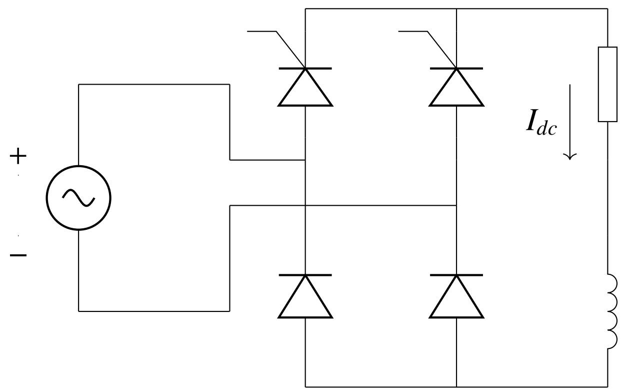
GATE: EE - 59.2022

EE23BTECH11013 - Avyaaz*

Question: For the ideal AC-DC rectifier circuit shown in the figure below, the load current magnitude is I_{dc} = 15 A and is ripple free. The thyristors are fired with a delay angle of 45°. The amplitude of the fundamental component of the source current, in amperes, is _________*(Round off to 2 decimal places).* (GATE 59 EE 2022)

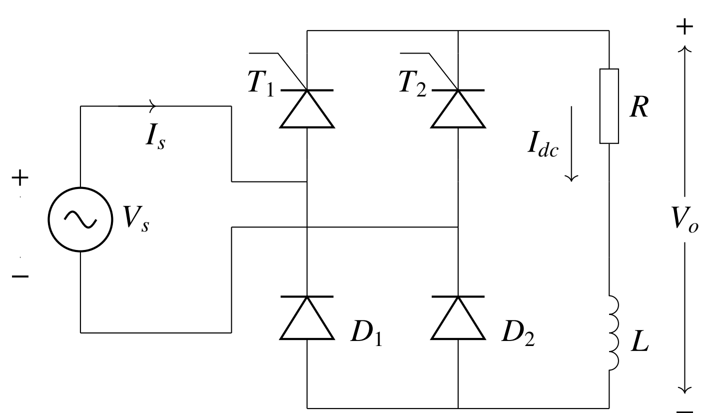


Solution:

Parameter	Description	Value
I_{dc}	Load current	15A
α	Firing angle	45°

TABLE 1

A symmetrical single phase semi converter is shown below,



The Fourier series representation of supply current is given by:

$$i_s(t) = a_o + \sum_{n=1}^{\infty} C_n \sin(n\omega t + \theta_n) \tag{1}$$

where,

$$a_o = \frac{1}{2\pi}\int_0^{2\pi} i_s(t)d\omega t \tag{2}$$

$$C_n = \sqrt{a_n^2 + b_n^2} \tag{3}$$

$$\theta_n = \tan^{-1}\left(\frac{a_n}{b_n}\right) \tag{4}$$

$$\implies a_o = \frac{1}{2\pi}\int_\alpha^{\pi} I_o d\omega t - \int_{\pi+\alpha}^{2\pi} I_o d\omega t = 0 \tag{5}$$

$$\implies a_n = \frac{1}{\pi}\int_\infty^{\pi} I_o \cos n\omega t d\omega t - \int_{\pi+\alpha}^{2\pi} I_o \cos n\omega t d\omega t \tag{6}$$

$$a_n = \begin{cases} \frac{-2I_o}{n\pi}\sin n\alpha & \text{for } n = 1,3,5... \\ 0 & \text{for } n = 2,4..... \end{cases} \tag{7}$$

$$\implies b_n = \frac{1}{\pi}\int_\alpha^{\pi} I_o \sin n\omega t d\omega t - \int_{\pi+\alpha}^{2\pi} I_o \sin n\omega t d\omega t \tag{8}$$

$$b_n = \begin{cases} \frac{2I_o}{n\pi}(1 + \cos n\alpha) & \text{for } n = 1,3,5... \\ 0 & \text{for } n = 2,4.... \end{cases} \tag{9}$$

From (3):

$$\therefore C_n = \frac{2\sqrt{2}I_o}{n\pi}\left(\sqrt{1+\cos n\alpha}\right) \tag{10}$$

$$\implies C_n = \frac{4I_o}{n\pi}\cos\frac{n\alpha}{2} \tag{11}$$

From (4):

$$\theta_n = \tan^{-1}\left(\frac{-\sin n\alpha}{1+\cos n\alpha}\right) \tag{12}$$

$$\implies \theta_n = \frac{-n\alpha}{2} \tag{13}$$

From (1),(11) and (13):

