Symbolic Logic
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Lecture – 24
Understanding the Rules
Strategic Tips for Doing Formal Derivation of Validity
Proof with All Rules

Hello, we are into module 24 of Symbolic Logic NOC course and we were working on the formal proof of validity. So, we will continue to do that.

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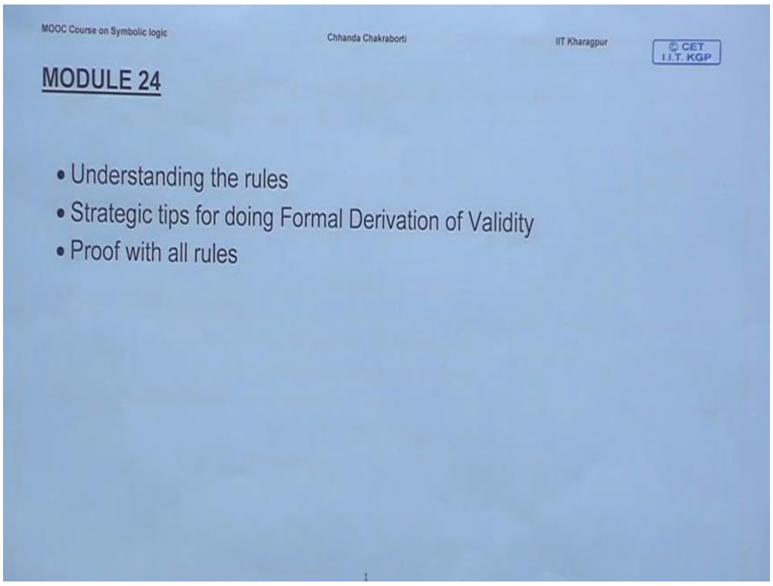

Last time we were talking about the equivalence rules, so we will continue understanding them a little bit. And then there will be some work related tips, if that is possible, some ideas that is, how to go ahead with the formal derivation of validity. And then we will combine all the rules, as in the rules of inference as well as the equivalence rules, to do the proofs.

So, once more, we take a look into the rules of replacement, because we were just beginning to look at them.

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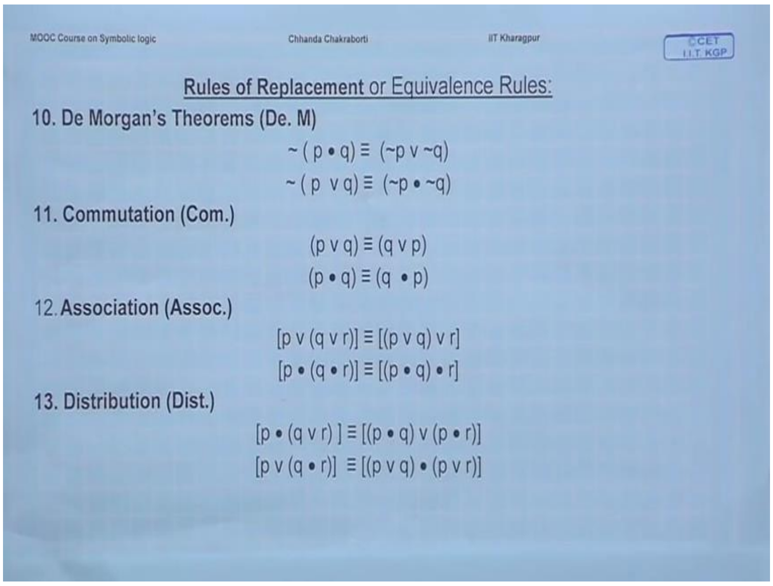

So, these four were introduced in the last module. See, all of them are about dot and wedges. So, what we are looking at is how to change the connective from one to the other in a way in a proof that could be very useful and also important to remember. For example, see here in the distribution what happened, a conjunction became a disjunction and the disjunction became a conjunction. And sometimes you know that there are rules which apply specifically to conjunction. For example, you want to get rid of the $(p \vee r)$. Right? And you know the there is a rule of inference called simplification which works only on the conjunction. So, there is benefit of looking closely into this equivalence rules I suggest that you also pay attention to the name that these are actually rules of replacement. So, in a way you are not deriving anything. What you are doing you are replacing one expression with a same meaning and same truth value expression. That's what you are doing. So, that is how the De Morgan's theorems, for example, has to be read. This is how the commutation has to be read. When you have a wedge and you can swap the position, when you have a dot you can swap the position. When you have Association, you can regroup them without disturbing the truth-value, though the emphasis changes in the Association. As I have shown you, $p \vee (q \vee r)$, where p is on… the first emphasis, whereas here the $p \vee q$ is the emphasis and then r is the other disjunct. So, this is Association and this is your Distribution.

