

Activation Functions

(Tanh, ReLU, Swish, Mish)

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 - Neural Network & Activation Functions
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Generality on supervised machine learning

In supervised machine learning, we are given a sample data $\{(X_i, y_i)\}_{i=1}^N$.

Goal

Find a function F such that

$$y = F(X)$$

for all (X, y) from the population of the given sample data.

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Approach

Approximating F by a function f using the sample data points, i.e.

$$f \approx F.$$

The function f is called **predictor function**.

Examples of approximation method

Linear functions are easy to parameterized, and that makes it easy to learn.

Definition (Linear Regression)

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One can also write the predictor function as a composition of functions i.e.

$$f = f_k \circ f_{k-1} \circ \cdots \circ f_1,$$

for some functions f_i .

This is the approach we use in Deep Learning ($k \geq 2$).

Neural Network & Activation Functions

Now we will use the approach of writing f as a composition of functions as follows

$$f = (g_k \circ f_k) \circ (g_{k-1} \circ f_{k-1}) \circ \cdots \circ (g_1 \circ f_1).$$

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Neural Network - Activation Function

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- f_i 's are linear functions,
- g_i 's are function that we set ourselves known as **activation functions**.

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- f_i 's are linear functions,
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For the approximation, we only adjust the weights of f_i 's.

Loss function

It is a distance function between f and F .

Gradient Descent

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A common algorithm to adjust weights of the predictor to get small loss is the Gradient Descent algorithm.

Gradient Descent

Let w be a weight of the predictor. To minimize the loss function ℓ w.r.t w , we can adjust w by

$$w \leftarrow w - lr \cdot \frac{\partial \ell}{\partial w}, \quad \text{for some } lr > 0.$$

Sigmoid function is one of the most popular activation function due to that fact that it can be interpreted as probability.

Definition (Sigmoid)

The sigmoid function and its derivative are given by:

$$\begin{aligned}\sigma(x) &= \frac{1}{1 + e^{-x}} \\ \Rightarrow \sigma'(x) &= \sigma(x)(1 - \sigma(x)).\end{aligned}$$

The sigmoid is a non zero-centered function. Its range lies between 0 to 1.

Hyperbolic Tangent

The tanh activation function has been conceived to overcome disadvantage of non-zero centered as in sigmoid.

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$$\begin{aligned}\tanh(x) &= \frac{e^x - e^{-x}}{e^x + e^{-x}} \\ \Rightarrow \tanh'(x) &= 1 - \tanh^2(x).\end{aligned}$$

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It can be seen as deformed sigmoid.

Property

We have

$$\tanh(x) = 2\sigma(2x) - 1.$$

Graphs of Hyperbolic Tangent and its derivative

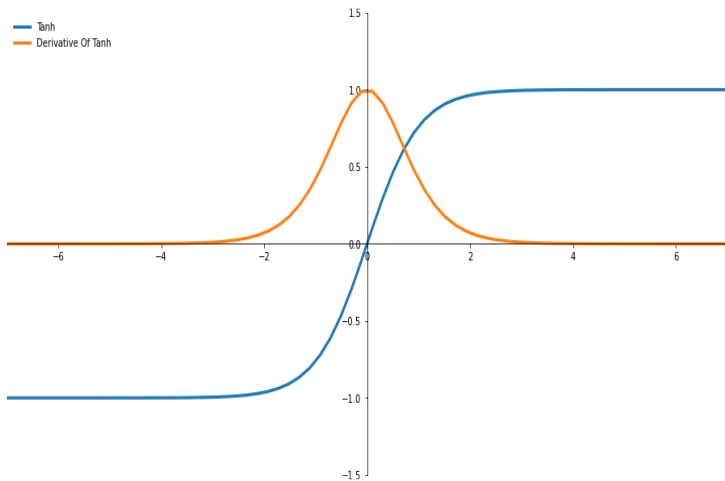


Figure: Hyperbolic tangent vs its derivative

Advantages - Disadvantages

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- Tanh is **continuously differentiable** and provides a smooth gradient, i.e., fast convergence.

