CS 373 Presentation

A proof of a context free language construction

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April 6, 2016

Overview

Overview

Let A be a regular language and B a context free language.
Is $A \cap B$ a context free language?

Solution

Solution

- **Yes, $A \cap B$ is a context free language**
- How will we prove it?
  - We will construct a PDA that simulates the action of a FA and a PDA

Construction

- Let M_A be a DFA such that $L(M_A) = A$
  - $M_A = (Q_A, \Sigma, \delta_A, q_{A0}, F_A)$
- Let M_B be a PDA such that $L(M_B) = B$
  - $M_B = (Q_B, \Sigma, \Gamma, \delta_B, q_{B0}, F_B)$.
- Define PDA $M = (Q_A \times Q_B, \Sigma, \Gamma, \delta, (q_{A0}, q_{B0}), F_A \times F_B)$

$$\delta(q_A, q_B, a, b) = \begin{cases} (\delta_A(q_A, a), \delta_B(q_B, a, b)) & a \neq \varepsilon \\ (q_A, \delta_B(q_B, a, b)) & a = \varepsilon \end{cases}$$

where q_A and q_B are states from Q_A and Q_B, a is from Σ_ε and b is from Γ_ε.

Proof

- Let $w \in A \cap B$.
- Then there are sequences of states from Q_A and Q_B such that the sequences start in the start states, end in an accept state, and do valid transitions.
- Thus both M_A and M_B end in accept states, and likewise the sequence in M starts in the start state, end in an accept state, and do valid transitions.
- Thus w is accepted by M and $A \cap B \subset L(M)$.

Proof cont.

- Let $w \in L(M)$.
- Then there is a sequence of states that M transitions through while processing w, starting in the start state, ending in an accept state, and performing valid transitions.
- By the definition of M, there are sequences from M_A and M_B that do the same. Thus w is accepted by M_A and M_B.
- The $w \in A$ and $w \in B$. Thus $L(M) \subset A \cap B$.
- Thus we have a PDA, M, that accepts $A \cap B$
- Therefore $A \cap B$ is context free.

Summary

Recap

- Given a regular language A and a context free language B
  - We want to show $A \cap B$ is a context free language
- Construction
  - We constructed M_A a DFA that recognizes regular language A
  - We constructed M_B a PDA that recognizes regular language B.
- Proof
  - We show that processing a $w \in A \cap B$ results in valid transitions leading to accept states in both M_A and M_B.
    - Therefore $A \cap B \subset L(M)$
  - We show that processing a $w \in L(M)$ results in valid transitions leading to accept states in M.
    - Therefore $L(M) \subset A \cap B$
- Therefore, M accepts $A \cap B$ so $A \cap B$ is context free.

Questions

Thank you.
Any questions?