

GATE: EE - 31.2021

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Question: The causal signal with Z transform $z^2(z - a)^{-2}$ is ($u(n)$ is unit step signal)

- 1) $a^{2n}u(n)$
- 2) $(n + 1)a^n u(n)$
- 3) $n^{-1}a^n u(n)$
- 4) $n^2 a^n u(n)$

(GATE EE 2021)

Solution:

Z-transform of a causal signal is,

$$X(z) = z^2(z - a)^{-2} = \frac{1}{(1 - az^{-1})^2}; |z| > |a| \quad (1)$$

The Z transform pair for $a^n u(n)$ signal is given by :

$$a^n u(n) \longleftrightarrow \frac{1}{1 - az^{-1}} \quad (2)$$

Using differentiation in z-domain property:

$$na^n u(n) \longleftrightarrow -z \frac{d}{dz} \left(\frac{1}{1 - az^{-1}} \right) \quad (3)$$

$$\Rightarrow na^n u(n) \longleftrightarrow \frac{az^{-1}}{(1 - az^{-1})^2} \quad (4)$$

Using time-shifting property:

$$(n + 1)a^{n+1}u(n + 1) \longleftrightarrow \frac{az^{-1}}{(1 - az^{-1})^2}z \quad (5)$$

$$(n + 1)a^n u(n + 1) \longleftrightarrow \frac{1}{(1 - az^{-1})^2} \quad (6)$$

From (1) and (6), Inverse Z transform is :

$$x(n) = (n + 1)a^n u(n + 1) \quad (7)$$

Sequence $u(n + 1)$ exist for $-1 \leq n < \infty$, but the factor $(n + 1)$ is zero for $n = -1$, so $x(n)$ may be expressed as a causal sequence.

$$x(n) = (n + 1)a^n u(n) \quad (8)$$

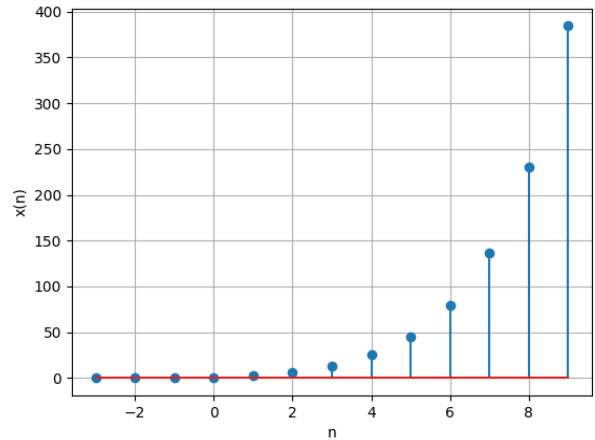


Fig. 1: $x(n)$ vs n using $a = 1.5$