Department of Mathematics
Indian Institute of Technology Kharagpur
End Semester Examination

Date of Exam.: .11.15 (FN/AN) Time: 3 Hrs Full Marks: 50 No. of Students: 60
Subject Name: Probability and Statistics Subject No.: MA41009
2 yr M.Sc. Maths & others

Instructions: Answer all questions. Marks are indicated at the end of each question. All parts of a question must be answered at one place. Statistical tables may be used.

1. (a) Suppose that X and Y be independent Binomial distributed random variables with parameters n_1 and $p_1 = 1/2$; and n_2 and $p_2 = 1/2$, respectively. Then show that the random variable $X - Y + n_2$ follow Binomial distribution with parameter $n = n_1 + n_2$ and $p = 1/2$.
   (b) Consider a random variable X with mean μ and standard deviation σ. Let the MGF of X be $M(t) = e^{4(e^t-1)}$. Find $P(|X - \mu| < 2\sigma)$. [8 M]

2. (a) Let (X, Y) have joint density $f(x, y) = x + y,\ \ 0 < x < 1,\ 0 < y < 1$. Find the mean and variance of conditional distribution of Y given $X = x,\ 0 < x < 1$.
   (b) Let (X, Y) have joint density

$$f(x,y) = \begin{cases} 6(1-x-y) & , \quad x+y<1,\ x>0,\ y>0 \\ 0 & , \quad \text{elsewhere} \end{cases}.$$

   Find Covariance between X and Y. [8 M]

3. Let X and Y be independent chi-square random variables with u and v degrees of freedom respectively.
   (a) Show that X/Y and $X + Y$ are independent.
   (b) Find the distribution of $X + Y$. [8 M]

4. (a) Let (X_1, X_2) have a bivariate normal distribution with parameters $\mu_1 = 3,\ \mu_2 = 1,\ \sigma_1^2 = 9,\ \sigma_2^2 = 16$ and $\rho = 3/5$. Determine $P(-2 < X_1 < 2)$ and $P(-2 < X_1 < 2 | X_2 = -3)$.
   (b) Let $X_1, \ldots, X_n$ be random sample from a distribution with density $f(x; a, b) = \frac{1}{b}\, e^{-\ (x-a)/b},\ x > a,\ -\infty < a < \infty,\ b > 0$. Find the MLE of a and b. [8 M]

5. (a) Let X be a single observation from a Bernoulli distribution with parameter p. If p is restricted so that $\frac{1}{2} \leq p \leq 1$. find the MLE of parameter p.
   (b) Let a random variable $F \sim F(u, v)$. Find the distribution of $1/F$. [6 M]

6. (a) A manufacturer produces piston rings for an automobile engine. It is known that the ring diameter is approximately normally distributed and has a standard deviation $\sigma = 0.01$ mm. A random sample of 10 rings has mean diameter as $\bar{x} = 80$ mm. Construct a 95% confidence interval on the mean piston ring diameter.
   (b) A random sample of sizes $n_1 = 20$ and $n_2 = 15$ are drawn from two independent normal populations. The sample means and variances are $\bar{x}_1 = 400,\ s_1^2 = 16$ and $\bar{x}_2 = 300,\ s_2^2 = 25$, respectively, from two populations. Assuming $\sigma_1^2 = \sigma_2^2$ construct a 90% confidence interval on difference of two means $\mu_1 - \mu_2$. [8 M]

7. The titanium content of an alloy is being studied and is assumed to follow a normal distribution. A random sample of 5 is chosen which has following titanium content (in %age)

   8, 9, 10, 8, 9.

   Is there any evidence that the mean titanium content is equal to 9 at level of significance $\alpha = 0.05$. [4 M]

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