System of Linear equation

→ A system of linear equation is a set of two or more linear equations involving the same variables.

Ex →

$$2x + 3y = 8$$
$$4x - y = 2$$

System of Sentences

System 1

The dog is black
The cat is orange.

↓

complete system
(2 unique fact)

↓

Non-Singular

System 2

The dog is black
the dog is black

↓

Reduntant system
(Repeats the same info)

↓

Singular

System 3

The dog is black
The dog is white

↓

Contradictory
(conflict in info)

↓

Singular

what is Linear Equations?

→ A linear eqn is a mathematical statements where:

- → variables are only multiplied by constants
- → variable added or subtracted, never squared, or multiplied together.
- → No fancy function like sin, cos, log, a^2 ---

Ex →

linear eq	non linear eq
(I) $a+b=10$	(I) $a^2+b=10$
(II) $2a+3b=15$	(II) $a \cdot b = 10$
(III) $3.4a+48.9b+2c=122.5$	(III) $\sin(a) = b$

what is a system of Linear eq?

A system = a group of Linear eqn involving the same variable

You analyze system to

- → find unique soln (one answer)
- → find infinite soln (many answer)
- → Discover no soln (contradiction)

Ex->

System 1 (unique sol^n) (Non singular)

$a+b=10$

$a+2b=12$

$b=2,\ a=10-2=8$

unique $\rightarrow$ complete system
$\downarrow$
Non-singular

System 2 (infinite sol^n) (singular - Redundant)

$a+b=10$

$2a+2b=20$

Note -> Second eq = 2 x First eq
$\rightarrow$ same info repeated

Result -> infinite sol^n

Ex -> (8,2), (5,5), (9,1) etc

System 3 (No-sol^n) (singular - contradictory)

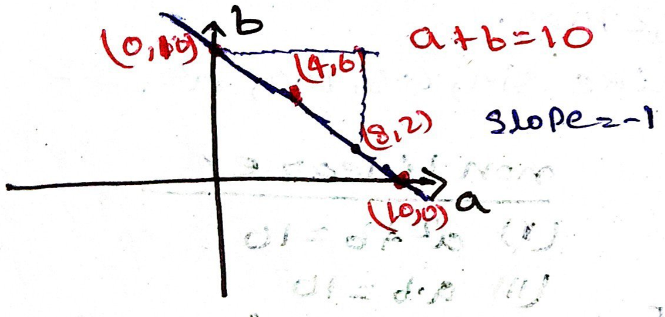
$a+b=10$

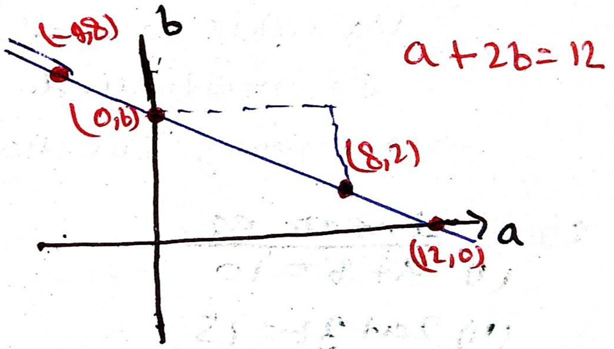
$2a+2b=24$

From first $2a+2b$ should $=20$
but it's 24 -> contradiction

Result :-> No sol^n
$\downarrow$
conflicting data
$\downarrow$
contradictory $\rightarrow$ singular

