AD

Indian Institute of Technology Kharagpur
Departments: MA, AR, BT and others.
MA30003 / MA41003 Linear Algebra
Autumn Mid Semester Examination, 2016 No. of Students: 110
Full Marks: 30, Time: 2 Hrs.

INSTRUCTION: Attempt all the questions. Each question carries equal marks.

1. (a) Let V be the set of all pairs (x, y) of real numbers, and let F be the field of real numbers. Examine whether V is a vector space over the field of real numbers or not?

Not a vector space,
(a+b).(x,y) != a.(x,y)+b.(x,y)

$$\begin{aligned}(x,y)+(x_1,y_1) &= (x+x_1, y+y_1)\\ c(x,y) &= (|c|x, |c|y).\end{aligned}$$

(b) Prove that the necessary and sufficient condition for a non-empty subset W of a vector space V over a field F to be a subspace of V is that W is closed under vector addition and scalar multiplication.

Just prove 0 belongs to W

(2+3 = 5 marks)

2. If S, T are subsets of $V(F)$, then show that

(i) $L(S \cup T) = L(S) + L(T)$,

(ii) S is a subspace of $V \Leftrightarrow L(S) = S$,

(iii) $L(L(S)) = L(S)$.

(5 marks)

3. (a) Find whether the vectors $2x^3 + x^2 + x + 1$, $x^3 + 3x^2 + x - 2$ and $x^3 + 2x^2 - x + 3$ of $P(X)$, the vector space of all polynomials over the real number field, are linearly independent or not.

(b) Construct three subspaces W_1, W_2, W_3 of a vector space $V(F)$ so that $V = W_1 \oplus W_2 = W_1 \oplus W_3$, but $W_2 \neq W_3$.

(a,b,c)=(a,b,0)+(0,0,c)=(a,b,c)+(0,tc,c)
(0,0,c)!=(0,tc,c)

(3+2 = 5 marks)

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4. (a) If W_1, W_2 are two subspaces of a finite dimensional vector space $V(F)$, then prove that

$$\dim\,(W_1 + W_2) = \dim\, W_1 + \dim\, W_2 - \dim\,(W_1 \cap W_2).$$

(b) Let V be the vector space of ordered pair of complex numbers over the real field $\mathbb{R}$, i.e., let V be the vector space $\mathbb{C}(\mathbb{R})$. Show that the set $S = \{(1,0), (\mathrm{i},0), (0,1), (0,\mathrm{i})\}$ is a basis for V.

(3+2= 5 marks)

5. (a) Find a basis and the dimension of the subspace W of $\mathbb{R}^3$, where

$$W = \{(x,y,z) \in \mathbb{R}^3 \mid x + 2y + z = 0,\ 2x + y + 3z = 0\}.$$

(b) Let $V = \mathbb{R}^3$ and W be a subspace of V generated by the vectors $(1,0,0), (1,1,0)$. Find a basis of the quotient space V/W. Verify that $\dim(V/W) = \dim V - \dim W$.

(2+3= 5 marks)

6. (a) A mapping $T : \mathbb{R}^3 \to \mathbb{R}^3$ is defined by

$$T(x_1, x_2, x_3) = (x_1 + x_2 + x_3,\ 2x_1 + x_2 + 2x_3,\ x_1 + 2x_2 + x_3), \forall\ (x_1, x_2, x_3) \in \mathbb{R}^3.$$

Show that T is a linear mapping. Find $\ker T$ and the dimension of $\ker T$.

(b) Let V and W be vector spaces over a field F. Let $T : V \to W$ be a linear mapping such that $\ker T = \{\theta\}$. Then show that the images of a linearly independent set of vectors $\{\alpha_1, \alpha_2, \ldots, \alpha_r\}$ in V are linearly independent in W.

(3+2= 5 marks)

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