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ABSTRACT

In the last few decades, we have witnessed numerous breakthroughs been made in Computational Electromagnetics (CEM). Together with advancements in computational hardware, today, it is quite common to solve full-wave three-dimensional electromagnetic wave problems with hundreds of millions or even billions unknowns. In the past, research was mainly focused on individual CEM technique, such as finite difference time domain (FDTD), finite element methods (FEMs), and the Integral Equation Methods or Method of Moments (MoM), to name a few. For FDTD, we have seen successful developments on dispersive material modeling, sub-gridding techniques, higher order finite difference schemes, higher order Absorbing Boundary Conditions (ABCs) and Perfectly Matched Layer (PML). On the FEM side, vector finite element, tree-cotree splitting, four potential AV formulations, multi-grid preconditioners, hybrid Finite Element Boundary Integral (FEBI) for mesh truncation, and of course the application of PML as well. On the IEs, there are the Rao-Wilton Glisson (RWG) conforming basis functions, the Calderon identities and the Calderon preconditioners, and most significantly, the research community has been awed by the fast computational techniques developed in IEs: such as FFT based methods, and the multi-level fast multipole method (MLFMM).

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CHAPTER 1
INTRODUCTION

1.1 Computational Electromagnetic

Computational Electromagnetics, or CEM, is the science of modeling how electromagnetic fields interact with the environment, with physical objects, and with each other. CEM uses powerful computers to perform calculations using Maxwell's equations and can work out radar cross sections, antenna performance and electromagnetic compatibility.

It is also described as the process of modeling the interaction of electromagnetic fields with physical objects and the environment. It typically involves using computationally efficient approximations to Maxwell's equations and is used to calculate antenna performance, electromagnetic compatibility, radar cross section and electromagnetic wave propagation when not in free space. A specific part of computational electromagnetics deals with electromagnetic radiation scattered and absorbed by small particles.

Computational models in electromagnetic compatibility, provide the foundation for numerous applications in both research and industry purposes. A powerful computer is an everyday tool of engineers and researchers and it is expected to become more powerful, allowing for widespread modeling of EMC problems. Computational models have become more and more important, especially when applied to problems that are not easily handled with experimental methods, like human exposure to electromagnetic fields. First of all we should have idea about EMC.

Electromagnetic compatibility (EMC) is the applied discipline within the science of electromagnetism including almost all relevant areas of theoretical (computational) and experimental electromagnetics. Theoretical methods in electromagnetics can be classified as analytical or numerical. Though many significant strides have been made in EMC modeling in the past few decades, progress in this research topic is expected to continue at a rapid pace. EMC is usually regarded as the ability of a device to function satisfactorily within its electromagnetic environment, that is a device, system, or equipment is assumed not only to be unaffected by external fields but also not to cause interference in sense of intolerable electromagnetic disturbances to a nearby system or anything in that environment. Satisfactory operation of a device, equipment, or system implies their functional work and immunity to certain interference levels which can be regarded as normal in the environment even under these circumstances. Therefore, the principal task of EMC is to suppress any kind of electromagnetic interference (EMI). The first request is often regarded as immunity testing, that is, once the device is constructed it is necessary to check if it can be a potential victim of EMI or if it satisfies the EMC request of being unaffected by an external source produced by its electromagnetic environment.

The second request raised in the design process that device is not a potential EMI source that is its normal operation does not interfere with other electrical systems, is referred to as the emission testing. In a theoretical sense, the aspects of immunity and emission testing, respectively, are related through the reciprocity theorem in electromagnetics. Generally, the methods used in EMC are not only to visualize electromagnetic phenomena but also to predict and suppress interferences can be regarded as either theoretical or experimental.

Electromagnetics as a rigorous theory started when James Clerk Maxwell derived his celebrated four equations and published this work in the famous treatise in 1865. In addition to Maxwell's equations themselves, relating the behavior of electromagnetic fields and sources, several other physical relationships are necessary for their solution. The most important are Ohm's law, the equation of continuity, and the constitutive relations of the medium and the imposed boundary conditions of the physical problem of interest. Before Maxwell, the science of electromagnetism had existed mostly as an experimental discipline for several centuries through the works of scientists such as Benjamin Franklin, Charles Augustin de Coulomb, Andre Marie Ampere, Hans Christian Oersted, and Michael Faraday. The early doubt about Maxwell's theory vanished in 1888 when Heinrich Hertz transmitted and received radio-waves, thus having demonstrated the validity of the Maxwell theory.

The early works on analytical solution methods in electromagnetics, based on Maxwell's equations, were mainly focused on the area of radio science. Some of such applications of the electromagnetic theory started to appear not long after Maxwell's treatise had appeared. Among the analyzed simple geometries were the fields radiated from the Hertzian dipole, an infinitely long straight circular wire and two coaxial cones. In most of the cases, the equations were solved as boundary value problems having yielded to the solution in terms of infinite series expansions.

CEM is a young discipline. It is still growing, in response to the steadily increasing demand for software for the design and analysis of electrical devices. Few years ago, most electrical devices were designed by building and testing prototypes, a process that is both costly and slow. Today the design can be made faster and cheaper by means of numerical computation. CEM has become a main design tool in both industrial and academic research. There are numerous application areas for CEM, viz. in electric power engineering, computation is well established for the analysis and design of electrical machines, generators, transformers, and shields. In applications to microwaves, CEM is a more recent tool, but it is now used for designing microwave networks and antennas, and even microwave ovens. The analysis and optimization of radar cross sections (RCS) for stealth devices has been the driving force for the development of many new techniques in CEM. The clock frequencies of modern microprocessors are approaching the region where circuits occupy a large fraction of a wavelength. Then ordinary circuit theory no longer applies and it may be necessary to solve Maxwell's equations to design smaller and faster processors. The increased demand for electromagnetic compatibility (EMC) also poses new computational problems. The performance of CEM tools is increasing rapidly. Most common applications of CEM are:

Finite element methods for microwave components

Finite element methods are used to simulate electromagnetic fields at microwave frequencies and design microwave components. Design, development and application of adaptive finite element methods for large-scale parallel and distributed computing environments; computational modeling, simulation and visualization of electromagnetic fields in microwave, optical and power frequency devices.

High performance computational electromagnetics

For large-scale electromagnetic simulations development of robust parallel 3-D automatic mesh generation procedures and solution strategies for adaptive finite element methods (AFEMs). For example, application of parallel and distributed simulation methods on emerging multi-core platforms and reconfigurable hardware to the development of accurate and efficient CAD tools for microelectronic systems performance.

Analysis of new antenna designs for specific applications. Examples include: broadband antennas for microwave breast scanning in bio medicals, compact antennas for hand-held devices.

Biomedical applications

Interaction of electromagnetic waves with tissues. Examples: microwave tumor detection and monitoring, light propagation through retinal photoreceptors, absorption of electromagnetic power from the cellular telephone by the human head tissues.

