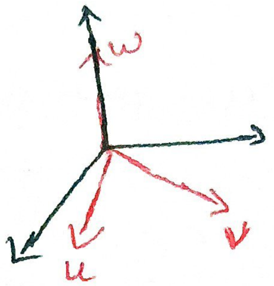
Overview of Linear algebra

Vector → matrices → subspaces

$\vec{u}, \vec{v}, \vec{w}$ $\boxed{\text{Linear comb}}$ - $x_1\vec{u} + x_2\vec{v} + x_3\vec{w} = b$

$x_1, x_2, x_3 \rightarrow$ scalar

$$\underbrace{\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{\text{these vector are Independent}}$$



$$Ax = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 - x_1 \\ x_3 - x_2 \end{bmatrix} = b$$

→ comb of column.

$$A\begin{bmatrix} 1 \\ 4 \\ 9 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

$b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ → now we want to find x_1, x_2, x_3

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_1 + b_2 \\ b_3 + b_1 + b_2 \end{bmatrix}$$

$$x = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

sum matrix
$S = A^{-1}$

↳ inverse matrix

$Ax = b \Rightarrow x = A^{-1}b$

It in inverse transform

A map $x \rightarrow y$
A^{-1} map $y - x$

→ It's a perfect reverse → only possible when the matrix is invertible.

$$Cx = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 - x_3 \\ x_2 - x_1 \\ -x_2 + x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$Cx = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} c \\ c \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

~~these~~ this has linear dependent columns

→ can't span R^3

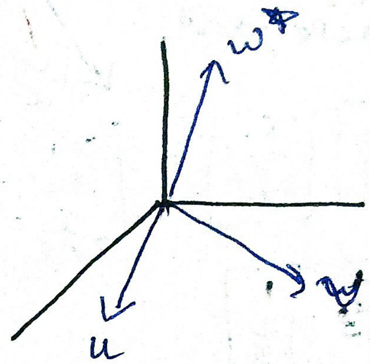
→ All combination lie in a plane

→ we can only solve $b_1 + b_2 + b_3 = 0$

$Cx = 0$

↳ dont exist inverse

Geomatrical view



w* → in the same plane
It gives nothing new

u, v, w* → dependent
↳ It gives plane

→ original u, v, w
↳ independent
↳ It gives whole space

u, v, w → basis
↳ ~~basis~~ For 5d space → 5 vector that independ
↓
It means their combination gives 5 demensional space
↓
invertible matrix

subspace

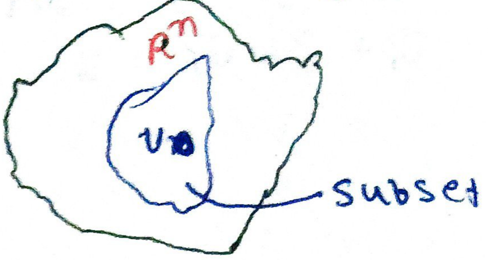
→ It contains 0 vector ~~means~~

→ Be closed under addition and scalar multiplication

Subspace of R^n

$\left\{ \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \; x_i \in R \;\; 1 \le i \le n \right\}$

$V \leftarrow$ subset of R^n



If V subspace of R^n

then

$\Rightarrow$ V contains $\vec{0} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$.

$\Rightarrow$ $\vec{x}$ in $V = c\vec{x}$ in V

↳ closour under scalar multiplication

$\Rightarrow$ $\left.\begin{matrix} \vec{a} \text{ in } V \\ \vec{b} \text{ in } V \end{matrix}\right\} \Rightarrow \vec{a} + \vec{b}$ in $\vec{V}$

↳ closure under addition

Ex $\rightarrow$ $V = \{0\} = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$ is a subspace of R^3 ?

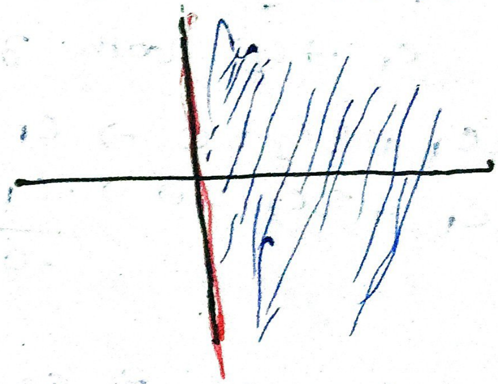
(I) Zero vector ✓

(II) $c\vec{V} = c\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \in V$ ✓

(III) $\vec{a}\vec{U} + \vec{b}\vec{U} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \in V$ ✓

$\rightarrow$ $S = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in R^2 \mid x_1 \ge 0 \right\}$

is S subpace of R^2 ?



$\rightarrow$ (I) contain $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ ✓

(II) $\begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} a+c \\ b+d \end{bmatrix} > 0$ ✓

close under addition

(III) $-c\vec{v} = \begin{bmatrix} -v_1 \\ v_2 \end{bmatrix} < 0$ $\rightarrow$ not close under multiplication

$\rightarrow$ S is not subspace of R^2