GATE: IN - 41.2022

EE23BTECH11013 - Avyaaz*

Question: A proportional-integral-derivative (PID) controller is employed to stably control a plant with transfer function

$$P(s) = \frac{1}{(s + 1)(s + 2)} \quad (1)$$

Now, the proportional gain is increased by a factor of 2, the integral gain is increased by a factor of 3, and the derivative gain is left unchanged. Given that the closed-loop system continues to remain stable with the new gains, the steady-state error in tracking a ramp reference signal   (GATE IN 2022)

Solution:

The transfer function of PID controller,

$$G_c(s) = K_p + \frac{K_I}{s} + sK_D \quad (2)$$

$$= \frac{s^2 K_D + sk_P + K_I}{s} \quad (3)$$

Overall loop-transfer function,

$$L(s) = G_c(s) \cdot P(s) \quad (4)$$

$$L(s) = \frac{s^2 K_D + sk_P + K_I}{s(s + 1)(s + 2)} \quad (5)$$

Steady-state error due to ramp signal,

$$e_{ss} = \frac{1}{K_v} \quad (6)$$

where,

$$K_v = \lim_{s \to 0} sL(s) \quad (7)$$

$$K_v = \frac{K_I}{2} \quad (8)$$

$$e_{ss} = \frac{2}{K_I} \quad (9)$$

Now, the proportional gain is increased by a factor of 2, the integral gain is increased by a factor of 3, and the derivative gain is left unchanged.

$$K'_P = 2K_P, K'_I = 3K_I \text{ and } K'_D = K_D \quad (10)$$

$$K'_v = \lim_{s \to 0} sL'(s) \quad (11)$$

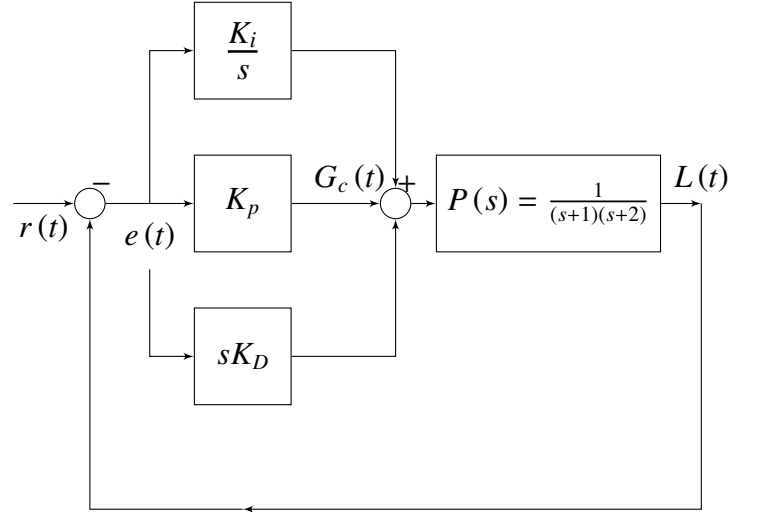$$K'_v = \frac{3K_I}{2} \quad (12)$$

$$e'_{ss} = \frac{1}{K'_V} = \frac{2}{3K_I} \quad (13)$$

Fig. 1: Block Diagram of System