NCERT 12.10 5Q

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Question: In Young's double-slit experiment using monochromatic light of wavelength λ, the intensity of light at a point on the screen where path difference is λ, is K units. What is the intensity of light at a point where path difference is $\lambda/3$?

Solution:

Parameter	Description	Value
$y_i(t)$	Equation of light from $S_{i\text{th}}$	$A\sin(\omega t - kx_i)$
I	Intensity of light at $\Delta x = \lambda$	K
k	Wave number	$\frac{2\pi}{\lambda}$
I	Intensity of wave	kA^2
$\Delta x = x_1 - x_2$	Path difference	λ
		$\frac{\lambda}{3}$

TABLE 1
Parameters

In YDSE, light waves coming from the source S fall on both S_1 and S_2 slits which behave like coherent sources since the light waves are coming from the same source implying that their frequency, wavelength, wave number and amplitude will be the same.

The equation of light wave coming out from the slit S_1 is:

$$y_1(t) = A\sin(\omega t - kx_1) \tag{1}$$

The equation of light wave coming out from the slit S_2 is:

$$y_2(t) = A\sin(\omega t - kx_2) \tag{2}$$

The superposition of the two waves is the sum of two individual waves:

From (1)&(2) $\implies$

$$y(t) = y_1(t) + y_2(t) \tag{3}$$

$$y(t) = A\sin(\omega t - kx_1) + A\sin(\omega t - kx_2)$$

$$y(t) = 2A\cos\left(\frac{k\Delta x}{2}\right)\sin\left(\omega t - \frac{k(x_1+x_2)}{2}\right) \tag{4}$$

From table (1) & equation(4) $\implies$

$$\therefore I = 4I_o\cos^2\left(\frac{k\Delta x}{2}\right) \tag{5}$$

When $\Delta x = \lambda$:
From table(1)& equation(5) $\implies$

$$K = 4I_1\cos^2\left(\frac{2\pi}{2}\right) \tag{6}$$

$$\therefore I_1 = \frac{K}{4} \tag{7}$$

When $\Delta x = \frac{\lambda}{3}$:
From table (1)&(5) $\implies$

$$\text{Resultant intensity} : I_r = 4I_1\cos^2\left(\frac{2\pi}{3}\right)$$

$$I_r = I_1 \tag{8}$$

From equation (7) $\implies$

$$\therefore I_r = I_1 = \frac{K}{4} \tag{9}$$

Hence, the Intensity of light at a point where path difference is $\frac{\lambda}{3}$ is $\frac{K}{4}$ units.