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1 Coloring

Vertex Coloring

Theorem 1.1 (Brook's Theorem). *In a connected graph in which every vertex has at most Δ neighbors, the vertices can be colored with only Δ colors, except for two cases, complete graphs and cycle graphs of odd length, which require $\Delta + 1$ colors.*

1.0.1 Chromatic Polynomial

$$P_G(k) = P_{G_1}(k) + P_{G_2}(k)$$

The first coefficient is always 1.

The degree of the first term is the $(|V|)$.

The second coefficient is always $-(|E|)$.

The final (constant) coefficient is always 0.

Definition 1.2. The chromatic polynomial of a complete graph K_n on n vertices is

$$P_{K_n} = k(k-1)(k-2)\dots(k-(n-1))$$

◇

Definition 1.3. The chromatic polynomial of a tree T_n on n vertices is

$$P_{T_n} = k(k-1)^{n-1}$$

◇

Edge Coloring

Definition 1.4. The chromatic index of a graph, χ' , is ...

◇

$$\text{number of edges in } L(G) = \sum_{i=1}^n \binom{d_i}{2}$$

Theorem 1.5. $\chi'(G) = \chi(L(G))$

Theorem 1.6 (Vizing's Theorem). *The chromatic index of simple undirected graph is either Δ or $\Delta + 1$.*

Theorem 1.7 (König's Line Coloring Theorem).

Continued line graph derivations of connected graphs

1. Paths
2. Cycles
3. $K_{1,3}$
4. All others grow

Theorem 1.8 (Whitney's Theorem). *Two connected graphs on at least 4 vertices are isomorphic if and only if their line graphs are isomorphic.*

Planar Duality

Definition 1.9 (Dual Graph). The dual graph of a plane graph G is a graph that has a vertex for each face of G . The dual graph has an edge whenever two faces of G are separated from each other by an edge. \diamond

Theorem 1.10. *A graph has the same number of edges as its dual.*

Theorem 1.11. *A graph with n vertices and f faces has a dual with f vertices and n faces.*

Proposition 1.12. *Wheels are self-dual.*

Proposition 1.13. *Duality is an involution.*

2 The Next Section

Hello this is another section.