Filter Design #114

EE23BTECH11013 - Avyaaz*

Design the equivalent FIR and IIR filter realizations for filter number 114. This is a bandpass filter with sampling rate $F_s = 48\text{kHz}$.

I. IIR Filter Design

A. The Analog Filter

We are designing filters whose stopband is *monotonic* and *passband equiripple*. Hence, we use the *Chebyschev approximation* to design our bandpass IIR filter.
The Low Pass Chebyschev Filter Paramters:

$$|H_{a,LP}(j\Omega_L)|^2 = \frac{1}{1+\epsilon^2 c_N^2(\Omega_L/\Omega_{Lp})} \tag{1}$$

Using,
$c_N(x) = \cosh(N \cosh^{-1} x)$ and the integer N, which is the order of the filter, and ϵ are design paramters. Since $\Omega_{Lp} = 1$:

$$|H_{a,LP}(j\Omega_L)|^2 = \frac{1}{1+\epsilon^2 c_N^2(\Omega_L)} \tag{2}$$

We obtain $0.3184 \le \epsilon \le 0.6197$. In Figure(1), we plot $|H(j\Omega)|$ for a range of values of ϵ, for $N = 4$. $\epsilon = 0.4$ is used for this IIR filter design.

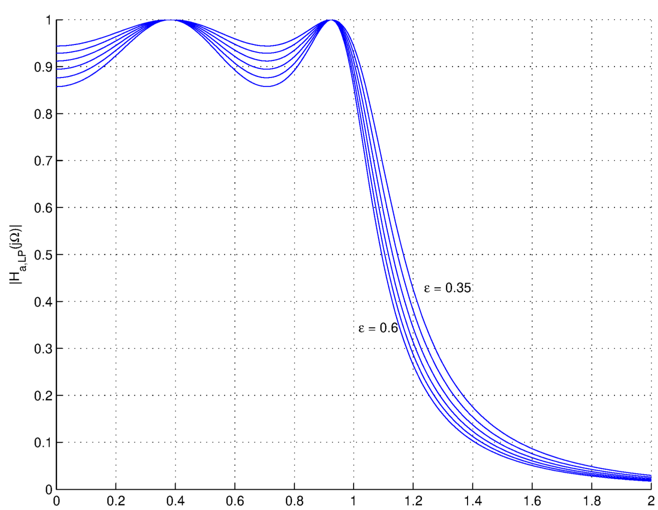

Fig. 1: The Analog Low-Pass Frequency Response for $0.35 \le \epsilon \le 0.6$

The Low Pass Chebyschev Filter:

$$|H_{a,LP}(j\Omega_L)|^2 = \frac{1}{1+0.16c_4^2(\Omega_L)} \tag{3}$$

where,

$$c_4(x) = 8x^4 + 8x^2 + 1. \tag{4}$$

The low-pass stable Chebyschev fiter, with a gain G has the form:

$$H_{a,LP}(s_L) = \frac{G_{LP}}{\prod_{k=1}^{\frac{N}{2}-1}(s_L^2 - 2r_1\cos\phi_k s_L + r_1^2\cos^2\phi_k + r_2^2\sin^2\phi_k)} \tag{5}$$

Substituting $N = 4$, $\epsilon = 0.5$ and $H_{a,LP}(j) = \frac{1}{\sqrt{1+\epsilon^2}}$, from (5), we obtain

$$H_{a,LP}(s_L) = \frac{0.3125}{s_L^4 + 1.1068s_L^3 + 1.6125s_L^2 + 0.9140s_L + 0.3366} \tag{6}$$

In Figure (2) we plot $|H(j\Omega)|$ using (3) and (6), thereby verifying that our low-pass Chebyschev filter design meets the specifications.

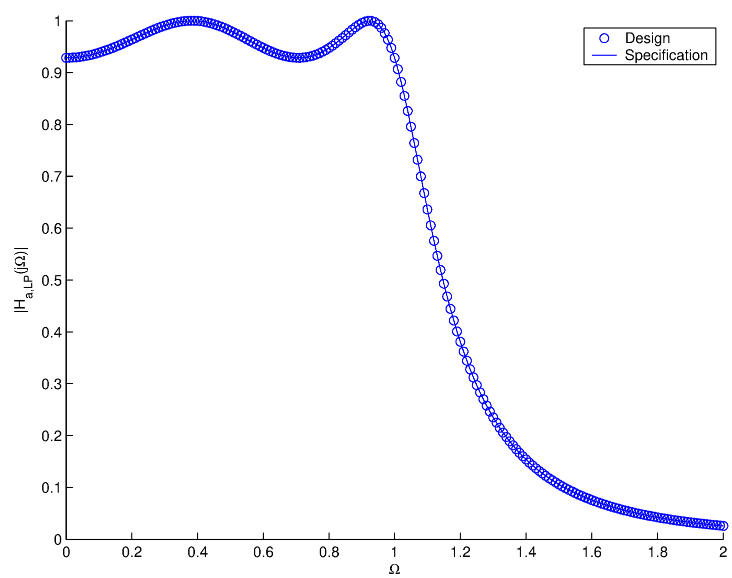

Fig. 2: The magnitude response plots from the specifications in Equation 3 and the design in Equation(6)

