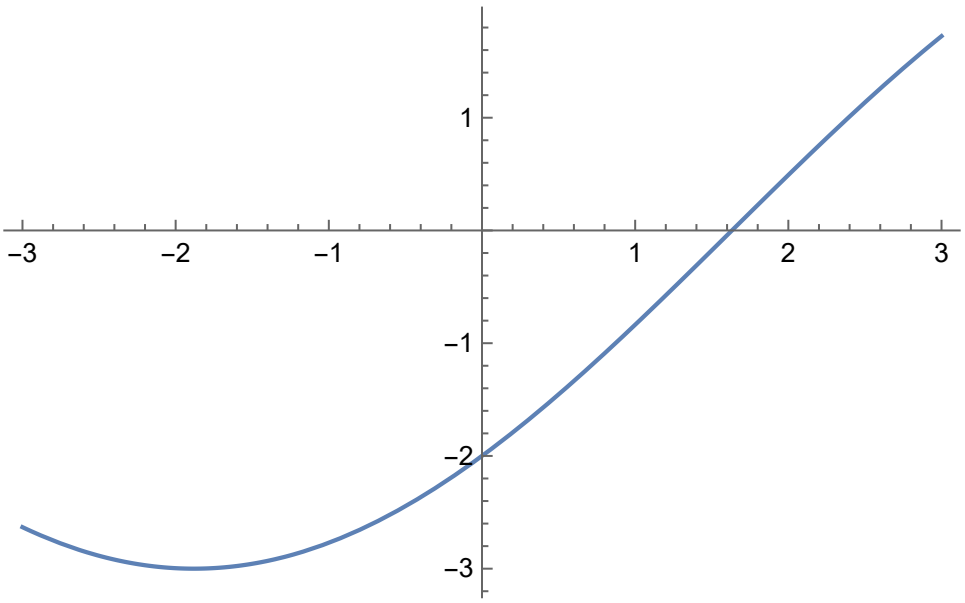


Example 5 :-

In[ ]:=
```
eqn  := 5 * Y''[X] + Y[X];
a = DSolve[eqn == 0, Y[X], X];
a1 = a /. {C[1] → -2, C[2] → √5}
Plot[Y[X] /. a1, {X, -3, 3}]
```

Out[ ]=
$$\left\{\left\{Y[X] \to -2\,\text{Cos}\left[\frac{X}{\sqrt{5}}\right] + \sqrt{5}\,\text{Sin}\left[\frac{X}{\sqrt{5}}\right]\right\}\right\}$$

Out[ ]=



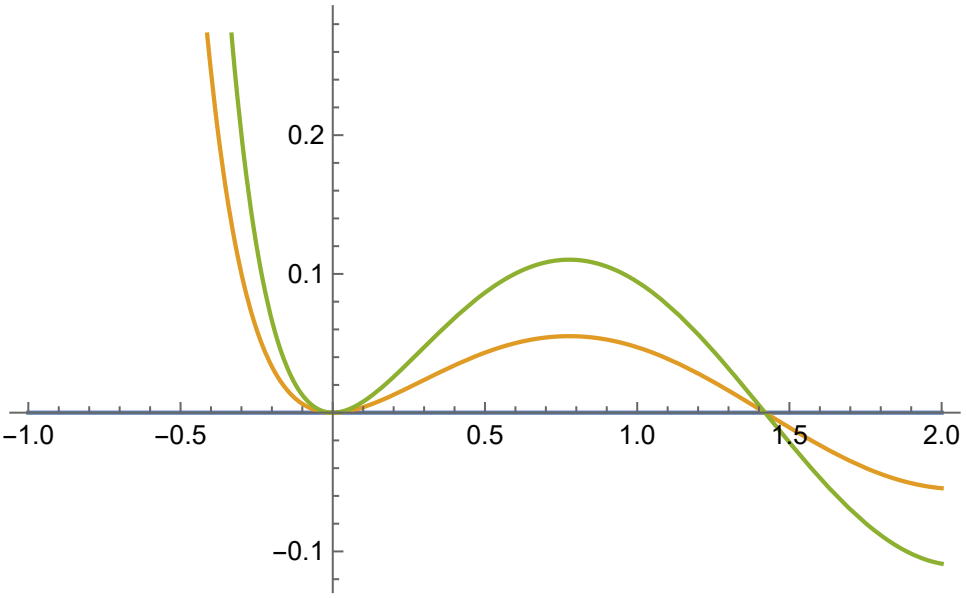
3. Plotting of Third Order Solution family of Differential Equation

