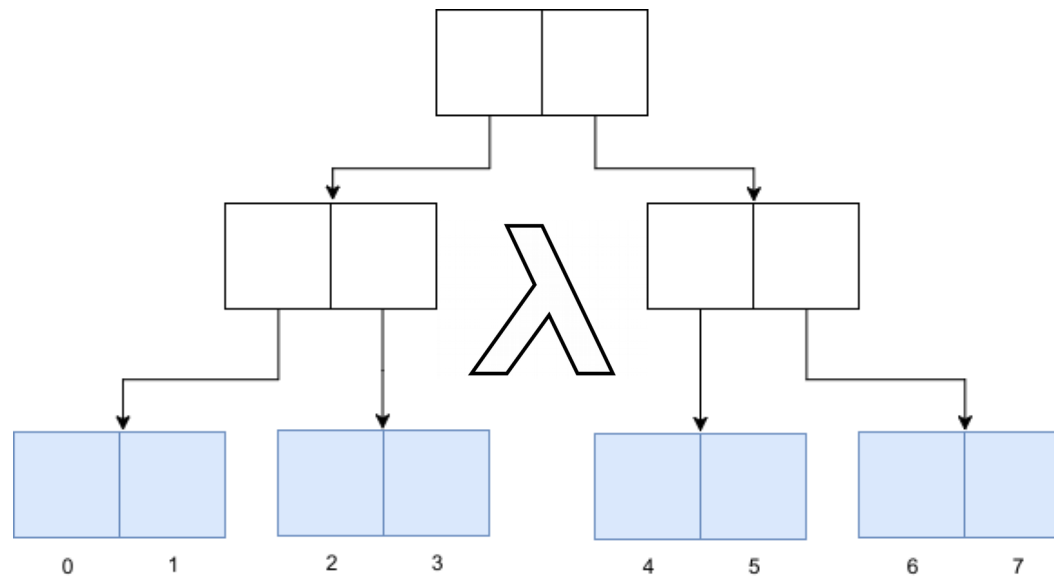


# Marvellous Functional Datastructures: The Vector





# Hello!

- Robert



# Hello!

- Robert
- Scala & Clojure



# Hello!

- Robert
- Scala & Clojure
- Obsessed with FP



# Vectors: An Introduction

- What are Vectors?



# Vectors: An Introduction

- What are Vectors?
- Properties:
  - Append (effectively) constant time ( $\log_{32} n$ )
  - Look-up (effectively) constant time ( $\log_{32} n$ )
  - Update (effectively) constant time ( $\log_{32} n$ )
  - Pop (effectively) constant time ( $\log_{32} n$ )
  - Iterate linear time



# Vectors: An Introduction

- What are Vectors?
- **H**ash **A**rray **M**apped **T**ries (HAMT)



# Vectors: An Introduction

- What are Vectors?
- **H**ash **A**rray **M**apped **T**ries (HAMT)
  - Tries?





# Vectors: An Introduction

- What are Vectors?
- **H**ash **A**rray **M**apped **T**ries (HAMT)
  - Tries?
  - Trie, Radix Tree, Prefix Tree



# Vectors: An Introduction

- What are Vectors?
- **H**ash **A**rray **M**apped **T**ries (HAMT)
  - Tries?
  - Trie, Radix Tree, Prefix Tree
  - Structured based on their contents



# Vectors: An Introduction

- Tries:
  - tee, try, add, aid, map, see, sit



# Vectors: An Introduction

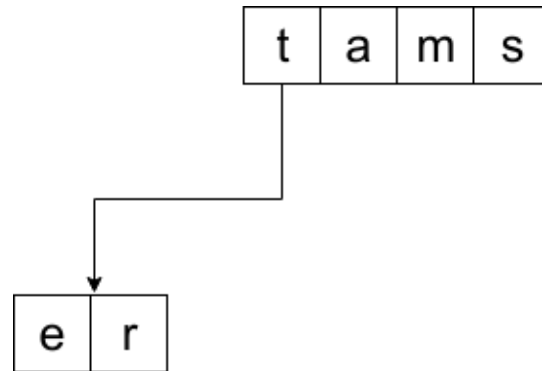
- Tries:
  - tee, try, add, aid, map, see, sit

t	a	m	s
---	---	---	---



# Vectors: An Introduction

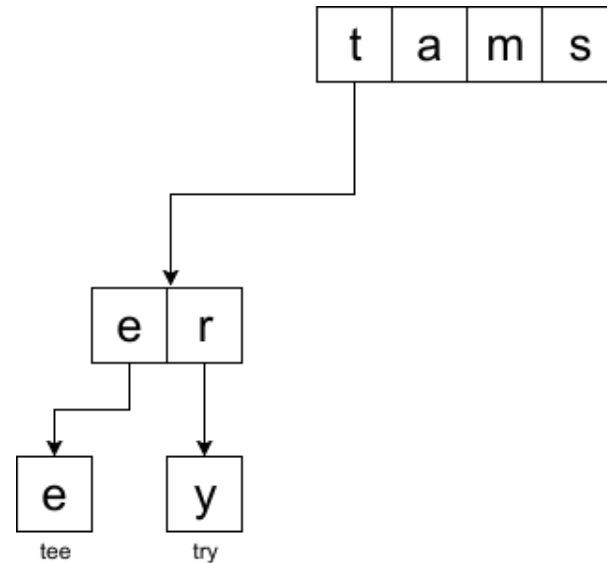
- Tries:
  - tee, try, add, aid, map, see, sit





# Vectors: An Introduction

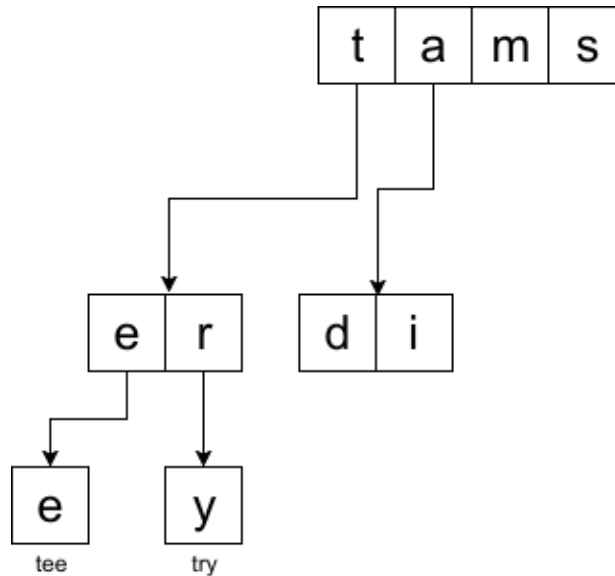
- Tries:
  - tee, try, add, aid, map, see, sit





# Vectors: An Introduction

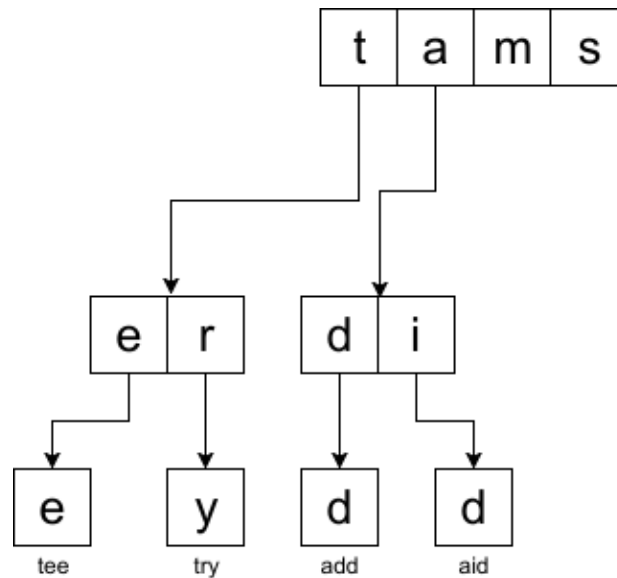
- Tries:
  - tee, try, add, aid, map, see, sit





# Vectors: An Introduction

- Tries:
  - tee, try, add, aid, map, see, sit

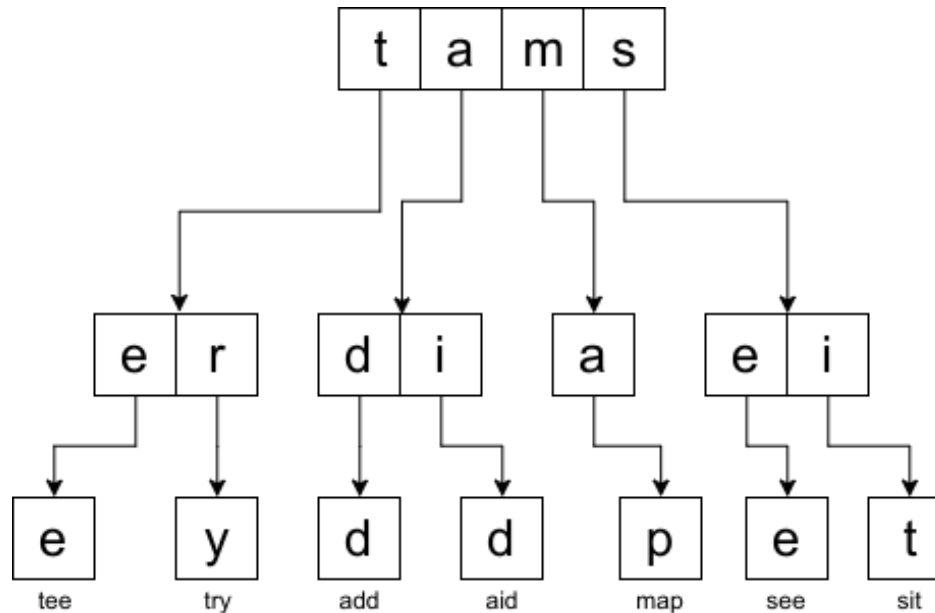






# Vectors: An Introduction

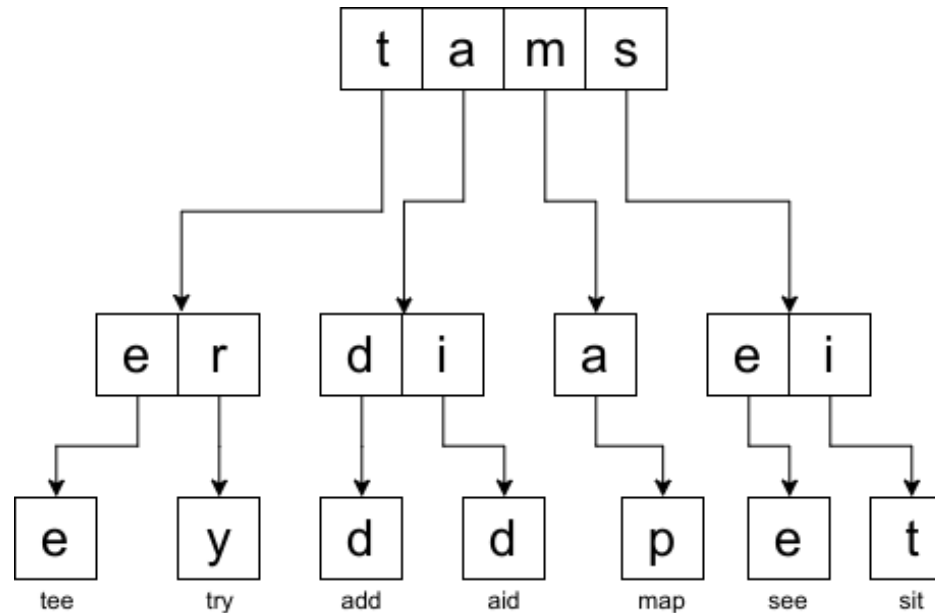
- Tries:
  - tee, try, add, aid, map, see, sit





# Vectors: An Introduction

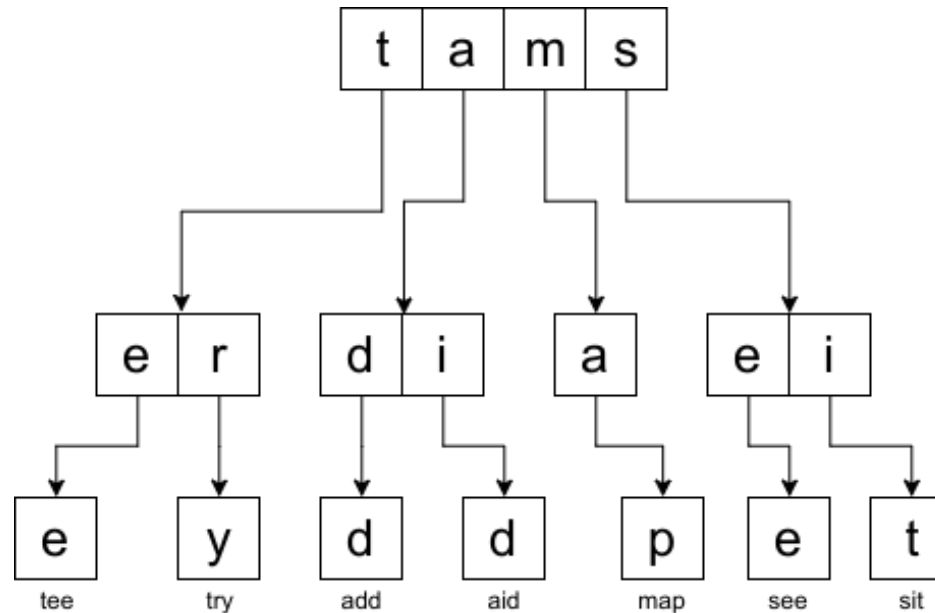
- Tries:
  - Assume we want to lookup “add”





# Vectors: An Introduction

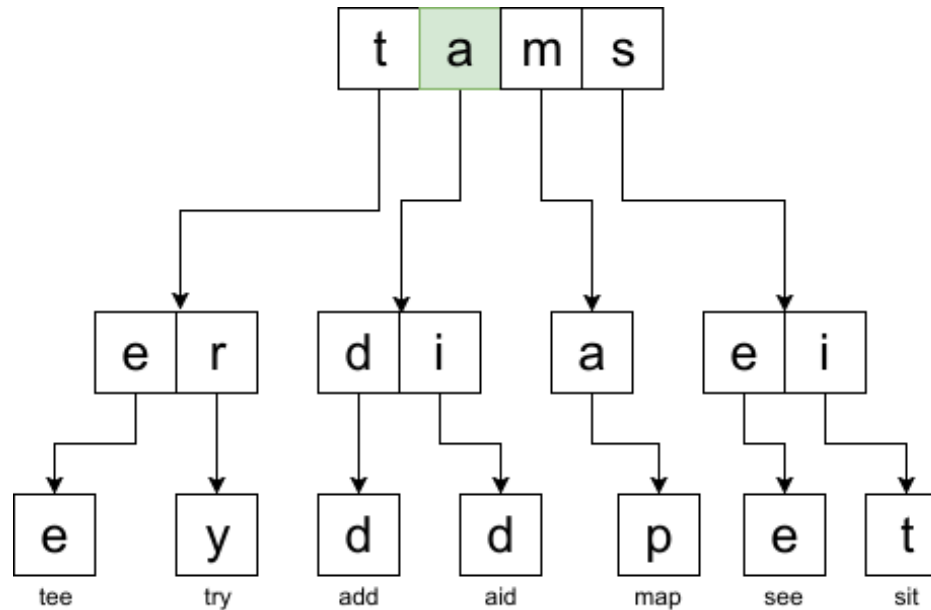
- Tries:
  - Assume we want to lookup “add”
    - Split: a - d - d





