GATE: IN - 41.2022

EE23BTECH11013 - Avyaaz*

Question: A proportional-integral-derivative (PID) controller is employed to stably control a plant with transfer function

$$P(s) = \frac{1}{(s+1)(s+2)} \tag{1}$$

Now, the proportional gain is increased by a factor of 2, the integral gain is increased by a factor of 3, and the derivative gain is left unchanged. Given that the closed-loop system continues to remain stable with the new gains, the steady-state error in tracking a ramp reference signal (GATE IN 2022)

Solution:

Parameter	Description
K_P	Proportional Gain
K_I	Integral Gain
K_D	Derivative Gain
$r(t)$	Reference Input
$G_c(t)$	Controller Output
$L(t)$	Plant Output
$e(t)$	Error Input

TABLE 1

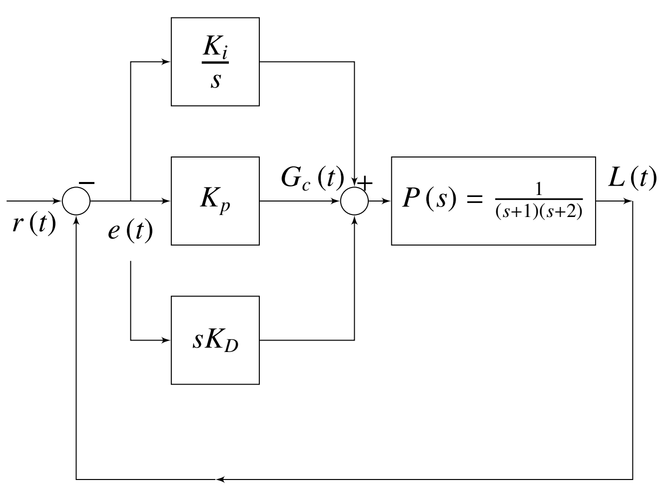

Fig. 1: Block Diagram of System

The transfer function of PID controller,

$$G_c(s) = K_P + \frac{K_I}{s} + sK_D \tag{2}$$

$$= \frac{s^2K_D + sK_P + K_I}{s} \tag{3}$$

Overall loop-transfer function,

$$L(s) = G_c(s) \cdot P(s) \tag{4}$$

$$L(s) = \frac{s^2K_D + sK_P + K_I}{s(s+1)(s+2)} \tag{5}$$

Steady-state error due to ramp signal,

$$e_{ss} = \frac{1}{K_v} \tag{6}$$

where,

$$K_v = \lim_{s\to 0} sL(s) \tag{7}$$

$$K_v = \frac{K_I}{2} \tag{8}$$

$$e_{ss} = \frac{2}{K_I} \tag{9}$$

Now, the proportional gain is increased by a factor of 2, the integral gain is increased by a factor of 3, and the derivative gain is left unchanged.

$$K_P' = 2K_P, K_I' = 3K_I \text{ and } K_D' = K_D \tag{10}$$

$$K_v' = \lim_{s\to 0} sL'(s) \tag{11}$$

$$K_v' = \frac{3K_I}{2} \tag{12}$$

$$e_{ss}' = \frac{1}{K_V'} = \frac{2}{3K_I} \tag{13}$$