MATH 327: Problem Set #6

Due on March 22, 2017 at 2:10pm

Professor Mei-Hsiu Chen

Tim Hung

Problem 22

You arrive at a bus stop at 10 oclock, knowing that the bus will arrive at some time uniformly distributed between 10 and 10:30. What is the probability that you will have to wait longer than 10 minutes?
Solution

$$P(X > 10) = 1 - P(X \leq 10) = 1 - \frac{10}{30 - 0} = \frac{2}{3}$$

If at 10:15 the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?
Solution

$$P(X \geq 25 | X \geq 15) = \frac{P(X \geq 25 \cap X \geq 15)}{P(X \geq 15)} = \frac{\frac{5}{30}}{\frac{15}{30}} = \frac{1}{3}$$

Problem 23

If X is a normal random variable with parameters $\mu = 10, \sigma^2 = 36$, compute

(a) $P\{X > 5\}$

$1 - \phi(\frac{-5}{6}) = \phi(\frac{5}{6}) = .80...$

(b) $P\{4 < X < 16\}$

$\phi(1) - \phi(-1) = 2\phi(1) - 1 = .68...$

(c) $P\{X < 8\}$

$\phi(\frac{-1}{3}) = .37...$

(d) $P\{X < 20\}$

$\phi(\frac{10}{6}) = .95...$

(e) $P\{X > 16\}$

$1 - \phi(1) = .16...$

Problem 24

The Scholastic Aptitude Test mathematics test scores across the population of high school seniors follow a normal distribution with mean 500 and standard deviation 100. If five seniors are randomly chosen, find the probability that

(a) all scored below 600

$\phi(1)^5 = .42...$

(b) all scored below 600 and exactly three of them scored above 640.

$10\phi(1.4)^2\phi(1.4)^3 = .00...$

Problem 26

The weekly demand for a product approximately has a normal distribution with mean 1,000 and standard deviation 200. The current on hand inventory is 2,200 and no deliveries will be occurring in the next two weeks. Assuming that the demands in different weeks are independent,

(a) what is the probability that the demand in each of the next 2 weeks is less than 1,100?

$P(D_1 < 1100) \times P(D_2 < 1100) = \phi(.5)\phi(.5) = .49...$

(b) what is the probability that the total of the demands in the next 2 weeks exceeds 2,200?

$P(D_1 + D_2 > 2000) = P(\frac{D_1+D_2-2000}{\sigma} > \frac{2200-2000}{\sigma}) = 1 - .76... = .24...$

Problem 27

A certain type of lightbulb has an output that is normally distributed with mean 2,000 end foot candles and standard deviation 85 end foot candles. Determine a lower specification limit L so that only 5 percent of the lightbulbs produced will be below this limit. (That is, determine L so that $P\{XL\} = .95$, where X is the output of a bulb.)

Solution

$$P\{XL\} = .95 = 1 - \phi(\frac{L-2000}{85})$$

$$\frac{L-2000}{85} = -.16... \to L = 1900...$$

Problem 28

A manufacturer produces bolts that are specified to be between 1.19 and 1.21 inches in diameter. If its production process results in a bolts diameter being normally distributed with mean 1.20 inches and standard deviation .005, what percentage of bolts will not meet specifications?

Solution

$$P\{|X1.2| > .01\} = 2P\{Z > \frac{.01}{.005}\} = 2(1\phi(2)) = .05...$$

Problem 30

A random variable X is said to have a lognormal distribution if log(X) is normally distributed. If X is lognormal with $E[log(X)] = \mu$ and $Var(log(X)) = \sigma^2$, determine the distribution function of X. That is, what is $P\{X \leq x\}$?

