No queries will be entertained during examination

Indian Institute of Technology, Kharagpur

Date ………FN/AN,	Time: 2 hrs,	Full Marks 30,	Deptt : Mathematics
No. of students 60	Year 2016		Mid Semester Examination
Sub. No.: MA31007	Sub. Name: Mathematical Methods		M. Sc./ M. Tech (Dual)

<u>READ THE INSTRUCTIONS CAREFULLY FOR EACH QUESTION AND FOLLOW THE EXACT STEPS ASKED FOR. ATTEMPT ALL QUESTIONS.</u>

1.(a) Find all the singular points of the differential equation and determine their nature

$$(x^5 + x^4 - 6x^3)y'' + x^2y' + (x-2)y = 0$$

(b) Use the method of Frobenius to find solutions of the differential equation

$$2x^2y'' + xy' + (x^2 - 3)y = 0$$

around $x = 0$. If R denotes the radius of convergence of your series solution, then write down the interval of convergence. **[2+4]**

2. (a) Prove that $J_n(x)$ and $J_{-n}(x)$ are linearly dependent, where n is an integer and $J_n(x)$ denotes the Bessel function of order n given by the following infinite series

$$J_n(x) = \sum_{\nu=0}^{\infty} \frac{(-1)^\nu}{\nu!\ \Gamma(\nu+n+1)} \left(\frac{x}{2}\right)^{2\nu+n}, \qquad n \geq 0.$$

(b) Derive the general solution of the following differential equation

$$9\,(x^2y'' + xy') + (9x^2 - 1)y = 0$$

around origin in terms of Bessel functions. **[2+4]**

3. (a) Find the general solution of Legendre equation of degree n

$$(1-x^2)y'' - 2xy' + n(n+1)y = 0$$

in the neighbourhood of $x = \infty$, where n is neither an integer nor an half integer.

(b) Write down the value of the integral of the product $P_m(x)$ and $P_n(x)$ with respect to x over $[-1,1]$, where $P_k(x)$ denotes the Legendre polynomial of degree k and m , n are integers.

(c) Using Rodrigues formula prove the following recurrence relation

$$(n+1)P_{n+1}(x) - (2n+1)xP_n(x) + nP_{n-1}(x) = 0$$

where $P_n(x)$ denotes the Legendre polynomial of degree n. **[5+2+3]**

4. (a) Check whether the following series is convergent

$$\sum_{n=1}^{\infty} \frac{(-1)^n \sin(nx)}{n^2}.$$

(b) Prove or disprove that the following harmonic series is convergent

$$\sum_{n=1}^{\infty} \frac{1}{n}.$$

(c) Obtain all criteria for convergence and divergence of the following hypergeometric series

$$1 + \frac{\alpha.\beta}{1.\gamma}x + \frac{\alpha(\alpha+1)\beta(\beta+1)}{1.2.\gamma.(\gamma+1)}x^2 + \frac{\alpha(\alpha+1)(\alpha+2)\beta(\beta+1)(\beta+2)}{1.2.3.\gamma.(\gamma+1)(\gamma+2)}x^3 + \dots\dots\dots\,, \qquad x\epsilon\mathbb{R}^+$$

for all values of the variable x and the parameters α , β , γ. **[1+2+5]**

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