Since we have done these rules earlier in the earlier module, we will proceed to no more. There will be 10 Rules of Replacement or equivalence rules on top of your 9 Rules of

Inference. So, all together in your data base or in your rule base, there will be 19 rules of derivation and you will see that each one has its own place and each one has its advantage to know. So, pay attention to each one of them as we go along. So, we have covered so far De Morgan's theorems which we will call De. M. We have Commutation which we will call in short Com. And we have Association which we will call Assoc, and this is Distribution. Distribution will be referred to as Dist. Apart from this, there are few others which we also have to sort of look into.

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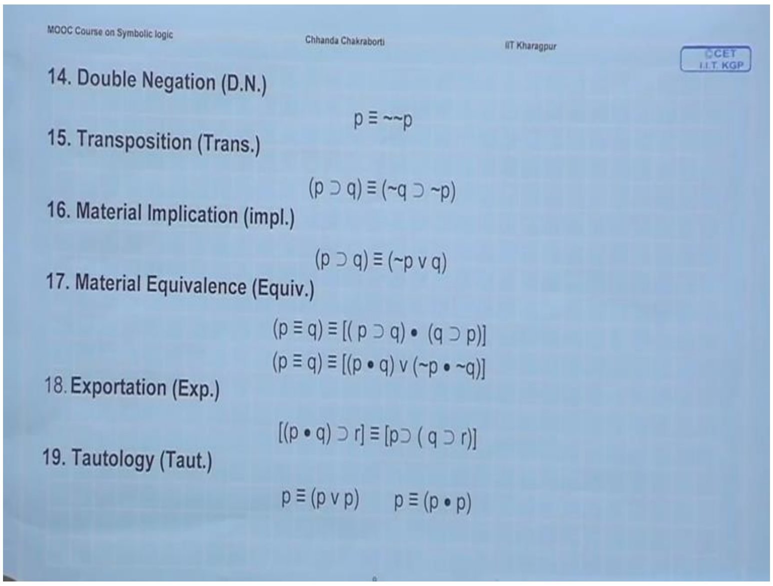

So, here are the remaining four. The first one is Double Negation. Intuitively obvious and very well known. All we are saying is that given an expression you can replace it with the negation of negation of that. And similarly when you have an expression like this ~~p, you can rewrite it as p. So, that is a rule. Without the rule, remember, you do not have the permission to replace that. So, the system which misses this double negation rule, for example, cannot do what you can so easily do. Though you may intuitively know, but since you are in a formal process, unless there is a rule that specifically says that you can do this, you can’t really take liberty. (Refer Time: 04:39).

This is, on the other hand, Transposition rule, Transposition rule. What you are doing? You are saying if p then q, and that is replaceable by if not-q then not-p, right? So, in a way we are re-stating what is expression now. Now if you… those of you who are conceptually would like to understand it, then what we are saying here is as equivalent to this. Why? Because here we said that p is sufficient condition and q is necessary condition. So, if q does not happen then p does not happen. This is exactly what we have said. Similarly when we have an expression like this, you can rewrite it as $p \supset q$. So, there you are. This is Transposition rule which allows you to swap the position and attach a tilde to the components.

This is, on the other hand, the material implication rule which follows from the very truth table of the horseshoe. You have already worked with it in the truth tree and I have mentioned it during your truth table time also. So, this is nothing but saying that if p is… horseshoe can be rewritten as a disjunction in this form. So, 'if p then q' is equivalent to saying ' either not-p or q' . Alright? Similarly, this disjunction is equivalent to saying that there is a horseshoe here. Depending upon what your proof needs , whether it needs a disjunction or whether it needs a horseshoe, you are going to use this.

This is Material Equivalence rules. They are also equivalence rules and again if you recall the truth table of triple bar, you will understand why this has to hold. What I am saying here that 'p triple bar q' is equivalent to 'if p then q and if q then p'. We have already established that. When I explained to you what if and only if meansthis is exactly what we referred to : That it is a conjunction of two conditionals. This is the reason that triple bar is called a bi-conditional. So, that is captured in this rule.

