MATH 327: Problem Set #4

Due on February 20, 2017 at 2:10pm

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Problem 21

Five men and five women are ranked according to their scores on an examination. Assume that no two scores are alike and all 10! possible rankings are equally likely. Let X denote the highest ranking achieved by a woman (for instance, $X = 2$ if the top-ranked person was male and the next-ranked person was female).

$$
\begin{aligned}
P\{1\} &= .5 \\
P\{2\} &= .28 \\
P\{3\} &= .14 \\
P\{4\} &= .06 \\
P\{5\} &= .02 \\
P\{6\} &= .00 \\
P\{t \geq 7\} &= 0
\end{aligned}
$$

Compute the expected value of the random variable X.

Solution

$$
\begin{aligned}
E[X] &= \sum_t tP\{t\} \\
&= 1(.5) + 2(.28) + 3(.14) + 4(.06) + 5(.02) + 6(.00) + 0 \\
&= 1.83\ldots
\end{aligned}
$$

Problem 22

Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times.
If the coin is assumed fair, for n = 3, what are the probabilities associated with the values that X can take on?

$$X = -3 + 2(k), \forall k \in \mathbb{N} \leq 3$$

$$
\begin{aligned}
k = 0, X &= -3 + 2(0) = -3 \\
k = 1, X &= -3 + 2(1) = -1 \\
k = 2, X &= -3 + 2(2) = 1 \\
k = 3, X &= -3 + 2(3) = 3
\end{aligned}
$$

Compute the exected value of the random variable X.

Solution

$E[X] = 0$, because the graph of X is symmetric and centered at 0.

Problem 23

Each night different meteorologists give us the probability that it will rain the next day. To judge how well these people predict, we will score each of them as follows: If a meteorologist says that it will rain with probability p, then he or she will receive a score of

$$\text{score} = \begin{cases} 1-(1-p)^2 & \text{if it does rain} \\ 1-p^2 & \text{if it does not rain} \end{cases}$$

We will then keep track of scores over a certain time span and conclude that the meteorologist with the highest average score is the best predictor of weather. Suppose now that a given meteorologist is aware of this and so wants to maximize his or her expected score. If this individual truly believes that it will rain tomorrow with probability p^*, what value of p should he or she assert so as to maximize the expected score?

Solution
The score function favors correct guesses. If the meteorologist is certain that it will rain with probability p^*, then to maximize his/her score, they should assert that it will rain with probability p^*.

Problem 25

A total of 4 buses carrying 148 students from the same school arrive at a football stadium. The buses carry, respectively, 40, 33, 25, and 50 students. One of the students is randomly selected. Let X denote the number of students that were on the bus carrying this randomly selected student. One of the 4 bus drivers is also randomly selected. Let Y denote the number of students on her bus.

(a) Which of $E[X]$ or $E[Y]$ do you think is larger? Why?

E[X] should tend to be bigger. A bus driver will be on a bus regardless of how many students are on it, but the more students are on a bus, the more likely a chosen student will be on it.

(b) Compute $E[X]$ and $E[Y]$.

$$\begin{aligned} E[X] &= \sum_t tP\{t\} \\ &= 40(\frac{40}{148}) + 33(\frac{33}{148}) + 25(\frac{25}{148}) + 50(\frac{50}{148}) \\ &= 39.3\ldots \end{aligned}$$

$$\begin{aligned} E[Y] &= \sum_t tP\{t\} \\ &= \frac{1}{4}40 + \frac{1}{4}33 + \frac{1}{4}25 + \frac{1}{4}50 \\ &= 37 \end{aligned}$$

Problem 27

The density function of X is given by

$$\text{f(x)} = \begin{cases} a + bx^2 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

