NCERT 10.5.3 17Q

EE23BTECH11013 - Avyaaz*

Question: In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class *XII*. There are three sections of each class. How many trees will be planted by the students?

Solution:

From the above question, the number of trees planted by the students is in AP.

$$3, 6, 9, 12....36 \tag{1}$$

Parameter	Description	Value
$x(0)$	First Term	3
d	common difference	6 – 3 = 3
$x(n)$	$(n+1)^{th}$ term	$(x(0)+nd)u(n)$
$y(n)$	sum of $(n+1)$ terms	

TABLE 1

From Table 1:

$$x(n) = (3n+3)u(n) \tag{2}$$

$$u(n) \xleftrightarrow{\mathcal{Z}} \frac{1}{(1-z^{-1})}; |z| > 1 \tag{3}$$

$$nu(n) \xleftrightarrow{\mathcal{Z}} \frac{z^{-1}}{(1-z^{-1})^2}; |z| > 1 \tag{4}$$

$$n^2u(n) \xleftrightarrow{\mathcal{Z}} \frac{z^{-1}\left(1+z^{-1}\right)}{(1-z^{-1})^3}; |z| > 1 \tag{5}$$

From (3) and (4):

$$X(Z) = \frac{3z^{-1}}{(1-z^{-1})^2} + \frac{3}{(1-z^{-1})}; |z| > 1 \tag{6}$$

$$Y(z) = X(z)U(z) \tag{7}$$

$$= \frac{3z^{-1}}{(1-z^{-1})^3} + \frac{3}{(1-z^{-1})^2} \tag{8}$$

$$= \frac{3}{2}\left(\frac{2z^{-1}+z^{-2}-z^{-2}}{(1-z^{-1})^3} + \frac{2}{(1-z^{-1})^2}\right) \tag{9}$$

$$= \frac{3}{2}\left(\frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3} + \frac{3z^{-1}-2z^{-1}+2}{(1-z^{-1})^2}\right) \tag{10}$$

$$= \frac{3}{2}\left(\frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3} + \frac{3z^{-1}}{(1-z^{-1})^2} + \frac{2}{(1-z^{-1})}\right)$$
$$; |z| > 1 \tag{11}$$

Taking reverse z-transform using (3) - (5):

$$y(n) = \frac{3}{2}\left(n^2+3n+2\right)u(n) \tag{12}$$

$$y(n) = 3\left(\frac{n(n+1)}{2} + (n+1)\right)u(n) \tag{13}$$

$$\therefore y(11) = 234 \tag{14}$$

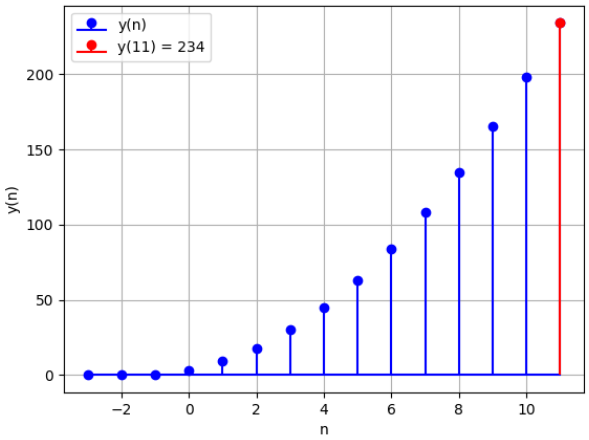

Fig. 1