This, on the other hand, follows from the very truth table of triple bar. ' p triple bar q' is equivalent to 'either p and q, or not-p and not-q' . This rule you have used and understood when we were doing the truth tree. So, this is… these two together are going to be referred to as Material Equivalence.

And then, comes Exportation. Just like exporting it out. You know, so, here is what you are saying is that ' if both p and q then r' , then it follows that ' if p then q then r' . Or, you go come the other way 'if p then if q then r' which means that 'if p and q then r'. Ok? So, these are interchangeable or replaceable by each other and the name of the rule is Exportation. The names you have to sort of remember and the rules also you need to remember. We… go back to your list again and again to see the name as well as what exactly it says.

This is the Tautology rule, last one which gives you these two tautologies. So, p can be replaced either by $p \vee p$ or p can be replaced by $p \bullet p$. Similarly $p \bullet p$ can be reduced to

p. p ∨ p can be reduced to p. Please note that these two are the *only two* tautologies that this derivation system will allow. No other tautology will be applied here.

So, let us look back into our rule list and once more learn the abbreviations. Because we… there is no need to refer to the rules by the whole name. But it's important that you know the rules exactly and as they are.

So, Double Negation will be referred to as D.N, Transposition as Trans, Material Implication as Impl, Material Equivalence as Equiv, Exportation as Exp, and Tautology as Taut. And this on top of your other four rules that we have covered already, that is De Morgan's, Commutation, Assoc and Distribution.

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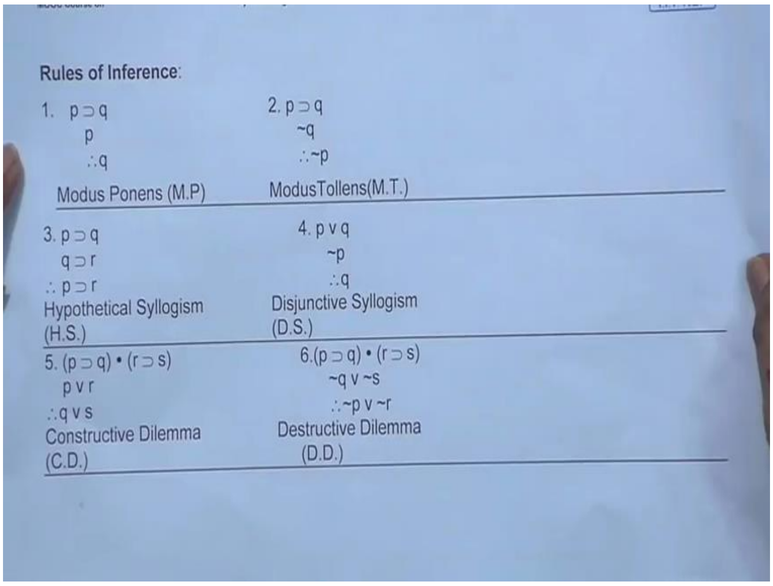

So, let's look into this totality of these rules. How many rules we have learned and what is the difference. Let me remind you once more. There are Rules of Inference. The first nine that you learned, these are Rules of Inference. So, one-directional and they have this requirement that you can derive this triple bar… triple dots actually referred to that you are deriving a conclusion. So, that's why they are Rules of Inference.

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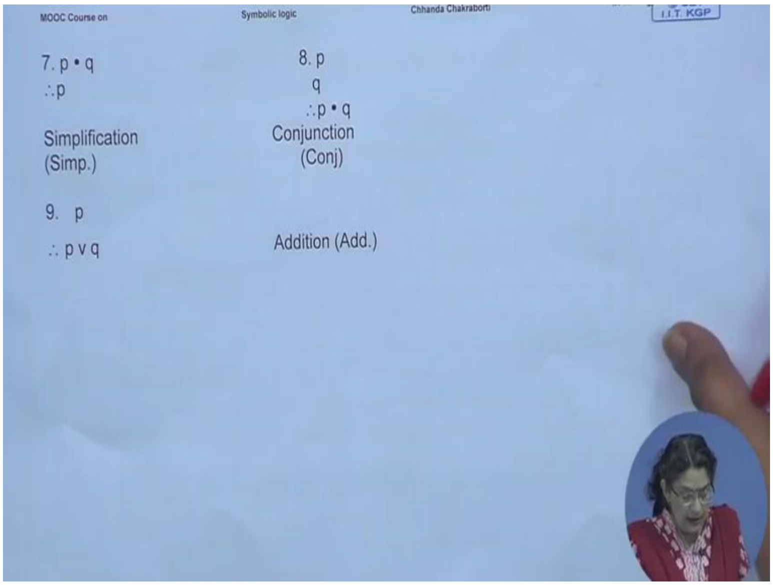

So, these 9 were our first 9 Rules of Inference which we are going to use. and then we have the Rules of Replacement. So, how many rules of replacement? You have 10, these 4 plus this 6. So altogether we have 19 rules at our disposal. And as I have already mentioned couple of times, but still it may help you to go through this, that the Rules of equivalence or Replacement rules are actually bi-directional. You… it comes you can use them any which way you want to.