Equation 6 :-

In[ ]:=
```
eqn := Y'''[X] + 7 * Y''[X] + 6 * Y'[X] + 42 * Y[X];
s = DSolve[{eqn == 0, Y[0] == 0, Y'[0] == 0, Y''[0] == A},
  Y[X], X]
Plot[Evaluate[Y[X] /. s /. A → Range[0, 2]], {X, -1, 2}]
```

Out[ ]=
$$\left\{\left\{Y[X] \to -\frac{1}{330} A e^{-7X}\left(-6 + 6 e^{7X} \text{Cos}\left[\sqrt{6}\, X\right] - 7\sqrt{6}\, e^{7X} \text{Sin}\left[\sqrt{6}\, X\right]\right)\right\}\right\}$$

Out[ ]=

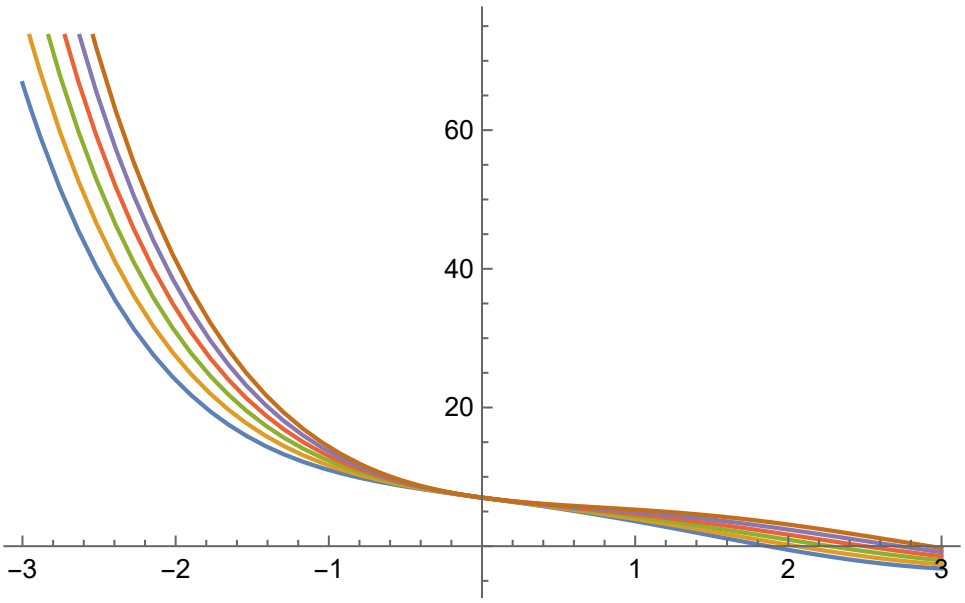


Equation 7 :-

In[ ]:=
```
eqn := Y'''[X] + Y''[X] + Y'[X] + Y[X];
s = DSolve[{eqn == 0, Y[0] == 7, Y'[0] == -3, Y''[0] == A},
  Y[X], X]
Plot[Evaluate[Y[X] /. s /. A → Range[0, 5]], {X, -3, 3}]
```

Out[ ]=
$$\left\{\left\{Y[X] \to -\frac{1}{2}\, e^{-X} \left(-7 - A - 7\, e^{X} \operatorname{Cos}[X] + A\, e^{X} \operatorname{Cos}[X] - e^{X} \operatorname{Sin}[X] - A\, e^{X} \operatorname{Sin}[X]\right)\right\}\right\}$$

