# MATH 327: Problem Set #10

Due on May 08, 2017 at 2:10pm

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# Problem 27

A sample of 10 fish were caught at lake A and their PCB concentrations were measured using a certain technique. The resulting data in parts per million were

$$\text{Lake A: } 11.5, 10.8, 11.6, 9.4, 12.4, 11.4, 12.2, 11, 10.6, 10.8$$

In addition, a sample of 8 fish were caught at lake B and their levels of PCB were measured by a different technique than that used at lake A. The resultant data were

$$\text{Lake B: } 11.8, 12.6, 12.2, 12.5, 11.7, 12.1, 10.4, 12.6$$

If it is known that the measuring technique used at lake A has a variance of .09 whereas the one used at lake B has a variance of .16, could you reject (at the 5 percent level of significance) a claim that the two lakes are equally contaminated?

**Solution**
Yes, you can reject a claim that the two lakes are equally contaminated.
Calculated $p$-value $\approx 0$

# Problem 28

A method for measuring the pH level of a solution yields a measurement value that is normally distributed with a mean equal to the actual pH of the solution and with a standard deviation equal to .05. An environmental pollution scientist claims that two different solutions come from the same source. If this were so, then the pH level of the solutions would be equal. To test the plausibility of this claim, 10 independent measurements were made of the pH level for both solutions, with the following data resulting.

TABLE GOES HERE

(a) Do the data disprove the scientists claim? Use the 5 percent level of significance.

No

(b) What is the p-value?

$$p\text{-value} = P(Z > 0.81) = .420$$

# Problem 29

The following are the values of independent samples from two different populations.

<div align="center">TABLE GOES HERE</div>

Let $\mu_1$ and $\mu_2$ be the respective means of the two populations. Find the p-value of the test of the null hypothesis

$$H0 : \mu_1 \leq \mu_2$$

versus the alternative

$$H1 : \mu_1 > \mu_2$$

when the population standard deviations are $\sigma_1 = 10$ and

  (a) $\sigma_2 = 5$

$$p\text{-value} = 0.004$$

  (b) $\sigma_2 = 10$

$$p\text{-value} = 0.018$$

  (c) $\sigma_2 = 20$

$$p\text{-value} = 0.092$$

# Problem 31

The viscosity of two different brands of car oil is measured and the following data resulted:

<div align="center">TABLE GOES HERE</div>

Test the hypothesis that the mean viscosity of the two brands is equal, assuming that the populations have normal distributions with equal variances.

**Solution**

$$p\text{-value} = 2P(T_11 > 1.75) = .420$$

# Problem 33

Twenty-five men between the ages of 25 and 30, who were participating in a wellknown heart study carried out in Framingham, Massachusetts, were randomly selected. Of these, 11 were smokers and 14 were not. The following data refer to readings of their systolic blood pressure.

TABLE GOES HERE

Use these data to test the hypothesis that the mean blood pressures of smokers and nonsmokers are the same.

**Solution**

$$p\text{-value} = 0.019$$

# Problem 35

A professor claims that the average starting salary of industrial engineering graduating seniors is greater than that of civil engineering graduates. To study this claim, samples of 16 industrial engineers and 16 civil engineers, all of whom graduated in 2006, were chosen and sample members were queried about their starting salaries. If the industrial engineers had a sample mean salary of \$72,700 and a sample standard deviation of \$2,400, and the civil engineers had a sample mean salary of \$71,400 and a sample standard deviation of \$2,200, has the professors claim been verified? Find the appropriate p-value.

**Solution**

$$p\text{-value} = P(T_{30} > 1.60) = .06$$

Claim is unverified. Not at 5% significance.

# Problem 38

To learn about the feeding habits of bats, 22 bats were tagged and tracked by radio. Of these 22 bats, 12 were female and 10 were male. The distances flown (in meters) between feedings were noted for each of the 22 bats, and the following summary statistics were obtained.

TABLE GOES HERE

Test the hypothesis that the mean distance flown between feedings is the same for the populations of both male and of female bats. Use the 5 percent level of significance.

**Solution**
Cannot reject null hypothesis
Test statistics = 1.15

# Problem 42

Ten pregnant women were given an injection of pitocin to induce labor. Their systolic blood pressures immediately before and after the injection were:

TABLE GOES HERE

Do the data indicate that injection of this drug changes blood pressure?

**Solution**

$$p\text{-value} = 2P(T_9 > 2.33) = .044$$

Data does not indicate the injection changes blood pressure: Rejected at 5% level of significance.

# Problem 47

A pharmaceutical house produces a certain drug item whose weight has a standard deviation of .5 milligrams. The companys research team has proposed a new method of producing the drug. However, this entails some costs and will be adopted only if there is strong evidence that the standard deviation of the weight of the items will drop to below .4 milligrams. If a sample of 10 items is produced and has the following weights, should the new method be adopted?

TABLE GOES HERE

**Solution**
Test H0 : $\sigma \geq .4$

$$\frac{9S^2}{0.4^2} = 9.252 \times 10^4$$

$$p\text{-value} = P(X_9^2 < .000925) < .0001$$

Null hypothesis is rejected.
New method should be adopted.

# Problem 48

The production of large electrical transformers and capacitators requires the use of polychlorinated biphenyls (PCBs), which are extremely hazardous when released into the environment. Two methods have been suggested to monitor the levels of PCB in fish near a large plant. It is believed that each method will result in a normal random variable that depends on the method. Test the hypothesis at the $\alpha = .10$ level of significance that both methods have the same variance, if a given fish is checked 8 times by each method with the following data (in parts per million) recorded.

TABLE GOES HERE

**Solution**

$$\frac{S_1^2}{S_2^2} = .5317$$

$$p\text{-value} = 2P(F_{7.7} < .5317) = .42$$

Hypothesis is accepted.

# Problem 49

In Problem 31, test the hypothesis that the populations have the same variances.

**Solution**

$$\frac{S_1^2}{S_2^2} = 14.05$$

$$p\text{-value} = 2P(F_{5.6} > 14.05) = 0.006$$

Hypothesis is rejected.

# Problem 58

A standard drug is known to be effective in 75 percent of the cases in which it is used to treat a certain infection. A new drug has been developed and has been found to be effective in 42 cases out of 50. Based on this, would you accept, at the 5 percent level of significance, the hypothesis that the two drugs are of equal effectiveness? What is the p-value?

**Solution**

$$p\text{-value} = 2P(Bin(50, .75) \geq 42) = .183$$

# Problem 59

Do Problem 58 by using a test based on the normal approximation to the binomial.

**Solution**

$$p\text{-value} = 2P\left( Z > \frac{41.5 - \frac{150}{4}}{\sqrt{\frac{150}{16}}} \geq 42 \right) = .183$$