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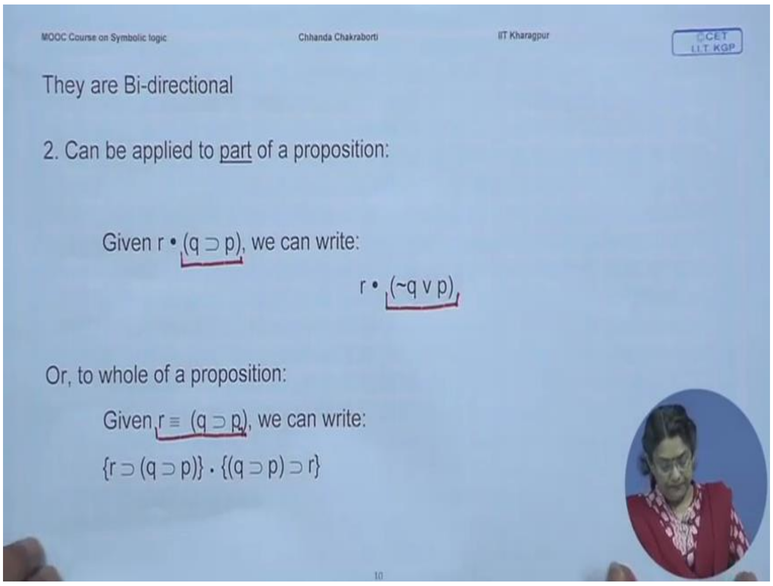

Moreover, they can be applied to part of a statement. Remember the Rules of Inference can be applied only to a stand-alone proposition as a whole. But these Rules of Replacement can be applied to a part of a proposition as well as to the whole of a proposition. So here you are.

This is for example, suppose you are given r • (q ⊃ p). You can rewrite it as r • (~q ∨ p). What did we do? What we did is to apply the Impl rule. Where? Here, this is the part that we have replaced it with. Remember the main connective is here is dot (•). Without touching it, you can still replace the parts of it. Similarly…. this doesn't mean that the Rules of Replacement only apply to part. But you can also apply to the whole of a statement. So there is no bar. For example, r ≡ (q ⊃ p), here triple bar (≡) is the main connective and you can apply the Material Equivalence rule to this triple bar to get this kind of horseshoe (⊃).

So, it is a conjunction of two horseshoes. {r ⊃ (q ⊃ p)} • {(q ⊃ p) ⊃ r} right? So, here we are applying it to the whole sentence and here we are applying it to the part.

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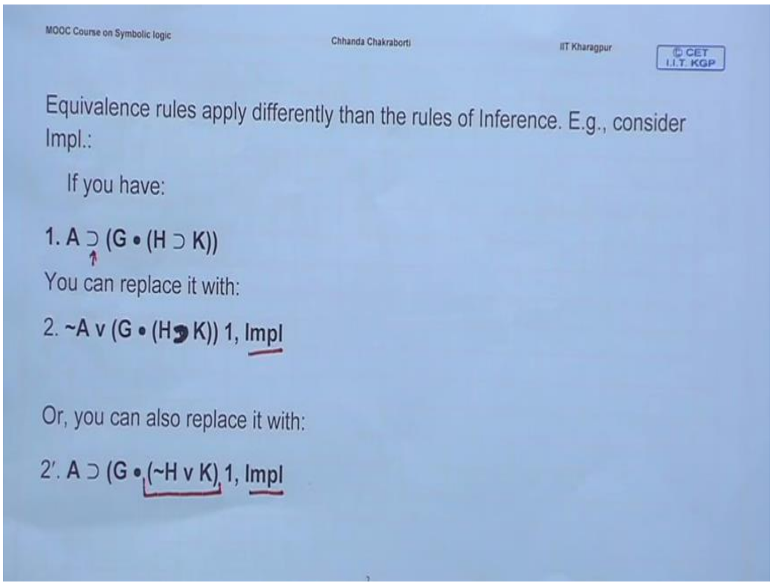

Now, same thing, same point, but let me elucidate that again further with another example. Suppose you have A ⊃ (G • (H ⊃ K)). Now you can do two things here, depending upon what you need in the proof you can work this rule. So if you want to replace this horseshoe for some reason for with a disjunction, then this is where you are going to apply. Right? Here you are and it nothing changes here. But this changes into ~A ∨ and the justification is by 1, Impl.