Out[ ]=



4. Solution of Differential Equation by Variation of Parameter method

Example 8 :-

In[ ]:=
```
homsol = DSolve[{y''[x] + y[x] - Tan[x] == 0}, y[x], x]
y1[x_] = Cos[x];
y2[x_] = Sin[x];
caps = {y1[x], y2[x]};
ws = Simplify[Det[{caps, ∂x caps}]]
f[x_] = Tan[x];
u1prime = -y2[x] * f[x] / ws;
u2prime = y1[x] * f[x] / ws;
u1[x_] = ∫ u1prime dx;
u2[x_] = ∫ u2prime dx;
yp[x_] = y1[x] * u1[x] + y2[x] * u2[x] // Simplify
```

Out[ ]=
$$\left\{\left\{y[x] \to c_1 \cos[x] + \cos[x] \log\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] - \cos[x] \log\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] + c_2 \sin[x]\right\}\right\}$$

Out[ ]=
$$1$$

Out[ ]=
$$\cos[x]\left(\log\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] - \log\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right]\right)$$

Example 9 :-

In[ ]:=
```
homsol = DSolve[{y''[x] + 3*y'[x] + 2*y[x] - e^(2*x)*30 == 0},
  y[x], x]
y1[x_] = e^-x;
y2[x_] = e^-2*x;
caps = {y1[x], y2[x]};
ws = Simplify[Det[{caps, ∂x caps}]]
f[x_] = 30*e^2*x;
u1prime = -y2[x]*f[x] / ws;
u2prime = y1[x]*f[x] / ws;
u1[x_] = ∫ u1prime dx;
u2[x_] = ∫ u2prime dx;
yp[x_] = y1[x]*u1[x] + y2[x]*u2[x] // Simplify
```

Out[ ]=
$$\left\{\left\{y[x] \to e^{-2x} c_1 + e^{-x} c_2 + \left(15\, e^{-2x} \left(-e^{x\,(2+2\,\text{Log}[e])} + 2\, e^{x+x\,(1+2\,\text{Log}[e])} - 2\, e^{x\,(2+2\,\text{Log}[e])}\,\text{Log}[e] + 2\, e^{x+x\,(1+2\,\text{Log}[e])}\,\text{Log}[e]\right)\right) \Big/ \left((1+\text{Log}[e])\,(1+2\,\text{Log}[e])\right)\right\}\right\}$$

Out[ ]=
$$-e^{-3x}\,\text{Log}[e]$$

Out[ ]=
$$\frac{5\,e^{2x}}{2\,\text{Log}[e]^2}$$

5. Solution of system of Ordinary Differential Equation

Example 10 :-

```
In[ ]:= DSolve[{y[x] == z'[x], z[x] == -3 * y'[x]}, {y, z}, x]
```

$$\text{Out[ ]=}\ \left\{\left\{y \to \text{Function}\left[\{x\}, c_1 \cos\left[\frac{x}{\sqrt{3}}\right] - \frac{c_2 \sin\left[\frac{x}{\sqrt{3}}\right]}{\sqrt{3}}\right],\right.\right.$$
$$\left.\left. z \to \text{Function}\left[\{x\}, c_2 \cos\left[\frac{x}{\sqrt{3}}\right] + \sqrt{3}\, c_1 \sin\left[\frac{x}{\sqrt{3}}\right]\right]\right\}\right\}$$

Example 11 :-

```
In[ ]:= DSolve[{y'[t] == Cot[t] * y[t], x[t] == -3 * x'[t]}, {x, y}, t]
```

$$\text{Out[ ]=}\ \left\{\left\{y \to \text{Function}[\{t\}, c_1 \sin[t]], x \to \text{Function}\left[\{t\}, e^{-t/3} c_2\right]\right\}\right\}$$

