Symbolic Logic
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Lecture – 29
More on Conditional Proof
Strengthened Version
Its Formats and Procedure

Hello, how are you today? We are going to start the lesson for today. This is our module number 29 for the Symbolic Logic course, the NOC course that we have been doing. You may remember that we were talking about the Limited Scope Assumption proofs and we have already looked in details at one of the procedures called the Indirect Proof.

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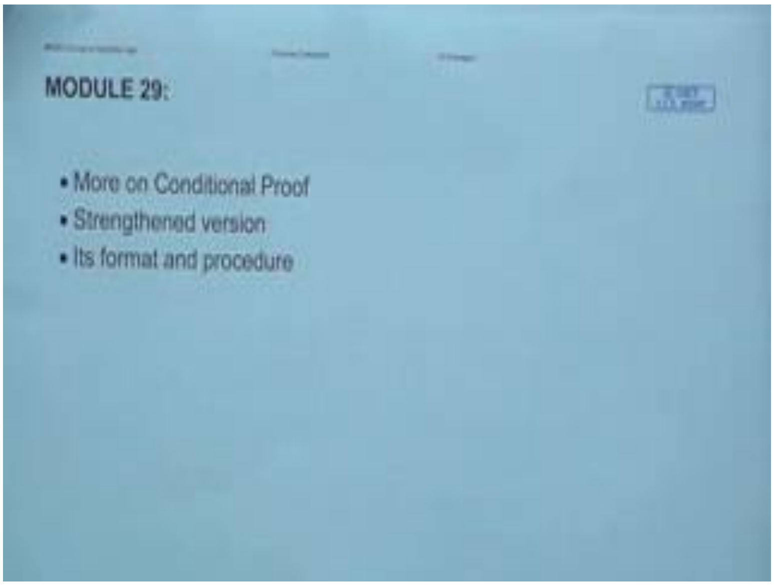

So today we are going to look into what is known as the Conditional Proof. We have already done this preliminary version anyway. So you have some idea about what the Conditional Proof is like, how it moves and what are its assumptions. But there is still more to learn, specially because we have said that there is a preliminary version; therefore, there is the anticipation for what we call the advanced or the Strengthened version of the Conditional Proof.

So that's what we are going to look at today. And, the strengthened version you will see, you I will try to highlight the differences and the… also the power of this procedure.

There is a reason why we have not stopped with the preliminary version. So, all that will be discussed and obviously, the new format and the procedure and so on. So this is on our agenda for today on module 29.

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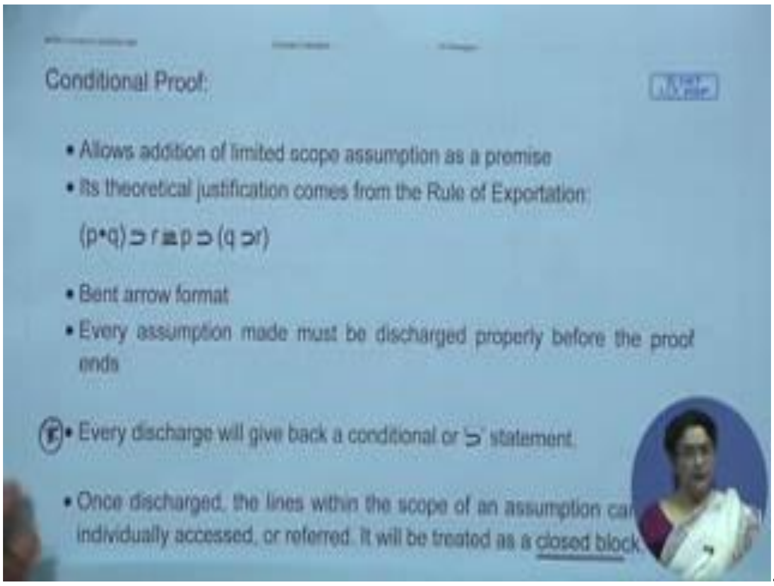

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Let me let us remind ourselves what is it that we have learnt so far. That in Conditional Proof we have learnt that this is one of the Limited Scope Assumption procedure which means that it allows insertion of a premise, insertion of a Limited Scope Assumption as a premise that we know. Now the theoretical justification, if you are thinking in… in that direction, then let me just point at this that the theoretical justification comes in a nutshell from the Rule of Exportation. Rule of Exportation, which you have encountered earlier when you were discussing the 19 Rules of Inference. And if you recall now then this is the nature of Exportation that we say that if $p \supset (q \supset r)$ is equivalent to saying $(p \bullet q) \supset r$.

