INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR
MID SEMESTER EXAMINATION

Date : - 2-2013 FN/AN   Time : 2 hours   Full Marks : 30
Spring Semester : 2012-2013.   Department : Mathematics
Subject No.:MA20104   Subject Name : Probability & Statistics
Course : B.Tech./M.Sc. 2nd Year (AG, BT, CE, CS, HS, IM, MA, MI, MF, QE, QM,
breadth, additional, backlog)   No. of students: 620

Instructions: Answer all questions. Marks are indicated at the end of each question.
Statistical tables may be used.

1. (i) A survey of people in given region showed that 25% drank regularly. The
    probability of death due to liver disease, given that a person drank regularly, was 6
    times the probability of death due to liver disease, given that a person did not drink
    regularly. If the probability of death due to liver disease in the region is 0.005, what
    is the probability of death due to liver disease given that a person drank regularly?

                                                                              3M

    (ii) A missile can successfully hit a target with probability 0.75. If three successful
    hits can destroy the target completely, how many missiles must be fired so that the
    probability of completely destroying the target is not less than 0.95?   2M

2. (i) A card is drawn at random from a well shuffled deck. If the number on the card is
    between 2 and 10, its score is that number. If the card drawn is a jack, queen or king,
    its score is 10. If an ace is drawn, its score is 20. Find the probability distribution of
    the score of the card drawn. Find its mean and the median.   3M

    (ii) Let the distribution of marks on a class test have mean 75 and s.d. 3. Use
    Chebyshev's inequality to show that the probability of a student having marks above
    90 or below 60 is at most 1/25.   2M

3. (i) A mechanical system consists of n components each of which functions
    independently with probability p. The entire system will be able to operate
    effectively, if at least one-third of its components function. For what values of p, a
    6-component system is more likely to operate effectively than a 3-component
    system?

                                                                              3M

    (ii) The number of times that an individual contracts viral infection in a given year is
    a Poisson random variable with parameter $\lambda = 5$. Seventy five percent of the
    population uses a new healthy life style and thereby reduces the Poisson parameter λ
    to 1. If an individual does not get viral infection for a year, what is the probability
    that he/she followed the new healthy life style?   2M

                                                                          P.T.O.

4. (i) The time to failure (in years), X, of the electronic tubes produced at two manufacturing plants I and II follows a gamma distribution. For the tubes produced at plant I, the mean is 2 and the variance 4, whereas for the tubes produced at plant II, the mean is 4 and variance 8. Plant II produces four times as many tubes as plant I. The tubes are intermingled and supplied. What is the probability that a tube selected at random will work for at least 6 hours? 3M

(ii) A small industrial unit has 20 machines whose lifetimes are independent exponentially distributed with mean 100 months. If all the machines are under use at a time, find the probability that even after 200 months there are at least two machines working. 2M

5. (i) Let Y_r denote the time for the rth occurrence (starting from 0 time) in a Poisson process with rate λ. Derive the density function of Y_r. 3M

(ii) The distance that an Olympic standard male long jumper clears at every attempt, is a normal random variable with mean 8 meters and standard deviation 10 cm. What is the maximum distance that he will clear with probability 0.95? What is the distance that he will clear only 10% of the time? 2M

6. (i) Let $X \sim \text{Bin}(n, p)$. Show that as $n \to \infty$ and $p \to 0$, so that $np \to \lambda$, the distribution of X converges to a Poisson distribution with parameter λ. 2M

(ii) Let $X \sim P(\lambda)$. Prove that $E(X^n) = \lambda E(X + 1)^{n-1}$. 1M

(iii) Derive the moment generation function of a $N(\mu, \sigma^2)$ distribution. 2M