Estimating pooled within-time series variograms with spatially shifted temporal points

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Introduction

Estimation of pooled within-time series (PTS) variograms is frequently done for geostatistical interpolation in spatially data-scarce regions [1]. The only available averaging empirical variograms (AEV) method averages semivariances that are computed for individual time steps over each spatial-lag within a pooled time series [2]. However, semivariances computed by a few paired comparisons in individual time steps are erratic and hence hamper precision of PTS variogram estimation.

Here, we outline an alternative method, i.e. "spatially shifting temporal points (SSTP)", for PTS variogram estimation that was developed using R open source software environment.

Methods

Spatially shifting temporal points

The temporal data point sets were spatialized, i.e. assigned to different coordinate clusters on the same space (Figure 1 and 2). s is a data point location vector with coordinate touples (x, y), t is a time vector, $Z(s, t)$ is the vector for a random process in data point s and year t, $||s_{i,t} - s_{j,t}||$ is the spatial-lag for the point pair s_i and s_j in year t; $i, j \in 1..n$. We first assigned the points from t_1 to its original coordinates x_{t_1}, y_{t_1}. The coordinates for the the latter years were calculated according to Eq. (1), when $(t_1 + 1) + 4n \leq t < (t_1 + 1) + 4(n + 1); n \in N$.

$$
\begin{aligned}
s_{(t_1+1)+4n} &= x_{(t_1+1)+4n} + (n+1)d, y_{(t_1+1)+4n} \\
s_{(t_1+1)+4n+1} &= x_{(t_1+1)+4n+1} - (n+1)d, y_{(t_1+1)+4n+1} \\
s_{(t_1+1)+4n+2} &= x_{(t_1+1)+4n+2}, y_{(t_1+1)+4n+2} + (n+1)d \\
s_{(t_1+1)+4n+3} &= x_{(t_1+1)+4n+3}, y_{(t_1+1)+4n+3} - (n+1)d
\end{aligned}
\qquad (1)
$$

