

3-dimensional plots in MATLAB

[1] 3D Trajectory of a Charged Particle in Magnetic Field:

Assume a charged (q) particle having velocity component both along z axis and x axis (i.e., $\vec{v} = v_x \hat{i} + v_z \hat{k}$) enters into a region where there is a magnetic field along z direction (i.e., $\vec{B} = B_z \hat{k}$). Because of the work-less magnetic force ($F = q \cdot \vec{v} \times \vec{B}$), the trajectory of the particle is given by the following equations,

$$x = x_0 \cdot \cos(\omega_c t)$$

$$y = x_0 \cdot \sin(\omega_c t)$$

$$z = v_z \cdot t$$

where, $\omega_c = qB_z/m$ is the cyclotron frequency. Plot the trajectory of the particle. This kind of trajectory is called helical trajectory or helix.

[Hint: Notice that it has only one independent parameter. Choose the values of the constants (x_0 , ω_c & v_z) as you wish]

[2] Quantum particle in a 2-d Box:

In quantum mechanics, it is assumed that a particle also exhibits wave-like behavior [wave-particle duality]. As a result, a quantum particle is always associated with a wavefunction (Ψ). The wavefunction of such a particle in a 2-dim (x - y plane) square box is given by,

$$\Psi_{n_x, n_y}(x, y) = \frac{2}{L} \cdot \sin\left(\frac{n_x \pi x}{L}\right) \cdot \sin\left(\frac{n_y \pi y}{L}\right)$$

where, L is the length of each side of the box, n_x & n_y are positive integers and $x, y \in [0, L]$. Plot the square of the wavefunction, i.e., $(\Psi_{n_x, n_y})^2$ for three cases, case-1: $n_x = 1$, $n_y = 1$, case-2: $n_x = 1$, $n_y = 2$ & case-3: $n_x = 2$, $n_y = 2$ in three separate figures.

[Note: Choose the x-limit and y-limit carefully. $|\Psi_{n_x, n_y}(x, y)|^2$ signifies the probability of finding that particle at that position (x, y)]

[3] Modes of a Circular Membrane:

Tabla is a well-known circular membrane. When not played, it can be considered a circular membrane in the x - y plane with zero vertical (z) displacement. Now when a musician plays it, he actually creates different vibrational modes, i.e, some particular patterns of displacement along z -direction. At a particular time, two such modes are given by,

$$z_0(x, y) = J_0(k_0 \cdot \sqrt{x^2 + y^2})$$

$$z_1(x, y) = J_1(k_1 \cdot \sqrt{x^2 + y^2}) \cdot \frac{y}{\sqrt{x^2 + y^2}}$$

where, J_0 and J_1 are 0'th and 1'st order Bessel functions respectively. Plot the two modes in two separate figures.

[Hint: Fortunately MATLAB includes a built-in Bessel function. `besselj(n,M)` computes the n 'th order Bessel function $J_n(M)$ for each element in array(or matrix) M . Use it. k_0 and k_1 are constants. Choose their values as you wish.]