Problem Set - 12

AUTUMN 2016

ANSWER/HINTS

MATHEMATICS-I (MA10001)

1. $-156 + 38i$ (the integral is path independent).
2. $\frac{4\pi e^2}{3}$ (use $|\int_\Gamma \frac{e^z}{z^2+1}dz| \leq \int_\Gamma |\frac{e^z}{z^2+1}|dz$ and $|\frac{e^z}{z^2+1}| \leq \frac{e^3}{3}$).
3. The maximum value of $\frac{1}{|z|^2}$ on C is 1 and the arc length of C is 2.
4. (i) (a) Alone the curve C in (a), $\frac{2}{3}(1+i)$ (parametrize the line segment C).
   (b) Along the curve C in (b), $1+i$.
   (ii) (a) $-1+i$ (parametrize the line segment).
   (b) $-1+i$.
5. (a) $\frac{8+i}{3}$ (parametrize the line segment C).
   (b) $\frac{70+91i}{30}$ (parametrize the line segment C).
6. By Cauchy's theorem.
7. 0 (use Cauchy's theorem).
8. $i-1$.
9. (a) 0
   (b) $4\pi i$.
10. (a) $\frac{-4+8i}{3}$.
11. (a) e^2 (use Cauchy integral formula)
    (b) 0 (use Cauchy't theorem).
12. (a) $-2\pi i$ (use Cauchy integral formula)
    (b) 0 (use Cauchy's theorem).
13. $-\pi i$ (use Cauchy integral formula).
14. 0 (use Cauchy integral theorem).
15. The maximum value of $|\frac{1}{\bar{z}^2+\bar{z}+1}|$ is$\frac{1}{5}$ on C and the arc length is $\frac{6\pi}{4}$.
16. Use Cauchy integral formula.
17. Apply Cauchy Integral theorem and the fact that e^z is entire function.
18. $\frac{\pi i}{2}$ (Apply Cauchy Integral formula).
19. $\frac{\pi i}{2}$ (Use Cauchy Integral formula)