MA10002 Mathematics-II : Tutorial Sheet - 8

1. Evaluate $\iint x^2y^2 \, dxdy$ over the circle $x^2+y^2 \leq 1$.

2. Evaluate $\iint\limits_R xy \, dxdy$, where R is the domain bounded by the x-axis, ordinate $x = 2a$, and the curve $x^2 = 4ay$.

3. Evaluate $\iint \frac{r \, dr d\theta}{\sqrt{a^2+r^2}}$ over loop of the lemniscates $r^2 = a^2 \cos \, 2\theta$.

4. Evaluate $\iint r^3 \, dr d\theta$ over the area included between the circles $r = 2a\cos\theta$, $r = 2b\cos\theta$, where $b < a$.

5. Evaluate $\int\limits_0^\infty \int\limits_0^\infty e^{-(x^2+y^2)} \, dydx$ by changing to polar coordinates. Hence, deduce that $\int\limits_0^\infty e^{-x^2} \, dx = \frac{\sqrt{\pi}}{2}$.

6. Evaluate $\iint \sqrt{\frac{a^2b^2-b^2x^2-a^2y^2}{a^2b^2+b^2x^2+a^2y^2}} \, dxdy$ over the positive quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$.

7. Use the transformation $x+y = u$ and $y = uv$ to show that $\int\limits_0^1 \int\limits_0^{1-x} e^{\frac{y}{x+y}} \, dydx = \frac{e-1}{2}$.

8. Changing the order of integration, find the value of the integral $\int\limits_0^\infty \int\limits_x^\infty \frac{e^{-y}}{y} \, dydx$.

9. Evaluate the following integrals by changing the order of integration.
   (i) $\int\limits_0^{4a} \int\limits_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx$ (ii) $\int\limits_0^1 \int\limits_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}} dydx$ (iii) $\int\limits_0^\infty \int\limits_0^x xe^{\frac{-x^2}{y}} \, dydx$.

10. Find the area lying between the parabola $y^2 = 4ax$ and $x^2 = 4ay$.

11. Find the area of the cardioid $r = a(1+\cos\theta)$.

12. Find the volume contained between the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} = 1$ and the cylinder $\frac{x^2}{a^2}+\frac{y^2}{b^2} = \frac{x}{a}$.

13. Find the volume bounded by the cylinder $x^2+y^2 = 4$ and the planes $y+z = 4$ and $z = 0$.

14. Find the area of the surface of the paraboloid $x^2+y^2 = z$, which lies between the planes $z = 0$ and $z = 1$.

15. Find the area of the paraboloid $2z = \frac{x^2}{a}+\frac{y^2}{b}$ inside the cylinder $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$.

16. Evaluate $\iiint \frac{dxdydz}{(x+y+z+1)^3}$ over a terahedron bounded by coordinate planes and the plane $x+y+z = 1$.

17. Evaluate the triple integral $\int\limits_0^a \int\limits_0^{\sqrt{a^2-x^2}} \int\limits_0^{\sqrt{a^2-x^2-y^2}} \frac{1}{\sqrt{a^2-x^2-y^2-z^2}} dzdydx$.

18. Evaluate $I = \iiint \sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}} \, dxdydz$ over the region $V = \{(x,y,z); x \geq 0, y \geq 0, z \geq 0, \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} \leq 1\}$.

19. Evaluate $I = \iiint\limits_V (x^2+y^2+z^2)^m \, dxdydz, m > 0$ over the region $V = \{(x,y,z); x^2+y^2+z^2 \leq 1\}$.

20. Find the volume of the portion cut off from the sphere $x^2+y^2+z^2 = a^2$ by the cylinder $x^2+y^2 = ax$.