If $E[X] = \frac{3}{5}$, find a and b.
Solution

$$\begin{aligned} E[X] = \frac{3}{5} &= \int_{-\infty}^{\infty} xf(x)dx \\ &= \int_0^1 x(a+bx^2)dx \\ &= \int_0^1 ax + bx^3 dx \\ &= \frac{a}{2}x^2 + \frac{b}{4}x^4\Big|_0^1 \\ &= \frac{a}{2}1^2 + \frac{b}{4}1^4 - (\frac{a}{2}0^2 + \frac{b}{4}0^4) \\ \frac{3}{5} &= \frac{a}{2} + \frac{b}{4} \end{aligned}$$

$$\begin{aligned} F(x) = 1 &= \int_{-\infty}^{\infty} f(x)dx \\ &= \int_0^1 (a+bx^2)dx \\ &= ax + \frac{b}{3}x^3\Big|_0^1 \\ &= a \cdot 1 + \frac{b}{3}1^3 - (a \cdot 0 + \frac{b}{3}0^3) \\ 1 &= a + \frac{b}{3} \end{aligned}$$

$$\begin{aligned} \frac{3}{5} &= \frac{a}{2} + \frac{b}{4} \\ \frac{3}{5} &= \frac{1 - \frac{b}{3}}{2} + \frac{b}{4} \\ \frac{3}{5} &= \frac{3-b}{6} + \frac{b}{4} \\ b &= \frac{6}{5} \\ a &= \frac{3}{5} \end{aligned}$$

Problem 30

Suppose that X has density function

$$\text{f(x)} = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Compute $E[X^n]$

(a) by computing the density of X^n and then using the definition of expectation

Solution

Let Y be X^n. $\forall\, 0 \leq y \leq 1$, let the cumulative distribution function of Y be

$$\begin{aligned} F_Y(y) &= P\{Y \leq y\} \\ &= P\{X^n \leq y\} \\ &= P\{X \leq \sqrt[n]{y}\} \\ &= \int_0^{\sqrt[n]{y}} 1\, dx \\ &= x\Big|_0^{\sqrt[n]{y}} \\ F_Y(y) &= \sqrt[n]{y} \end{aligned}$$

Now we differentiate $F_Y(y)$ to get the density function of Y.

$$\begin{aligned} f_Y(y) &= \frac{d}{dy} F_Y(y) \\ &= \frac{d}{dy} \sqrt[n]{y} \\ &= \frac{y^{\frac{1}{n}-1}}{n} \\ f_Y(y) &= \frac{1}{n} y^{\frac{1-n}{n}}, 0 \leq y \leq 1 \end{aligned}$$

Problem 30 continued on next page...

Now we use the definition of expectation to find $E[Y]$.

$$\begin{aligned}
E[Y] &= \int_{-\infty}^{\infty} y f_Y(y)\,dy \\
&= \int_0^1 y\left(\frac{1}{n}y^{\frac{1-n}{n}}\right)\,dy \\
&= \frac{1}{n}\int_0^1 y^{\frac{1-n}{n}+1}\,dy \\
&= \frac{1}{n}\int_0^1 y^{\frac{1}{n}}\,dy \\
&= \frac{1}{n}\left(\frac{n}{1+n}\right) y^{\frac{1+n}{n}}\Big|_0^1 \\
&= \frac{1}{n+1}1^{\frac{1+n}{n}} \\
E[Y] = E[X^n] &= \frac{1}{n+1}
\end{aligned}$$

(b) by using Proposition 4.5.1

If X is a continuous random variable with probability density function f(x), then for any real-valued function g,

$$E[g(X)] = \int_{-\infty}^{\infty} g(x)f(x)dx$$

Solution

$$\begin{aligned}
E[g(X)] &= \int_{-\infty}^{\infty} g(x)f(x)dx \\
E[X^n] &= \int_{-\infty}^{\infty} x^n f(x)dx \\
&= \int_0^1 x^n 1\,dx \\
&= \frac{x^{n+1}}{n+1}\Big|_0^1 \\
&= \left(\frac{1^{n+1}}{n+1}\right) - \left(\frac{0^{n+1}}{n+1}\right) \\
E[X^n] &= \frac{1}{n+1}
\end{aligned}$$

Problem 32

If $E[X] = 2$ and $E[X^2] = 8$, calculate

(a)

$$\begin{aligned}
E[(2+4X)^2] &= E[4 + 16X + 16X^2] \\
&= E[4] + E[16X] + E[16X^2] \\
&= E[4] + 16E[X] + 16E[X^2] \\
&= 4 + 16 \cdot 2 + 16 \cdot 8 \\
&= 164
\end{aligned}$$

