**Swarup Panda** Assignment-I

Note: Justification for each question should not be more five lines.

1. Consider the vector space $\mathbb{V} = \{f \in C[0,1] : f(1) = 0\}$ over the field $\mathbb{R}$, where $C[0,1]$ is the set of all continuous functions from $[0,1]$ to $\mathbb{R}$. Which of the following statement(s) is(are) correct?Justify your answer.

   (a) $\mathbb{V}$ is finite dimensional.

   (b) The cardinality of each basis of $\mathbb{V}$ is countable.

   (c) The cardinality of each basis of $\mathbb{V}$ is uncountable

2. Consider the vector space $\mathbb{V} = \mathbb{R}^n$ over the field $\mathbb{R}$. Let $||x|| = \max\{|x_1|, |x_2|, \ldots, |x_n|\}$ for all $x \in \mathbb{R}^n$ be a norm on $\mathbb{R}^n$. Which of the following statement(s) is(are) correct?Justify your answer.

   (a) There is a unique inner product on $R^n$ which induces the above norm $||.||$.

   (b) There are infinitely many inner product on $R^n$ which induce the above norm $||.||$

   (c) There is no inner product on $R^n$ which induces the above norm $||.||$.

3. Let $\mathbb{V}$ be a vector space of dimension 2 over a filed $\mathbb{F}$ and let $|\mathbb{F}| = 2$. Which of the following statement(s) is(are) correct?Justify your answer.

   (a) $\mathbb{V}$ has exactly one basis.

   (b) $\mathbb{V}$ has infinitely many bases.

   (c) $\mathbb{V}$ has exactly three bases.

   (d) $\mathbb{V}$ has exactly four bases.

4. Consider the vector space $\mathbb{V} = \mathbb{M}_{3\times 2}(\mathbb{R})$ (set of all $3 \times 2$ real matrices) over the field $\mathbb{R}$. Let $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ in $\mathbb{R}^2$. Let $\mathbb{W} = \{A \in \mathbb{V} : Ax = 0\}$. Which of the following statement(s) is(are) correct?Justify your answer.

   (a) Dimension of $\mathbb{W}$ is 2.

   (b) Dimension of $\mathbb{W}$ is 6.

   (c) Dimension of $\mathbb{W}$ is 3.

5. Let $\mathbb{V}$ be a vector space over the filed $\mathbb{R}$. Let $u_1$, $u_2$ and $u_3$ be linearly independent vectors in $\mathbb{V}$. Consider $v_1 = u_1 + 2u_2 + 3u_3$, $v_2 = au_2 + 5u_3$ and $v_3 = 2u_3$. Which of the following statement(s) is(are) correct?Justify your answer.

   (a) $v_1, v_2$ and $v_3$ are linearly independent if and only if $a = 0$.

(b) $v_1, v_2$ and $v_3$ are linearly independent if and only if $a \neq 0$.

(c) $v_1, v_2$ and $v_3$ are linearly independent if and only if $a$ is any real number.

6. Consider the vector space $\mathbb{V} = \mathbb{M}_{n\times n}(\mathbb{R})$ over the field $\mathbb{R}$. Let $\langle A, B\rangle = trace(AB^t)$ be an inner product on $\mathbb{V}$. Consider $\mathbb{W} = \{A \in \mathbb{V} : A$ is an upper triangular and $trace(A) = 0\}$. Which of the following statement(s)is(are) correct?Justify your answer.

   (a) Dimension of the orthogonal complement of $\mathbb{W}$ is $n^2 - n$.

   (b) Dimension of the orthogonal complement of $\mathbb{W}$ is $\frac{n^2-n}{2}$.

   (c) Dimension of the orthogonal complement of $\mathbb{W}$ is $\frac{n^2+n-2}{2}$.

7. Let $(\mathbb{V}, \langle , \rangle)$ be an inner product space over the filed $\mathbb{K}$. Let $||x|| = \sqrt{\langle x, x\rangle}$ be the norm on $\mathbb{V}$ induced by $\langle , \rangle$. Which of the following statement(s) is(are) correct?Justify your answer.

   (a) $||x + y||^2 = ||x||^2 + ||y||^2 \implies x$ is orthogonal to $y$

   (b) $||x + y||^2 = ||x||^2 + ||y||^2 \implies x$ is orthogonal to $y$ if $\mathbb{K} = \mathbb{C}$.

   (c) $||x + y||^2 = ||x||^2 + ||y||^2 \implies x$ is orthogonal to $y$ if $\mathbb{K} = \mathbb{R}$.

8. Consider the subspace $\mathbb{W} = \{(x_1, x_2, \ldots, x_n\} : \sum_{i=1}^{n} x_i = 0\}$ of $\mathbb{R}^n$. Which of the following statement(s) is(are) correct?Justify your answer.

   (a) $\mathbb{W}$ has two virtually disjoint complements.

   (b) $\mathbb{W}$ does not have virtually disjoint complements.

   (c) Dimension of each complement of $\mathbb{W}$ is always greater than 1.

9. Let $(\mathbb{V}, \langle , \rangle)$ be a nontrivial inner product space over the filed $\mathbb{K}$. Which of the following statement(s) is(are) correct?Justify your answer.

   (a) $\{\phi\}^\perp = \mathbb{V}$, $\{\phi\}$ is an empty set.

   (b) $\{\phi\}^\perp = \{0\}$, $\{\phi\}$ is an empty set.

   (c) $\{\phi\}^\perp$ is a proper subspace of $\mathbb{V}$, $\{\phi\}$ is an empty set

10. Suppose $C[-1, 1]$ is the vector space of continuous real-valued functions on the interval $[-1, 1]$ with inner product given by

$$\langle f, g\rangle = \int_{-1}^{1} f(x)g(x)dx$$

.

Let $\mathbb{U} = \{f \in C[-1, 1] : f(0) = 0\}$ be the subspace of $C[-1, 1]$. Which of the following statement(s) is(are) correct?Justify your answer.

   (a) $C[-1, 1] = \mathbb{U} \oplus \mathbb{U}^\perp$

   (b) $\mathbb{U}^\perp = \{0\}$

   (c) $\mathbb{U}^\perp$ is a proper and non-trivial subspace of $C[-1, 1]$