PENTAGAME

The Book

Jan Suchanek

2018

27. Oktober 2018

Wenn wir nun auch die Idee des Spieles als eine ewige und darum längst vor ihrer Verwirklichung schon immer vorhandene und sich regende erkennen, so hat ihre Verwirklichung in der uns bekannten Form doch ihre bestimmte Geschichte, von deren wichtigsten Etappen wir kurz zu berichten versuchen wollen.

In the understanding that the idea of the game is eternal and has thus existed and roused itself long before its realisation, nevertheless its realisation in the form we know today has its concrete history, the most important stages of which we shall try to report briefly.

Hesse: *Das Glasperlenspiel*

pauca sed matura

OPENTALPHA

pentagame.org
jan@pentagame.org

Inhaltsverzeichnis

Foreword . . . 5
1 Setting up . . . 8
2 Moving and winning . . . 10
3 Special situations . . . 12
4 How the idea was born . . . 14
5 Developing play . . . 16
6 A game of its own kind . . . 19
7 The ancient game petteia . . . 20
8 Is Pentagame petteia? . . . 24
9 A vanished game? . . . 25
10 The golden proportion . . . 28
11 Pentagrams in the sky . . . 32
12 The wild history of a beautiful shape . . . 33
13 Pentagrams in the East . . . 37
14 A Judeo-Christian view . . . 39
15 Solving the board . . . 41
16 Theory of play and game theory . . . 46
17 The 2-player game . . . 48
18 The 3-player game . . . 52
19 The 4-player game . . . 54
20 Some remarks on strategy . . . 57
21 Competitions and tournaments . . . 61
22 A learning curve . . . 65
23 Myths . . . 66
Regulae officialis latinae . . . 70
Notation of moves . . . 71
Acknowledgements . . . 72
Notes . . . 72
Index . . . 76
List of Figures, List of Tables . . . 80

Foreword

PENTAGAME is a game with a simple structure, very little material and easy rules. But behind its simplicity lurks complexity: there are more possible games than stars in the sky. And while the image of the board is a simple geometric shape, it nevertheless has a rich history. This book deals with all this, and with more.

I try to tell a story in this book. For completeness, I begin with explaining the game itself again in simple prose, a little more extensive than the nutshell rules that you find in your game box. Then the book becomes continuously more abstract.

First I explain how the idea of Pentagame was born and the story of its development. Naturally, everything that is related to games and to pentagrams was interesting, and I present you my finding along with what I have made out of them. From that I move on to discuss some of the many questions that the game so developped poses; I investigate its properties—geometric properties on the one hand side, and properties as a game on the other. This is still an almost unkonwn continent, but I present approaches and results. This part should be instrutive for the Pentagame fanatic who is as eager as me to expand the borders of our understanding of the game.

Many times have I been ask why I write in English which is not my mother tongue. In particular, the question came up which were the 'official' rules. I have decied to render them in Latin, which should be equally clear for every reader; they top up this collection of thoughts.

Originally, this book was meant to be a little brochure to accompany the game. It somehow grew, but still is short everywhere. There sure are gems, and stumbling stones, galore. I wish you a pleasant journey through these pages.

Pentagame

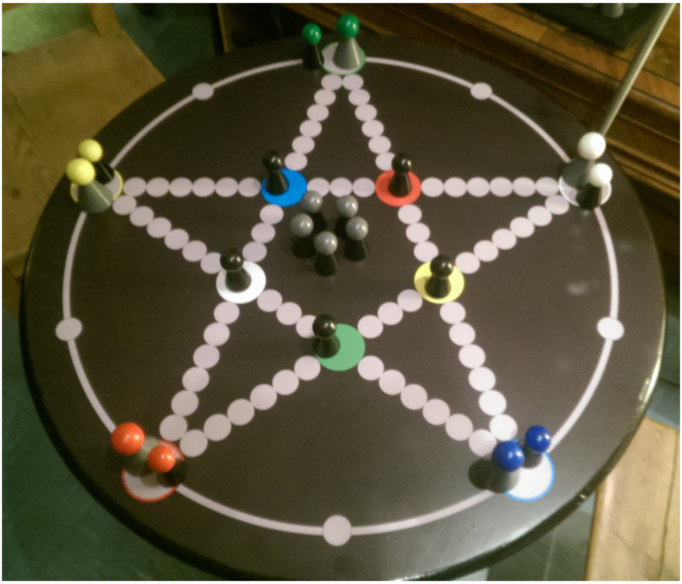

Figure 1: Set-up for two players
small & black vs. large & gray.
3 or 4 playes would begin with 3 or 4 pieces per corner.

1 Setting up

The board sports a pentagram in a circle. On the rim, the five *corners* (the points of the star) are coloured white, blue, red, yellow and green. In the middle, there are the five *crossings* (where the lines forming the star bisect), these are also coloured white, blue, red, yellow and green. For each corner of the star there is a corresponding crossing of the same colour—exactly opposite and furthest from it.

On the the lines of the pentagram are additional *round stops:* two times six linking each corner to each of its two adjacent crossings and three linking each crossing to each of its neighbours. On the rim, there are *three stops* between adjacent corners. We'll call corners, crossings and all other stops together *stops.* In total this makes 100 stops, of which five corners and five crossings, coloured.—

The pieces Every player commands a *set* of five *pieces.* The pieces within a player's set are distinguished by having the same shape or ornament. However, each piece in a player's set has different colour: a white, a blue, a red, a yellow and green.

For examples: the first player may be 'stars' and have five stars—each a different colour, the second plays using five moons of different colours (and so on for circles, triangles, or squares etc.). It is simply necessary for a player's set to be clearly distinguished.

In addition to the player's pieces there are five *black blocks* placed on on the five crossings in the middle of the board. The blocks are passive pieces, moved by any player when they land on them (page 10).

Also there are five *grey blocks*; these are also passive pieces. Park them in the centre for later.

Objective All coloured pieces start at the corner of their colour; each player will have a white piece starting at the white corner, a blue piece that starts at the blue corner, and so on.

Pieces strive to reach the *crossing* that matches their colour.

The corresponding crossing is always exactly opposite the corner from which pieces of a colour originate.

Who first manages to bring *three* of her pieces to these goals wins.

2 Moving and winning

You can move in any direction on the star or the rim as far as you please, as long as the path is free. On a free path you may turn at any corner without stopping. But you cannot jump over anything.

You can, however, take the place of a piece or block obstructing.

If you take the place of a black block, take its place and replace it on a free stop of your choice. Think carefully—this is a major strategic part of the game!

If you take the place of another of your own pieces, you put that second piece where you have just come from. In such a move you thus swap the positions of two of your own adjacent pieces.

If you take the place of a adjacent piece of another player, you place it on the stop from which you have just come, swapping the position of your own piece with the adjacent foreign piece. The other player can reverse this move. However, if they do you are not allowed to repeat the exact same move a second time, but must choose an alternative. (This prevents stalemates.)

Of course, you can also just simply move to an unoccupied stop.

These are the ways you can move.

Only at the start of the game can there be more than one piece on a (corner) stop.—If you move to a corner occupied by multiple pieces, you must choose one of these pieces to swap position with.

When a piece reaches its destination, rejoice! – Then, you remove that winning piece from the board, parking it in the centre of the board. For this you receive a grey block, which you may place on any free stop of your choice.[1] These grey blocks are, similar to the black blocks, just passive pieces and simply blocks.—Remember the black blocks? When you 'beat' them, you replace them somewhere else... But when you take the place of a grey block, that grey block simply leaves the board again (park it in the centre again). They are 'one time blocks'.

In the rare case that all grey blocks are already in the game and you gain

Figure 2: Setup for four players
Photo: Jan Fels

the right to place one more, move one of the existing ones to the location of your choice.

Whoever gets three pieces ‘out’ (off the board) is the winner, with a score of three.

Continue the game to complete the last round so all players have had the same number of moves. With other words: If one player has moved their third piece out, the others can still complete the round and see if they can tie the game. A player’s score is the number of their pieces ‘out’ at the end of that last round.

And that is all there is to it, this is the whole of the law.

The rest is etiquette: never to mention open possibilities to the player who is to move is probably the most eminent.

There are only very few necessary clarifications for uncommon situations; there are some tips for special situations, too: these we will briefly go through in the following section.

3 Special situations

Abracadabra. If you can reach a goal and find it occupied by a black block, you remove that black block, move out and gain a gray block. This allows you to position both a back and a gray block in one single move, which is great and a challenge at once. This move is called 'abracadabra'.

You can moves a piece of another player to its destination. When you swap a piece with another player's it may end up on its final destination. When this happens it moves 'out' on the other player's turn, not before. Moving out is an action!

More than two players. The rules for two, three and four players are exactly identical. Just give the additional player her own set of five coloured figures and put these her pieces also on the five corners to start.

Three players. A three player game should take about 45 minutes.
Because the last round always gets played out, there is no huge first mover advantage. It is more decisive in which *order* players sit and move. Make sure that you play equally often clockwise and widdershins.—You can play twice with three players, then (in case of a draw) have the leading two play a two-player.

Four players.

- The simplest way is to play with four players is to take the normal, two-player, rules and form two teams. This is quick and a good way to introduce the game.
- Or you play everyone against everyone, each player commanding a set of pieces. This takes maybe 85 minutes.
- You can also have each player have her own set of pieces, but nevertheless form teams. Players sitting opposite each other should form a team. Then every player has one partner (the player sitting opposite) and two opponents. The team that has a total of 5 pieces out first wins.

Five players. Five players *can* play, even though you only have four sets of pieces in your box. To do this, simplify the standard rules as follows. Give each player all pieces *of one colour* regardless of shape, so one player gets all red, another all blue, another all white, another all green or all yellow pieces. Have them place their pieces on the corners of their colour. Then all players can start their pieces at the corner where they sit. All other rules remain unchanged. The winner is who brings out three pieces (out of four).

Playing inside out. If you want, you can play from crossings to corners instead of playing from corners to crossing; because the board is isomoroph, this works just as well.

Fine tuning score. Consider that a score won seated after a strong player must count more than the same score won seated after a weak player. If, for example, players A, B and C have played 3:3:2 and were moving in the order $A \to B \to C$, then player B is a stronger player than player A.

Time limits. You may want to limit the time per move with an egg timer. In two-player and in three-player settings, about one minute per move is practical. If you move before your minute runs out, your opponent can use that extra time before you can turn the egg timer on her. A player who does not move in time loses turn.

In four-player games, players may need more than one minute. If you play in teams, you can use a more sophisticated clock and give each party total consideration time.

Talking. Etiquette demands not to distract the player who has to move, and never to tell anyone about any possibilities you spot on the board.

If you want to know *why* all these rules are the way they are and learn more about the past, present and future of Pentagame, read on.

4 How the idea was born

When we look at strategic board games, many of which are classics that have stood the test of time, we find that they have two characteristics in common. One is simplicity of their rules. The other is that they seem to always be played on fundamental geometrical shapes. These are based on natural numbers; there is the linear structure, there are triangles and squares:

n	Form	Examples
1, 2	linear, circular	Pachisi, Ludo, Mancala, Backgammon, etc.
3	triangular	Chinese Checkers, Gipf, etc.
4	square	Chess, Go, Othello, Draughts etc.
5	**pentagonal**	– **?** –
6	hexagonal	Abalone, Hex, Settlers etc.

Table 1: Geometric boards of strategy games

Why is there a gap at five? Can you not possibly also play on the five-fold (pentagram) shape? What would that game be? Has there never been such a game? And if there might have been, why was it lost?

The quest was to find a reasonable structured board and compellingly simple rules for a convincingly complex game. Pentagame is the result, a special case of reverse engineering.

We tried to find a game that would fulfil the following:

- Be played on a pentagram shape
- Rules be few and general with few or no exceptions
- Restrictions to moving be minimal
- Complex and interesting game play should emerge
- The game should always come to an end and stalemates be rare.

The whole development from the first idea to the game we have now took more than twenty years. Countless people have contributed.

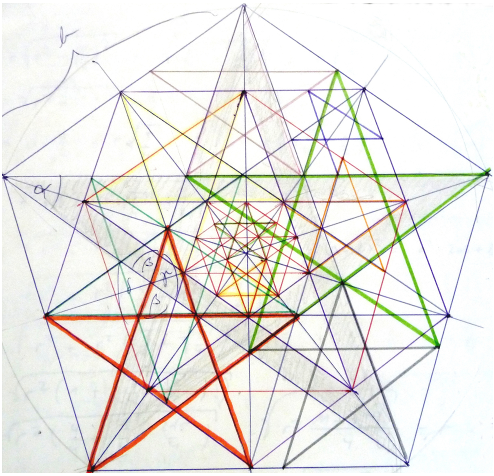

Figure 3: Author's first study (1996)

The first and rather complex challenge was the board design. Firstly, regular pentagons do not tile a plain surface,[2] so it had to be stops on a shape rather than a surface. But how do you segment a pentagram into stops?

It is possible, of course, to draw a Pentagame board roughly, say with chalk, but it is surprisingly hard to construct it *exactly*. This is because the lines in a pentagram are incommensurable. To have stops on a pentagram poses a little geometric challenge, a riddle which kept us occupied for some time; compare Figure 3, the author's first study. We shall present the exact solution in Section 15. At the beginning, we just drew a pentagram and marked a number of stops on it, focusing on developing a playable rule set.

First prototypes were played with number dice; this was tedious, as it demanded quite some computation capacity on behalf of players. The idea then laid dormant for a number of years.

Then we learned of the interpretations of the pentagram, of the five elements of antiquity and their association with colours, which inspired us to *colour* the stops.

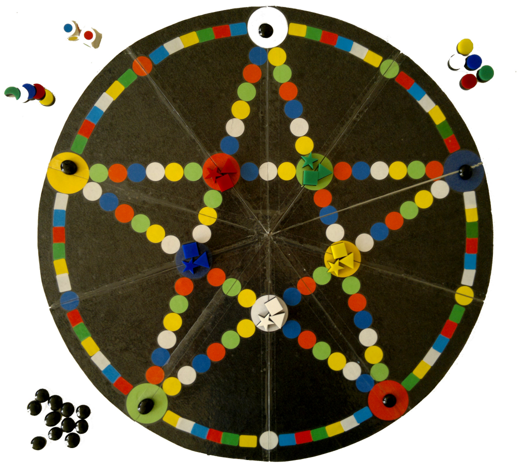

Figure 4: An early Pentagame prototype
The colour scheme repeats throughout.

5 Developing play

Coloration of Pentagame was a large step forward. It turned out that there is only one way to continue a scheme of five colours all through the lines: there must be exactly 3 stops between the crossings in the center, and 6 between a center and a corner (Figure 4).