$$where, d > 2(max||s_{i,t} - s_{j,t}||)$$

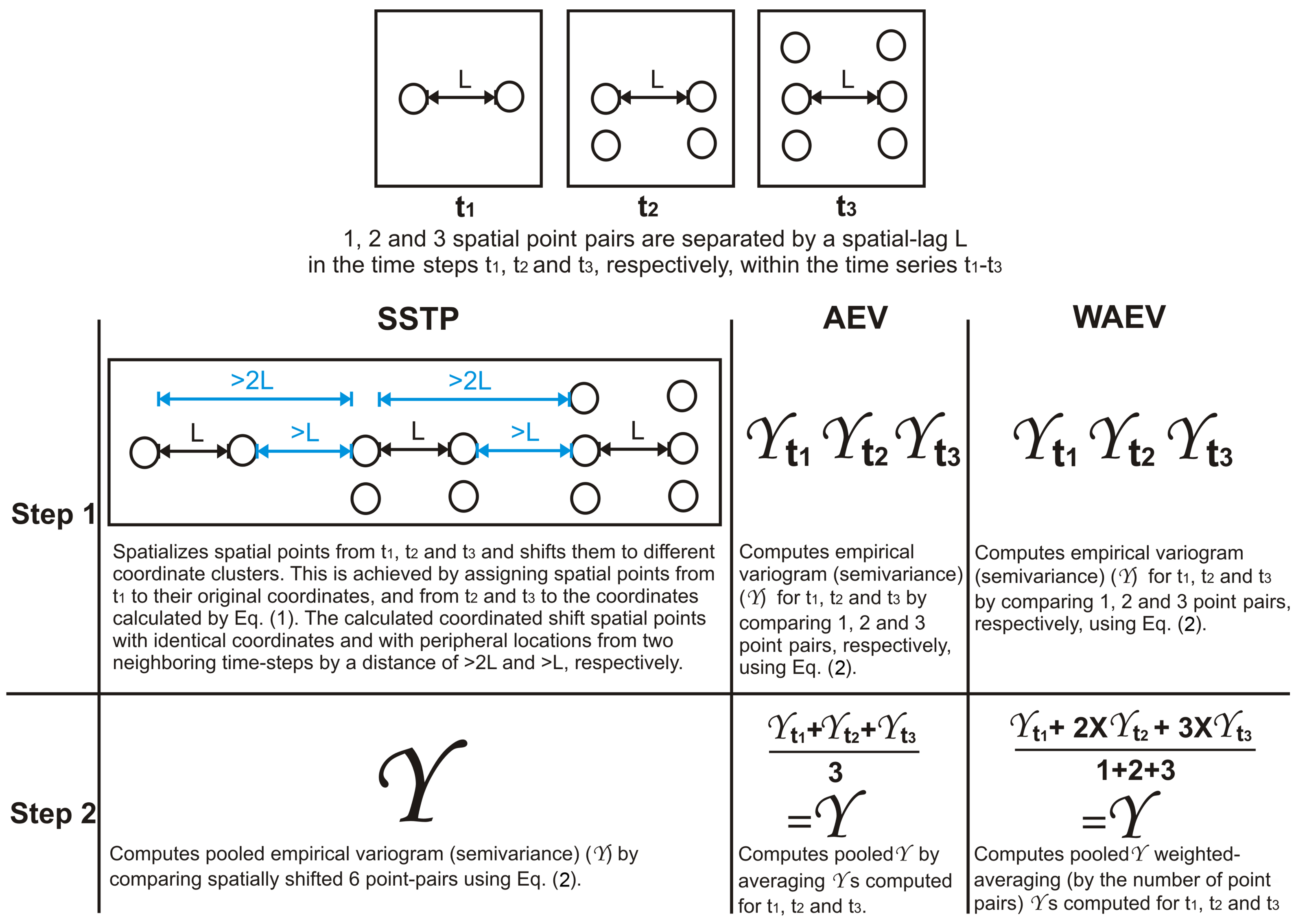

Figure 1: Work-flows of spatially shifting temporal points (SSTP), averaging empirical variograms (AEV) and weighted AEV methods.

Estimation of pooled within-time series variograms

The empirical variograms were computed by simulatenous comparison of spatially shifted points using the commonly applied Methods of Moments (MoM) [3]. For the point pair s_i and s_j, the pooled semivariance $\gamma||s_i - s_j||$ was computed by Eq. (2).

$$\gamma||s_i - s_j|| = \frac{1}{2M||s_i - s_j||} \sum_{i,j} (Z(s_i) - Z(s_j))^2 \qquad (2)$$

$$where, ||s_1 - s_2|| = min||s_{i,t} - s_{j,t}||; ||s_{n-1} - s_n|| = max||s_{i,t} - sj, t||$$

We checked for anisotropy and computed the ratio between the major and minor axes ($A : B$) of the anisotropy ellipse and the anisotropy angle (ϕ).

The available variogram models were fitted to the computed semivariances by a weighted least square approach providing $\frac{M||s_i - s_j||}{(s_i - s_j)^2}$ as weights. The nugget, partial sill, and range (a) parameters were extracted.

Precision of the PTS variogram estimation was evaluated by (i) variogram model-fit (weighted mean of squared error (MSE)) and (ii) cross-validation of a universal kriging (UK) interpolation (root means squared error (RMSE) and Nash-Sutcliffe efficiency (NSE)) using the best-fit models.

Study area and data

SSTP was applied to the PTS variogram estimation for "annual total precipitation in hydrological wet days (PRCPTOT)" in Bangladesh (Figure 2). Currently, 32 rain-gauges report daily precipitation in Bangladesh, classifying the country as data scarce. Moreover, the numbers of data points exhibit an increasing coverage from 8 in 1948 to 32 in 2007, indicating variable lengths of the time-series. PTS variograms were also estimated using the available AEV and weighted AEV (WAEV) (see Figure 1 for details) and the precision statistics were compared with the SSTP estimated variograms.

