ASSIGNMENT - 3

Numerical Solutions of Ordinary and Partial Differential Equations

1. Determine the interval by absolute stability of the following implicit method when applied to the test equation $y' = \lambda y, \lambda < 0$;

$$u_{n+1} = u_n + \frac{h}{4}(K_1 + 3K_2); \quad K_1 = f(t_n, u_n); \quad K_2 = f\left(t_n + \frac{h}{3}, u_n + \frac{h}{3}(K_1 + K_2)\right)$$

2. Determine the interval of absolute stability of the following implicit method when applied to the test equation $y' = \lambda y, \lambda < 0$;

$$u_{n+1} = u_n + \frac{1}{4}(3K_1 + K_2); \quad K_1 = hf(t_n + \frac{h}{3}, u_n + \frac{K_1}{3}); \quad K_2 = hf(t_n + h, u_n + K_1).$$

Using this method, find $y(1.1)$ from $y' = t^2 + y^2, y(1.0) = 2, h = 0.1$(use Newton-Raphson iteration wherever required).

3. Determine the constants α, β and γ so that the difference approximation

$$y_{n+2} - y_{n-2} + \alpha(y_{n+1} - y_{n-1}) = h[\beta(f_{n+1} + f_{n-1}) + \gamma f_n]$$

for $y' = f(x, y)$ will have the order of approximation 6.

4. Find the local truncation error of the method

$$y_{n+1} = \frac{18}{19}(y_n - y_{n-2}) + y_{n-3} + \frac{4h}{19}(f_{n+1} + 4f_n + 4f_{n-2} + f_{n-3})$$

for solving the differential equation

$$y' = f(x, y),\ y(x_0) = y_0.$$

5. Show that the **order** of the linear multistep method

$$u_{j+1} + (\alpha - 1)u_j - \alpha u_{j-1} = \frac{h}{4}[(\alpha + 3)u'_{j+1} + (3\alpha + 1)u'_{j-1}]$$

is TWO if $\alpha \neq -1$ and is THREE if $\alpha = -1$. Find the values of α for which the **root condition** is satisfied.

6. Solve numerically the equation $y' = x + y$ with the initial conditions $x(0) = 0, y(0) = 1$ by Milne's method for $x = 0.4$ with $h = 0.1$.

7. Solve the differential equation $y' = x^3 - y^2 - 2$ using Milne's method for $x = 0.3(0.1)(0.6)$. Initial value $x = 0, y = 1$. The values of y for $x = -0.1, 0.1$ and 0.2 are to be computed by third order Taylor series expansion .

8. Use Milne's method to solve $\frac{dy}{dx} = y + x$, with initial condition $y(0) = 1$, from $x = 0.20$ to $x = 0.30$ with $h = 0.1$.

9. Given $y' = 2 - xy^2$ and $y(0) = 10$. Show by Milne's method, that $y(1) = 1.6505$ taking $h = 0.2$.

10. Solve $y' = -y$ with $y(0) = 1$ by using Milne's method from $x = 0.5$ to $x = 0.8$ with $h = 0.1$

11. Solve the initial value problem $\frac{dy}{dx} = x - y^2, y(0) = 1$ to find $y(0.4)$ by Adams-Moulton method. With $y(0.1) = 0.9117, y(0.2) = 0.8494, y(0.3) = 0.8061$.

12. Using the Adams-Bashforth formula, determine $y(0.4)$ given the differential equation $\frac{dy}{dx} = \frac{1}{2}xy$, and the data

x	0	0.1	0.2	0.3
y	1	1.0025	1.0101	1.0228

13. Find the value of α with which the linear multistep method

$$u_{n+1} = u_n + \frac{h}{2}(5u'_n + \alpha u'_{n-1})$$

is consistent.

14. Solve the differential equation $y' = x^2 + y^2 - 2$ using the Modified Euler predictor-corrector method (Euler as predictor and Euler-Cauchy as corrector) for $x = 0.1$ given the initial value $x = 0, y = 1$.

15. Using the Adams-Bashforth-Moulton predictor-corrector formulas, evaluate $y(1.4)$, if y satisfies

$$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2} \text{ and } y(1) = 1, y(1.1) = 0.996, y(1.2) = 0.986, y(1.3) = 0.972.$$