

ASSIGNMENT – 1

Solve manually the following problems:

1. Given $\frac{dy}{dx} = \frac{1}{x^2 + y}$, $y(4) = 4$, find $y(4.2)$ by Taylor's series method of order 2, taking $h=0.1$.
2. Solve $\frac{dy}{dx} = 3x + y^2$, $y=1$, when $x = 0$, numerically for $x = 0.1$ by Taylor's series method of order 2.
3. Solve the differential equation $\frac{dy}{dx} = 2y + 3e^x$ with $x_0 = 0, y_0 = 0$, using Taylor's series method of order 2 to obtain and check the value of y for $x = 0.1, 0.2$.
4. Given $\frac{dy}{dx} = y - x$, where $y(0) = 2$, find $y(0.1)$ and $y(0.2)$ by Euler's method up to two decimal places.
5. Solve $y' = x - y^2$, by Euler's method for $x=0.2$ to 0.6 with $h = 0.2$ initially $x = 0, y = 1$.
6. Given that $\frac{dy}{dx} = x + y^2, y(0)=1$, find $y(0.2)$, by backward Euler's method.
7. Given $\frac{dy}{dx} = -\frac{y-x}{1+x}$, with initial condition $y(0) = 1$, find approximately y for $x = 0.1$, by backward Euler's method (two steps).
8. Use modified Euler's method with one step to find the value of y at $x = 0.1$ to five significant figures, where $\frac{dy}{dx} = x^2 + y, y=0.94$, when $x = 0$.
9. Using modified Euler's method, solve numerically the equation $\frac{dy}{dx} = x + |\sqrt{y}|$ with initial condition $y = 1$ at $x = 0$ for the range $0 \leq x \leq 0.4$ in steps of 0.2 .
10. Use Runge-Kutta method of order 2 to solve $y' = xy$ for $x = 1.4$, initially $x = 1, y = 2$ (by taking step-length $h = 0.2$).
11. Solve the differential equation $\frac{dy}{dx} = \frac{1}{x+y}$ for $x = 2.0$ by fourth-order Runge-Kutta method, given that $y(0)=1$, interval length $h = 0.5$.
12. Use fourth-order Runge-Kutta method to solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}, 0.1$, with $y(0)=1$ at $x = 0.2, 0.4$.
13. Using fourth-order Runge-Kutta method compute $y(0.2), y(0.4)$ from $10 \frac{dy}{dx} = x^2 + y^2, 0.1$, taking $h=0.1$.

Write computer code / use MATLAB software or any other software to solve the following problems: Also plot the solution curves.

1. Find $y(1)$ by Euler's method from the differential equation $\frac{dy}{dx} = \frac{-y}{1+x}$ when $y(0.3) = 2$.

Convert up to four decimal places taking step length $h = 0.1$.

2. Given $\frac{dy}{dx} = x^2 + y$, with $y(0) = 1$, evaluate $y(0.02)$, $y(0.04)$ by backward Euler's method.

3. Find $y(4.4)$, by modified Euler's method taking $h = 0.2$ from the differential equation

$$\frac{dy}{dx} = \frac{2-y^2}{5x}, \text{ given that } y=1 \text{ when } x=4.$$

4. For the equation $\frac{dy}{dx} = 3x + \frac{y}{2}$, $y(0)=1$, find y at $x=0.1, 0.2$ with step-length 0.1, using mid-point method.

5. Use the Runge-Kutta method of order 2 to approximate y at $x = 0.1$ and $x = 0.2$ for the equation $\frac{dy}{dx} = x + y$.

6. Use implicit Runge-Kutta method with 2 slopes to calculate the value of y at $x = 0.1$, to five decimal places after a single step of 0.1, if $\frac{dy}{dx} = 0.31 + 0.25y + 0.3x^2$ and $y = 0.72$ when $x = 0$.

7. Find by implicit Runge-Kutta method with 2 slopes, an approximate value of y for $x=0.8$, given that $y=0.41$ when $x=0.4$ and $\frac{dy}{dx} = \sqrt{x+y}$. Take $h=0.4$.

8. Solve the equation $\frac{dy}{dx} = x - y^2$, $y(0)=1$ for $x = 0.2$ and 0.4 to 4 decimal places by fourth-order Runge-Kutta method.

9. $\frac{dy}{dx} = -\frac{y^2 - 2x}{y^2 + x}$, use fourth-order Runge-Kutta method to find y at $0.1, 0.2, 0.3, 0.4$, given that $y=1$ when $x=0$.

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