CS 375: Theory Assignment #1

Due on February 12, 2016 at 2:20pm

Professor Lei Yu Section B1

I have done this assignment completely on my own. I have not copied it, nor have I given my solution to anyone else. I understand that if I am involved in plagiarism or cheating I will have to sign an official form that I have cheated and that this form will be stored in my official university record. I also understand that I will receive a grade of 0 for the involved assignment for my first offense and that I will receive a grade of F for the course for any additional offense.

Tim Hung

Problem 1

(10 points) Given the pseudo code below for bubble sort:

```
function BubbleSort(A)
    for i = 1 to (length[A] − 1) do                ▷ store next smallest element in A[i]
        for j = length[A] downto (i + 1) do
            if A[j] < A[j − 1] then
                swap A[j] and A[j − 1]
            end if
        end for
    end for
end function
```

a) (5 points) Let length[A] = n. What is the count for BubbleSort(A)? Show the steps necessary to derive your final answer. This question requires you to use the instruction count method from the textbook (also introduced in lecture 2 slides). Answers using asymptotic notations will receive 0 point.

Solution

My Solution here.

b) (5 points) Show the worse case and best case time complexity in term of instruction counts.

Solution

My solution here.

Problem 2

2. (28 points) Fill in all the missing values. For the f(n) column, you need to compute the sums and fill in the exact format of f(n) for the last two rows. For the last three columns, you need to fill in each cell with either yes or no.

$f(n)$	$g(n)$	$f(n) = O(g(n))$	$f(n) = \Omega(g(n))$	$f(n) = \Theta(g(n))$
$n^{2.125}$	$n^2 lg(n)$	yes	no	no
$\sqrt{n}$	n	yes	no	no
$n!$	$(n+1)!$	yes	yes	yes
$2^{n/2}$	2^n			
$\sum_{i=1}^{n} i =$	n^2			
$\sum_{i=0}^{n-1} 4^i =$	$n4^{(n-1)}$			

Problem 3

(10 points) Order the functions below by increasing growth rates (no justification required).

$$n^n \quad n \ln n \quad n^\epsilon (0 < \epsilon < 1) \quad 2^{\lg n} \quad \ln n \quad 10 \quad n! \quad 2^n$$

Problem 3 continued on next page...

Let $g_i(n)$ be the ith function from the left after the ordering (the leftmost function has the slowest growth rate). In the order, $g_i(n)$ should satisfy $g_i(n) \in O(g_{i+1}(n))$.

Solution

$$10 \quad \ln n \quad n^{\epsilon}(0 < \epsilon < 1) \quad n \ln n \quad 2^{\lg n} \quad 2^n \quad n! \quad n^n$$

Problem 4

(12 points) Let $f(n)$ and $g(n)$ be asymptotically positive functions. Prove or show a counter example for each of the following conjectures.

a) $f(n) \in O(g(n))$ implies $2^{f(n)} \in O(2^{g(n)})$.

b) $f(n) \in O(g(n))$ implies $g(n) \in \Omega(f(n))$.

Solution a
my proof

Solution b
my proof

Problem 5

(10 points) Prove $n^2 - 3n - 20 \in \Theta(n^2)$ using the original definition of Θ.

Problem 6

(10 points) Disprove $n^3 \in O(n^2)$ using the original definition of O.

Problem 7

(10 points) Prove $n = \Omega(\lg n^2)$ using limit.

Problem 8

(10 points) Prove $n^a = \Omega(\lg^k n)$, where $k > 0, a > 0$ using limit.