GATE: AG - 14.2023

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Question: $y = ae^{mx} + be^{-mx}$ is the solution of the differential equation

a) $\frac{dy}{dx} - my = 0$
b) $\frac{dy}{dx} + my = 0$
c) $\frac{d^2y}{dx^2} + m^2y = 0$
d) $\frac{d^2y}{dx^2} - m^2y = 0$

Solution:

Let us assume $y = ae^{mx} + be^{-mx}$ is the solution of the differential equation :

$$\frac{d^2y}{dx^2} - m^2y = 0 \tag{1}$$

According to the property of the Laplace transform for derivatives :

$$\mathcal{L}\left\{\frac{d^2y}{dx^2}\right\} = s^2Y(s) - s \cdot y(0) - y'(0) \tag{2}$$

So, the Laplace transform of the differential equation becomes:

$$s^2Y(s) - s \cdot y(0) - y'(0) - m^2Y(s) = 0 \tag{3}$$

$$\therefore Y(s) = \frac{s \cdot y(0) + y'(0)}{s^2 - m^2} \tag{4}$$

We Know That,

$$Y(s) = \frac{s \cdot y(0) + y'(0)}{s^2 - m^2} \tag{5}$$

$$= \frac{\frac{m \cdot y(0) + y'(0)}{2m}}{s - m} + \frac{\frac{-m \cdot y(0) + y'(0)}{-2m}}{s + m} \tag{6}$$

$$= \frac{1}{2m}\left(\frac{m \cdot y(0) + y'(0)}{s - m} - \frac{m \cdot y(0) - y'(0)}{s + m}\right) \tag{7}$$

Taking the Inverse Laplace transform of $Y(s)$:

$$\implies \frac{1}{2m}(m \cdot y(0) + y'(0))e^{mx} - (m \cdot y(0) - y'(0))e^{-mx}) \tag{8}$$

$$\implies \frac{1}{2}\left(m \cdot y(0)(e^{mx} + e^{-mx}) + y'(0)(e^{mx} - e^{-mx})\right) \tag{9}$$

Parameter	**Value**
$y(0)$	$a + b$
$y'(0)$	$m(a - b)$

TABLE 1: Input Parameters

From Table 1:

$$y = ae^{mx} + be^{-mx} \tag{10}$$