To find the then required exact *sizes* of stops turned out to be a challenge, and solving this rather intricate mathematical riddle took us a while; the solution can be found below in Section 15. For the time being, we continued experimenting with approximate boards.

On a board coloured this way we could replace the number dice with colour dice. Rolling a specific colour would now allow a player to move any or her pieces to the nearest stop of that colour. This much improved the game, but still rather complicated rules about allowed directions were necessary and game duration was rather long.

Then we took a closer look on the rules of existing classics.

In today's Backgammon, pieces start their way already half way into the board. So instead of having to return each piece to its origin, you now only needed to go to each opposite node, which made all directions equally possible.[3] Interestingly, one simplification allowed for another.

In Chess, pieces were originally only allowed a few steps per move; this was only changed during the Renaissance.[4] Accordingly, we allowed to move to any reachable stop of that colour. Suddenly, pieces could move long distances, and players had more freedom, which was definitely more fun.

But then pieces could also move out at once, after the first roll. To impede this we introduced the black blocks.[5] We also introduced the idea to insert blocks upon leaving the board, because otherwise the boards becomes soon too empty.[6]

But what to do if there was another player's piece blocking, which could force a stalemate?—The idea to swap neighbouring pieces came up in Glastonbury,[7] so we call it 'Glastonbury-rule'.

At that stage we played the game with two colour dice, so players would have to find the best combination. This makes already an entertaining game.

However, not everyone is a friend of dice. We tried cards, so you would draw a colour rather than roll a colour. You could collect these cards in your hand and play them later. This was fun as well, but required quite some material; it added something that wasn't quite required. Then it dawned on us that the colour restriction *in general* (be it by dice or cards) was only *limiting* player's options without any true justification, and that we actually need none of that at all. Instead, a player can now move on *any* stop within reach. Thus, we arrived at rules much simpler than before; and it was always our intention to create a *simple* game, happy to eliminate and reluctant to introduce rules.

The more pieces a player has brought to their destinations, the less she has remaining on the board. Her choice thus diminishes as she wins, favouring the catch-up of her opponents: a form of 'winner's curse'. This is compensated by the extra move to freely position the extra block (before the extra block would just appear at the junction where the player moves out). Still, having less pieces is a disadvantage, and once you only have one single piece remaining, end games become cumbersome. But if you play ‘three out wins’ rather than ‘all out’, you can strategically focus on a part of your cohort.

We test-played with more and more players, and observed that the board can become rather crowded and blocked up as more and more blocks enter the game. That is when we decided to distinguish between black and grey blocks; a distinction not really necessary in a two-player setting, but very helpful in three and four player rounds. And of course we like to have the same rules for any setup.

EVERY game developer knows that a good game has to be designed so that the race is mostly open until close to the end, but also in such a way that allows the better player to win, and not to allow the losing party to be able to easily and always force a draw. Pentagame fulfils these conditions. Nevertheless, depending on how you treat certain moves, stalemates can happen more or less often.

Of course, a stalemate will always be possible if both players agree on a draw. The other draw is when the game enters a loop of continuous repetition. In the other classic board games, we know of two principal ways to deal with this: Chess simply calls it a draw after three repetitions; in Go a player is not allowed to enact repetitions, a rule known as ‘Ko’. In Pentagame the Chess rule of draw is not fitting, since it is comparatively easy for a player losing by an edge to make a move that forces the leading player into a loop.

This leaves the Ko-rule. There are two versions of this: either you say

a) *do not re-do what your oppenent has just done to you,* or

b) *do not repeat the very same move you have done before.*

In Ko-rule (a), player A can effectively deprive player B of an option. Also, if A is the first to move to a stop and than gets ‘taken’ by B, A has a disadvantage from her original advance. But being the first to move anywhere should not be a disadvantage; an advantage should never turn into a disadvantage. In contrast, version (b) is a rule that only affects one of the players and avoids the formal issues of the first option. This is why we settle on rule (b). It could be phrased as *Do not try the same thing twice.*

6 A game of its own kind

PENTAGAME is the only game we know of playable on a pentagram shape. The rules fit on a single sheet, and it can be explained in a minute. It thus has what it takes for a classic. It also has some further unique qualities that sets it apart from all other game of pure strategy.

- It can be played not only with two, but also with three, four, or even five players. This is noteworthy, since almost all other pure strategy games are for two players only, and it is sometimes erroneously assumed that games of pure strategy be limited to two players.
- Players can sit at any angle to the board; you can indeed while playing turn the board, if you so please. You can also play it inside-out.
- Pentagame is perfectly fair for all numbers of players. (Few notice that for example Ludo is unfair when played with three players.)
- The pentagram is the most simple regular shape of radial symmetry allowing the specific Pentagame rules; for example, a hexagon cannot serve because it has no central crossings opposite its points.
- The game is cooperative; in contrast to many other games, there is no 'killing' of pieces. All pieces stay on the board.
- It is probably the only purely strategic race game.

The game has been played thousands of times. It never seems to wear out; instead, it remains interesting; we will return to the subject of the many possible ways the game can take and the learning curve later (Section 16). We conclude that Pentagame closes the surprising gap in the family of classic board games; and that it has indeed all it takes to be a classic.

But the question remains: Could it have existed before?

Figure 5: Achilles & Aiax at play. EXEKIAS (ca. 540 BC)

7 The ancient game petteia

A game played on five lines is mentioned in ancient European texts. The most detailed description provides IULIUS POLLUX in the 2nd or 3rd century AD. Pollux compiled an encyclopedia of the name 'Onomasticon', in which he describes a game called petteia played with five pieces per player on five lines. Pollux' style is more poetic than precise (and was already criticised heavily in antiquity), and he neither explains the rules nor does he provide details of the board. He mentions a middle line called the holy line. Then he quotes the much earlier SOPHOCLES (5th century BC) calling *Palamedes the inventor*

> 'καὶ πεσσὰ πεντέγραμμα και κύβων βολαί'
> *'of both petteia's five lines [pentegramma] and of dice-throwing'*[8]

Since this short quote is our main source on the ancient game, it is worth studying each word.

πεσσὰ *pessa* pessoi are 'pebbles', and petteia the act of playing with them: name of a board game. *Petteia* can thus either be a word for one specific board game perceived as 'the' board game; or it can be a general term for board games as such, just like players would later play 'board' when playing what we now call Backgammon. For many centuries, the science of antique studies has interpreted *petteia* as name of one specific game. Some that have opted to not translate the term, considering *petteia* the name of a profoundly lost game; others have speculated that it was the predecessor of one of today's well known games like Chess[9] or similar square games.[10] LAMER and AUSTIN presented comprehensive overviews on the sources, but remained inconclusive to the nature of *petteia*.[11] MURRAY assumed it to have been of the pentagram shape, but presented little evidence.[12] Since this cannot quite be resolved, today *petteia* is mainly interpreted as a general term for board game.

πεντέγραμμα *pentegramma* means simply 'five lines', which alone does not say how these five lines are arranged. The expression appears exclusively in context of this game. Given today's meaning of 'pentagram', it seems appropriate to assume that this would indeed imply a pentagram shaped board; since the pentagram is the only shape requiring strictly five lines. Board games on parallel lines can of course have any number of lines.

κύβων βολαί *kyblon bolai* means 'to throw dice'.

καὶ **...** καὶ *kai...kai* means 'both...and...'. Sophocles contrasts *petteia* with dice play. If this is for reasons other than rhyme, then petteia must have been a diceless game, or at least without much scope for gambling.

According to HOMER, *petteia* was already played before and during the Trojan war by PALAMEDES and others.[13] PLATO has SOCRATES credit THOT, the Egyptian HERMES, TRISMEGISTOS, as inventor of petteia.[14] These shows the high esteem in which both petteia and dice play were held. PLATO equates *petteia* in to the rule of reason, to dialectics, and to geometry, saying it needed a lifetime to master.[15] Much earlier, and in a famous and famously obscure passage, HERACLITUS compared *petteia* to the world: Αἰών παίς ἔστι παίζων, πεσσεύων· παιδὸς ἡ βασιλῄη.[16] 'Time is a child at play, moving pieces in a board game (πεσσεύων); the kingly power is a child's.'

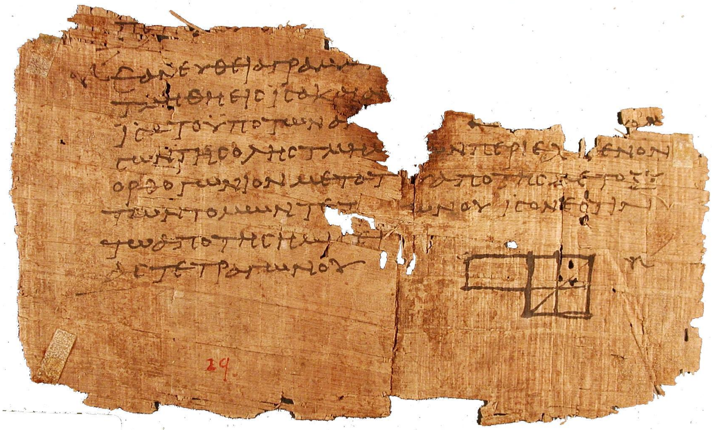

Figure 6: A typical fragment of ancient literature
part of the 2nd book of the Elements of Euclid.
Photo: Jite Niesen

We also have about a dozen places where petteia is mentioned in other contexts, for example when Polybius writes that Hannibal's father Hamilcar

> *like a good draughts-player [petteia-player], by cutting off and surrounding large numbers of the enemy, destroyed them without their resisting.*[17]

The ancient Greek were fond of playing games; Johan Huizinga famously stated that their entire culture was based on playful competition, and that the large advances of science they have bestowed upon us are a consequence thereof. However, in the haze of history many riddles are buried, and it is not quite easy to discern how things really were.

When speaking of antiquity, we are talking about a period stretching from maybe 1600 BC to about 400 AD, so about a period of two thousand years that is bygone for almost two 1.500 years. So much has already changed within that period of antiquity. In addition, much has happened since: in particular, in late antiquity and the middle or 'dark' ages much of what there once was got obliterated. Consequently, ancient archaeology is piecemeal. All we have are suspicions and fragments of literature. AUSTIN put it nicely: *The study of the Greek games is, in fact, a journey into complete darkness.* The amount of texts from 'back then' is limited (Figure 6). Estimates about the amount of books that were available in antiquity—e.g. in the famed library of Alexandria—bear in themselves large degrees of uncertainty; but clearly, today we only possess a tiny fraction of what there was. For instance, we know that SUETONIUS (69-122) wrote a book on Greek games, and this book is lost.

Another source on petteia is the byzantine bishop EUSATHIUS of Thessalonica, who apparently still had access to the book of SUETONIUS. Eustathius explains that petteia was somehow similar to *tavli* or Backgammon, probably in that it was played on a board. This has lead many to believe (especially in Greece!) that the original *petteia* was a forerunner of Backgammon; thus many older translators render *petteia* as 'drafts'. However, the fact that today's Greeks call Backgammon 'tavli', which cleary is a Latin lean word from 'tabula', speaks for Roman origins of that game. The archaeological record shows that Backgammon has been played throughout the Empire on *twelve* lines (hence its roman name ' duodecim scripta'),[18] related certainly to six-sided dice;[19] there is no reason to assume a five-line ancestor of Backgammon, or to assume that the mysterious *petteia* is somehow related to Backgammon at all, unless, of course, all boasrd games are *petteia.*[20]

The ancient five-line *petteia* may have been a thoroughly *lost* game of its own right, relatively unrelated to the games we play today and not resembling any of the surviving classics. It *may* have just been like Pentagame.

8 Is Pentagame petteia?

Under the assumption that all ancient descriptions rely to *one* game, we can add up the ancient sources on *petteia* as follows:

- There were five lines ('pente grammai').[21]
- There were five pieces per player, each with its own origin.[22]
- There was a line dubbed 'holy line'.[23]
- Dice were not a key feature.[24]
- It was a race game of some sort.[25]
- Blocking was key, pieces had to be coordinated in a strategic fashion with pieces not be isolated.[26]
- There was an element of 'taking' 'unyoked' pieces (ἄζυγες).[27]
- It was complex but simple enough to survive for a long time.

Pentagame fits all of the above well—with the notable exception of the line dubbed 'holy' line, which is difficult to account for. Nevertheless, for Alcaeus a player moved 'from the holy line' and by doing so was 'gaining the upper hand'.[28] Maybe the expression 'to move from the holy line' signified to move out of the game; then the pentagram itself is the (one) holy middle line, which—given the religious context of a pentagram—makes some sense. Then Pentagame is a *plausible* solution to the riddle of the assumed lost antique game of five lines *petteia*, which has thus here been re-engineered. Or to be more precise: Pentagame is *possibly* a (somehow close) re-creation of *petteia.*

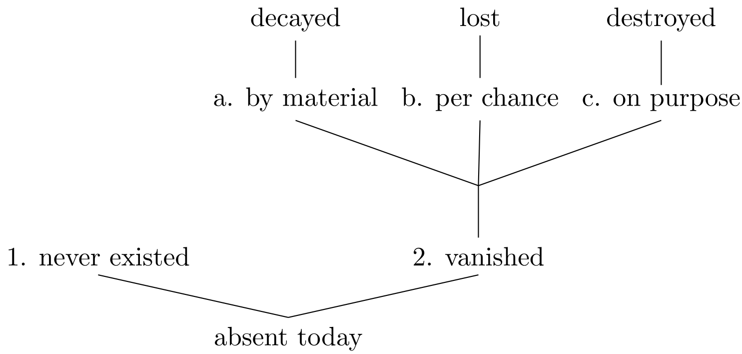

Figure 7: Why is there no trace?

9 A vanished game?

NOT a single pentagram shaped antique game board exists today.[29] Why? Figure (7) plots possible causes for the absence of such findings. (1) There may not be such objects because they have simply never existed. (2) Such objects could have existed but they have completely disappeared. This could be for a number of reasons: (a) they were of such poor material that they have vanished; (b) they vanished for no reason, by pure chance; or (c) they were destroyed on purpose. Of course, a combination of these three resaons could establish the cause (2).

1. It never existed: There are no pentagram shaped boards, but there are at least eleven boards with five *parallel* lines from at least two sites (Vari, Kerameikos, etc.), and some additional boards which have been expanded from five to eleven lines; along with these were found dice which must have been used with them.[30] Based on this, Ulrich SCHÄDLER, today's leading expert on ancient games, has recently proposed a dice game which beautifully combines the descriptions of POLLUX and others with these boards.[31] In contrast to Pentagame this proposal accounts for the 'middle line'—that 'holy line' which was apparently a core feature of the game described by POLLUX.[32] However, while this would explain the 'middle line', *parallel* lines seem a weak explanaition for a game to have earned the name ‘pentagramma’; since every rules that fit a board of five lines can equally fit a board with seven, nine, or indeed any odd number of lines.

