Test - I Answers
Mathematical Methods

23.9.20

Q1. (a)
$$\frac{dy}{dx} - p(x)y = q(x) \qquad y(x_0) = a$$
$$y' - \tan x \, y = 3x \qquad y(2\pi) = 0$$

$p(x)$ has discontinuities at $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$
$q(x)$ is always continuous. $\therefore$ Sol^n. of this IVP is guaranteed to exist uniquely on any interval containing $x_0 = 2\pi$ but not containing any of the discontinuities. The largest of such interval is $\left(\frac{3\pi}{2}, \frac{5\pi}{2}\right)$

(b) With similar argument $(-9, 9)$

(c) $(9, \infty)$

(d) $(-\infty, -9)$

(e) No assurance of a unique solution

Q2. Two L.I. sol^n. $y_1 = e^t$, $y_2 = te^t$

$y_p(t) = -\frac{1}{2}e^t \ln(1+t^2) + te^t \tan^{-1} t + \frac{3}{2}t^2e^t$ [I may be wrong To check this]

Q3.
$$q(x) = \frac{a_2(x)}{a_0(x)} \exp\left\{\int \frac{a_1(x)}{a_0(x)} dx\right\}$$
$$x^{-2}\frac{d^2y}{dx^2} - 2x^{-3}\frac{dy}{dx} + 2x^{-4}y = 0$$
$$(\cos x)y'' - (\sin x)y' + (\cos x)y = 0$$

Q4. (a) $y_1 = x^2$, $y_2 = x^2 \log x$ $\quad W(x) = x^3 \neq 0$, x is not a pt. in the domain of $\log x$ $\therefore$ L.I.
$$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 0$$

(b) $W = 2x^3$ is not zero everywhere
$$x^3\frac{d^3y}{dx^3} - 3x^2\frac{d^2y}{dx^2} + 6x\frac{dy}{dx} - 6y = 0$$

Q5.
$$G(x,t) = \begin{cases} -\dfrac{\sinh x \sinh(t-1)}{\sinh 1} & 0 \le x < t \\ -\dfrac{\sinh t \sinh(x-1)}{\sinh 1} & t < x \le 1 \end{cases}$$

Sol^n. $y = \dfrac{\sinh x}{\sinh 1} - x$