Analysis and design of electrical machines and low frequency systems

The development of new and efficient optimization process for electromagnetic systems using virtual prototyping techniques based on advanced computer based simulations. Examples include: induction and permanent magnet machines, micro-electromechanical systems (MEMS); generators for renewable energy (Wind, tidal, etc.), sensors and actuators.

Intelligent autonomous design systems

The development of computer systems based on artificial intelligence techniques, for the automatic design of low frequency electromagnetic systems. Examples include: applications of expert system technology; case-based reasoning; neural networks; blackboard systems and constraint propagation techniques.

Application area of CEM is so vast. Each application requires specific method for computation. Or we can say in real time it is difficult have a only method by which all the problems or application can be solved. So according to the nature of the problem a particular method is to be selected. In short we can say that these methods are application specific,

The following methods are among the most commonly methods used as numerical methods in EM

a) Finite differences (FD) (usually in the time domain),

b) Finite element method (FEM),

c) Boundary element method (BEM), which is usually referred to, for historical reasons, as the method of moments (MoM).

Finite difference methods are more or less straight forward discretizations of Maxwell's equations in differential form, using the field components, or the potentials, on a structured grid of points as unknowns. Finite differences in general , and the finite-difference time-domain (FDTD) method in particular, are very efficient and require few operations per grid point. The

FDTD is one of the most widespread methods in CEM, and it can be applied to a large variety of microwave problems. One drawback of finite difference methods is that they work well only on uniform Cartesian (structured) grids, and typically use the so-called staircase approximation of boundaries not aligned with a grid.

Finite element methods in which the computational region is divided into unstructured grids (typically triangles in two dimensions and tetrahedral in three dimensions) can approximate complex boundaries much better, but are considerably slower in time-domain calculations. The FEM is mainly used for time-harmonic problems, and it is the standard method for eddy current calculations.

The MoM discretizes Maxwell's equations in integral form, and the unknowns are sources such as currents or charges on the surfaces of conductors and dielectrics. This method is advantageous for problems involving open regions, and when the current-carrying surfaces are small. The MoM is often applied to scattering problems. We will discuss how the three types of methods, FD, FEM, and MoM, can be applied to different electromagnetic problems, in both the time domain and the frequency domain (time-harmonic fields and currents). Before proceeding we must have a little idea about the field theory.

1.2 Review of Electromagnetic Theory

Electromagnetic field theory is a study of an electric charge at rest and in motion. It encloses the analysis, synthesis, physical interpretation, and application of electric and magnetic fields. It is a branch of physics in which both electric and magnetic phenomena are studied. From elementary physics we know that there are too kind of charges positive and negative. Both positive and negative charges are sources of electrical field. Moving charge produce current, this gives rise to magnetic field. A field is a spatial distribution of quantity, which may or may not be function of time. Time varying field is accompanied by magnetic field and vice versa. In other words, time varying electric and magnetic field are coupled, resulting in an electromagnetic field.

Electromagnetics is of fundamental importance to physicists and to electrical and computer engineers. Electromagnetic theory is indispensable in understanding the principle of atom smashers, cathode ray oscilloscope, radar, satellite communication, television reception, remote sensing, radio astronomy, microwave devices, optical fiber communication, transient in transmission lines, electromagnetic compatibility. Circuit concepts represents restricted version, a special case of electromagnetic concept.

Electromagnetic Field Theory has applications in analyzing and designing of communication system and various other fields:

- Satellite Communication

The fastest growing and most recent field of communication involves the use of various satellite relays. Let us discuss the space wave communication. In this mode of propagation, electromagnetic waves from the transmitting antenna reach the receiving antenna either directly or after reflections from ground in the earth's troposphere region. Troposphere is that portion of the earth which extends up to 16 km from the earth surface. It means in the former, wave reaches directly from the transmitting antenna to receiving antenna and in later, the wave reaches the receiving antenna after reflection from the ground, where the phase change of 180 degree is also introduced due to reflection at the ground, in the ground reflected wave. Although both the waves leave the transmitting antenna at the same time with the same phase but may reach the receiving antenna either in the phase or out of the phase, because the two wave travel different path lengths. The strength of the resultant waves, thus, at the receiving point may be stronger or weaker than the direct path alone depending upon whether the two waves are adding or opposing in phase. At receiving point the signal strength is the vector addition of direct and indirect waves. Space wave propagation is also called as tropospheric propagation because space wave propagates through troposphere. Space wave propagation is mainly in VHF and higher frequencies because at such frequencies sky wave and ground wave propagation both fail. Beyond 30 MHz sky wave fails as the wavelength becomes too shorts to be reflected from ionosphere and ground waves are propagating close to the antenna only, as attenuation is very high. Therefore just after few hundred feet ground wave also die due to attenuation and wave tilt. Space wave propagation is also called as the line of sight propagation because at VHF, UHF and microwave frequencies, this mode of propagation is limited to the line of sight distance and is also limited by the curvature of earth. Although in actual particle space wave propagates even slightly beyond the line of sight distance due to the refraction in the atmosphere of the earth. In line of sight distance transmitting antenna and receiving antenna can usually see each other. In fact, the line of sight distance i.e. range of communication can also be increased by increasing the heights of transmitting and receiving antennas. The curvature of earth and the height of the transmitting and receiving antennas determines maximum range of communication through direct waves. In fact, the line of sight distance has now been extended by what is known as Space Communication or specially Satellite communication which has facilitated trans-oceanic propagation of microwaves with the potentiality of large bandwidth. By space communication we mean the radio traffic between a ground station and satellite or space probe, between satellites or space probes and also between the ground station itself via man made communication satellites or natural space body(e.g. the sun, the moon, the venus etc.). Earlier it was not possible to propagate beyond the radio horizon and hence it revolutionized the field of communication engineering and it is possible to show that three geosynchronous satellites can establish communication over entire world. Role of electromagnetic waves can be seen by studying the different bands available for satellite communication

- Wireless Communication

  Low level radio frequency fields are emitted by cellular phones, base tower, tv, radio signal, Bluetooth device wifi. These all come under wireless communication
- This theory is also used in analysis and designing of antenna, transmission lines and wave guides.
- Bio-medical system

  Magnetic sensors and magnetic actuators are used in biomedical. Magnetic Sensors are used in situ measurement of the mass evolution of cell culture, test of blood coagulation, Sensor system for early detection of heart valve bio prostheses failure

  Magnetic Actuators is used in Magnetic endoluminal artificial urinary sphincter and Hyperthermia HeLa cell treatment with silica-coated manganese oxide nano-particles.
- Weather forecast and Remote sensing

  Remote sensing refers to the activities of obtaining information about an object by a sensor without being in direct contact with the object. Information needs a physical carrier to travel from the object to the sensor through an intervening medium. In remote sensing, the information carrier is the electromagnetic radiation. Radar is a ground-based and active remote sensing equipment. It emits microwave radiation from a fixed location into the atmosphere and receives the reflected radiation called echoes from water droplets in the air. Microwave is not intense in the solar radiation and the earth's emission spectrum. Therefore the background radiation level in the microwave frequencies is not high and it usually does not affect the operation of the radar. Microwave frequencies can be divided into a number of frequency bands. Table 4 contains a list of band designation in the microwave frequencies.