# Vectors: An Introduction

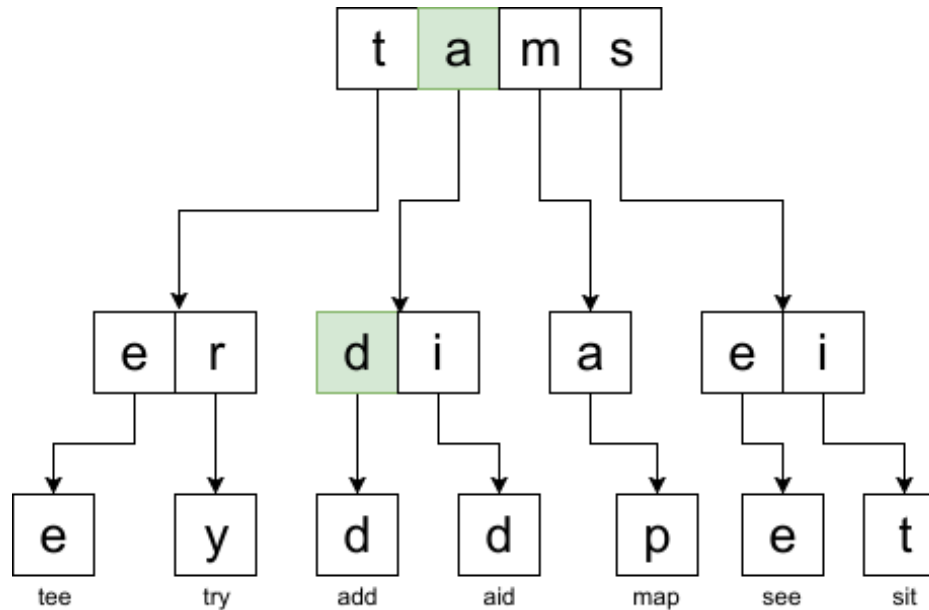
- Tries:
  - Assume we want to lookup “add”
    - Split: a - d - d





# Vectors: An Introduction

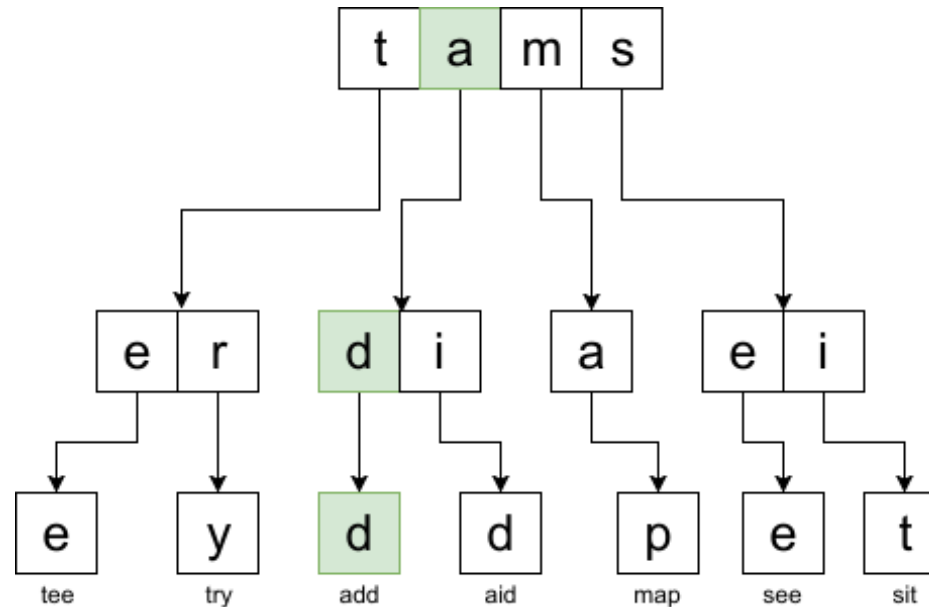
- Tries:
  - Assume we want to lookup “add”
    - Split: a - d - d





# Vectors: An Introduction

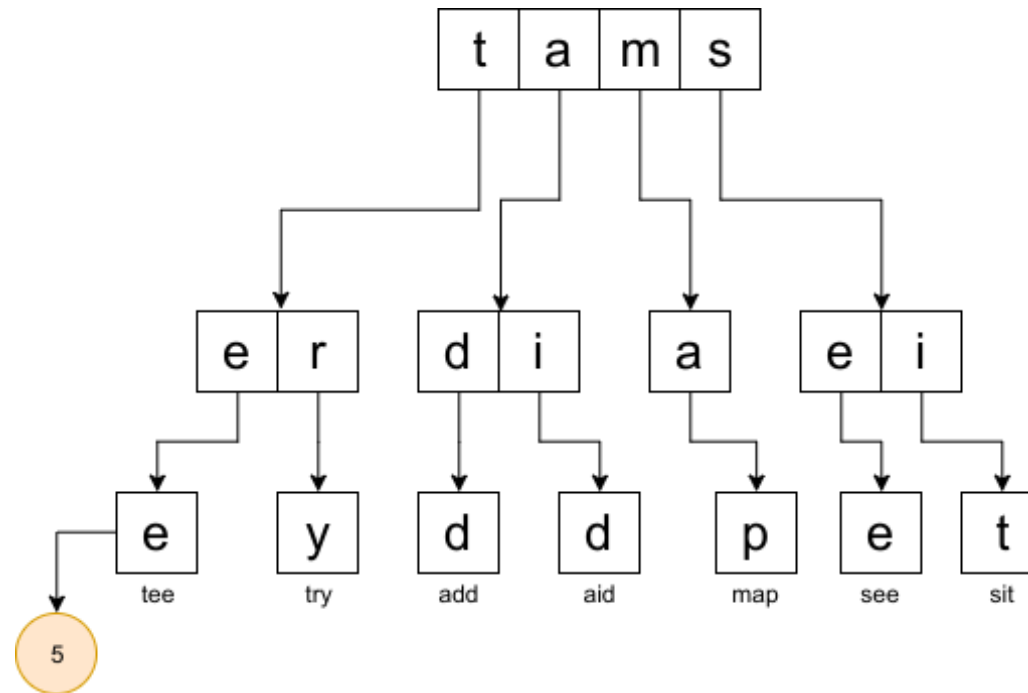
- Tries:
  - Assume we want to lookup “add”
    - Split: a - d - d





# Vectors: An Introduction

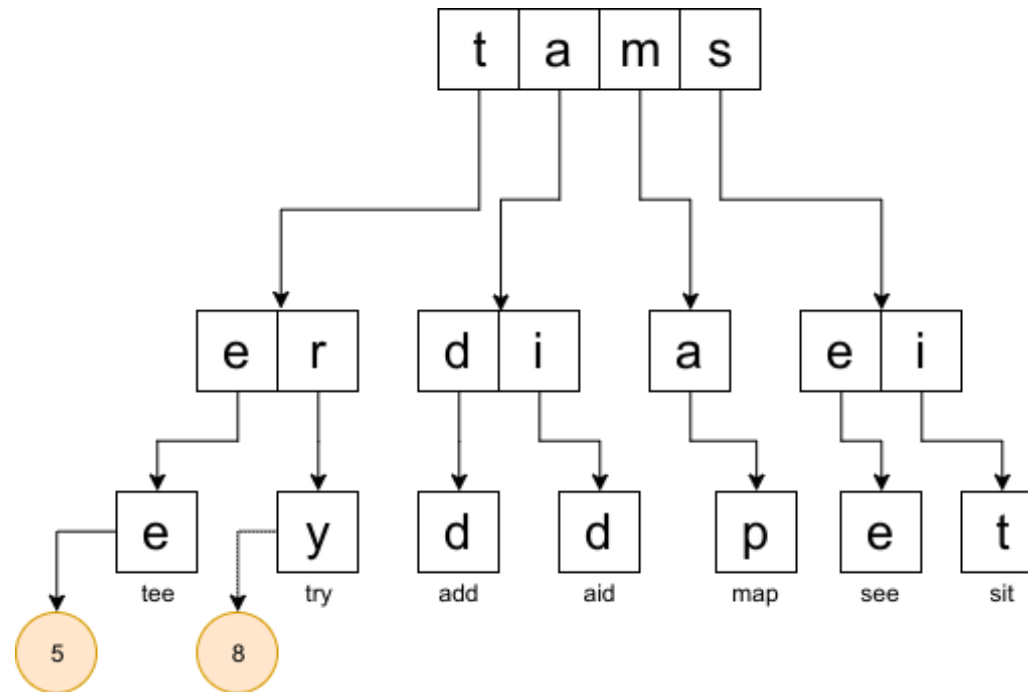
- Trie Maps:
  - Associate value with leaf node





# Vectors: An Introduction

- Trie Maps:
  - Associate value with leaf node

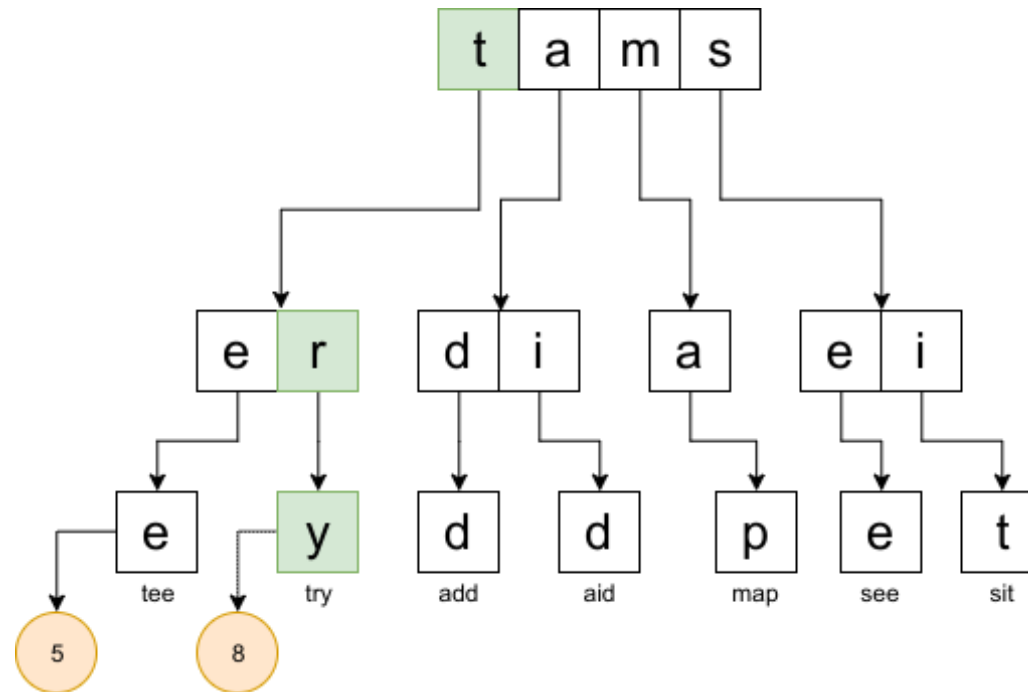






# Vectors: An Introduction

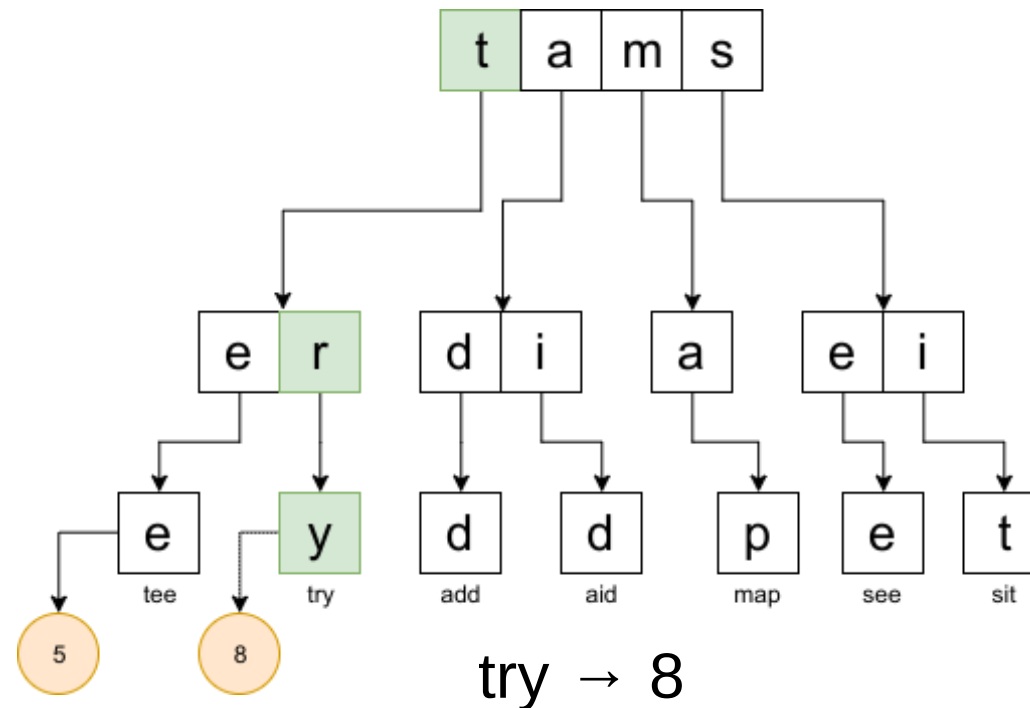
- Trie Maps:
  - Associate value with leaf node
  - Look-up: “try”





# Vectors: An Introduction

- Trie Maps:
  - Associate value with leaf node
  - Look-up: “try”





# Vectors: An Introduction

- Hash Array Mapped Tries:
  - Tries with hash ranges



# Vectors: An Introduction

- Hash Array Mapped Tries:
  - Tries with hash ranges
  - Hash-Maps!



# Vectors: An Introduction

- Hash Array Mapped Tries:
  - Tries with hash ranges
  - Hash-Maps!
- Vectors:
  - HAMTs with ranges of integers



# Vectors: An Introduction

- Indexing a trie:



# Vectors: An Introduction

- Indexing a trie:
  1. Fixed branching factors (power of 2):  
 **$M = 2^b$**



# Vectors: An Introduction

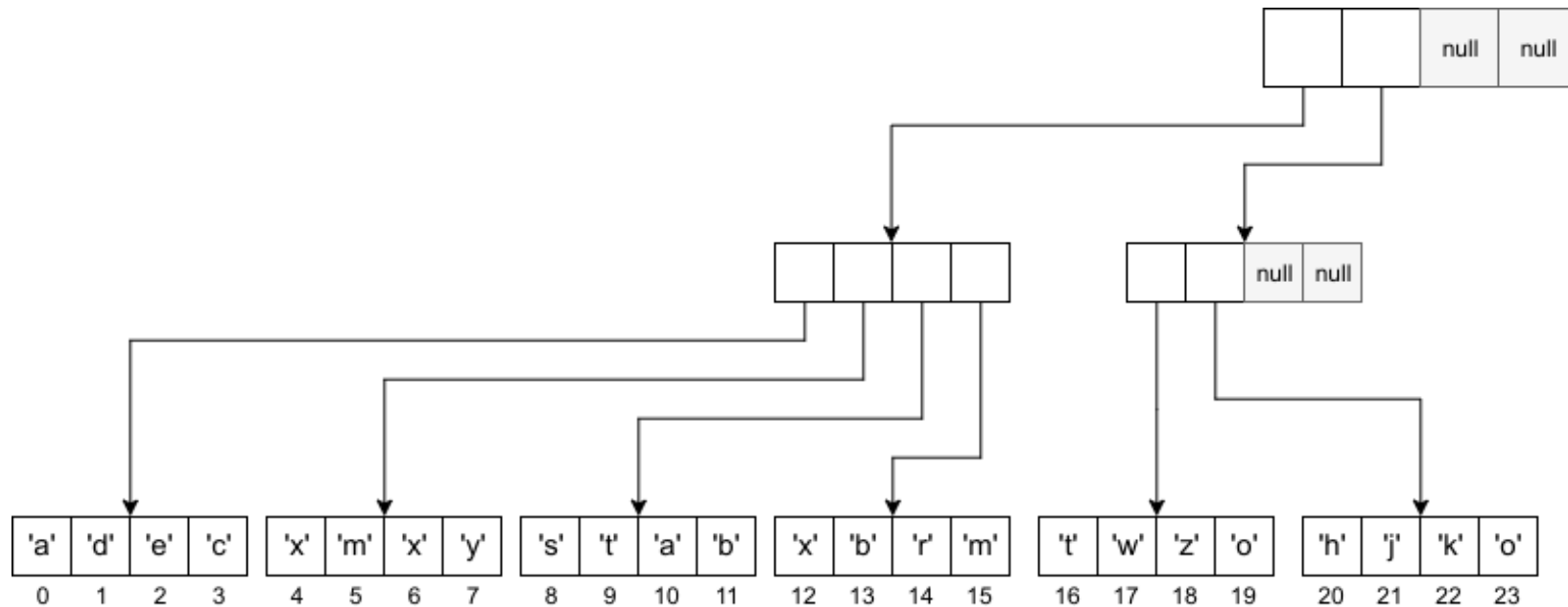
- Indexing a trie:
  1. Fixed branching factors (power of 2):  
 **$M = 2^b$**
  2. Indexes are binary => Trie is bit-mapped