If we buy into the argument that it has never existed, then of course all further speculations are futile. In other words: if we assume that had such a game existed then there would be physical evidence, then we must conclude that it has not existed. Nevertheless, there are some good reasons why such boards and pictures of them may have been lost.

2. It vanished. Admittedly, *‘a pentagram shaped game has existed’* is unproven, speculative, and not even likely. What we say is that *‘it was possibly there and all specimen got destroyed’* is a possibility of a likelihood greater than nil. Suppose that 1 out of 1000 artefacts were a game, and suppose further that today less than 1/1000 of all artefacts be games. Then we would assume that the rate of diminishing was not random, but that games have been particularly destroyed; this could be for material reasons (games were mostly drawn on the floor and not durable) or for ideological reasons (destruction focused on games more than on other artefacts). You could also state that there were multiple different games (one of which in pentagram form), but only specific types exist today, and start to argue why this could be the case (pentagrams were especially destroyed or pentagram shaped games particularly perishable).

2a. Decay. The pentagram is difficult to produce precisely on hard material. The very nature of the pentagram makes it thus susceptible to decay. But it is easy to sketch with chalk etc. on the occasion, which would explain why no board has been found. Pentagrams are also particularly difficult to draw in perspective. Thus it makes sense to not depict it in an artistic setting where lines drawn with rulers are uncommon; that game boards, where depicted on ancient frescoes, paintings, mosaics or vases, are mostly given sidewise (cf. Figure 5) could thus be explained.

2b. Chance. What we have from antiquity is a tiny sample of what there was. Since the finding of the Anitkythera mechanism, we are more aware how advanced ancient science was, and also how much was lost, and that surprising new evidence can surface. The filter 'chance' should not be underestimated.

2c. Destruction. The pentagram and golden section were important in ancient culture; however, not only have we no pentagram shaped board game, but we have nothing in the form of a pentagram at all. In the special case of the pentagram, the surprising lack of archaeological findings in this shape *can be explained* by what went on later. We mean 'can be explained' in a non-casualistic way: we wish to express 'there is A that may be the reason for B'. This should not be confused with 'thus it is proven that B is only because of A'. A possible cause is not a necessary condition.

As we will show below (Section 10), the golden section and the pentagram played a major role in ancient Greek culture.[33] Nevertheless, nothing pentagonal has survived. The surprising lack of the five-fold game as shown in Table (1) in conjunction with the name *'pente grammai'* plus the rich and somehow wild history of the pentagram (Section 12) lead us to the suspicion that a pentagram-shaped game may have existed and was suppressed or forgotten. We know that the ancients knew enough about geometry to have studied and used the pentagram; there is, however, not only no pentagram shaped board game, but there are no ancient pentagrams left over *at all.* Our 'evidence' is the stunning *absence* of evidence. Albeit, of course, such evidence is not a proof.

Thus, there are two equally plausible explanations, or alternative models, of why there is no ancient pentagram shaped board to be found: (1) that it has never existed; (2) that it got lost. Consequently, the cause for its present non-existence cannot be finally identified from the evidence at hand.[34] We leave to the reader to subsume likelyhood to each of the propositions above, and to possibly come to your own verdict. For the quality of the game, however, the question of a possible lost history is of course irrelevant.

10 The golden proportion

PYTHAGORAS was the first man to call himself a philosopher, a 'friend of wisdom' (rather than a sage), and the one who coined the term *cosmos* 'that which is adorned'. He was a believer in the harmony of the spheres, laid the foundations of acoustics, and greatly advanced mathematics. He taught numbers to be the essence of all things. Some of his disciples he called 'acousmathicoi', those who learn from hearsay, as opposed to the 'mathematicoi', those who study.[35] Only the latter were given 'esoteric', or secret, instructions. Among these was mathematics.

It was known by then that the five-pointed star can be drawn using ruler and compass alone (Figure 9 on page 29). Much harder, however, was to measure its lines. These relate to each other in a proportion called 'golden' or 'divine' proportion. If you can construct the golden proportion, you can draw a pentagram, and vice versa.

Once the discovery was made how to construct the golden proportion and its unique fractal qualities (see below) were discovered, it became a core feature in Greek aesthetics, particularly in architecture and in sculpture. For the classic Greek, beauty and truth were a inseparable (cf. καλοκἀγαθία, the unity of that what is good and that what is beautiful). This explains why we call this proportion 'divine'. The most prominent example for this importance is, of course, the Parthenon in Athens, which is a temple for the Goddess of Wisdom, Knowledge, and Strategey, Athena, and which was built from about 485 AD. Its architects were IKTINOS and KALLIKRATES; the artistic supervision had PHIDIAS.

At around that time HIPPASOS, a disciple of PYTHAGORAS, made an astonishing observation: The existence of 'irrational' numbers. With other words: that there are mathematically well defined proportions which cannot be measured with the same yard stick by natural or rational numbers.[36] This discovery of the 'incommensurability' was of course a milestone on the intellectual journey of mankind.[37] How this discovery was made is not reported, however it is particularly easy to demonstrate it on a pentagram; compare

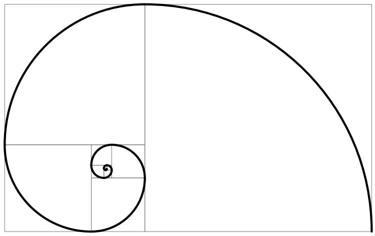

Figure 8: The Fibonacci spiral
A logarithmic spiral in a 'golden' rectangle

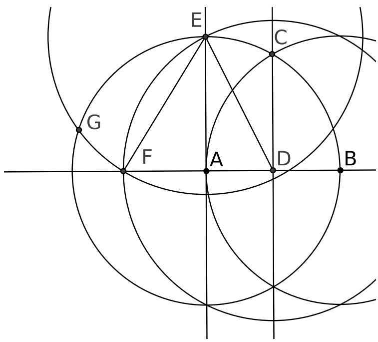

Figure 9: Pentagon construction
E and G are the first two points of a pentagon or pentagram

Figure (3). One could imagine HIPPASOS trying what the author has tried: to find a way to put stops on a pentagram to use it as a board game. [38] Shorter and longer lines in a pentagram are incommensurable; it is not possible that all stops on a board like Pentagame have the exact same size.

The golden proportion appears in many natural structures from microscopic shells to the solar system (page 32). It has been used in architecture and fine arts, and considered a beautiful proportion for centuries.[39] That this particular relation deeply governs all proportions of the pentagram makes it so particularly charming.

The golden proportion as a ruling principle can be found in Fibonacci sequences[40] and in many growth related structures. It makes for a beautiful paper format, and gives rise to a golden spiral (Figure 8 on page 29), a form of which can also be drawn within a pentagram.[41] Pentagonal forms can be found in fauna and flora; think sea-stars and the five fingers all land chordata share. A five-fold structure is naturally more stable than anything that can be cut more easily in two equal halfs.

IAMBLICHOS accounts the story of a disciple of PYTHAGORAS who fell ill on a journey and of an the inn keeper who took care of him regardless of costs. Before meeting his end, the disciple drew a symbol on a table of the inn. Later, after the disciple had died, another follower of PYTHAGORAS saw this sign and paid the inn keeper his expenses.[42] If we read this with what LUCIAN writes about the importance of health and the pentagram for Pythagoreans, this was likely a pentagram: symbol of being learned, and of health. Even today, provide anyone with a ruler and a compass and ask her to draw a pentagram: only those with some geometrical training will be able to comply.

So let us have a look at this magical, or divine, proportion and find out why it is unique. Figure (10) shows the proportion, and also the simplest way to find it. Two lengths, a longer one $a = \overline{AB}$ and a shorter one $b = \overline{BC}$, are in golden proportion if and only if their sum $a + b = \overline{AC}$ relates to the longer length a like a to b .

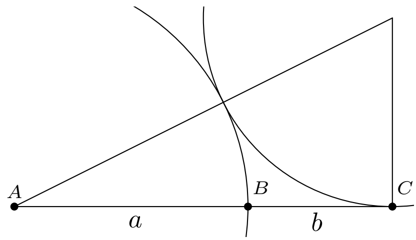

Figure 10: Golden section
The sides of the triangle are $1, 2$, and $\sqrt{5}$.
$a + b = 2$ and $a = \varphi = 1.618\ldots b$.

The golden proportion is defined by the relation

$$\frac{a+b}{a} = \frac{a}{b} \tag{1}$$

Solving this for a yields a quadratic polynomial; you get the algebraic solution $a = \varphi b$ with φ *phi* (named after PHIDIAS[43]) being

$$\varphi = \frac{1+\sqrt{5}}{2} = 1.61803... \tag{2}$$

Interestingly, if you now subtract b from a, the proportion reoccurrs!

$$\frac{b}{a-b} = \frac{a}{b} \tag{3}$$

And this goes on and on and on. φ is really a 'fractal' number. It can also be expressed in form of a chain fraction, which demonstrates nicely its fractal quality and proves its irrational nature directly. You may compare Figure 3 on page 15:

$$\varphi = 1 + \frac{1}{1 + \frac{1}{1+...}} \tag{4}$$

The golden section is the marriage of simplicity with complexity. It is a proportion at once familiar and strange.

11 Pentagrams in the sky

When you want to practice astrologogy, or astronomy for that matter, measuring angles is of course of upmost importance. So first of all you need to segment a circle. It is easy to segment a circle in two, three, four or six parts with ruler and compass. As a result, you get a dial of twelve. But only if you manage to draw a pentagram, you can then proceed to the 60 minutes of a circle, and from there on to 360° or seconds.[44]

Geometrical knowledge and astronomy / astrology is thus intrinsically linked. You can simply not draw a horoscope (with ruler and compass) without knowledge of drawing a pentagram. So if you equate astrology and superstition, you can easily mistake the pentagram for a sign of superstition, all the while it actually is a symbol of science.

If one marks the points of largest elongation or of transits of Venus in the zodiac, in four years a pentagram results. This is because the orbital periods of Earth (365.256 day) and Venus (224.701 days) relate to each other almost perfectly in the golden proportion:[45] $365.256 : 224.701 = 1.6255... \approx \varphi$. We will explain the maths of the golden proportion below in Section 10; for now keep that this is the hard reason why the pentagram is associated to the planet Venus, at times the brightest star after the Sun and the Moon.[46]

Because of this relation a Venuvian calendar produces five Venus periods in eight earth years, or a similar constellation every four years.[47] The most ancient Greek calendars were Venuvian calendars, which is likely the root of the Olympic four-year cycle and another hint on the importance of the pentagram for ancient culture.[48]

Venus as Morning Star is called $\varphi o\sigma\varphi o\rho o\varsigma$ 'Phosphoros' in Greek and 'Luzifer' in Latin, where both words simply mean 'light-bringer'.

12 The wild history of a beautiful shape

THE fact that the pentagram was so important for the Pythagoreans (see above) could be a reason why we find very few ancient pentagrams in total.[49] Members of their school or brotherhood were persecuted as a heretic sect, which helps explain the vanishing of anything ancient and pentagrammiform. That late Pythagoreans, such as IAMBLICHOS and PORPHYRY, whose works have survived, explicitly presented pythagoreanism as an alternative to Christianity, did certainly not help either.[50]

Crisis and destruction leave few traces, but only gaps, and an archaeology of ‘gaps’ is still in its infancy, if not outright impossible. As as a result, we know too little of the victims of history; and with the victims, the culprits are forgotten. Not much is known about the actual process of destruction of ancient thought, ancient images and ancient artefacts: We don’t even know what exactly happened to the Library of Alexandria, who has burnt it down, or if it was destroyed in a lengthy progress.

The change from paganism to the monotheistic religions accompanied iconoclasm, and there were waves of such in times of political instability which are destructive in their own right. *Damnatio memoriae* includes the process of the destruction just as much as that which has been destroyed.[51] In Europe, much was destroyed as pagan artefacts by Church and Christian rulers fighting Gnosis, Manichaeism, Bogumilism, Katharers and so forth, who had kept a good deal of pre-Christian ancient mysticism. About all these movements very little detail is known.

If the pentagram is a symbol of five—not four—elements and spirit is shown on the same plane as the material elements, then it may appear a heresy.[52] If it was proof of someone practising astrology, it was a proof of heresy. These were indeed the times in which science as such were often thought of as heresy, as medieval theologians considered seeking knowledge as vanity and all science was supposedly opposed to revealed knowledge.

Medieval knowledge was mostly confined to monasteries, which on the one hand side collected and kept works on Aristotelian logic and his categories and

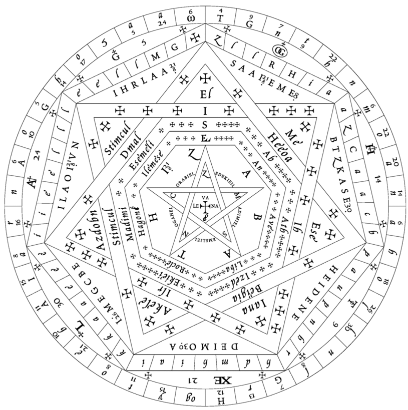

Figure 11: Sigilum Dei
as found in the *Liber iuratis Honorii*, a medieval grimoire

were relatively open to revelation and prophecy (e.g. Hildegard of Bingen), but to whom on the other hand the concept of progress of science through pure reasoning and empirical studies was suspicious.

Science was mainly the art of collecting and classifying knowledge (many books on the 'art of memory' survive), and classification often followed Porphyry (who was a staunch Pythagorean!). His *Isagoge,* also known as *quinque voces,* is structured in a a five-fold scheme following Aristotle's five *kategorema* or *predicabilis* (in *Topics*): *genus, species, proprium, differentia* and *accidens.* This scheme has had a massive influence on Diderot, d'Alembert and Linnaeus to name a few. A five-fold scheme of categories is also found in Plato's *Sophistes.* Objects were sorted by genus and accidents, and anything 'watery' would be sorted into the same category.[53] For example Paracelsus speaks of Elementary spirits, one type per element.

Thus it comes to no surprise that geometric diagrams to illustrate such systems were widespread. The pentagram was one of them, but of course not the only one. It makes a prominent appearance in the 13th century grimoire *Liber iuratis Honorii,* where it appears as part of a larger construction of polygons and letters (Figure 11). Such a device could then symbolise the cosmos, and contain all characteristics of God.[54] We see that medieval thought was predominantly magical, in that analogy ruled; this heremeutic *'as above, so below'* is the very essence of magical thinking, which is of course today discredited, although today for other reasons: not because it contradicts revelation, but because it contradicts science. Humans see patterns and construct reasons too easily where there are often only coincidences.[55]

The pentagram also symbolised Five Virtues or the Five Wounds of Christ.[56] It can be found on many medieval churches,[57] and is prominent in freemasonry, where inter alia it is a symbol of engineering knowledge. It was also known and used as badge by guilds of medicals as symbol of Hygeia, the goddess of health, and PYTHAGORAS was held in high esteem in the middle ages.