- Electric motors

  Finite Element Method has been proved as valuable tool for solving different electromagnetic problems inside electrical machines. Calculation of magnetic flux density and its distribution in machine cross-section is difficult to be calculated by analytical methods. Therefore Finite Element Method is implemented for solving set off Maxwell equation which enables precise calculation of electromagnetic field and magnetic flux density in three different electrical machines

- In mechanical workshop

  Induction heaters, Dielectric heating, Joining and Sealing, and Soldering

- Reducing acidity in vegetables to improve taste

  Except these applications EM has some more area of application these are Lasers, Masers, Radio astronomy radars, Plasmas, Radiation therapy, Surface hardening, and Annealing.

The whole subject of EM unfolds as a logical deduction from eight postulated equations, namely as Maxwell's equations.

1.2.1 MAXWELL EQUATIONS

James Clerk Maxwell is regarded as the founder of electromagnetic theory in its present form. Through his theoretical effort over about 5 years Maxwell published the first unified theory of electricity and magnetism. The theory comprised all previously known results both experimental and theoretical on electricity and magnetism. It further introduced displacement current and predicated the existence of electromagnetic waves. Maxwell understood the significance of Faraday's work and realized that the speed of electromagnetic waves travelled at the speed of light. As a result, he was able to incorporate light, magnetism and electricity into a single theory. Maxwell further concluded that light propagated in electric and magnetic waves, which he believed would vibrate perpendicular to one another. Maxwell's electromagnetic theory of light propagation eventually paved the way for a number of major technological innovations.

Maxwell's equations were not fully accepted by many scientists until they were later confirmed by Heinrich Rudolf Hertz. in 1888, when Heinrich Hertz used Maxwell's theory to create instruments capable of sending and receiving radio pulses. This discovery, contributed to the creation of the television and the microwave and without Maxwell's tireless efforts, many of the modern conveniences upon which society has come to depend would not exist.

The integral form of Maxwell's equation despite the underlined physical laws where as the differential form is used more frequently in solving problems.

For a field to be qualified as an electromagnetic field it must satisfy all four Maxwell's equations. The importance of Maxwell equation cannot be overemphasized because they summarize all known laws of electromagnetism.

Physical Significance of Maxwell’s Equations

For static EM fields

$$\nabla \mathrm{x} \mathrm{H} = \mathrm{J} \quad \textbf{(1.1)}$$

But the divergence of the curl of any vector field is identically zero

Hence,

$$\nabla . (\nabla \mathrm{x} \mathrm{H}) = 0 = \nabla . \mathrm{J} \quad \textbf{(1.2)}$$

Now by current continuity equation

$$\nabla . \mathrm{J} = -\frac{\partial \rho_v}{\partial t} \neq 0 \quad \textbf{(1.3)}$$

Thus equation 1.2 and 1.3 are obviously incompatible for time-varing conditions. We must modify equation 1.1 to agree with equation 1.3. to do this, we add a term to equation 1.1 so it becomes

$$\nabla \mathrm{x} \mathrm{H} = \mathrm{J} + \mathrm{J}_d \quad \textbf{(1.4)}$$

Again taking divergence of equation 1.4 we have

$$\nabla . (\nabla \mathrm{x} \mathrm{H}) = \nabla . \mathrm{J} + \nabla . \mathrm{J}_d \quad \textbf{(1.5)}$$

Now,

$$\nabla . \mathrm{J}_d = \nabla . \frac{\partial \mathrm{D}}{\partial t}$$

Or $$\mathrm{J}_d = \frac{\partial \mathrm{D}}{\partial t} \quad \textbf{(1.6)}$$

From equation 1.6 and 1.4

$$\nabla x H = J + \frac{\partial D}{\partial t} \quad \textbf{(1.7)}$$

This is Maxwell’s equation for time-varying field. The term J_d is called as displacement current density and J is the conduction current density. The insertion of J_d into equation 1.2 was on of the major contribution of Maxwell. Without this term, electromagnetic wave propagation would be impossible. At low frequencies, J_d is usually neglected compared with J. however at radio frequencies, the two terms are comparable.

Differential form of Maxwell equations

$$\nabla \ \vec{E} = -\frac{\partial \vec{B}}{\partial} \quad \textbf{(1.8)}$$

$$\nabla \ \vec{H} = \vec{J} + \frac{-\partial \vec{D}}{\partial} \quad \textbf{(1.9)}$$

$$\nabla . \vec{D} = \rho \quad \textbf{(1.10)}$$

$$\nabla . \vec{B} = 0 \quad \textbf{(1.11)}$$

where

$\vec{E}$ is the electric field intensity,

$\vec{H}$ is the magnetic field intensity,

$\vec{B}$ is the magnetic flux density,

$\vec{D}$ is the electric flux density,

$\vec{J}$ is the current density,

p is the charge density,

φ is the electric scalar potential,

A is the magnetic vector potential; and these fields are governed by physical laws expressed mathematically by four Maxwell equations.

Physical significance of Maxwell's 1st equation

It shows that with time varying magnetic flux, electric field is produced in accordance with Faraday is law of electromagnetic induction. This is a time dependent equation.

Physical significance of 2nd equation

This is a time dependent equation which represents the modified differential form of Ampere's circuit law according to which magnetic field is produced due to combined effect of conduction current density and displacement current density

Physical significance of 3rd equation

According to this total electric flux through any closed surface is $1/\varepsilon_0$ times the total charge enclosed by the closed surfaces, representing Gauss's law of electrostatics, As this does not depend on time, it is a steady state equation. Here for positive ρ, divergence of electric field is positive and for negative ρ divergence is negative. It indicates that ρ is scalar quantity

Physical significance of 4th equation

It represents Gauss law of magnetostatic as ∇.B = 0 resulting that isolated magnetic poles or magnetic monopoles cannot exist as they appear only in pairs and there is no source or sink for magnetic lines of forces. It is also independent of time i.e. steady state equation.

Integral form of Maxwell equations

$$\oint_l \vec{E}.\boldsymbol{d} = 0 \quad \textbf{(1.12)}$$

$$\oint_l \vec{H}.\boldsymbol{d} = \oint_s \vec{J}.\boldsymbol{d} \quad \textbf{(1.13)}$$

$$\oint_S \vec{D}.\boldsymbol{d}\vec{S} = \int_v \rho_v \boldsymbol{d} \quad \textbf{(1.14)}$$

$$\oint_S \vec{B}.\boldsymbol{d} = 0 \quad \textbf{(1.15)}$$

The solution of the four Maxwell equations in either differential or integral form is possible only if additional constitutive equations are available connecting $\vec{D}$ to $\vec{E}$, $\vec{J}$ to $\vec{E}$ and $\vec{H}$ to $\vec{B}$ such as

$$\vec{D} = \varepsilon\vec{E} \quad \textbf{(1.16)}$$

$$\vec{J} = \sigma\vec{E} \quad \textbf{(1.17)}$$

$$\vec{B} = \mu\vec{H} \quad \textbf{(1.18)}$$

for a linear medium, where ε is the permittivity, σ is the conductivity, and μ is the permeability of a medium, or whatever forms apply for a nonlinear medium.