# How to vector

Branching factor: 4  
Height: 2

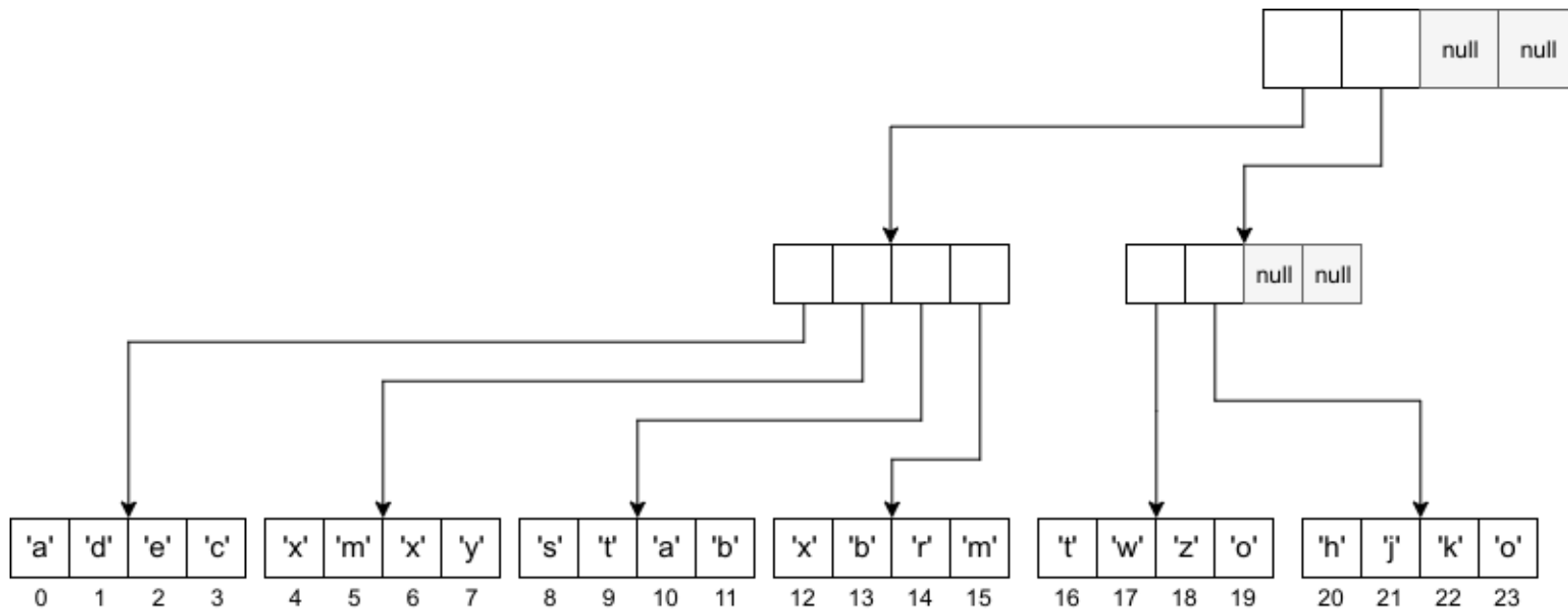




# How to vector

Branching factor: 4  
Height: 2

Look-up: 17

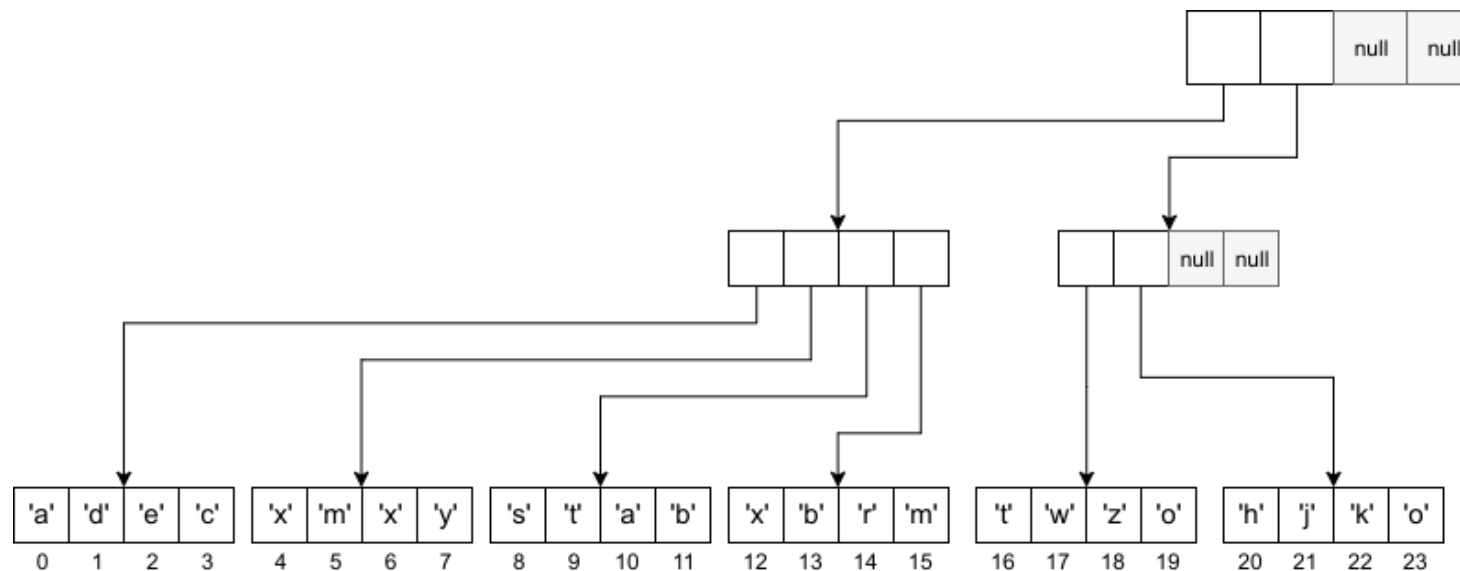




# How to vector

Branching factor: 4  
Height: 2

Look-up: 17



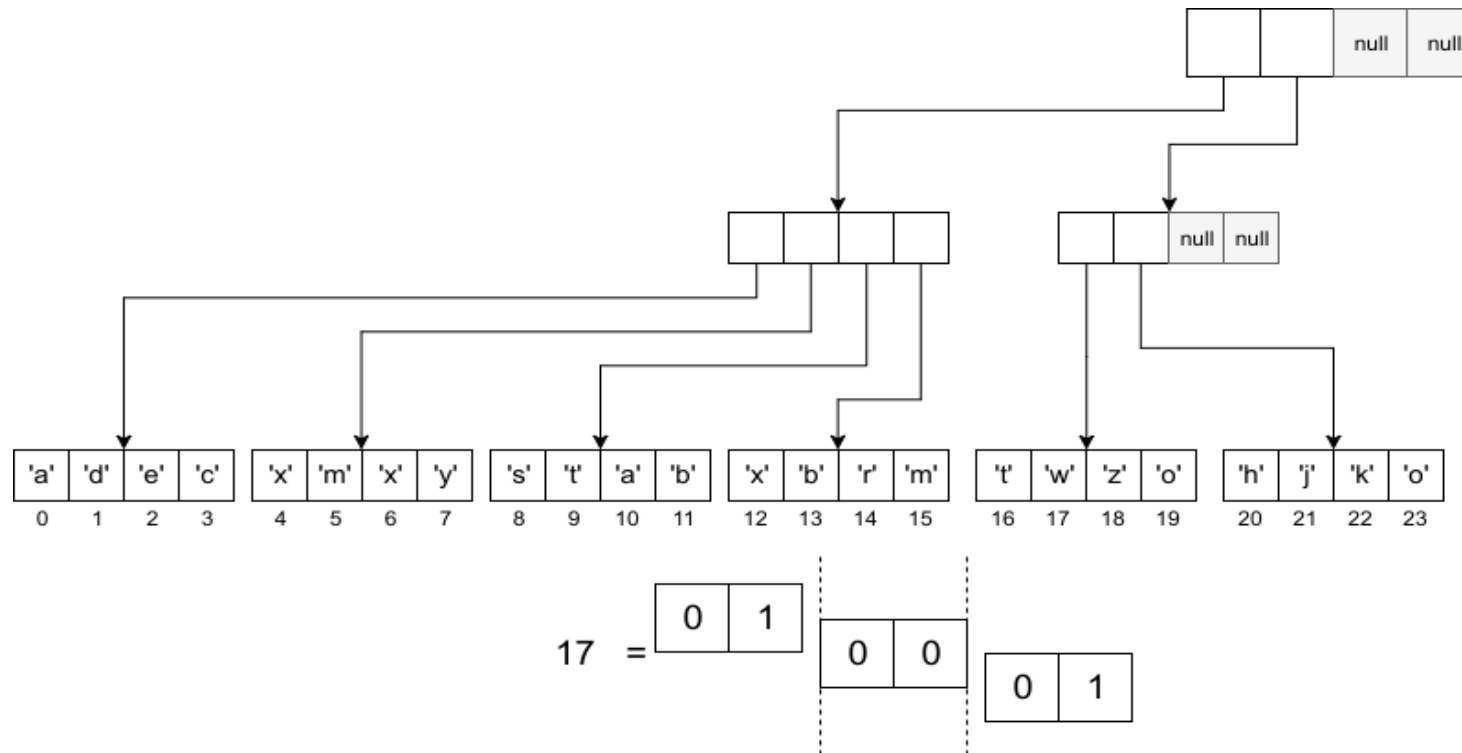
$$17 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$



