INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Date———FN/AN 2 Hrs. Full Marks: 30 No. of Students 80

Mid Spring Semester 2014-2015 Deptt: MATHEMATICS Sub No: MA 20013 & MA 21014

——Yr. B.Tech.(H)/B.Arch.(H)/M.Sc. Sub. Name: Discrete Mathematics

Instruction: Answer all questions, which are of equal values

1. According to a survey among 160 college students, 95 students take course in English, 72 take course in French, 67 take a course in German, 35 take a course in English and in French, 37 take a course in French and in German, 40 take a course in German and in English, and 25 take a course in all three language. Find the number of students in the survey who take a course in:

   (i) English, French, or German.

   (ii) English and French but not German.

2. Show that the distinct equivalence classes of an equivalence relation on A provide us a decomposition of A as a union of mutually disjoint subsets. Conversely, given a decomposition of A as a union of mutually disjoint, nonempty subsets, show that we can define an equivalence relation on A for which these subsets are the distinct equivalence classes.

3. (a) Construct the true table of $(p \vee q) \Leftrightarrow (p \wedge q)$.

   (b) Determine whether or not $[(p \vee q) \wedge (\sim q)] \rightarrow p$ is a tautology.

4. (a) Construct the Hasse diagram and hence the greatest and least element (if exists) for the poset $(A, |)$, where $A = \{1, 2, 3, 6, 9, 18\}$ and $|$ denotes the divisibility relation.

   (b) Find the number of relations which are both reflexive and symmetric that can be defined on a set with n elements.

5. (a) Let R be transitive relation on A, then show that $R^n \subseteq R$ for every positive integer n.

   (b) Find the connectivity relation of the relation on $\{a, b, c\}$ with the adjacency matrix

$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

6. (a) Let $f : X \mapsto Y$ be function. Prove that f is injective iff $\forall A \subset X$, $f^{-1}(f(A)) = A$. Show that for any map $A \subset f^{-1}(f(A))$ and also show that proper inclusion can occur.

   (b) Let $f : X \mapsto Y$ be function. Show that f is onto iff $f(f^{-1}(B)) = B \ \forall B \subset Y$. Show that for any map $f(f^{-1}(B)) \subset B$ and also show that proper inclusion can occur.