GATE: EE - 31.2021

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Question: The causal signal with Z transform $z^2(z-a)^{-2}$ is ($u(n)$ is unit step signal)

1) $a^{2n}u(n)$
2) $(n+1)a^n u(n)$
3) $n^{-1}a^n u(n)$
4) $n^2 a^n u(n)$

(GATE EE 2021)

Solution:

Z-transform of a causal signal is,

$$X(z) = z^2(z-a)^{-2} = \frac{1}{(1-az^{-1})^2}; |z| > |a| \tag{1}$$

The Z transform pair for $a^n u(n)$ signal is given by :

$$a^n u(n) \longleftrightarrow \frac{1}{1-az^{-1}} \tag{2}$$

Using differentiation in z-domain property:

$$na^n u(n) \longleftrightarrow -z\frac{d}{dz}\left(\frac{1}{1-az^{-1}}\right) \tag{3}$$

$$\implies na^n u(n) \longleftrightarrow \frac{az^{-1}}{(1-az^{-1})^2} \tag{4}$$

Using time-shifting property:

$$(n+1)a^{n+1}u(n+1) \longleftrightarrow \frac{az^{-1}}{(1-az^{-1})^2}z \tag{5}$$

$$(n+1)a^n u(n+1) \longleftrightarrow \frac{1}{(1-az^{-1})^2} \tag{6}$$

From (1) and (6), Inverse Z transform is :

$$x(n) = (n+1)a^n u(n+1) \tag{7}$$

Sequence $u(n+1)$ exist for$-1 \le n < \infty$, but the factor $(n+1)$ is zero for $n=-1$, so $x(n)$ may be expressed as a causal sequence.

$$x(n) = (n+1)a^n u(n) \tag{8}$$

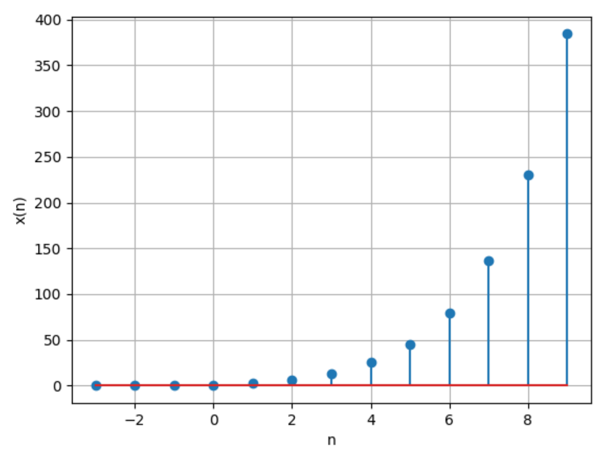

Fig. 1