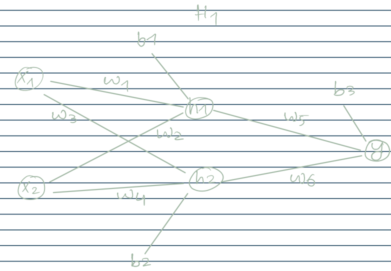
Input $(x) = [x_1, x_2]$

target $(Y) = [1]$

$h1 = \text{sigmoid}(w_1 * x_1 + w_2 * x_2 + b1)$

$h2 = \text{sigmoid}(w_3 * x1 + w_4 * x_2 + b2)$

$\text{output} = \underline{\text{sigmoid}(w_5 * h1 + w_6 * h2 + b_3)}$



MSE loss:

$\text{loss} = 0.5 * (\text{output} - \text{target})^2$

$\text{loss} = \frac{1}{2}(\hat{y} - y)^2$

$$\frac{d(\text{loss})}{d(\hat{y})} = \frac{d}{d\hat{y}}\left(\frac{1}{2}(\hat{y} - y)^2\right)$$

$$= \frac{1}{2} \times 2(\hat{y} - y)$$

$$= (\hat{y} - y)$$

Backward pass__

Computing Gradients:

- $\frac{d(\text{loss})}{d(\text{output})} = \underline{\text{output} - \text{target}}$ → constant

- $\frac{d(\text{output})}{d(w_5)} = h1 * \text{output}(1 - \text{output})$

- $\frac{d(\text{output})}{d(w_6)} = h2 * \text{output}(1 - \text{output})$

- $\frac{d(\text{output})}{d(h1)} = w_5 * \text{output}(1 - \text{output})$

- $\frac{d(\text{output})}{d(h2)} = w_6 * \text{output}(1 - \text{output})$

- $\frac{d(h1)}{d(w1)} = x1 * h1(1 - h1)$

- $\frac{d(h1)}{d(w2)} = x2 * h1(1 - h1)$

- $\frac{d(h2)}{d(w3)} = x1 * h2(1 - h2)$

- $\frac{d(h2)}{d(w4)} = x2 * h2(1 - h2)$

Sigmoid func^n:

$$S(x) = \frac{1}{1 + e^{-x}}$$

$$\text{output} = \frac{1}{1 + e^{-(w_5 * h1 + w_6 * h2 + b_3)}}$$

$$\frac{d(\text{output})}{d(w_5)} = -\left(1 + e^{-(w_5 * h1 + w_6 * h2 + b3)}\right)^{-2} * \left(-e^{-(w_5 * h1 + w_6 * h2 + b3)}\right) * h1$$

$$= S(x) \cdot (1 - S(x)) \text{ ; simplified}$$

⊘ loss func^n was chosen for easy derivatives.

⊘ $\frac{d}{dw}(\text{loss}) = \frac{d(\text{loss})}{d(a)} * \frac{d(a)}{d(b)} * \frac{d(b)}{d(w)}$;

a, b is intermediate value.

Back-propagating Gradients:

- updating using learning rate:

$$w1 = w1 - \text{learning rate} * \frac{d(\text{loss})}{d(\text{output})} * \frac{d(\text{output})}{d(h1)} * \frac{d(h1)}{d(w1)}$$

$$w2 = w2 - \text{learning rate} * \frac{d(\text{loss})}{d(\text{output})} * \frac{d(\text{output})}{d(h1)} * \frac{d(h1)}{d(w2)}$$