Example 12 :-

```
In[ ]:= DSolve[{3 * y'[t] == Sin[t] + Cos[t], 4 * x'[t] == 8 * x[t]},
  {x, y}, t]
```

$$\text{Out[ ]=}\ \left\{\left\{y \to \text{Function}\left[\{t\}, c_1 - \frac{\cos[t]}{3} + \frac{\sin[t]}{3}\right],\right.\right.$$
$$\left.\left. x \to \text{Function}\left[\{t\}, e^{2t} c_2\right]\right\}\right\}$$

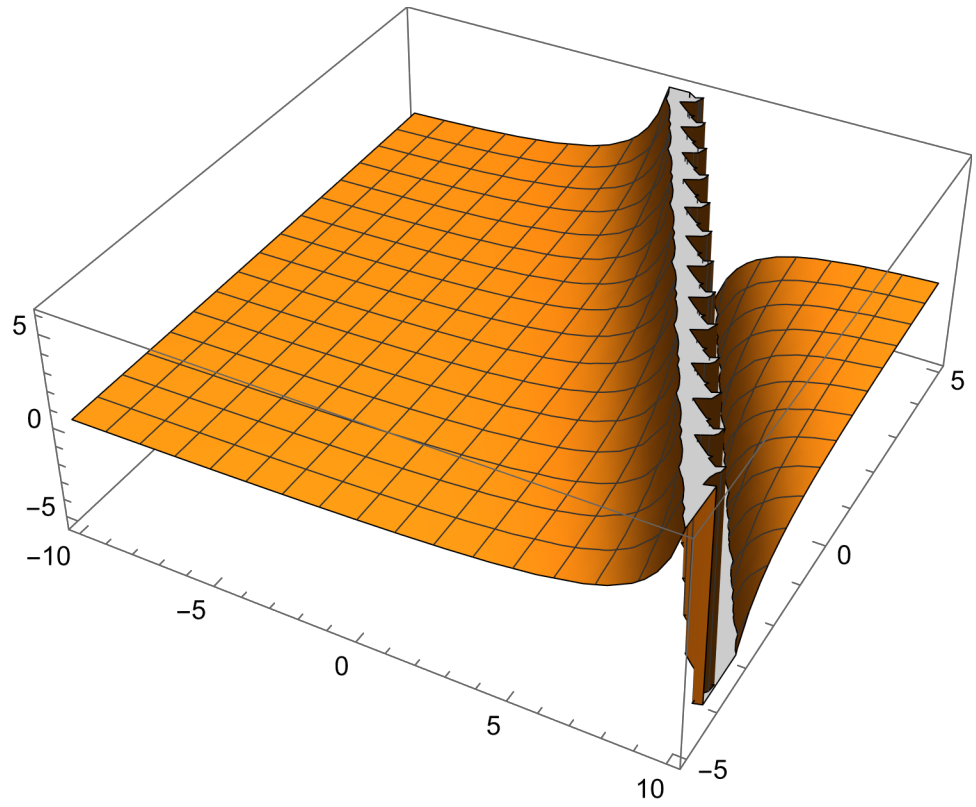
6. Solution of Cauchy problem for First Order Partial differential equation

Example 13 :-

In[4]:=
```
eq1 = D[2 * u[x, y], x] + D[4 * u[x, y], y] == u[x, y] * u[x, y];
Sol1 = DSolve[{eq1, u[x, -x] == 1}, u[x, y], {x, y}]
Plot3D[Sol1[ [1, 1, 2]], {x, -10, 10}, {y, -5, 5}]
```

