[1] **Projectile Motion: Effect of Launch Angle**

A particle is projected with an initial speed v at an angle θ above the horizontal. Neglecting air resistance, its trajectory follows the equations:

$$x(t) = vt\cos(\theta),\ \ y(t) = vt\sin(\theta) - \frac{1}{2}gt^2$$

where g=9.8 m/s^2 is the acceleration due to gravity.

Tasks:

1. Write a MATLAB script to simulate the projectile motion for a fixed initial speed (e.g., v=20 m/s) and for a fixed angle θ = 45° **until the projectile returns to the ground**.
   - Compute the trajectory (x(t), y(t)).
   - Plot x(t) vs t, y(t) vs t and x(t) vs y(t) in a single plot [Use subplot].
2. Write a MATLAB script to simulate the projectile motion for a fixed initial speed (e.g., v=40 m/s) and a range of launch angles θ = {0 °, 15°, 30°, 45°, 60°, 75°, 90°} **until the projectile returns to the ground**.
   - For each launch angle:
     - Compute the trajectory (x(t), y(t)).
     - Plot all trajectories on a single figure, clearly indicating which curve corresponds to which angle.
   - For each angle, compute and display the **maximum height** reached by the projectile.

(This kind of projectile motion modeling is fundamental in classical mechanics, with applications ranging from ballistics to sports physics and orbital launches.)

[2] **<u>Damped Harmonic Oscillator:</u>**

(a) Exponential Decay of Amplitude:

A mass–spring system experiences an exponentially decaying oscillation when a small damping force is present. If the damping coefficient γ is sufficiently small ($\gamma<\omega$), the system remains **underdamped**, and its displacement from equilibrium follows:

$$x\,(t) = Ae^{-\gamma t}\cos\,(\omega t)$$

Tasks:

1. For a given parameter set (e.g., $A=1$, γ $=0.2$, $\omega=2$ rad/s), compute and plot the displacement x(t) as a function of time over a suitable interval $0 \leq t \leq T$.
2. Keeping γ and ω fixed, repeat the above computation for multiple values of A (e.g., A=0.5, 1.0, 1.5, 2.0, 5.0), and plot the resulting trajectories on the same figure.

Ensure you label both axes clearly.

(This type of **damped oscillatory motion** is ubiquitous in physical systems, from **mechanical resonators** losing energy through friction to **electrical RLC circuits** dissipating energy via resistive elements.)

(b) [Optional] Transition Across Damping Regimes:

A damped harmonic oscillator exhibits qualitatively different behavior depending on the relation between the damping coefficient γ and the natural frequency ω_0. The general solution to the equation

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = 0$$

falls into three distinct regimes:

- Underdamped ($\gamma < \omega_0$): decaying oscillations.

- Critically damped ($\gamma = \omega_0$): fastest return to equilibrium without oscillation.

- Overdamped ($\gamma > \omega_0$): slow return to equilibrium, no oscillation.

Tasks

• Fix $\omega_0 = 2$ rad/s.

• For three values of γ: 0.5, 2.0, and 4.0, determine the damping regime.

• **Use the appropriate analytical solution for each case:**

- Underdamped:

$$x(t) = e^{-\gamma t} [\cos(\omega_d t) + (\gamma / \omega_d) \sin(\omega_d t)], \quad \text{where } \omega_d = \sqrt{(\omega_0^2 - \gamma^2)}$$

- Critically damped:

$$x(t) = (1 + \gamma t)\, e^{-\gamma t}$$

- Overdamped:

$$x(t) = D_1\, e^{-\lambda_1 t} + D_2\, e^{-\lambda_2 t} \quad \text{where } \lambda_{1,2} = \gamma \pm \sqrt{(\gamma^2 - \omega_0^2)}$$

with D_1 and D_2 determined from:

$$D_1 + D_2 = 1$$

$$-\lambda_1 D_1 - \lambda_2 D_2 = 0$$

• Compute and plot the solution $x(t)$ for each γ over a common time interval.

• Display all three responses on the same figure and clearly indicate the damping regime for each.

(This classification of damping is foundational in understanding how physical systems respond to dissipation, with applications ranging from mechanical suspension design to transient response in electrical circuits.)

[3] **Bar Chart: Energy Measurements Over Time**

In many experimental setups—such as solar cells, oscillating systems, or thermal devices—discrete energy measurements are taken at regular intervals across multiple trials or days. A bar chart provides a clear visual comparison of how these values vary across time or conditions.

Problem Statement:

A device is tested over five consecutive days, and three energy measurements are taken each day under identical conditions.

1. Create a 5×3 matrix to store the measured energy values (e.g., in arbitrary units). You may use randomly generated values or manually input data.
2. Plot the data using a **grouped bar chart**, where each group represents a day and each bar within a group corresponds to one of the three measurements.
3. Clearly label the x-axis as "Day", the y-axis as "Energy (arb. units)", and include a legend identifying the three measurements.

Find out the day with the **highest total energy output** across all three measurements.

(This type of data representation is frequently used in experimental physics and engineering to identify trends, anomalies, or systematic shifts across repeated measurements.)