GATE: IN - 24.2023

EE23BTECH11013 - Avyaaz*

Question: The number of zeroes of the polynomial $P(s) = s^3 + 2s^2 + 5s + 80$ in the right side of the plane? (GATE IN 2023)

Solution:

General n^{th}-order characteristic polynomial :

$$a_0 s^n + a_1 s^{n-1} ...... + a_{n-1} s^1 + a_n s^0 \tag{1}$$

s^n	a_0	a_2	a_4	...
s^{n-1}	a_1	a_3	a_5	...
s^{n-2}	$b_1 = \frac{a_1 a_2 - a_3 a_0}{a_1}$	$b_2 = \frac{a_1 a_4 - a_5 a_0}{a_1}$	...	..
s^{n-3}	$c_1 = \frac{b_1 a_3 - b_2 a_1}{b_1}$	$\vdots$		
$\vdots$	$\vdots$	$\vdots$		
s^1	$\vdots$	$\vdots$		
s^0	a_n			

TABLE 1: Routh Array

$$P(s) = s^3 + 2s^2 + 5s + 80 \tag{2}$$

$$x(n) = (n^3 + 2n^2 + 5n + 80)u(n) \tag{3}$$

From Table 1 and equation (2):

s^3	1	5
s^2	2	80
s^1	$\frac{2 \times 5 - 80 \times 1}{2} = -35$	
s^0	$\frac{-35 \times 80}{-35} = 80$	

TABLE 2

From Table 2:

Since there are 2 sign changes in the first column of the Routh tabulation. So, the number of zeros in the right half of the s-plane will be 2.

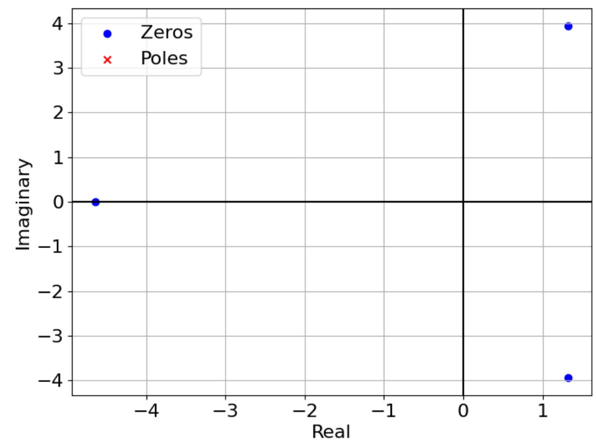

Fig. 1: Pole-Zero Plot of the Polynomial

From equation (3) and Appendix (??) - (??):

$$X(z) = \frac{z^{-1}\left(1 + 4z^{-1} + z^{-2}\right)}{(1 - z^{-1})^4} + \frac{2z^{-1}\left(z^{-1} + 1\right)}{(1 - z^{-1})^3} + \frac{5z^{-1}}{(1 - z^{-1})^2} + \frac{80}{1 - z^{-1}}; |z| > 1 \tag{4}$$

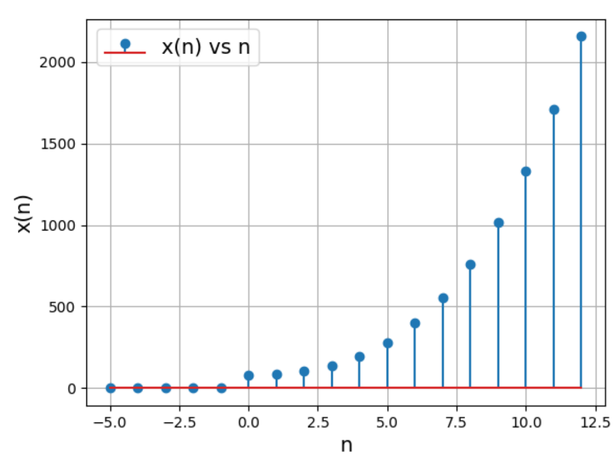

Fig. 2