No queries will be entertained during examination

Indian Institute of Technology, Kharagpur

Date ....FN/AN, Time: 2 hrs, Full Marks 30, Deptt : Mathematics

No. of students 60 Year 2014 Mid Semester Examination

Sub. No.: MA31007 Sub. Name: Mathematical Methods M. Sc./ M. Tech (Dual)

Group I

10 Marks

1. Check whether following series converge [4M]
   a) $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\cdots$
   b) $\sum 1/n$
   c) $1+\frac{1}{2!}+\frac{1}{3!}+\cdots$
   d) $\sum\ u_n\ , u_n=\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$

2. [4M]
   a) State D'Alembert's ratio test and Raabe's test for the convergence of infinite series.
   b) Test the convergence of the series

$$\frac{\alpha}{\beta}+\frac{1+\alpha}{1+\beta}+\frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)}+\cdots$$

3. Find the radius of convergence of following power series [2M]
   a) $1+2x+3x^2+4x^3+\cdots$
   b) $\frac{1}{2}x+\frac{1.3}{2.5}x^2+\frac{1.3.5}{2.5.8}x^3+\cdots$

Group II

20 Marks

Attempt any four questions

4.
   a) Define ordinary point, regular singular point and irregular singular points of second order ordinary differential equation.
   b) What are the regular singular point(s) in the finite domain of the following ODE

$$(1-x^2)y''-xy'+4y=0$$

   Obtain series solution of this ODE around $x=0$.

P.T.O.

5. Find the singular point(s) of the equation

$$(x + x^2 + x^3)y'' + 3x^2y' - 2y = 0$$

Obtain two linearly independent solutions around $x = 0$.

6.
   a) Solve the Legendre equation of order n around $x = 0$. Show that Legendre polynomial $P_n(x)$ is one solution when n is positive integer or zero.
   b) Establish Rodrigue's formula for $P_n(x)$.

7.
   a) Check whether $x = \infty$ is a regular singular point of the hypergeometric equation

$$x(1 - x)y'' + [\gamma - (\alpha + \beta + 1)x]y' - \alpha\beta y = 0$$

   b) Find two linearly independent solutions around $x = 0$, where $1 - \gamma \neq$ integer or zero.

8. Prove the following orthogonal property of Legendre polynomials

$$\int_{-1}^{1} P_m(x)\, P_n(x)\, dx = 0, \qquad m \neq n$$

$$= \frac{2}{2n+1},\ m = n$$

9.
   a) Solve the hypergeometric equation around $x = \infty$ and write down the solutions in terms of hypergeometric functions.
   b) Show that Legendre equation can be transformed into hypergeometric equation.

10. Prove the following recurrence relations for Legendre polynomials
   a) $P'_{n+1}(x) - xP'_n(x) = (n + 1)P_n(x)$
   b) $(n + 1)P_{n+1}(x) - (2n + 1)xP_n(x) + nP_{n-1}(x) = 0$
   c) $xP'_n(x) - P'_{n-1}(x) = nP_n$

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