Def: A vector is a tuple of numbers.
Def: The dimension is the size of this tuple.
Def: A matrix is a 2-dimensional grid of numbers.
Def: $\mathbb{R}^{m\times n}$ denotes all $m \times n$ matrices with field $\mathbb{R}$.
Def: $\mathbb{C}^{m\times n}$ denotes all $m \times n$ matrices with field $\mathbb{C}$.
Def: vector-matrix product: $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax+by \\ cx+dy \end{bmatrix}$ (Inner dimensions must agree. Outer dimensions remain.)
$\begin{bmatrix} ax+by \\ cx+dy \end{bmatrix} = x \begin{bmatrix} a \\ c \end{bmatrix} + y \begin{bmatrix} b \\ d \end{bmatrix}$: Express matrix-vector multiplication as a linear combination of the matrix.
Def: Inner product $< \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} >= \begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = x_1y_1 + x_2y_2 + \cdots + x_ny_n$
Def: Matrix Multiplication: $A \in \mathbb{R}^{m\times k}, B \in \mathbb{R}^{k\times n}$, $C = AB \in \mathbb{R}^{m\times n}$ and $C_{ij} = \sum_{k=1}^{n} A_{ik}B_{kj}$.
Def: span of vectors $v_1, v_2, \ldots, v_n$ is the set of vectors that can be obtained as linear combination of the vectors $v_1, v_2, \ldots, v_n$:
$span\{v_1, v_2, \ldots, v_n\} = a_1v_1 + a_2v_2 + \cdots + a_nv_n$ for $a_1, a_2, \ldots, a_n$ are scalars