- Electrostatic field
- Magnetostatic field
- Time varying field

Electrostatic field is an electric field produced by static electrical charges. The charges are starting in the sense of charge amount and their positions in the space. Due to its simple nature, the electrostatic field or it's visible manifestation electrostatic force has been observed long time ago. Electrostatic field plays an important role in modern designing of electromagnetic devices whenever a strong magnetic field appears. For example, an electrical field is of paramount importance for the design of X-Ray device, lightning protection equipments in the high voltage components for electric power transmission system, and hence and analysis of electrostatic field is needed. In industry electrostatics is applied in a variety of forms such as paint spraying electrodeposition, electrochemical machining and separation of fine particles. Electrostatic is used in agriculture to sort seeds direct sprays to plants, measure the moisture content of crops spin cotton and smoking of meat.

A definite link between electrical and magnetic field was established by Oersted in 1820. We know that an electrostatic field is produced by static or stationary charges. If the charges are moving with constant velocity, a static magnetic or magnetostatic field is produced. A magnetostatic field is produced by a constant current flow. This current flow may be due to magnetization current as in permanent magnets, electron beam currents as in vacuum tubes or conduction current as in current carrying wires. The study of magnetostatic field is necessary as the developments of motors, transformers, microphones compasses, telephone bells ringers, televisions focusing control, advertising displays involves magnetic phenomena and plays an important role in our daily life. There are two major laws governing magnetostatic field i.e. Biot- Savart’s law and amperes circuit law.

The electric field produced by a changing magnetic field and thje magnetic field is produced by a changing electric field. The first of these concepts resulted from experimental research of Michael Faraday. Due to the first experiment of Faraday, we can say that a time-varying magnetic field produces an electromotive force (emf) which may establish a current in a suitable closed circuit. In general, Faraday's law manifests itself in either or both a stationary circuit linked by a time varying magnetic flux, such as a transformer, or the magnetic flux may be static, but the circuit is moving relative to the flux in such a way as to produce a time varying flux enclosed by the circuit. A rotating machine generates an emf by the latter mechanism.

Electromagnetics deal with space concepts and required thinking in the three dimension of real world. Hence we must understand the three dimensions co-ordinate system that is rectangular (cartesian), cylindrical and spherical system. Before understanding electrical and magnetic fields as vector quantities, it is worthwhile to review the simple concepts of vector algebra, vector calculus i.e. differentiation and integration of vectors, line, surface and volume integrals, del operator, gradient, divergence and curl operations and the three co-ordinate system cartesian, cylindrical and spherical

Vector analysis is a mathematical tool with which electromagnetic concepts are most conveniently expressed. A vector quantity has both magnitude and direction.

In physical problems, to describe vector accurately, some specific length, direction, angle, projections, or component must be given. There are many methods by which this can be done, but there are three simple methods, called coordinate systems. These coordinate systems are cartesian coordinate system, cylindrical coordinate system, and spherical coordinate system. The simplest being cartesian coordinate system. These coordinate systems are orthogonal. It is necessary to choose a co-ordinate system for a problem. A hard problem in one coordinate system may turn out to be easy in another system. Sometimes it is necessary to transform points and vectors from one system to another system.

The rate of change of vector field is complex and is defined by two types

- Divergence of vector
- Curl of a vector

Divergence is a scalar and bears similarity to the derivative of a function. When the divergence of a vector field is non zero, that region is said to contain sources or sinks, sources when the divergence is positive, sinks when negative. In static electric fields there is a correspondence between positive divergence, sources, and a positive electrical charge Q. Electric flux by definition originates on positive charge. Thus, a region which contains positive charges contains the source of electric flux. The divergence of the electric flux density will be positive in this region. The similar correspondence exists between negative divergence, sinks, and negative electric charge.

The curl of vector field B is a vector whose magnitude is maximum net circulation of B per unit area as the area tends to zero and whose direction is normal direction of the area when area is oriented to make the net circulation maximum.

1.3 COMPUTATIONAL ELECTROMAGNETICS METHOD

There are many numerical methods that have been developed to solve Maxwell’s equations. The three most-widely used methods are:

a) Method of Moments (MoM) / Boundary Element Method (BEM)

b) Finite Element Method (FEM)

c) Finite-Difference Time-Domain (FDTD) Method

Many commercial CEM codes have been developed around particular methods, and, of course, each method has its strengths and weaknesses. Historically, the codes themselves had their strengths and weaknesses that were associated with the strengths and weaknesses of underlying numerical methods.

Recognizing these differences, software developers have been addressing the shortcomings of their numerical techniques of choice. This work has led to an effective blurring of the differences between various commercial simulation packages. Furthermore, many companies started to apply multiple methods in their packages, sometimes combining two or more methods into hybrid methods. In addition, complementary simulation techniques have been developed to address the weaknesses of some methods, most notably, effective formulation of unbounded (open) problems.

As a result, detailed comparison of the various methods, which used to be an important factor in selecting software packages, is much less important now. Below is a reference list of various numerical methods used for solving Maxwell’s equations.

Methods of moments / boundary element method (BEM) is a numerical computational method of solving linear partial differential equations which have been formulated as integral equations (i.e. in boundary integral form). It can be applied in many areas of engineering and science including fluid mechanics, acoustics, electromagnetics and fracture mechanics

Finite element method

In mathematics, the finite element method (FEM) is a numerical technique for finding approximate solutions to boundary value problems for partial differential equations. It uses subdivision of a whole problem domain into simpler parts, called finite elements, and variational methods from the calculus of variations to solve the problem by minimizing an associated error function. Analogous to the idea that connecting many tiny straight lines can approximate a larger circle, FEM encompasses methods for connecting many simple element equations over many small sub domains, named finite elements, to approximate a more complex equation over a larger domain.

Finite-difference time-domain (FDTD)

Finite-difference time-domain (FDTD) is a numerical analysis technique used for modeling computational electrodynamics (finding approximate solutions to the associated system of differential equations). Since it is a time-domain method, FDTD solutions can cover a wide frequency range with a single simulation run, and treat nonlinear material properties in a natural way.

The FDTD method belongs in the general class of grid-based differential numerical modeling methods (finite difference methods). The time-dependent Maxwell's equations (in partial differential form) are discretized using central-difference approximations to the space and time partial derivatives. The resulting finite-difference equations are solved in either software or hardware in a leapfrog manner: the electric field vector components in a volume of space are solved at a given instant in time; then the magnetic field vector components in the same spatial volume are solved at the next instant in time; and the process is repeated over and over again until the desired transient or steady-state electromagnetic field behavior is fully evolved.

CHAPTER -2
LITERATURE SURVEY

Computational electromagnetics has had a long history. Computational electromagnetic (CEM) aims to solve Maxwell's equations using various numerical algorithms. It has evolved rapidly in the past decades to stand as an important tool for electrical engineers to solve radiation and scattering from complex objects that cannot be handled before. However, although tremendous progress has been witnessed after many years fruitful study, there is still a long way to go before it is brought to the same confidence level as that taken by circuit simulation and enjoys the same pervasiveness in engineering design as does circuit simulation.