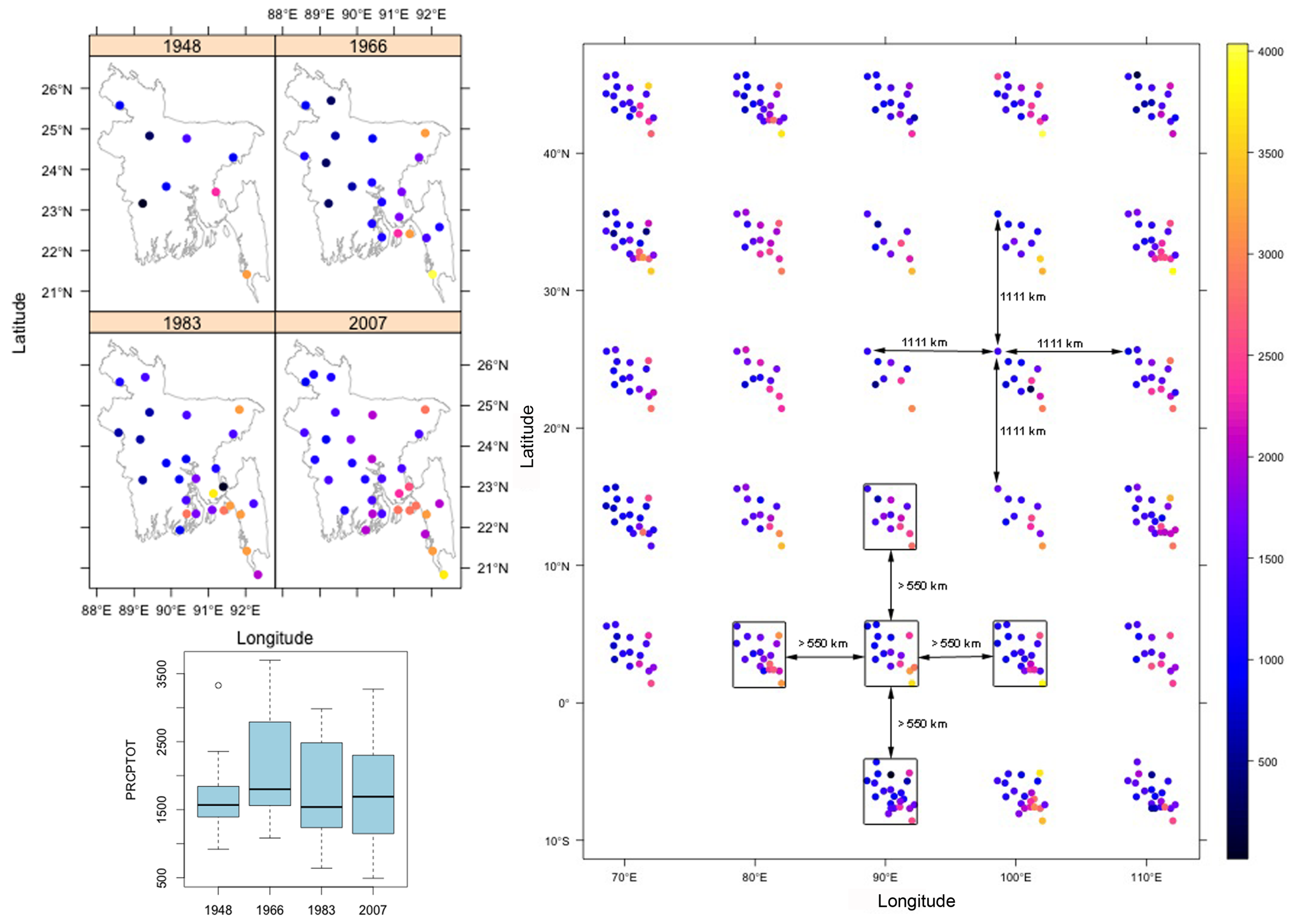

Figure 2: PRCPTOT points & distribution in four representative years (left) and SSTP for 1948-1975 series (right).

Results and Conclusion

"Power model" showed the best-fit for all methods and pooled series except for SSTP in 1948-2007. SSTP computed semivariances exhibited much less noise than the AEV and WAEV, and consequently PTS variograms estimated by SSTP showed lower MSE and RMSE, and higher NSE indicating higher precision than AEV and WAEV (Figure 3). SSTP reduces uncertainty for variogram modelling while preserving spatiotemporal properties. It can be improved by including temporal autocorrelation, external variables and expert elicitation.

