Symbolic Logic
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Lecture – 09
Truth-Functional Connectives
Propositional Variables
Propositional Constants

Hello! So we are into module 9 of the NOC course in Symbolic Logic. I have already introduced you to the connectives of the PL, and we are going to do in module 9 symbolization with the use of the connectives.

So, we are going to learn, this is our first lesson in symbolization. So, we are going to first understand what the propositional variables are, what propositional constants are. Remember, this is syntax, so there is going to be certain guide lines for what to use and what not to use. And ultimately, our goal is to learn how to symbolize with the PL connectives. So that's what is in the agenda for today. But before I go and start talking about the symbolization, let me just quickly find out whether and how much of the discussion on connectives you remember. So, we will start with a small and simple problem.

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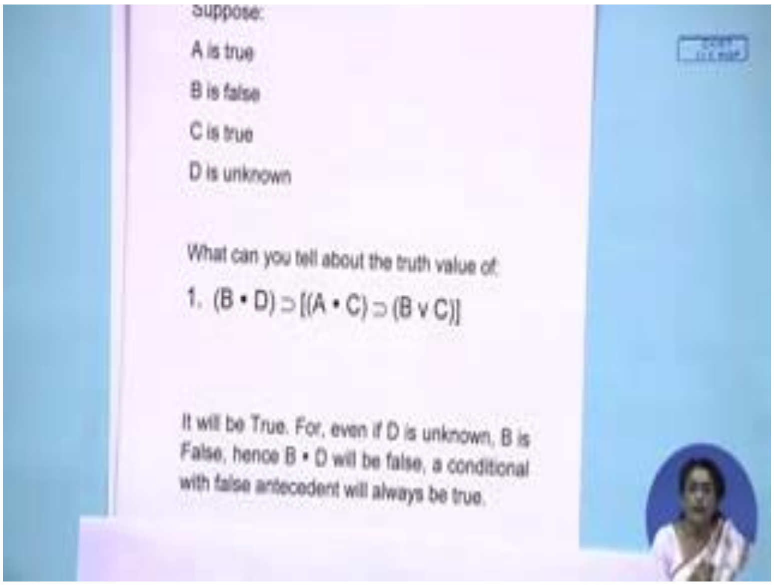

Suppose I tell you that we do not know anything more than this: That we know that the proposition A is true, proposition B is false, proposition is C is true and D, note, is unknown. Right? So these are what is given, and then I am going to ask you : What can you tell me about the truth-value of, say, this compound proposition? This is:

$$(B \bullet D) \supset [(A \bullet C) \supset (B \vee C)]$$

This whole thing is the consequent and $(B \bullet D)$ is your antecedent. So plugging this value, can you tell me what is going to be the truth value of this? I will give you a minute to think about. Remember A is true, B is false, C is true and D is unknown.

Some of you, I am sure, have already figured it out, and some, in case you are in trouble ,or in case you are like little bit confused, I am going to explain this. But this should tell you how much you remember from the previous discussion of the connectives. Because you know unless you know how the dot works, the horseshoe works and the wedge works, you can't really do the computation of the truth-value of this compound.

So, if you recall that, this is actually the test of that, that knowledge that we have already discussed. So, what is the truth value of this whole compound? Can you tell us? The answer is : Yes, we can tell. Because these are all truth functional connectives, and we can figure out only if we can compute the value of the components. So the answer is : Computable.

Then the question is : What is the truth-value of this compound? Now, there are more than one ways to do this answer. But, no matter which method you follow, the answer is going to be : the truth-value of this compound is going to be always true. Now let's see how. I will explain. See, though I understand, you will say that D is unknown, but even if D is unknown, there is a way to find this out. For example, if you start at the antecedent. See, B is false and D is unknown. No matter what the value of D is, B is false. Therefore, $(B \bullet D)$ is going to be false. And once you know the $(B \bullet D)$ is false, you can say that the whole horseshoe proposition is going to be true. Why? Because horseshoe ($\supset$) with false antecedent is always true.

Now, this is one way to go about it. The antecedent you figured out. Those of you who are saying : But D is unknown, what do you… how can you know the value of that? The question is: First of all, even if it unknown, remember, the system is such that it believes in only two truth-values. So either D can be true, or D has to be false. So that's one way, that's one answer right there: That D is unknown doesn't mean that it is completely unknown. You can still make the possibilities clear.