But there are two horseshoes here. One is here, one is here. So, you can apply it here in the main connective or you can do it in this part. Without changing this horseshoe, you can selectively use it on this horseshoe. The rule here, Impl is a rule of equivalence and that's why you could do this. But you cannot do this with a Rule of Inference. So, you need to know which one is a Rule of Inference and which one is a Rule of Equivalence. Because there are things that you can do with Rules of Equivalence which is not permissible if you are handling a Rule of Inference.

So, let's come to now how to do the proof, how to do the derivations. We have seen some examples already. But now we are going to slowly move onto doing more and as you have more rules now, so the proofs are going to be somewhat not so elementary and so on.

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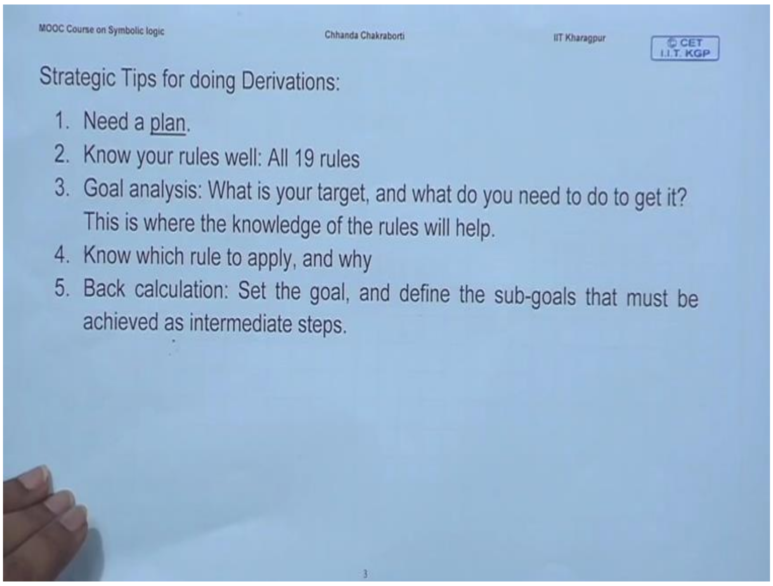

So, first of all what is that that you need to know? How to prepare for doing derivations? I have already told you that you need a plan. There is nothing haphazard (Refer Time: 13:45) or nothing random. It's not mindless like doing a truth table : That I start somehow and automatically it will be finished. In proofs it's like a little puzzle. You know the answer is there, but answer may not be very obvious or explicitly present. So, which is why you need to think a little and have a game plan before you can even start the proof. So, first is a plan. How do you formulate a plan? As I have told you that you do a rough work before you will get ahead and do a rough work on this piece paper to think how to get at the conclusion.

In order to have a plan, it is mandatory, essential , that you know your rules. *All* the rules. So, you have total 19 rules and it is important that you know them. Because otherwise you do not know which inferences are possible and which ones are not, and which rule can help you and in which way. So, this is why you need to practice a little with the rules to see how far you can go with these rules. Now the other thing is that when you are looking and making a plan, the rules will be there to give you the leverage, to give you the strategic advantage. But what is also needed is some sort of goal analysis. So, ask yourself what is it that you want? What do you want to derive? And in order to get that, what else do you need to get in the proof? So, that way you know this is my ultimate goal, but before that I have to meet some of the sub-goals; otherwise I cannot reach my target.

The target obviously, ultimate target is the conclusion that you are going to establish, right? Of the given argument, the conclusion. But in order to get there, you need to probably arrive at some of the sub-goals. So, this is where the knowledge of the rules is going to immensely make a difference. Because many of the steps, if you don't know the rules, you might think that this is not even possible. Whereas there are very efficient ways to get at them, provided you know the rules. And when you are applying the rules, know which one you are applying and why. Just like in the truth tree case, I always said that you should go with an open mind and open eye. So, you should know why you are picking this rule out of the rule base and not the other. Obviously, there is a relevance to what is it that you are trying to do in the proof, so that *why* is going to be a crucial*why* for you to keep in mind.

