The Transportation Model

Formulations

The Transportation Model

The transportation model is a special class of LPPs that deals with transporting(=shipping) a commodity from **sources** (e.g. factories) to **destinations** (e.g. warehouses). The objective is to determine the shipping schedule that minimizes the total shipping cost while satisfying supply and demand limits. We assume that the shipping cost is proportional to the number of units shipped on a given route.

We assume that there are m sources 1,2, …, m and n destinations 1, 2, …, n. The cost of shipping one unit from Source i to Destination j is c_{ij}.

We assume that the availability at source i is a_i (i=1, 2, …, m) and the demand at the destination j is b_j (j=1, 2, …, n). We make an important assumption: the problem is a **balanced** one. That is

$$\boxed{\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j}$$

That is, total availability equals total demand.

We can always meet this condition by introducing a dummy source (if the total demand is more than the total supply) or a dummy destination (if the total supply is more than the total demand).

Let x_{ij} be the amount of commodity to be shipped from the source i to the destination j.

Thus the problem becomes the LPP

Minimize
$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to
$$\sum_{j=1}^{n} x_{ij} = a_i \quad (i = 1, 2, \ldots, m)$$

$$\sum_{i=1}^{m} x_{ij} = b_j \quad (j = 1, 2, \ldots, n)$$

$$x_{ij} \geq 0$$

Thus there are $m \times n$ decision variables xij and m+n constraints. Since the sum of the first m constraints equals the sum of the last n constraints (because the problem is a balanced one), one of the constraints is redundant and we can show that the other m+n-1 constraints are linearly independent. **Thus any BFS will have only m+n-1 nonzero variables.**

Though we can solve the above LPP by Simplex method, we solve it by a special algorithm called the transportation algorithm. We present the data in an $m \times n$ tableau as explained below.

Destination

		1	2	.	.	n	Supply
S	1	c_{11}	c_{12}			c_{1n}	a_1
o	2	c_{21}	c_{22}			c_{2n}	a_2
u r	.						
c	.						
e	m	c_{m1}	c_{m2}			c_{mn}	a_m
Demand		b_1	b_2			b_n	

Destination

		Denver	Miami	Supply
Source	Los Angeles	80	215	1000
	Detroit	100	108	1300
	New Orleans	102	68	1200
	Dummy	0	0	200
	Demand	2300	1400	

We write inside the (i,j) cell the amount to be shipped from source i to destination j. A blank inside a cell indicates no amount was shipped.

Destination

Source	Denver	Miami	Supply
Los Angeles	80	M	1000
Detroit	100	108	1300
New Orleans	102	68	1200
Dummy	200	300	200
Demand	2300	1400	

Note: M indicates a very "big" positive number.
In a software it is denoted by "infinity".

Destination
Distribution Area

Source

Refinery	1	2	3	Supply
1	12	18	M	6
2	30	10	8	5
3	20	25	12	8
Demand	4	8	7	

The problem is a balanced one. M indicates a very "big" positive number.

The total cost will be 10* $\sum_{i=1}^{3}\sum_{j=1}^{3} c_{ij} x_{ij}$

Destination

Distribution Area

Source Refinery

	1	2	3	Dummy	Supply
1	12	18	M	15	6
2	30	10	8	22	5
3	20	25	12	0	8
Demand	4	8	4	3	

M indicates a very "big" positive number.

The total cost will be 10* $\sum_{i=1}^{3}\sum_{j=1}^{3} c_{ij}x_{ij}$

Destination
Retailer

Source — Orchard

	1	2	3	4	Dummy	Supply
1	1	2	3	2	0	350
2	2	4	1	2	0	400
3	1	3	5	3	M	250
Demand	150	150	400	100	200	