RA INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Date———FN/AN 2 Hrs. Full Marks: 30 No. of Students 75

Mid Spring Semester 2011-2012 Deptt: MATHEMATICS Sub No: MA 51002/MA50002

——Yr. B.Tech.(H)/B.Arch.(H)/M.Sc. Sub. Name: Measure Theory & Integration

Instruction: Answer all questions, which are of equal values

1. (a) Show that the interval (a, ∞) is measurable.

   (b) Show that $\chi_{A\cap B} = \chi_A.\chi_B$ and $\chi_{A\cup B} = \chi_A + \chi_B - \chi_A.\chi_B$

2. (a) Show that if $F \in \mathcal{M}$ (measurable sets) and $m^*(F\Delta G) = 0$, (Δ denotes the symmetric difference) then G is measurable.

   (b) Show that there exists an uncountable set of measure zero.

3. (a) Let f be defined on $[0,\ 1]$ by $f(0) = 0, f(x) = x \sin \frac{1}{x}$ for $x > 0$. Find the measure of the set $\{x : f(x) \geq 0\}$.

   (b) Show that if f_n are sequence of measurable functions (with same domain of definition), then $\limsup f_n$, $\liminf f_n$ are measurable.

4. (a) Let f be a nonnegative measurable function. Show that $\int f = 0$ imply $f = 0$ *a.e.*.

   (b) Show that there exist a non-measurable set on $[0,\ 1)$.

5. (a) Let $\{f_n\}$ be a sequence of non-negative measurable functions. Then show that

$$\int \sum_{n=1}^{\infty} f_n\, dx = \sum_{n=1}^{\infty} \int f_n\, dx$$

   (b) Show that if f is measurable function, then $|f|$ is measurable but not conversely.

6. (a) Let f be a non-negative function which is integrable over a set E. Then show that for given $\epsilon > 0$, $\exists \delta$ such that for every set $A \subset E$ with $mA < \delta$

$$\int_E f < \epsilon.$$

   (b) Show that $\lim_{n\to\infty} \int \frac{nx}{1+n^2x^2}\, dx = 0$.