The Importance of the Feature Importance

Mathematical Foundations of Machine Learning Course

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Random Forest and Wavelet Decomposition

The Classic Random Forest [Breiman,1996]

$T = \{x_i, f(x_i) = y_i\}_{i=1}^{N} \in (\Omega_0 \in \mathbb{R}^n, \mathbb{R})$.

- Draw B bootstrap samples $\{T_j\}_{j=1}^{B}$ of $0 < P < 100$ percents of T
- Fit a decision tree for each one, consider m covariates in each split
- Average all tree predictions

Denote $\Omega \subseteq \Omega_0$, $c_\Omega = \frac{1}{\#\{x_i \in \Omega\}} \sum_{x_i \in \Omega} f(x_i)$. The next split:

$$\min_{\Omega' \cup \Omega'' = \Omega} \sum_{x_i \in \Omega'} \left(f(x_i) - c_{\Omega'}\right)^2 + \sum_{x_i \in \Omega''} \left(f(x_i) - c_{\Omega''}\right)^2$$

Splitting Criteria Equivalence [Elisha and Dekel, 2016]

$$\max_{\Omega' \cup \Omega'' = \Omega} \|\psi_{\Omega'}\|_2^2 + \|\psi_{\Omega''}\|_2^2$$

where $\psi_{\Omega'} = I_{\Omega'}(c_{\Omega'} - c_\Omega)$ and $\|\psi_{\Omega'}\|_2^2 = \|c_{\Omega'} - c_\Omega\|_2^2 \# \left\{x_i \in \Omega'\right\}$.

The Wavelet Representation of Random Forest

$$\tilde{f}(x) = \frac{1}{B} \sum_{j=1}^{B} \sum_{\Omega \in t_j} \psi_{\Omega}(x)$$

A "pruned" representation of the ensemble [Elisha and Dekel, 2016]:

$$\|\psi_{\Omega_{k1}}\|_2 \geq \|\psi_{\Omega_{k2}}\|_2 \geq \|\psi_{\Omega_{k3}}\|_2 \dots$$

Final prediction: $\hat{f}_M(x) = \frac{1}{B} \sum_{m=1}^{M} \psi_{k_m}(x)$.

Representation Size Selection

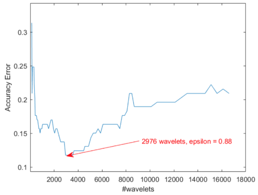


Figure 1: Illustration of the choice of M on the validation set [Elisha and Dekel]

Approaches of Feature Importance

Approaches of Feature Importance

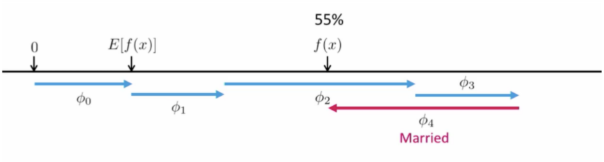
Global Feature Importance of feature i:

- **Gain:** Reduction of loss/impurity contributed by all splits by i
- **Split Count:** Number of times feature i was used to split the trees
- **Permutation:** Randomly permute the values of i in the test set
- **Wavelet decomposition based:** choose $\epsilon = \|\psi_{k_{M^*}}(x)\|_2$

$$s_i^\tau = \frac{1}{B}\sum_{j=1}^{B} \sum_{\Omega \in t_j \wedge v_i, \|\psi_\Omega\| \geq \epsilon} \|\psi_\Omega\|_2^\tau$$

Global&local Feature Importance of feature i:

- **SHAP:** Given a feature i and an observation x, $E[f(x) \mid x_i]$.
  The global FI is the mean of absolute values of the SHAP values.



Modeling Methodology

Modeling Methodology

Model parameters are chosen using 5-fold CV over a grid of parameters.

We consider the following Feature Importance methods:

- Gain on RF and GB
- SHAP on RF and GB
- Permutation FI on RF
- Wavelet-based FI

We compare the top k features selection using varied models:

- **Regression:** SVM, RF, linear regression
- **Classification:** SVM, RF, logistic regression

Regression Task

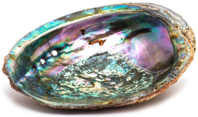
Regression Task

Abalone Dataset

Abalone's age (number of rings) $\approx$ physical measurements

Data description - number of rings with respect to:

- Sex
- Length
- Diameter
- Height
- Whole weight
- Shucked weight
- Viscera weight
- Shell weight



Classification Task

Classification Task

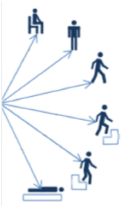
Human Activity Recognition Data

HAR $\approx$ accelerometer and gyroscope data

The data is originated in recordings of 30 subjects performing activities of daily living.

HAR classes:

- walking
- walking upstairs
- walking downstairs
- sitting
- standing
- laying



Results

Results

In order to further investigate the results, use the following **link** and explore our Shiny app.

For an HTML view of the results, use this **link**, or visit the github repo using this **link** for the full code.