With the Renaissance individual inquiry became again more prominent, certainly inspired to a good degree by the study of ancient originals such as EUCLIDS *Elements,* which still today serves as the ultimate example for a axiomatically inductively structured mathematical text book, and PORPHYRY's *Isagoge* in the translation of BOETHIUS. Only in the 13th century the first university was founded in Bologna. Nevertheless, it took some further centuries for the scientific methods we use today to become firmly established.[58] Until then, scientists were burnt at stake for their use of reason.

Still today, the pentagram is surprisingly rare in elementary mathematical text books, even though it is so basic and perfect for explaining irrational numbers. Books containing pentagrams were certainly often censored, the Vatican denying them Imprimatur. Not even the famous 'De divina proportione' from PACIOLI (1498) contains a pentagram!

A prominent victim was GIORDANO BRUNO. He wrote an interesting treaty on the classification of magic, where he uses 'magic' in places we would today read 'science', and 'astrology' for 'mathematics'.[59] For him 'mathematics' is the use of symbols to describe the world and of course a form of magic.[60]

In the early modern times, characters of the like of John DEE, Jakob BÖHME and Athanasius KIRCHER knew the pentagram as a centre part of the 'Sigilum Dei' (Figure 11) and sign of Jesus in the Christian Kabbalah. A popular design can be found in a fine illustration to Agrippa (Figure 12). Back then

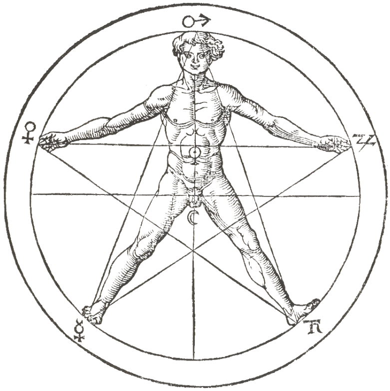

Figure 12: Human in a pentagram
anonymous illustration in AGRIPPA

science was still interested in the supernatural—because that was a time before statistical testing became a bullet proof scientific method.

Today we know that divination does not work—albeit cold reading and free association do. The human mind is built to associate, draw parallels, and see all sorts of connections where there is mere similitude. This is the fallacy of false analogies.

For long, folk magic has seen the pentagram as a banishing sign. It appears as such in GOETHE's *Faust*, where it has lost half of its power due to not been drawn correctly.[61]

The pentagram regained prominence in the 19th century with the first wave of 'occultism'. Rudolf STEINER recommended it (in various places) for meditation on the human nature. This is a counter movement to enlightenment; since then, an up-pointing pentagram is called a symbol of the rule of reason, and a down-pointing of the opposite.[62] That is fitting for a time when the idealism–materialism debate was in its heyday.[63]

Ritualists today in the neo-pagan, occult or Wicca tradition distinguish the two directions in which it can be drawn, clockwise being held as 'banishing' and counter-clockwise as 'invoking'.

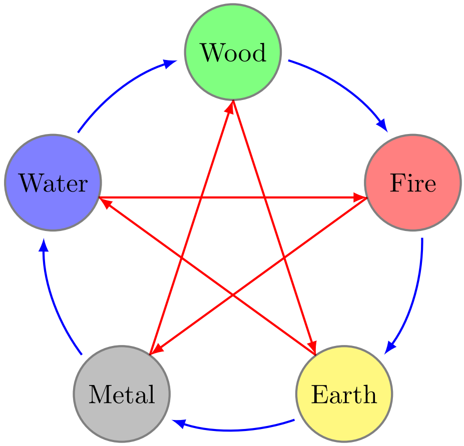

Figure 13: Wu Xing
Blue circle: generating. Red star: overcoming.

13 Pentagrams in the East

IN the Far East a concept of five elements is known as 五行 *Wu Xing*. While in the West 'element' is something substantial (or at best, a *state* of matter), 'Xing' is dynamic, a movement or a shift.[64] Each *xing* is thought to have one that generates it and one which it generates. It is thus instrumental—or better: fundamental—to arrange them in a circular fashion, where all lines bear significance, and so does the pentagram in its centre. In a typical arrangement, the *xing* change into each or 'generate' each other clockwise; if you connect them creating a pentagram, the lines of the star signify 'overcoming' (again clockwise); see Figure (13). This map is fundamental to Feng shui and Traditional Chinese Medicine practice, inter alia.

Table (2) presents the most commonly associated colours, plus the five Chinese seasons, which defines the core of the concept. The elements *('phases')* are **metal, water, wood, fire** and **earth**. Colours associated with them vary a little. This table is often extended to include not only planets, but body parts, senses, characters and so forth, similar to the ancient European association tables; and again, there is much variation from school to school.

colour	行 *xíng* phase	season	planet
white / black	金 *jīn* metal	autumn	Venus
black / blue	水 *shuǐ* water	winter	Mercury
green / blue	木 *mù* wood	spring	Jupiter
red	火 *huǒ* fire	summer	Mars
yellow	土 *tǔ* earth	dry season	Saturn

Table 2: Colours, phases, seasons, planets (trad. East Asia)

The Wu Xing is of enormous importance for East Asian culture. A clear difference to the Western tradition is that the order of elements here are truly fixed, and that there is meaning associated to the lines connecting them in a circle and a pentagram; in the West, the association of pentagram corners with elements is incoherent in time and does not convey much meaning.

Since we find the concept of five elements in both the occident and the orient, we can speculate that this is a very old concept. Pythagoras is said to have studied in Asia.[65] However the pentagram is first and foremost a very simple geometric shape and a fundamental graph. It will appear wherever one maps the complete relations between five elements of which kind ever.

The colours we have settled on for the use in Pentagame have nothing to do with any of these schemes, but are purely chosen for visual reasons. Of course any five colour scheme—or indeed context scheme—could be used.

⋆

14 A Judeo-Christian view

PENTAGAME is a parlour game played on a pure geometric shape. Some people interpret this beautiful shape spiritually. Some Christians seem to see the sign of the Antichrist; some neo-pagans accuse me of 'mocking' their holy symbol. Both see something 'occult' in the sign of the pentagram and feel estranged by it published and played in public.

The term occult 'that which is hidden' resonates with PYTHAGORAS' distinction of *acousmaticoi* and *mathematicoi,* those who hear, and those who learn, and between *exoterioi* and *esoterioi:* that what is spoken about in public, and that what is taught individually.

Mathematics seems to some esoteric. Some see the devil in geometry, others speak of 'sacred geometry'. Geometry measures the world (hence the name), and 'the world belongs to Satan' (2 Cor 4:4). But the world is Gods creation. If we look at its mathematical beauty, we see the Creator.

'Every good gift and every perfect gift is from above, coming down from the Father of lights, with whom there is no variation or shadow due to change.' (James 1:17) 'So, whether you eat or drink, or whatever you do, do all to the glory of God.' (1 Cor 10:31).

We should be proud of our godly ability to see the light, to reason. *Sapere aude*—use your own judgement. Test everything, keep what is good (1 Thess 5:21). You shall see the truth, and the truth shall set you free (John 8:32).

Satan is mentioned many times in the Bible as personification of evil. The figure or concept of the devil is a mix of Greek, Roman and Pagan myths. 'Lucifer' is originally the Latin name of the planet Venus as morning star, which means 'bearer of light'; and as we have shown above (page 32), Venus is associated to the pentagram. It is a twist that the advent of light has become associated with the advent of darkness. But in the New Testament, the Morning Star, the bringer of light, is JESUS himself (2 Petr 1:19).

Evil is not what goes into a person, but what comes out of them (Mt 15:11). Thus, it is not the pentagram that is evil, but the thoughts it may evoke. If you think of harmony—then it is a good sign. If you think otherwise—you should try to find out why that is so.

Someone came up with '*Pentagame—Satan approved*' as a sales slogan! I thought about that statement and concluded that it actually is accurate... Of course, Satan would *approve* Pentagame; that is because the devil is the *inability* to distinguish good and evil. God, said the serpent in 1 Mos. 2, has the capability to distinguish between good and evil. And since the Fall of Man we share this ability. Humans can and must find their best guess on good and evil all the time and decide for their best knowledge, much like you ought to seek the best move in a game.

The devil, in contrast, is unconcerned with truth: he is the confusion of truth and lie (John 8:44). *Satan is impartial* to good and evil. It is bad not to tell good from evil. Not knowing, not telling good from bad, while having all capability to do, that is evil. Ignorance, relativism, is evil. The very word 'diabolo' means 'the one who throws things (into disarray)'. The very concept of the devil is that he confuses good and evil, and lures you into approving anything. Mixing up a pretty geometric shape with gross superstition is right up this path.

Compare 2:16 and Mos 3:1 in Genesis: The serpent denied the godly distinction between what is good and what is bad for Man, suggesting either anything to be fine, or everything forbidden. Satan is the approver of *all* things, and denier of all things as well. All the while God lets His sun shine, and let come rain, over good and evil alike (Mt 5:45). He is the enlightening source of distinction between what is good and what is not. Satan, in contrast, who is not in truth, the father of lies, is the voice of relativism. Here is indifference, and at the end short-sighted selfishness; the darkness which confuses.

This game will never harm anybody. That I can clearly see that Pentagame is good: a good game, offering nice encounters, and training your mind. Like any game, Pentagame is purely leasure.

15 Solving the board

HERE is how to compute the exact sizes of the stops on a Pentagame board. We have three sizes that we wish to find: the radius of the star stops s, the radius of the corner stops c, and the radius of the crossings or junction stops j. The size of the board will follow. We need, and will find, two conditions.

Definitions: We remember φ as explained above, Equation 2. Since we investigate the angles of the pentagram we define $\vartheta = \frac{\pi}{10} = 18°$. Let the radius of each stop on the lines $s = 1$ be of unit length. On the lines, for reasons of game play we want three stops on the short lines $k = 3$. On the long lines, we count $l = 6$ stops.[66]

First Condition: In a pentagram long and short lines are in golden proportion. Thus we know the following, where the l.h.s. is the long line and the r.h.s. the short line:

$$c + j + 2l = \varphi(2j + 2k) \quad \text{with} \quad \varphi = \frac{\sqrt{5}+1}{2} \tag{5}$$

This can be solved for j as follows:

$$j = \frac{c + 9 + \sqrt{5}}{\sqrt{5}} \tag{6}$$

Second Condition: Now comes the gist. Take a closer look at the corner (Figure 14). There we have three circles: the actual corner circle c and two adjacent stops $s_{1,2} = 1$. All three are tangential to each other. In addition both stops s are tangential to our angle $\vartheta = \frac{\pi}{10} = 18°$. We have four conditions for three circles. The proportion $s \mapsto c$ is fully determined by the angle, and independent of the number of stops on the board. We can find it:

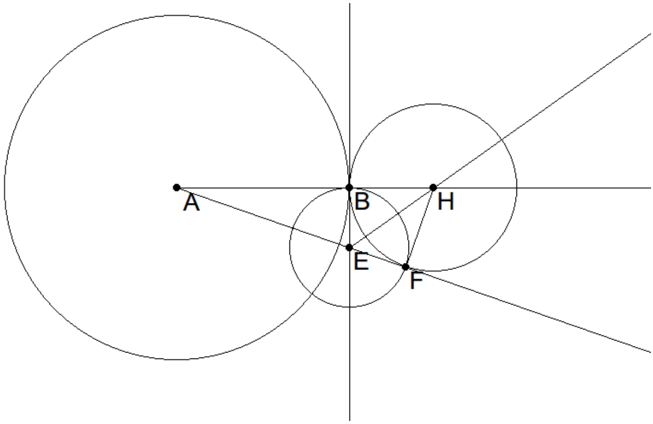

Figure 14: Corner condition

AFH is a triangle with a right angle $\angle AFH$; thus

$$\frac{FH}{AH} = \sin(\vartheta) \tag{7}$$

We know that $s = 1 = \overline{FH}$ and $\overline{AH} = c + 1$, thus

$$c = \frac{1}{\sin(\vartheta)} - 1 \tag{8}$$

Comfortably, all sines of the angles in a pentagram are algebraic:

$$\sin(\vartheta) = \sin(\frac{\pi}{10}) = \frac{\varphi}{2} \tag{9}$$

which means that equation (8) can be solved as

$$c = \frac{2}{\varphi} + 1 = \sqrt{5} = 2.23...s. \tag{10}$$

Corollary: $c \neq j \neq s \quad \forall\, k, j \in \mathbb{N}$; there are necessarily three different sizes of stops, no matter how many k and l.

Proof: If we set $j = \sqrt{5}$ or $j = 1$ then we can solve Equation (5) for $\sqrt{5}$ so that the r.h.s. contains only natural numbers, fractions and $k, l \in \mathbb{N}$; it would then follow that $\sqrt{5} \in \mathbb{Q}$ which is a contradiction: if $\sqrt{5} = \frac{p}{q}$ then $p^2 = 5q^2$, where the l.h.s. is even and the r.h.s. is odd $\forall p, q \in \mathbb{N}$. $\square$

Solution: We have already found $c = \sqrt{5}$. To find j we simply plug $k = 3$ and $l = 6$ into Equation (6) and see that in this case the junctions are of radius

$$j = \frac{9 - 2\sqrt{5}}{\sqrt{5}} = 2.02606...s. \tag{11}$$

Now for the board dimensions. The diameter d from the centre of one corner to another consists of two long lines and one short line:

$$d = 2(c + 12 + j) + (2j + 6) \tag{12}$$

The relation of the diameter of a pentagram to its radius is

$$r = \frac{d}{\sqrt{\varphi + 2}} \tag{13}$$

Insertion leads to a complicated term which can be simplified and solved to

$$r = \frac{2}{5}\sqrt{1570 + 698\sqrt{5}} = 22.38133...s. \tag{14}$$

Of course the entire board is slightly larger for the protruding corner stops:

$$R = r + \sqrt{5} = 24.61740...s. \tag{15}$$

Now we can draw the board precisely (Figure 15), *quod erat faciendum.*

$\square$

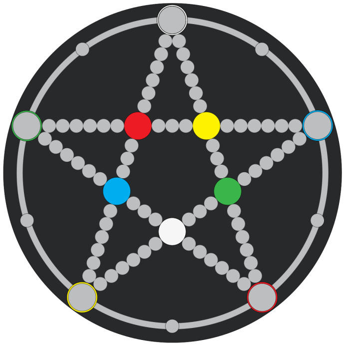

Figure 15: The board today

There are 100 stops on the board, of which 10 are nodes, of which 5 are corners and 5 are crossings. There are 20 lines, 5 between the crossings bearing 3 stops each; $5 \cdot 2$ linking corners and junctions, bearing 6 stops each; and 5 connecting the corners, also bearing 3 stops each. That makes a total of $2 \cdot 5 + 2 \cdot 5 \cdot 3 + 2 \cdot 5 \cdot 6 = 100$ stops. The is isomporphy; you could as well play inside-out as outside-in. It can be proven that any other (small) numbers of stops result in large difference of size between corners and crossings. The three stops on the rim between corners are equal; the middle one is round only for decorative reasons.