The other thing is that you really don't need to know what the value of D is, if you follow this kind of method to figure out the value. Or, some of you may have looked here: B ∨ C. Both are known values, B is false and C is true. If that happens then B ∨ C, the vel (∨), the way it works, disjunction, it's going to be true. And when you know that, you know that any horseshoe statement with a true consequent is going to be true. So this is true, and which happens to be the consequent of the whole compound proposition. So, if the consequent is true, then same logic applies to this horseshoe, the whole compound is going to be true. Alright?

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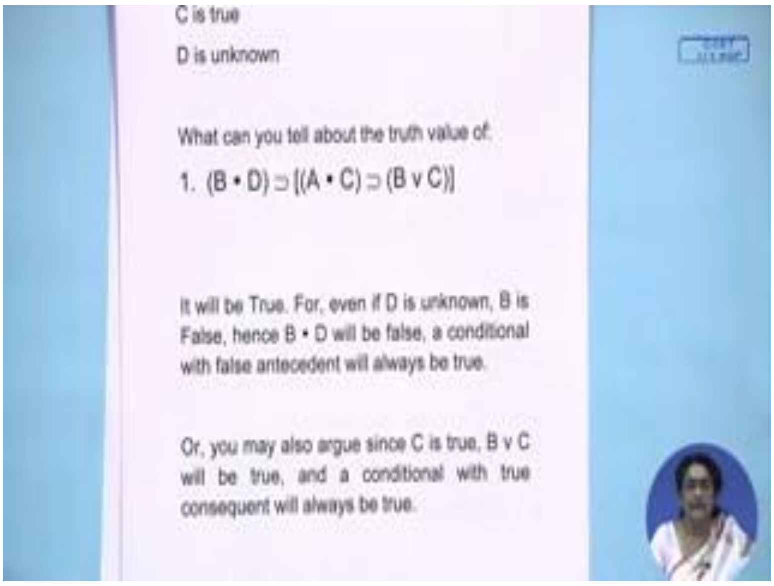

So, this is one way to figure out how you can know that the sentence has to be true from the given values. As I said, this is just an exercise; sort of a reminder to yourself, whether you know the meaning of the connectives that we explained earlier in terms of the truth tables. So, that is the test that was about. If you thing that you are still unsure, I suggest that you look it up, that you come to know about that as confidently as possible. Otherwise, the more you progress towards this PL system; you are going to be slightly unsure about your moves. So that was our first round. But as I said our goal is to learn how to do the symbolization today.

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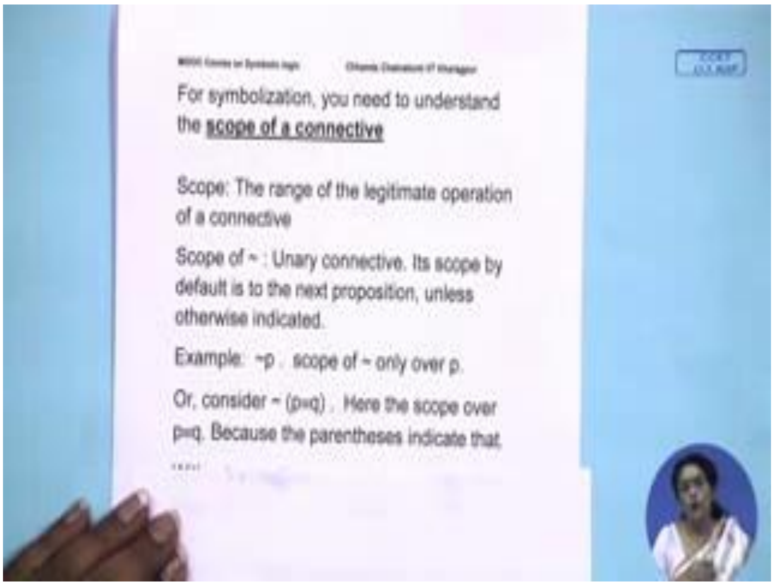

When you are going to do symbolization, my first step would be with the *scope of a connective*. I have introduced already the connectives. You know that there are five connectives. But the point is that the five connectives are actually at par. There is no weightage. There is no relative weightage by which you can say this connective is going to rule over the other connectives. It's not exactly like algebra. So, all connectives are at par. Therefore there has to be other mechanisms to know which one to compute first, which one to do, to save for the later, and so on and so forth. And for that the concept of scope of connective is going to be very very handy.

