No queries will be entertained during examination

Indian Institute of Technology, Kharagpur

Date ………FN/AN,	Time: 2 hrs,	Full Marks 30,	Deptt : Mathematics
No. of students 60	Year 2015		Mid Semester Examination
Sub. No.: MA31007	Sub. Name: Mathematical Methods		M. Sc./ M. Tech (Dual)

READ THE INSTRUCTIONS CAREFULLY FOR EACH QUESTION AND FOLLOW THE EXACT STEPS ASKED FOR. ATTEMPT ALL QUESTIONS.

1. For the following series, do the following [3+3=6M]

   a) $S = \sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)}$, b) $T = \sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

- Find the expressions for partial sums S_m, T_m in terms of m, using mathematical induction
- Then find the limits of S_m, T_m as $m \to \infty$ to find the sums S and T.

  [**No marks for finding sums by any other means. Convergence test is not asked and marks will be deducted for such unnecessary answer.**]

2. A bouncing ball rises each time to $2/3$ of the height of the previous bounce and the ball is originally dropped from a height of 1m. [5M]

   a) Express the heights by an infinite sequence.

   b) Express the total distance S the ball goes by an infinite series with proper justification.

   c) Compute partial sum S_n by using the formula $S_n = a(1-r^n)/(1-r)$, and finally find the total distance S by taking limit of S_n as $n \to \infty$.

   [**No marks for finding S_n and S by any other means.]**

3. Express the following repeating decimal numbers as an infinite series and then find the fractions that are equivalent to them by computing the sums of the infinite series. (The formula $S = a/(1-r)$, $|r| < 1$ for infinite G.P. series may be used) :--- [3+3=6M]

   a) $A = 0.55555\cdots$, b) $B = 0.576923076923076923\cdots$

   [**No marks for direct writing the fractions by guess or by any other means.**]

4. Let $y_1(x)$ and $y_2(x)$ be two linearly independent solutions of the equation $y''(x) + P(x)y'(x) + Q(x)y = 0$. The Wronskian $W(y_1, y_2)$ of two solutions is defined by

$\left(y_1(x), y_2(x)\right) \coloneqq y_1 y_2' - {y_1}' \, y_2$, which is not identically zero because of independence. Suppose one solution $y_1(x)$ is known. [2+1+1+1+1+1=6M]

a) Then construct the second solution $y_2(x)$ from $y_1(x)$ by using the following steps: ---

   i) Differentiating $W(x)$ with respect to x, obtain a differential equation for W, and integrating that equation, derive $W(x) = A \exp\left[-\int P(x)dx\right]$, where A is an arbitrary constant.

   ii) Writing $W(x)$ with justification in the form $W(x) = {y_1}^2 (y_2/y_1)'$, construct a differential equation for (y_2/y_1).

   iii) Integrating above equation, derived in ii), show that

$$y_2(x) = y_1(x) \left(B + A \int {y_1}^{-2}(x) \exp\left[-\int P(x)dx\right] dx\right),$$

   where B is another arbitrary constant.

   iv) Finally, justify that A and B may be dropped to obtain following final form for $y_2(x)$ as

$$y_2(x) = y_1(x) \int {y_1}^{-2}(x) \exp\left[-\int P(x)dx\right] dx \, .$$

[**No marks for finding $\boldsymbol{y_2(x)}$ by any other means.**]

b) Apply the above formula for $y_2(x)$ to obtain second solution of linear oscillator equation $y'' + y = 0$ from the known solution $y_1(x) = \sin x$.

[**No marks for direct solving the differential equation.**]

5. Find a series solution of odd powers of x of the equation [6M]

$$(1 - x^2) y''(x) - 3x y'(x) + n(n+2) y(x) = 0$$

by assuming the solution as $y(x) = \sum_{\lambda=0}^{\infty} a_\lambda x^{k+\lambda}$, $a_0 \neq 0$ and by choosing the appropriate root of the indicial equation (for k). Write the general term of the series. Finally choose a class of values for the parameter n to convert the infinite series into a polynomial. Express the polynomial as a compact summation form.

[**No marks for taking the solution without $\boldsymbol{k}$. Two solutions are not asked and massive marks will be deducted for such blindfold answer.**]

6. Write few lines (point wise) in plain English about the innovative part of this question paper according to your opinion. No mathematical symbols and/or equations will be allowed here. [1M]

[**There may be significant negative impression for not answering this question.**]

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