- The *analog passband* frequencies are $\Omega_{p1} = 0.5095$, $\Omega_{p2} = 0.4142$

- The *analog stopband* frequencies are $\Omega_{s1} = 0.5345$, $\Omega_{s2} = 0.3914$
- If Ω be the *analog bandpass* frequency and Ω_L be the corresponding *analog lowpass* frequency,
$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega}. \tag{7}$$
- $\Omega_0 = \sqrt{\Omega_{p1}\Omega_{p2}} = 0.4594$ and $B = \Omega_{p1} - \Omega_{p2} = 0.0953$

The Band Pass Chebyschev Filter:

The analog bandpass filter is obtained from (6) by substituting $s_L = \frac{s^2+\Omega_0^2}{Bs}$. Hence

$$H_{a,BP}(s) = G_{BP}H_{a,LP}(s_L)\Big|_{s_L=\frac{s^2+\Omega_0^2}{Bs}}, \tag{8}$$

where G_{BP} is the gain of the bandpass filter. After appropriate substitutions, and evaluating the gain such that $H_{a,BP}(j\Omega_{p1}) = 1$, we obtain

$$H_{a,BP}(s) = \frac{2.7776 \times 10^{-5} s^4}{s^8 + 0.1055s^7 + 0.8589s^6 + 0.0676s^5 + 0.2735s^4 + 0.0143s^3 + 0.0383s^2 + 0.001s + 0.002} \tag{9}$$

In Figure (3), we plot $|H_{a,BP}(j\Omega)|$ as a function of Ω for both positve as well as negative frequencies. We find that the passband and stopband frequencies in the figure match well with those obtained analytically through the bilinear transformation.

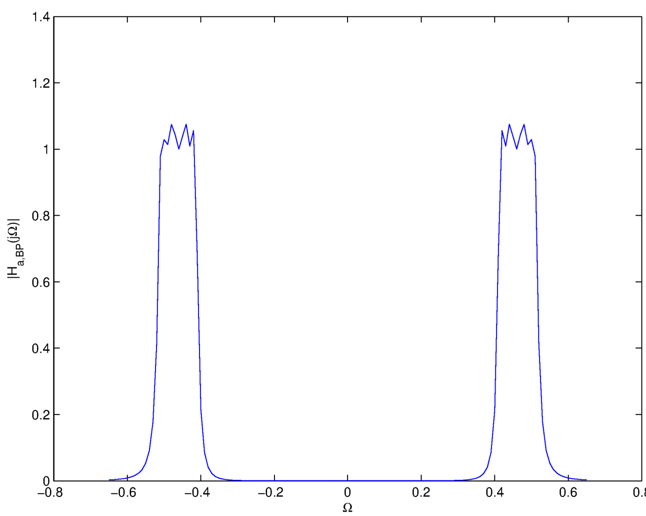

Fig. 3: The analog bandpass magnitude response plot from Equation 9

B. The Digital Filter

$$H_{d,BP}(z) = G\frac{N(z)}{D(z)} \tag{10}$$

where, $G = 2.7776 \times 10^{-5}$,

$$N(z) = 1 - 4z^{-2} + 6z^{-4} - 4z^{-6} + z^{-8} \tag{11}$$

and

$$D(z) = 2.3609 - 12.0002z^{-1} + 31.8772z^{-2} - 53.7495z^{-3} + 62.8086z^{-4} - 51.4634z^{-5} + 29.2231z^{-6} - 10.5329z^{-7} + 1.9842z^{-8} \tag{12}$$

The plot of $|H_{d,BP}(z)|$ with respect to the normalized angular freqency (normalizing factor π) is available in Figure 4. Again we find that the passband and stopband frequencies meet the specifications well enough.