# How to vector

Branching factor: 4  
Height: 2

Look-up: 17

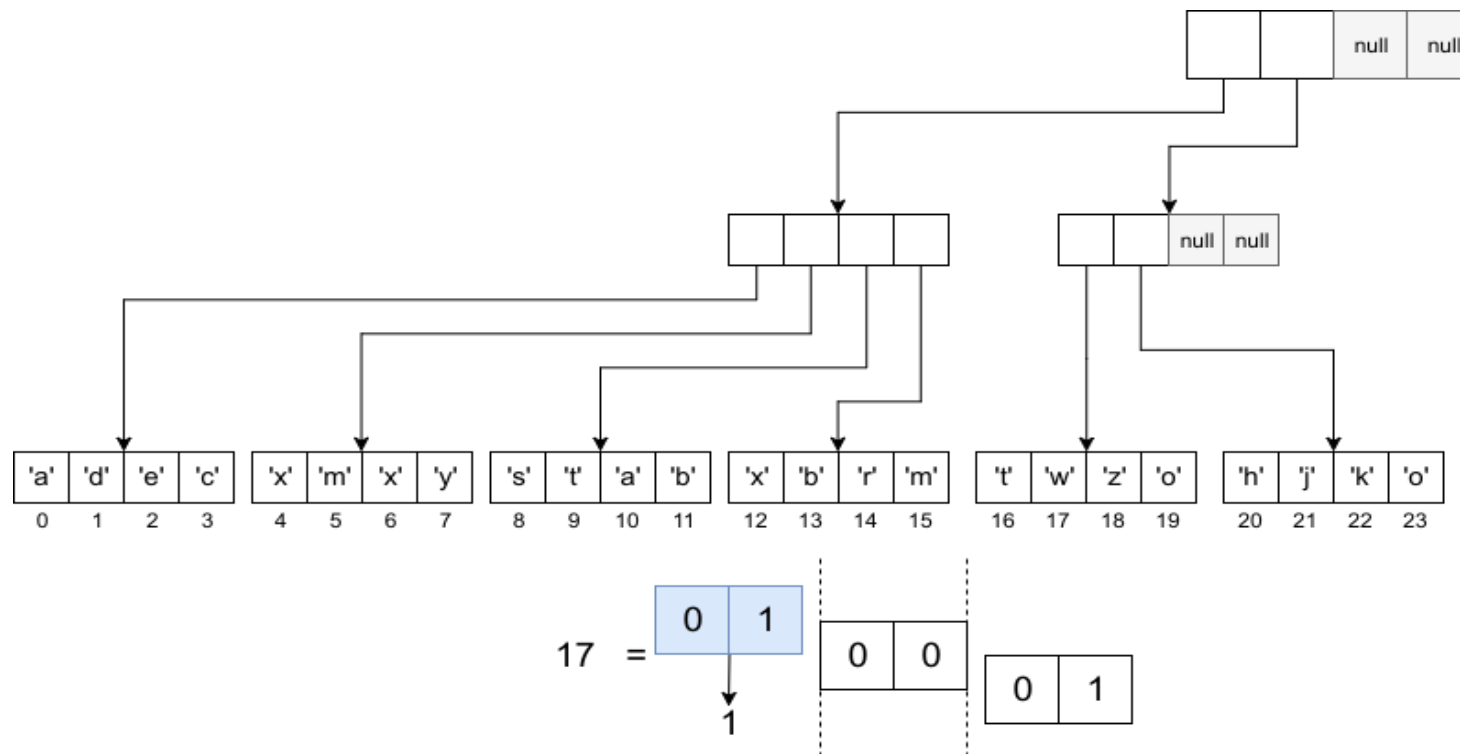




# How to vector

Branching factor: 4  
Height: 2

Look-up: 17

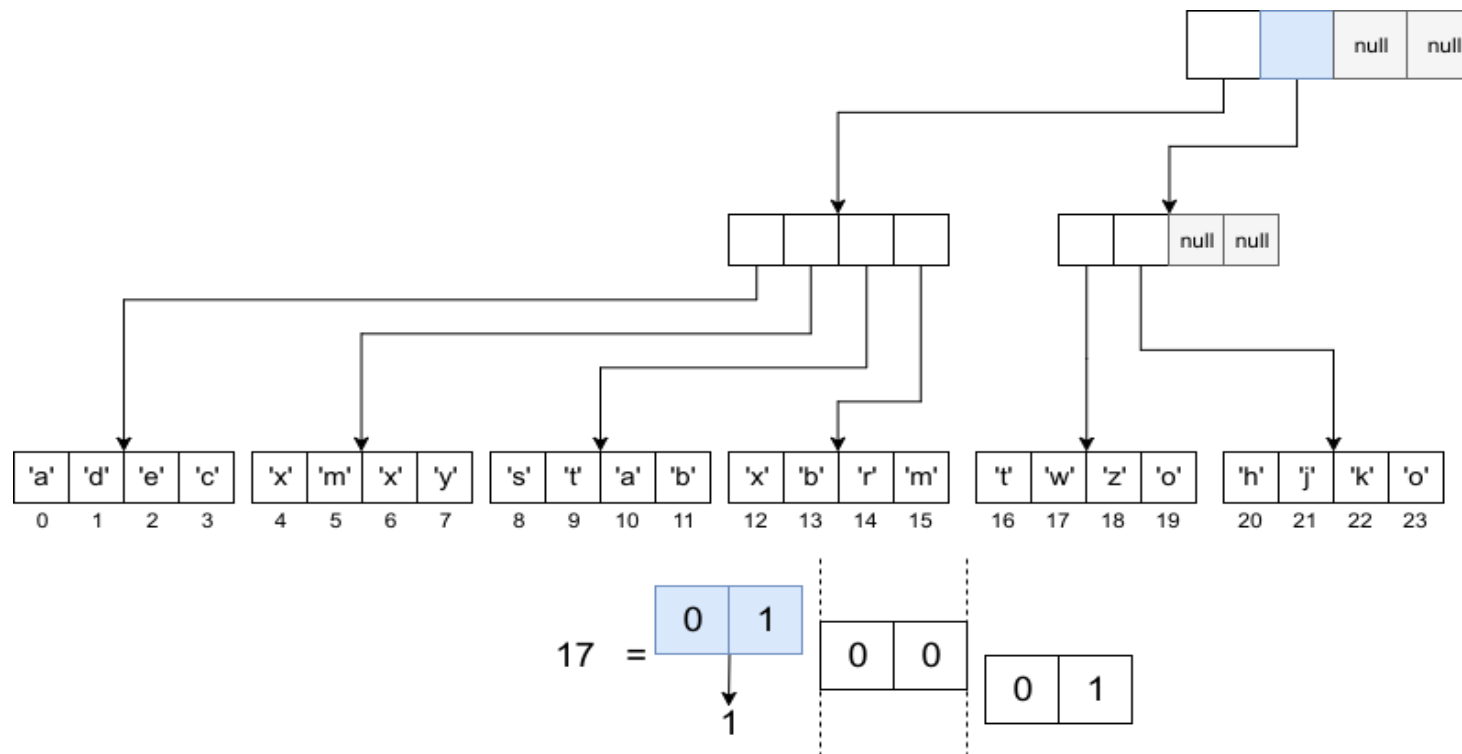




# How to vector

Branching factor: 4  
Height: 2

Look-up: 17

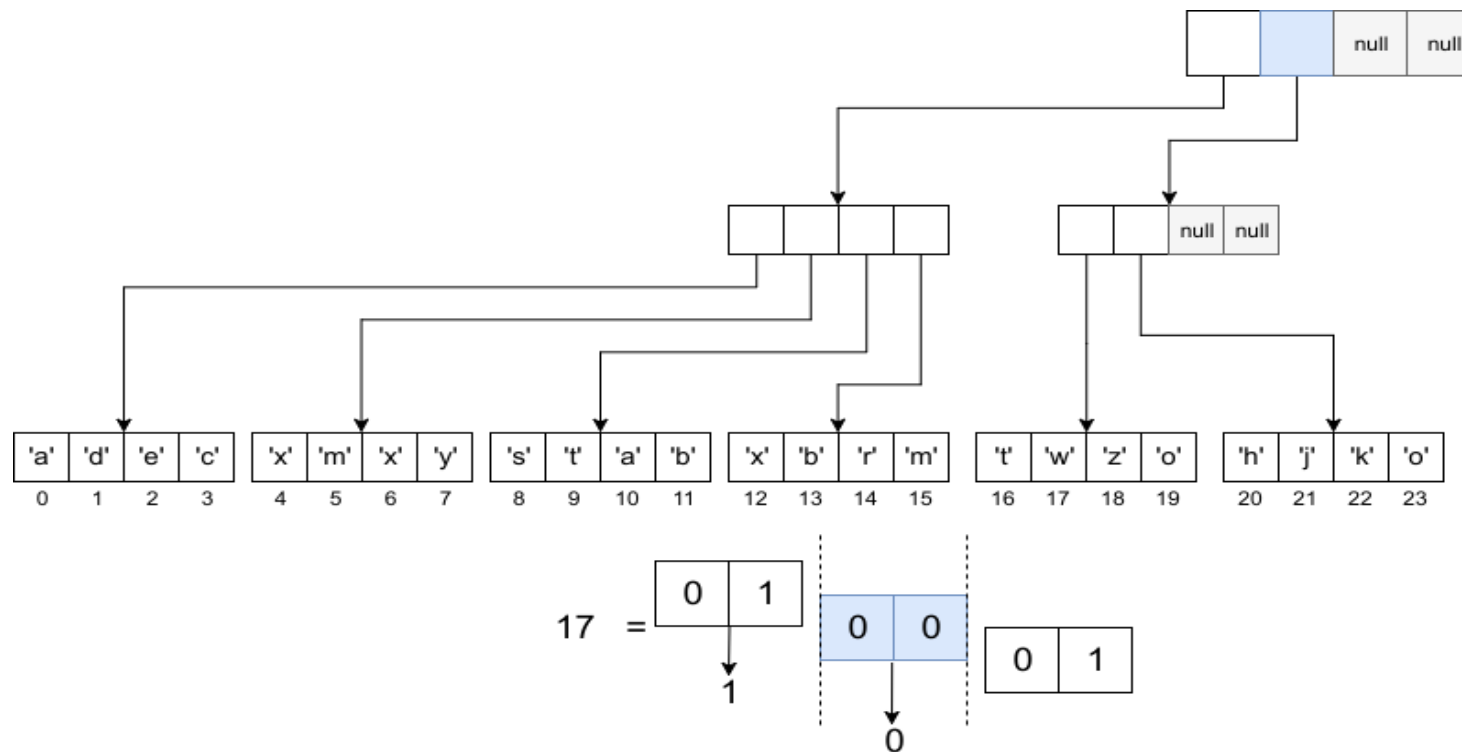




# How to vector

Branching factor: 4  
Height: 2

Look-up: 17

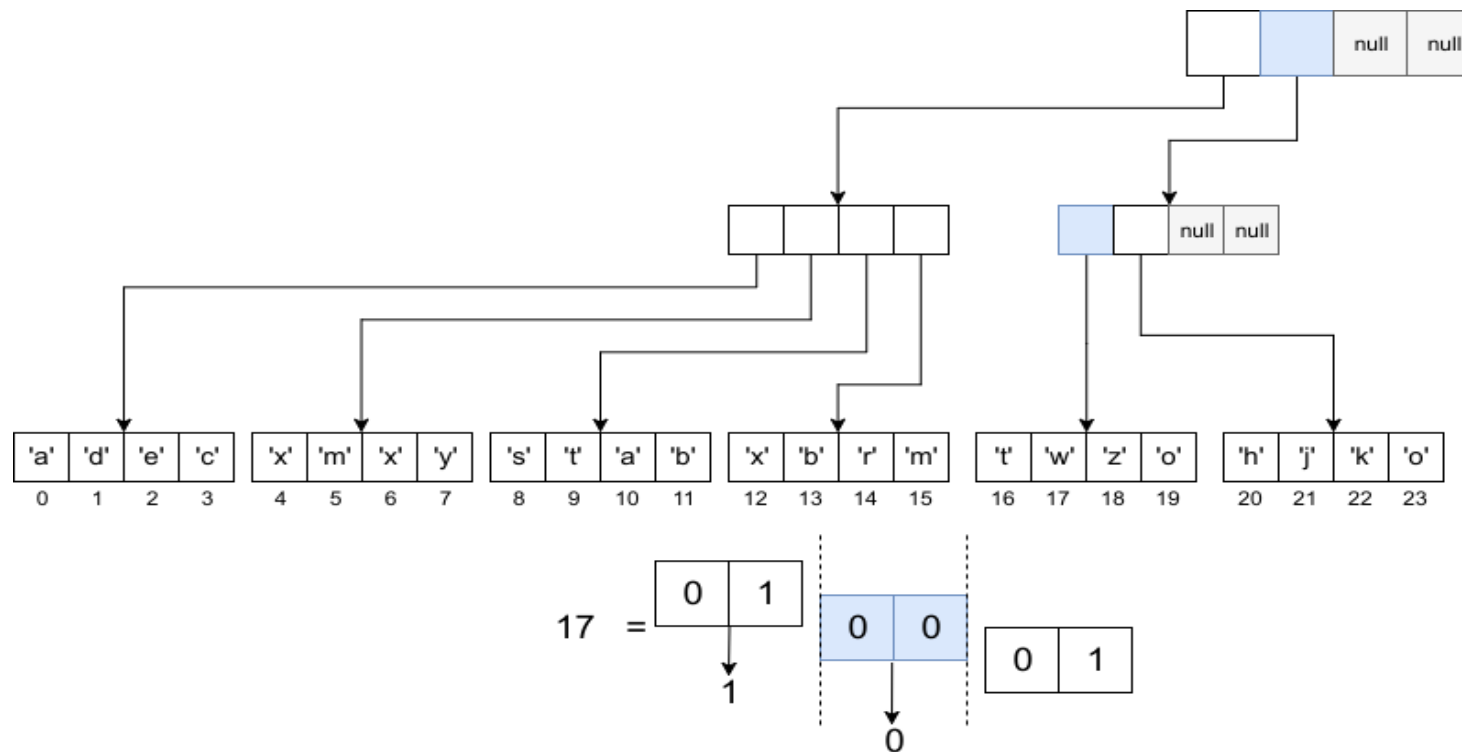




# How to vector

Branching factor: 4  
Height: 2

Look-up: 17



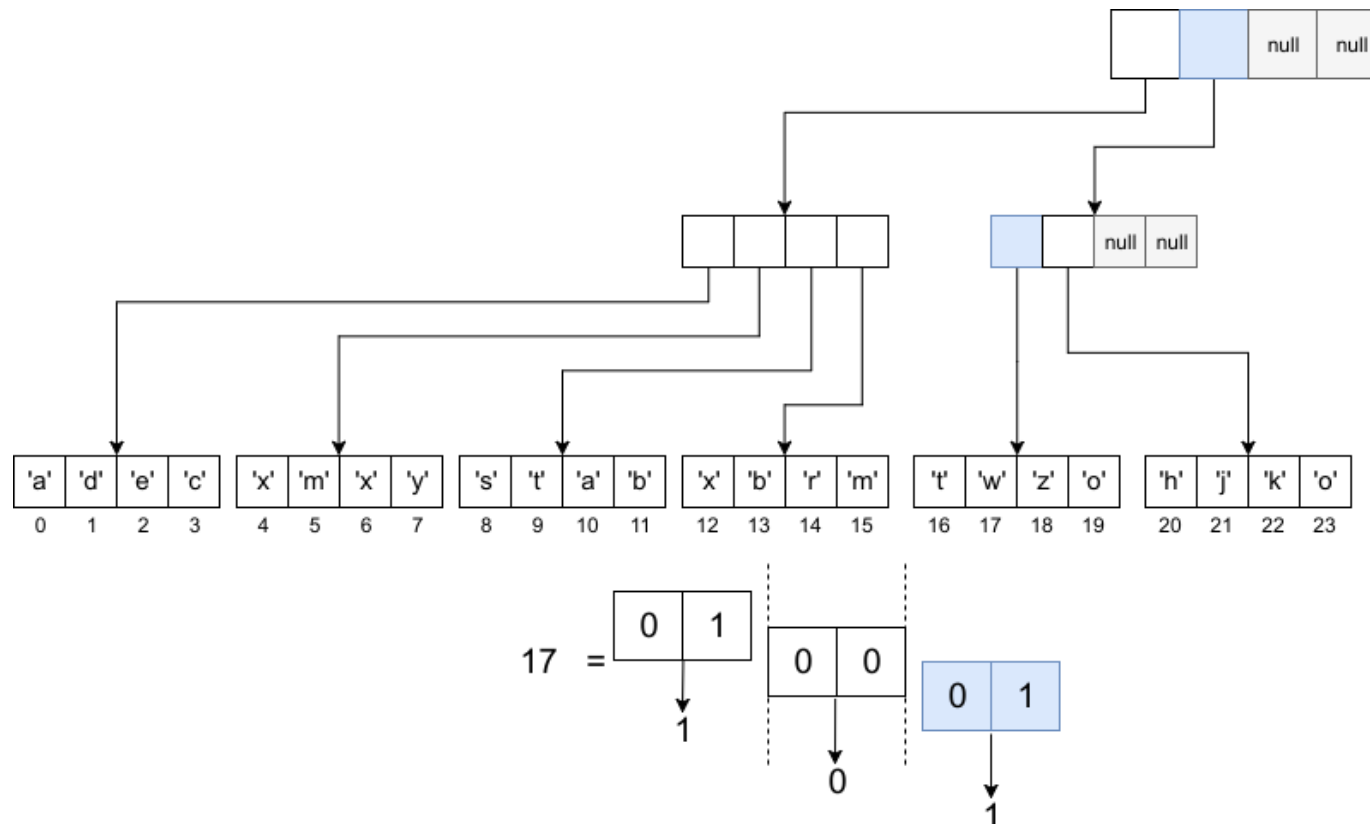




# How to vector

Branching factor: 4  
Height: 2

Look-up: 17

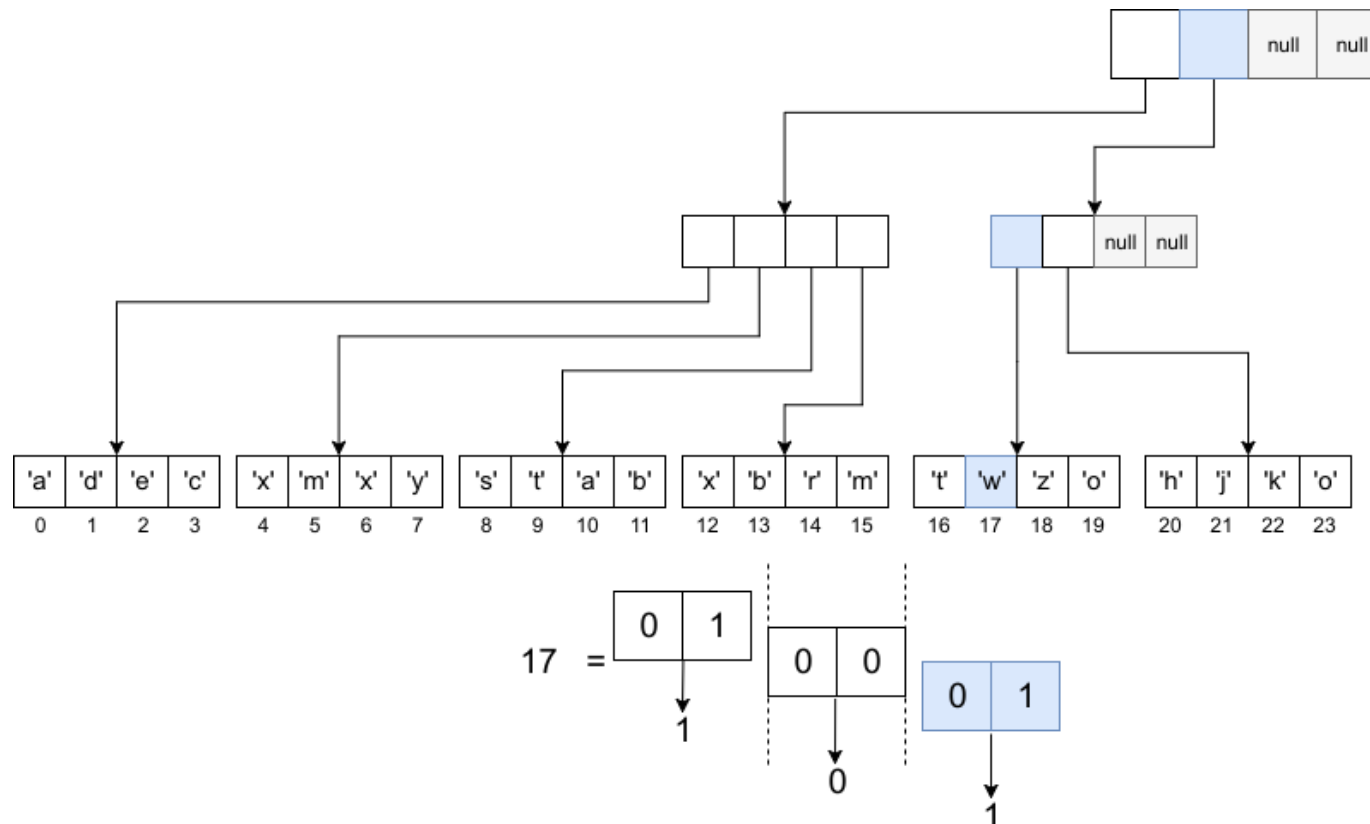




# How to vector

Branching factor: 4  
Height: 2

Look-up: 17

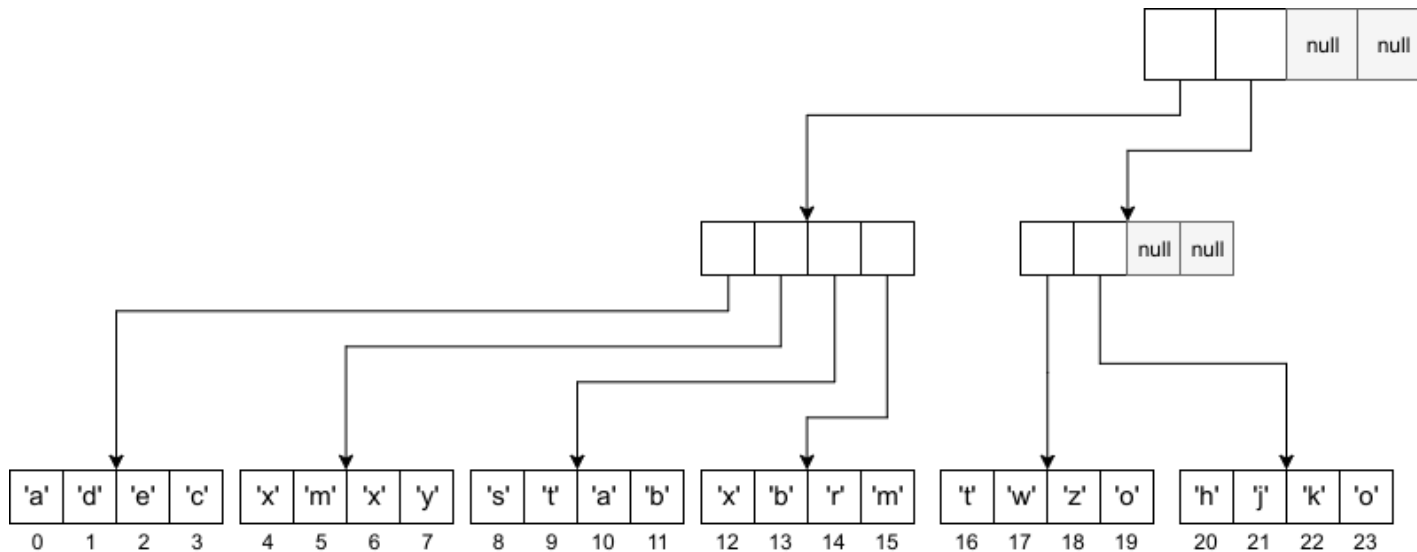




# How to vector

Branching factor: 4  
Height: 2

Append: 'n' → index 24



'n' = 24

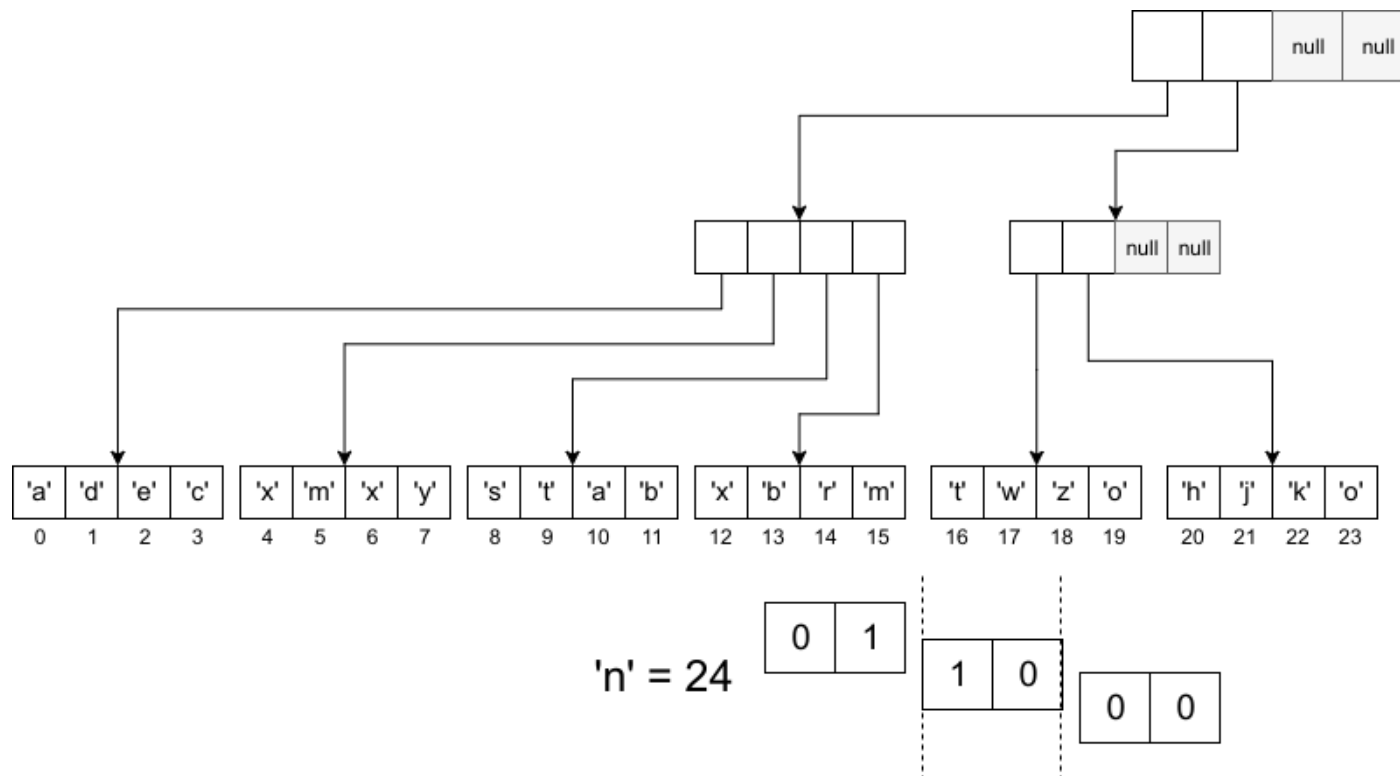


