4 COLOR THEOREM	If graph is planer then we can always color it using 4 colors
ADACENT EDGE AND NODES	We say that two vertices v and $w$ of a graph $G$ are <b>adjacent</b> if there is an edge $vw$ joining them, and the vertices v and $w$ are then <b>incident</b> with such an edge. Similarly, two distinct edges $e$ and/are adjacent if they have a <b>vertex</b> in common
ADJENCY MATRIX AND INCIDENT MATRIX	If $G$ is a graph with vertices labelled $\{1,n\}$ , its adjacency <b>matrix</b> A is the $n \times n$ matrix whose ij'-th entry is the number of edges joining vertex $i$ and vertex $j$ . If, in addition, the edges are labelled $\{1,, m\}$ , its <b>incidence matrix M</b> is the $n \times m$ matrix whose $ij$ -th entry is 1 if vertex $i$ is incident to
BIPARTITE GRAPHS	edge;, and 0 otherwise.  If the vertex set of a graph <i>G</i> can be split into two disjoint sets <i>A</i> and <i>B</i> so that each edge of <i>G</i> joins a vertex of <i>A</i> and a vertex of <i>B</i> , then <i>G</i> is a <b>bipartite graph</b> . Alternatively, a bipartite graph is one
BRIDGE	whose vertices can be coloured black and white in such a way that each edge joins a black vertex (in $A$ ) and a white vertex (in $B$ ). Denote bipartite graph of $r$ black vertices and $s$ vertices as $K_{r,s}$ If cutset has one edge e then it is called a bridge.
CALEY THEOREM	There are $n^{n-2}$ distinct labels in a tree with n vertices.
CHROMATIC NUMBER	If G is a graph without loops, then G is $k$ -colourable if we can assign one of $k$ colours to each
CINOMATIC NOMBER	vertex so that adjacent vertices have different colours. If G is $k$ -colourable, but not $(k-1)$ - colourable, we say that G is $k$ -chromatic, or that the chromatic number of G is $k$ , and write $\chi(G) = k$
COMPLEMENT GRAPH	If $G$ is a simple graph with vertex set $V(G)$ , its <b>complement</b> $\bar{G}$ is the simple graph with vertex set $V(G)$ in which two vertices are adjacent if and only if they are <i>not</i> adjacent in $G$ .
COMPLETE BIPARTITE GRAPH	Each vertex in A is joined to vertex B with just one Edge. (From bipartite graphs)
COMPLETE GRAPH	Simple graph with each pair of vertices have an edge between them
COMPONENTS	Each connected subgraph in a Graph is called a component
CONNECTED GRAPH	A graph is <b>connected</b> if and only if there is a path between each pair of vertices
CONNECTIVITY OR K CONNECTED	connectivity $\kappa(G)$ is the size of the smallest separating set in G. Thus $\kappa(G)$ is the minimum number of vertices that we need to delete in order to disconnect G.
CUBES	The <b>k-cube</b> $Q_k$ is the graph whose vertices correspond to the sequences $(a_1,a_2,\ldots,a_k)$ where each $a_i=0$ or $1$ and whose edges join these sequences that differ in just one place. $Q_k$ has $2^k$ vertices and $k2^{k-1}$ edges, regular in degree k
CUTSET	A disconnecting set whose no proper subset is a disconnecting set.
CUT-VERTEX	If a separating set contains only one vertex v, we call v a cut-vertex
CYCLE	A walk starting and ending to the same edge
CYCLE GRAPH	Every node in this graph has a degree 2. $C_n$
DEGREE OF A VERTEX	Number of edges with vertex as an end point. The degree of a vertex v of $G$ is the number of edges incident with v, and is written deg(v). In calculating the degree of v, we usually make the convention that a loop at v contributes 2 (rather than 1) to the degree of v.
DEGREE SEQUENCE	The degree sequence of a graph consists of the degrees written in increasing order, with repeats where necessary
DIGRAPH OR DIRECTED GRAPH	A directed graph, or digraph, $D$ consists of a non-empty finite set $V(D)$ of elements called vertices, and a finite family $A(D)$ of ordered pairs of elements of $V(D)$ called arcs. We call $V(D)$ the vertex set and $A(D)$ the arc family of $D$ .
DISCONNECTED GRAPH	A graph more than one piece
DISCONNECTING SET	Set of edges whose removal disconnects G
EDGE CONNECTIVITY OR K-EDGE	$\lambda(U)$ is the size of smallest cutset in G. Thus $\lambda(G)$ is the minimum number of edges that we need
CONNECTED	to delete in order to disconnect G
EULERIAN GRAPHS	A connected graph G is <b>Eulerian</b> if there exists a closed trail containing every edge of G. Such a trail is an <b>Eulerian trail</b> . A connected graph G is Eulerian if and only if the degree of each vertex of G is even
FLUERY ALGORITHM	Let G be an Eulerian graph. Then the following construction is always possible, and produces an Eulerian trail of G.  Start at any vertex u and traverse the edges in an arbitrary manner, subject only to the following
	rules: (i) erase the edges as they are traversed, and if any isolated vertices result, erase them too;
FOREST	(ii) at each stage, use a bridge only if there is no alternative
FOREST	A <b>forest</b> is a graph that contains no cycles
GRAPH SUBTRACTION	If S is any set of vertices in G, we denote by G - S the graph obtained by deleting the vertices in S and all edges incident with any of them.
HAMILTION GRAPHS	Graph containing closed trail that includes every vertex exactly once ending at initial vertex. Such a cycle is called Hamilton cycles.
HANDSHAKING LEMMA	Sum of all vertex degrees is an even number
ISOLATED AND END VERTEX	A vertex of degree 0 is an isolated vertex and a vertex of degree 1 is an end-vertex

