NCERT 12.10 5Q

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Question: In Young's double-slit experiment using monochromatic light of wavelength λ, the intensity of light at a point on the screen where path difference is λ, is K units. What is the intensity of light at a point where path difference is $\lambda/3$?

Solution:

Parameter	Description	Value
λ	Wavelength of light	
$y_i(t)$	Displacement produced by $S_{i^{th}}$	
ω	Angular frequency	
I	Intensity of light at $\Delta x = \lambda$	K
k	Wave number	$\frac{2\pi}{\lambda}$
$I_{\text{net}}, I_{\text{R}}$	Net Intensities of resulting waves	kA^2
$\Delta x = x_1 - x_2$	Path difference	λ
		$\frac{\lambda}{3}$
A	Amplitudes of light waves	$A_1 = A_2$

TABLE 1
Parameters

In, Young's double-slit experiment the light waves coming out from the source S fall on both S_1 and S_2 slits which behave like coherent sources since the light waves coming from both slits are from the same original source. Hence, the light waves coming out from the slits are coherent.

The equation of light wave coming out from the slit S_1 is:

$$y_1(t) = A_1 \sin(\omega t - kx_1) \tag{1}$$

The equation of light wave coming out from the slit S_2 is:

$$y_2(t) = A_2 \sin(\omega t - kx_2) \tag{2}$$

Since, the waves coming out from the slits are from the same source their frequency, wavelength, wave number and amplitude will be the same.

$$A_1 = A_2$$

The resultant wave resulting from the superposition of the two waves is the sum of two individual waves:

From (1)&(2) $\implies$

$$y_{res}(t) = y_1(t) + y_2(t) \tag{3}$$

$$y_{res}(t) = A\sin(\omega t - kx_1) + A\sin(\omega t - kx_2)$$

$$\because \sin A + \sin B = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$\implies y_{\text{res}} = 2A\cos\left(\frac{k\Delta x}{2}\right) \sin\left(\omega t - \frac{k(x_1+x_2)}{2}\right) \tag{4}$$

The amplitude of the resulting wave is:

$$A_{\text{net}} = 2A\cos\left(\frac{k\Delta x}{2}\right) \tag{5}$$

From table (1) $\implies$

$$\therefore I = K\left(2A\cos\left(\frac{k\Delta x}{2}\right)\right)^2$$

$$\therefore I = 4I_o\cos^2\left(\frac{k\Delta x}{2}\right) \tag{6}$$

From table(1) $\implies$

When $\Delta x = \lambda$

From, (6) $\implies$

$$\therefore K = 4I_1\cos^2\left(\frac{2\pi}{2}\right)$$

$$K = 4I_1$$

$$\therefore I_1 = \frac{K}{4} \tag{7}$$

From table (1)&(6) $\Longrightarrow$

When $\Delta x = \dfrac{\lambda}{3}$:

Resultant intensity:

$$I_\mathrm{R} = 4I_1 \cos^2\left(\frac{2\pi}{3}\right)$$

$$I_\mathrm{R} = 4I_1 \left(\frac{-1}{2}\right)^2$$

$$I_\mathrm{R} = I_1 \tag{8}$$

From the above result (7) $\Longrightarrow$

$$i.e., \quad I_1 = \frac{K}{4}$$

$$\therefore I_\mathrm{R} = \frac{K}{4} \tag{9}$$

Hence, the Intensity of light at a point where path difference is $\dfrac{\lambda}{3}$ is $\dfrac{K}{4}$ units.