What is scope? Scope here means the range of legitimate operation of a connective. How far can it legitimately operate on? What is its power ? What is the limit of its power? That's what scope indicates. Now if you are dealing with Unary connectives, remember these are the ones that can pick up at most one proposition when it is connecting.

So there is only one Unary connective, that's your tilde. And the tilde, the way it works, its default scope is only up to the adjacent proposition. So up to the next proposition. Unless there are some other indications in the proposition. Right? Let's see what we mean.

So suppose you find ~P. That's a unary connective. As I said, default is that the scope starts from here and it goes only up to the adjacent proposition which is P. So the scope starts here and it ends here, only over P.

Now you compare that with, say, this statement. This says tilde (~) and then there is a bracket, a parentheses starting, within the parentheses there is p ≡ q. Again the same logic applies that the scope start here, it goes up to the adjacent proposition. But here the adjacent proposition is not single P. The adjacent proposition because of the presence of the parentheses is p ≡ q. So the range of this tilde (~) will start here, and will go till the end of the proposition. That is what this parenthesis does. It indicates that the tilde's scope goes over the entire thing within the parentheses. Ok? So this. Now, if we had missed out doing the parentheses and we just left it like this. That's an important lesson for you also. Suppose we don't use the parentheses here and we just leave it like this, then what happens? Then the answer is that this tilde (~) the range of it will start here; it will go only up to the adjacent P. It will not range over the whole proposition.

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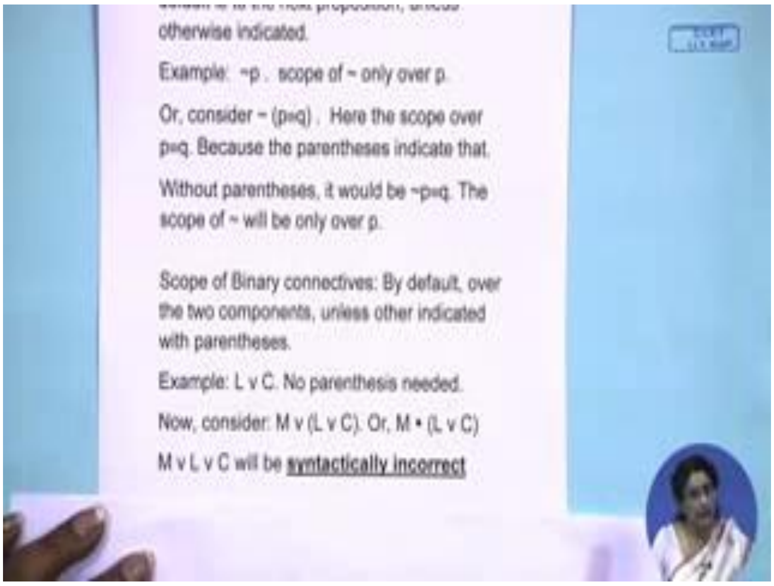

So what do you have? We have two completely different kinds of propositions. This one is a negation. Its main connective is tilde, remember main connective means though one with the largest scope; whereas here you are dealing with a triple bar proposition. Why? Because the triple bar, the dyadic connective, it the range starts from here it goes up to here. Tilde is only starting here; it ranges only over the P. Alright? The the truth tables will differ, the meanings will differ, therefore you need to understand the scope very carefully and wherever possible, if you want to mean it in a certain sort of way, use the mechanism called parentheses or the brackets.

Now, talking about the Binary connectives. So far we saw the Unary connective. Binary connective means that they are going to connect two, at most two. By default, the range of the dyadic connective is going to be over the two components, unless you indicate it in a different sort of way. So let's take an example.

For example here, L wedge C. Because 'wedge' is a dyadic or a binary connective, no need, if you want this to be the main connective, no need to have any parentheses. Because, default is that it will go over the…both the components that it is connecting. Now suppose you have more than two, and this is where you will understand why it is important to indicate the scope. See here, for example, you have M wedge (L wedge C). Same connective, but you need to remember that each vel or "∨" is a dyadic connective. So at a time it is going to only pick two. Therefore, there is need to group it in such a way that you have… ultimately you have two disjuncts. So here is this wedge, and this is where the range ends, it goes up to here and goes up to here. This one ,on the other hand, is the main connective. But here is the disjunct and the whole thing here is a disjunct.

Now many of you will think that 'but the truth value is the same, why do I need the parentheses?' Please remember that what you talking about here is the *emphasis*. How we are disjoining it? We are saying here 'Either M, or L wedge C'. . So, the emphasis is that this is my one option, and the other option is an 'either-or'. If you regroup it in the different sort of way, for example, if you want to put the brackets here, between M and L and you say 'Either M or L, or C', the emphasis is different and the meaning also changes. Therefore, remember the grouping is important.

