R.M.

Indian Institute of Technology
Department of Mathematics
Spring End Semester Examination-2016
Subject Name: Discrete Mathematics
Subject No: MA20013
No. of students: 69 Time: 3 hrs F.M. 50

Instructions: Answer ALL questions. Numerals in righthand margin indicate marks. No query on this question paper will be entertained in the examination hall.

1. Answer ALL parts. 2M×8=16M

   (a) Let a and b be atoms of a Boolean algebra such that $ab \neq 0$. Then prove that $a = b$.

   (b) Find conjunctive normal form of $f(x_1, x_2, x_3) = (x_1'x_2)'(x_1 + x_3)$.

   (c) At a party, five gentlemen check their hats. In how many ways can their hats be returned so that no gentlemen gets the hat with which he arrived?

   (d) Find the sequence for which $\frac{1}{x^2-5x+6}$ is the generating function.

   (e) Whether the statements $(p \implies q) \vee (p \implies r)$ and $p \implies (q \vee r)$ are logically equivalent?

   (f) Find the truth value of $(q \implies (r \implies s)) \wedge ((p \implies s) \implies (\sim t))$, if p and q are true statements, and r, s and t are false statements.

   (g) Find the maximum number of vertices in a binary tree of height h.

   (h) Draw the graph of a cube and then find two different Hamiltonian cycles in it.

2. Answer ALL parts. 3M×6=18M

(i) Give a Boolean expression for majority voting of three persons and then draw its circuit diagram using gates.

(ii) State and prove the absorption property in a Boolean algebra.

(iii) Is the following a valid argument?

I will become famous or I will not become a writer.
I will become a writer.

$\therefore$ I will become famous.

(iv) A ship carries 48 flags, 12 each of the colors red, white, blue and black. Twelve of these flags are placed on a vertical pole in order to communicate a signal. How many of these signals use an even number of blue flags and an odd number of black flags.

(v) Solve the following recurrence relation by generating function method: $a_n - 7a_{n-1} + 10a_{n-2} = 0,\ n \geq 2,\ a_0 = 10,\ a_1 = 41.$

(vi) Solve the following non-homogeneous recurrence relation:

$$a_n - 6a_{n-1} + 8a_{n-2} = 3^n,\ n \geq 2,\ a_0 = 3,\ a_1 = 7.$$

3. Answer ALL parts $4M \times 4 = 16M$

(i) State and prove the principle of inclusion and exclusion.

(ii) What is strong pigeonhole principle? Applying this principle prove that in any group of six people there are at least three mutual friends or at least three mutual strangers.

(iii) Applying Karnaugh map technique find minimal form of $f(x, y, z, w) = x'z'w' + xy'w' + x'yw + x'y'zw' + x'yzw' + xyz'w + xyzw$.

(iv) Prove that every finite Boolean algebra is isomorphic to a power set Boolean algebra.

****************THE END****************