(b) $E[X^2 + (X+1)^2]$

$$\begin{aligned}
E[X^2 + (X+1)^2] &= E[X^2 + X^2 + 2X + 1] \\
&= E[2X^2 + 2X + 1] \\
&= 2E[X^2] + 2E[X] + E[1] \\
&= 2 \cdot 8 + 2 \cdot 2 + 1 \\
&= 21
\end{aligned}$$

Problem 34

If X is a continuous random variable having distribution function F, then its median is defined as that value for m for which

$$F(m) = \frac{1}{2}$$

Find the median of the random variables with density function

(a) $f(x) = e^{-x}, x \geq 0$

$$\begin{aligned} F(x) &= \int_{-\infty}^{x} f(x)dx \\ &= \int_{0}^{x} e^{-x}dx \\ &= -e^{-x}\Big|_{0}^{x} \\ &= (-e^{-x}) - (-e^{0}) \\ F(x) = \frac{1}{2} &= 1 - e^{-x} \\ \frac{1}{2} &= e^{-x} \\ \ln(2^{-1}) &= \ln(e^{-x}) \\ \ln(2) &= x \end{aligned}$$

(b) $f(x) = 1, 0 \leq x \leq 1$

$$\begin{aligned} F(x) &= \int_{-\infty}^{x} f(x)dx \\ &= \int_{0}^{x} 1dx \\ &= x\Big|_{0}^{x} \\ F(x) = \frac{1}{2} &= x \end{aligned}$$

Problem 36

We say that m_p is the $100p$ percentile of the distribution function F if

$$F(m_p) = p$$

Find m_p for the distribution having density function

$$f(x) = 2e^{-2x}, x \geq 0$$

Solution

$$\begin{aligned}
F(x) &= \int_{-\infty}^{x} f(x)dx \\
&= \int_{0}^{x} 2e^{-2x}dx \\
&= -e^{-2x}\Big|_{0}^{x} \\
&= (-e^{-2x}) - (-e^{-2*0}) \\
&= 1 - e^{-2x} \\
F(x) &= 1 - e^{-2x} \\
F(m_p) = p &= 1 - e^{-2m_p} \\
1 - p &= e^{-2m_p} \\
\ln(1-p) &= -2m_p \ln(e) \\
\frac{-\ln(1-p)}{2} &= m_p
\end{aligned}$$

Problem 38

Compute the expectation and variance of the number of successes in n independent trials, each of which results in a success with probability p. Is independence necessary?

Solution

Let random variable X represent the number of successes in n independent trials.

Each trial can be a success (1) or a failure (0). We can denote the success of an arbitrary i^{th} trial with $X_i = 0$ or 1.

If each trial has a success chance of p and there are n trials, then the expectation of X must be

$$E[X] = \sum_{i=1}^{n} p = np$$

To calculate variance, we must first calculate the variance of one arbitrary trial X_i.

$$\begin{aligned} Var(X_i) &= E[X_i^2] - (E[X_i])^2 \\ &= E[(0 \text{ or } 1)^2] - (p)^2 \\ &= E[X_i] - p^2 \\ &= p - p^2 \end{aligned}$$

The sum of the variances for each trial X_i is the variance of X.

$$Var(X) = \sum_{i=0}^{n} p - p^2$$
$$Var(X) = n(p - p^2)$$

Independence is necessary for $\sum Var(X_i) = Var(X)$ to hold.

Problem 44

Let X_i denote the percentage of votes cast in a given election that are for candidate i, and suppose that X_1 and X_2 have a joint density function

$$f_{X_1,X_2}(x,y) = \begin{cases} 3(x+y), & \text{if } x \geq 0, y \geq 0, 0 \leq x+y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the marginal densities of X_1 and X_2.

$$\begin{aligned}
f_{X_1}(x) &= \int_{-\infty}^{\infty} f(x,y)dy \\
&= \int_0^{1-x} (3x+3y)dy \\
&= 3xy + \frac{3y^2}{2}\Big|_0^{1-x} \\
&= 3x(1-x) + \frac{3(1-x)^2}{2} - (3x(0) + \frac{3 \cdot 0^2}{2}) \\
&= 3(x-x^2) + \frac{3(1-2x+x^2)}{2} \\
&= \frac{3(2x-2x^2+1-2x+x^2)}{2} \\
f_{X_1}(x) &= \frac{3(1-x^2)}{2}
\end{aligned}$$