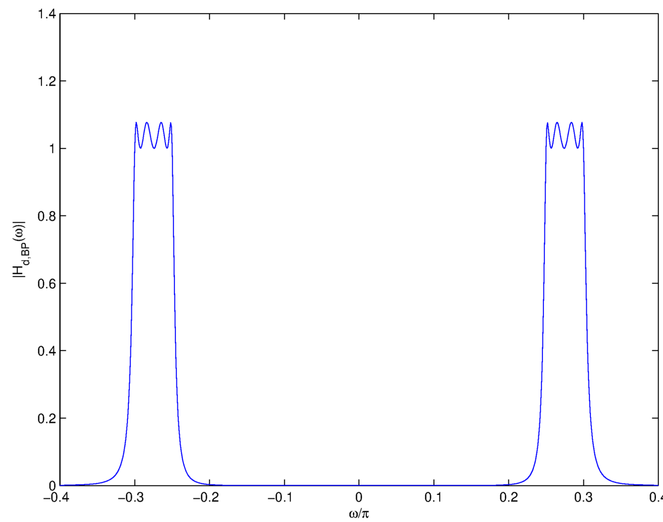

Fig. 4: The magnitude response of the bandpass digital filter designed to meet the given specifications

II. FIR Filter Design

We design the FIR filter by first obtaining the (non-causal) lowpass equivalent using the Kaiser window and then converting it to a causal bandpass filter.

FIR Filter Transfer Function Realization Using Kaiser Window:

- The equivalent *lowpass* filter has *passband frequency* $\omega_l = \frac{\omega_{p1}-\omega_{p2}}{2} = 0.025\pi$.
- The centre of the passband of the desired bandpass filter is $\omega_c = \frac{\omega_{p1}+\omega_{p2}}{2} = 0.275\pi$
- For the given specifications, the *Kaiser* window reduces to the *rectangular* window of *length* ≥ 97.

- The desired *lowpass filter impulse response*

$$
\begin{aligned}
h_{lp}(n) &= \frac{\sin(\frac{n\pi}{40})}{n\pi} && -100 \leq n \leq 100 \\
&= 0, && \text{otherwise}
\end{aligned}
$$

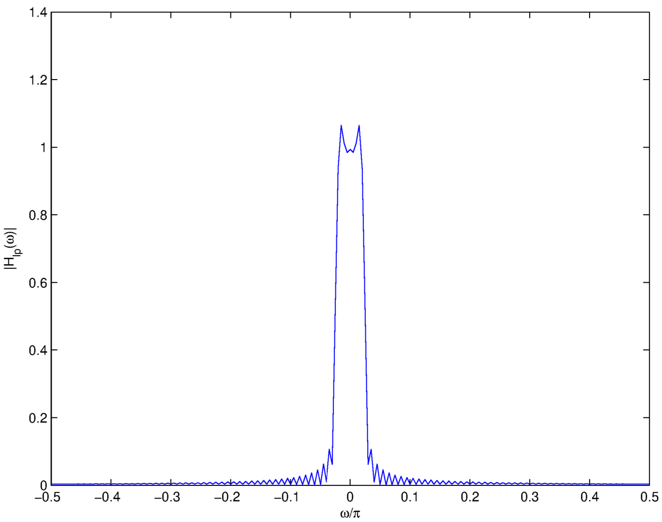

Fig. 5: The magnitude response of the FIR lowpass digital filter designed to meet the given specifications

- The desired *bandpass filter impulse response*

$$
\begin{aligned}
h_{bp}(n) &= \frac{2\sin(\frac{n\pi}{40})\cos(\frac{11n\pi}{40})}{n\pi} && -100 \leq n \leq 100 \\
&= 0, && \text{otherwise}
\end{aligned}
$$

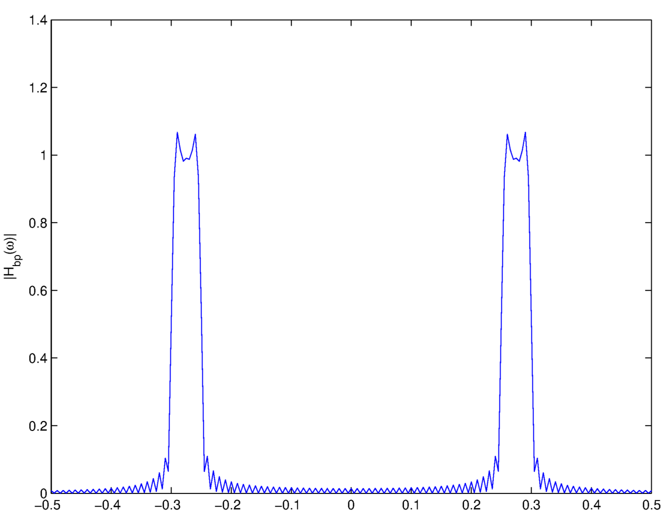

Fig. 6: The magnitude response of the FIR bandpass digital filter designed to meet the given specifications