Problem Set - 7

AUTUMN 2016

ANSWER/HINTS

MATHEMATICS-I (MA10001)

1. (i) Linear, second order, degree one.
   (ii) Non linear, second order, degree one.
   (iii) Non linear, first order, degree one.
   (iv) Non linear, first order, degree one.
   (v) Non linear, first, degree two.
   (vi) Non linear, second order, degree two.

2. (i) $y'' + m^2 y = 0$.
   (ii) $y'' - 3y' + 2y = 0$.
   (iii) $(1 + (y')^2)^3 = r^2 (y'')^2$.
   (iv) $(3x^2 - y^2) y y' = x(3y^2 - x^2)$.

3. (i) $y = Ax^{b/a}$.
   (ii) $y = Ae^{-x} - 1$.
   (iii) $y = \frac{A}{(x^2+x+1)^3}$.
   (iv) $\frac{x}{y^3} = Ae^{3x}$.

4. (i) Set $u = y'$, then $\frac{du}{u} + \frac{dx}{x} = 0$
   $\Rightarrow y = A \ln x + B$.
   (ii) Set $u = e^y y y'$
   $\Rightarrow u' = 0$,
   $\Rightarrow (y - 1)e^y = Ax + B$.

5. (i) $(y^2 + 4y) = 2(1 - \cos x)$.
   (ii) $y = (\ln x)^2$.

6. (i) Set $v = 2x - y + 1$, then $\frac{dv}{2-v^2} = dx$
   $\Rightarrow (2x - y + 1 + \sqrt{2}) = A(\sqrt{2} - 2x + y - 1)e^{2\sqrt{2}x}$.
   (ii) Set $y = vx \Rightarrow \frac{dv}{(v-1)^2} = \frac{dx}{x}$,
   $\Rightarrow (x - y)\ln(Ax) - x = 0$.
   (iii) $y' = \frac{3y^2+2xy}{x^2+2xy}$, set $y = vx \Rightarrow \frac{(1+2v)dv}{v^2+v} = \frac{dx}{x}$, so $y^2 + xy = Ax^3$.
   (iv) $y' = \frac{3-4x-2y}{2x+y-1}$, set $2x + y = v \Rightarrow (v - 1)dv = dx$, so $(2x + y)(2x + y - 2) - 2x = A$.
   (v) $y' = \frac{y+3}{x+y+2}$, set $x = X + h$, $y = Y + k$ such that $k + 3 = 0$, $h + k + 2 = 0$
   $\Rightarrow \frac{dY}{dX} = \frac{Y}{X+Y}$. Set $Y = vX$,
   $\Rightarrow \frac{(1+v)dv}{v^2} = -\frac{dX}{X} \Rightarrow y + 3 = Ae^{\frac{x-1}{y+3}}$.
   (vi) $\frac{dx}{dy} = \frac{x^2+y^2 e^{\frac{x^2}{y^2}}}{xy}$, set $x = vy$ then $\frac{vdv}{e^{v^2}} = \frac{dy}{y}$
   $\Rightarrow \ln y^2 + e^{\frac{x^2}{y^2}} = A$.

7. (i) $\frac{\partial M}{\partial y} = 1$, $\frac{\partial N}{\partial x} = 1 + y$, Not exact.
   IF$=\frac{1}{xy}$,
   Solution is $xye^y = A$.
   (ii) $\frac{\partial M}{\partial y} = -\sinh x \sin y$, $\frac{\partial N}{\partial x} = -\sinh x \sin y$
   $\Rightarrow$ Exact, $\exists\ f(x, y)$ such that $df = Mdx + Ndy$.
   $\frac{\partial f}{\partial x} = M \Rightarrow f(x, y) = \cosh x \cos y + h(y)$
   $\frac{\partial f}{\partial y} = -\cosh x \sin y + h'(y) = N$
   $\Rightarrow h'(y) = 0 \Rightarrow h(y) = A$,
   so $f(x, y) = \cosh x \cos y + A$.
   (iii) Exact, $f(x, y) = e^{xy} + y^2 + A$.
   (iv) Exact, $f(x, y) = (e^{2y} + 1)\sin x + A$.

8. M(x,y)dx+N(x,y)dy=0 is exact iff $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$
   iff $\frac{\partial}{\partial y}[M(x, y) + g(x)] = \frac{\partial}{\partial x}[N(x, y) + h(y)]$.