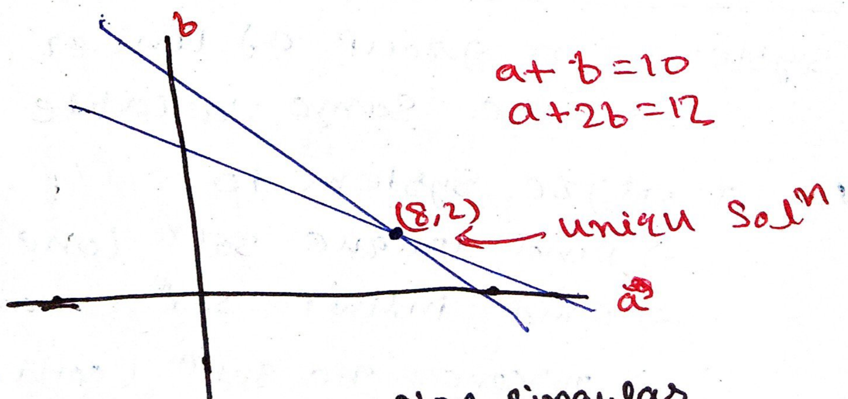
Linear equation - Line





$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{0-2}{10-8} = \frac{-2}{2} = -1$$

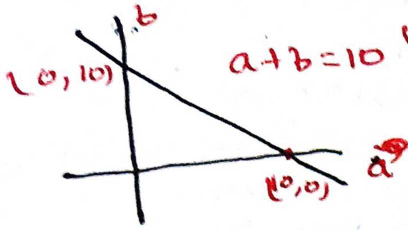
$$\text{Slope} = \frac{0-2}{12-8} = \frac{-2}{4} = -\frac{1}{2} = -0.5$$

