MATH 327: Problem Set #8

Due on April 05, 2017 at 2:10pm

Professor Mei-Hsiu Chen

Tim Hung

Problem 1

Let $X_1, ..., X_n$ be a sample from the distribution whose density function is

$$f(x) = \begin{cases} e^{(x)} & x \geq \theta \\ 0 & otherwise \end{cases}$$

Determine the maximum likelihood estimator of θ.

Solution
The smallest value of x

Problem 3

Let $X_1, ..., X_n$ be a sample from a normal μ, σ^2 population. Determine the maximum likelihood estimator of ^{2} when μ is known. What is the expected value of this estimator?

Solution

$$\sigma^2$$

Problem 5

Suppose that $X_1, ..., X_n$ are normal with mean μ_1; $Y_1, ..., Y_n$ are normal with mean μ_2; and $W_1, ..., W_n$ are normal with mean $\mu_1 + \mu_2$. Assuming that all $3n$ random variables are independent with a common variance, find the maximum likelihood estimators of μ_1 and μ_2.

Solution

$$\mu_1 = \frac{2\sum x_i + \sum w_i - \sum y_i}{3n}$$
$$\mu_2 = \frac{-\sum x_i + \sum w_i + 2\sum y_i}{3n}$$

Problem 8

An electric scale gives a reading equal to the true weight plus a random error that is normally distributed with mean θ and standard deviation $\sigma = .1mg$. Suppose that the results of five successive weighings of the same object are as follows: 3.142, 3.163, 3.155, 3.150, 3.141.

(a) Determine a 95 percent confidence interval estimate of the true weight.

$$(3.06..., 3.24...)$$

(b) Determine a 99 percent confidence interval estimate of the true weight

$$(3.03..., 3.27...)$$

Problem 9

The PCB concentration of a fish caught in Lake Michigan was measured by a technique that is known to result in an error of measurement that is normally distributed with a standard deviation of $.08ppm$ (parts per million). Suppose the results of 10 independent measurements of this fish are

$$11.2, 12.4, 10.8, 11.6, 12.5, 10.1, 11.0, 12.2, 12.4, 10.6$$

(a) Give a 95 percent confidence interval for the PCB level of this fish.

$$(11.43..., 11.53...)$$

(b) Give a 95 percent lower confidence interval.

$$(-\infty, 11.52...)$$

(c) Give a 95 percent upper confidence interval.

$$(11.44..., \infty...)$$

Problem 11

Let $X_1, ..., X_n, X_{n+1}$ be a sample from a normal population having an unknown mean μ and variance 1. Let $\overline{X_n} = \sum_{i=1}^{n} \frac{X_i}{n}$ be the average of the first n of them.

(a) What is the distribution of $X_{n+1}\overline{X_n}$?

Normal distribution

(b) If $\overline{X_n} = 4$, give an interval that, with 90 percent confidence, will contain the value of X_{n+1}.

Problem 13

A sample of 20 cigarettes is tested to determine nicotine content and the average value observed was 1.2 mg. Compute a 99 percent two-sided confidence interval for the mean nicotine content of a cigarette if it is known that the standard deviation of a cigarettes nicotine content is $\sigma = .2mg$.

Solution

$$(1.08..., 1.32...)$$

Problem 14

In Problem 13, suppose that the population variance is not known in advance of the experiment. If the sample variance is .04, compute a 99 percent two-sided confidence interval for the mean nicotine content.

Solution

$$(1.07..., 1.33...)$$

Problem 15

In Problem 14, compute a value c for which we can assert with 99 percent con- fidence that c is larger than the mean nicotine content of a cigarette.

Solution

$$c = 1.31$$

Problem 18

The following are scores on IQ tests of a random sample of 18 students at a large eastern university.

$$130, 122, 119, 142, 136, 127, 120, 152, 141, 132, 127, 118, 150, 141, 133, 137, 129, 142$$

(a) Construct a 95 percent confidence interval estimate of the average IQ score of all students at the university.

$$(128.14, 138.30)$$

Problem 21

A standardized test is given annually to all sixth-grade students in the state of Washington. To determine the average score of students in her district, a school supervisor selects a random sample of 100 students. If the sample mean of these students scores is 320 and the sample standard deviation is 16, give a 95 percent confidence interval estimate of the average score of students in that supervisors district.

Solution

$$(316.82.., 323.18...)$$

Problem 23

A random sample of 300 CitiBank VISA cardholder accounts indicated a sample mean debt of $1,220 with a sample standard deviation of $840. Construct a 95 percent confidence interval estimate of the average debt of all cardholders.

Solution

$$1220 \pm \frac{1.97 * 840}{\sqrt{300}}$$