ASSIGNMENT - 4

Numerical Solutions of Ordinary and Partial Differential Equations

1. Find the growth parameters corresponding to the roots of the reduced characteristic equation $\rho(\xi) = 0$ for the linear multistep method

$$u_{n+3} = u_n + \frac{3h}{8}[u'_n + 3u'_{n+1} + 3u'_{n+2} + u'_{n+3}].$$

2. Find the interval of absolute stability for the linear multistep method

$$u_{n+1} = u_n + \frac{h}{12}[5u'_{n+1} + 8u'_n - u'_{n-1}]$$

used to solve the IVP $y' = -y, y(x_0) = y_0$.

3. Find the general solution of the difference equations

$$(i)\Delta^2 u_n - 3\Delta u_n + 2u_n = 0$$

$$(ii)\Delta^2 u_n + \Delta u_n + \frac{1}{4}u_n = 0$$

$$(iii)\Delta^2 u_n - 2\Delta u_n + 2u_n = 0$$

$$(iv)\Delta^2 u_{n+1} - \frac{1}{3}\Delta^2 u_n = 0$$

4. Find the range for α so that the roots of the characteristic equation of the difference equations

$$(i)(1-5\alpha)y_{n+2} - (1+8\alpha)y_{n+1} + \alpha y_n = 0$$

$$(ii)(1-9\alpha)y_{n+3} - (1+19\alpha)y_{n+2} + 5\alpha y_{n-1} - \alpha y_n = 0$$

are less than one in magnitude.

5. Show that all solutions of the difference equation

$$y_{j+1} - 2\lambda y_j + y_{j-1} = 0$$

are bounded when $j \to \infty$ if $-1 < \lambda < 1$, while for all other complex values of λ there is atleast one unbounded solution.

6. Show that the two step method

$$u_{j+1} = \frac{4}{3}u_j - \frac{1}{3}u_{j-1} + \frac{2}{3}hu'_{j+1}$$

when applied to

$$u' = \lambda u, \lambda < 0$$

is A-stable.

7. Find the condition on $'a'$ and $'b'$ for which the linear multistep method

$$u_{j+1} - (1+a)u_j + au_{j-1} = h[\{\frac{1}{2}(1+a) + b\}u'_{j+1} + \{\frac{1}{2}(1-3a) - 2b\}u'_j + bu'_{j-1}]$$

when applied to the test equation

$$u' = \lambda u, \lambda < 0$$

is A-stable.

8. Consider the family of linear multistep methods

$$u_{n+1} = \alpha u_n + \frac{\alpha}{2}\left(2(1-\alpha)f_{n+1} + 3\alpha f_n - \alpha f_{n-1}\right)$$

where α is a real parameter. Analyze consistency and order of the method as function of α, determining the value of α^* for which the resulting method has maximal order. Further, using MATLAB draw its region of absolute stability in the complex plane for $\alpha = \alpha^*$.