If graph is planer then we can always color it using 4 colors

**DEFINATION** 

TERM

**4 COLOR THEOREM** 

ISOMORPHISM	Two graphs $G_1$ and $G_2$ are isomorphic if there is a one-one correspondence between the vertices of $G_1$ and those of $G_2$ such that the number of edges joining any two vertices of $G_1$ is equal to the
	number of edges joining the corresponding vertices of $G_2$
LINKAGE	Let A be any labelled tree in which $deg(v) = k - 1$ . The removal from A of any edge $wz$ that is not incident with v leaves two subtrees, one containing v and either $w$ or z ( $w$ , say), and the other containing z. If we now join the vertices v and z, we obtain a labelled tree $B$ in which $deg(v) = k$ (see Fig. 10.4). We call a pair (A, $B$ ) of labelled trees a <b>linkage</b> if $B$ can be obtained from $A$ by the above construction.
LOOP	edge of a node to itself
MATRIX-TREE THEOREM	Let G be a connected simple graph with vertex set $\{v_1, v_n\}$ and let $M = (m_{ij})$ be the n x n matrix in which $m_{ii} = \deg(v_i)$ , $m_{ij} = -1$ if $v_i$ and $v_j$ are adjacent, and $m_{ij} = 0$ otherwise. Then the number
	of spanning trees of G is equal to the cofactor of any element of M.
MINIMUM SPANNING TREE	Choose edges of minimum weight in such a way that no cycle is created
MULTIPLE EDGES	2 or more edges from same set of vertices
NULL GRAPH PATH	Graph whose edge-set is empty  A walk whose all edges and all vertices are distinct is called a path
PATH GRAPH	
PETERSON GRAPH	A graph obtained from $C_n$ by removing one of its edge. $P_n$
	Regular with degree 3
PLANER GRAPH	Graph which can be redrawn to avoid crossings with redrawn edge in the same place is called planer graph.
PLATONIC GRAPHS	Formed by vertices and edges of five regular(Plantonic) Solids: tetrahedron, octahedron, cube, icosahedron and dodecahedron
REGULAR GRAPH	Graph with each vertex having same degree. If degree is r, r-regular
SAME GRAPHS	If two vertices are joined by an edge in one graph if and only if the corresponding vertices are joined by an edge in the other
SEMI-EULERIAN GRAPH	A connected graph there exists a trail through all edges but not closed in nature. A connected graph is semi-Eulerian if and only if it has exactly two vertices of odd degree.
SEPERATING SET	A separating set in a connected graph G is a set of vertices whose deletion disconnects G
SIMPLE GRAPH	having no loops or multiple edges
SIMPLE GRAPH G VERTICES V AND	A simple graph G consists of a non-empty finite set V(G) of elements called vertices (or nodes),
EDGES E	and a finite set $E(G)$ of distinct unordered pairs of distinct elements of $V(G)$ called <b>edges</b> . We call $V(G)$ the vertex set and $E(G)$ the edge set of $G$ .
SPANNING FOREST, CYCLE RANK AND CUTSET RANK	If G is an arbitrary graph with $n$ vertices, $m$ edges and $k$ components, then we remove edges from each cycle such that there are no cycles left resulting in a <b>spanning forest</b> , and the total number of edges removed in this process is the <b>cycle rank</b> of G, denoted by $\gamma(G)$ . Note that $\gamma(G) = m - n + k$ . <b>Cutset rank</b> of G is number of edges in a spanning forest, denoted by $\xi(G) = n - k$
SPANNING TREE	Given any connected graph G, we can choose a cycle and remove any one of its edges, and the
	resulting graph remains connected. We repeat this procedure with one of the remaining cycles, continuing until there are no cycles left. The graph that remains is a tree that connects all the vertices of G. It is called a <b>spanning tree</b> of G
STRONGLY CONNECTED DIGRAPH	A directed graph is <b>strongly connected</b> if, for any two vertices v and w of D, there is a path from v to w
SUBGRAPH	Every vertices and edges of a subgraph of a graph $G$ is a graph belongs to $V(G)$ and $E(G)$
TRAIL	A walk whose all edges are distinct is called a trail.
TREES	Connected graphs with exactly one path between each pair of vertices or also can be interpreted as connected forest.
TREES EQUIVALENT DEFINATIONS	Let T be a graph with n vertices. Then following statements are equivalent:  i. T is a tree  ii. T contains no cycle and has n-1 edges  iii. T is connected and has n-1 edges  iv. T is connected and each edge is a bridge  v. Any two paths are connected by exactly one path  vi. T contains no cycles, but any new edge creates exactly one cycle
UNION	$G_1 = (V(G_1), E(G_1))$ and $G_2 = (V(G_2), E(G_2))$ , where $V(G_1)$ and $V(G_2)$ are disjoint, then their <b>union</b> $G_1 \cup G_2$ is the graph with vertex set $V(G_1) \cup V(G_2)$ and edge family $E(G_1) \cup E(G_2)$

A walk is a 'way of getting from one vertex to another', and consists of a sequence of edges, one following after another. The number of edges in a walk is called its **length** 

Graph obtained by  $\mathcal{C}_{n-1}$  by joining each vertex to a new vertex.  $\mathcal{W}_n$ 

ullet If G is a forest with n vertices and k components, then G has n-k edges

WALK

WHEEL GRAPH

**ADDITIONAL RESULTS** 

 $\bullet$  If G is a simple connected graph which is not a complete graph, and if the largest vertex-degree of G is  $\Delta$  (> 3), then G is  $\Delta + 1$ -colourable.