This, on the other hand, you are dealing with two kinds of connectives. Here is a 'wedge ', here is a 'dot'. And you need to indicate which one is your main connective. This one is a conjunction. So here is a conjunct ,and then there is other conjunct which happens to be your disjunction by itself.

So this is how we are going to read. What we will not allow syntactically is to leave this kind of sentences stand alone. Though it is the same connective, if you do not indicate what grouping you are intending it for, or which to are the disjuncts, then just by itself this is going to give you a syntax error. Therefore the mechanism of using the parentheses is going to be quite important for you to learn at this point.

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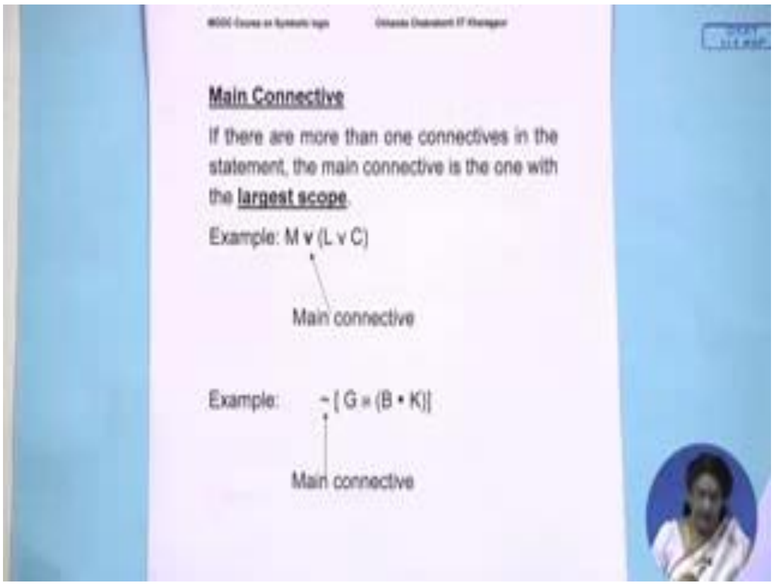

Now main connective. I have already mentioned this, but let me take examples so that we are here. Whenever you have more than one connective in the statement, remember the main connective is the one that I said will have the largest scope. The largest scope. So the subsequent connectives are going to be sub-connectives, but the one which has the maximum scope within the proposition is going to be the main connective. For instance here, as I was saying, if you group it like thi,s and the brackets are here, parentheses are here, then what you indicating this is your main connective. Though the 'wedge' is here, the 'wedge' is here also, please note the range of this 'wedge' , as I said, starts from L and ends with C. That's a sub-connective. This one on the other hand, has a larger scope. It starts from M, it goes all the way over to the end of the proposition. Hence this is your main connective.

Let me take another example to make this point clearer, say here. There is tilde, there is triple bar, and there is dot. Which one is the main connective? The answer is : The tilde. Why? Because every other, if you take every other connective, the tilde has the maximum scope. This dot, it ranges over B and ends with K. This triple bar, it starts with G and ends with this sentence. But this tilde, because of the way we have used the brackets it starts from here, beginning of the proposition, and goes until the end.

So this is how to understand what is your main connective. Why knowing main connective is important? Because soon we are going to go and talk about truth table and other things, you will see that the main connective holds a primary position. How we understand, how we approach the proposition, you will see that the main connective tells you what kind of a proposition you are dealing with, what truth-table you are going to use, etcetera. So at this time please pay attention to the main connective, how to understand the main connective.

Now we are in a position to talk about the propositional variables. Remember we said we are going to understand the propositional variables ? So here we are.