Out[5]=
$$\left\{\left\{u[x, y] \to -\frac{6}{-6 + x + y}\right\}\right\}$$

Out[6]=

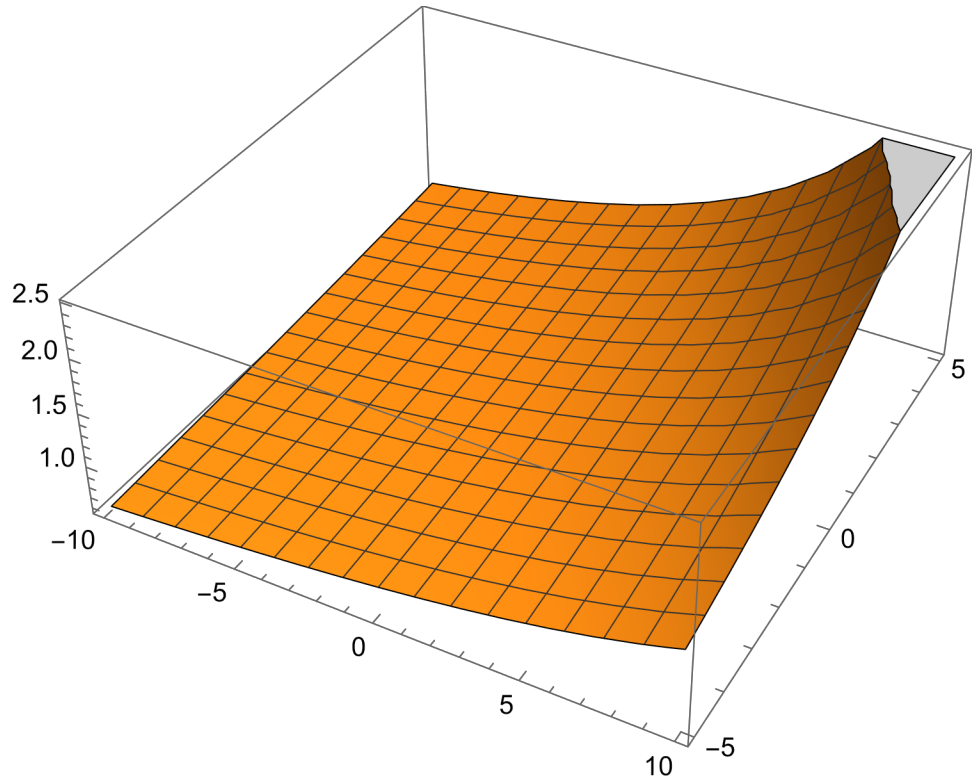


Example 14 :-

In[ ]:=
```
eq2 = D[u[x, y], x] + 5 * D[4 * u[x, y], y] == u[x, y] * u[x, y];
Sol2 = DSolve[{eq2, u[x, -x] == 1}, u[x, y], {x, y}]
Plot3D[Sol2[ [1, 1, 2]], {x, -10, 10}, {y, -5, 5}]
```

