DEPARTMENT OF MATHEMATICS, IIT - Kharagpur

End Semester Examination 2016

MA21007 Design and Analysis of Algorithm

No. of students: 205 Total Points: 50 DURATION: 3 Hours

Answer **ALL QUESTIONS**. All the notations are standard and no query or doubts will be entertained. If any data/statement is missing, identify it in your answer script. Marks are indicated at the end of each question.

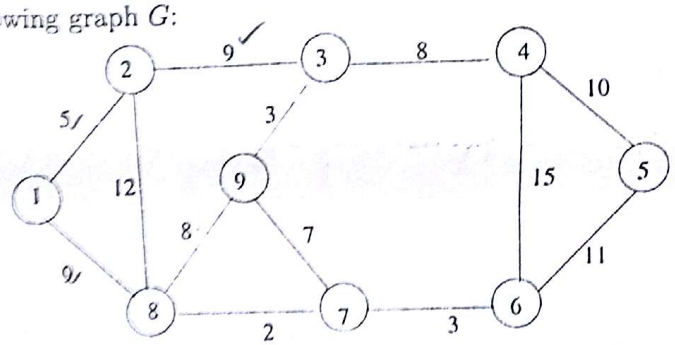
1. (a) Insertion sort can be expressed as a recursive procedure as follows. In order to sort $A[1\ .\ .\ n]$, we recursively sort $A[1\ .\ .\ n-1]$ and then insert $A[n]$ into the sorted array $A[1\ .\ .\ n-1]$. Write a recurrence for the running time of this recursive version of insertion sort.

(b) Describe a $\Theta(n \lg n)$-time algorithm that, given a set S of n integers and another integer x, determines whether or not there exist two elements in S whose sum is exactly x. [6]

2. a) Write an algorithm for inserting items in a Red-Black tree. What is the computing time of your algorithm?

b) Start with an empty Red-Black tree and insert the following keys in the given order using your algorithm: $80, 100, 140, 60, 84, 30, 40, 50, 54, 52, 120, 110$. [6]

3. Consider the following graph G:

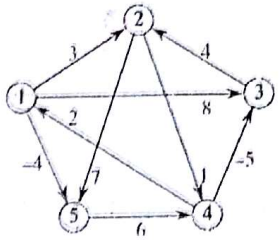


[8]

a) Find a Minimal Spanning Tree in G using Prim's algorithm.

b) Find the shortest path spanning tree from vertex 1 using Dijkstra's algorithm.

4. Write FLOYD-WARSHALL procedure and run the algorithm on the weighted, directed graph below. Show the matrix $D^{(k)}$ that results for each iteration.



[4]

5. a) Write Bellman-Ford Algorithm for solving shortest path problem.

b) Find a feasible solution or determine that no solution exists for the following system of difference constraints using the Bellman-Ford shortest path algorithm:

$$x_1 - x_2 \leq 4,\ x_1 - x_5 \leq 5,\ x_2 - x_4 \leq -6,\ x_3 - x_2 \leq 1,\ x_4 - x_1 \leq 3,$$
$$x_4 - x_3 \leq 5,\ x_4 - x_5 \leq 10,\ x_5 - x_3 \leq -4,\ x_5 - x_4 \leq -8$$ [6]

---P.T.O.---

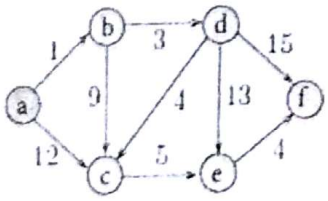
$c[i][j] = c[i-1][j-1]+1$ if $A[i]==A[j]$
$c[i-1][j]+1$ if

6. Find the longest common subsequence between the following strings: [5]

X: AAACCGTGAGTT

Y: CACCCCTAAGGTA

7. Consider the following directed graph:



[5]

Then the weight of the shortest path distances from start vertex 'a' to other vertices 'b', 'c', 'd', 'e', 'f' respectively are?

8. TRUE OR FALSE? If the statement is correct, briefly state why. If the statement is wrong, explain why. [10]

1. Building a max heap on n elements takes Θ(n log n) time. T
2. Van Emde Boas data structure is used to solve the sorting problem F
3. Given an array of n elements, median can be found in O(n) time in the worst case. approx?
4. Given two vertices in a graph s and t, both BFS and DFS can be used to find if there is path from s to t. ~~True~~ F X ✓
5. Runtime of merge sort on an array of size n which is already sorted is O(n) F
6. Heapsort can be used as the auxiliary sorting routine in radix sort, because it operates in place. T
7. Linear probing satisfies the assumption of uniform hashing.
8. Any comparison based sorting algorithm can be made to be stable, without affecting the running time by more than a constant factor. F
9. Consider a modification to QUICKSORT, such that each time PARTITION is called, the median of the partitioned array is found (using the SELECT algorithm) and used as a pivot. The worst case running time of this algorithm is O(n^2). F
10. Consider a modification to QUICKSORT with n elements, such that each time PARTITION is called, using the n/3-th element of the partitioned array as a pivot. The worst case running time of this algorithm is O(n^2) T

——-The End———