MA 20013

Indian Institute of Technology, Kharagpur

Date..................... FN/AN Time: 2 Hrs Full Marks: 30 No. of Students: 77

Mid ~~End~~ (Spring) Semester 2017-18 Subject Name: Discrete Mathematics

Deptt: MA/CE/MF/HS/EX/AE/EE

Instruction: Answer all questions. Notations used are as explained in the class.

Question 1 [$2+2+2+3=9$ marks]

a) Solve the following recurrence using the method of characteristic roots:

$$a_n = 7a_{n-1} - 12a_{n-2} + 3n4^n, \text{ and } a_0 = 0, a_1 = 2.$$

b) Determine a recurrence relation for $f(n)$, the number of regions into which the plane is divided by n circles, each pair of which intersect in exactly two points and no three of which meet in a single point.

c) Express the following sequence in terms of the Fibonacci numbers, where c is a given constant.

$$b_0 = 0, b_1 = 1, b_{n+2} = b_{n+1} + b_n + c$$

d) Solve the difference equation

$$r_n = \sqrt{r_{n-1} + \sqrt{r_{n-2} + \sqrt{r_{n-3} + \sqrt{\cdots}}}}$$

given that $r_0 = 4$.

Question 2 [$3+2+2=7$ marks]

a) Find a simple, closed-form expression for the exponential generating function if we have p types of objects, each in infinite supply, and we wish to choose k objects, at least one of each kind, and order matters.

b) In how many ways 6 apples, 1 orange, 1 pear, 1 peach, 1 plum, 1 strawberry and 1 grape can be divided among 3 people? There is no restriction on the distribution; a person may get all of these items or none of these.

c) Given a sequence of p integers $a_1, a_2, \ldots, a_p$, show that there exist consecutive terms in the sequence whose sum is divisible by p. That is, show that there are i and j with $1 \le i \le j \le p$, such that $a_i + a_{i+1} + \cdots + a_j$ is divisible by p.

——P.T.O.——

Question 3 [3 + 3 = 6 marks]

a) Use the principle of inclusion and exclusion to prove that the chromatic polynomial $P(G, x)$ of a graph G is a polynomial.

b) Let W_5 be the wheel of 6 vertices illustrated in the following figure. Compute the

   (i) chromatic number of W_5,

   (ii) chromatic polynomial of W_5.

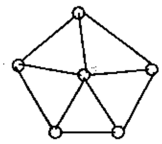

Figure 1: Wheel W_5

Question 4 [3 + 3 + 2 = 8 marks]

a) Formulate and prove by induction a rule for the sum

$$\frac{1^3}{1^4+4} - \frac{3^3}{3^4+4} + \frac{5^3}{5^4+4} - \cdots + \frac{(-1)^n(2n+1)^3}{(2n+1)^4+4}$$

b) Show that the set S of all infinite binary sequences is uncountable.

c) Is the set $Q - N$ copuntable or uncountable? What about the set $R - Z$ (No credit will be given for answer without justification.)

——-The End———