# Problem Set - 10

**AUTUMN 2016**

**ANSWER/HINTS**

MATHEMATICS-I (MA10001)

a. (Hint. Find $y_1, y_2$ solutions of Homogeneous equation then find $y_p = v_1 y_1 + v_2 y_2$).

(i) $y = C_1 \cos 2x + C_2 \sin 2x - \cos 2x \ln(\sec 2x + \tan 2x)$

(ii) $y = C_1 e^x + C_2 e^{-x} - 1 - xe^x + (e^x - e^{-x}) \ln(1 + e^x)$

(iii) $y = C_1 e^x + C_2 e^{2x} + e^x \ln(e^{-x} + 1) - e^{-x} - xe^{2x} + e^{2x} \ln(1 + e^x)$

(iv) $y = C_1 + C_2 e^{2x} - \frac{1}{2} e^x \cos x$

b. (Hint. Put $z = \ln x$)

(i) $y = C_1 + C_2 \ln x + 2(\ln x)^3$

(ii) $y = (C_1 + C_2 \ln x)x + \frac{C_3}{x} + \frac{\ln x}{4x}$

(iii) $y = x^2(C_1 \cos(\ln x)) + C_2 \sin(\ln x) - \frac{\ln x}{2} x^2 \cos(\ln x)$

(iv) $y = (C_1 + C_2 \ln x) \cos(\ln x) + (C_3 + C_4 \ln x) \sin(\ln x) + (\ln x)^2 + 2(\ln x) - 3$

(v) $y = y_h + y_p$, $y_h = C_1 e^{(2+\sqrt{3}) \ln x} + C_2 e^{(2-\sqrt{3}) \ln x}$
$y_p = \frac{e^{-\ln x}}{6} + e^{-\ln x} \left[ \frac{5 \ln x \sin(\ln x) + 6 \ln x \cos(\ln x)}{61} + \frac{382 \cos(\ln x) + 54 \sin(\ln x)}{61} \right]$

c. (i) $x = (C_1 + C_2 t)e^t$, $y = -\frac{1}{2} e^x (C_1 + C_2 t) - \frac{C_2}{4} e^t$

(ii) $y = C_1 + C_2 e^{-2x} + e^x$, $z = C_1 - C_2 e^{-2x} + e^x$

(iii) $x = x_c + x_p$, $x_c = C_1 e^{-3t} + C_2 e^{-2t}$, $x_p = \frac{e^{3t}}{30} + \frac{2e^{3t}}{15}(t - \frac{11}{30}) - \frac{1}{12} - \frac{5}{104} \sin 2t - \frac{1}{104} \cos 2t$
$y = -\frac{dx}{dt} - 4x + te^{3t}$

(iv) $x = \frac{1}{2} e^{2t} - \frac{1}{11} e^t + C_1 e^{-\frac{6}{5} t}$, $y = \frac{9}{5} e^{2t} - \frac{8}{55} te^t + C_2 e^t + \frac{8C_1}{121} e^{-\frac{6}{5} t}$

d. (i) $y(x) = -e^{-3x}(\frac{1}{30} \cos x + \frac{13}{30} \sin x) + \frac{\sin 2x}{15} + \frac{\cos 2x}{30}$

(ii) $y(x) = -\frac{1}{32} e^{4x} + \frac{1}{32} e^{-4x} + \frac{1}{4} xe^{4x}$

(iii) $y(x) = C_1 e^{2x} + C_2 e^{-x} + C_3 e^x$

(iv) $y(x) = C_1 + C_2 x + C_3 x^2 + C_4 e^x + \frac{e^{2x}}{8}$

(v) $y(x) = e^x(C_1 \cos x + C_2 \sin x) + e^x(x^2 - 2)$

(vi) $y = C_1 + C_2 x^2 + \frac{x^3}{3} + \frac{1}{2} x^2 \ln x,\ Hint.:\ Put\, z = \ln x$

(vii) $y(x) = xe^{-2x} - e^{-2x} \ln x$

(viii) $y(x) = C_1 e^x + C_2 e^{-4x} - \frac{14}{100}(8 \sin 2x + 6 \cos 2x)$

(ix) $y = C_1 x^2 + C_2 x^3 + \frac{1}{2x^4},\ Hint.:\ Put\, z = \ln x$

(x) $y = C_! e^x + C_2 xe^x + C_3 e^{2x} - (x + 4)$

e. (i) $y = y_h + y_p,\ y_h = C_1 \cos x + C_2 \sin x,\ y_p = -\frac{x^2}{3} \sin 2x - \frac{8}{9} \cos 2x + \frac{26}{27} \sin 2x$

(ii) $y = C_1 e^{-x} + e^x (C_2 \cos 2x + C_3 \sin 2x) + \frac{xe^x}{16} \sin 2x$

(iii) $y = C_1 + C_2 e^{-2x} + C_3 e^{3x} - \frac{1}{6}(\frac{x^3}{3} - \frac{x^2}{6} + \frac{25}{8}x)$

(iv) $y = (C_1 + C_2 x)e^x + (C_3 + C_4 x)e^{-x} + \frac{\cos x}{4}$

(v) $y = C_1 e^{-x} + C_2 \cos x + C_3 \sin x + \frac{1}{15}(2\cos 2x - \sin 2x)$

(vi) $y = (C_1 + C_2 x)e^x + 1 + \frac{1}{2}e^{-x} + \frac{1}{9}e^{-2x}$

(vii) $y = e^{\frac{2}{3}x}(C_1 \cos \frac{\sqrt{11}}{3}x + C_2 \sin \frac{\sqrt{11}}{3}x) + \frac{e^x}{4} - \frac{2e^{2x}}{9} + \frac{3e^{3x}}{20}$

(viii) $y(x) = C_1 e^x + C_2 e^{-x} + \frac{3}{4}xe^{2x} + xe^{-2x}$

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