# How to vector

Branching factor: 4

Height: 2

Append: 'n' → index 24

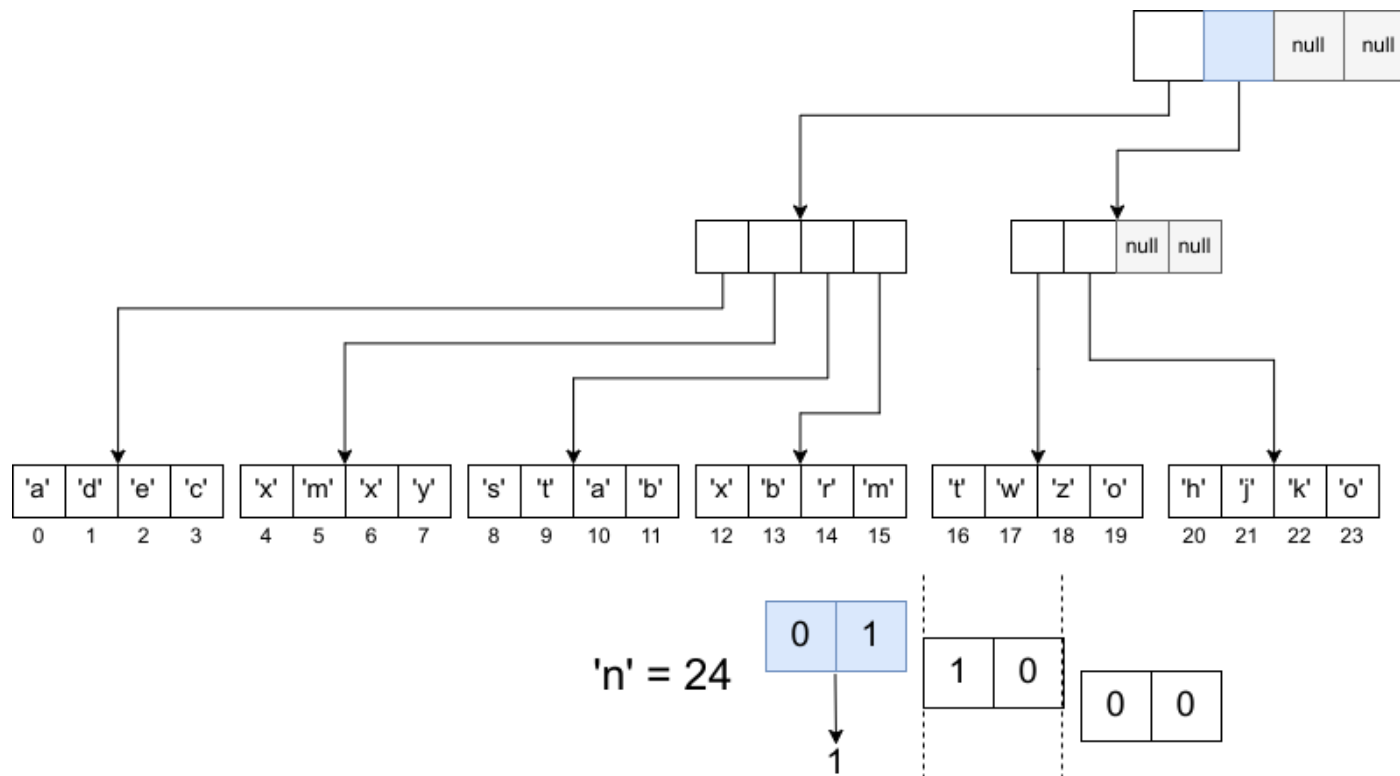




# How to vector

Branching factor: 4  
Height: 2

Append: 'n' → index 24

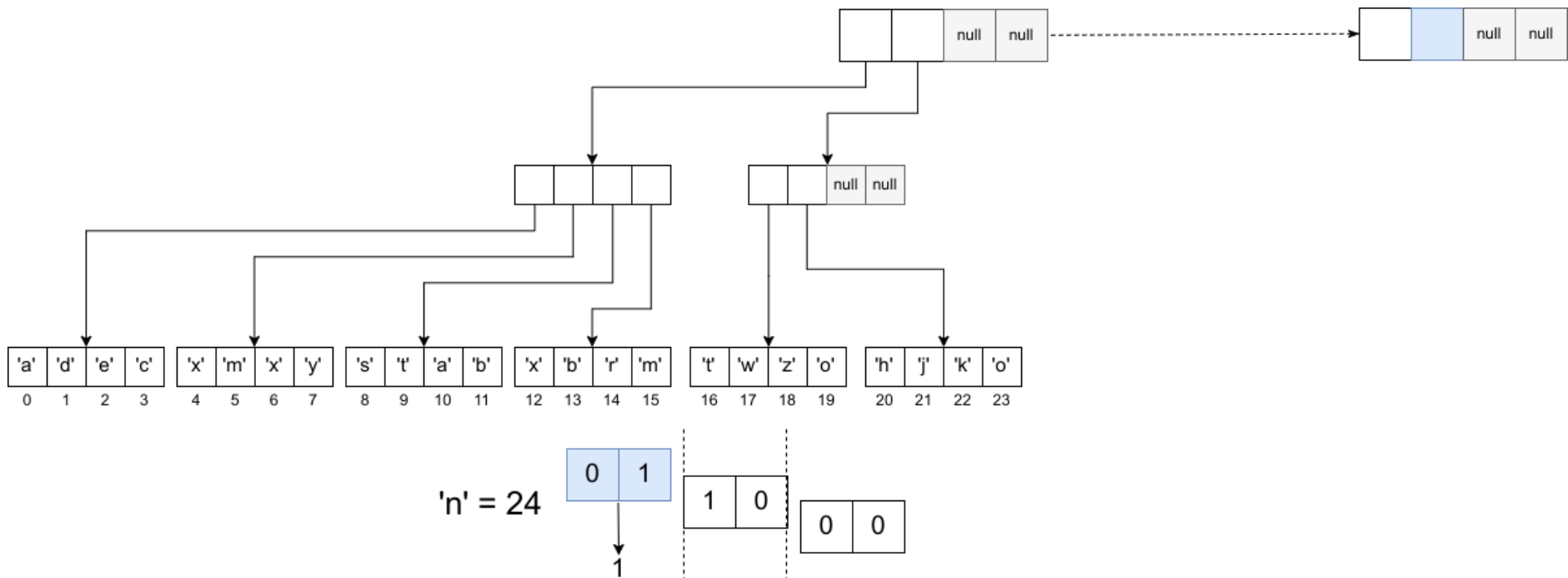




# How to vector

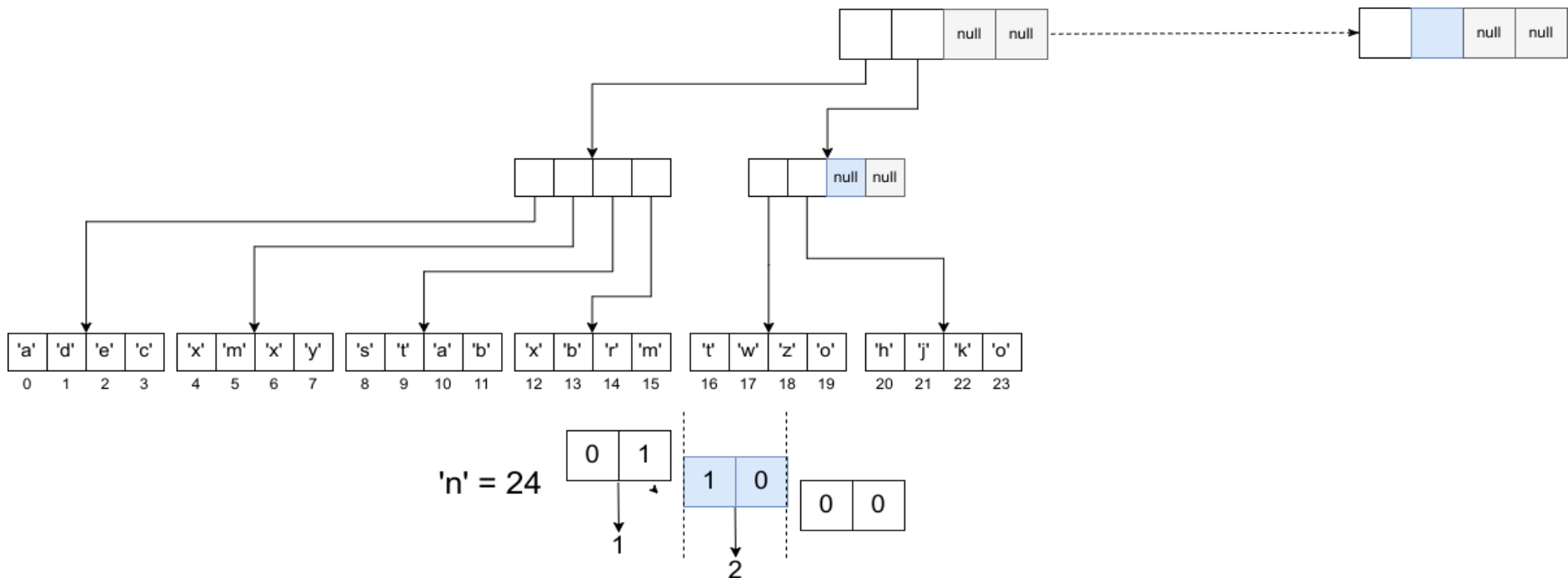
Branching factor: 4  
Height: 2

Append: 'n' → index 24





Append: 'n' → index 24

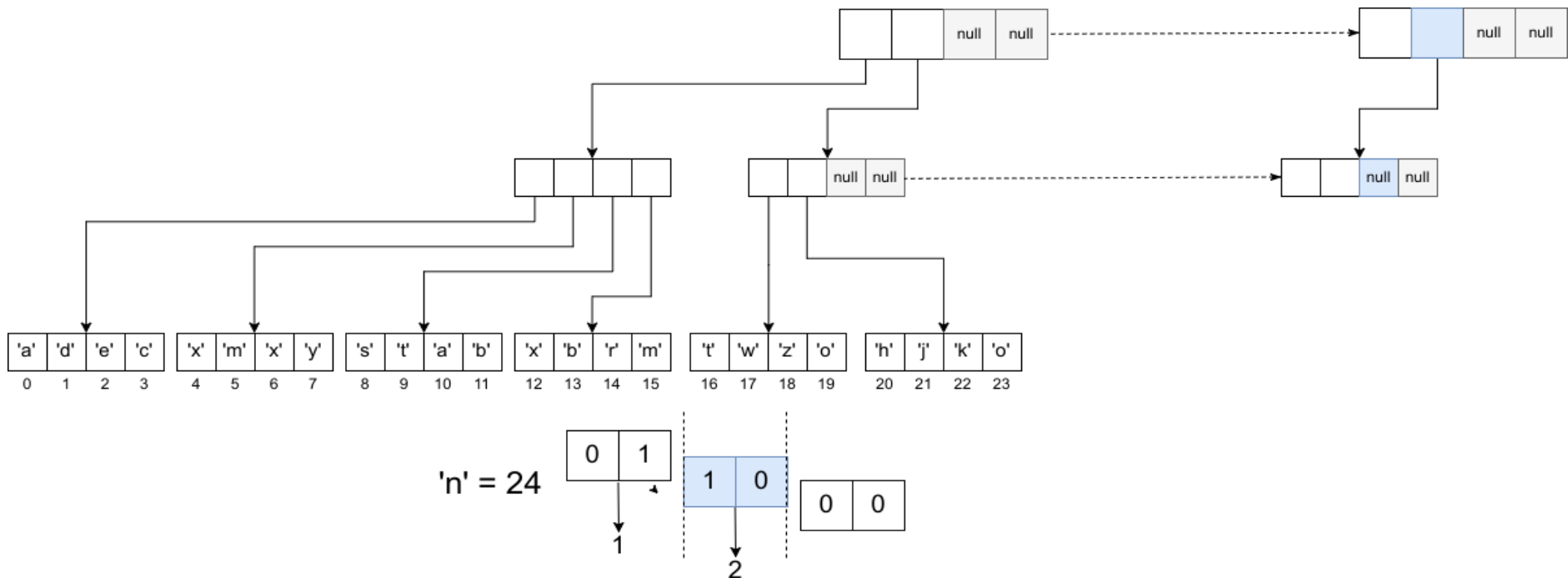




# How to vector

Branching factor: 4  
Height: 2

Append: 'n' → index 24



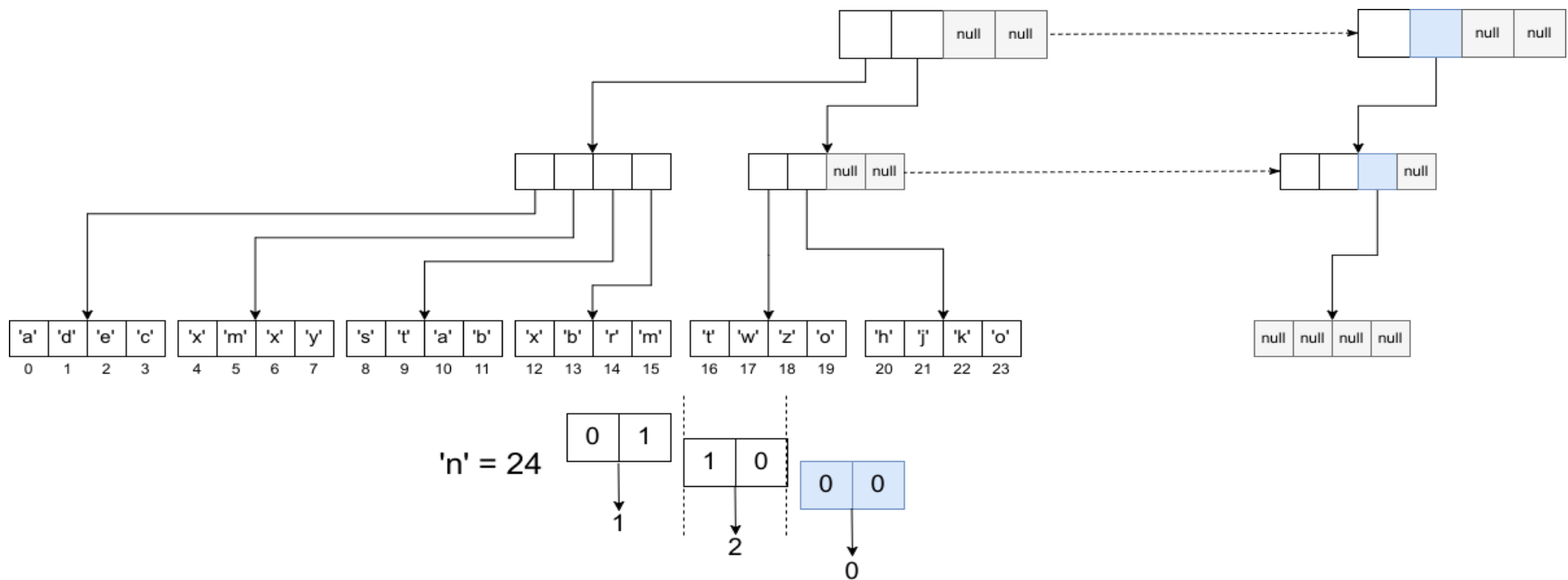




# How to vector

Branching factor: 4  
Height: 2

Append: 'n' → index 24

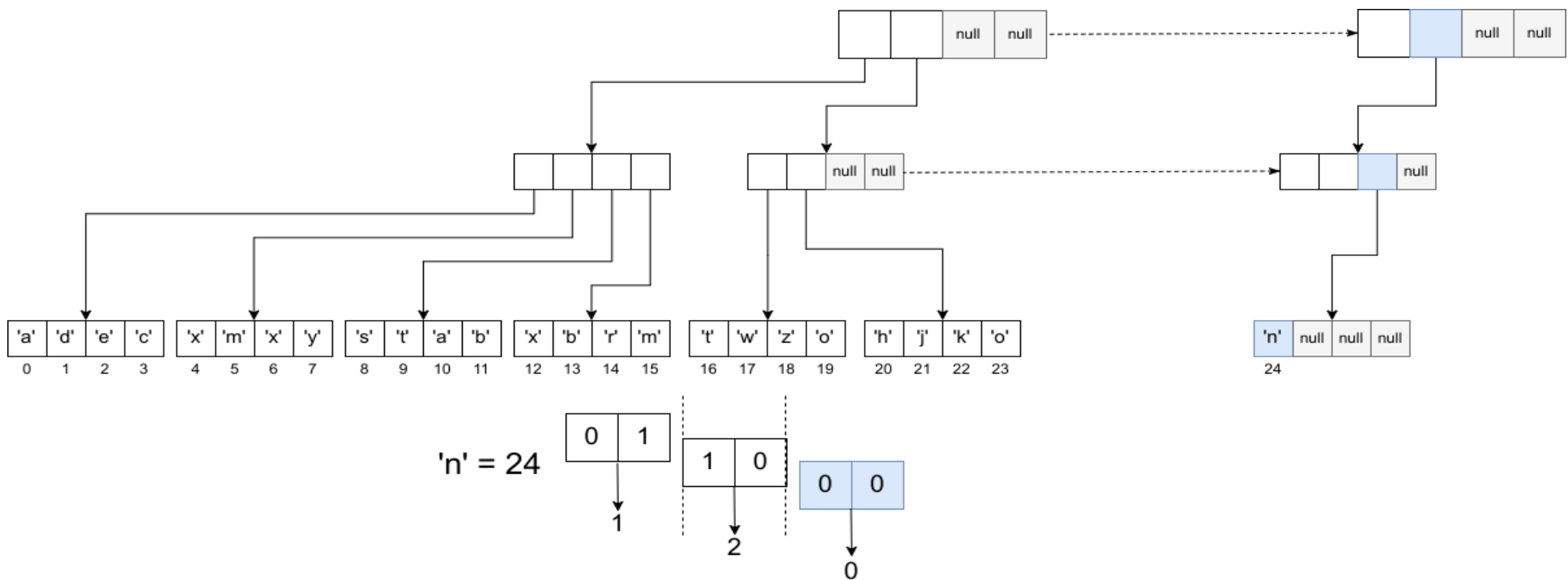




# How to vector

Branching factor: 4  
Height: 2

Append: 'n' → index 24

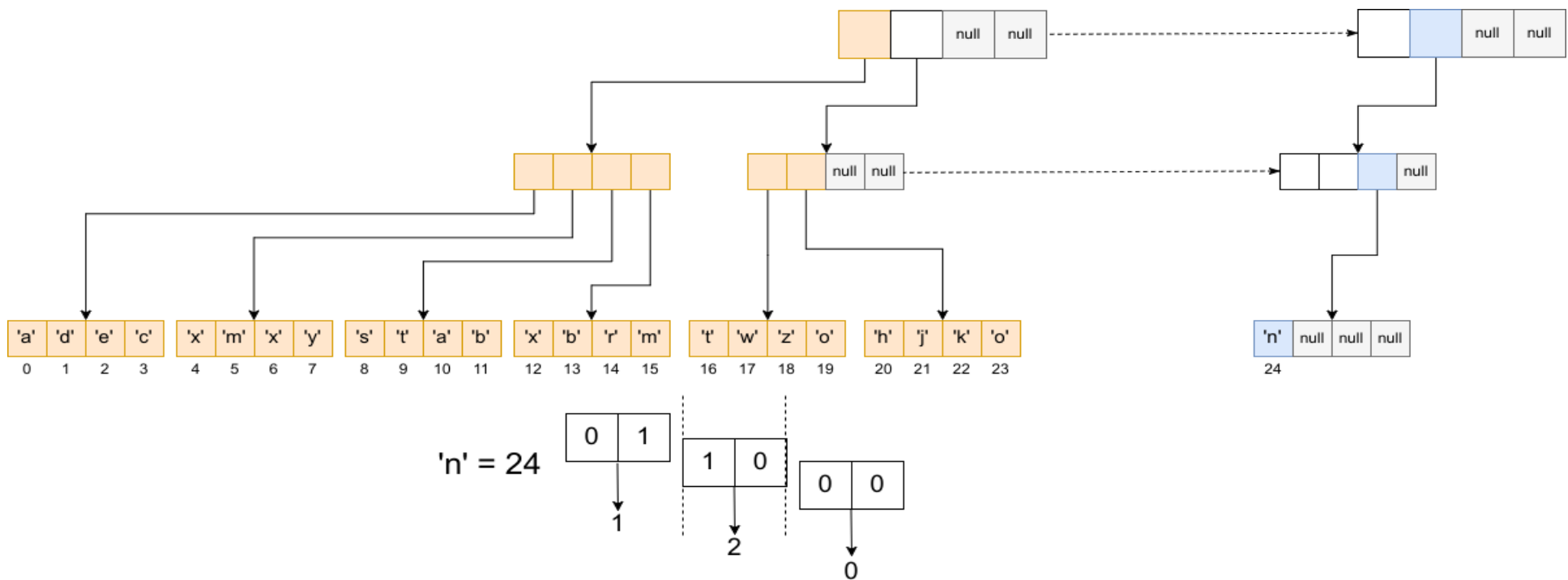




# How to vector

Branching factor: 4  
Height: 2

Append: 'n' → index 24