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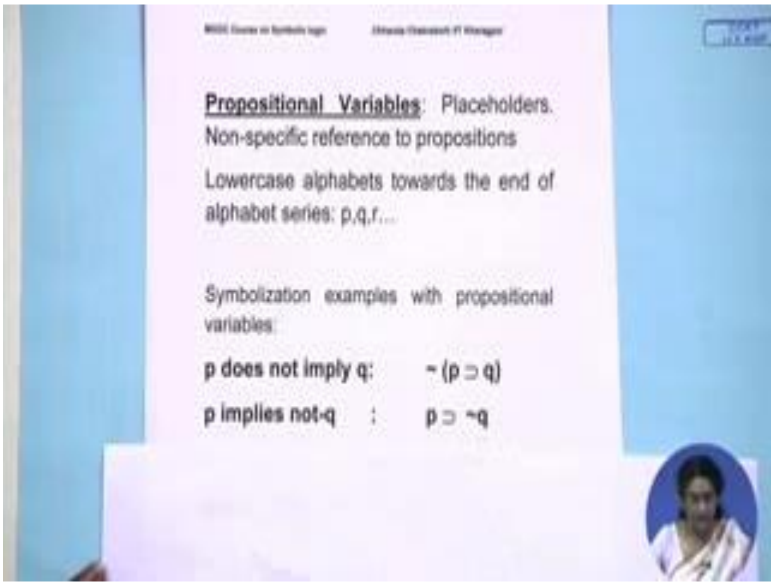

Variables, I am sure you have been exposed to the idea of variables earlier; if not, there is always a first time. What variables are, are place holders. Place holders. They hold a position, and there is no… some other thing can go into that place; so they just occupy space. When you want to use this kind of variables with proposition’s reference, what you are doing is making a non-specific reference to propositions. You don’t mean a particular proposition, like ‘today is Tuesday’, or something like that. You just have *any* proposition; when we want to talk about *any* proposition.

Then symbolically we say, this is case-sensitive, choose lower case alphabets; lower case alphabets towards the end of the alphabet series. In fact we choose *p, q, r* etcetera. Please note, I am going to remind you again, the *lower case*. So if you are using propositional variables, symbolization might work like this : Say we want to say for *any* two propositions p and q, any proposition p and q, suppose we want to say ‘p does not imply q'. I have already introduced you to the connectives. How to read this? We are talking about given *any two propositions*, p and q, it is not the case that p implies qSo the translation is going to be tilde, it is not the case that p horseshoe q. Alright? Similarly, when we are saying *p implies not q*; again, we are using propositional variables, not actual propositions. What we mean is that p implies that it's not the case that q. So p implies, if p, then its not the case that q. p horseshoe tilde q, $(p \supset \sim q)$

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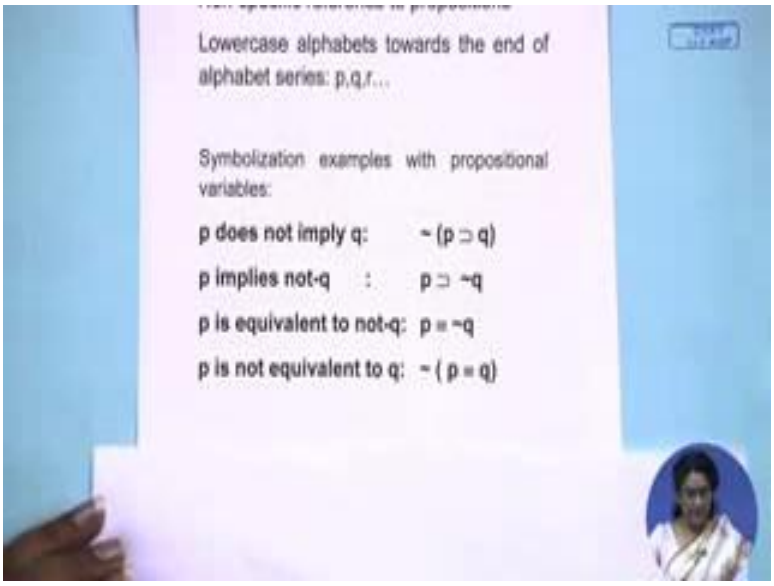

You can try this yourself so that your grasp over symbolization grows a little bit. Here is another kind of translation, see? p is *equivalent* to not-q. So the main connective is this, p is *equivalent* to not q. So here is p *triple bar* tilde q, $p \equiv \sim q$. Which one is the main connective? The equivalence or the triple bar.

Last one here: p is not equivalent to q. Very different from what I am saying here. What I am saying here? It is not the case that p is equivalent to q. Try to read it and you will see why the translation has to follow in this way. It is not the case that p triple bar q, $\sim(p \equiv q)$. The tilde attaches and rules over the entire sentence. So take a look, the main connective here is the tilde. The main connective here is the tilde. In order to indicate its large scope, we need to use these parentheses. Alright?

Let's now move to propositional constants. So propositional variables, we saw what they are and how they work with them. Remember the lower case alphabets towards the end of the series.