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Disadvantages

- Saturation leads to vanishing gradient
- Computationally expensive

Rectified Linear Unit - ReLU

The ReLU outperforms Tanh during learning due the fact of non-vanishing Gradient.

Definition (Rectified Linear Unit)

$$\begin{aligned}\text{ReLU}(x) &= \max(0, x) = x \max\left(0, \frac{x}{|x|}\right) \\ \Rightarrow \text{ReLU}'(x) &= \begin{cases} 0 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}^a.\end{aligned}$$

^abased on this link

Graph of ReLU and its derivative

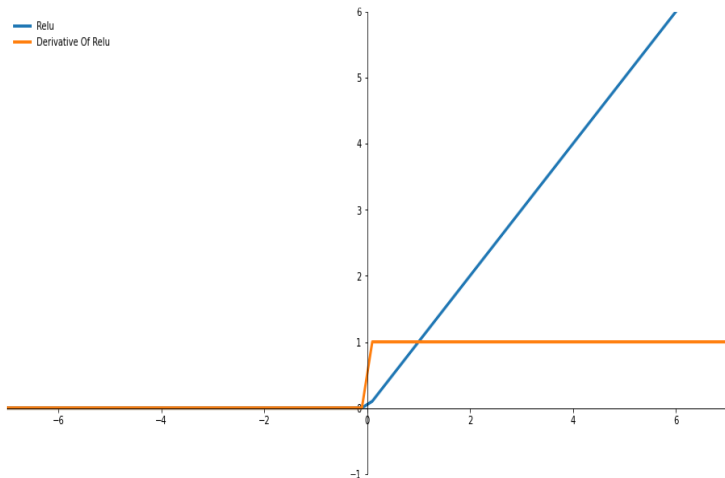


Figure: ReLU vs its derivative

Advantage and Disadvantage of ReLU

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- Cheap computation

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- The gradients for negative input are zero

A lookalike ReLU parameterized activation function named *swish* was proposed in 2017 by Ramachandran et.al., which is defined as follow

Definition (swish)

Swish function and its derivative :

$$\begin{aligned}\text{swish}(x; \beta) &= x \cdot \sigma(\beta x), \forall x \in \mathbb{R}, \beta \text{ is a constant} \\ \implies \text{swish}'(x; \beta) &= \beta \cdot \text{swish}(x; \beta) + \sigma(\beta \cdot x) (1 - \beta \cdot \text{swish}(x; \beta))\end{aligned}$$

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Properties

$$\begin{aligned}\lim_{\beta \rightarrow 0} \text{swish}(x; \beta) &= \frac{x}{2} \\ \lim_{\beta \rightarrow +\infty} \text{swish}(x; \beta) &= \text{ReLU}(x)\end{aligned}$$