And then some people do back calculation also. For example. I mean, just like we did in the last examples in the last module that you know we were working back. So, if the goal is like this, and in order to get at the goal what is it that I need to have, and then in order to get that what else intermediaries' steps do we need to take, and so on and so forth. These are tips. But what is required is that you do some practice. You actually try to solve some problems with the rules. It's not that you have to memorize the rules, because the more you use them, it will automatically stay in your memory. But what is needed is that you pay rather close attention to the rules; the name as well as how they move, what do they allow, what are the legitimate conclusions that they allow.

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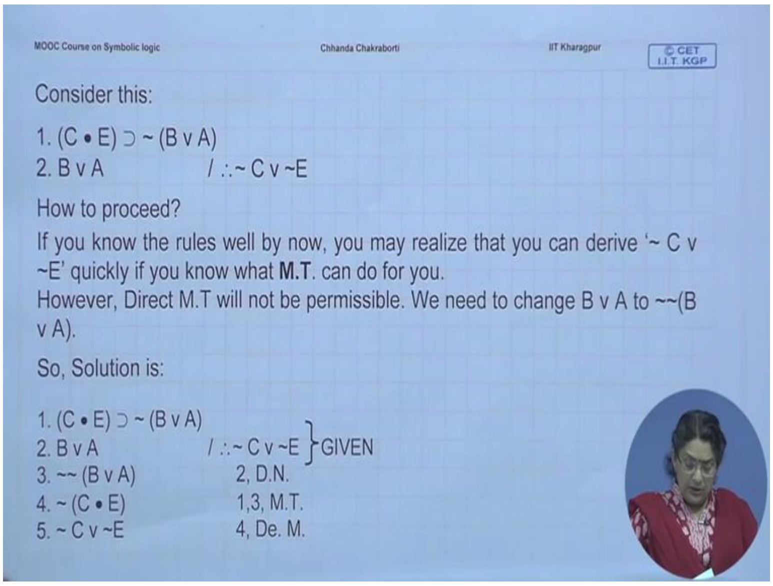

Now, consider this for example. Let's see whether we can proceed in this way. Suppose you have a proof like this. Now the claim is that this is a valid conclusion. Your job is to show how to get here. Now the premises are true and you can see that not ~C ∨ ~E is not obviously present in this. But clearly there is a way to derive that. That's the claim. Now you have to explicate that. So how to go about that? And, this is where the planning is necessary. What do you need to do? Now one thing you may probably have noticed. Always pay attention to the premises. So one thing you must have noticed is that you have B ∨ A here whereas you have ~(B ∨ A) here. So, in a way that should start some sort of thinking on your part that what exactly is to be done here.

Second is that this ~C ∨ ~E and then you look into this first premise and you have C • E. You should immediately connect these two that there is a relationship here. Namely, that if I negate this, I will probably get that. So, somewhere we need to work this out. Now main thing is that to have a plan. There are more than one ways you can go around. But here for example, you see that there is a possibility of doing Modus Tollens. (Refer Time: 19:04) How? B ∨ A is actually the negation of the ~ (B ∨ A), right? So, because we can rewrite this immediately with the double negation rule as ~~(B ∨ A), and that would give you the ~(C • E). Correct?

So, that unless you know what Modus Tollens (Refer Time: 19:26) can do for you, how would you even think in this way? That's what I meant that you need to do this. Now directly we cannot apply Modus Tollens here. I mean though it may intuitively seem to you that B $\vee$ A is the negation of the negation, but in the formal logic parlance you need to convert it into what is known as double negation of B $\vee$ A. So, that we will be requiring. Once that is done then the rest is very easy. So, if you have finished that thinking, then doing the proof is not really a problem. This is what is given and then we start adding lines. So, first line as we said is to convert this line into double negation step. And the justification appears here. Every time you are adding a line, just as in the truth tree, in formal proof of validity you have to justify and that will show up on the right hand side.

This is then the Modus Tollens (Refer Time: 20:30) result. We applied 1 and 3, Modus Tollens(Refer Time: 20:34) and we got this thing as a negation out. Why this is important? Because now you know that we want to go here and we can do this. By which rule? We can do it by the De Morgan rule. So, if you don't know what De Morgan does for you, then you cannot see this step very well. But if you do, then it is a matter of just a second to come from step 4 to step 5. Ok?

So, this is how we have established that this has to be valid.