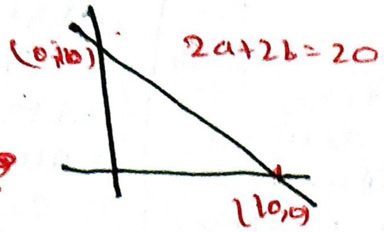


Non-singular

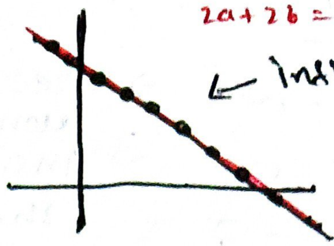
complete

System-2





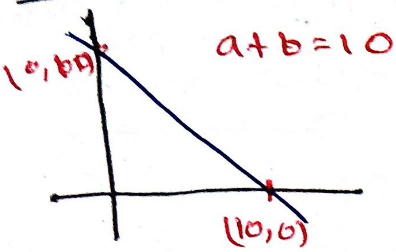
$a+b=10$
$2a+2b=20$

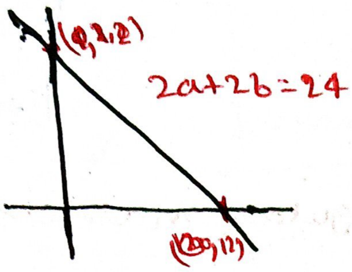


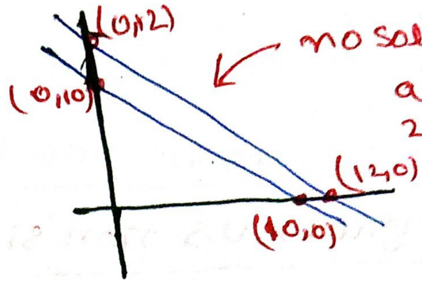
← infinitely many soln

→ Redundant
→ singular

System-3







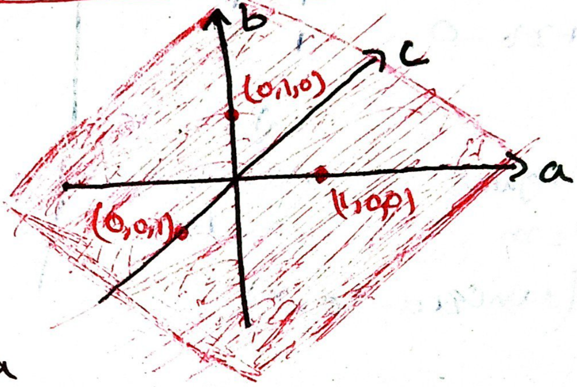
no soln

$a+b=10$
$2a+2b=24$

→ contradictory
→ singular

Linear equation in 3 variables as a plane

$a+b+c=1$
$1+0+0=1$
$0+1+0=1$
$0+0+1=1$



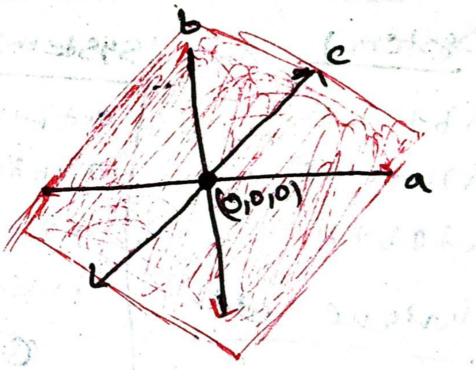
these 3 Points are non-collinear, so they define a unique plane.

→ Planes through origin

$3a-5b+2c=0$

since R.H.S $=0$, the plane passed through the origin $(0,0,0)$

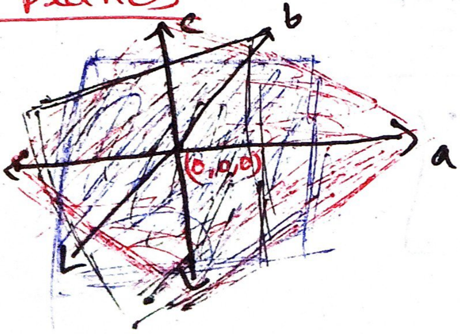
$3(0)+5(0)+2(0)=0$



→ intersecting planes

System 1

$a+b+c=0$
$a+2b+c=0$
$a+b+2c=0$



→ Each is a plane through origin
→ All 3 plane intersect at a single point $(0,0,0)$
→ non-singular → unique soln

System-2

$a+b+c=0$
$a+b+2c=0$
$a+b+3c=0$

→ three plane intersect along a line
→ infinite soln on that line
→ singular system

System 3

$a+b+c=0$
$2a+2b+2c=0$
$3a+3b+3c=0$

→ All eqn are multiple of each other
→ so they define the same plane
→ infinite many soln, the entire plane
→ singular system

Singular vs non singular matrices

System 1

$a+b=0$
$a+2b=0$

a	b
1	1
1	2

↓ non-singular system (unique soln)

↓ non-singular matrix

System-2

$a+b=0$
$2a+2b=0$

a	b
1	1
2	2

↓ singular system (infinite many sol^n)

↓ singular matrix

→ constants don't matter for singularity

System 1

$a+b+c=0$
$a+2b+c=0$
$a+b+2c=0$

unique soln

$a=0$
$b=0$
$c=0$

System 2

$a+b+c=0$
$a+b+2c=0$
$a+b+3c=0$

infinite soln

$c=0$
$a+b=0$
i.e, $a=-b$

System 3

$a+b+c=0$
$a+b+2c=0$
$a+b+3c=0$

System-4

$a+b+c=0$
$2a+2b+2c=0$
$3a+3b+3c=0$

$a+b+c=0$

$\boxed{\text{i.e } c=-a-b}$

$$\begin{bmatrix}1&1&1\\1&2&1\\1&1&2\end{bmatrix}$$
↓ non-singular

$$\begin{bmatrix}1&1&1\\1&1&2\\1&1&3\end{bmatrix}$$
↓ singular

$$\begin{bmatrix}1&1&1\\2&2&2\\3&3&3\end{bmatrix}$$
↓ singular

Linear dependence

Non-singular

$a+b=0$
$a+2b=0$

a	b
1	1
1	2

→ no eqn is a multiple of the other one

→ Rows are linearly independent.

Singular

$a+b=0$
$2a+2b=0$

a	b
1	1
2	2

×2

2× First row = second row

↓

rows are dependent each other

→ Rows are linearly dependent

linear independence and dependance

→ $a=1$
$b=2$
$a+b=3$

$$\begin{array}{r} a+0b+0c=1 \\ +\,0a+b+0c=2 \\ \hline a+b+0c=3 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Row 1 + Row 2 = Row 3.

