Indian Institute of Technology, Kharagpur

Date..... FN/AN Time: 2 Hrs Full Marks: 30 No. of Students: 61
Mid (Spring) Semester 2016-17 Subject Name: Discrete Mathematics Deptt: MA

Instruction: Answer **all** questions. Notations used are as explained in the class.

Question 1 [$2 \times 5 = 10$ marks]

a) It is conjectured, but not proved, that there is always a prime between every two consecutive squares n^2 and $(n+1)^2$ for $n \geq 1$. Prove the weaker statement that there are infinitely many n such that there is a prime between n^2 and $(n+1)^2$.

b) Let $E(x_1, x_2, x_3) = \overline{(\overline{x_1 \vee x_2}) \vee (\overline{x}_1 \wedge x_3)}$ be a Boolean expression over the two-valued Boolean algebra. Write $E(x_1, x_2, x_3)$ in both dnf and cnf.

c) Prove or disprove: The NAND operator is associative.

d) Prove that if n is an integer and $(3n+2)$ is even, then n is even using

(i) a proof by contraposition.

(ii) a proof by contradiction.

e) Determine whether the following is valid. If valid, what rule of inferences being used? If not, what logical error occurs?

Some alligators are friendly and sociable.
All alligators which are friendly live in a zoo.
Therefore, some alligators which live in a zoo are sociable.

Question 2 [$2 + 3 = 5$ marks]

a) State the resolution principle and use it to check the validity of the following argument. Do not use truth table.

If the accused lives at a small rate, his economy is evidence of his poverty.
If he has maintained a large expenditure, that must have impoverished him.
But, either he lives at a small rate, or has maintained a large expenditure.
Therefore, he is poor, and is incapable of paying heavily to the king.

b) (i) What does it mean for a set of operators to be functionally complete?

(ii Is the set $\{+, .\}$ functionally complete? Give justification.

(iii) Are there sets of a single operator that are functionally complete? Give justification.

——-P.T.O.———

Question 3 [5 + 2 + 3 = 10 marks]

a) (i) What is *don't care* condition.

(ii) Explain how K-map can be used to simplify *product-of-sums* expansion in four variables.

(iii) Use a K-map to simplify the Bollean function F together with the don't care conditions d in *product-of-sums* form.

$$F(w, x, y, z) = \sum(0, 2, 4, 9, 12, 15),$$

$$d(w, x, y, z) = \sum(1, 5, 7, 10).$$

b) Implement the following function with two-input NAND gates. Assume that both the normal and complement inputs are available.

$$(AB + \overline{A}\ \overline{B})(C\overline{D} + \overline{C}D)$$

c) Use the Quine-McCluskey method to simplify the *sum-of-products* expression for

$$f(x, y, z) = xy\overline{z} + x\overline{y}z + x\overline{y}\ \overline{z} + \overline{x}yz + \overline{x}\ \overline{y}z$$

Question 4 [1 + 1 + 1 + 2 = 5 marks]

a) Distinguish between paradox and dilemma.

b) Translate the following proposition into symbolic form using two-place predicates. Define predicates used and where necessary, define the universe of discourse.

"Everybody loves somebody."

c) Using the rules of inference, prove that $q \wedge [(p \vee q) \wedge (\overline{\overline{q} \wedge \overline{p}})]$ is logically equivalent to $q \wedge (q \vee p)$.

d) A library has two kinds of employees. A person from the first kind always tells truth while the other kind of person always lies. When you visit library to find whether Discrete Mathematics book is available (on shelf), person of library at counter Mr. X answers as

"The Discrete Mathematics book is available if and only if I always tell truth."

How will you confirm whether the book is available or not?
Formalize the problem in propositional logic and find the solution using a truth table.

——-The End———