So this is the modus operandi of the Conditional Proof, that you are already given some… some premises and we add to that an assumption called q. If together they show that r, then what you have shown is that 'if p then if q then r' $(p \supset (q \supset r))$. So the r derivation depends upon conditionally upon q along with p. So that is the… in a very brief manner what would be the justification for this Conditional Proof procedure. As always the Limited Scope Assumption means that you are going to follow the bent arrow format. We have already seen how it works, so I am not going to spend time on that and you are going to see examples of that today also. All you need to remember is that the assumptions are limited scope, which means they have a specific purpose and they have a specific beginning point and an end point. Within the proof you have to show that every single assumption made has been discharged.

So no assumption should be unclosed or undischarged. So when the proof ends, the proof stands on its own feet. But what we have learnt is that somehow in the Conditional Proof, every assumption that you make and then when you close it, you are going to get back a conditional or horse shoe type statement. And I have said this earlier, I will repeat it again is that you better know what to do with this conditional statement in the proof. Right? There is a… there is a reason why you are taking up the Conditional Proof and that objective should match with what you are going to obtain after the discharge. And just reminding you that once the assumption block is closed, there is no way you can access the individual lines in it, nor can you refer, nor can you utilize any of the lines inside. So this closed block situation is what we need to remember.

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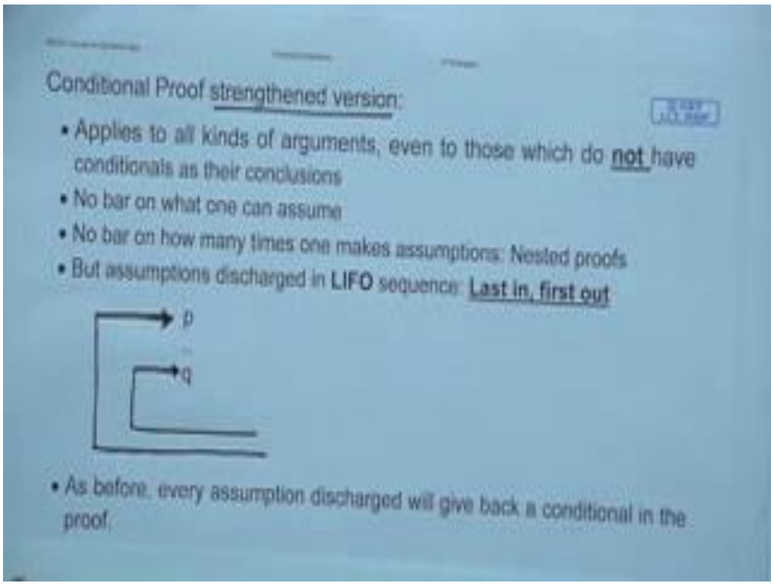

Let us now…so that was our just recap of what we already knew namely, how the Conditional Proof works. But now it's time to come to the Conditional Proof of Strengthened version. So how is it Strengthened and what are its strong points? This is what we are going to pick up. First notice that the strength comes from its wider application. The Conditional Proof in this version applies to *every* kind of argument. So irrespective of whether the conclusion is the conditional statement or not. It doesn't matter. Remember preliminary version has limited application. It applies only if you are lucky enough to have the conclusion as a conditional. But look at this. This says that it doesn't matter what argument you are dealing with, and whether the conclusion is a conditional statement or not, it doesn't matter, you can still apply this. How? You are going to learn. So that's the first strong point about this procedure.

Second, note that there is no restriction on what you can assume. Anything that you find necessary for your proof, you are entitled to assume, right? So earlier in the preliminary version what did we know? That only the antecedent of the conclusion can be assumed, then you solve for the consequent. But here you are being given a lot of freedom. Lot of freedom. No bar on what you can assume and I mean it. So you will see soon that this means that how strong you are strategically. You need to take a good look at your proof to decide what is it that you do not have and therefore, whether you can assume that. Alright? No bar.

Also, there is no bar on how many times you want to make an assumption. So there can be number of assumptions you can start with. One after the other; not simultaneously, but one after the other. So you are going to see nested proofs; meaning, that there the first assumption starts, and then later on you feel you want to add a second assumption, and that starts and so on. So you will see a structure that is more complex than the preliminary version. So the nesting of the sub-derivations within the derivation, and so on, and that is how it is going to be. When you are making several assumptions, that is, one after the other, please note, but there is a certain order in which the assumption are to be discharged. And that sequence is the LIFO, that is, *last in first out*. So the latest assumption that you have made is the first one to go; and the earliest assumption, that you have made the first assumption, is going be the last one to go, will have the major scope, the maximum scope, will be the first assumption. So remember that. So that when you are drawing these lines, the assumption bent arrow lines, *the wires are not going to cross*, right? So this sequence is something to remember when you are doing this, when you are specially handling more than one assumption.