⋆

The above shows as a corollary that the pentagame board can be constructed using ruler and compass alone, because all trigonometric functions of the angles have algebraic solutions and all roots are square roots. We also see that such a construction is not straightforward; that is clear because of the complexity of the above.

That to draw such a board precisely is non-trivial could help explain why there are no *petteia* boards of old; while you can easily draw a close enough approximation of a board like this using chalk whenever you need one, you are in a tight spot when it comes to fabricating a precisely cut artefact.

⋆

Since ancient times was known that a pentagram can be constructed using ruler and compass alone. But nobody could ever find a way to construct a polygon with seven sides. The riddle was finally solved by Carl Friedrich GAUSS in 1796. Using complex numbers he managed to proof that constructing a heptagon precisely with ruler and compass is impossible.[67]

⋆

Complex numbers are particularly helpful regarding circular structures, and hence also when it comes to constructing the board. With their help the (cartesian) coordinates of the five corners are easy to find.

A 'root of unity' is any number z which when raised to a higher power n equals 1: $z^n = 1$; this is always true for the real number 1 which in the complex plane is $1 + 0i$. But on the complex plane $\mathbb{C}$ there are actually five such numbers $z \in \mathbb{C}$ such that $z^5 = 1$. From EULER's identity

$$e^{i\varphi} = \cos\varphi + i\sin\varphi \tag{16}$$

follows that the five roots of unity are:

$$\left\{e^{\frac{2\pi ik}{5}} \mid k \in \{1,2,3,4\}\right\} = \left\{\frac{u\sqrt{5}-1}{4} + vi\sqrt{\frac{5+u\sqrt{5}}{8}} \mid u, v \in \{-1,1\}\right\}$$

Plugging in $u, v \in \{-1, 1\}$ into the r.h.s. gives us the desired coordinates of the corners in a unit circle in cartesian coordinates, and plugging $k \in \{1,2,3,4\}$ into the l.h.s. gives us polar coordinates.

16 Theory of play and game theory

OUR perception and comprehension of games have changed over the millennia. In ancient Greek there was no word 'game', but ἀγών 'agon', describing a competition,[68] or παίζων 'paidson', which is non-rule-based child's play. Latin 'ludus' equates pretty much to what we call 'game' in English, albeit 'game' comes from hunting. Latin also knows 'alea', which means dice throwing[69] and the betting on chances.

Betting on chances was and is legally restricted for a number of reasons; this was already the case in the Roman Empire. And for long, the Church was suspicious of chance and therefore of games in general.

Since probability calculation was established by PASCALE, FERMAT and LAPLACE, the philosophical discussion between Bayesians and Frequentists is still largely unsettled. With other words, we do not quite know what chance exactly is. It is an interesting question how even in games with perfect information there remains a factor of 'chance' in the uncertainty of the player's moves. These are deep philosophical questions linked to terms like time and freedom.

Creating and playing games is a particular human quality: as beautifully as nightingales may sing, composing remains human;[70] to agree on certain rules and to act within such a self-chosen framework *is* civilisation. Humans are about the only species that *agree on games*, and for sure the only ones to invent new games every now and then, and to teach them one another.

HUIZINGA,[71] then CAILLOIS[72] laid the foundations to our present view of the importance of rule-based games in culture.[73] There are physical games like football or theatre ('circensic' and 'scenic' games, as these were called), there are board games, card games, computer games and so forth. Board games train the player, his abstraction or dexterity, but first of all also his ways to interact while abiding rules. Playing is a highly regulated and separate human interaction. It has much to do with etiquette, and also with the ability to communicate.

TODAY, there are two widely independent approaches on games and play: 'game theory' and 'theory of play'. Game theory is the mathematical branch; theory of play looks at player's motivation, and the social role of play.

G. W. LEIBNIZ was fascinated by games and wrote worth quoting:

> Saepe notavimus, nusquam homines quam in ludicris ingeniosiores esse: atque ideo ludos mathematicorum curam mereri, non per se, sed artis inveniendi causa.
>
> '*We have often said that humans are never as inventive as when amused* (ludicris)*: thus, games deserve the attention of mathematicians, not just for themselves, but for the art of invention'.*[74]

Game theory is the science of the ideal moves when the choices of the participants are limited by strict rules.[75] It was established by VON NEUMANN[76] and NASH,[77] among others. There is literature applying game theory to many things ranging from theology[78] to economics[79] and evolution.[80]

With the advent of big data and massive multi-player online games, the science of games reaches new heights. Insights from both game theory and the study of play gain traction in many areas these days in a process called 'gamification',[81] fundamentally the approach to design any rules in accordance to the insights from these fields.

But what are the best rules, and how to we find and establish them? The ultimate test for the quality of a game is the quality of the play. Social choice theory looks at how individuals agree on rules—thus, the negotiation of and agreement on an optimum rule set can itself be understood a game. Participants want a maximum amount of freedom and maximum chances to win. This is the quest for the best rules, which must thus be fair and efficient: a minimum of rules to create the most marvellous possible outcomes. Nevertheless, in society we often observe 'regulatory creep'; the introduction of additional rules seems much easier than their simplification. Ever growing bureaucracy and an ever growing body of laws is a troubling consequence.

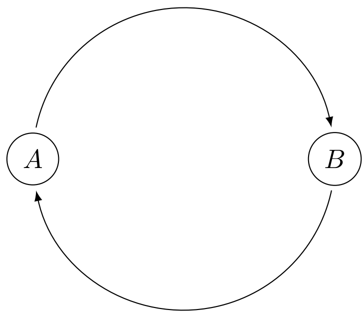

Figure 16: Two players A and B

17 The 2-player game

When you play one against one, two players compete with ten pieces. That plus the five impediments (black blocks) makes fifteen pieces, which are five more than there are nodes on the board. This number should be almost constant until the game is over.

In a two-player, after two undisturbed moves a player will be able to move a piece out in her next move, which is something the other player usually tries to prevent. So in practice, a block gets taken in the fourth (semi-)move at the latest.

At the beginning, every piece is on a node; this only changes after a significant number of moves. From a node a piece can move in four cardinal directions; thus, at first sight every player has $5 \cdot 4 = 20$ reasonable moves. Ten of these are *swaps,* which always are symmetrical; thus remain 15 reasonable moves, of which 10 are *take & close*. Every time a player plays *take & close*, he closes one of 20 lines; thus there are actually more than $10 * 20 + 5 = 205$ reasonable moves every player can consider once it is her turn. Let us try to lower that bound and speak of realistically only 20 different options. As we observe, a game consists of about 20 moves; thus we can compute that there must be more than $N = 20^{20}$ different games. Written out this is

N =104,857,600,000,000,000,000,000,000 or about 100 *septillion* games. Of course, many of these will be symmetric in space or time (permutations); for instance the same position may be reached by the same moves in a different order.

A reasonable player will take all possible answer moves of his opponent into account when choosing her move; thus consider her own 20 possible moves and the 20 possible answers, which totals 40 moves. Lets assume we think about each for a second; then 40 seconds suffice for each move. Let us round this up to 1 minute per move. A game will then take about 20 minutes.

As pieces move out, a player's options diminish; with already two pieces out a player only commands three more pieces. Moving out is thus a 'bad move' for the remaining pieces, which is some sort of 'winner's curse'.

Games can be described as 'trees' with 'branches' leading to victory; the amount of 'leaves' is the amount of possible games.[82] The game tree of Pentagame grows quickly.

Let us draw a tree branch to be read from top to bottom, starting with the initial move on the top and writing the possible answers underneath. Thus, the first 'row' of the tree shows the action of the first player; the second 'row' the answer of the second; and then the third row shows the second move of the first player. The numbers on the branches count the variations available. This presentation is called the 'extensive form' of a game in game theory. At the final end of each branch would be a 'leaf' showing the winner.

At its root, just two types of opening moves make sense: Either (1) you swap two of your pieces on the rim *(swap)*, or (2) you move in, taking a block *(take & close)*. It does not make any real difference which particular piece begins, and whether you move to the left right, because at the beginning everything is still symmetrical.

Lets begin with the first branch, where the beginning player A begins with a swap ('open games'). In practice, swapping is a popular opening, since A moves to pieces at once without committing to a direction of play. Beginners should pay considerable attention to these openings.

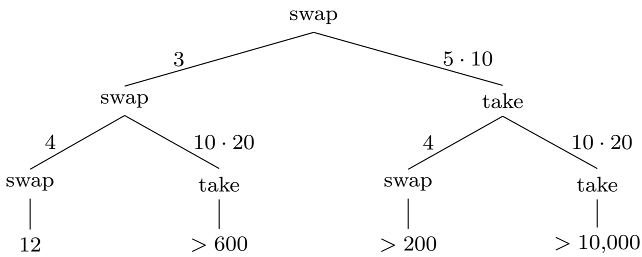

Figure 17: The branch of 'open games'

In Figure (17), player A begins with swap; then player B has a choice between 3 swaps or 5 in-moves, each of which has him choose one of 20 lines to close. These are actually only 10 distinct options since the initial swap retains axial symmetry.—And so forth. We calculate on the bottom the amount of possible positions, using the sign $>$ to account for the fact that there are actually 90 stops, rather than just 20 lines, to place a taken block.

The ability to limit another player's options is of course strategically attractive. Coordination and development is more important than rapid progress of a single piece.

When we look at the start of any Pentagame, we see that to move any one piece out will need at lest 3 moves. Thus, moving out could potentially happen two rows down from the bottom row of the tree above. If player B wants to impediment this, she *must* opt for taking a block in the next round.

We call two games are symmetric in time or permutations if the same position can be reached by a different order of moves. This effect is pronounced in open games since whatever the answer of B to the initial swap cannot hinder A to execute whichever move she prefers to make next.

The second possible opening is called ‘take & close’. Once you take and thus replace a block, you close one of the 20 lines, strategically adding structure (‘closed games’). If player A has begun with ‘take & close’ and set her block, only axial symmetry remains. Player B can then either swap, or also ‘take & close’:

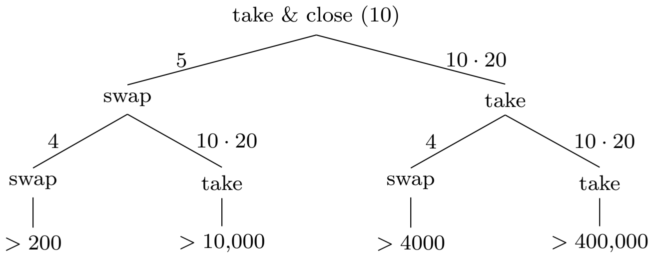

Figure 18: The branch of ‘closed games’

As Figure (18) illustrates, take & close unfolds far more options than swap. Taking a block—and thus closing a line!—offers many more options to influence the direction of play. That is, while a player opting for a ‘take & close’ does not move two pieces at once, she influences the structure of play more.

Thus after the first three moves, there are almost $\frac{1}{2}$ *a million* different possible Pentagame positions. As demonstrated above, the total number of possible games must be significantly bigger than $20^{20} \approx 1.05 \cdot 10^{26}$. For comparison, a human being consists of approximately $2 \cdot 10^{27}$ atoms, there are an estimated $2.5 \cdot 10^{11}$ stars in the Milky Way, and about $2 \cdot 10^{11}$ galaxies in the universe. This is considerably smaller than the number of positions in Go or Chess, but should still be enough to never get bored.

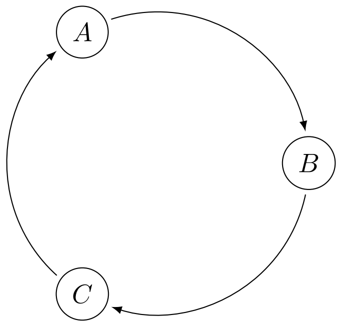

Figure 19: Three players A, B and C

18 The 3-player game

In a three player, every player faces two very distinct opponents: one always moving before her, and one always moving after her; see Figure (19). Her relationship to each of the two will fundamentally differ. We use the German names 'vorhand' for the player playing before and 'hinterhand' for the player moving after.[83]

Again, a player must consider her own reasonable moves (which maybe somewhere in the order of 25). But before choosing a move a rational player B would first of all check the options of hinterhand C; if C can move a piece out, then B is pretty much forced to hinder her. This 'zugzwang' is of course a fact which vorhand A can take into account. Assuming everyone at least trying to play perfect and everything else being equal, then every player will play primarily against her hinterhand. From the direction of play results an intransitive order of adversary, if not dominance.

This makes immediately clear that a single three player game with players of different characteristics cannot be fair. It is a disadvantage to play after a strong player and an advantage to play before a weak player.

However, this 'natural order of adversary' (in a Nash equilibrium) in a sequentially ordered game is not stable; it only holds 'ceteris paribus', everything else being equal. If we abstract from personal preferences (which in the real world can indeed be an issue),[84] the only thing that can be not equal is the likelihood of a player to win. This is clearly measurable by pieces brought out. It is thus quite common that as soon as one of the player takes the lead (thus 'outing' herself as in a better position than the others) the two loosing parties will develop partisanship against that leader; such a coalition is usually stable until someone else has taken the lead, which has the other two gang up, and so on until the very end. It is thus quite normal that a three player game (among almost equal opponents) can be won only by an edge.[85]

The most extreme form of such partisanship is known as 'kingmaker effect'. This is when a party who cannot win a game (anymore) by her actions decides who else wins. This is a fundamental property of multi-player games (and a surprisingly little researched area, so it seems). The only appropriate way to level out such partisan effects is of course repeated play.

Repeated play is also the remedy against the unfairness of the order, as the order can simply be reversed. (There are of course $m = (n-1)!$ possible orders.) Thus it is reasonable to always play *two* three player games, one clockwise and one counterclockwise. Such a match can already produce a clear winner; if it doesn't, let the two leaders in score play a two player to settle the issue.

The time that such a three player game takes is of course longer than that for a two player, and not just linearly longer. Again, a player A will think about her own possible ~ 25 moves; but she will consider not just the moves of B, but also those of C. Thus the number of players has an approximately quadratic influence on the average game duration. If we follow the approach from the above section, the time for such a game will be about 45 minutes.