Visualizations of swish with different values of β

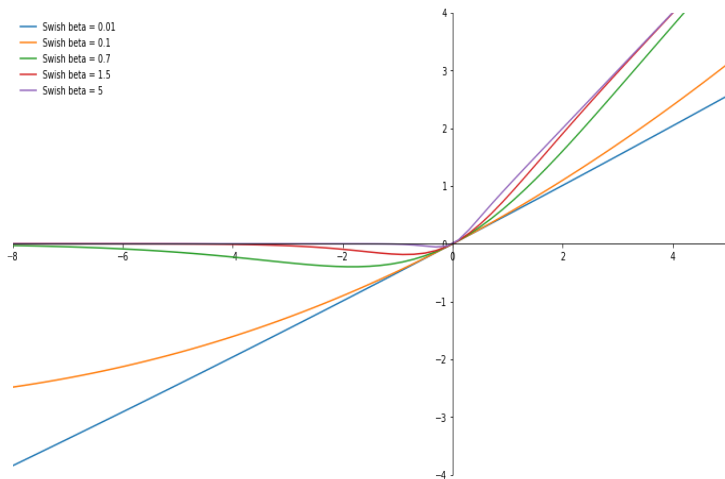


Figure: Swish and its derivatives with different values of β

Visualizations of swish and its first derivative

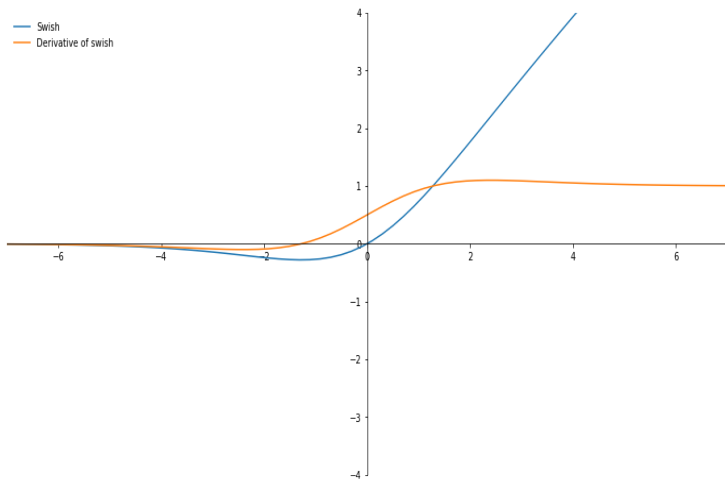


Figure: Swish and its derivatives for $\beta = 1$

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- Self-gated

Disadvantage

- Computationally expensive

Definition (Mish activation function)

$$\begin{aligned}\text{softplus}(x) &= \ln(1 + e^x) \\ \text{mish}(x) &= x \cdot \tanh(\text{softplus}(x))\end{aligned}$$

The first derivative of Mish is given by

Derivative

$$\begin{aligned}\text{mish}'(x) &= \frac{\text{mish}(x)}{x} + x \cdot \sigma(x) (1 - \tanh^2(\text{softplus}(x))) \\ &= \frac{\text{mish}(x)}{x} + \text{swish}(x) \cdot \text{sech}^2(\text{softplus}(x))\end{aligned}$$

Visualizations of mish and its first derivative

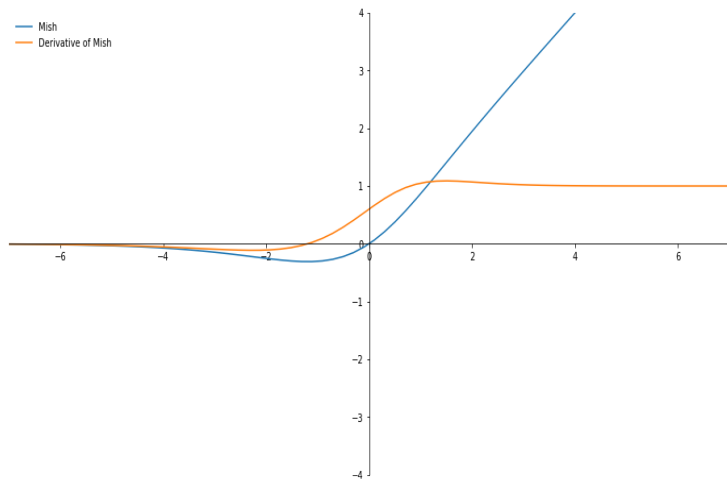


Figure: Mish vs its derivative

Difference of mish versus swish

Properties

- $\forall x \in \mathbb{R}, \tanh(\text{softplus}(x)) \geq \sigma(x)$
- $\forall x \geq 0, \text{mish}(x) \geq \text{swish}(x) \geq 0$
- $\forall x < 0, \text{mish}(x) < \text{swish}(x) < 0$

Mish vs Swish

- This last inequality implies that swish is more regularized than mish.
- Harder to compute the gradient compared to swish.

