1. The minimax loss function in GAN promotes an adversarial approach to improving the generator and discriminator aspects. The value function of GANs is established as a two-player minimax game represented by the equation below.

   $$\min_{G} \max_{D} V(D,G) = \mathbb{E}_{x\sim p_{data}(x)}[\log D(x)] + \mathbb{E}_{z\sim p_z(z)}[1 - \log D(G(z))]$$

   The discriminator D aims to maximize classification accuracy by giving high scores to real samples and low scores to generated (fake) samples. At the same time, G tries to minimize this objective, allowing it to produce outputs that fool D. Each iteration of the training updates D (to separate better real from fake) and G (to mimic real data better). The networks start competing with each other, improving little by little. The hope is that they will reach a Nash equilibrium, at which point the generated samples cannot be distinguished from real ones.

2. Mode collapse can be observed when a generator correlates many different input z to one output (or a few outputs). The generator sacrifices diversity to produce a few "safe" samples that trick the discriminator and score well. This distorted behavior happens since the discriminator's gradients will not punish the lack of variation enough. Thus, the generator converges to a degenerate equilibrium with limited modes of the data distribution, which is not very useful. Researchers adopt many stabilization methods to prevent mode collapse. Batch normalization normalizes the intermediate activations to smooth training, while minibatch discrimination encourages diversity by comparing the samples within the minibatch and penalizing ones with very similar outputs. Wasserstein-based approaches (WGAN) rework the loss so as to provide more informative gradients, even when support sets do not overlap. The combination of these above methods helps maintain a healthy adversarial balance and leads to diverse samples.

3. In adversarial training, the discriminator is a learned critic that gives a probability D(x) deciding whether the input x comes from the real data distribution or the generator. The discriminator continually improves its decision boundary. It seeks to maximize its ability to classify the outputs of the generator and the real samples. Thus, it scores high for real samples, but low for the generator outputs. The gradients flowing back through the discriminator tell the generator where its outputs do not look real, which gives the generator precise update signals. In short,

the generator's improvement is caused by the evolving judgment of the discriminator, which makes the discriminator essential to the GAN.

4. The Inception Score (IS) assesses the output of GAN by using a pretrained Inception v3 network. It computes the conditional label distribution p(y|x). IS assesses whether the quality of the image samples is good, in that they produce a low entropy p(y|x), meaning that samples receive confident class (y) and meaningful class (y) on average. IS also assesses sample diversity. In particular, the marginal p(y) across all samples has high entropy, meaning we see a wide spread of classes (y). It uses the mean KL divergence between p(y|x) and p(y), where higher scores lead to a more appropriate mix of realism and variety.

   Unlike the previous measure, the Fréchet Inception Distance (FID) compares the real and generated data distributions in the feature space of the same pretrained network. To compute the Fréchet distance equation, FID models each set of activations as a multivariate Gaussian with mean μ and covariance σ.

   When the FID values are lower, they show that the generated samples have closer feature means and feature covariances to the data distribution of actual samples. Thus, FID is important for estimating GAN samples' overall fidelity and diversity.

Sources:

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