Test – I

Mathematical Methods

23. 9. 20

Time – 1hr

FM – 15

Instructions are given at the end. Please read that first.

Q1. Consider the following problems and in each case find the largest interval for existence of unique solution.

(a) $y' - (\tan x)\, y = 3t, \quad y(2\pi) = 0$ — 1M

(b) $(x^2-81)\, y' + 5e^{3x} y = \sin x \quad y(1) = 10\pi$ — $\frac{1}{2}$M

(c) $(x^2-81)\, y' + 5e^{3x} y = \sin x \quad y(10\pi) = 1$ — $\frac{1}{2}$M

(d) $(x^2-81)\, y' + 5e^{3x} y = \sin x \quad y(-100) = 5$ — $\frac{1}{2}$M

(e) $(x^2-81)\, y' + 5e^{3x} y = \sin x \quad y(-9) = 88$ — $\frac{1}{2}$M

Q2. Find a particular integral of the ODE

$$y'' - 2y' + y = \frac{e^t}{1+t^2} + 3e^t$$ 2M

Q3. Consider the 2nd order linear ODE

$$a_0(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)\, y = 0$$

where $a_0(x), a_1(x), a_2(x)$ are continuous in $[a,b]$, $a_0(x) \neq 0$. Suppose that it can be transformed into equivalent self adjoint eqn. $\frac{d}{dx}\left\{p(x)\frac{dy}{dx}\right\} + q(x)\, y = 0$

where $p(x) = \exp\left\{\int \frac{a_1(x)}{a_0(x)}\, dx\right\}$. Then find $q(x)$. 1M

Apply this theory to transform the following equations into equivalent self adjoint equation

(i) $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$ — 1M

(ii) $\frac{d^2y}{dx^2} - (\tan x)\, y' + y = 0$ — 1M

Q4. Consider the following set of functions in appropriate domain. Justify with reasons if they can be linearly independent solutions of an ordinary differential equation. If yes, determine the ODE.

(a) $x^2, \quad x^2 \log x$

3M

(b) $x, \; x^2, \; x^3$

Q5. Find Green's function for the BVP

$y'' - y = x \qquad y(0) = y(1) = 0$ 4M

Using that, find the solution of the BVP.

*** The End ***

Instructions

Start solving the questions from the 2nd page and write all the answers in the 1st page, only answers. While solving, you may jump steps; but you have to show the solution. At the end, take photo of all the pages and upload.