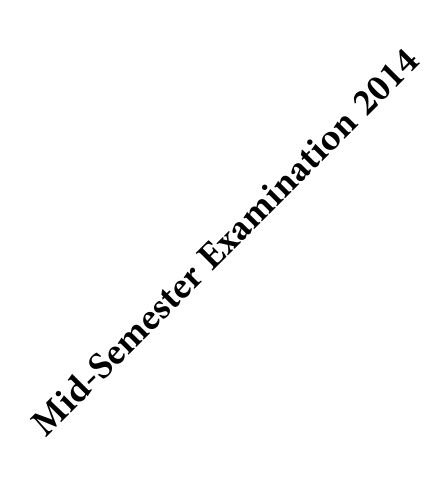
- 1(a) Apply mean value theorem to prove that $\left(1-\frac{1}{x}\right)^x$, x>0 is an increasing function.
- 1(b) Is Rolle's theorem applicable to the function

$$f(x) = \begin{cases} 1 - x^2, & x \le 0\\ \cos x, & x > 0 \end{cases}$$

in $\left[-1,\frac{\pi}{2}\right]$? Justify your answer with proper arguments. If Rolle's theorem is applicable, then find $c\in\left(-1,\frac{\pi}{2}\right)$ such that f'(c)=0.

[2+2]



2(a) Expand the function $f(x,y)=x^y$ in powers of (x-1) and (y-1) upto terms including 2^{nd} order (without remainder). Use the result to compute the approximate value of $(1.1)^{1.02}$.

2(b) Find
$$\lim_{x\to 0} \left(\frac{1}{2x} - \frac{1}{x(1+e^x)}\right)$$
.

[2+2]

Mid. Semester Examination 2014

3(a) Find the intervals for which the function $f(x) = \ln(x^2 + 1)$ is convex upwards (concave downwards) and convex downwards (concave upwards). Further, find the point of inflexion(s).

3(b) Using
$$\epsilon - \delta$$
 approach, show that $\lim_{(x,y) \to (1,1)} 2x^2 + 3y = 5$.

[2+2]

Mid. Semester Examination 2014

4(a) Find the radius of curvature of the curve $x = 6t^2 - 3t^4$, $y = 8t^3$ at an arbitrary point t. Evaluate its maximum value over $t \in [0, 1]$.

4(b) Find the asymptote(s) of the curve $x^3 - x^2y + axy + a^3 = 0$.

[2+2]

Mid. Semester Examination 2014

5(a) Is the function

$$f(x,y) = \begin{cases} \frac{2x^3 + 3y^3}{x^2 + y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

differentiable at the origin? Justify your answer.

5(b) Is the partial derivative with respect to x of the function

$$f(x,y) = \begin{cases} \frac{x^3y}{x^6 + y^2}, & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

continuous at the origin? Justify your answer.

Mid-Semester Examination 201A [3+2] 6(a) Let

$$u = \sin^{-1}\left(\frac{x^{\frac{5}{2}} - y^{\frac{5}{2}}}{x^2 + y^2}\right), (x, y) \neq (0, 0)$$

Find the value of $x^2u_{xx} + y^2u_{yy} + 2xy u_{xy}$ as a function of u.

6(b) If z = f(u, v), u = x + 4y, v = -x - 4y and f has continuous first and second order partial derivatives, then find the relation between z_{xx} and z_{yy} .

[2+2]

Mid. Semester Examination 201A

7(a) Using the Lagrange multiplier method, find the minimum value of the function

$$f(x,y) = x^2 + y^2$$

subject to

$$x^2 - xy + y^2 = 48.$$

7(b) If the curves f(x,y)=0 and $\varphi(x,y)=0$ touch each other at the point P, then evaluate

$$\frac{\partial f}{\partial x}\frac{\partial \varphi}{\partial y} - \frac{\partial f}{\partial y}\frac{\partial \varphi}{\partial x}$$

at the point P.

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