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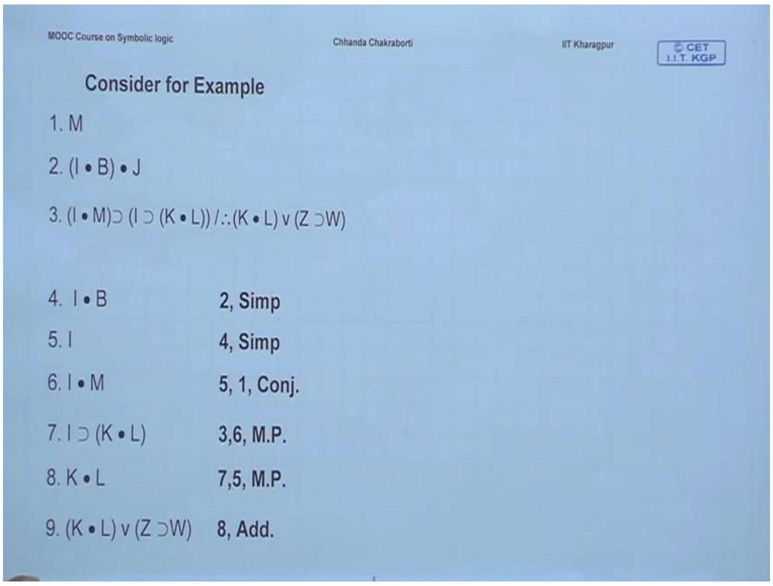

Here is another example. There! Which has three premises and this is the conclusion. Take a good look at the premises. Form a plan, a strategy and then the solution is worked out already. I will explain it. But why don't you just take a look and try to have a plan how to go about so that we have this as a result?

Again, I need to remind you that you have to know the rules to see what you can achieve. Any idea? Ok, let us see that. See this is your target. Now $Z \supset W$ is nowhere here, correct? So, somewhere we have to bring that in. On the other hand what is there in the proof is $(K \bullet L)$, which is present in line number 3, correct? Now, therefore, what we need is somehow if we can have I, then we can have this provided we already have this part derived from line number 3. So, your sub-goal is somehow to get $I \bullet M$, and there is a possibility that we can see in lines 1 and 2 to get $I \bullet M$, can you see it? Here is M and here is somewhere I, right? So, that should tell you which rules to apply, correct?

So, if you have figured that out then the matter is very simple. Namely, we will just go like this. This is my line 4, what did I do? We just took line 2 and we chopped the J out. Remember, J is not even relevant for this. So, better to simplify. And then, further simplification. Why? Because all I need is $I \bullet M$. I see M and here is I, stand-alone I. How did I derive that? From 4, by further simplification on that line. Now the job is to join this and the rule that allows you to do that is called Conjunction. So, have to use that, 5 and 1 gives you $I \bullet M$. The order is important, I have always said. You wanted $I \bullet M$. Now M appears first, I appears here in line 5. That is not the point. When you are joining them, you are joining in this kind of sequence. You are joining 5 with 1. So, 5 is I and 1 is M and we put them together this is 5, 1 conjunction.

Once you have $I \bullet M$, it is a matter of just taking out $I \supset (K \bullet L)$. And you have I, so you can get $K \bullet L$ without any further problem. But is this the conclusion? Not yet, right? But you have this. So, how can I have this? Now take a look at the main connective that you need. And if you know your rules very, very well, then you see the possibility of using a rule from the rule of inference. Which one is that? That is the rule of Addition. So, this is your p, to which you have added q. Can I do that? Of course! What is being said in the rule is that if the statement is true, then you can add with it anything you want, with a disjunction. If this is true, there is no way this can be false. This is what Addition rule guarantees.

So, this is how we have derived this problem. Does that make sense? So, in a way, you know, if you do not know what the Addition rule can do for you, you cannot anticipate the line in our mind. Alright? So, that is what I mean. I mean you need to think along with the rules and arrange your thinking as far as the rules that allow you to do. Let's take one other example and then we'll be finished. Here we are.

