MATH 327: Problem Set #7

Due on March 29, 2017 at 2:10pm

Professor Mei-Hsiu Chen

Tim Hung

Problem 1

Suppose that X1, X2, X3 are independent with the common probability mass function

$$P\{X_i = 0\} = .2, \quad P\{X_i = 1\} = .3, \quad P\{X_i = 3\} = .5, \quad i = 1, 2, 3$$

(a) Plot the probability mass function of $\overline{X_2} = \frac{X_1+X_2}{2}$.

(b) Determine E[$\overline{X_2}$] and Var($\overline{X_2}$).

$$\mathrm{E}[\overline{X_2}] = 1.8$$
$$\mathrm{Var}(\overline{X_2}) = 0.8...$$

(d) Determine E[$\overline{X_3}$] and Var($\overline{X_3}$).

$$\mathrm{E}[\overline{X_3}] = 1.8$$
$$\mathrm{Var}(\overline{X_3}) = 0.5...$$

Problem 2

If 10 fair dice are rolled, approximate the probability that the sum of the values obtained (which ranges from 10 to 60) is between 30 and 40 inclusive.

Solution

$$\begin{aligned} P\{30 \leq X \leq 40\} &= P\{29.5 \leq X \leq 40.5\} \\ &\cong P\left\{\frac{29.5 - 35}{\sqrt{\frac{350}{12}}} \leq Z \leq \frac{40.5 - 35}{\sqrt{\frac{350}{12}}}\right\} \\ &= \Phi(1.02) - \Phi(-1.02) = .6922 \end{aligned}$$

Problem 3

Approximate the probability that the sum of 16 independent uniform (0, 1) random variables exceeds 10.

Solution

$$P\{S > 10\} \cong 1 - \Phi\left(\frac{2}{\sqrt{\frac{4}{3}}}\right) = .42$$

Problem 5

A highway department has enough salt to handle a total of 80 inches of snowfall. Suppose the daily amount of snow has a mean of 1.5 inches and a standard deviation of .3 inches.

(a) Approximate the probability that the salt on hand will suffice for the next 50 days.

$$\Phi\left(\frac{80-50(1.5)}{.3\sqrt{50}}\right) = \Phi(2.36...) = .99...$$

(b) What assumption did you make in solving part (a)?

That the snowfall is independent of each other every day.

(c) Do you think this assumption is justified? Explain briefly

Probably not, because snowstorms can last for multiple days etc. Weather is pretty cyclic.

Problem 9

The lifetime of a certain electrical part is a random variable with mean 100 hours and standard deviation 20 hours. If 16 such parts are tested, find the probability that the sample mean is

(a) less than 104;

$$P\{\overline{X} < 104\} \cong \Phi\left(\frac{52-60}{1.5\sqrt{52}}\right) = .23$$

(b) between 98 and 104 hours.

$$.79 - \Phi(\frac{-8}{40}) = .443$$

Problem 10

A tobacco company claims that the amount of nicotine in its cigarettes is a random variable with mean 2.2 mg and standard deviation .3 mg. However, the sample mean nicotine content of 100 randomly chosen cigarettes was 3.1 mg. What is the approximate probability that the sample mean would have been as high or higher than 3.1 if the companys claims were true?

Solution

$$1 - \Phi(\frac{9}{.3}) = 0$$

Problem 12

An instructor knows from past experience that student exam scores have mean 77 and standard deviation 15. At present the instructor is teaching two separate classes one of size 25 and the other of size 64.

(a) Approximate the probability that the average test score in the class of size 25 lies between 72 and 82.

$$\Phi(\frac{25}{15}) - \Phi(\frac{-25}{15}) = .90$$

(b) Repeat part (a) for a class of size 64.

$$\Phi(\frac{40}{15}) - \Phi(\frac{-40}{15}) = .99$$

(c) What is the approximate probability that the average test score in the class of size 25 is higher than that of the class of size 64?

$$.5$$

(d) Suppose the average scores in the two classes are 76 and 83. Which class, the one of size 25 or the one of size 64, do you think was more likely to have averaged 83?

