1. (10 points)

Carl Friedrich Gauss (1777-1855) is considered to be one of the greatest mathematicians of all time. He made significant contributions to almost every area of mathematics. Not much is really known about his early childhood, though legends abound. According to one of them, as an eight-year-old boy, he was very mischievous and often disruptive in classroom. In order to keep him occupied, his math teacher asked him to add the numbers from 1 to 100. Before the teacher walked back from his desk to the blackboard, Carl yelled out the answer 5,050. The teacher's reaction has not survived to our time. It is definitely untrue that he dropped in an apoplectic fit…

Your first puzzle for today is a little, but only a little, harder. You are to find the sum of all the numbers from 1 to 1000, except for all the perfect squares. (Yes, to be more precise, you are to add all the numbers from 2 to 1000 skipping all the perfect squares – 1 being a perfect square.) The first few terms of your sum are:

$$2 + 3 + 5 + 6 + 7 + 8 + 10 + 11 + 12 + 13 + 14 + 15 + 17 + \cdots$$

2. (20 points)

Winnie an immortal inchworm, requiring no nourishment, can move at a constant speed of one inch per second. One beautiful sunny day at exactly noon, Winnie stumbled upon an infinitely elastic string (a piece of an ideal rubber band, if you must know) exactly one-foot long. Excitedly, Winnie stepped onto the string and within a second covered 1/12 of its length. Alas, just then, a malicious, evil, and equally immortal wizard Salamander pulled the other end of the string instantaneously stretching it uniformly by one foot, thus making it 2 feet long. Unperturbed, Winnie moved on and covered another inch of the now two-foot long string in one second flat. But then again, Salamander instantaneously stretched the string by one foot. This game of wizard and worm went on for a very long time. Each second Winnie traversed one inch of the string only to have it uniformly stretched by one foot every second on the second. When did Winnie step off the other end of the string, if ever? And if she did, how long was the string at the time?

3. (20 points)

$$\sqrt{2018}$$

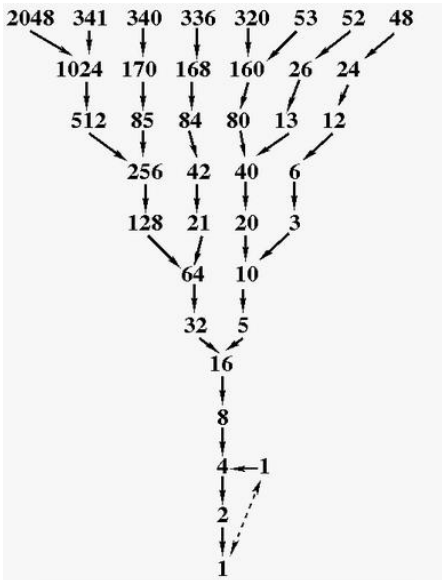
Irrational numbers are those real numbers that cannot be expressed as fractions. It is well known that $\sqrt{2}$ is an irrational number. Ancient Greeks had a big problem with this fact, allegedly even executing some people by drowning, for making this claim (cf. Hippasus of Metapontum). Regardless, we have decent approximations of $\sqrt{2}$ by fractions. For example, 3/2 – which is off by less than 0.1. The approximation often used in the Jewish Talmud is 7/5 – which is off by less than 0.02. The best ancient approximation was the one discovered by the Babylonians. It is accurate to six decimal places and is given by the following expression.

$$1+\frac{24}{60}+\frac{51}{60^2}+\frac{10}{60^3}=\frac{30{,}547}{21{,}600}$$

Which only goes to show you, that one should never underestimate the people using the Sexagesimal counting system (base 60).

Putting all history associated with the square root of 2 aside, your task is to come up with the best approximation to $\sqrt{2018}$ by a fraction with the denominator no larger than 20,180. It better be better than $\frac{269}{6}$ which happens to be off by less than one tenth.

4. (40 points)



A Collatz Tree, also known as a Collatz sequence, Collatz Conjecture, or as a 3n+1 puzzle, is a famous unsolved mathematical problem. The statement of the problem is quite simple. Starting with an arbitrary natural number (positive integer), if it happens to be even, then divide it by two. If it is not even, then multiply it by 3 and add 1 (thus getting an even result). Continue this process. Show that eventually the sequence has to end in an infinite cycle $4 \rightarrow 2 \rightarrow 1 \rightarrow 4 \rightarrow 2 \rightarrow 1 \ldots$

Restated formally, we define a function f on natural numbers as follows:

$$f(n) = \begin{cases} n/2, & if\ n\ is\ even \\ 3n+1, & if\ n\ is\ odd \end{cases}$$