$$\begin{aligned}
f_{X_2}(x) &= \int_{-\infty}^{\infty} f(x,y)dx \\
&= \int_0^{1-x} (3x+3y)dx \\
&= 3yx + \frac{3x^2}{2}\Big|_0^{1-y} \\
&= 3y(1-y) + \frac{3(1-y)^2}{2} - (3y(0) + \frac{3 \cdot 0^2}{2}) \\
&= 3(y-y^2) + \frac{3(1-2y+y^2)}{2} \\
&= \frac{3(2y-2y^2+1-2y+y^2)}{2} \\
f_{X_2}(y) &= \frac{3(1-y^2)}{2}
\end{aligned}$$

Problem 44 continued on next page. . .

(b) Find $E[X_i]$ and $\text{Var}(X_i)$ for $i = 1, 2$.

$$\begin{aligned}
E[X_i] &= \int_{-\infty}^{\infty} x f_{X_i}(x) dx \\
&= \int_0^1 x \frac{3(1-x^2)}{2} dx \\
&= \int_0^1 \frac{3(x-x^3)}{2} dx \\
&= \frac{3}{2}(\frac{x^2}{2} - \frac{x^4}{4})\Big|_0^1 \\
&= \frac{3}{2}\left((\frac{1^2}{2} - \frac{1^4}{4}) - (\frac{0^2}{2} - \frac{0^4}{4})\right) \\
&= \frac{3}{2}(\frac{1}{2} - \frac{1}{4}) \\
&= \frac{3}{4} - \frac{3}{8} \\
E[X_i] &= \frac{3}{8}
\end{aligned}$$

$$\begin{aligned}
Var(X_i) &= \int_{-\infty}^{\infty} x^2 f_{X_i}(x) dx - E[X_i]^2 \\
&= \int_0^1 x^2 \frac{3(1-x^2)}{2} dx - \left(\frac{3}{8}\right)^2 \\
&= \int_0^1 \frac{3(x^2-x^4)}{2} dx - \frac{9}{64} \\
&= \frac{3}{2}(\frac{x^3}{3} - \frac{x^5}{5})\Big|_0^1 - \frac{9}{64} \\
&= \frac{3}{2}\left((\frac{1^3}{3} - \frac{1^5}{5}) - (\frac{0^3}{3} - \frac{0^5}{5})\right) - \frac{9}{64} \\
&= \frac{3}{2}\left((\frac{1}{3} - \frac{1}{5})\right) - \frac{9}{64} \\
&= \frac{1}{5} - \frac{9}{64} \\
Var(X_i) &= \frac{19}{320}
\end{aligned}$$

Problem 45

A product is classified according to the number of defects it contains and the factory that produces it. Let X_1 and X_2 be the random variables that represent the number of defects per unit (taking on possible values of 0, 1, 2, or 3) and the factory number (taking on possible values 1 or 2), respectively. The entries in the table represent the joint possibility mass function of a randomly chosen product.

X_1 \ X_2	1	2
0	$\frac{1}{8}$	$\frac{1}{16}$
1	$\frac{1}{16}$	$\frac{1}{16}$
2	$\frac{3}{16}$	$\frac{1}{8}$
3	$\frac{1}{8}$	$\frac{1}{4}$

(a) Find the marginal probability distributions of X_1 and X_2.

$$P_{X_1}(x) \begin{cases} \frac{3}{16} & \text{if } x = 0 \\ \frac{2}{16} & \text{if } x = 1 \\ \frac{5}{16} & \text{if } x = 2 \\ \frac{6}{16} & \text{if } x = 3 \end{cases}$$

$$P_{X_2}(x) = \frac{1}{2}$$

(b) Find $E[X_1]$, $E[X_2]$, $\text{Var}(X_1)$, $\text{Var}(X_2)$, and $\text{Cov}(X_1, X_2)$.

