NCERT 12.10 5Q

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Question: In Young's double-slit experiment using monochromatic light of wavelength λ, the intensity of light at a point on the screen where path difference is λ, is K units. What is the intensity of light at a point where path difference is $\lambda/3$?

Solution:

Given,

Parameter	Description
λ	Wavelength of monochromatic light
K	Intensity of light at path difference λ
Δx	Path difference
A, A_1, A_2	Amplitudes of light waves
ω	Angular frequency
k	Wave number
ϕ, ϕ_1, ϕ_2	Phases
$\Delta\phi$	Phase differences between two waves
$y_1(t)$	Displacement produced by S_1
x_1, x_2	Distace travelled by the respective waves
$y_2(t)$	Displacement produced by S_2
$I_1, I_2, I_{\text{net}}, I_{\text{R}}$	Intensities of coherent waves

TABLE 1
Parameters

Path difference $= \lambda$

The general equation of light wave is:

$$y(t) = A\sin(\omega t - kx) \tag{1}$$

where,

$$\text{phase} = \phi = \omega t - kx$$
$$\text{wave number} = k = \frac{2\pi}{\lambda}$$

In, Young's double-slit experiment the light waves coming out from the source S fall on both S_1 and S_2 slits which behave like coherent sources since the light waves coming from both slits are from the same original source. Hence, the light waves coming out from the slits are coherent.

The equation of light wave coming out from the slit S_1 is:

$$y_1(t) = A_1\sin(\omega t - kx_1) \tag{2}$$

The equation of light wave coming out from the slit S_2 is:

$$y_2(t) = A_2\sin(\omega t - kx_2) \tag{3}$$

where,

$$\therefore \phi_1 = \omega t - kx_1 \tag{4}$$
$$\phi_2 = \omega t - kx_2 \tag{5}$$

Subtracting equations (4) *and* (5) $\Rightarrow$

$$\phi_1 - \phi_2 = \omega t - kx_1 - (\omega t - kx_2)$$
$$\Delta\phi = k(x_2 - x_1)$$
$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x \tag{6}$$

$$\text{Phase difference} = \frac{2\pi}{\lambda} \times \text{Path difference}$$

The variation of distance covered by two waves from their sources to the point where they meet is known as Path difference.

The disparity in phases of two particles at any two moments where their position and motion are the same is known as Phase difference.

$\because$ path difference $= \lambda$

$$\Delta\phi = \frac{2\pi}{\lambda} \times \lambda$$
$$\text{Phase difference} = \Delta\phi = 2\pi \tag{7}$$

The intensity of light is defined as the energy transmitted per unit area in one unit of time. The square of the wave's amplitude is generally determined as the intensity of the light. Let at an arbitrary point the phase difference between the two displacements produced by the waves be y_1 and y_2 be ϕ. Thus, the displacement produced by y_1 is given by:

$$y_1(t) = A\sin(kx_1 - \omega t + \phi) \tag{8}$$

The displacement produced by y_2 is given by:

$$y_2(t) = A\sin(kx_2 - \omega t) \tag{9}$$

The resultant wave resulting from the superposition of the two waves is the sum of two individual waves:

$$y_{res}(t) = y_1(t) + y_2(t) \tag{10}$$

$$y_{res}(t) = A\sin(kx_1 - \omega t + \phi) + A\sin(kx_2 - \omega t)$$

$$\because \sin A + \sin B = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$\Rightarrow y_{\text{res}} = 2A\sin\left(\frac{k(x_1+x_2)}{2} - \omega t + \frac{\phi}{2}\right)\cos\left(\frac{k(x_1-x_2)}{2} + \frac{\phi}{2}\right) \tag{11}$$

$$\Rightarrow y_{\text{res}} = 2A\cos\left(\frac{k\Delta x}{2} + \frac{\phi}{2}\right)\sin\left(\frac{k(x_1+x_2)}{2} - \omega t + \frac{\phi}{2}\right) \tag{12}$$

Here, The amplitude of the resulting wave is:

$$A_{\text{net}} = 2A\cos\left(\frac{k\Delta x}{2} + \frac{\phi}{2}\right) \tag{13}$$

$$\because I \propto |\overrightarrow{A}|^2$$

$$I = KA_{\text{net}}^2 \tag{14}$$

$$\therefore I = K\left(2A\cos\left(\frac{k\Delta x}{2} + \frac{\phi}{2}\right)\right)^2 \tag{15}$$

$$I = 4KA^2\cos^2\left(\frac{k(x_1-x_2)+\phi}{2}\right) \tag{16}$$

$$\therefore I = 4I_o\cos^2\left(\frac{k(x_1-x_2)+\phi}{2}\right) \tag{17}$$

Let,

$$\left(\frac{k(x_1-x_2)+\phi}{2}\right) = \Delta\phi \tag{18}$$

$$\therefore I = 4I_o\cos^2\left(\frac{\Delta\phi}{2}\right) \tag{19}$$

In the general case of two waves of different amplitude interfering, then the amplitude of the net resulting wave is the vector addition of individual amplitude:

$$\because \overrightarrow{A} = \overrightarrow{A_1} + \overrightarrow{A_2}$$

$$|\overrightarrow{A}|^2 = |\overrightarrow{A_1}|^2 + |\overrightarrow{A_2}|^2 + 2|\overrightarrow{A_1}||\overrightarrow{A_2}|\cos\phi \tag{20}$$

$$\because I \propto |\overrightarrow{A}|^2 \tag{21}$$

Let I_1 and I_2 be the intensity of two coherent waves. The resultant intensity is given by:

$$I_{\text{net}} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi \tag{22}$$

Here, ϕ is the phase difference between two light waves.

Intensities are equal for monochromatic light waves.

$$I_1 = I_2 \tag{23}$$

$$\therefore I_{\text{net}} = I_1 + I_1 + 2\sqrt{I_1 I_1}\cos\phi$$

$$I_{\text{net}} = 2I_1 + 2I_1\cos\phi \tag{24}$$

$\because I_{\text{net}} = K$

$$\therefore K = 2I_1 + 2I_1 \cos 2\pi$$
$$K = 4I_1$$
$$\therefore I_1 = \frac{K}{4} \qquad (25)$$

When path difference $= \dfrac{\lambda}{3}$

$$\Delta\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{3}$$
$$\text{Phase difference} = \Delta\phi = \frac{2\pi}{3} \qquad (26)$$

Hence, Resultant intensity,

$$I_{\text{R}} = 2I_1 + 2\sqrt{I_1 I_1} \cos \frac{2\pi}{3}$$
$$I_{\text{R}} = 2I_1 + 2I_1 \left(\frac{-1}{2}\right)$$
$$I_{\text{R}} = I_1 \qquad (27)$$

From the above result,

$$I_1 = \frac{K}{4}$$
$$\therefore I_{\text{R}} = \frac{K}{4} \qquad (28)$$

Hence, the Intensity of light at a point where path difference is $\dfrac{\lambda}{3}$ is $\dfrac{K}{4}$ units.