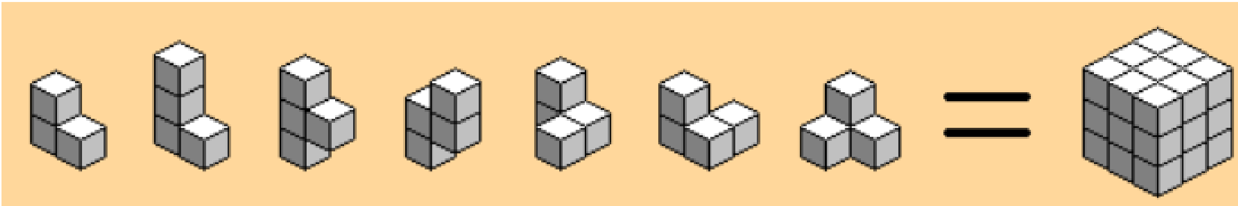
The conjecture is that successive application of f to any natural number has to terminate in the 1, 4, 2, 1, 4, 2, 1… cycle

Thus, for example,

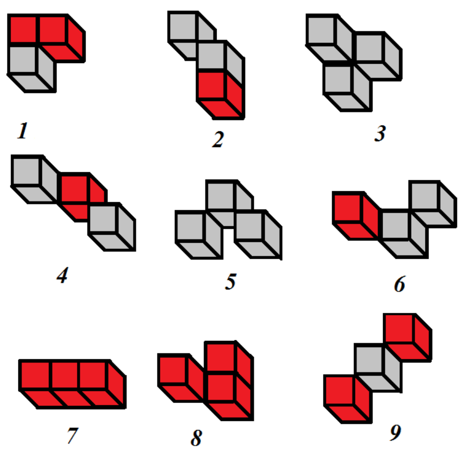
$f(340) = 170$	$f(170) = 85$	$f(85) = 256$	$f(256) = 128$
$f(128) = 64$	$f(64) = 32$	$f(32) = 16$	$f(16) = 8$
$f(8) = 4$	$f(4) = 2$	$f(2) = 1$	

Your mission, should you choose to accept it, is to calculate the number of times this function has to be applied to the consecutive numbers in the Collatz sequence beginning with two sextillion, five hundred eighty one quintillion, three hundred eighty three quadrillion, one hundred seventy four trillion, two hundred forty one billion, eight hundred fifty four million, one hundred sixty thousand and eight hundred ninety six (written out numerically, for those who like numbers better than words, this is 2,581,383,174,241,854,160,896) before the number 1 is reached. In the Illustration above the function had to be applied 11 times starting with 340 before reaching 1.

5. (40 points)



In the year 1936, while attending a lecture on Quantum physics by Werner Heisenberg, a Danish author Piet Hein allegedly came up with a dissection of a 3x3x3 cube into 7 polycube pieces as shown above. The puzzle is known as SOMA cube. Not counting symmetric solutions, the polycubes can be arranged into a 3x3x3 cube in 240 ways. One of the polycubes consists of 3 unit cubes, the remaining six consist of four cubes each. A more interesting puzzle consists of dissecting the cube into 9 pieces of 3 unit cubes each (cf. figure below). Unlike Hein's polycubes which consist of unit cubes joined side to side, these pieces consist of unit cubes that are often joined merely along their edges.

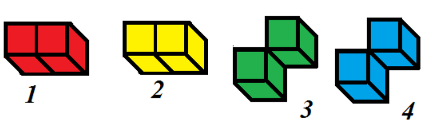


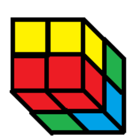
If the unit cubes in a 3x3x3 cube are described as ordered triples of their coordinates, then these pieces can be described as the following sets of unit cubes

1: {(1,1,1), (1,2,1), (2,2,1)} 2: {(1,3,1), (2,1,1), (2,2,1)}
3: {(1,2,2), (1,1,1), (2,2,1)} 4: {(1,2,2), (2,2,1), (3,1,1)}
5: {(1,1,1), (2,2,1), (3,1,1)} 6: {(1,1,2), (2,1,1), (3,2,1)}
7: {(1,1,1), (2,1,1), (3,1,1)} 8: {(1,1,2), (2,1,1), (2,2,1)}
9: {(1,1,1), (2,2,2), (3,3,3)}.

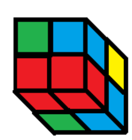
Your task is to arrange these pieces into a 3x3x3 cube. A photograph of a sample solution appears at left. Describe your solution as a 3x3x3 array of integers 1 through 9, where each of the 27 unit cube coordinates contains the ordinal number of one of the pieces above.

For example, for a 2x2x2 cube, given four pieces of two cubes each as shown below left, {{{3,4},{4,3}},{{1,1},{2,2}}} is not a solution because pieces 3 and 4 cannot intersect on the bottom layer of the cube (below right, left cube) On the other hand, {{{3,4},{2,2}},{{1,1},{3,4}}} is a solution (as depicted below right).