Out[ ]=
$$\left\{\left\{u[x, y] \to -\frac{21}{-21 + x + y}\right\}\right\}$$

Out[ ]=

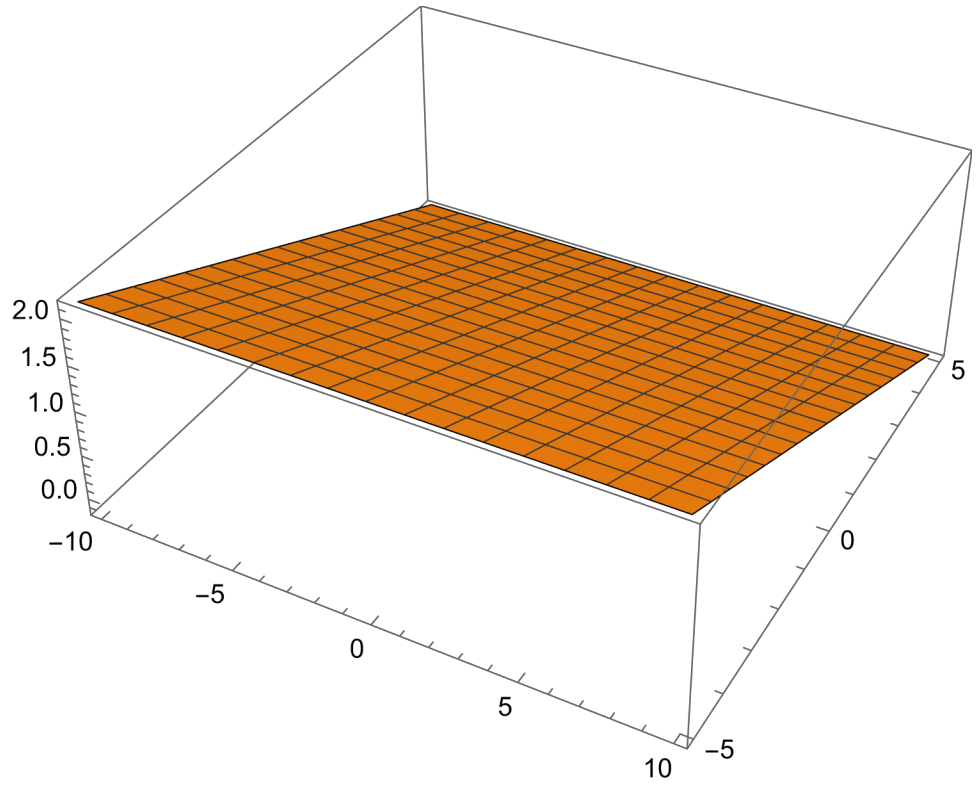


Example 15 :-

In[ ]:=
```
eq3 = D[14 * u[x, y], x] - 7 * D[5 * u[x, y], y] == 8;
Sol3 = DSolve[{eq3, u[x, 0] == 1}, u[x, y], {x, y}]
Plot3D[Sol3[ [1, 1, 2]], {x, -10, 10}, {y, -5, 5}]
```

