Indian Institute of Technology Kharagpur QUESTION-CUM-ANSWERSCRIPT	Stamp/Signature of the Invigilator
MID-SEMESTER EXAMINATION	**SEMSETER (Spring-2017)**

Roll Number											**Section**		**Name**	
Subject Number	**M**	**A**	**2**	**0**	**1**	**0**	**2**	**Subject Name**	**NUMERICAL SOLUTION OF ODE & PDE**					
Department/Centre/School														

Important Instructions and Guidelines for Students

1. You must occupy your seat as per the Examination Schedule/Sitting Plan.
2. Do not keep mobile phones or any similar electronic gadgets with you even in the switched off mode.
3. Loose papers, class notes, books or any such materials must not be in your possession; even if they are irrelevant to the subject you are taking examination.
4. Data book, codes, graph papers, relevant standard tables/charts or any other materials are allowed only when instructed by the paper-setter.
5. Use of instrument box, pencil box and non-programmable calculator is allowed during the examination. However, the exchange of these items or any other papers (including question papers) is not permitted.
6. Write on both sides of the answer-script and do not tear off any page. Use last page(s) of the answer-script for rough work. Report to the invigilator if the answer-script has torn or distorted page(s).
7. It is your responsibility to ensure that you have signed the Attendance Sheet. Keep your Admit Card/Identity Card on the desk for checking by the invigilator.
8. You may leave the Examination Hall for wash room or for drinking water for a very short period. Record your absence from the Examination Hall in the register provided. Smoking and consumption of any kind of beverages is strictly prohibited inside the Examination Hall.
9. Do not leave the Examination Hall without submitting your answer-script to the invigilator. **In any case, you are not allowed to take away the answer-script with you**. After the completion of the examination, do not leave your seat until invigilators collect all the answer-scripts.
10. During the examination, either inside or outside the Examination Hall, gathering information from any kind of sources or exchanging information with others or any such attempt will be treated as '**unfair means**'. Don't adopt unfair means and also don't indulge in unseemly behavior.
11. Please see overleaf for more instructions

Violation of any of the above instructions may lead to severe punishment.

Signature of the Student

To be Filled by the Examiner

Question Number	1	2	3	4	5	Total
Marks Obtained						

Marks Obtained (in words)	Signature of the Examiner	Signature of the Scrutineer

Instructions and Guidelines to the Students appearing in the Examination

1. The question-cum-answer booklet has 28 pages and 5 questions.
2. All questions are compulsory.
3. Answer each question in the space provided below to that question only.
4. No additional answer sheet will be provided.
5. Use the space for rough work given in the booklet only.
6. After the completion of the examination do not leave the examination hall until the invigilator collects the booklet.

Space for Rough Work

1 a) Derive the Forward and backward Euler methods to solve the Initial value problem $\frac{dy}{dx} = f(x,y),\ y(x_0) = y_0$ in the interval $[x_0, b]$. Give their geometrical interpretation. Find the Truncation error of these methods.

[3]

1 b) Discuss the absolute stability of both the Forward and Backward Euler methods when applied to the test equation $\frac{dy}{dx} = \lambda y,\ y(x_0) = y_0, \lambda < 0.$

[3]

1 c) Using the Backward Euler method, solve the initial value problem $\frac{dy}{dx} = 3x + y^2,\ y(0) = 1$ in the interval $[0, 0.2]$ by taking the step size $h = 0.1$. Use Newton-Raphson method to solve the algebraic equations and obtain the solution accurate up to the third decimal place. [3]

2) Using the fourth order explicit Runge-Kutta method, find a numerical solution at $x = 0.2$ of the 2nd order I.V.P $\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 4y = 0$, $y(0) = 1, \frac{dy(0)}{dx} = 2$ by taking the step size $h = 0.2$. [Hint: Reduce the equation into a first order system] [6]

3) Derive the Picard's method of successive approximations for the I.V.P. $\frac{dy}{dx} = f(x,y),\ y(x_0) = y_0$ and find the second iterative approximation for the solution $y(x)$ of $\frac{dy}{dx} = x + y^2,\ y(0) = 0$. [2]

4) A linear multistep method to solve the I.V.P. $\frac{dy}{dx} = f(x,y),\ y(x_0) = y_0$ is given by $y_{n+1} = y_n + \frac{h}{12}[\,23\,y'_n - 16\,y'_{n-1} + 5\,y'_{n-2}\,] + T_{n+1}(h)$, where $T_{n+1}(h)$ is the local truncation error. [4+1+3]

a) If $T_{n+1}(h) = a\,h + b\,h^2 + c\,h^3 + d\,h^4$, then find a, b, c, d

b) Find the order of this method

c) Apply this method to the I.V.P. $\frac{dy}{dx} = -y^2,\ y(0) = 1$ with step size $h = 0.1$ to get the solution $y(0.3)$. Calculate the other initial value(s) using appropriate order method.

5) Find $y(0.4)$ from the I.V.P. $\frac{dy}{dx} = -2xy^2, y(0) = 1, h = 0.2$ using the following Predictor – Corrector set:

$$P: y_{j+1} = y_j + \frac{h}{2}(3f_j - f_{j-1}),$$

$$C: y_{j+1} = y_j + \frac{h}{2}(f_{j+1} + f_j).$$

Use only three iterations for corrector. Also, use the 3rd order Taylor series method to calculate $y(0.2)$. [5]