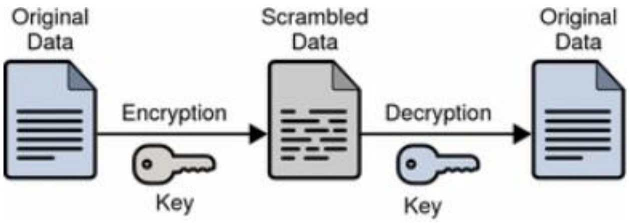


Impossible



Solution

6. (40 points)



A simplistic encryption scheme uses XOR (bitwise exclusive disjunction) applying it repeatedly between every byte of the file and the bytes comprising the key (password). To decrypt the output, merely reapplying the XOR function with the key will remove the encryption. For example, encrypting string “hackathon” with key “PSWD” results in the string “824/1’?+>” as shown below. To decrypt, just apply XOR to the string “824/1’?+>” using the same key “PSWD”.

	h	a	c	k	a	t	h	o	n
	01101000	01100001	01100011	01101011	01100001	01110100	01101000	01101111	01101110
	P	S	W	D	P	S	W	D	P
	01010000	01010011	01010111	01000100	01010000	01010011	01010111	01000100	01010000
^	00111000	00110010	00110100	00101111	00110001	00100111	00111111	00101011	00111110
	8	2	4	/	1	'	?	+	>

This simplistic scheme was used to encrypt a Microsoft Word file containing an English sentence. The password used was 8 bytes long and consisted only of upper and lower case letters. Armed with this information, it should not be too difficult to decrypt the encrypted file WORDFILEENCR.DOCX, which is provided. If you are unable to decrypt the file, you will get partial credit for determining the password used.

7. (40 points)

There are many different compression algorithms.

https://en.wikipedia.org/wiki/Data_compression

https://www.youtube.com/watch?v=xyKA4arxQ5I

https://www.geeksforgeeks.org/lzw-lempel-ziv-welch-compression-technique/

You are provided with a jpeg file, File2Compress.jpg . Your task is to write two programs: one to compress this file, the other to decompress its compressed version. You will have to demonstrate both programs and you will be judged on the size of your compressed file.

8. (40 points)

This is a GUI challenge with a choice. You can implement either one of the two problems described below. However, whichever problem you select, your solution should consist of a dynamic graphic presentation (an animation) as for example in the following

A) https://en.wikipedia.org/wiki/File:Eight-queens-animation.gif
B) https://en.wikipedia.org/wiki/File:Knights-Tour-Animation.gif

Problem A is known as the Eight Queens Puzzle. It was first proposed in 1848 by Max Bezzel. It requires placing 8 chess queens on an 8x8 chessboard in such a way that no two queens attack one another. Probably because nobody cared, the first solution was not published until 1850 (by Franz Nauck). Nauck later generalized the puzzle to the n queens problem, with n queens on a chessboard of n × n squares.

Since then, many mathematicians, including Carl Friedrich Gauss (remember Gauss?), have worked on both the eight queens puzzle and its generalized n-queens version.

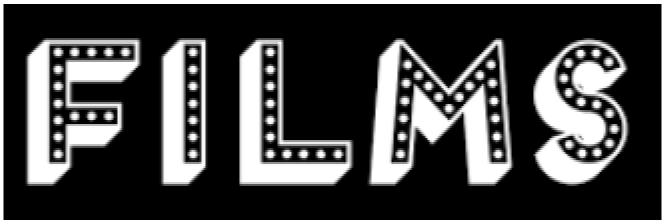
In 1972, Edsger Dijkstra used this problem to illustrate the power of what he called structured programming. He published a highly detailed description of a depth-first backtracking algorithm.

Your solution should demonstrate dynamic queen placement on a 7x7 board, not unlike that on the first link, above.

Problem B is known as a Knight's Tour. It is of a much older origin, going back to the 9th century. It requires moving the chess knight on a chessboard in such a way that the knight visits each and every square once. Many chess players used to demonstrate a solution blindfolded, by calling out successive squares visited by the knight.

Your solution should demonstrate dynamic tour starting on the square b1 – initial square of the white queen's knight on the chessboard.

9. (50 points) Loosely defined problem with lots of partial credit. You will be asked to explain your solution and submit the code.



Targeted advertising is a form of advertising where online advertisers use various heuristic methods to target the most receptive audiences with certain traits, based on the product the advertiser is promoting. Cablevision uses this technique to recommend films based on the movies you have watched. Your goal is to come up with a suggestion of three movies for a middle age male who has watched the following 10 films during the prior 30 days:

Star Wars the Last Jedi,
Blade Runner,
The Martian,
Good Will Hunting,
the Bourne Supremacy,
Edge of Tomorrow,
Oblivion,
Minority Report,
Vanilla Sky,
Rain Man

You are encouraged to mine any online movies database and to implement several characteristics of available films. Using these characteristics applied to the list above, select three additional films with characteristics coming closest to the ones of the listed films.