So in a way if we do it pictorially, then this is how it is going to sort of look like. This is where… this is an arbitrary random example of a proof, where the first assumption has been made on this line and this is p. And then you have done the proof and you felt the need for q. Look, this has been started and therefore, q is the first one to be discharged. After that you are entitled to discharge p, not before that. Alright?

So this is the sequence in which we are going to work. And, as before what is to be remembered and this I say with full indication for what is to come is that, as you can see, in the Strengthened version, you have been given a lot of freedom. You can assume anything you want, as many times you want and so on. But with freedom comes responsibility. So the responsibility is here, that you are entitled to assume anything, but remember that you have to discharge each and every one of these assumptions, and when you do that, you are going to get back a conditional in the proof and that conditional you should be able to utilize in the proof.

So this is the crux of the Strengthened version. There is a lot of freedom, but there is also a lot of strategy as you can see. Till the end, you need to know what is it that you are going to do. So have this in front of you as we are going to try this Strengthened version of Conditional Proof.

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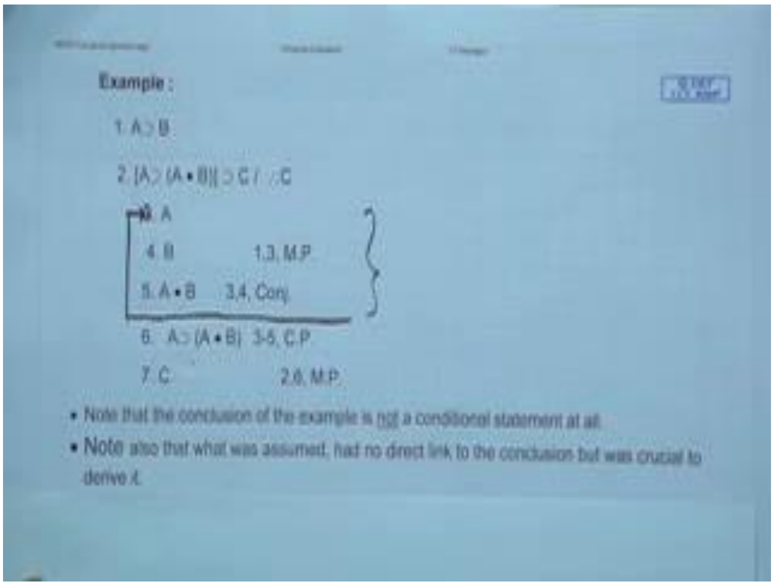

We’ll start by looking at some examples. So here we are. Remember, this is one small proof that we are going to try, where there are two premises and here is C, alright? The question is we are now applying the Strengthened version. Thank God, because in preliminary version there is no way you can apply the preliminary version of Conditional Proof on this. Why? Because C is just standalone C, it is not a conditional statement. But in Strengthened version, there is no restriction. So we can start thinking what is it that we need? And what is it that if we have it, we can come to C? Now, your best bet from these two premises is that if we can have this, then we need to have C. Then we can easily get the Modus Ponens and do C.

But then, what is it that we have to assume? Should we assume this whole thing? Or should we work a little differently? This is where your strategy comes. What is it that we are going to work on? And how I am going to utilize that? Remember, whatever you assume, you are going to soon have horseshoe in your proof and that horseshoe should be useful for you. So if you, for example, start with this whole thing and then you solve C , what will you get back? This whole thing, but is that your conclusion? Conclusion is C and that is what we need. So you need to think a little. What is it that we can have?

On the other hand, and this is where you need to reorient yourself, that there is no need that we solve immediately for this, but it's good enough if by the procedure we can simply get this part. Then, we apply that to line 2 and we get C out. The question is what is it that we need to assume in order to get this? And, take a look at here. This is $A \supset (A \bullet B)$.

So we need to work on that. So we can assume A, and then we have to solve for $A \bullet B$, right? Only then you are going to get back $A \supset (A \bullet B)$. Can we do that? So let's start slowly. This is my beginning point, ok? And there is a reason why we have chosen this. The moment you assume A, the bent arrow starts. Now, what can we do with A? Well, we can easily plug it in with 1 and get B. Why? Because then we can put them together to get $A \bullet B$, right? So let's try that. So here is B, from 1 and 3 Modus Ponens.