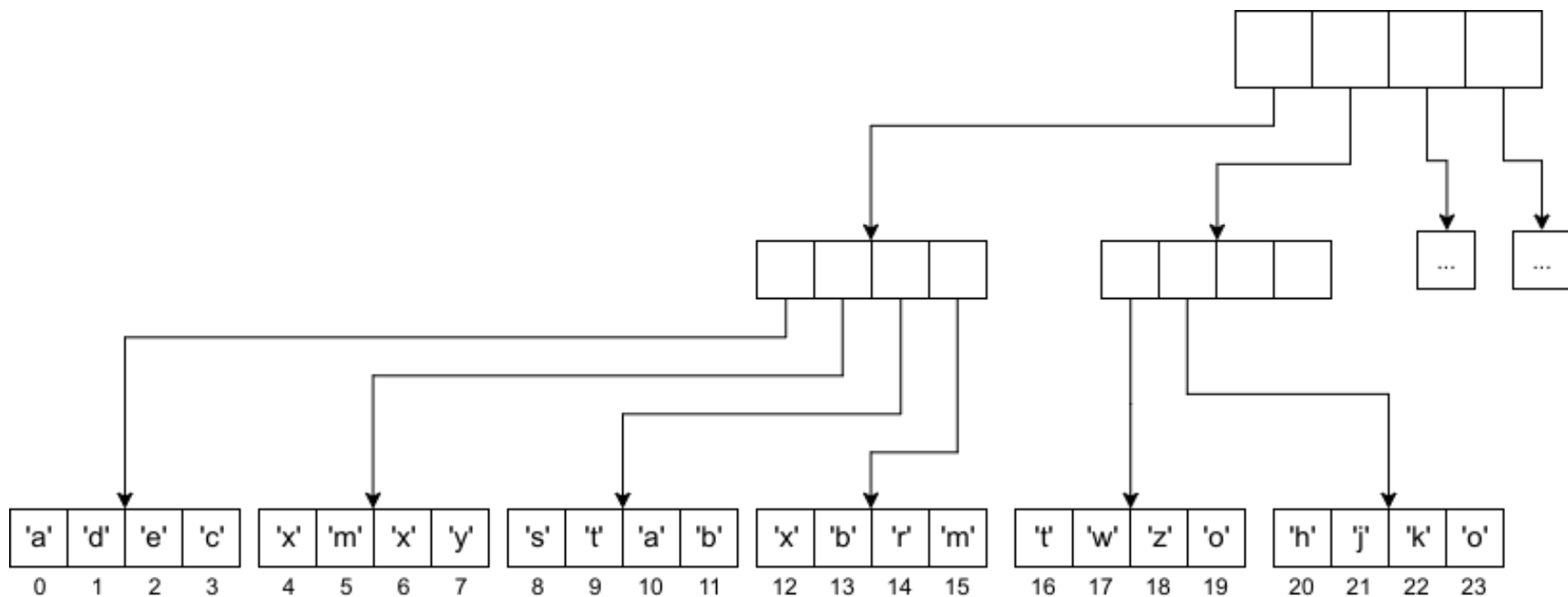


# How to vector

Branching factor: 4

Height: 2

Append when full





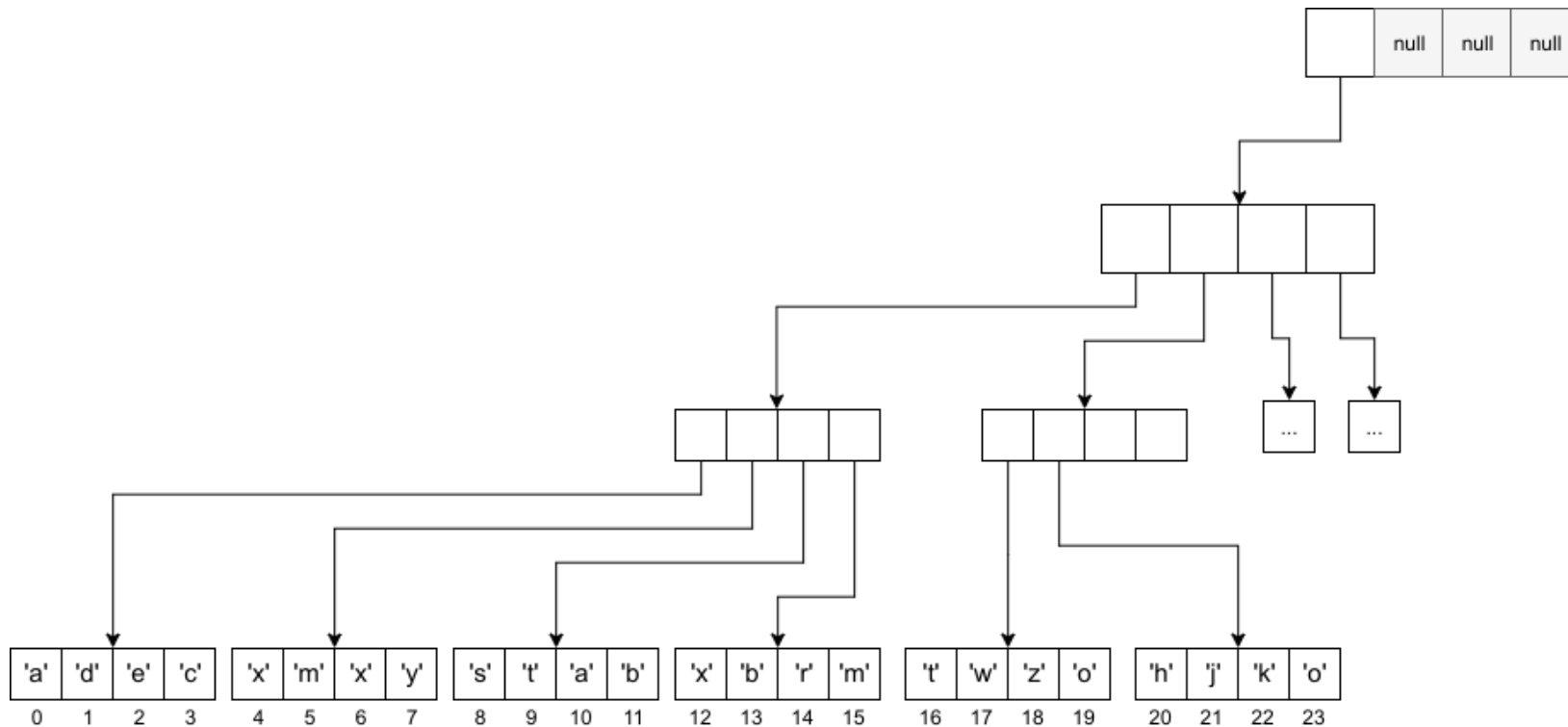


# How to vector

Branching factor: 4

Height: **3**

Append when full

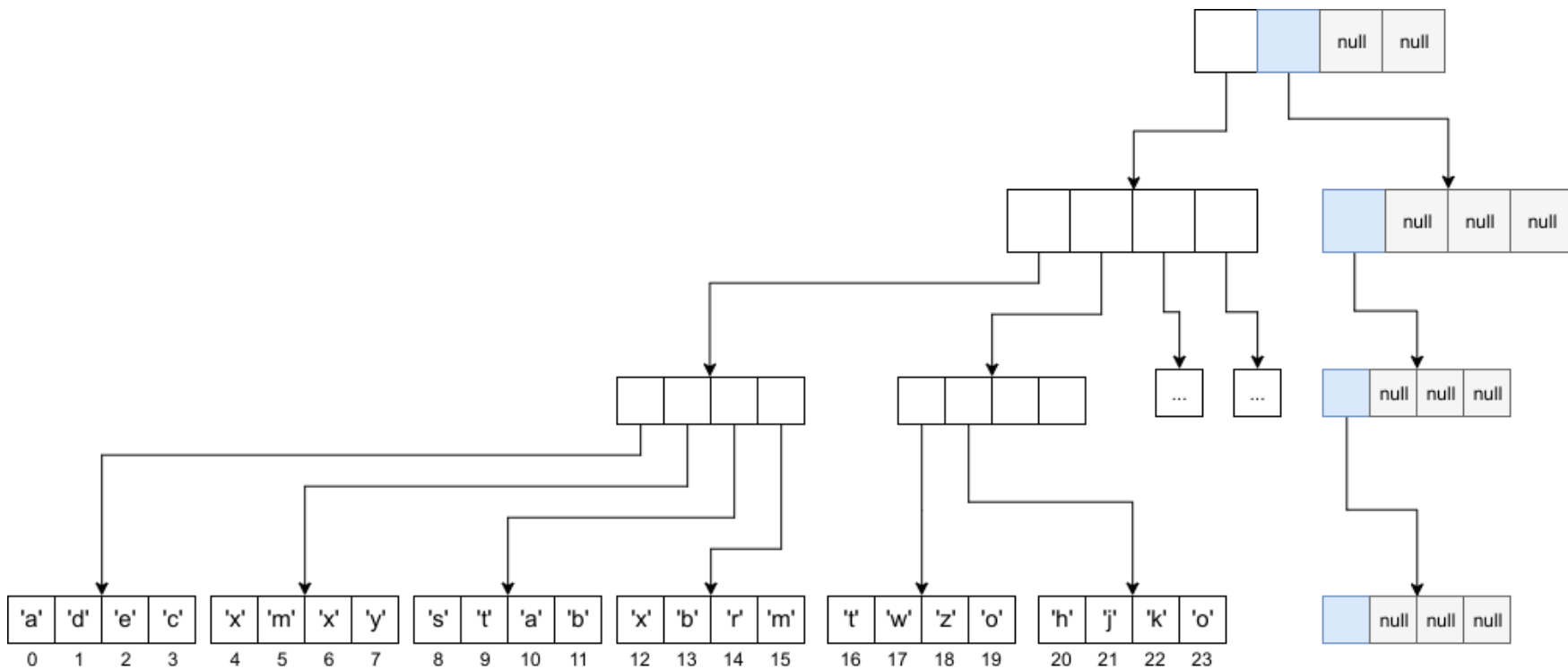




# How to vector

Branching factor: 4  
Height: **3**

Append when full



# How to vector (properly this time)



- You don't implement binary nonsense



# How to vector (properly this time)



- You don't implement binary nonsense
- Calculate with bitwise operations



# How to vector (properly this time)

- You don't implement binary nonsense
- Calculate with bitwise operations
- Index with formula:

$$f(i, h) = (i \gg \text{shift}) \& (M - 1)$$

$$\text{shift} = h * \mathbf{b} \quad (M = 2^{\mathbf{b}})$$

$M$  = branching factor



# Vectors: In the wild

- Why  $\log_2$ ?