Etiquette demands again to leave the player in peace who is about to move. If you feel like talking, speak to the other idle player. And again: never mention possibilities. Doing so is really bad style, and bad style can disqualify.

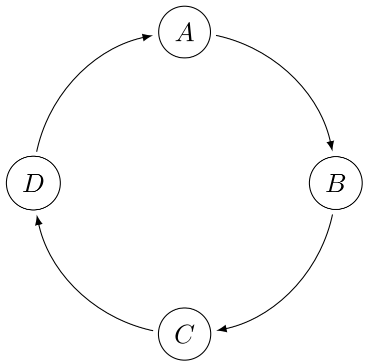

Figure 20: Four players A, B, C and D

19 The 4-player game

THINGS get vastly more complicated when four players play. This can be done everyone against everyone, or cooperative by forming teams. These are two very different modes of playing altogether.

Consider everyone-against-everyone. Again the sitting order is very important, plus dynamics of partisanship. In such an almost chaotic situation there is not much pure playing strategy for a player; politics becomes more important than the quality of action. For proper planning simply too much happens before it is a player's turn again. Thus such games can be quite entertaining and interesting from an interactive point of view, but they are less interesting if you cease talking.

To level their chances players must vary the order in which they play; thus six games $m = (n-1)!$ make a match. It it obvious that such a match would be a lengthy affair—and still be prone of other factors of unfairness. Thus such a setup is only common in social settings, usually with beginners in Pentagame.

It thus makes far more sense, and a far better game, when two players each form a party and play a *cooperative four-player*. This is well the ultimate form (*Königsdisziplin*) of playing Pentagame, and it is certainly the most

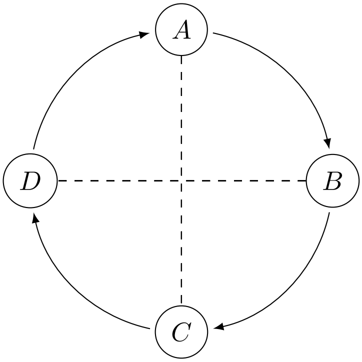

Figure 21: A and C play against B and D

challenging. Most commonly, partners face each other (A teams with C while B teams with D in Figure 21).

When you play alone, you can practically either *swap* or *take.* But when players form teams, a new type of move enters play. A player can now also *cooperate* by swapping strategically with a partner. Because my partner will always move next before I do, cooperation allows for a considerable gain in speed. As a result, in such a setting a party can practically move their first piece out already in the *second* round. This considerable gain in speed makes *cooperate* a very strong move; players will thus have an incentive to make such moves possible, trying to spread their pieces to allow for more.

Again can we attempt to draw the extensive form—the tree form. The first row again is the initial move by player A; the second row the next move of player B, and so forth. Instead of two, here we have three branches per move. Figure (22) illustrates the ramification of a cooperative four-player and shows the main threat: to move out in the second move.

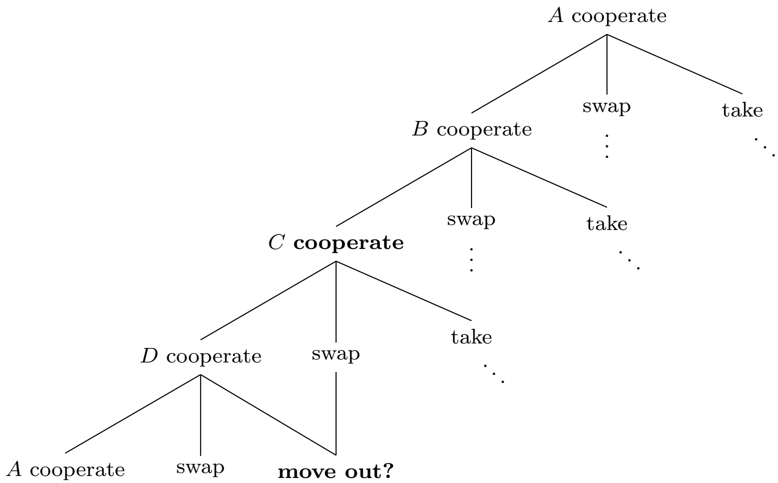

Figure 22: A cooperative four-player game

From this is should be clear that such a cooperative 4-player has a vastly more complex game tree altogether. In such a game, players must take great care to consider both their own cooperative possibilities and those of their opponents. And because the cooperative swap is such a strong move, players should seek to position their pieces to allow for such moves.

Nevertheless, due to the complexity such a game requires some concentration and can hence take quite some time. Once again it is sensible to somehow limit the time for each move. The winning team usually is the one that brings out any 5 of their pieces.

20 Some remarks on strategy

Nodes (corners and crossings) are generally much more attractive than stops on the lines, since from a node you always have four, not just two, ways to go. Consequently, the stops on the lines immediately adjacent to a node are somehow stronger than those further away, since with only one move you can reach a node. And if you plan to block another piece, you will generally want to set the block as close to that piece as you can.

We have already spoken about the three most common types of moves: *swap, take & close* and—in the cooperative four-player game—*cooperate.* In terms of speed, *cooperate* beats *swap* beats *take & close.*

But there are more moves than that. Not uncommon is to move one piece on a free stop on a line or node just to stand in the way of your opponent. Such a move results in a mutual unsolvable blockade. Another very strategic move is to move from a specific place just to make room; this is often a side effect of a take & close move. And finally there is the swap with a piece of the other party, which rarely is a good idea.

In the long run, strategy will prevail. Just like in Chess, it is not advantageous to run forward with one particular piece. A good strategist will position her pieces so that she has more opportunities than her opponent, and so that her own pieces allow for coordinated action, rather than standing alone. It is well worth to have two pieces next to each other, so that one retains the *option* or threat to swap them. It is important that you keep your own neighbourhood; Pentagame is a spacial game, and much can be gained by gaining the upper hand over entire areas.

When you play a cooperative four-player, you should position your pieces to allow *cooperate* moves. Winning within a team—and against a team!—is entirely a different strategic challenge than winning alone.

As you play, you will notice the massive strategic difference between the black blocks and the grey blocks. The latter are far more annoying. Black blocks hinder you, but still grant you to obstruct your opponent. A grey block

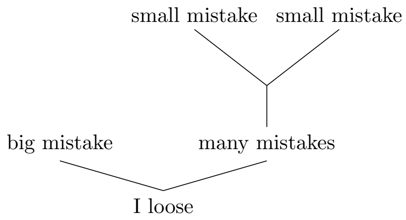

Figure 23: A negative causality tree

simply makes you loose a turn without gaining a block. This may allow your opponent to exercise an important move.

Game theory defines strategy as a rule which allows you to have an answer to any move your opponent may enact. The easiest such rule is *'make the move that improves your position and deteriorates the other player's position'*. The question for strategy thus reduces to the evaluation of positions.

Pentagame is a game with perfect information (that is, without any hidden information), and hence Pentagame can only have one outcome when players play perfectly. Knowing this outcome equals knowing the perfect strategy. That would make Pentagame a solved game. At the time of writing, Pentagame remains unsolved.

So far, we have looked at foresight. Foresight asks: *What can follow if I do this or that?* But strategic analysis can also go backwards using hindsight. Hindsight asks: *What was the reason for the given result?* Have I lost because of a single mistake while generally playing well; or have I lost because of continuously inferior play? What was the decisive moment, what the decisive decision? Which were the crucial paths and nodes? What was the decisive difference in overall strategy?[86]

One can attempt to draw an extensive form tree upside-down and try to find all variants that would have ended in the same result: compare Figure (23). It demonstrates that backwards reasoning is fundamentally different from forward planning. Hindsight simplifies matters dramatically. Psychologically, we often focus more on the negative than on the positive and ask more often

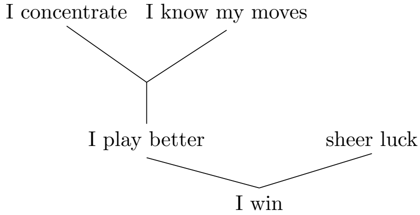

Figure 24: A positive causality tree

for the causes of failure than for the reasons of success. Notice that the reason *for* something is why I have decided to do it, while the causes *of* something is another concept altogether. We see mistakes in hindsight where at the time we actually made the best move we could think of. A good strategist will use foresight and hindsight

> *like the owner of a house who brings out of his storeroom new treasures as well as old.*[87]

‘Mistake’ in games is a problematic term: there is no opposite of ‘mistake’. In theory there will always be the best move, and always acting that one infallibly leads to winning the game. Su Tzu stated the obvious when he wrote that

> *making no mistakes is what establishes the certainty of victory, for it means conquering an enemy that is already defeated.*[88]

Nevertheless, the quality of a move depends on the reactions (and hence quality of moves) of the opponent; a strategy can only be a winning strategy if it wins over the strategy of the opponent. And such strategies may be temporary; the switching from one strategy to another (so called mixed strategy) within a game can be a (meta-)strategy, and so forth. Every player is probably imagining what her opponent would consider the best next move; the technical term here is ‘fictitious play’. It has been shown that such fictitious play also converges to Nash equilibria (under certain constraints).

At any moment a game will *tend* to be won by one party or the other; one of the players will be in the lead (‘$\succ$’), usually she who needs less moves to

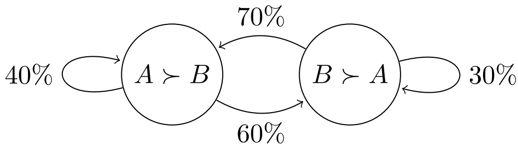

Figure 25: A Markov chain state diagram
(numbers arbitrary)

win if everything else remains as it is, or she who controls more territory. Yet as long as the outcome is not yet crystal clear the game will somehow oscillate between ‘$A \succ B$’ and ‘$B \succ A$’. This can be visualised as a Markov chain; Figure (25) illustrates this, where the numbers on the arrows represent the ‘odds’ of the following state at a time. Each individual move changes not necessarily the probable outcome per se, but the probabilities of the system change state. Of course such a chain can be ‘solved’ towards some ‘odds’ of one party or the other winning; this would in theory equal the rate in which bystanders would bet on these outcomes, given a state of the game.

Note that in our example situation in Figure (25) the game will likely often change from state “$A \succ B$’ to ‘$B \succ A$’ and vice versa. In a relatively balanced game (and thus at the beginning of every game) this is realistic, since having to move is always an advantage. A part of the joy of Pentagame stems from the fact that we often feel in the lead after having made a move—a joy which often only lasts until our opponent has found her answer. Nevertheless, the overall constellation and the ‘odds’ to win will have changed as well.

Put simply, the numbers representing probabilities in our graph could be interpreted as ‘available moves leading in a certain direction’. Considering this we see intuitively that the strategy ‘*gain more good options and diminish the options of the other*’ is at least in a zero-sum game the dominant strategy, since this means influencing these ‘odds’ of winning.

⋆

21 Competitions and tournaments

PENTAGAME never wears out and thus leads itself to repeated play, where players can become quite competitive. We have already discussed above how to score. However, to organise an evening of Pentagame with many participants in a logical way is a little challenging, especially since the game can be played with two, three, four (or even five) players. Let us take a look at games, repeated games, and game durations. For clarity, we first need to agree on some words.

A **game** is just what it is: some n people playing once.

By virtue of the rules, every player has the same amount of moves in a game. Because of this and because there are only two reasonable opening moves, it does not matter much who begins; there is no large first mover advantage.

But the *sitting order* of the n players makes a difference for three or more players, because it is a disadvantage to play after a very strong player, etc. So the first thing to do is to swap seats or change direction. Thus three players play *twice:* clockwise, and widdershins. Four and five players have more possible sitting combinations and would have to play many more times. We call this a **match** and a match consists of games with all possible *orders* of players. The number of games in each match m grows clearly factorially and is $m = (n-1)!$

When you have everyone start once *and* change the sitting order in the above fasion, you play a **round.** The number of games per round r is $r = n!$ which can be a rather large number. But alas—there is no much first-mover advantage, so we don't actually have to do rounds. We can stick with matches.

Multiples of matches (or rounds, for that matter) we call a **competition.** A competition is thus the 'mini-tournament' you typically have on one table on the course of one evening.

If you then also have opponents change from table to table and find new challengers, you enter the realm of **tournaments.**

Table (3) sums up the above—and more than that.

		2-player	3-player	4-player round-robin	4-player 2 vs. 2
Game	games required	1 by definition			
	duration (ca.)	20min	40min	1h 20min	
Match	games required	1	2	6	2
change order	duration (ca.)	20min	1h 20min	8h	2h 40min
Round	games required	2	6	24	8
+everyone to start	duration (ca.)	40min	4h	2d	16h

Table 3: Number of games and duration of play
assuming 20 (semi-)moves per game and $\frac{1}{2}$ minute to think per set of pieces.

THE number of required games for fair play grows factorially with player number. In consequence, matches or rounds with many players per table require many many games. But the duration of each individual game increases with player number as well, and more than exponentially:

Playing time would be linear with player number if both (1) the total amount of moves required to finish and (2) each player's consideration time be independent of player number. This is, however, not realistic. Realistically, a player who is to move will think about *each piece* on the board for some split-seconds; so the consideration time she requires increases *also* linear with player number. That means that the duration of a *game* $T_g(n)$ grows at least quadratic $T_g(n) \in \mathcal{O}(n^{\mathbf{2}})$. We say *at least* because we can safely assume that the amount of moves grows *somehow* also.

Consequently, the duration of a *round* for n players will then be growing (at least) quadratically *and* factorially: $T_r(n) \sim n^{\mathbf{2}} \cdot n!$

As a result, the necessary length of a round of 4 players playing round-robin would be somthing like eight hours, and a competition even take *two days.*

Table (3) reveils some further interesting information. Notice that three players play 2 games in about $1h20min$, while two players would play 3 games in the same time? This allows us to find a 'tweak' which allows for the comparison of two and three player games!

The gist is that we consider points *per time*. We can squarely compare two and three player games if we multiply the points of the three player table by three, and those of the two player table by two.

Table (3) shows also that a four-player round-robin, though fun and playable, is virtually useless in a competitive setup due to time restraints. But the table also lists another setup: the '2 vs. 2' setup, which some consider the most noble form to play Pentagame. This form is very playable, because in this setup four players consitute just two parties, so a match (of two games, one clockwise, one anti-clockwise) can be done in maybe $2h40min$. In that time span a three-player table will play 2 matches (4 games) and a two-player table will play 8 matches (8 games), so at least in theory a four-player *could* be integrated in an overall tournament. To sum up:

- Competitions or tournaments with games with more than 3 players per table are lengthy.
- Two and three player games can be played in the same tournament relatively easily.

