INDIAN INSTITUTE OF TECHNOLOGY
Department of Mathematics
Time : 3 hrs. Full Marks : 50
No. of Students : 90 END-AUTUMN, 2018
Subject: MA 41007 Functional Analysis

Instruction: "No queries will be entertained during examination". Answer all the questions.

1. State and prove Hahn-Banach extension theorem. **[7 Marks]**

2. Stae and prove uniform boundedness principle. **[7 Marks]**

3. Let X be a normed linear space and $P : X \to X$ be a projection. Then show that P is a closed operator if and only if the null space $Z(P)$ and the range space $R(P)$ are closed subspaces of X. Hence prove P is continuous, if X is a Banach Space. **[7 Marks]**

4. Let X and Y be Banach spaces and $F : X \to Y$ be a linear map which is closed and surjective. Then prove that F is continuous and open map. **[7 Marks]**

5. Let X_0 be a subspace of normed linear space X and Y be a Banach space. Let and $A : X_0 \subset X \to X$ be a bounded and closed operator. Then prove that X_0 is a closed subspace of X. **[5 Marks]**

6. (a) State and prove Bessel's inequality.
   (b) Show that $C[a, b]$ is not an inner product space, hence show that $C[a, b]$ is not a Hilbert space. Is it a Banach space?
   (c) Let E be any orthonormal set in a Hilbert space X and for $x \in X$, let $E_x = \{u \in E\ : < x, u > \neq 0\}$. Then show that E_x is countable.
   (d) State and prove Riesz representation theorem.

[5 + 4 + 3 + 5 Marks]

***** GOOD LUCK*******