INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Date of Examination:.....-02-2013, FN/AN, Time: 2 Hours, Full Marks: 30

Mid-Spring Semester 2013, Dept. of Mathematics, Branch: B.Tech/M.Sc

Subject No.:MA30014, Subject Name: Operations Research

Number of Students: 83, Instructions: Answer all (3 x 10= 30 marks).

If needed Graph papers will be supplied in the Examination Hall.

Q1. A 24-hour super market has the following minimal requirement for salespersons. Period 1 follows immediately after Period 6. A salesperson works eight consecutive hours starting at the beginning of one of the six periods. Formulate a linear programming model for a daily salesperson worksheet which satisfies the requirement with least number of personnel.

Period	1	2	3	4	5	6
24 Hour Clock	24--4	4--8	8--12	12--16	16--20	20--24
Minimum No.	4	6	8	10	8	6

Q2.Find the optimal solution(s) of the given linear programming problem:

$$\text{Min: } Z = 6x_1 + 4x_2 + 10x_3$$

$$\text{subject to } \quad 3x_1 + 2x_2 + 5x_3 \geq 90, \quad x_1, x_2, x_3 \geq 0.$$

How many optimal solution(s) can be obtained ? Present the optimal solution(s) mathematically.

Q3. Find the optimal solution(s) of the linear programming problem:

$$\text{Min: } Z = 3x_1 + 51x_2 + 18x_3$$

$$\text{subject to } \quad x_1 + 17x_2 + 6x_3 \geq 24, \quad x_1, x_2, x_3 = 0, 1, 2, 3, \ldots.$$

How many optimal solution(s) can be obtained? Find the optimal solution(s).

Q4. Prove that feasible region of the given LPP form a convex set.

$$\text{Min: } Z = C^T X, \quad \text{subject to } \quad AX \geq b,\ X \geq 0.$$

Q5. Solve the LPP by Simplex method ~~method~~:

$$\text{Max: } Z = 3x_1 + 13x_2 + 13x_3$$

$$\text{subject to } \quad x_1 + x_2 \leq 7, \quad 2x_2 + 3x_3 \leq 15$$

$$x_1 + 3x_2 + 2x_3 \leq 15, \quad x_1, x_2, x_3 \geq 0$$

Q6. Solve the LPP by Two-Phase Simplex method:

$$\text{Min: } Z = 12x_1 + 13x_2 + 14x_3$$

$$\text{Subject to } \quad 2x_1 + 6x_2 + 5x_3 \geq 120$$

$$4x_1 + 2x_2 + x_3 \geq 100, \quad x_1, x_2, x_3 \geq 0$$

Q7. State the Basic difference between Simplex method and revised simplex method. Explain with an example.

Q8. Find the product form of inverse of the matrix M,

where $M=\begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

Show that $M^{-1} = E_3 E_2 E_1$.

Q9. Find the Dual of the given LPP:

Max: $Z = c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4$

Subject to $a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 = b_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 \le b_2$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 \ge b_3$

x_1, x_2, x_3, x_4 are free variables.

Q10. Max: Z= $x_1 + x_2 - x_3$

Subject to

$3x_1 - 3x_2 + 9x_3 \le 3$

$3x_1 + 5x_2 - 3x_3 \le 1$

$6x_1 + 2x_2 - 2x_3 \le 1$

$x_1, x_2, x_3 \ge 0$

Is $(\frac{3}{16}, \frac{5}{16}, \frac{3}{8})$ this point an optimal solution of the LPP ?

Justify with your answer.

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