INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

End Semester Exam-Spring, 2017

Department of Mathematics

Time : 3 hrs. Total Marks : 50,

Subject: MA 41002/MA 30002, Modern Algebra

Instruction: "No queries will be entertained during the examination".

Answer all the questions.

(1) State weather the following statements are true or false with justification.

(a) Every abelian group is cyclic.

(b) $\mathbb{Z}[x]$ is a PID.

(c) $7x$ is irreducible polynomial over $\mathbb{Z}[x]$.

(d) Let $R = C([0,1], \mathbb{R})$ be the ring of all real valued continuous functions defined on $[0,1]$. Then R is an integral domain.

(e) $\mathbb{Z}[\sqrt{-3}]$ is an UFD.

(f) $(1+2i)$ is a Gaussian prime.

$[2 \times 6 = 12]$

(2) Let G be a finite abelian group and n be a positive integer relatively prime to $|G|$. Is the mapping $f : G \longrightarrow G$ defined by $f(a) = a^n$ is an automorphism? Justify your answer. [4]

(3) Let G be a finite group of order pqr where p, q, r are primes and $p < q < r$. Is G a simple group? Justify your answer.

[5]

(4) Let p be a prime number. Find upto isomorphism all abelian groups which have order p^5 but whose elements has order atmost p^3. [5]

(5) Let R be the ring of continuous functions from $[0,1]$ to $[0,1]$ and $I = \{f \in R |\ f(1/2) = 0 \text{ and } f(1/3) = 0\}$. Show that I is an ideal of R. Is I a prime ideal? Justify your answer. [2 + 2]

PTO

(6) Let R be a commutative ring such that for every $x \in R$ there is an integer $n > 1$ (depending on x) such that $x^n = x$. Show that every prime ideal of R is maximal. [4]

(7) Is $\mathbb{Z}[x, y]$ is an UFD? Is it a PID ? Is it an ED? Justify your answer. [4]

(8) Is (5) a maximal ideal in $\mathbb{Z}[i]$? Justify your answer. [4]

(9) Is 3 irreducible element in $\mathbb{Z}[\sqrt{-5}]$? Is it a prime element? Justify your answer. [4]

(10) State Gauss' Lemma. Is the polynomial $x^7 - 10x^6 + 5x^2 - 25x + 20$ is irreducible over $\mathbb{Z}[x]$? Is it irreducible over $\mathbb{Q}[x]$? [2 + 2]