$$\begin{aligned} E[X_1] &= (0)\frac{3}{16} + (1)\frac{2}{16} + (2)\frac{5}{16} + (3)\frac{6}{16} = \frac{30}{16} \\ E[X_2] &= (1)\frac{1}{2} + (2)\frac{1}{2} = \frac{3}{2} \end{aligned}$$

$$\begin{aligned} Var(X_1) &= \sum_{x=0}^{3} P_{X_1} x^2 - (E[X_1])^2 \\ &= (0)^2\frac{3}{16} + (1)^2\frac{2}{16} + (2)^2\frac{5}{16} + (3)^2\frac{6}{16} - (\frac{30}{16})^2 \\ &= \frac{76}{16} - (\frac{30}{16})^2 = \frac{79}{64} \end{aligned}$$

$$\begin{aligned} Var(X_2) &= \sum_{x=1}^{2} P_{X_2} x^2 - (E[X_2])^2 \\ &= (1)^2\frac{1}{2} + (2)^2\frac{1}{2} - (\frac{3}{2})^2 \\ &= \frac{5}{2} - \frac{9}{4} = \frac{1}{4} \end{aligned}$$

$$\begin{aligned} cov(X_1, X_2) &= \sum_{0\le x\le 3, 1\le y\le 2} (x - E[X_1])(y - E[X_2])P(x, y) \\ &= (0 - \frac{30}{16})(1 - \frac{3}{2})(\frac{1}{8}) + (1 - \frac{30}{16})(1 - \frac{3}{2})(\frac{1}{16}) + (2 - \frac{30}{16})(1 - \frac{3}{2})(\frac{3}{16}) + (3 - \frac{30}{16})(1 - \frac{3}{2})(\frac{1}{8}) + \\ &(0 - \frac{30}{16})(2 - \frac{3}{2})(\frac{1}{16}) + (1 - \frac{30}{16})(2 - \frac{3}{2})(\frac{1}{16}) + (2 - \frac{30}{16})(2 - \frac{3}{2})(\frac{1}{8}) + (3 - \frac{30}{16})(2 - \frac{3}{2})(\frac{1}{4}) \end{aligned}$$

Problem 46

Find $\text{Corr}(X_1, X_2)$ for the random variables of Problem 44.

$$f_{X_1,X_2}(x,y) = \begin{cases} 3(x+y), & \text{if } x \geq 0, y \geq 0, 0 \leq x+y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

$$E[X_i] = \frac{3}{8}$$
$$Var(X_i) = \frac{19}{320}$$

Solution

$$Corr(X_1, X_2) = \frac{cov(X_1, X_2)}{\sigma_{X_1}\sigma_{X_2}} = \frac{E[X_1X_2] - E[X_1]E[X_2]}{\sqrt{Var(X_1)}\sqrt{Var(X_2)}}$$

We must calculate the joint expectation of X_1 and X_2 to finish our calculation for the correlation!

$$\begin{aligned}
E[X_1X_2] &= \int\int xyf_{X_1,X_2}(x,y)\,dx\,dy \\
&= \int_0^1\int_0^{1-y}(xy)3(x+y)\,dx\,dy \\
&= 3\int_0^1 y\int_0^{1-y} x^2 + xy\,dx\,dy \\
&= 3\int_0^1 y\left(\frac{x^3}{3} + \frac{x^2y}{2}\Big|_0^{1-y}\right)dy \\
&= 3\int_0^1 y\left(\left(\frac{(1-y)^3}{3} + \frac{(1-y)^2y}{2}\right) - \left(\frac{0^3}{3} + \frac{0^2y}{2}\right)\right)dy \\
&= 3\int_0^1 y\left(\frac{(1-y)^3}{3} + \frac{(1-y)^2y}{2}\right)dy \\
&= 3\int_0^1 y\left(\frac{-y^3+3y^2-3y+1}{3} + \frac{(y^2-2y+1)y}{2}\right)dy \\
&= 3\int_0^1 \frac{-y^4+3y^3-3y^2+y}{3} + \frac{y^4-2y^3+y^2}{2}\,dy \\
&= \int_0^1 -\frac{2y^4}{2} + \frac{6y^3}{2} - \frac{6y^2}{2} + \frac{2y}{2} + \frac{3y^4}{2} - \frac{6y^3}{2} + \frac{3y^2}{2}\,dy \\
&= \frac{1}{2}\int_0^1 y^4 - 3y^2 + 2y\,dy \\
&= \frac{1}{2}\left(\frac{y^5}{5} - y^3 + y^2\right)\Big|_0^1 \\
&= \frac{1}{2}\left(\frac{1^5}{5} - 1^3 + 1^2\right) - \left(\frac{0^5}{5} - 0^3 + 0^2\right) \\
&= \frac{1}{2}\left(\frac{1}{5} - 1 + 1\right) \\
&= \frac{1}{10} - \frac{1}{2} + \frac{1}{2} \\
E[X_1X_2] &= \frac{1}{10}
\end{aligned}$$

