MATH 327: Problem Set #3

Due on February 13, 2017 at 1:10pm

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Problem 1

Five men and 5 women are ranked according to their scores on an examination. Assume that no two scores are alike and all 10! possible rankings are equally likely. Let X denote the highest ranking achieved by a woman (for instance, $X = 2$ if the top-ranked person was male and the next-ranked person was female). Find $P\{X = i\}, i = 1, 2, 3, ..., 8, 9, 10$.

Problem 2

Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times. What are the possible values of X ?

Problem 3

In Problem 2, if the coin is assumed fair, for n = 3, what are the probabilities associated with the values that X can take on?

Problem 4

The distribution function of the random variable X is given

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x}{2} & 0 \leq x < 1 \\ \frac{2}{3} & 1 \leq x < 2 \\ \frac{11}{12} & 2 \leq x < 3 \\ 1 & 3 \leq x \end{cases}$$

(a) Plot this distribution function.

(b) What is $P\{X > \frac{1}{2}\}$?

(c) What is $P\{2 < X \leq 4\}$?

(d) What is $P\{X < 3\}$?

(e) What is $P\{X = 1\}$?

Problem 5

Suppose the random variable X has probability density function

$$f(x) = \begin{cases} cx^3, & \text{if } 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the value of c.

(b) Find $P\{.4 < X < .8\}$.

Problem 9

A set of five transistors are to be tested, one at a time in a random order, to see which of them are defective. Suppose that three of the five transistors are defective, and let N_1 denote the number of tests made until the first defective is spotted, and let N_2 denote the number of additional tests until the second defective is spotted. Find the joint probability mass function of N_1 and N_2.

Problem 10

The joint probability density function of X and Y is given by

$$f(x,y) = \frac{6}{7}(x^2 + \frac{xy}{2}), 0 < x < 1, 0 < y < 2$$

(a) Verify that this is indeed a joint density function.

(b) Compute the density function of X.

(c) Find $P\{X > Y\}$.

Problem 12

The joint density of X and Y is given by

$$f(x,y) = \begin{cases} xe^{-(x+y)}, & x > 0, y > 0 \\ 0, & \text{otherwise} \end{cases}$$

(a) Compute the denisty of X.

(b) Compute the density of Y.

(c) Are X and Y independent?

Problem 13

The joint density of X and Y is given by

$$f(x,y) = \begin{cases} 2, & 0 < x < y, 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) Compute the denisty of X.

(b) Compute the density of Y.

(c) Are X and Y independent?