Indian Institute of Technology, Kharagpur

Date..... FN/AN Time: 3 Hrs Full Marks: 50 No. of Students: 46
End (Spring) Semester 2013-14, Deptt: SI/EX/MI/PH/CH/EE/IE/IM/MT/AT/MA
Sub. No. MA 60002 Subject Name: Data Structures and Algorithms

Instruction: Answer **all** questions.

Question 1 [6 + 3 + 3 + 3 = 15 marks]

a) Consider the following sorting methods: Insertion Sort, Selection Sort, Merge Sort, and Quick Sort. What is the running time using O-notation for each method?

(i) When all the the array values are equal?

(ii) When the values are in order?

(iii) When the values are in reverse order?

Explain your answers.

b) Suppose you are given a sequence S of n elements each of which is an integer in the range $[0, n^2 - 1]$. Design an algorithm for sorting S in $O(n)$ time.

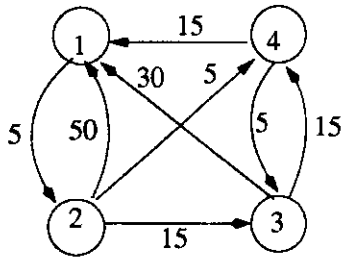
c) Write a pseudo-code for finding the k-th largest element in an array of n elements in linear time. Illustrate your algorithm on the following sequence by finding the 3-rd largest element:

$$3.72, 4.11, 5.34, 6.25, 7.76, 2.66, 1.83, 0.41, 9.10$$

d) Explain why the average computing time for your algorithm in part (c) is linear?

Question 2 [3 + 2 + 2 + 3 = 10 marks]

a) Run the Floyd-Warshall algorithm on the following weighted, directed graph. Show the matrices $D^{(k)}$ and $\Pi^{(k)}$ those result for each iteration of the outer loop.



b) Explain the reweighting procedure of Johnson's algorithm. What is the computing time of this algorithm?

c) Give an $O(|V| + |E|)$ algorithm that tests whether an undirected graph $G = (V, E)$ is connected. The graph is given in adjacency list representation and has $|V|$ vertices and $|E|$ edges.

d) Distinguish between DFS and BFS.

——-P.T.O.———

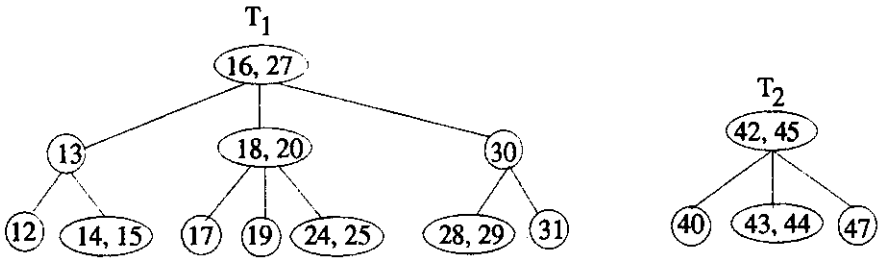
Question 3 [5 + 2 = 7 marks]

a) Find the optimal matrix ordering that produces the minimum number of integer multiplications to compute the matrix product $ABCDEF$ with the following dimensions:

Matrix	Dimension
A	2×4
B	4×8
C	8×16
D	16×32
E	32×2
F	2×256

b) Obtain a set of Optimal Huffman codes for the messages $(M_1, \ldots, M_7)$ with probabilities $(p_1, \ldots, p_7) = (2/33, 5/66, 7/66, 4/33, 5/33, 6/33, 10/33)$. Draw the decode tree for this set of codes.

Question 4 [2 × 5 = 10 marks]
Consider the following two 2-3 trees T_1 and T_2:



a) Draw the tree after inserting 26 in T_1.

b) Draw the tree after deleting 19 from T_1.

c) Join T_1 and T_2 with new data item 35 to form a single 2-3 tree.

d) Split T_1 in two new trees A and B where all items in A are < 25 and all items in B are > 25.

e) Draw a Red-Black tree equivalent to the 2-3 tree T_1.

Question 5 [3 + 3 + 2 = 8 marks]

a) Write an algorithm for inserting items in a Red-Black tree. What is the computing time of your algorithm?

b) Start with an empty Red-Black tree and insert the following keys in the given order using your algorithm: 40, 50, 70, 30, 42, 15, 20, 25, 27, 26, 60, 55

c) Starting with an empty AVL tree, insert the following strings (in this order) using the usual English alphabetical ordering: while, for, int, do, if.
(Draw the AVL tree following each insertion and state the rotation type (if any) for each insert.)

——-The End———