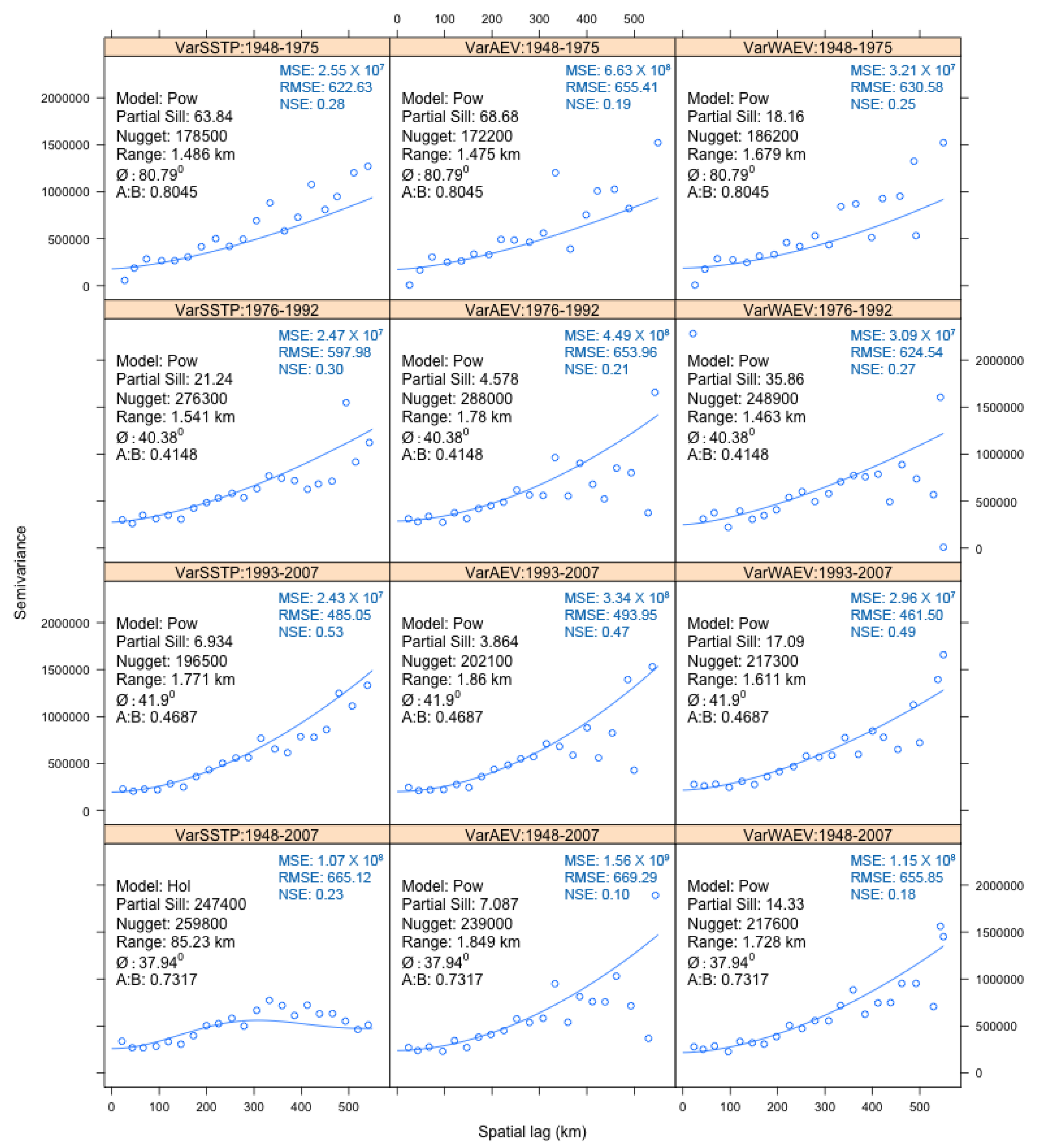

Figure 3: Estimated PTS variograms by SSTP, AEV and WAEV methods with model parameters and precision statistics.

References

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