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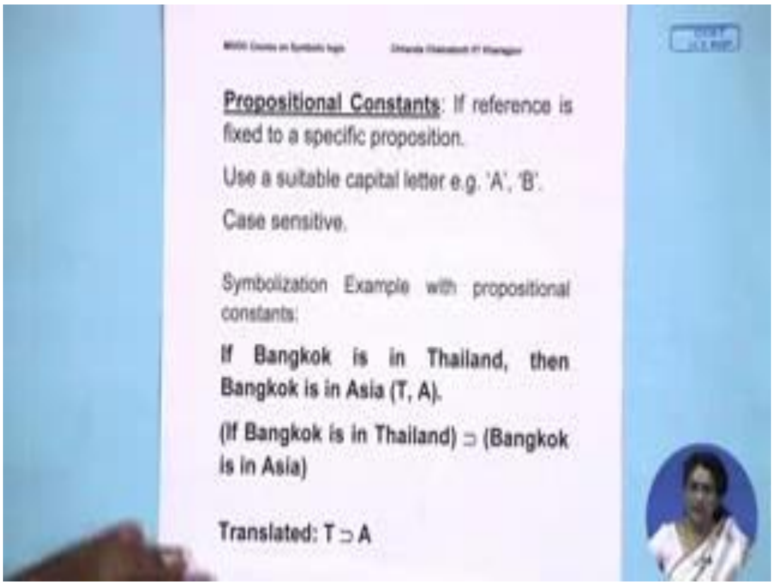

Propositional constants, on the other hand, are like numerical constants. Within the context they refer to once specific proposition. So they remain constantly referred to to some actual proposition. Symbolically saying, we need to use *capital letter* and with respect to the proposition, you will see soon when we go into actual problems, that we need to find the suitable capital letters; something that reminds you of that proposition. But, the point here to remember that you need to use capital letters. So, something like A, B. The inverted commas are not required, but this is capital letter that you need to. So if you are using capital letter, we understand that you are referring to actual propositions; if you are using lower case, we know that you are talking in terms of variables. So, case sensitivity is something to remember by.

Here is example of symbolization with propositional constants. Suppose we have ‘if Bangkok is in Thailand then Bangkok is in Asia’. Here is a proposition, here is a proposition, and because these are actual propositions, we will pick up capital letters and suitable capital letters in the sense that in order to remind us, we are talking about this proposition which has Thailand in it, we choose the T, the capital T. This one has the reference to Asia. So we have chosen capital letter A. This is an 'if then' sentence. So very easy to understand that the main connective is going to be horseshoe ‘⊃’.

So this is how to read, and then, once you are practiced, this is the translation that should be of that proposition. But remember these are all constants and you are talking about T ⊃ A as the symbolization for an actual proposition such as this.

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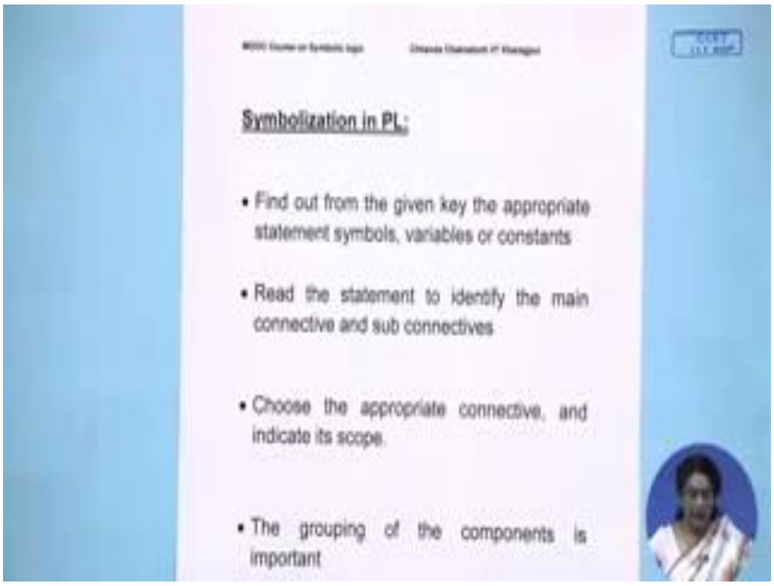

So in general, therefore, how to do symbolization in PL? And we will see more symbolization in the next module. But, remember that, first of all, you need to choose the statement symbol. If the key is already given in which symbols are mentioned, then your job is done, you just follow those symbols. But the main point is whether you are using the variables or constants. By now you should know what we are talking about and there is a difference. But most of the time, along with the problem, you will be given the statements symbols to use for with clear indication what the symbol stand for.

