Elimamation with matrix

3 equation and 3 unknown

$$x + 2y + z = 2$$
$$3x + 8y + z = 12$$
$$4y + z = 2$$

$Ax = b$

Elimation Process

1st Pivot →

$$\left[\begin{array}{ccc|c} \boxed{1} & 2 & 1 & 2 \\ 3 & 8 & 1 & 12 \\ 0 & 4 & 1 & 2 \end{array}\right] \xrightarrow{(2,1)} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & \boxed{2} & -2 & 6 \\ 0 & 4 & 1 & 2 \end{array}\right] \xrightarrow{(3,2)} \left[\begin{array}{ccc|c} \boxed{1} & 2 & 1 & 2 \\ 0 & \boxed{2} & -2 & 6 \\ 0 & 0 & \boxed{5} & -10 \end{array}\right]$$

2nd pivot → 2

A b — Agumented matrix

3rd Pivot → 5

u → upper traingular

e

$\boxed{\text{Pivot can't be zero}}$

$\boxed{Ab \to Uc}$

→ How caunt this will fail ?

→ in case zero pivot, we exchange row and solve

→ If last row become 000 then ~~the~~ matrix wounldnt be invertible.

Back subsituation

$$x + 2y + z = 2$$
$$2y - 2z = 6$$
$$5z = -10$$

$x + 2 - 2 = 2 \Rightarrow x = 2$

$2y - 2\cancel{x} - 2 = 6 \Rightarrow 2y = 2 \Rightarrow y = 1$

$\boxed{z = -2}$

matrices

subtract 3 × row1 from row 2

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_{21}} \begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix}$$

E_{21} → Elementry matrix

Step-2 → subtracted 2 × row2 from row 3

$$E_{32} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 5 \end{bmatrix}$$

$E_{32}(E_{21} A) = U$

$A \to U$

$\underline{(E_{32} E_{21})} A = U$

↳ Assosiative Law

Permutation ~~mat~~

→ Exchange row 1 and row 2

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} c & d \\ a & b \end{bmatrix}$$

$\vec{P}$ → permutation

→ exchange col 1 and col 2

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} b & a \\ d & c \end{bmatrix}$$

$(E_{32} E_{21}) A = U$

$U \to A$

↳ inverse

inverses

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

E^{-1} $\quad E \quad = I$

↳ inverse