Problem Set-8

AUTUMN 2016

MATHEMATICS-I(MA10001) *October 3, 2016*

1. Find the integrating factor and hence solve the following ordinary differential equations:

(i) $\left(y+\frac{y^3}{3}+\frac{x^2}{2}\right)dx+\left(\frac{x}{4}(1+y^2)\right)dy=0.$

(ii) $dx+e^{x-y}dy=0.$

(iii) $\left(x^3+y^3+1\right)dx+xy^2dy=0.$

(iv) $\left(2y^3xe^y+y^2+y\right)dx+\left(y^3x^2e^y-xy-2x\right)dy=0.$

(v) $y(1+xy^2)dx+2(x^2y^2+x+y^4)dy=0.$

(vi) $\left(x^2+y^2\right)dx-2xydy=0.$

(vii) $y^2dx+x(x-y)dy=0.$

(viii) $\frac{y}{x}dx-dy=0.$

2. Show that $F(x,y)$ is integrating factor of $M(x,y)dx+N(x,y)dy=0$ if and only if

$$\left(M\frac{\partial F}{\partial y}-N\frac{\partial F}{\partial x}\right)+\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)F=0.$$

3. Verify that $e^{\int p(x)dx}\left[dy+\left(p(x)y-q(x)\right)dx\right]=0$ is exact and hence solve it.

4. Solve the following ordinary differential equations:

(i) $x^2y'-y=2sin\frac{1}{x}.$

(ii) $y'+2xy=e^{-x^2}, \quad y(0)=1.$

(iii) $(1+x^2)y'+2xy=xsinx.$

(iv) $xy'-3y=x^4\left(e^x+cosx\right)-2x^2, \quad y(\pi)=\pi^3e^\pi+2\pi^2.$

5. Solve the following ordinary differential equations:

(i) $x\frac{dy}{dx}-y+xy^2=0.$

(ii) $x^3y^{'}-x^2y+y^4cosx=0.$

(iii) $y^{'}+y=xy^{5/3}.$