Now we put 3 and 4 together and we get $A \bullet B$. And if we close our assumption here, because this is what I needed, we are going to get back what? $A \supset (A \bullet B)$. By what? By 3 through 5, CP. Once more reminding you this is an entirely an assumption block. So we have to refer to it as a block. Ok? There is no way we can refer to the individual lines. Is this where we stop the proof? No. There is a further line. 2 and 6 will give us what we are looking for; namely, C. Please note that line C is no longer dependent upon your A, right? The assumption has been closed.

So, this is a way to appreciate what the Strengthened version of CP can do for you. Did you understand this? So even when you are looking at this, ask yourself, what is it that we can assume? And because we are all beginners, so it is better to be a little cautious and try out on the margin to see if I assume this and if I solve for this, remember, when I close it I am going to get a horseshoe back. And do I know what to do with that horse shoe? You know, sometimes we are too eager to jump into the proof without thinking it thoroughly. So here is a point of caution, that instead of doing that, just, you know, work it out a little bit in the rough work and then come back and do the proof.

So this is one example of how it is done. Please note that apparently that we have done Conditional Proof on an argument which does not have a conditional as a conclusion. Point taken? So now, our scope, the possibility of application has really grown, increased. Ok? Second is that what was assumed, apparently is no part of the conclusion. It’s a purely convenient assumption with very pragmatic objective that we need to have A so that we can have A • B. This thinking is something that you need to get used to now. So what is it that I need in order to get C out? That is the kind of operational, pragmatic consideration that should guide you in the choice of your assumption, alright? And you are not going to get any clue from the premises, like in preliminary version you know there is a definite starting point; namely, the antecedent. There is no definite starting point here.(Refer Time: 16:08) But there is a lot of strategy and somewhat thinking involved in it. Alright?

And we will take a look into another example as we go along with this. So and this is when you should also start realizingthat you are now almost in a sort of a more advanced and mature way of doing proofs, right? Because CP has given you so much possibility now open to you. The proofs that you …that was so difficult to do earlier, now probably with the CP, you will be able to it in much faster way and more efficiently. So I will show you another example and then again I will ask you to do this, work this out on your own so that you get some habit or some practice with it.

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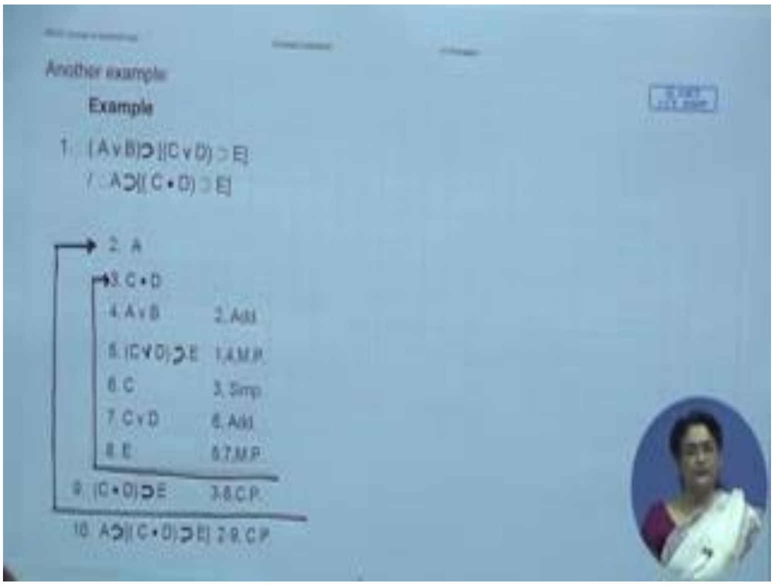

So here is the proof. One premise and that is this, and here is the conclusion. Now this conclusion is a conditional one. If you want, you can choose to have the preliminary version also. That is, you start with A and you solve for E. The Strengthened version does not limit you to just A. What you need to think is what is more efficient to assume here or what will give me the result quickly and correctly?

So take a good look, have a strategy first of all, and the place to work with the strategy is not in the proof space. Always do it in the margin. Draw a line and think a little about the proof how to go about it, what is it that you need? And I am sure most of you are already thinking that I will start with A. The question is why? You need to work not from the conclusion, but from the premises that you have. A will give you A $\vee$ B, right? Why? Why do you need that? You need because so that you can derive (C $\vee$ D) $\supset$ E. But my point is that what you need here… there is a slight difference between C $\vee$ D and this C $\bullet$ D. So you need to work on that a little bit before you can get the E and remember this is Conditional Proof. So if you let go, let go of the assumption, you need to remember that you are going to get a horseshoe statement back in the proof, right?