TOURNAMENTS can be organised in different way; there are three chief tournament systems. One is the round-robin or 'league' system, where everyone plays everyone else once (or twice). It has the disadvantage to be lengthy, plus you need very committed players. To run such a system with both two and three player tables is complicated. This is why we prefer the second system, which is the Swiss system tournament, prominently in use in Chess and many other 'non-sports' game tornaments. (The third system would be the knock out tournanment, with we hereby omit.)

In the Swiss system tournament the winner of every game plays next against the next highest-ranking player against whom she has not yet played. The beauty of this is that everyone generally encounters players of similar ability, and you need much less games in total than in a round-robin.[89] Usually you begin a Swiss system tournament with some prior ranking, typically the result of the previous tournament, or a pre-round.

To not give an incentive for parties to agree on 3:3 draws, make each game a *zero-sum game*. A simple rule to do is called the 'dropped score'-method; it states 'everyone gets as many points as she has moved out pieces, plus the amount of pieces the opponent has *failed* to move out'. Then the possible results in a two-player become 3:3, 4:2, 5:1 or 6:0, always summing up to 6 points in total, making this a zero-sum game as intended.

To sum up, a Pentagame tournament can be organised as follow:

- Organise your competition in turns of ca. 40*min*. Let the three player tables play two games per turn, one clockwise and one anti-clockwise. In that period two-player tables should finish three games.
- Make the score of each game a zero-zum using the 'dropped score' method.
- A the end of the turn, multiply the scores of the three player games by 3 and those of the two players by 2.
- Rank players per score. Working top-down, pair each player with the next player down the list against whom she has not yet played.
- Make sure to record ranking, score and number of turns.
- You can compare average scores per game and match.

And of course, you should insist on a pleasurable setting; provide sufficient light to see the colours properly, provisions for player's physical needs, a promotion for the winner and accolades for all parties.

		first out				
		white	blue	red	yellow	green
second out	white	A	B	C	D	E
	blue	F	G	H	I/J	K
	red	L	M	N	O	P
	yellow	Q	R	S	T	U
	green	V	W	X	Y	Z

Table 4: Interpreting colours as letters

22 A learning curve

Just how 'steep' is the 'learning curve' in Pentagame?—We observe that to grasp the rules is a very simple exercise for most people, and that players are in full comprehension after their introductory game. We observe further that initiates' strength grows quickly. The more experienced and hence stronger player seems to almost always win with an edge. But after many hundred of games, we are still not done with Pentagame.

Players deserve praise: those winning because they are the better players; but more so the weaker player, because the weaker player always plays at the hight of her ability, while the winner usually has easy play.

The outcome of every game are the scores of the players, but also the specific combination of pieces they have moved out. You can assign letters to the colour combination of the winning pieces using the scheme of Table (4) and write these down as well. If you play infinitely often, you will eventually create all literature ever written.[90] Similarly, if you play a finite amount of games and record the letters thus produced, when thinking long enough about it you will always find therein some meaning.

23 Myths

ACKNOWLEDGING that any of the above could be fake news, here we collect some alternative stories of the origin of Pentagame. It should be clear from what was said above that this is pure fiction. But fiction sometimes contains more truth than history, said ARISTOTLE.

I.

ONCE upon a time in the *Walpurgisnacht*, members of the Five Guilds of the Elements travel to the mighty Blocksberg in Germany's Harz mountains, and they ascend its slopes to gather on its peak. The paths are narrow, so there is no overtaking; and here and there opponents charms get in the way. But hurry they must, since once three of a guild have made it to the peak, the gates are closed, and the ceremony starts.

When the mountain became inaccessible due to the iron curtain separation of Germany, this ancient ritual was only preserved in the form of Pentagame, which was kept secret until finally released to the greater public by some magister ludi *Joseph Knecht*, who probably also is just a legend. It comes as a gift to mankind in the attempt of the illuminati to teach us that productive interaction can happen both without balls and without beating each other up.

There are people who believe that every specific sequence of moves works like a key to a door. Once we have found the perfect match, Barbarossa will rise again and re-erect the Empire. Others believe it is some sort of mirror, and from the movements of the pieces anything that happens in the world can be deduced, if we only knew how.

Others believe that the Five Guilds of the Elements still exist as secret societies; rumours have it that there are at least 25 different ranks, all of which are so secret that even their members are unaware of their cosmic relevance.

⋆

II.

Once upon another time a space-station was abandoned on planet Earth. Aeons went by, and time covered the station with sand and weed deep within the subsoil. Dinosaurs rampaged, mammals evolved, ice-ages went by, hunter-gatherers wandered about, then settlers arrived. Parts of the ship visible overground were hidden by the still-functioning stealth system, as culture evolved, palaces and churches got erected, empires rose and fell, and a town started spreading upon the remnants of the station. In 1969 the stealth system failed, revealing the towering antenna. In the 1990s, a group of nerds started unearthing parts of the space ship, baptising it `c-base`.

Among the alien artefacts discovered was an object that resembled a pentagonal board game, but which may actually be an interface to the board computer, in particular the astro-navigational system. The setting and moving of pieces on this board causes strange distortions of space-time; as a result, copies of Pentagame were sent backwards in time and played in antiquity, of which we have some evidence. The finding of the Latin rules in 2018 furthers the likelihood of this story.

It is of course also possible that Pentagame was developed here and now and by the distortions mentioned sent into the future, from where copies crashed in the past, which are the artefacts we have found.[91]

There are people who believe that every specific sequence of moves works like a key to a door. Once we have found the perfect match, we will unlock the propulsion system of the space station and finally move into orbit. Others believe it is some sort of screen, and that from the movements of the pieces anything that happens in the universe can be deduced, if we only knew how.

Time will tell.

⋆

Annex

Regulae officialis latinae

Tabula pentagrammiforma, quattuor cohortes quinorum peditum coloratorum, quina impedimenta nigra et cana Pentaludum componunt.

Lusor quisque agit pedites quinos, quibus est eadem forma (aut stella aut luna &c.), sed sunt diversi colores. Sic tua cohors peditem caeruleum, peditem rubrum continet &c.

In quinque nodis positis in circulo, qui circumvenit pentagramma, ponite pedites eiusdem coloris: pedites albos in nodo albo, caeruleos in nodo caeruleo &c.

Praeter illa decem loca quae nodos formant, existunt alter octoginta aedicula in viis, ubi pedites in ludo etiam possunt collocari.

Sunt etiam impedimenta nigri et impedimenta cana.

Impedimenta nigra ponite in nodis centri. In centro retinete impedimenta cana.

Destinatio per unum quemque peditem, qui occupat nodum externi circuli, est nodus eiusdem coloris positus in interno pentagrammatis. Albo est eundum ad album, rubro ad rubrum &c.

Lusor victor qui primus duxit tres pedites ad nodum aequi coloris.

I. Duc peditum tuorum unum in quamvis directionem sequens viam circuli aut vias stellae, quoad tibi paret et per regulas potes occupare illud aediculum.

II. In quodlibet aediculo libero, ubi duae lineae conveniunt, tibi licet de via recta declinare sine retentione.

III. At numquam tibi licet transcendere nec impedimenta nec pedites alios.

IV. Potes vero ingredi in aediculum occupatum:

a. Se ibi stat impedimentum nigrum, id pone in aediculo libero ad libitum.

b. Se ibi iam stat pedes alius, pedites positiones suas inter se mutare debent. Sed nota regulam V.

c. Se ibi sunt plures pedites (non potest fieri, nisi initio ludi), elige unum mutandum.

d. Ita potes etiam mutare positiones duorum peditum tuorum.

V. Eundem motum non licet iterum: bis idem ne cana.

VI. Pedes progressus ad finem ludo abit. Pone eum in centrum. Prehende per illo impedimentum canum et id colloca in aediculo, quod tibi paret.

VII. Quando captas tale impedimentum canum, remove a tabula.

VIII. Lusor ille, qui primus tres pedites ad finem duxit, ludo victor.

Notation of moves

The notation of moves in Pentagame is complex, since coordinates of stops in its pentagonal architecture are not easy to express. But any stop can be described relatively with these parameters: (1) The node of origin n; (2) the node towards which to go d; (3) the amount of stops to go s. Thus any stop can be called in the form (n, d, s). Consequently, each stop can be addressed in two relative ways.

Let us note the outer nodes with their colour and an underline $\underline{w}, \underline{b}, \underline{r}, \underline{y}, \underline{g}$ and the inner nodes simply w, b, r, y, g. Then the stop $(\underline{w}, b, 3)$ is the stop on the line leading from the white corner towards the blue node (necessarily a crossing) number three. For convenience, drop s when a node is reached, so e.g. $(\underline{w}r7) \equiv (\underline{w}r)$. Let us use $\times$ as symbol for 'take' followed by a placement and s for a swap, followed by ! if the player swaps a *foreign* piece, and z for leaving the board followed by the placement of the grey block.

A move description consists of the origin of the piece (n_0, d_0, s_0) followed by $\times, s$ or $s!$ and the target (n_1, d_1, s_1) plus in case of $\times$ a colon : as separator and the new position for the block (n_2, d_2, s_2). This seems more complicated than it is in most cases, since many moves do not require all that information, so their notation is straightforward.

Take the following opening of a closed game as an example: player A begins with a take & close, player B answers with a swap:

$(\underline{w} \times r : \overline{g2w})$ read 'with the piece from the white corner she takes the block on red and places it on stop 2 between green and white on the circle', or shorter: 'white takes red puts stop 2 between green and white on circle.'

$(\overline{gsw})$ read 'with the piece from green on the circle she swaps the piece on white', or shorter: 'green swaps white on circle'.

The beauty of this notation and the proof of its adaptability lies in the fact that one can deduce the coordinates of a stop from its name with a function, and that there is no difference between calling a move and calling a stop.

Acknowledgements

The development, or re-development, or discovery, of Pentagame has been a lengthy process. The analysis of the game is an ongoing endeavour. Thanks go to everyone who has contributed kindness, ideas, insights and knowledge.

Special thanks go to my father, my family and my friends, notably to Gerhard Suchanek, Tim Grünewald, Familie Spiegelhalder, Billy Smith, John Martineau, Dr. Robert Chapman, Dr. Jörg Grimm, Dr. Ulrich Schädler, and of course to all players world wide.

A propos contribution: Since this is the first edition of this book, I shall be grateful for further comments and hints. The vastness of the topic makes it impossible to not make mistakes, and in many areas research has just begun.

Berlin, October 2018

J.S.

Notes

[1]This is so that the total number of pieces on the board does not diminish.
[2]Roger Penrose has discovered the Penrose-tiling, which makes use of the golden section and tiles the plain surface in a non-periodic way.
[3]Thanks to my brother Tim Grünewald né Suchanek upon his visiting me in Leamington Spa.
[4]H.J.R. Murray: *A History of Chess* (1913)
[5]An idea of course inspired by Malefiz.
[6]Thanks to Dirk Suchanek for the suggestion.
[7]Thanks to John Martineau.
[8]Iulius Pollux (2. century AD), *Onomastikon* 9,97; Sophokles (497–406 BC.), *Fragment 429;* this is a fragment from the lost play 'Nauplios'.
[9]E.g. Celio Calcagnini (1479–1541): *Opera Aliquot*
[10]L. Becq de Fouquières: *Les jeux des anciens* (1869), probably the first work on (ancient) board games not written in Latin.
[11]Lamer, Hans: *Lusoria tabula.* In: Pauly's Realencyclopaedie der classischen Altertumswissenschaft (RE). Austin, Robert G.: *Greek Board Games* (1940): *"[...] it is clear from both Pollux and Eustathius that each player had five lines, but do they mean five vertical and five horizontal lines, or two sets of five lines running in the same direction? If the latter, why the name πεντεγραμμα ? Finally, what was the 'sacred line'? Did it run between two sets of five lines (thus making 11 altogether), or was it the middle one of each set, or can we infer from Eustathius that the board did in fact have five lines each way, and that the 'sacred line' was the middle one in each direction? [...] No answer seems possible to any of these problems."*
[12]Murray, Harold: *A History of Board-games Other Than Chess* (1952)
[13]Homer (ca. 850 BC), *Odyssey,* I,107.
[14]Plato (428–348 BC), *Phaidros, 274D.*
[15]Plato: *Politeia,* 333B, 374D, 487B; cf. *Nomoi* 7:820d: τε πεττεία καὶ ταῦτα ἀλλήλων τὰ μαθήματα οὐ πάμπολυ κεχωρίσθαι.
[16]Heraclitus (520-460), B:52
[17]Polybius (200-120): *Historiae.* I, 84; Trad.: Thompson
[18]Later it was sometimes called 'Nerdiludum' in Latin, from Persian 'nerd' for dice.
[19]Admittedly, in antique times, many games were played with Astragaloi; these generate four possible outcomes, rather than six.
[20]It is unlikely that all board games were *petteia* because *petteia* is sometimes contrasted to other board games such as *poleis* (a surrounding game as is Go) or *latrunculi* (which seem to have been similar to Ludo).
[21]In any other case it is not clear why the board could not have been extended to seven, eight, twelve or any number of lines. Schädler shows that such games have existed in good number. Schädler, ibd.
[22]Each must have had it own origin since otherwise, the game could as well have been played with three, or twelve, or any number of pieces. Note that in Ludo or Backgammon the number of pieces per player is random.
[23]This was called 'holy line'. To 'move a piece from the holy line' has been proverbial in ancient times, though the meaning of this phrase is relatively obscure; cf. Plato: *Nomoi* 5.739a1 etc. There is also evidence that the 'holy line' provided some sort of safe place for pieces; maybe the game was played inside-out (instead of *outside* → *in* like Pentagame). The line must clearly have been an addition to the five constituent lines, and of a different

quality. Note that throughout history, the perfect circle has long been associated with holiness.

[24]We cannot assume that all the iconic depictions of players like the one by Exekias shown in figure (5) depict the same game, be it a board or dice game; the image may just be generic. Clearly the ancients played more than one game, and rules can vary, just as it *is* possible to play a game of dice on Pentagame.

[25]The travelling of the pieces was compared to the motion or 'dance' of stars (cf. Kidd, op. cit.). An ancient Greek calendar was based on Venus cycles, which draw in the Zodiac a pentagram. A comparison to horoscopes would suggest the game was somehow circular (page 32).

[26]Polybius (200-120): *Historiae.* I, 84 Cf. Aristotle: *Politeia* 1253a.

[27]Kidd, op. cit. These could have been 'unconnected' pieces of a player (as in Backgammon), or special stones which are not 'tied' to any particular player, as the blocks in Pentagame.

[28]Stephen Kidd, op. cit.

[29]With the possible exception of some dubious inscriptions on the roof slabs on the temple of Kurna in Egypt. There seem to be a number of game boards, among them Nine Men's Morris, but also some general glyphs. There are pentagrams which have been interpreted as board games (Murray, op. cit.).