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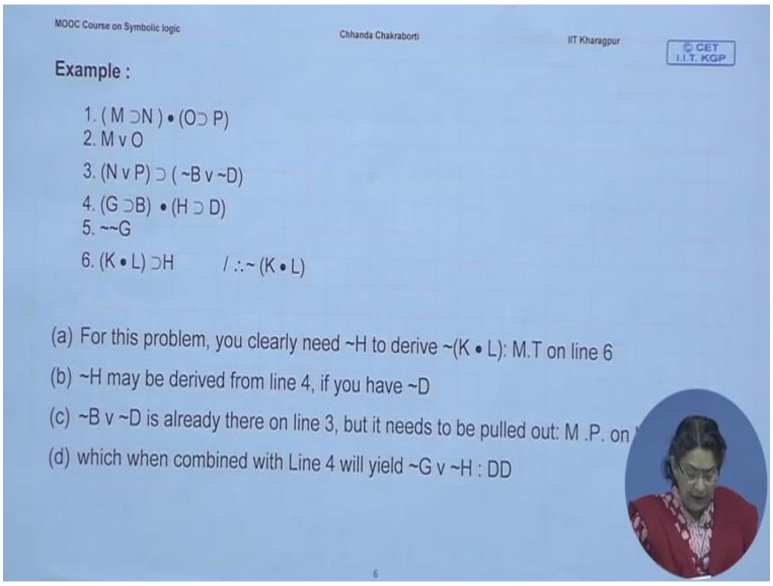

So, here is an argument and this has about 6 premises and here is the conclusion, right. Now this K • L, negation of that, is right here. So, unless you have not-H, there is no way you can take it out. So, your main idea is first of all to get this, but in order to get this we need ~H. So, how from here we can get not-H? Take a look into the premises very, very carefully.

Here is a sort of a giveaway that you can get N ∨ P very easily from 1 and 2, except you need to know the rule which allows you to do that, right? Once you have N ∨ P, you can get ~B ∨ ~D out, right? And then if you can plug that into line number 4, it will give you ~G ∨ ~H, provided you know which rule to use. And you have ~~G. Once you have used ~~G on ~G v ~H, we are going to have ~H. So, if we put it all together , all those thinking together, then here comes what will be known as the whole proof. Take a look.

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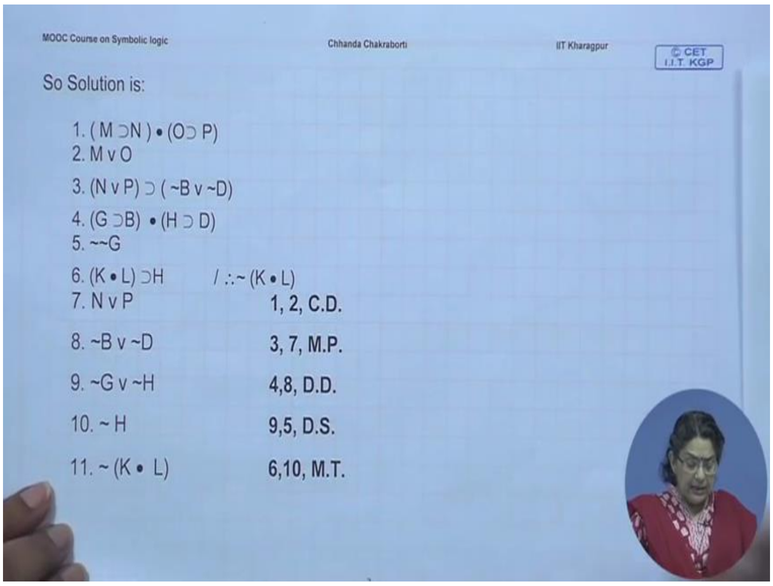

What we have said we can put it into this way. What did we do? We used 1 and 2, C.D. This is the rule, unless you know the rule, there is no way you can use it. This is Modus Ponens on 3 and 7, right? And here is the D.D. Destructive Dilemma. If you don't know what Destructive Dilemma does, you don't anticipate this line that you can use. We are using it on 4 and 8 to get line number 9. Once you are here and you have line number 5, you can apply them together and you can get ~H. Why do we need ~H? To get that ~(K • L) and that is my goal. Ok?

So, this should give you an idea about how to go about with the proofs. You cannot afford not to know the rules. So that first has to be sort of settled that you have to have a grasp over them. Then take a very detailed attention to the given premises, because the whole game here is that there is an underlying claim that within the premises somehow the conclusion is contained. All you need to do is to explicate it, derive it and then by step by step you will arrive there. So, there is a guarantee that the conclusion is there somewhere, but you need to find, just like in a puzzle, the ways to connect the premises with the rules so that you can arrive at the conclusion.

So, give yourself some practice. I think you will succeed absolutely without any problem. So, this is where I am going to end this module.

Thank you very much.