So have you figured it out where to start? Well, we'll start with A, if you wish we'll start with A. And the second step is obvious or not? When you are a little bit experienced in this, I will say that you need to think a little before. This is what I am saying that you do think on your margin. And all the assumptions that you need, you will learn soon that you can stack them right at the beginning; or you assume them only when you absolutely have to. So if you want, you can do this line 4 here instead of line 3 and another assumption you can do $A \vee B$ here also. But I chose to do it before because I knew sort of what I need to do. So this is actually the line that after 2 you can do. This is why you needed $A \vee B$. That $A \vee B$ will give you, as I said, $(C \vee D) \supset E$, right?

So how do I come from that, $(C \vee D) \supset E$ to $C \bullet D$? Think in a different way. Why do you have to come from $C \vee D$ to $C \bullet D$? Rather can we not go from $C \bullet D$ to $C \vee D$? That's easier. So this is the reason why the second assumption has made: $C \bullet D$. So that once you have obtained, this is 1 and 4 of course, no problem. This is 1 and 4 got you $(C \vee D) \supset E$. But now you also have 3, $C \bullet D$, from which we can do a simplification on that, and then add this. This is the most interesting line here. That on line 6 we can add this D, $C \vee D$ appears. Once $C \vee D$ appears, E is just a matter of Modus Ponens.

And now you see what happens. Now you see the power of this proof procedure. Once you have this, we close. What shall we get back? Well, the last in first out. So we are going to get back $(C \vee D) \supset E$ and here is 3 through 8, Conditional Proof.

Are we done? No, there is one unclosed or undischarged assumption. Now we are going to discharge that and if you discharge that this is how it's going to show up. But this is exactly what we need as the conclusion. Got it?

So this is how the Conditional Proof moves. If you were troubled and if you… if you still wondering that why I couldn't see that $C \bullet D$ can be assumed, that is because the procedure is very, very new. And you still have not realized the kind of power that you can wield using this procedure, right? So this is why I said that, you know, you take it in and it's a different kind of thinking process that you have to get used to. But you know there is always a beginning, there is always a first and so on. But this is what the Strengthened version of Conditional Proof is like. Ok?

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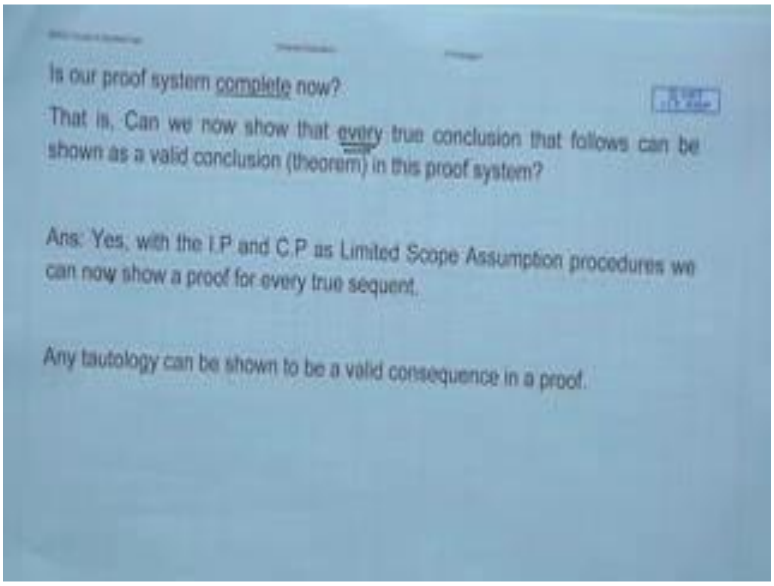

So we come back to the final point that the reason that why we included IP and CP was with the question is our proof system complete? And, we found out earlier that it was *not*. With the 19 rules it was not. And, that is the reason why we added these proof procedures IP and CP.

So with IP and CP, now we are complete. So our claim is now give us *any* true conclusion, including all tautologies, we should be able to give you a proof. And, that is what the whole enterprise was all about. This is why we came here. So that remains to be shown that give us any tautology, we can show it as a valid consequence in a proof. But that will be discussed in our next module. Right now, you concentrate on mastering this Conditional Proof and its Strengthened version, alright? So with that I will close this module. Thank you very much and thanks for your time.