Solution

$$P\{X \leq x\} = P\{log(X) \leq log(x)\} = \phi(\frac{[log(x) - \mu]}{\sigma})$$

Problem 32

The sample mean and sample standard deviation on your economics examination were 60 and 20, respectively; the sample mean and sample standard deviation on your statistics examination were 55 and 10, respectively. You scored 70 on the economics exam and 62 on the statistics exam. Assuming that the two histograms of test scores are approximately normal histograms,

(a) on which exam was your percentile score highest?

$$\frac{70-60}{20} < \frac{62-55}{10}$$

Therefore, you scored highest on your statistics exam

(b) approximate the percentage of the scores on the economics exam that were below your score.

$$P(X < 70) = P(Z < .5) = .69...$$

(c) approximate the percentage of the scores on the statistics exam that were below your score.

$$P(X < 62) = P(Z < .7) = .76...$$

Problem 35

The height of adult women in the United States is normally distributed with mean 64.5 inches and standard deviation 2.4 inches. Find the probability that a randomly chosen woman is

(a) less than 63 inches tall

$$\phi(\frac{-1.5}{2.4}) = .27...$$

(b) less than 70 inches tall

$$\phi(\frac{5.5}{2.4}) = .99...$$

(c) between 63 and 70 inches tall

$$\phi(\frac{5.5}{2.4}) - \phi(\frac{-1.5}{2.4}) = .72...$$

(d) Alice is 72 inches tall. What percentage of women is shorter than Alice?

$$\phi(\frac{7.5}{2.4}) = 1.0...$$

Problem 38

The number of years a radio functions is exponentially distributed with parameter $\lambda = \frac{1}{8}$. If Jones buys a used radio, what is the probability that it will be working after an additional 10 years?

Solution

$$e^{\frac{-10}{8}} = .29...$$

Problem 39

Jones figures that the total number of thousands of miles that a used auto can be driven before it would need to be junked is an exponential random variable with parameter $\frac{1}{20}$. Smith has a used car that he claims has been driven only 10,000 miles. If Jones purchases the car, what is the probability that she would get at least 20,000 additional miles out of it?

Solution

$$e^{-1} = .37...$$

Repeat under the assumption that the lifetime mileage of the car is not exponentially distributed but rather is (in thousands of miles) uniformly distributed over (0, 40).

Solution

$$\frac{1}{4} + \frac{3}{4} = \frac{1}{3}$$

Problem 43

If X is a chi-square random variable with 6 degrees of freedom, find

(a) $P\{X \leq 6\}$

```
pchisq(6, df=6)
.5770
```

(b) $P\{3 \leq X \leq 9\}$

```
pchisq(3, 9, df=6)
.6354
```

Problem 44

If X and Y are independent chi-square random variables with 3 and 6 degrees of freedom, respectively, determine the probability that X + Y will exceed 10.
Solution

```
ans = rchisq(10^7, df = 3) + rchisq(10^7, df=6)
length(ans[ans > 10]) / length(ans)

.3504
```

Problem 46

If T has a t-distribution with 8 degrees of freedom, find

(a) $P\{T1\}$

.1732

(b) $P\{T2\}$

.9597

(c) $P\{1 < T < 1\}$

.6536

Problem 48

Let be the standard normal distribution function. If, for constants a and $b > 0$

$$P\{X \leq x\} = \phi(\frac{x-a}{b})$$

characterize the distribution of X.

Solution
Not sure, but it is normally distributed with $\bar{x} = a$, and $\sigma = b$?

In class homeworks

Problem 1

If $X \sim N(\mu, \sigma^c)$

1. Find $P[\mu - 1.51\sigma < X \leq \mu + 1.51\sigma]$

   $\phi(1.51) - \phi(-1.51) = 2\phi(1.51) - 1$

2. Find c

   $P[\mu - c\sigma < X \leq \mu + c\sigma] = .90$

   $2\phi(c) - 1 = .90 \rightarrow c = \phi^{-1}(.85)$

Problem 2

$\chi \sim \chi_1^2$ (use A1 Normal Table) $P[X < 0.7809] =?$
$X \sim t_6$
$P[-x < X < x] = .95$