GATE: EE - 11.2022

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Question: The transfer function of a real system $H(S)$ is given as:

$$H(s) = \frac{As + B}{s^2 + Cs + D}$$

where A, B, C and D are positive constants. This system cannot operate as

(A) Low pass filter
(B) High pass filter
(C) Band pass filter
(D) An Integrator

(GATE EE 2022)

Solution:

The transfer function $H(s)$ is given by:

$$H(s) = \frac{As + B}{s^2 + Cs + D} \tag{1}$$

Put $s = j\omega$ in (1):

$$H(j\omega) = \frac{A(j\omega) + B}{(j\omega)^2 + C(j\omega) + D} \tag{2}$$

$$|H(j\omega)| = \frac{\sqrt{(A\omega)^2 + B^2}}{\sqrt{(D - \omega^2)^2 + (\omega C)^2}} \tag{3}$$

Parameter	Description
Low Pass Filter	The gain should be finite at low frequency
High Pass Filter	The gain should be finite at high frequency
Band Pass Filter	Finite gain over frequency band
Integrator	Transfer function should have at least one pole at origin

TABLE 1: Conditions

(i) From Table 1 and equation (3):
At low frequency ($\omega = 0$):

$$|H(\omega = 0)| = \frac{B}{D} \tag{4}$$

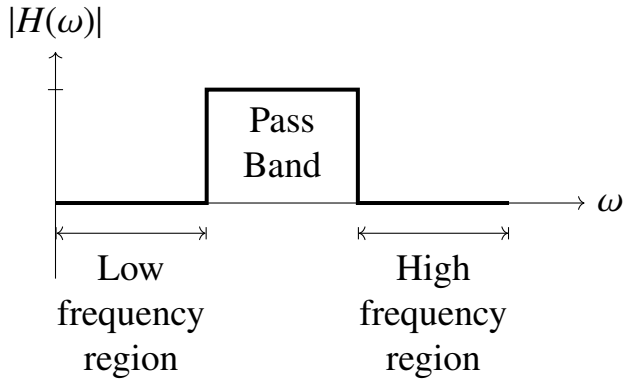
(ii) From Table 1 and equation (3):
At high frequency ($\omega = \infty$):

$$|H(\omega = \infty)| = 0 \tag{5}$$

(iii) Assuming B is a very less positive valued constant as compared to others:

$$|H(j\omega)| = \frac{(A\omega)}{\sqrt{(D - \omega^2)^2 + (\omega C)^2}} \tag{6}$$

$$\implies |H(\omega = 0)| = 0 \text{ and } |H(\omega = \infty)| = 0 \tag{7}$$



Hence, H(s) passes frequency between low and high frequencies.

(iv) At very high value of frequency($\omega \to \infty$):

$$H(s) \approx \frac{As}{s^2} \approx \frac{A}{s} \tag{8}$$

The pole of the transfer function cannot exist at origin as $D > 0$.
From equations (4),(5),(7) and (8):
The Transfer function $H(s)$ cannot be operated as a High pass filter and an Integrator.