INDIAN INSTITUTE OF TECHNOLOGY

Department of Mathematics

Date : Time : 3 hrs. Full Marks : 50, No. of Students : 25

Spring Semester, 2012, I Yr. M. Tech (CSDP) Sub. No. MA 60002

Subject Name : **Data Structures and Algorithms**

Answer all the questions. Marks corresponding to each question are shown at the end of each question. Follow standard notations that are defined in the lectures. There are 2 pages.

1. Solve the recurrence relation $t_n = 2t_{n-1} - t_{n-2} + 2^n$ for $n \geq 2$ and $t_0 = 1, t_2 = 2$ and estimate the upper bound as close as possible. [4]

2. An undirected graph containing nodes numbered 1 ... 11 has the following edges: (1, 4), (6,4), (1,6), (8, 7), (5, 7), (10, 9). Use $find(x)$ and $merge(a, b)$ functions of the disjoint set structures and find the number of connected components in the given graph. You must show the working of $find(x)$ and $merge(a, b)$ functions. [4]

3. Consider the 0-1 knapsack problem where the weights and the corresponding values of five objects respectively are given by w_i : 20, 10, 40, 50, 30; v_i: 100, 60, 140, 230, 120 . If the capacity of the knapsack is 60, find the optimal solution using dynamic programming. Show the table only for capacities in multiples of 10s. Compare the solution with the solution obtained by the greedy method. [4]

4. The inorder and postorder traversals of a binary tree are given by Y X W Z U R S V TO and X Z U W Y T O V S R respectively. What will be the corresponding binary tree that is unique. [4]

5. Find all possible solutions of $a^2 + b^2 + c^3 + d^3 = 101$ such that $a \geq b \geq c \geq d$, using back-tracking. Show the working. [4]

6. Let G be a digraph with positive edge weights c_{ij} corresponding to an edge (i, j) . Let P be the shortest path between two nodes u and v of the graph when c_{ij} are the costs. Prove or disprove that P is still the shortest path between u and v if all the edge costs are incremented by 1 unit. [4]

7. Consider the dynamic programming approach to compute the optimal number of multiplications required in performing the matrix chain multiplication of size 5 given by $A(30; 25)B(25; 15)C(15; 5)D(5; 20)E(20; 20)$. The numbers shown in the parenthesis denote the dimension of the respective matrix. Also, the entries in the table m_{ij} corresponding to $j = i + 1$ and $i = 1, 2, 3, 4$ are given by 1250 : 1875: 1500 : 2000. These are the optimal number of multiplications required corresponding to the sub chains $M_i \ldots M_j$. Compute the final solution by showing intermediate steps. [4]

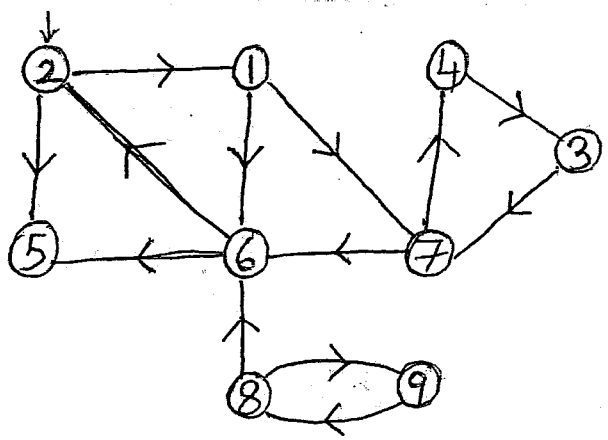
8. Given the following list of elements: 132, 6, 124., 45, 67, 12, 150, 18, 87, 4, 99, 5, 11, 29, partition the list using 67 as pivot so that all the elements to the left of 67 are smaller than 67 and all the elements to the right are greater or equal to 67. Show your working clearly. Can you use this partition technique to find the median of a given list of

elements? Suggest a method if so. [5]

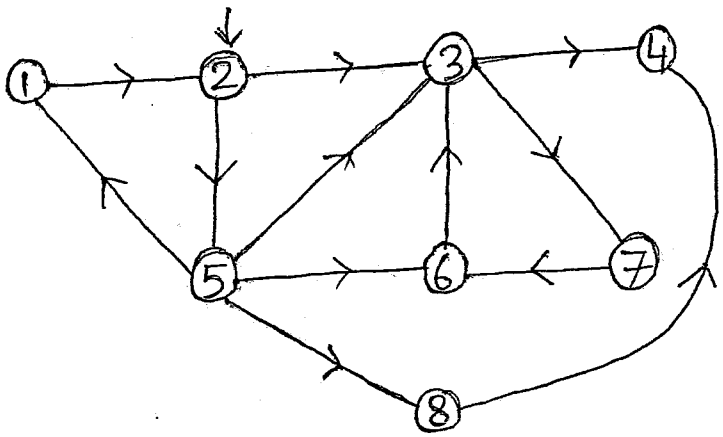
9. Consider a list of n numbers that may include negatives also. $Sum(i;j); 1 \leq i \leq j \leq n$ is summing all values from index i to j. Write a linear algorithm that computes i and j such that $Sum(i;j)$ is maximum. [5]

10. State Master's method to solve recurrence relations that generally occur in divide and conquer technique. Verify if this method could be applied to solve the following recurrence relations. (i) $T(n) = 16T(n/4) + n^{3.5}$, (ii) $T(n) = 25T(n/5) + 10n^3$. [6]

11. (i). With respect to DFS traversal on node 2, of the digraph given in Fig.1, classify the edges. (ii). Consider the digraph given in Fig. 2. Classify the edges using BFS-traversal starting at node 2. Identify all the articulation points. List all the strongly connected components. Shrink the graph such that the graph is acyclic. Note: Follow increasing numerical order for traversal both in (i) and (ii). [6]



← Fig.1



← Fig.2

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