Visualizations of mish versus swish

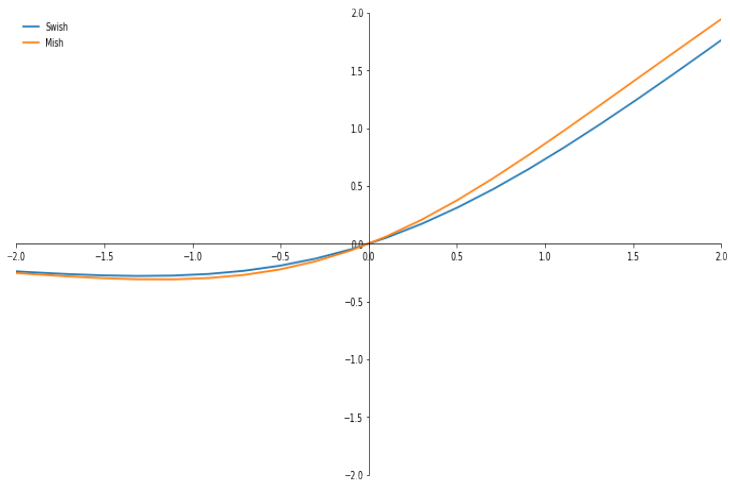


Figure: Mish vs swish

Visualizations of mish versus swish

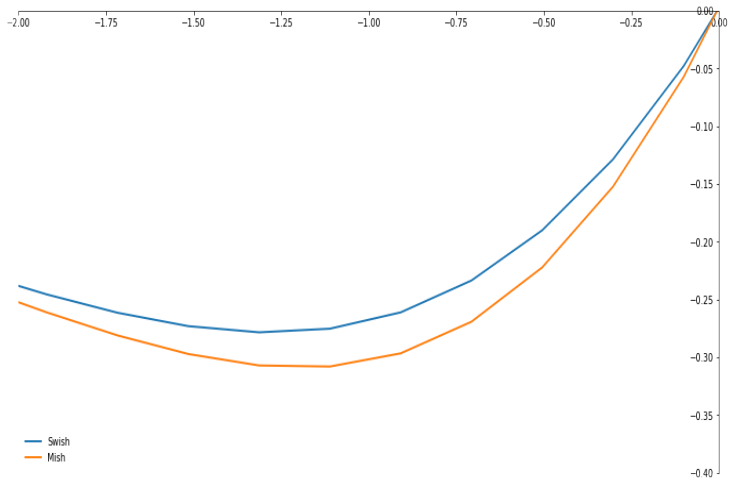


Figure: Zoom in the negatives values

Presentation of the used data and model architecture

For the experiment, we used the MNIST dataset.

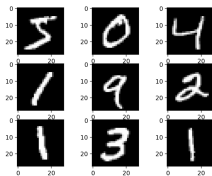


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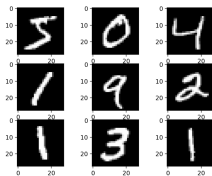
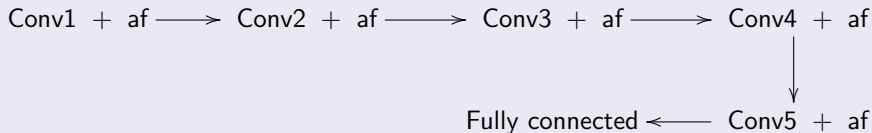


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The **model architecture** is as followed :



Experimental result

Model parameters

batchsize : 64

optimizer : SGD

lr : 0.01

momentum : 0.9

loss function : cross entropy

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Activation function	Num epochs	5	10	30
Tanh		97.69%	98.32%	98.74%
ReLU		98.21%	11.35%	98.78%
Swish		97.99%	98.50%	98.67%
Mish		98.01%	98.49%	98.90%

Table: Test Accuracy.

Observations

We observe the following in general.

- Tanh performs the least.

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- Tanh performs the least.
- Mish performs the most.
- ReLU have better performance than swish unless it faces the zero gradient too early.





Conclusion

We conclude the following :

Tanh < Swish < ReLU (risky) < Mish.

Thank you for listening ...

References

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