# Vectors: In the wild

- Why  $\log_2$ ?
  - Branching factor influences:



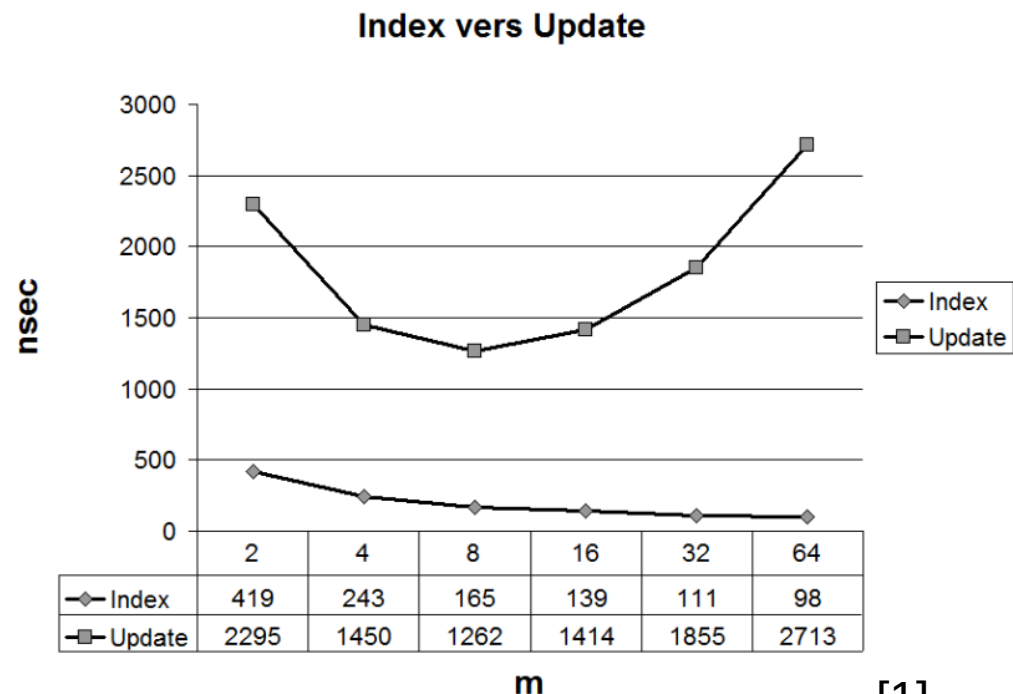
# Vectors: In the wild

- Why  $\log_2 32$ ?
  - Branching factor influences:
    - Look-up, iteration
    - Appension, update



# Vectors: In the wild

- Why log32?
  - Branching factor influences:
    - Look-up, iteration
    - Appension, update





# Vectors: In the wild

- Scala Vector:
  - (on average)  $O(1)$  update
  - (on average)  $O(1)$  look-up



# Vectors: In the wild

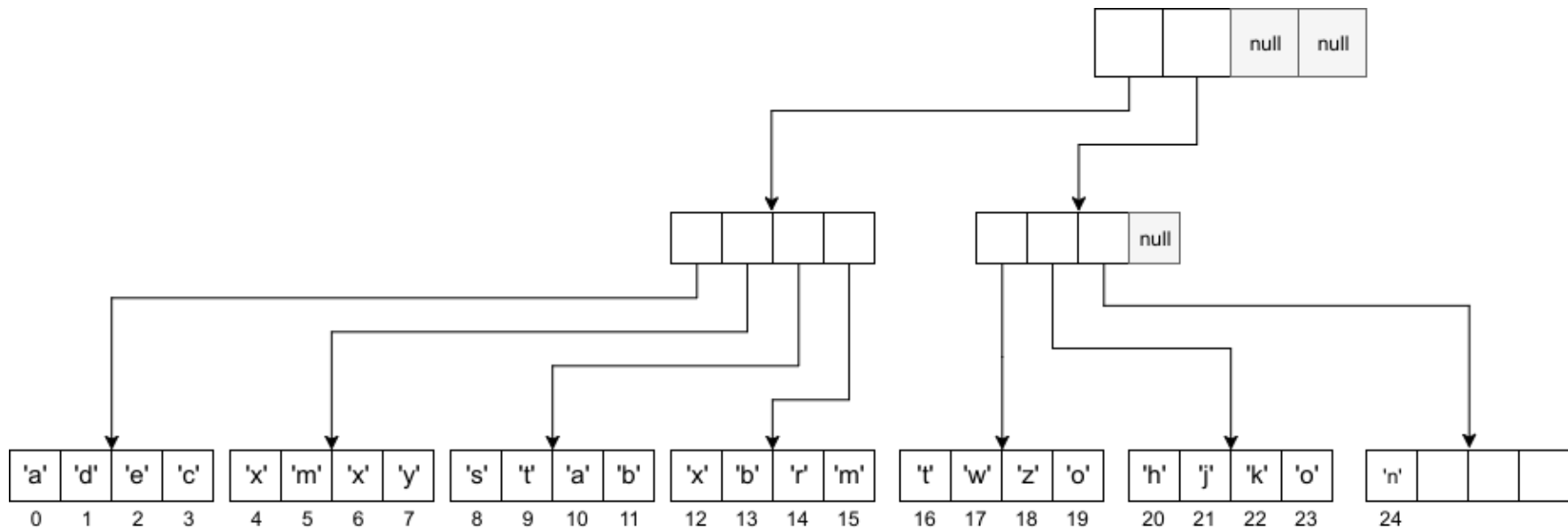
- Scala Vector:
  - (on average)  $O(1)$  update
  - (on average)  $O(1)$  look-up
- Clojure Vector:
  - $O(1)$  appension
  - $O(1)$  last
  - $O(1)$  pop





# Closure Vector

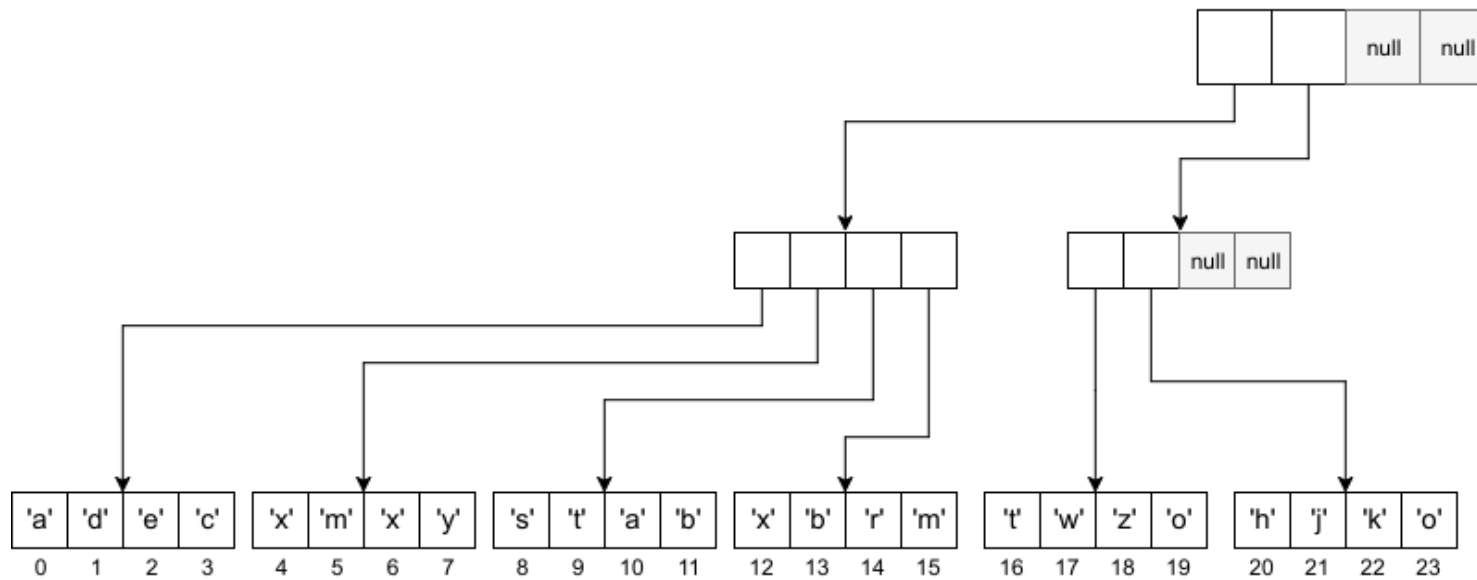
- Tail





# Closure Vector

- Tail

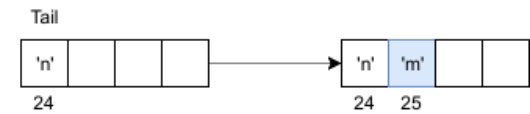
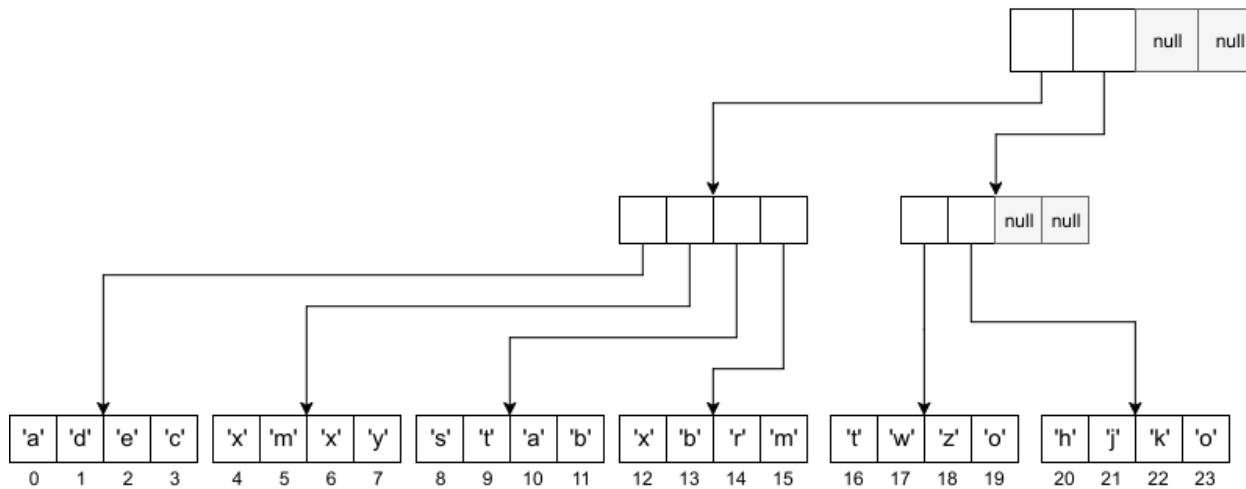