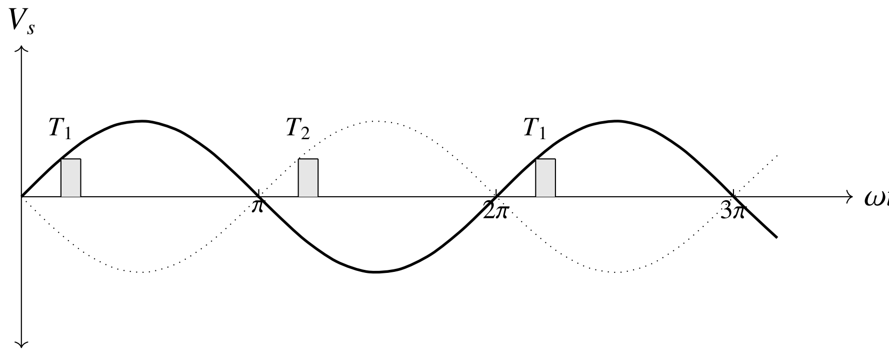
$$I_s(t) = \sum_{n=1,3,5...}^{\infty} \frac{4I_o}{n\pi}\cos\frac{n\alpha}{2}\sin\left(n\omega t - \frac{n\alpha}{2}\right) \tag{14}$$

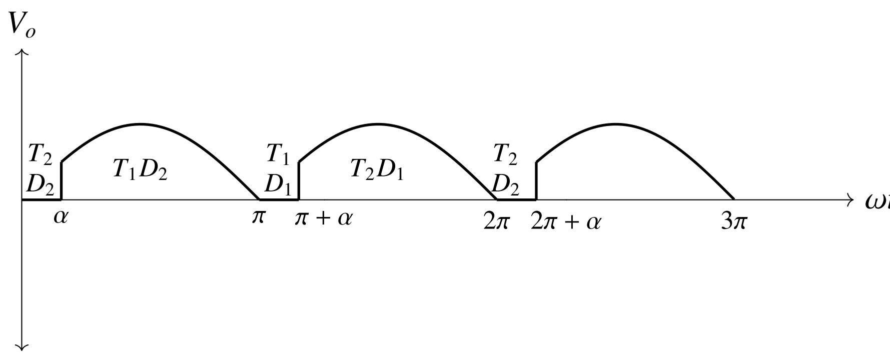
From Table 1:

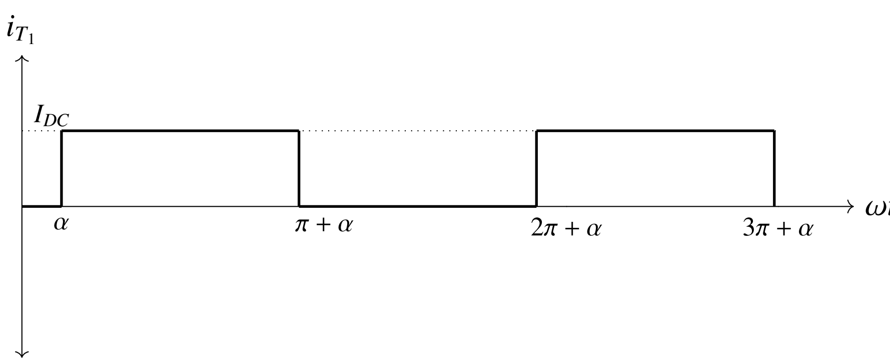
$$(I_{s_1})_{peak} = \frac{4I_{dc}}{\pi}\cos\left(\frac{\alpha}{2}\right) \tag{15}$$

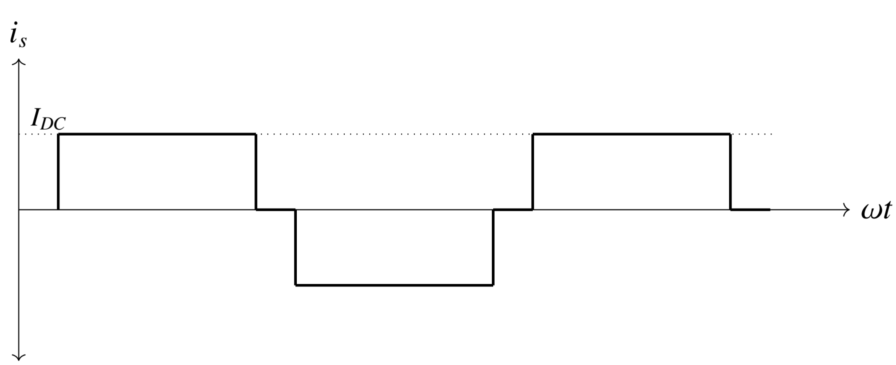
$$= \frac{4\times 15}{\pi}\times\cos\frac{45^\circ}{2} \tag{16}$$

$$= 17.64A \tag{17}$$


V_s
T_1
T_2
T_1
ωt
π
2π
3π


V_o
T_2 D_2
$T_1 D_2$
T_1 D_1
$T_2 D_1$
T_2 D_2
ωt
α
π
$\pi + \alpha$
2π
$2\pi + \alpha$
3π


i_{T_1}
I_{DC}
ωt
α
$\pi + \alpha$
$2\pi + \alpha$
$3\pi + \alpha$


i_s
I_{DC}
ωt