Hints and Answers of Tutorial Sheet-1, MATHEMATICS-II Spring 2017

1. (i) Not a vector space. Scalar multiplication is not distributive over vector addition.

   (ii) Not a vector space. Vector addition is not associative.

   (iii) Vector space. Verify all the properties.

   (iv) Not a vector space. Closure property does not hold for addition.

   (v) Vector space. If p_1, p_2 are periods of the functions $f_1, f_2 \in V$, then $lcm(p_1, p_2)$ is the period of $f_1 + f_2$. Verify all the properties.

2. (i),(ii),(iii) Subspace. Verify $\alpha v_1 + \beta v_2 \in S$, for scalars α, β and vectors $v_1, v_2 \in S$.

   (iv),(v) Not a subspace. Closure property does not hold for addition.

   (vi) W is a subspace of $\mathbb{R}^3$ but not a subspace of $\mathbb{C}^3$. In $\mathbb{R}^3$, $W = \{(a, b, c) \in \mathbb{R}^3 : a = b\}$ is a subspace. But in $\mathbb{C}^3$, $W = \{(a, b, c) \in \mathbb{C}^3 : a = b, a = \omega b, a = \omega^2 b, \omega^3 = 1\}$, then the closure property does not hold for addition.

3. If S is a subspace then $0(x) \in S$, where $0(x) = 0, \forall x \in [0, 1]$. This implies $b = 0$. Conversely prove that $S = \{f \in C[0, 1] : \int_0^1 f(x)dx = 0\}$ is a subspace of $C[0, 1]$.

4. Let $g(x) = \alpha f_1(x) + \beta f_2(x)$ for $\alpha, \beta \in \mathbb{R}$ and $f_1, f_2 \in S$. Try to show that $g'(-1) = 3g(2)$ i.e. $g(x) \in S$.

5. (a) $E = 2A - B + 2C$. Let $E = \alpha A + \beta B + \gamma C$ and solve for α, β, γ.

   (b) $p = \frac{1}{2}(p_1 - p_2 + p_3)$. Let $p = \alpha p_1 + \beta p_2 + \gamma p_3$ and solve for α, β, γ.

   (c) (i),(ii),(iv). Let $(4, 2, 6) = \alpha u + \beta v$ and solve for α, β. Do similar for (ii), (iii) and (iv).

6. Try to show that u_3 is a linear combination of u_1, u_2, i.e. $u_3 = \alpha u_1 + \beta u_2$ for some scalars α, β.

7. (a) If $spanW = V$ there exist α, β, γ such that $\alpha(v_1 - v_2) + \beta(v_2 - v_3) + \gamma(v_3 - v_4) + \delta v_4 = av_1 + bv_2 + cv_3 + dv_4$, for some fixed $a, b, c \in F$, then try to find α, β, γ in terms of a, b, c .

   (b) Similar to part (a).

8. (a) Linear independent. Consider the relation $c_1(4,-4,8,0)+c_2(2,2,4,0)+c_3(6,0,0,2)+c_4(6,3,-3,0)=0$. Try to solve for $c_1=c_2=c_3=c_4=0$

   (b) Linearly dependent. Consider the relation $c_1.2+c_2.(4\sin^2 x)+c_3.(\cos^2 x)=0$. Differentiating it successively twice try to solve for non zero c_1, c_2, c_3.

   (c) Linearly dependent. Consider the relation $c_1(t^3-5t^2-2t+3)+c_2(t^3-4t^2-3t+4)+c_3(2t^3-7t^2-7t+9)=0$. Try to solve for non zero c_1, c_2, c_3.

   (d) $f_1,\ f_2$ are linear dependent in $[a,b]$ if $f_1(t)=\alpha f_2(t)\ \forall t\in[a,b]$. In $[-1,0]$, $f_1(t)=-f_2(t)\ \forall t\in[-1,0]$. In $[0.1]$, $f_1(t)=f_2(t)\ \forall t\in[0,1]$. But in $[-1,1]$ there does not exist such unique α.

   (e) $\{1+i,1-i\}$ is independent in $\mathbb{R}^2$, but dependent $\mathbb{C}$. Since $1(1+i)-i(1-i)=0$.

   (f) Linearly independent. Since for $x=2$ we can get $c_j\neq 0$ for $j=0,1,\ldots,m$ such that $\sum_{j=0}^{m} c_j p_j(2)=0$. As $p_j(2)=0$, for $j=0,1,\ldots,m$