→ Row 3 dependent on row 1 and 2

→ Rows are linearly dependent

→ it means it is singular

→ $a+b+c=0$
$a+2b+c=0$
$a+b+2c=0$

→ no relation btw eqn

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$ → No reation btw rows

→ Rows are linearly independent

determinant

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ → If this matrix is singular

$[a \;\; b] \cdot k = [c \;\; d]$

$ak = c$

$bk = d$

$\frac{c}{a} = \frac{d}{b} = k$

$ad = bc$

$\boxed{ad - bc} = 0$

↳ Determinant

Determinant

$\boxed{ad - bc}$

Ex→ (i) $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

Determint

$\boxed{1}\boxed{2} - \boxed{1}\boxed{1}$

$= 1\cdot2 - 1\cdot1 = 1$

(ii) $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$

Determinant

$\boxed{1}\boxed{2} - \boxed{1}\boxed{2}$

$= 1\cdot2 - 2\cdot1 = 0$

→ If Determint = 0

↳ matrix will be singular

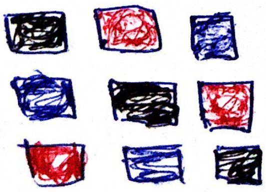
→ Determint ≠ 0

↳ matrix will be non-singular

Ex→ (i) $\begin{bmatrix} 5 & 1 \\ -1 & 3 \end{bmatrix}$ Det = 15 + 1 = 16 → non-singular

(ii) $\begin{bmatrix} 2 & -1 \\ -6 & 3 \end{bmatrix}$ Det = 6 − 6 = 0 → singular

→ Diagonals in a 3×3 matrix



$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

$+ 1\cdot2\cdot2 \quad + 1\cdot1\cdot1 \quad + 1\cdot1\cdot1$

$- 1\cdot2\cdot1 \quad - 1\cdot1\cdot1 \quad - 1\cdot1\cdot2$

Det = 4 + 1 + 1 − 2 − 1 − 2

= 1

Ex-> (i) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 3 & 3 & 3 \end{bmatrix}$ $Det = 1(3-0) - 0 + 1(0-3)$

$= 3 - 3 = 0$ -> singular

(ii) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix}$ $det = 1(-1+0) - 1(-1+0) + 1(0)$

$= -1 + 1 = 0$ -> singular

(iii) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ $det = 1(6-0) - 1(0-0) + 1(0+0)$

$= 6$ -> non-singular

(iv) $\begin{bmatrix} 1 & 2 & 5 \\ 0 & 3 & -2 \\ 2 & 4 & 10 \end{bmatrix}$ $det = 1(30+8) - 2(0+4) + 5(0-6)$

$\Rightarrow 38 - 8 - 30$

$= 0$ -> singular

Q-> $A = \begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix}$ $det(A) = 8 - 6 = 2$ -> non-singular

Q-> $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ $det = 3 - 4 = -1$ -> non-singular

-> Linearly independent

Q-> $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ -1 & 2 & 1 \end{bmatrix}$ $det = 1(1-2) - 2(2+1) + 1(4+1)$

$= -1 - 6 + 5$

$\Rightarrow -7 + 5 = -2$

Q-> $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 2 & 1 \end{bmatrix}$ $det = 1(2-2) - 2(3-2) + 3(6-4)$

$= 0 - 2 + 6$

$= 4$

Q-> $\begin{bmatrix} 2 & 1 & 5 \\ 1 & 2 & 1 \\ x & y & z \end{bmatrix}$ $det = 2(2z-y) - 1(z-x) + 5(y-2x) = 0$

$\Rightarrow 4z - 2y - z + x + 5y - 10x = 0$

$\Rightarrow 3z + 3y - 9x = 0$

$\Rightarrow -3x + y + z = 0$

$\Rightarrow y + z = 3x$

10) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 1 & 4 & 5 \end{bmatrix}$

$det = 1(10-8) - 2(0-2) + 3(0-2)$

$\Rightarrow 2 + 4 - 6$

$= 0$