Once you have the symbols, then read the statement carefully. Why? So that you can identify the connectives; especially what is going to be the main connective. There may be sub-connectives also, but this symbolization means you are translating from one language to another. So there will be English words and then you need to figure out what will be the corresponding PL connective to use in place of that English word. That is going to be how the symbolization is going to be done. And the thing is, finally, is that once you have chosen the appropriate connective in each case; remember to check its scope. Because if there is an error to approach the scope, you are changing the meaning of the proposition. Your job is to translate, not distort; not completely change the meaning of the proposition itself.

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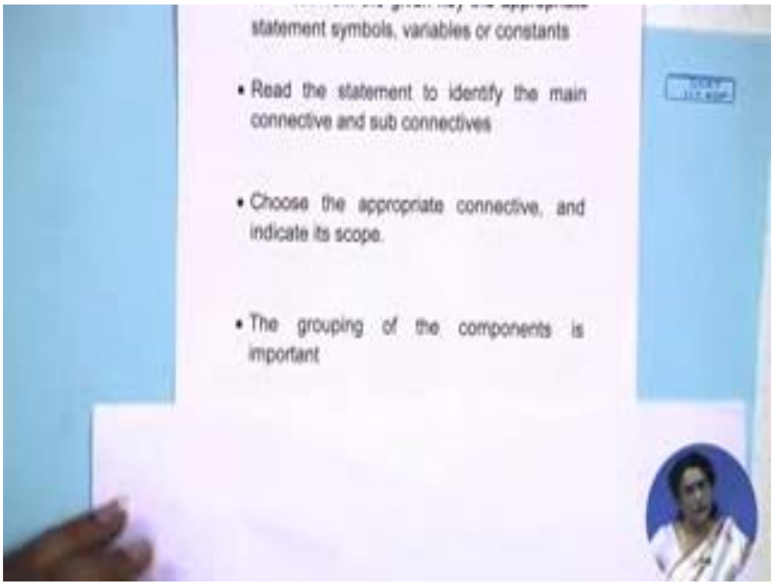

And the grouping of the components is going to be important also and this is what you learnt in terms of use of the mechanism of parentheses. So how the intended proposition is meant, that grouping you have to sort of keep as is. Once this is done, then we are in a position to talk about the actual symbolizations. So I am going to try out certain things with you.

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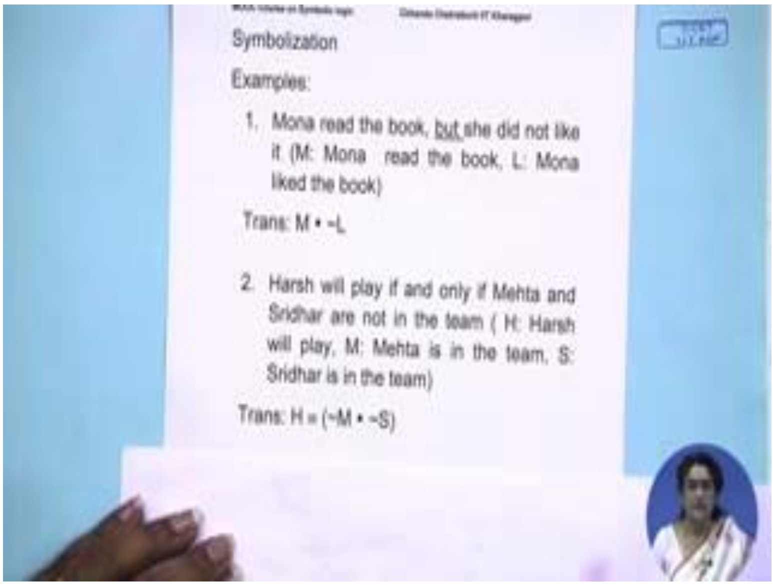

Here is an English sentence. ‘Mona read the book, but she did not like it’. Two sentences or two propositions, and here is the given key. M stands for ‘Mona read the book’, L for ‘Mona likes the book’ and here is “but”. This ; ‘but’ is to be translated as what in PL? Its corresponding is the ‘and’, or the conjunction or the ‘•’. So if you have done that correctly, then the translation is going to be very easy ‘M dot not-L’, (M •~L). Where is this ‘not’ coming from? The tilde is coming from ‘not’. ‘did *not* like it’. Remember the key is ‘Mona likes the book’. So if you want to say ‘Mona did not like the book’, this is how it is going to appear.