Out[ ]=
$$\left\{\left\{u[x, y] \to \frac{1}{35}\ (35 - 8\ y)\right\}\right\}$$

Out[ ]=

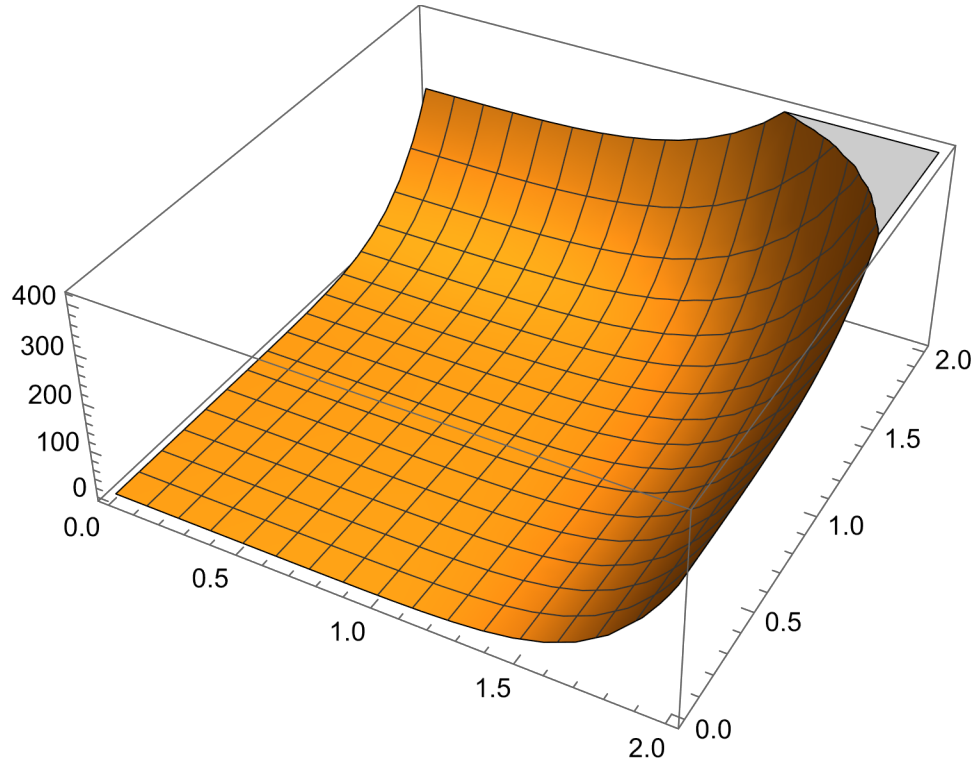


7. Plotting the Characteristics of the First Order Partial Differential Equations

Example 16 :-

In[ ]:=
```
p = Plot3D[(x^4 + y^4)^(2), {x, 0, 2}, {y, 0, 2},
  PlotPoints → 20]
```

Out[ ]=

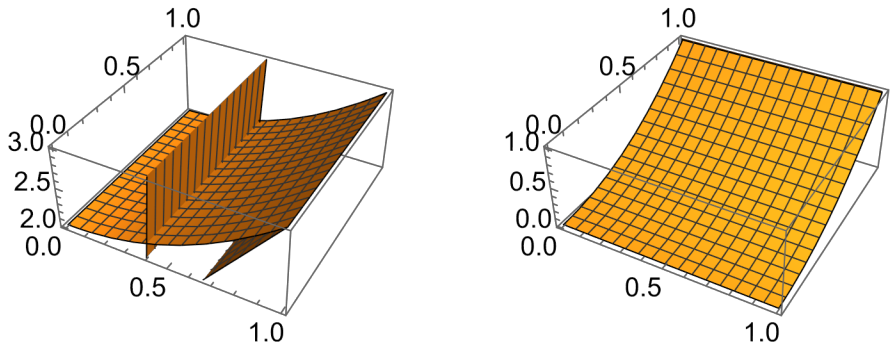


Example 17 :-

In[ ]:=
```
f0 = Plot3D[2 + x^2, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
f1 = Plot3D[x * 3, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
f2 = Plot3D[15 - 5 * x^(-1), {x, 0, 1}, {y, 0, 1},
   PlotPoints → 10];
g1 = Show[f0, f1, f2];
h0 = Plot3D[ y^(2), {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
h1 = Plot3D[2 - y^5, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
h2 = Plot3D[10 - y^4, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
g2 = Show[h0, h1, h2];
Show[GraphicsArray[{g1, g2}]]
```

GraphicsArray: GraphicsArray is obsolete. Switching to GraphicsGrid.

Out[ ]=



8. Plot the integral surfaces of First Order Partial Differential Equations with initial data

Example 18 :-

In[ ]:=
```
u[s_] := s^3 - 2 * s^2 + 1;
x[s_, t_] := s + t * s^3 - 3 * s^2 * t + 4 * t;
y[s_, t_] := 2 * s + 3 * t^4;
h0 = ParametricPlot[{x[s, t], u[s]}, {s, 0, 2},
   {t, 0, 5}, PlotRange → {0, 1}];
h1 = ParametricPlot[{x[s, t], u[s]}, {s, 0, 2}, {t, 0, 5},
   PlotRange → {0, 1}];
h2 = ParametricPlot[{x[s, t], u[s]}, {s, 0, 2}, {t, 0, 5},
   PlotRange → {0, 1}];
h3 = ParametricPlot[{y[s, t], u[s]}, {s, 0, 1}, {t, 0, 4},
   PlotRange → {0, 2}];
h4 = ParametricPlot[{y[s, t], u[s]}, {s, 0, 1}, {t, 0, 4},
   PlotRange → {0, 2}];
Show[GraphicsArray[{{h0, h1, h2}, {h3, h4}}],
 FrameTicks → None, Frame → False]
```

••• GraphicsArray: GraphicsArray is obsolete. Switching to GraphicsGrid.

Out[ ]=