INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR Date of Examination:.....-02-2017, FN/AN, Time: 2 Hours, Full Marks: 30 Mid-Spring Semester 2017, Department of Mathematics, Branch: All

Subject No.:MA30014, Subject Name: Operations Research Number of Students: 130, Instructions: Answer all the questions(6 x 5=30).

Q1. (a) Find all the Basic Solutions and Basic Feasible Solutions of the system:

$$x_1 + 2x_2 + x_3 + 2x_4 + 3x_5 = 18$$
  
 $x_1 + 2x_2 + 3x_3 + x_4 + 2x_5 = 24$ 

(b) Find the optimal solution of the LPP using the Basic Feasible Solutions.

Max: 
$$Z = x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5$$
  
Subject to
$$x_1 + 2x_2 + x_3 + 2x_4 + 3x_5 = 18$$

$$x_1 + 2x_2 + 3x_3 + x_4 + 2x_5 = 24, \quad x_1, x_2, x_3, x_4, x_5 \ge 0$$

Q2. Solve the LPP by Simplex method( use condensed Tableau):

Max: 
$$Z = 3x_1 + 5x_2 + 4x_3$$
  
Subject to  $x_1 + 2x_2 + 3x_3 \le 72$   
 $3x_1 + x_2 + 2x_3 \le 66$ ,  $3x_1 + 2x_2 + x_3 \le 60$ ,  $x_1, x_2, x_3 \ge 0$ 

Q3. Solve the LPP by Big-M (Charne's Penalty) method:

Min: 
$$Z = 3x_1 + 2x_2 + x_3$$
  
Subject to  $2x_1 + x_2 + x_3 \ge 20$ ,  $x_1 + 3x_2 + x_3 \ge 30$   
 $4x_1 + x_2 + 2x_3 \ge 40$ ,  $x_1, x_2, x_3 \ge 0$ 

Q4. Solve the LPP by Two- Phase Simplex method. Then state the values of the optimal primal variables. Use condensed Tableau only.

Min: 
$$Z = 5x_1 + 6x_2 + 8x_3$$
  
Subject to  $2x_1 + 6x_2 + 5x_3 \ge 40$   
 $4x_1 + 2x_2 + x_3 \ge 50, x_1, x_2, x_3 \ge 0.$ 

- Q5. Establish the relationship between a Primal Linear Programming Problem (P) and a Dual Linear Programming Problem (D) using Lagrange multiplier method.
- Q6. Prove that minimum value of linear function  $Z = C^T X$ , defined over a convex region  $AX \ge b, X \ge 0$  if exists to be found at the extreme point(s) of the convex region.