Differential evolution has also been implemented to help CEM researchers to realize their dream. It is used to trace the optimal higher-order Whitney element to enhance convergence for the vector finite element modeling of microwaves and antennas.

2.1 Background:

Several real-world electromagnetic problems like electromagnetic scattering, electromagnetic radiation, modeling of waveguides etc., are not analytically calculable, for the multitude of irregular geometries found in actual devices. Computational numerical techniques can overcome the inability to derive closed form solutions of Maxwell's equations under various constitutive relations of media, and boundary conditions. This makes computational electromagnetics (CEM) important to the design, and modeling of antenna, radar, satellite and other communication systems, nano-photonic devices and high speed silicon electronics, medical imaging, cell-phone antenna design, among other applications.

CEM typically solves the problem of computing the E (Electric), and H (Magnetic) fields across the problem domain (e.g., to calculate antenna radiation pattern for an arbitrarily shaped antenna structure). Also calculating power flow direction (Pointing vector), a waveguide's normal modes, media-generated wave dispersion, and scattering can be computed from the E and H fields. CEM models may or may not assume symmetry, simplifying real world structures to idealized cylinders, spheres, and other regular geometrical objects. CEM models extensively make use of symmetry, and solve for reduced dimensionality from 3 spatial dimensions to 2D and even 1D.

An eigen value problem formulation of CEM allows us to calculate steady state normal modes in a structure. Transient response and impulse field effects are more accurately modeled by CEM in time domain, by FDTD. Curved geometrical objects are treated more accurately as finite elements FEM, or non-orthogonal grids. Beam propagation method (BPM) can solve for the power flow in waveguides. CEM is application specific, even if different techniques converge to the same field and power distributions in the modeled domain.

2.2 HISTORY OF NUMERICALMETHODS

Electromagnetics, the study of the solution methods to solving Maxwell's equations, and the application of such solutions for understanding and engendering new technologies, has a long history of over a hundred years. But the analysis method with Maxwell's equation is constantly evolving over the years. In the beginning, there was the age of simple shapes: during this period, roughly between the late 19^{th} century to 1950s, solution methods, such as the separation of variables, harmonic analysis, and Fourier transform methods were developed to solve for the scattering solution from simple shapes. We can identify the names of Sommerfeld, Rayleigh, Mie, Debye, Chu, Stratton, Marcuvitz, and Wait for contributions during this era.

Despite the successful closed form solutions for simple geometries, the solutions available were insufficient to analyze many electromagnetic systems. Hence, scientists and engineers started to seek approximation solutions to Maxwell's equations. This was the age of approximations, roughly between 1950s and 1970s. During this period, asymptotic and perturbation methods were developed to solve Maxwell's equations. The class of solvable problems for which approximate solutions exist, was greatly enlarged.

Actually, quite sophisticated mathematical techniques were used to analyze electromagnetic problems because electromagnetic theory was predated by the theory of fluid and theory of sound. These fields were richly entwined with mathematics with the work of famous mathematicians such as Euler, Lagrange, Stokes, and Gauss. Moreover, many of the mathematics of low Reynold number flow in fluid theory and scalar wave theory of sound can be transplanted with embellishment to solve electromagnetic problems.

Examples of problems solved during the age of simple shapes are the Mie and Debye scattering by a sphere and Rayleigh scattering by small particles. Rayleigh also solved the circular waveguide problem for electromagnetic waves because he was well versed in the mathematical theory of sound, having written three volumes on the subject while sailing down the Nile River. Sommerfeld solved the half plane problem as far back as 1896 because the advanced mathematical techniques were available then. He also solved the Sommerfeld half space problem in 1949 in order to understand the propagation of radio waves over the lossy half-earth. The problem was solved in terms of, what is now known as, the Sommerfield integrals

Evaluating the Sommerfeld integrals was impossibility during his time, but it is a piece of cake now in the modern era. Subsequently, approximation techniques, such as the stationary phase method, the method of steepest descent, and the saddle point methods were used to derive approximations to the Sommerfeld integrals.

However, even though electromagnetics has been intimately entangled with mathematics, a student of electromagnetics has to be able to read the physics into the mathematical expressions that describe the solutions of Maxwell's equations. Approximate methods generally help to elucidate the physics of the wave interaction with complex geometry.

The physical insights offered by approximate solutions spurred the age of approximations, roughly between 1950s and 1970s. A large parameter such as frequency is used to derive asymptotic approximations. Moreover, heuristic ideas were used to derive the physical optics approximation, Kirchhoff approximation, and various geometrical optics approximations. These approximations eventually lead to the geometrical theory of diffraction and the uniform asymptotic theory of diffraction. The applications of these approximate methods to scattering by complex structures are usually ansatz based. The coefficients of the ansatzcan be found from canonical solutions such as the Sommerfeld half plane problem, or scattering by a sphere or a cylinder, followed by the use of Watson transformation. The use of approximate solution enlarges the class of solvable problems, but the error is usually not controllable. Asymptotic series are semi-convergent series; hence there is not a systematic way to reduce the solution error by including more terms in the ansatz. Moreover, the range of application is limited because the frequency has to be sufficiently high before the ansatz forms a good approximation.

However, the limited range of approximate solutions of Maxwell's theory still could not meet the demand of many engineering and system designs. As soon as the computer was developed, numerical methods were studied to solve Maxwell's equations. This was the age of numerical methods (1960s+). Method of Moments (MoM), finite difference time domain method (FDTD), and finite element method (FEM) were developed to solve problems alongside with many other numerical methods. In particular, Harrington was noted for popularizing MoM among the electromagnetics community, while it is known as the boundary element method (BEM) in other communities. Yee developed FDTD, for solving Maxwell's equation. Finite element has been with the structure and mechanics community, and Silvester was an early worker who brought its use into the electromagnetics community. Other names commonly cited in this field are: Wilton, Mittra, and Taflove.

The advent of the transistorized computer in the 1960s almost immediately brought about the birth of numerical methods for electromagnetics. The method of moments (MoM) was popularized among the electromagnetics community by Harrington in the 1960s. The method is integral-equation-based, and is versatile for solving problems with arbitrary geometries. It entails a small number of unknowns since the unknown is the current, but unfortunately, the pertinent matrix equation is dense. The finite-difference time-domain method was proposed by Yee in the 1960s for solving Maxwell's equations in its partial differential form. The method is extremely simple, and gives rise to a sparse matrix system. Since the field is the unknown to be solved for, the drawback is that it entails a large number of unknowns. Moreover, the field is always propagated from point to point via a numerical grid, hence yielding grid dispersion error, which accentuates with increasing problem size.