The class of size 25.

Problem 14

Each computer chip made in a certain plant will, independently, be defective with probability .25. If a sample of 1,000 chips is tested, what is the approximate probability that fewer than 200 chips will be defective?

Solution

$$P\{X < 199.5\} \cong \Phi\left(\frac{-50.5}{\sqrt{\frac{3000}{16}}}\right) = \Phi(-3.7...) = .0001$$

Problem 15

A club basketball team will play a 60-game season. Thirty-two of these games are against class A teams and 28 are against class B teams. The outcomes of all the games are independent. The team will win each game against a class A opponent with probability .5, and it will win each game against a class B opponent with probability .7. Let X denote its total number of victories in the season.

(a) Is X a binomial random variable?

It is not a binomial random variable.

(b) Let X_A and X_B denote, respectively, the number of victories against class A and class B teams. What are the distributions of X_A and X_B?

Both are binomially distributed.

(c) What is the relationship between X_A, X_B, and X?

X is the sum of the other two random variables.

(d) Approximate the probability that the team wins 40 or more games.

$$P\{X > 39.5\} \cong .148$$

Problem 17

Use the text disk to compute $P\{X \leq 10\}$ when X is a binomial random variable with parameters n = 100, p = .1. Now compare this with its

(a) Poisson approximation

$$.583$$

(b) Normal approximation

$$.566$$

In using the normal approximation, write the desired probability as $P\{X < 10.5\}$ so as to utilize the continuity correction.

Problem 18

The temperature at which a thermostat goes off is normally distributed with variance σ^2. If the thermostat is to be tested five times, find

(a) $P\{\frac{S^2}{\sigma^2} \leq 1.8\}$

$$P\{\chi_4^2 \leq 7.2\}$$

(b) $P\{.85 \leq \frac{S^2}{\sigma^2} \leq 1.15\}$

$$P\{3.4 \leq \chi_4^2 \leq 4.2\}$$

where S^2 is the sample variance of the five data values.

Problem 20

Consider two independent samples the first of size 10 from a normal population having variance 4 and the second of size 5 from a normal population having variance 2. Compute the probability that the sample variance from the second sample exceeds the one from the first. (Hint: Relate it to the F-distribution.)
Solution

$$P\{S_2^2 > S_1^2\} = P\{F_{9,4} < 0.5\}$$

Problem 21

Twelve percent of the population is left-handed. Find the probability that there are between 10 and 14 left-handers in a random sample of 100 members of this population. That is, find $P\{10 \leq X \leq 14\}$, where X is the number of left-handers in the sample.
Solution

.56...

Problem 23

The following table gives the percentages of individuals of a given city, categorized by gender, that follow certain negative health practices.

	Sleeps 6 Hours or Less per Night	Smoker	Rarely Eats Breakfast	Is 20 Percent or More Overweight
Men	22.7	28.4	45.4	29.6
Women	21.4	22.8	42.0	25.6

Suppose a random sample of 300 men is chosen. Approximate the probability that

(a) at least 150 of them rarely eat breakfast

.6711

(b) fewer than 100 of them smoke

.9735

Problem 25

(Use the table from Problem 23.) Suppose random samples of 300 women and of 300 men are chosen. Approximate the probability that more women than men rarely eat breakfast.

Solution

$$P\{X - Y > 0\} \cong \Phi(-.84) = .2005$$

Problem 28

The sample mean and sample standard deviation of all San Francisco student scores on the most recent Scholastic Aptitude Test examination in mathematics were 517 and 120. Approximate the probability that a random sample of 144 students would have an average score exceeding

(a) 507

$$.84$$

(b) 517

$$.02$$

(c) 537

$$.00$$

(d) 550

$$.06$$

Problem 29

The average salary of newly graduated students with bachelors degrees in chemical engineering is $53,600, with a standard deviation of $3,200. Approximate the probability that the average salary of a sample of 12 recently graduated chemical engineers exceeds $55,000.

Solution

$$.06$$