Here is another sentence. ‘Harsh will play if and only if Mehta and Sridhar are not in the team’. Alright? Now, as you can see, start reading it with me so that you can see the there are three components in it. One is ‘Harsh will play’, and that is your H, the other one is ‘Mehta is not in the team’. There is no sentence called ‘Mehta and Sridhar’. Obviously ‘Mehta is in the team’, that is your simple proposition. ‘Sridhar is in the team’ also a component. So, M is for ‘Mehta is in the team’; S is for ‘Sridhar is in the team’. Now we are in a position to find out how to translate. Which one is the main connective you think? And you will ….if you are reading it properly, you will say the ‘if and only if’ is the main connective. Once you know that the rest is easy; slowly will go. So this is our one of the component H, and then this is going to be triple bar and here comes the next one, which is ‘Mehta and Sridhar are not in the team’. So Mehta is not in the team Sridhar is not in the team.

Now how to express that ? And you see that you need to translate it like this. One is ‘Harsh will play if an only if’ and then you are saying ‘Mehta and Sridhar are not in the team’. So neither Mehta is in the team nor Sridhar is in the team. It is not the case that Mehta is in the team *and* it is not the case that Sridhar is not in the team. Right?

You need to work a little bit with problems such as this to see how the translation is happening here.

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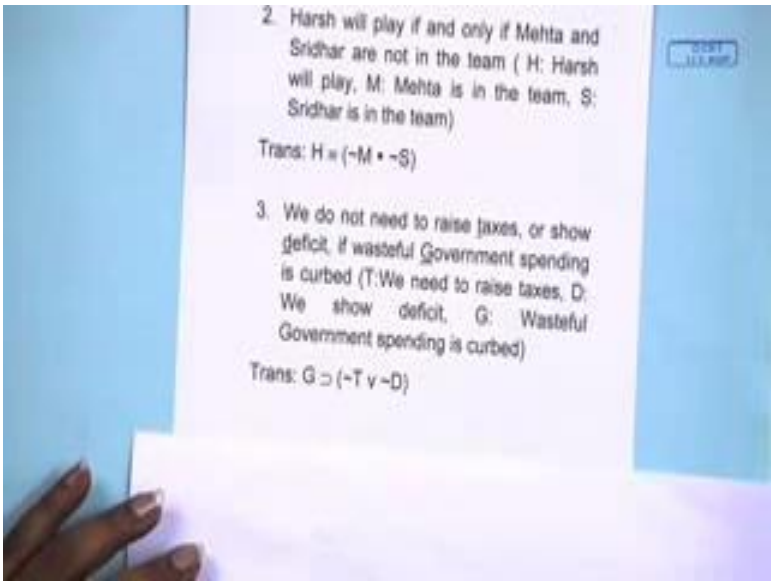

Let me go through another example here. This is "we do not need to raise taxes or show deficit, if wasteful government spending is curbed". Clear indication that we have one, two, three; three components here. But T stands for 'we need to raise taxes'. Remember, usually the key is given in terms of simple proposition. Wherever you have 'not', you have a compound sentence in your hand.

So T stands for we need to raise taxes, D is for 'we show deficit', G is for 'wasteful government spending is to be curbed'. And the translation happens like this. That G… if G, then not-T or not-D. How did we get this 'if-then?' Well, here, take a look. 'We do not need to raise taxes or show deficit if…' whatever. Now…follows 'if', that is your antecedent. How do we know this is our, our main connective? Read again, there is a comma. So it separates out the component clearly. This is your antecedent and this whole thing is your consequent. So this translation is to be done carefully with this kind of parsing.

With that, we are going to end this, but not without giving you a little bit of help here is that, you know, don't get discouraged in the very beginning that if you are missing out translation, or 'I do not know how to do this' etcetera. Give yourself a little practice. For example, don't look at the cues I have given. Try to do this on your own and see whether you come up with the same result or not. Sometimes we are impatient in reading the proposition. We just did not get it; we just think that in one reading we will be able to answer… understand it. Not necessarily. So give yourself a little bit of a practice. Here, for example, if you missed out this ‘if’, then you are missing out the whole structure of the proposition. So a little bit of a careful reading, careful understanding, and a little patience in the beginning is going to help you a little. With that I am going to end this module, but we will come back with more symbolization in the next module.

Thank you very much.