Now we will solve for the correlation.

$$
\begin{aligned}
Corr(X_1, X_2) &= \frac{E[X_1X_2] - E[X_1]E[X_2]}{\sqrt{Var(X_1)}\sqrt{Var(X_2)}} \\
&= \frac{\frac{1}{10} - \frac{3}{8}\frac{3}{8}}{\sqrt{\frac{19}{320}}\sqrt{\frac{19}{320}}} \\
&= \frac{\frac{1}{10} - \frac{9}{64}}{\frac{19}{320}} \\
Corr(X_1, X_2) &= -\frac{13}{19} = -0.68\ldots
\end{aligned}
$$

Problem 52

If X_1 and X_2 have the same probability distribution function, show that

$$\mathrm{Cov}(X_1 - X_2, X_1 + X_2) = 0$$

Note that independence is not being assumed.

Solution

$$
\begin{aligned}
Cov(A, B) &= E[AB] - E[A]E[B] \\
Cov(X_1 - X_2, X_1 + X_2) &= E[X_1 - X_2X_1 + X_2] - E[X_1 - X_2]E[X_1 + X_2] \\
&= E[X_1 - X_2X_1 + X_2] - (E[X_1] - E[X_2])(E[X_1] + E[X_2]) \\
&= E[0\,(X_1 + X_2)] - (0)(0) \\
&= 0
\end{aligned}
$$

Problem 53

Suppose that X has density function

$$f(x) = e^{-x}, x > 0$$

Compute the moment generating function of X and use your result to determine its mean and variance. Check your answer for the mean by a direct calculation.

Solution

$$\begin{aligned}
M_x(t) &= \int_{-\infty}^{\infty} e^{tx} f(x)\, dx \\
&= \int_{0}^{\infty} e^{tx} e^{-x}\, dx \\
&= \int_{0}^{\infty} e^{tx-x}\, dx \\
&= \int_{0}^{\infty} e^{x(t-1)}\, dx \\
&= \frac{e^{x(t-1)}}{t-1}\bigg|_0^\infty \\
&= \left(\frac{e^{\infty(t-1)}}{t-1}\right) - \left(\frac{e^{0(t-1)}}{t-1}\right) \\
M_x(t) &= \frac{-1}{t-1}
\end{aligned}$$

Calculating the mean

$$\begin{aligned}
\mu &= \frac{d}{dt} M_x(t=0) \\
&= \frac{d}{dt} - (t-1)^{-1} \\
&= (t-1)^{-2} \cdot 1 \\
&= (t-1)^{-2} \\
&= (0-1)^{-2} \\
\mu &= 1
\end{aligned}$$

Calculating the variance

$$\begin{aligned}
E[X^2] &= \frac{d^2}{dt} M_x(t=0) \\
&= \frac{d^2}{dt} - (t-1)^{-1} \\
&= -2(t-1)^{-3} \\
&= -2(0-1)^{-3} \\
E[X^2] &= 2
\end{aligned}$$

$$\begin{aligned}
\sigma^2 &= E[X^2] - (E[X])^2 \\
&= 2 - (1)^2 \\
\sigma^2 &= 1
\end{aligned}$$

Problem 57

Let X and Y have respective distribution functions F_X and F_Y , and suppose that for some constants a and $b > 0$,

$$F_X(x) = F_Y(\frac{x-a}{b})$$

Hint: X has the same distribution as what other random variable?

(a) Determine $E[X]$ in terms of $E[Y]$.

$$\begin{aligned}
F_X(x) &= F_Y(\frac{x-a}{b}) \\
P(X \leq x) &= P(Y \leq \frac{x-a}{b}) \\
P(X \leq x) &= P(bY \leq x-a) \\
P(X \leq x) &= P(bY + a \leq x) \\
X &= bY + a \\
E[X] &= E[bY + a]
\end{aligned}$$

(b) Determine Var(X) in terms of Var(Y).

$$Var(X) = Var(bY + a)$$