In the finite difference method based on differential equations, the solution region is subdivided into rectangular mesh (a discretization, and thus, an approximation) and the values of a scalar potential field are sought at all grid points. Engineers, with intuition derived through experience in circuit theory, were able to use network analogous models and generalize the finite difference method to inhomogeneous problems with graded meshes in such a way as to be easily understood. Initially, solutions were obtained for small problems by hand relaxation, which in turn gave way to bigger problems by transferring to digital computers. The disadvantage of this method is that curved boundaries and varying sources cannot be modeled using simple, general data structures, so that general purpose software is difficult to write for complex problems.

Moreover, open boundaries have to be modeled by a close boundary with artificial boundary conditions. Recent years have witnessed the development of very general finite difference schemes that overcome most of these deficiencies. These elegant variants, however, require complex programming efforts and special data structures.

There are other numerical methods which were developed from variational methods, based on the so-called Euler´s brachistochrone problem [Gould 1957]. This problem gives us that particular shape of a curve between two points in space, along which a mass may slide down without friction in the shortest time. A variational scheme may commonly be described as differential in that we replace the condition of the satisfaction of a differential equation governing an unknown function by the equivalent requirement that an integral function of the unknown function shall be at a minimum. Indeed, integral variational principles are also used where we use a functional that satisfies an integral relationship at its minimum.

Already in the early 1950s differential variational schemes have been used to analyze electromagnetic devices, e.g. waveguides. These schemes are remarkably similar to the technique of modern finite element analysis. They relied upon a Rayleigh-Ritz scheme, which assumed the solution to be a sum of coordinate functions, with their weights to be determined by variational calculus [van Bladel 1964]. These variational schemes differed from present-day finite elements only in that the solution domain was not split into sub domains over each of which a differential trial function may be assumed. As a consequence of their trial solution being applicable to the whole domain, the choice of coordinate functions was critical to the accuracy of the solution. Thus, their methods are use only in a few problems for which a good estimate of the solution may be made.

With the development of the computer, statistical methods involving long solution times were also employed for the solution of field problems. The Monte Carlo simulation technique [Ehrlich 1959] is used to find the potential at one point within a device that has been divided into a mesh. We start at the point where we want the solution and “walk” through the nodes until we reach a boundary. The solution at the starting node is expressed as a statistical formula in terms of the charge densities at the nodes through which the walk is performed. Because this is a statistical formula, its accuracy increases with the number of walks we perform. Thus, for convergence of the solution several walks must be made. What we finally have is the solution at just one node. The validity of the method for general gradient (Neumann) boundary conditions has not been thoroughly investigated. It has also been found that the time taken to get the potential at one node is much greater than that for identifying the potential everywhere in the mesh by relaxation [Binns/Lawrenson/Trowbridge 1992]. As a result, the Monte Carlo method in electromagnetics has fallen into general disuse. However, the method is still extremely useful in other disciplines such as statistical chemistry where alternative techniques are hardly available.

Real numerical modelling of the continuum may by broadly divided into four groups:

- Differential methods
- Integral methods
- Variational methods
- Asymptotic methods

Differential equation methods (FDM, FDTD, etc.) are based on direct discretization of the differential governing field equations. Their most appealing characteristic relates to their ease of implementation. But they are also dispersive and costly (e.g., typically 10 – 20 points per wave length). Moreover they are restricted in applicability to complex geometries, especially in high-order implementations.

Integral equation methods (BEM, VIEM, MoM, Fast Multipole Methods, Adaptive Integral Methods, FFT-based Methods, Charge Simulation Methods, etc.) are based on discretizations of integral equation formulations of the governing equations. Similarly to variational approaches, they are versatile and flexible. They can also be made to be very efficient, particularly in applications involving piecewise homogeneous structures where they lead to a lower dimensional problem (posed on the interfaces separating different media). But they lead to full matrices and thus can only be made competitive through the use of mechanisms that accelerate the evaluation of fields. In addition, the singular character of the integrals imposes substantial challenges which typically result in low-order implementations, with consequently large computational costs.

Variational methods (FEM, Finite Volume Methods, etc.) are based on the variational formulation of Maxwell´s equations. They lead to algorithms that present several favourable properties, including great applicability and flexibility, a natural setting for adaptivity and parallelization, sparse matrices, etc. But these approaches typically require large (volumetric) computational domains and the use of approximate radiation conditions within a finite computational domain, which may lead to high computational costs and large memory requirements; in addition, low-order implementations (e.g. finite volume) suffer from significant dispersion and dissipation errors.

Asymptotic methods (ray-tracing, etc.) do not solve the full Maxwell model (in contrast with the methods above), but rather an approximation of it. Of particular interest from the point of view of applications (e.g. radar) are those that relate to the high-frequency (geometrical or physical optics) limit of Maxwell´s equations. Such methods are extremely efficient since, in contrast with methods that solve the full Maxwell model, they do not involve the resolution of the fields in the scale of the wavelength of radiation. But, they are asymptotic in nature and therefore are not error-controllable; as a result, they can give rise to significant inaccuracies for finite (but large) frequencies.

As any review of the literature will reveal, schemes from each of these classes have been very successful at resolving a variety of problems. The same review will also discover that some methods may be better adapted to specific applications, and that no method can be considered "universally" superior. More importantly perhaps, one will also find that all available methods have very definite limitations in spite of continuous advances in the capabilities of computational algorithms and hardware. In fact, a number of applications continue to challenge every approach, and some lie well-beyond today´s capabilities (e.g. the rigorous prediction of scattering returns at very high frequencies).

And there is still another aspect which should be taken into account during evaluation of the numerical methods. Usually, it is not so clear how to classify the different methods following the classes given above. There are several modifications of some basic methodologies of numerical treatment of Maxwell´s equations where it is not possible to say precisely this is a differential method or that is a variational methods, etc. There are a number of rather new developments which have led to sometimes called "hybrid" methods, which not always means that there are combined two methods on a certain interconnection between two domains of solutions space, but they use combined discretization strategies in the same equations.

The variational methods are really based on the differential or integral form of the equation to be solved. Integral schemes for materially homogeneous problems have long been known and used. The equation for integration naturally follows from the potential due to a unit point charge being given at a certain distance (source point). Thus, any charge cluster of given density may be subdivided into small volumes so that the charge in that volume is effectively a point charge causing a potential in the considered field point. On the basis of the superposition principle, we may integrate such effects at any field point and get the potential there. When materials in homogeneities are involved, this scheme cannot be used. To overcome this in electrostatics, we may resort to dipole moment techniques whereby we eliminate an inhomogeneous region and replace the effects coming from inside the eliminated region by a surface charge. Because these charges are secondary sources, i.e. they only "appear" if primary sources generating the field are present, this method is called the method of secondary sources [Tozoni 1975]. In magnetics the analogous idea can be realized by introducing fictitious magnetic poles to replace the magnetized material.

