INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR
MID SEMESTER EXAMINATION

Date : -2-2012 FN Time : 2 hours Full Marks : 40
Spring Semester : 2011-2012 Department : Mathematics
Subject No.:MA20104 Subject Name : Probability & Statistics
Course : B.Tech./M.Sc. 2nd Year (AG, BT, CE, CS, HS, IM, MA, MI, MF, breadth, additional)
No. of Students : 600

Instructions: Answer all questions. Marks are indicated at the end of each question.

1(a) Football clubs F_1 and F_2 are set to play a series of three games against each other to decide the league champion. The probabilities of Club F_1 winning, drawing and losing a game against Club F_2 are $1/2$, $1/8$ and $3/8$ respectively. A club gets 3 points for a win, 1 for a draw and 0 for a loss. What is the probability that (i) Club F_1 wins the league (ii) Club F_2 wins the league, (iii) the league will end in a tie? 3M

(b) Let B and C be disjoint events. If A and B are independent, and A and C are independent, prove that A and $B \bigcup C$ are independent. 1M

(c) Let A and B be independent, B and C be independent. If A and $B \bigcap C$ are independent show that C is independent of $A \bigcap B$. 1M

2(a) In a production line ICs are packed in vials of 5 and sent for inspection. The probabilities that the number of defectives in a vial is 0,1,2,3 are $1/3$, $1/4$, $1/4$, $1/6$ respectively. Two ICs are drawn at random from a vial and found to be good. What is the probability that all ICs in this vial are good? 3M

(b) Two decks of 52 cards are mixed and well shuffled. The cards are randomly distributed to two players A and B, each one getting 52 cards. What is the probability that the player B does not get a king of hearts given that Player A gets it? 2M

3(a) Let X be a continuous random variable with the pdf

$$f(x) = \begin{cases} \dfrac{b\,x}{(1+x^2)}, & \sqrt{e-1} < x < \sqrt{e^2-1}, \\ 0, & \text{otherwise.} \end{cases}$$

Find $b, P(2 < X^2 < 2.5)$ and $V(X)$. 3M

(b) Let X be a random variable with moment generating function

$$M_X(t)=\left(\frac{2e^t}{3-e^t}\right)^4.$$

Find $P(5\le X\le 7)$ and $E(X)$. 2M

4(a) Let X be a continuous random variable with pdf

$$f(x)=\begin{cases} px, & 0<x\le 1,\\ p, & 1<x\le 2,\\ p(3-x), & 2<x<3,\\ 0, & \text{otherwise.}\end{cases}$$

(i) Find the value of p. (ii) Find the cdf of X. (iii) Suppose X_1, X_2, X_3 are three independent observations of X. What is the probability that exactly one of them is greater than $3/2$? 3M

(b) Use Chebyshev's inequality to determine how many times a fair coin be tossed so that the probability that the proportion of the number of heads lies between 0.45 to 0.55 is at least 0.95. 2M

5(a) The average number of accidents during 9:00 a.m. to 9:00 p.m. is thrice the average number during 9:00 p.m. to 9:00 a.m. at a traffic junction. Given no accidents are recorded during a day (24 hour period), the conditional probability that the recording time was between 9:00 p.m. to 9:00 a.m. is thrice the conditional probability that the recording time was between 9:00 a.m. to 9:00 p.m. What is average number of accidents during the day (24 hour period)? 3M

(b) Questions are asked to Girish in a quiz competition one by one until he fails to answer. The probability of his answering a question is p. The probability that he will quit after answering an odd number of questions is 0.9. Find the value of p. 2M

6(a) The lives (in years) of components in a mechanical system are independently distributed with the i^{th} component life having density function

$$f_i(x)=\frac{1}{2^i}\exp\left(-\frac{x}{2^i}\right),\quad x>0,\quad i=1,2,3.$$

What is the probability that exactly two systems are working at the end of 8 years? 3M

(b) State and prove the memoryless property of an exponential distribution. 2M