ASSIGNMENT - 5

Numerical Solutions of Ordinary and Partial Differential Equations

1. Find the solution of the BVP using the shooting method

$$x^2y'' - 2y + x = 0,\ y(2) = 0,\ y(3) = 0.$$

Using Taylor series method of order 2, with $h = \frac{1}{4}$ to solve the resulting initial value problems (IVPs).

2. Find the solution of the BVP

$$y'' = 2y - y',\ y(1) = 2e + e^{-2},\ y(2) = 2e^2 + e^{-4}$$

using shooting method. Use the Taylor series method of order 3 with $h = \frac{1}{3}$ to solve the resulting IVPs.

3. Solve the BVP using shooting method

$$y'' = y,\ y'(0) = 3,\ y'(1) = e + \frac{2}{e}.$$

Use the Taylor series method of order 3 with $h = \frac{1}{4}$ to solve the resulting IVPs.

4. Using shooting method solve the BVP

$$y'' = xy + 1,\ y(0) + y'(0) = 1,\ y(1) = 1.$$

Use Taylor series method of order 3 with $h = \frac{1}{4}$ to solve the resulting IVPs.

5. Use shooting method to solve the BVP

$$y'' = 6y^2,\ y(0) = 1,\ y\left(\frac{3}{10}\right) = \frac{100}{169}.$$

Use Taylor series of order 3 with $h = \frac{1}{10}$ to solve the resulting IVPs and the secant method for iteration. Take $s^{(0)} = -\frac{9}{5}$, $s^{(1)} = -\frac{19}{10}$ and perform two iterations of the secant method. Compare the numerical results with the exact solutions $y(x) = \frac{1}{(1+x)^2}$.

6. Solve the following boundary value problem using the shooting method

$$u'' = 2uu',\ 0 < x < 1 \qquad u(0) = 1/2, \quad u(1) = 0.5.$$

Use the Taylor series method of second order to solve the initial value problem and Newton's method for iteration using the initial approximation $u'(0) = s^{(0)} = 0.09$. Assume $h = 0.25$ and perform one iteration.

7. Use shooting method to solve the BVP

$$y'' = (3/2)y^2,\ y(0) = 1,\ y\,(1) = 4.$$

Use Runge kutta method of order 2 with $h = \frac{1}{4}$ to solve the resulting IVPs and the Newton's method for iteration. Take $s^{(0)} = 0.9$, and perform one iterations.

8. Use second order finite difference method to solve the following boundary value problems;

i. $y'' = y + x,\ y(0) = 0,\ y(1) = 0$ with (a) $h = \frac{1}{2}$, (b) $h = \frac{1}{3}$, (c) $h = \frac{1}{4}$.

ii. $x^2y'' = 2y - x,\ y(2) = 0,\ y(3) = 0$ with (a) $h = \frac{1}{2}$, (b) $h = \frac{1}{3}$.

iii. $y'' - 3y' + 2y = 0$, with $2y(0) - y'(0) = 1$, $y(1) + y'(1) = 2e + 3e^2$ and $h = \frac{1}{2}$.

iv. $y'' = 2yy'$, with $y(0) = \frac{1}{2}$, $y(1) = 1$ and

(a) for $h = \frac{1}{2}$, take $y_1^{(0)} = \frac{1}{4}$,

(b) for $h = \frac{1}{3}$, take $y_1^{(0)} = \frac{4}{5}$, $y_2^{(0)} = \frac{3}{5}$.

v. $y'' = \frac{3}{2}y^2$, with $y(0) = 4$, $y(1) = 1$ and

(a) for $h = \frac{1}{2}$, take $y_1^{(0)} = \frac{7}{2}$,

(b) for $h = \frac{1}{3}$, take $y_1^{(0)} = 2$, $y_2^{(0)} = 3$.

Note: Taylor series method of order p

$$y_{i+1} = y_i + hy_i' + \frac{h^2}{2}y_i'' + \cdots + \frac{h^p}{p!}y_i^{(p)},$$
$$y_{i+1}' = y_i' + hy_i'' + \frac{h^2}{2}y_i''' + \cdots + \frac{h^{p-1}}{(p-1)!}y_i^{(p)}.$$

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