In the late 1960s the finite element method was first applied to electromagnetics [Winslow 1967]. The finite element method is a general method for the solution of differential equations. Here, we subdivide the solution region into sub domains, called elements, and postulate a trial function ((with free parameters) over each of the elements. It is commonly found convenient to have interpolation nodes on the element and to define the trial function in terms of the unknown values of the unknown variable of the differential equation at the node. As a result, the nodal values become the free parameters. The finite element method essentially consists of finding the values of the free parameters with respect to some optimality criterion such as minimum error, energy extremum, functional orthogonality, etc. This method, now widely accepted as one of the most powerful numerical schemes available, is one that engineers may take true pride in, for having founded intuitively, leaving for later the rigorous justifications of the method, i.e. there was a time when engineers used the method on the grounds simply that it worked. Their mathematical verification was studied much later.

2.3 Overview of methods:

One approach is to discretize the space in terms of grids (both orthogonal, and non-orthogonal) and solving Maxwell's equations at each point in the grid. Discretization consumes computer memory, and solving the equations takes significant time. Large-scale CEM problems face memory and CPU limitations. As of 2007, CEM problems require supercomputers, high performance clusters, vector processors and/or parallelism. Typical formulations involve either time-stepping through the equations over the whole domain for each time instant; or through banded matrix inversion to calculate the weights of basis functions, when modeled by finite element methods; or matrix products when using transfer matrix methods; or calculating integrals when using method of moments (MoM); or using fast fourier transforms, and time iterations when calculating by the split-step method.

2.4 Choice of methods:

Choosing the right technique for solving a problem is important, as choosing the wrong one can either result in incorrect results, or results which take excessively long to compute. However, the name of a technique does not always tell one how it is implemented, especially for commercial tools, which will often have more than one solver.

Davidson gives two tables comparing the FEM, MoM and FDTD techniques in the way they are normally implemented. One table is for both open region (radiation and scattering problems) and another table is for guided wave problems.

2.5 Maxwell's equations in hyperbolic PDE form:

Maxwell's equations can be formulated as a hyperbolic system of partial differential equations. This gives access to powerful techniques for numerical solutions.

It is assumed that the waves propagate in the (x,y)-plane and restrict the direction of the magnetic field to be parallel to the z-axis and thus the electric field to be parallel to the (x,y) plane. The wave is called a transverse magnetic (TM) wave. In 2D and no polarization terms present, Maxwell's equations can then be formulated as:

$$\frac{\partial}{\partial}\bar{u} + \mathbf{A}\frac{\partial}{\partial}\bar{u} + \mathbf{B}\frac{\partial}{\partial}\bar{u} + \mathbf{C}\bar{u} = \bar{g}$$

where u, A, B, and C are defined as

$$\bar{u} = \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\epsilon} \\ 0 & \frac{1}{\mu} & 0 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 0 & 0 & \frac{-1}{\epsilon} \\ 0 & 0 & 0 \\ \frac{-1}{\mu} & 0 & 0 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} \frac{\sigma}{\epsilon} & 0 & 0 \\ 0 & \frac{\sigma}{\epsilon} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

In this representation, $\bar{g}$ is the forcing function, and is in the same space as $\bar{u}$. It can be used to express an externally applied field or to describe an optimization constraint. As formulated above:

$$\bar{g} = \begin{pmatrix} E_{x,}\ C \\ E_{y}, C \\ E_{z}, C \end{pmatrix}$$

$\bar{g}$ may also be explicitly defined equal to zero to simplify certain problems, or to find a characteristic solution, which is often the first step in a method to find the particular inhomogeneous solution.

Electromagnetic theory was fully formulated by James Clerk Maxwell in 1864 in terms of the Maxwell's equations. Even though it has been around for over a hundred years, scientists and engineers are continuously pursuing new methods to solve these equations. The reason is that Maxwell's equations govern the law for the manipulation of electricity. Hence, many branches of electrical engineering are directly or indirectly related to the electromagnetic theory. Scientists and engineers solve these equations in order to gain a better understanding of and physical insight into systems related to the use of electromagnetic fields and waves.

The solutions of Maxwell's equations can also be used to predict design and experimental outcomes. Electromagnetics has persisted as a vibrant field despite it being over a hundred year old is because many electrical engineering technologies depend on it. To name a few, these are: physics based signal processing and imaging, computer chip design and circuits, lasers and optoelectronics, MEMS (micro-electromechanical sensors) and microwave engineering, remote sensing and subsurface sensing and NDE (non-destructive evaluation), EMC/EMI (electromagnetic compatibility/electromagnetic interference) analysis, antenna analysis and design, RCS (radar cross section) analysis and design, ATR (automatic target recognition) and stealth technology, wireless communication and propagation, and biomedical engineering and biotechnology.

CHAPTER – 3
BOUNDARY ELEMENT METHOD (BEM)

The method of moment is a general procedure for solving integral or intro-differential equations. It is a numerical computational method of solving linear partial differential equations which have been formulated as integral equations (i.e. in boundary integral form). This method owes its name to the process of taking moments multiplying with appropriate weighing functions. It can be applied in many areas of engineering and science including fluid mechanics, acoustics, electromagnetic and fracture mechanics. Method of moment has its origin in Russian literature. The origin and development of the moment method are fully documented by Harrington in 1960s.

Boundary integral equations are a classical tool for the analysis of boundary value problems for partial differential equations. The term "boundary element method" (BEM) denotes any method for the approximate numerical solution of these boundary integral equations. The approximate solution of the boundary value problem obtained by BEM has the distinguishing feature that it is an exact solution of the differential equation in the domain and is parameterized by a finite set of parameters living on the boundary.

The procedure for applying MOM to solve any integral problem involves four steps

a) Derivation of the appropriate integral equation (IE),
b) Conversion of the IE into a matrix equation using basic function and weighing functions,
c) Evaluation of the matrix elements,
d) Solving the matrix and obtaining the parameter of interest.

3.1 Advantages:

The BEM have some advantages over other numerical methods like finite element methods (FEM) or finite differences:

1. Only the boundary of the domain needs to be discretized. Especially in two dimensions where the boundary is just a curve this allows very simple data input and storage methods.

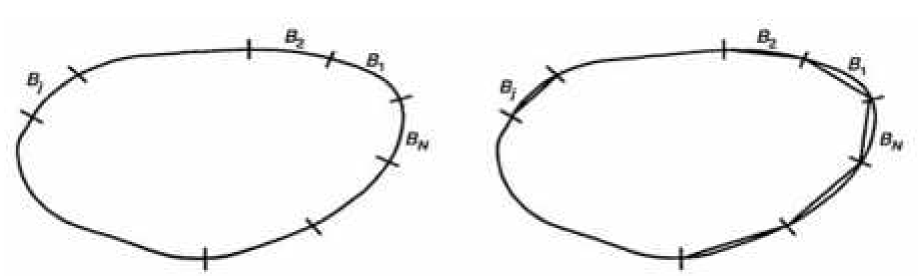
2. Exterior problems with unbounded domains but bounded boundaries are handled as easily as interior problems.

3. In some applications, the physically relevant data are given not by the solution in the interior of the domain but rather by the boundary values of the solution or its derivatives. These data can be obtained directly from the solution of boundary integral equations, whereas boundary values obtained from FEM solutions are in general not very accurate.