[30]I owe this information to the kind correspondence with Dr. Ulrich Schädler.

[31]Schädler, Ulrich: *Pente grammai – the ancient Greek board game Five Lines* (2009)

[32]Recently, Stephen Kidd has improved Schädler's attempt, adding a strategic element and providing another good overview of the ancient literature. Kidd, Stephen: *Pente Grammai and the 'Holy Line'* (2017)

[33]Herz-Fischer, Roger: *A Mathematical History of the Golden Number* (1998)

[34]For a definition of the identifiability of a causal effect cf. Pearl, Judea: *Causality* (2009), p. 77.

[35] Prophyry: op. cit.

[36]Cassiodor: *Institutiones* II, 4.2.

[37]Cf. Plato, *Nomoi*, 7:820.

[38]Lore goes that Hippasos met death by drowning in conncetion to this discovery. von Fritz, Kurt: *The discovery of incommensurability by Hippasos of Metapontum;* Annals of Mathematics 46 (1945)

[39]Pacioli, Luca (ca. 1447 – 1517): *De Divina Proportione,* 1509, with Leonardo da Vinci (1452 – 1519); cf. Olson, Scott: *The Golden Section. Nature's Greatest Secret.*

[40]The Fibonacci sequence is defined as $F_n = F_{n-1} + F_{n-2}$

[41]You can construct one logarithmic spiral based on a golden rectangle and another on a golden triangle as found in the pentagram; these two are not identical, albeit both *are* related to the golden section. Contrary to popular believe, most logarhitmic spirals in nature (galaxies, nautilus...) in fact have nothing to do with the golden section.

[42]Iamblichos, op. cit. He speaks of a 'symbol'; this could also have been a sentence of some kind.

[43]Phidias, ca. 500–430 BC, architect of the Parthenon.

[44]Because you will need the factor 5 as $60 = 2 \cdot 2 \cdot 3 \cdot 5$.

[45]Martineau, John: *A Little Book on Coincidence in the Solar System* (2001). A possible explanation is the Kolmogorow-Arnold-Moser theorem.

[46]Some say this dates back to Babylonian times, but the godess Ishtar is often depicted with a four or eight pointed star (again, no pentagrams survive). This can however possibly be because of the four-year Venuvian calendar.

[47] 224.710 days is the *siderian* Venus period; the *synodic* Venus period (time between conjunctions etc.) is approximately 583,92 days, and $584 \cdot 5 = 2920 = 365 \cdot 8$.
[48] The Mayan calendar is another prominent Venus based calendar.
[49] There is nothing major, only some coins and amulets.
[50] The Synod of Elvira (ca. 300 AD) banned board games (Can. LXXVIIII).
[51] MacMullen, Ramsey: *Christianizing the Roman Empire A.D. 100-400* (1984)
[52] Still today in India a typical Puja-thali holds symbols of five elements.
[53] On associative thinking see Claude Lévi-Strauss: *La pensée sauvage* (1962); also (though clearly outdated) James Gregor Frazer: *The Golden Bough* (1890).
[54] The idea that unity (be it Aum, cosmos, God) unfolds into duality, trinity and so forth is called 'emanation' and became a large topic in middle and late neo-platonism, most prominently in Proclus. It has been rebuked staunchly by catholoic dogma.
[55] Gestalt psychology etc.
[56] E.g. *Sir Gavain and the Green Knight* (late 14th century).
[57] Marktkirche in Hannover (1388) is an early example.
[58] René Decartes 1637 *'Discours de la methode'*, explicitly inspired by Euclid
[59] Mathematics was the general term for the Quadrivium encompassing Arithmetics, Geometry, Music and Astronomy—all sciences related to measure.
[60] Giordano Bruno: *Magia* (1590): 'Quinto cum his adduntur verba, cantus, rationes numerorum et temporum, imagines, figurae, sigilla, characteres seu litterae; et haec etiam est magia media inter naturalem et extranaturalem vel supra, quae proprie magia mathemathica inscriberetur, et nomine occultae philosophiae magis congrue inscriberetur.'
[61] Goethe, Johann Wolfgang (1749–1832) : *Faust. Der Tragödie erster Teil. Szene 8.*
[62] Éliphas Lévi (1810–1875). Giordano Bruno shows a pentagram with a man looking to the front and opposes it to a pentagram with a man looking backwards, thus 'flipping' the pentagram rather than rotating it.
[63] James Webb: *The flight from reason.* (1970)
[64] Marcus Aurelius (121–180 AD) though seems to have had the same conception: *Meditationes,* 2:17 and 4:46.
[65] Porphyry: op. cit.
[66] Other lengths are conceivable, but, as we will see, $l \approx \varphi k$ so that $c \approx j$.
[67] Gauß also proved that a 17-sided polygon *can* be drawn; such a polygon adorns his tombstone.
[68] Literally, 'competition' means 'to strive together'; thus here too a certain rule guaranteeing equality in the competition is necessary.
[69] The actual dice were known as *tesserae.*
[70] Nicolaus Cusanus (1401-1464), *Dialogus de ludo globi liber primus*: Nulla bestia talem habet cogitationem inveniendi ludum novum. 'No animal has the intellect to invent a new game.'
[71] Huizinga, Johan: *Homo ludens, proeve eener bepaling van het spel-element der cultuur.* (1938)
[72] Caillois, Roger: *Les Jeux et les Hommes.* (1952)
[73] Cf. Kurke, Leslie: Ancient Greek Board Games and How to Play Them. (1999)
[74] Leibnitz: *Misc. Berolinienseа: Annotatio de quisquam ludibus* (1710).
[75] Thus, the German term *Spiel* grasps the concept perfectly.
[76] von Neumann, John and Oscar Morgenstern: *Theory of Games and Economic Behavior.* (1944)
[77] Nash, John: *Non-cooperative games.* (1950)
[78] Brams, Steven: *Superior Beings. If They Exist, How Would We Know?* (1983)
[79] E.g. Tirole, Jean and Drew Fudenberg: *Game Theory.* (1991)

[80]E.g. John Maynard Smith: *Evolution and the Theory of Games.* (1982)

[81]McGonigal, Kelly: *Reality is Broken. Why Games can Make us Better and How They Can Change the World.* (2011)

[82]This depiction is called 'extensive form' of a game in Game Theory.

[83]Vorhand and hinterhand are terms from the German three player card game Skat, one of the few card games playable with three players. In game theory, the terms usually are also applied vorhand meaning 'the player initiating an action' and hinterhand 'the player to react' (per round).

[84]The technical term is *hedonic games* where player have preferences with whom to form coalitions.

[85]*Not even Hercules can win against two;* Suda

[86]For an introduction to causality analysis read Judea Pearl: *The Book of Why* (2018).

[87]*Matthew 13:52.*

[88]Su Tzu: *The Art of War* (ca. 510BC)

[89]If you have much time or few players, the Swiss system becomes a league system.

[90]This is called the 'infinite monkey theorem', first mentioned by Émile Borel (1913).—If you play just a few times, you will of course create gibberish. Nevertheless, it can be interesting to seek patterns in the colours winning.

[91]If you find this too confusing, please join the Campaign for Real Time. Douglas Adams: *The Hitchhiker's Guide Through the Galaxy* (1979–1992).

[92]This is a completely random footnote that has no connection to any of the above. It is an 'Easter egg'. Will you find the others?

Index

Abalone, 14
Abracadabra, 12
acousmaticoi, 28, 39
acoustics, 28
agon, 46
alea, 46
Anitkythera, 26
anonymous, 36
Antichrist, 39
architecture, 30
Aristotle, 66
Asia, 38
astrologogy, 32
astronomy, 32
Austin, 21

Böhme, Jakob, 35
Backgammon, 14, 17, 23
Bayesians, 46
bible, 39, 40
black blocks, 9, 17, 57
blocks, 9
Blocksberg, 66
board, 8, 41
board games, 46
Bogumilism, 33
Bologna, 35
Bruno, 35

c-base, 67
Caillois, 46
calendar, 32
card games, 46
cards, 17
chance, 46
Chess, 14, 17, 18, 21, 51, 57
Chinese Checkers, 14
Christ, 35
Christianity, 33
Church, 33, 46
circensic games, 46
classic games, 14
closed games, 51
colour scheme, 16
combination *of c.*, 65
complexity theory, 62
computer games, 46
cooperate, 55
cooperative play, 54
corners, 57
cosmos, 28
Creator, 39
crossings, 57

damnatio memoriae, 33
Dee, John, 35
destinations, 9
devil, 39, 40
diabolo, 40
dice, 16, 24
disciple, 30
Draughts, 14
dropped score rule, 64
duodecim scripta, 23

Easter egg, 76
edge cases, 12
esoteric, 28, 39
Euclid, 22
Euclids, 35
Euler, 45
Eusathius, 23
evil, 39
Exekias, 20
extensive form, 49

Fall of Man, 40
fauna and flora, 30
Faust, 36
Feng shui, 37
Fermat, 46
Fibonacci sequence, 30
fictitious play, 59
fine arts, 30
fine tuning, 13
five players, 13
Five Virtues, 35
Five Wounds, 35
foresight, 58
four players, 12, 54
freemasonry, 35
Frequentists, 46

game, 61
Game theory, 46, 47
game tree, 49
games, number of, 51
gamification, 47
Genesis, 40

Giordano, 35
Gipf, 14
Glastonbury, 17
Gnosis, 33
Go, 14, 18, 51
God, 40
Goethe, J.W., 36
golden proportion, 28
golden section, 27, 28
golden spiral, 30
grey blocks, 9, 18, 57
grimoire, 34

Hannibal, 22
harmony, 28
Heraclitus, 21
heremeutics, 35
heresy, 33
Hermes, 21
Hex, 14
Hildegard, 34
hindsight, 58
hinterhand, 52
Hippasos, 28
Homer, 21
horoscope, 32
Huizinga, 22, 46
Hygeia, 35

Iamblichos, 30, 33
iconoclasm, 33
idealism, 36
ignorance, 40
illuminati, 66
incommensurability, 28
incommensurable, 30
inside out, 13
irrational numbers, 28
isomorphy, 13
Iulius Pollux, 20

Jesus, 35, 39

Königsdisziplin, 54
Kabbalah, 35
kalokagathia, 28
Katharers, 33
Kircher, Athanasius, 35
Knecht, Joseph, 66
Ko, 18

Lamer, 21
Laplace, 46
Latin, 23
league system, 63
learning curve, 65
Leibniz, 47
letters, 65
Liber iuratus, 34
Lucian, 30
Lucifer, 39
Ludo, 14

Mancala, 14
Manichaeism, 33
Markov chain, 60
materialism, 36
mathematicoi, 28, 39
mathematics, 28
measure, 28
Milky Way, 51
minutes, 32
mistakes, 59
mixed strategy, 59
Morning Star, 32, 39
multiplayer, 12
Murray, 21
myths, 39

Nash equilibrium, 52, 59
Nash, John, 47
neo-paganism, 36
von Neumann, 47
New Testament, 39
nightingales, 46
Nodes, 57
notation, 71

objective, 11
occultism, 36
Olympic cycle, 32
online games, 47
Onomasticon, 20
open games, 50
opening moves, 49
options, 50
Othello, 14

Pachisi, 14
Pacioli, L., 35
paidson, 46
Palamedes, 20, 21
Paracelsus, 34
partisanship, 54
Pascale, 46
pentagram, 27, 30, 32, 37
perfect information, 58
permutations, 49
Petteia, 23, 44
petteia, 21
Phidias, 31
philosopher, 28
Plato, 21
Polybius, 22
Porphyry, 33
Porphyry's, 35
positions, 51, 58
pre-round, 63
proportions, 28
prototypes, 15
Pythagoras, 28, 30, 35, 38
Pythagoreans, 30, 33

ranking, 64
rational numbers, 28
reasons vs. causes, 59
relativism, 40
Renaissance, 17
Roman Empire, 23, 46
round-robin, 63
ruler and compass, 28, 32, 44
rules *(explained)*, 9
rules *(official)*, 70

sacred, 39
Satan, 39, 40
scenic games, 46
Schädler, 25
science, 32
scoring, 11, 63
seconds, 32
sets *(of pieces)*, 9
Settlers, 14
setup, 9
Sigilum Dei, 35
sizes of stops, 41
Social choice theory, 47
Socrates, 21
solved games, 58
Sophocles, 20
space-time, 67
stalemates, 18
Steiner, 36
stops, *types of*, 9
strategy, 57, 58
Su Tsu, 59
subsoil, 67
superstition, 32, 40
swap, 49, 50
Swiss system, 63

take & close, 49, 51
talking, 13
Tavli, 23
TCM, 37
teams, 54
Thot, 21
three players, 12
time limits, 13
tournament systems, 63
Trismegistos, 21
truth, 39, 40
turns, 64

universe, 51

Vatican, 35
Venus, 32, 39
vorhand, 52

Wicca, 36
winner's curse, 17, 49
Wu Xing, 37

xing, 37

zero-sum game, 64
zodiac, 32
zugzwang, 52

List of Figures

1 Set-up for two players 8
2 Setup for four players 11
3 Author's first study (1996) 15
4 An early Pentagame prototype 16
5 Achilles & Aiax at play. Exekias (ca. 540 bc) 20
6 A typical fragment of ancient literature 22
7 Why is there no trace? 25
8 The Fibonacci spiral 29
9 Pentagon construction 29
10 Golden section 31
11 Sigilum Dei 34
12 Human in a pentagram 36
13 Wu Xing 37
14 Corner condition 42
15 The board today 44
16 Two players A and B 48
17 The branch of 'open games' 50
18 The branch of 'closed games' 51
19 Three players A, B and C 52
20 Four players A, B, C and D 54
21 A and C play against B and D 55
22 A cooperative four-player game 56
23 A negative causality tree 58
24 A positive causality tree 59
25 A Markov chain state diagram 60

List of Tables

1 Geometric boards of strategy games 14
2 Colours, phases, seasons, planets (trad. East Asia) 38
3 Number of games and duration of play 62
4 Interpreting colours as letters 65

Simple rules create a complex game. This book tells you all about Pentagame you could possibly want to know—and perhaps more.

‘Pentagamer’s vade mecum’

‘A Game Changer’

‘A Hell of a Game’

Pentagame is just an abstract board game—and at the same time, it is so much more. The author explains not only the game, but how he came to develop it, and relays many highly interesting discoveries from his intellectual journey through places as distinct as ancient literature, astronomy, geometry and game theory—to name a few. While such topics may at first sight seem just remotely connected, they are indeed shown interwoven, just like the lines in Pentagame. The result is a captivating read.

Pre-print, rendered 2018-10-27