Tail  
'n'  
24



# Clojure Vector

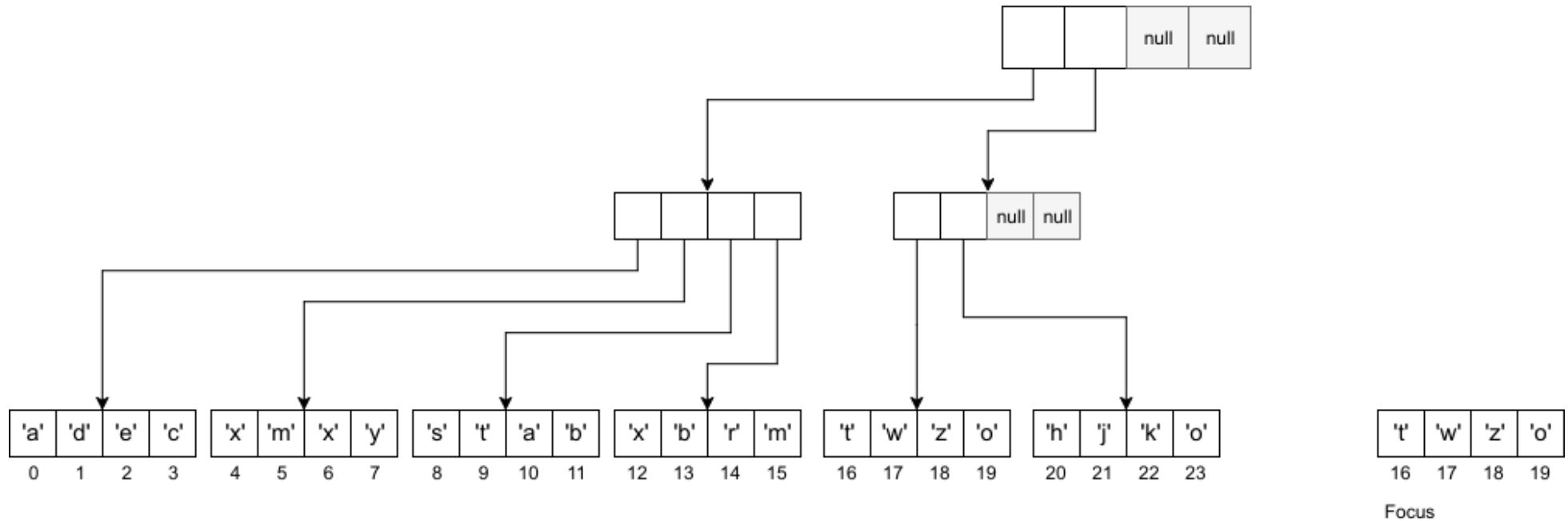
- Tail





# Scala Vector

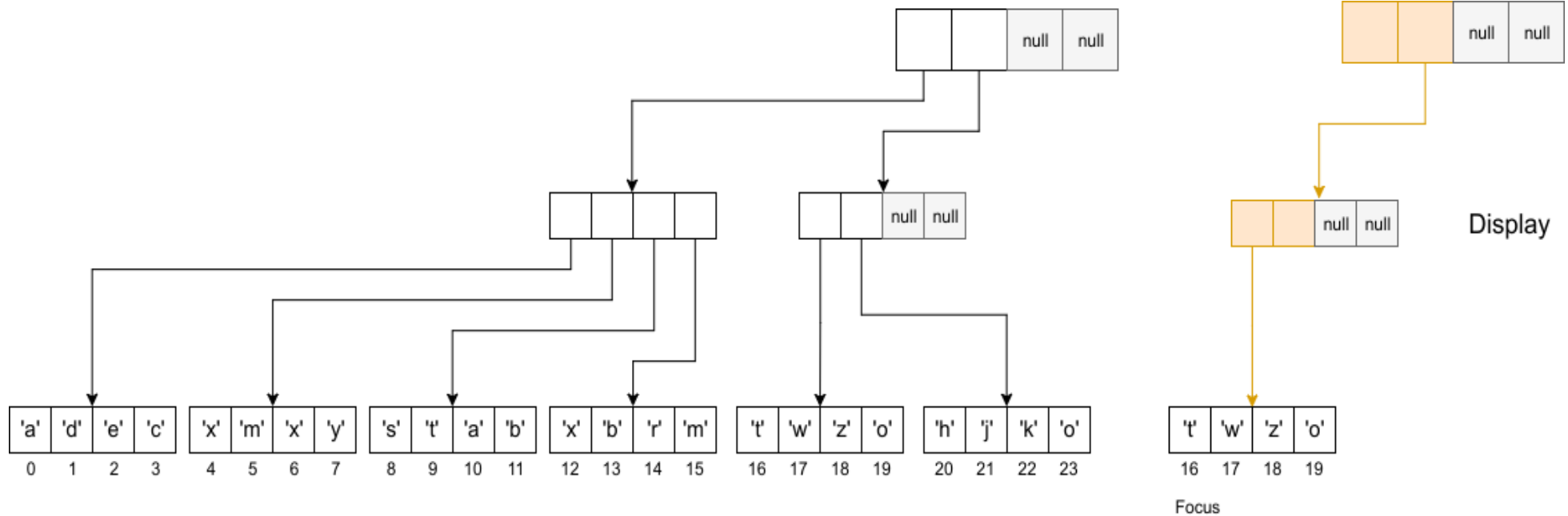
- Focus





# Scala Vector

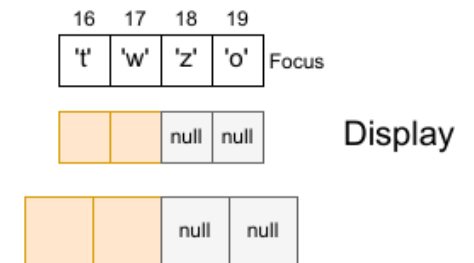
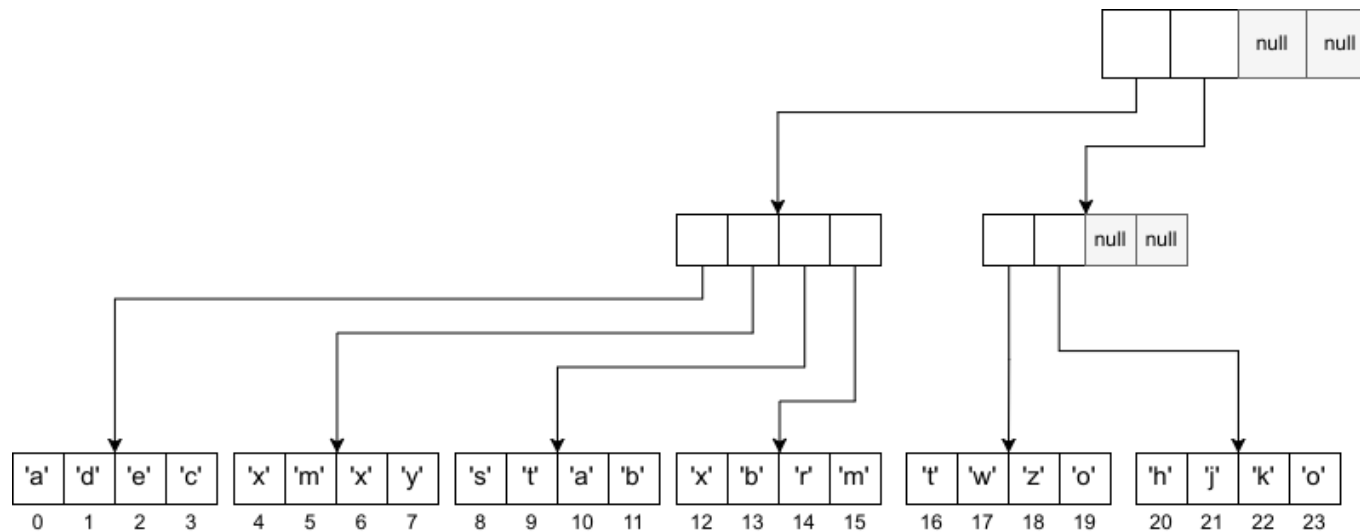
- Focus with Display





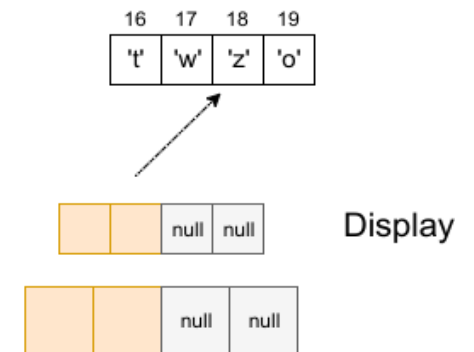
# Scala Vector

- Focus with Display





- [illegible]



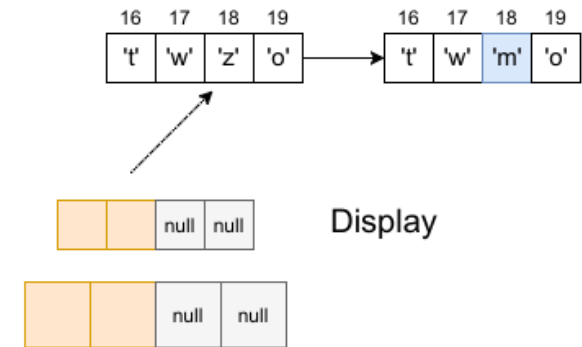
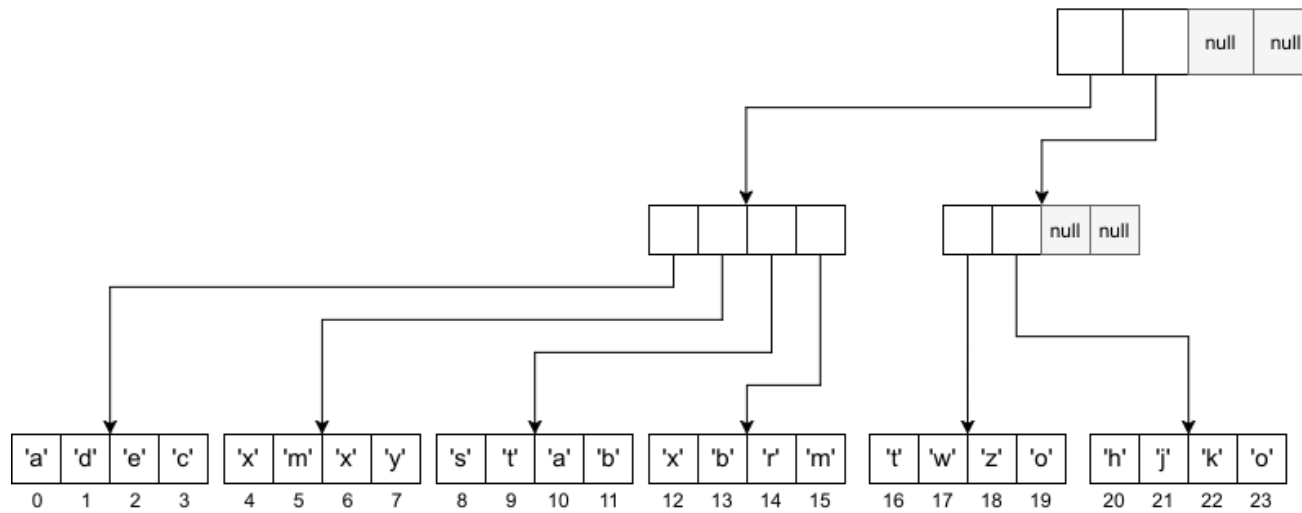






# Scala Vector

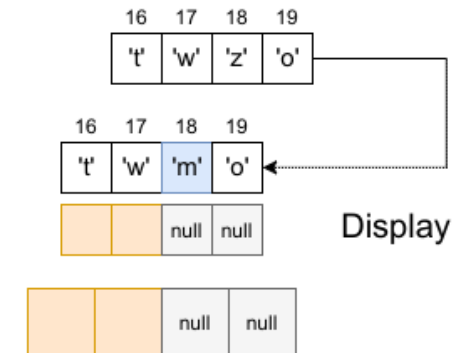
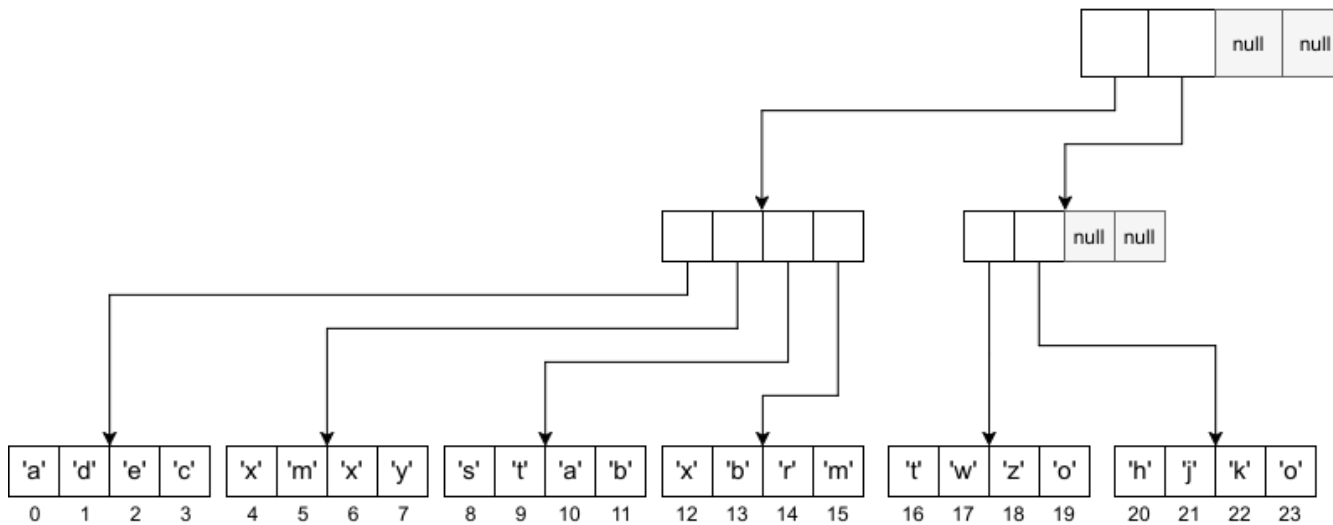
- Focus with Display





# Scala Vector

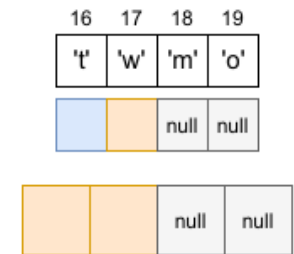
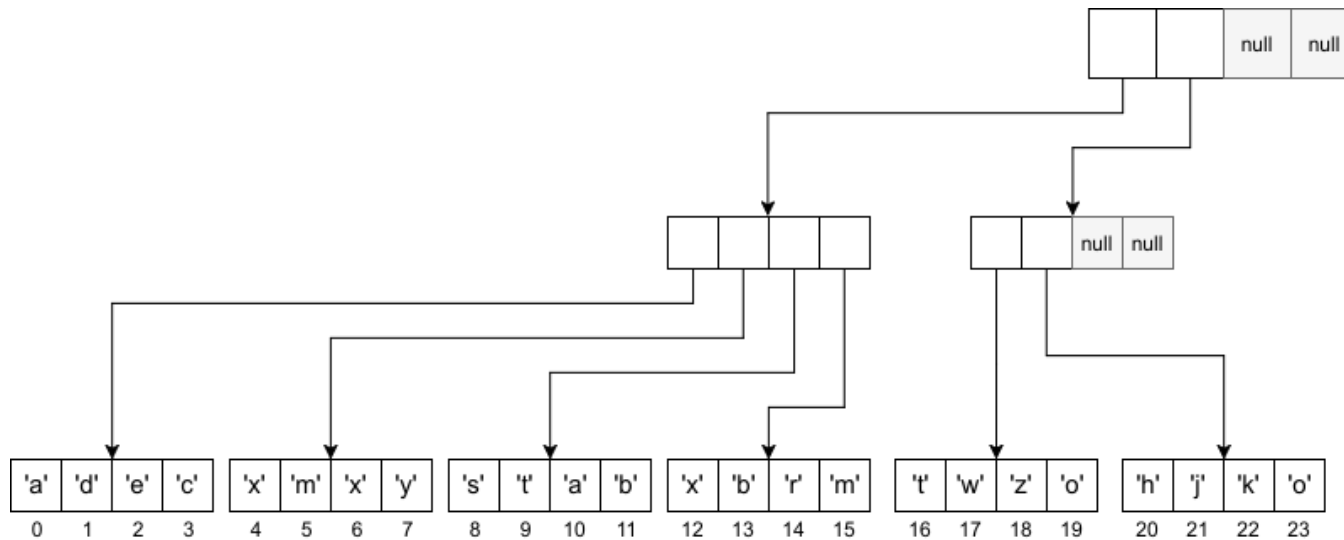
- Focus with Display



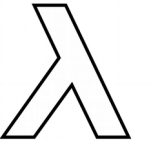


# Scala Vector

- Focus with Display



Display



# Vectors: An Implementation



Thank you!

**Presentation:** <https://github.com/AvramRobert/marvellous-functional-datastructures>

[1] <https://infoscience.epfl.ch/record/169879/files/RMTrees.pdf>