4. The solution in the interior of the domain is approximated with a rather high convergence rate and moreover, the same rate of convergence holds for all derivatives of any order of the solution in the domain. There are difficulties, however, if the solution has to be evaluated close to, but not on the boundary.

3.2 Numerical Foundation

Numerical solutions to the boundary integral equations are provided by the Boundary Element Method. In this, the boundary, B, is divided into parts, B_j , $j= 1.2 .... N$, which are termed the elements as shown in Figure 3.1(a). Over each element the unknown function and the boundary itself are represented by mathematical expressions. Such representations can be quite complex in advanced formulations. Initially however the boundary will be taken as a *straight line* in each element. B_j , as in Figure 3.1(b) and the unknown function will be assumed to be *constant* over each element. Such a linear geometry and constant function formulation is the simplest boundary element formulation. A natural extension is to assume that the unknown function is also linearly represented. This formulation, which is termed isoparametric.



(a) (b)

Figure 3.1 Boundary elements for an interior problem: (a) boundary divided into elements; and (b) elements represented by straight lines.

3.3 Linear Approximation

Since the representation of the boundary is crucial to the method a separate treatment of this is given. A single parametrisation of the boundary *B* will not always be possible or even desirable. Much depends on how it is assumed that *B* is defined. A few examples are shown in Fig. 3.2. A procedure which can be used in all these examples is to define the boundary by a series of point values which

in example (a) could be calculated from the single parametrization of the ellipse,

in example (b) could be calculated from the geometry of various parts of the boundary,

in example (c) could be taken from measurements on the engineering drawing and

in example (d) would simply be taken as the given points or nodes.

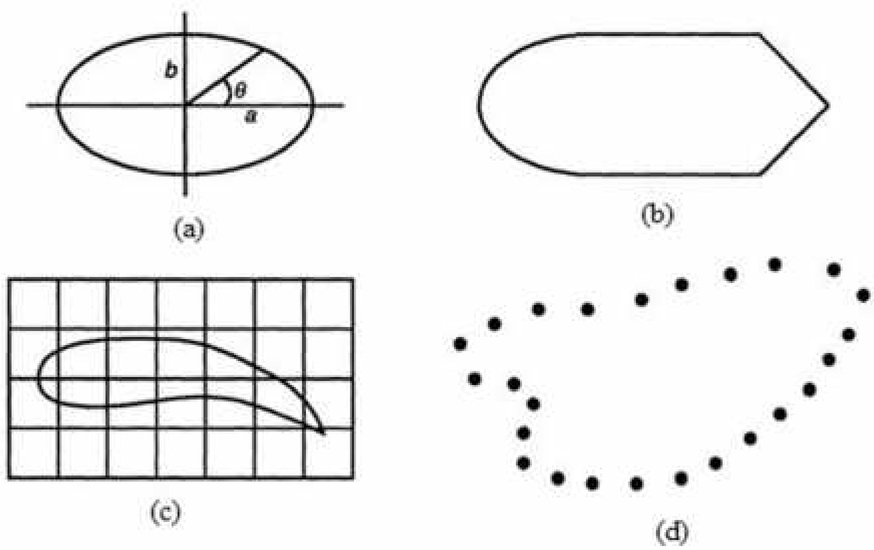


Figure 3.2 Examples of the definition of object boundaries: (a) ellipse, defined by a single parameter; (b) object defined in terms of combination of simple curves ; (c) aerofoil,defined from a drawing; and (d) arbitrary object, defined by a set of points.

Thus in all cases the boundary would be defined as in example (d) and information would be needed about the shape of the boundary between the nodes. This process may be described as approximation or representation of the boundary. In example (d) the representation would give information which was not previously known about the boundary. In the other examples it would create a boundary which was not the same as the one specified, but in any reasonable scheme the boundary would be close to the original.

The simplest representation of the boundary is the linear one which joins the nodes by straight lines as in Fig. 3.1(b). Points are represented by position vectors. As can be seen from Fig. 3.3 the vector joining points x_1 and x_2 is leading to the following equation of the straight line, x(t), between them

$$x(t) = x_1 + t(x_2 - x_1) \qquad \textbf{(3.1)}$$

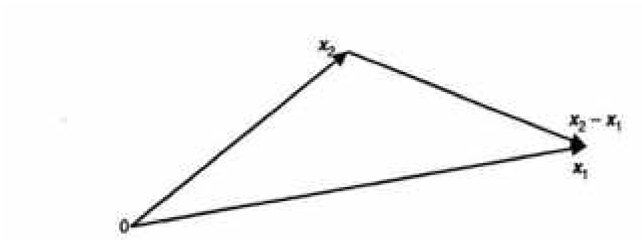


Figure 3.3 Vector joining points X_2 - X_l.

Thus when $t = 0$, $x(t) = X_l$ and when $t = 1$, $x(t) = x_2$. The range of values of t which give the line from X_2 to X_2 is thus [0,1]. Other parameter ranges are easy to construct simply by making a change of variable and it will be seen later that the range [-1,1] is the most useful. This range is obtained by changing t to $(t+ 1)/2$ so that

$$x(t) = x_1 + \frac{t+1}{2}(x_2 - x_1) \qquad \textbf{(3.2)}$$

and now, when $t=-1$, $x(t) = X_1$ and when $t = 1$, $x(t) = X_2$, so that the parameter range is [-1,1]. Representation (3.2) may also be written as

$$x(t) = \frac{t+1}{2}x_2 + \frac{1-t}{2}x_1 \qquad \textbf{(3.3)}$$

$$= M_1(t)x_1 + M_2(t)x_2$$

$$= \sum_{\alpha=1}^{2} M_\alpha(t)x_\alpha$$

The linear functions $M_1(t)$,
$M_2(t)$ are thus

$M_1(t) \;=\frac{1-t}{2}$, $\;M_2(t) \;=\frac{1+t}{2}$ **(3.4)**

The function *M* 1*(t)* thus has the property of being unity at the point whose position vector it multiplies (node 1) and zero at the other node. The function $M_2(t)$ also has the property of being unity at the point whose position vector it multiplies (node 2) and zero at the other node. Functions with this property are widely used in the Finite Element Method as well as in the Boundary Element Method and are called *shape functions.*

3.4 Integration on a Curve

Before looking in detail at the Boundary Element Method of solution it is neccessary to consider how to deal with the curvilinear integrals along the boundary. To carry out curvilinear integration, the curve must be expressed in terms of a parameter *t*, so that

$$x = x(t)$$

Then for integration of a general integrand *F(x, x')* over one of the elements *Bj*,

$$\int_{B_j} F(x, x')d\ (x) = \int_{t_1}^{t_2} F(x(t), x')\frac{d\ (x)}{d}d$$

where t_1, t_2. are the values of the parameter *t* corresponding to the ends of the element *Bj.* Also

$$\frac{d\ (x)}{d}d \;= J(t) \qquad \textbf{(3.5)}$$

where *J(t)* is a Jacobian associated with the change of variable from *x* to *t.* Considering the elemental triangle in Figure 3.4, it can be seen that

$$[dS(x)]^2 = dx^2 + dy^2$$

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