Problem Set - 7

AUTUMN 2016

MATHEMATICS-I (MA10001)

September 19, 2016

1. Classify the following differential equations and find their order and degree:
   (i) $y'' + 6y' + 4y = \sin x$.
   (ii) $x^2y'' + xy' + 4y = 10x$.
   (iii) $yy' + 3xy = 0$.
   (iv) $\sqrt{1+x^2}dx + \sqrt{1+y^2}dy = 0$.
   (v) $\sqrt{1+(y')^2} = x^2 + y$.
   (vi) $(y'')^2 + y' + 4 = 0$.

2. Form the ordinary differential equation by eliminating the arbitrary constants A and B.
   (i) $y = A\cos mx + B\sin mx$, m is fixed constant.
   (ii) $y = Ae^x + Be^{2x}$.
   (iii) $(x-A)^2 + (y-B)^2 = r^2$, r is fixed constant.
   (iv) $x^2 - y^2 = A(x^2+y^2)^2$.

3. Find the general solution of the following ordinary differential equations:
   (i) $axy' = by$, $a \neq 0$.
   (ii) $y' + y + 1 = 0$.
   (iii) $(x^2 + x + 1)y' + (6x+3)y = 0$.
   (iv) $(y - 3xy)dx - 3xdy = 0$.

4. Solve the following ordinary differential equations by reducing their order:
   (i) $xy'' + y' = 0$.
   (ii) $yy'' + (1+y)(y')^2 = 0$.

5. Solve the following initial value problems for ordinary differential equations:
   (i) $(y+2)y' = \sin x, \quad y(0) = 0$.
   (ii) $(x\ln x)y' = 2y, \quad y(2) = (\ln 2)^2$.

6. Solve the following ordinary differential equations:

   (i) $y' = (2x - y + 1)^2$.

   (ii) $x^2y' + xy = x^2 + y^2$.

   (iii) $(2xy + x^2)y' = 3y^2 + 2xy$.

   (iv) $(2x + y - 1)dy + (4x + 2y - 3)dx = 0$.

   (v) $(x + y + 2)dy = (y + 3)dx$.

   (vi) $x^2dy - xydx + y^2e^{x^2/y^2}dy = 0$.

7. Verify that the following ordinary differential equations are exact and find the corresponding solution:

   (i) $ydx + x(1 + y)dy = 0$.

   (ii) $\sinh x \cos ydx - \cosh x \sin ydy = 0$.

   (iii) $(xe^{xy} + 2y)dy + ye^{xy}dx = 0$.

   (iv) $(e^{2y} + 1)\cos xdx + 2e^{2y} \sin xdy = 0$.

8. Show that $M(x, y)dx + N(x, y)dy = 0$ is exact iff $[M(x, y) + g(x)]dx + [N(x, y) + h(y)]dy = 0$ is exact.