Symbolic Logic
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Lecture - 13
Using Truth Table
Testing A Set of Propositions for Consistency and Inconsistency and for Logical Equivalence

Hello, this is module 13 of our NOC (Refer Time: 00:24) course on symbolic logic, and we are going to do more of truth table and learn something new; namely, how to determine validity and invalidity of an argument.

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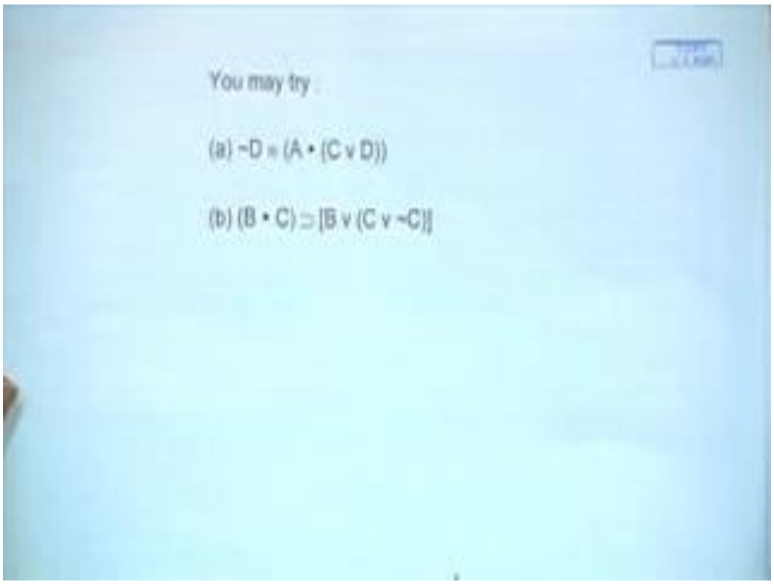

But before that, you may remember that, in our previous module we said, you know, why don't you try doing these problems to see whether this is tautology, contradiction or contingent. So have you given some try to see whether this, you can do the truth table of this? So otherwise we will be looking at the result for how to do this, but the whole idea is that you will also try out, and then you will look at the result that I have produced and sort of see from that whether and how far your work is matching mine and so on and so forth.

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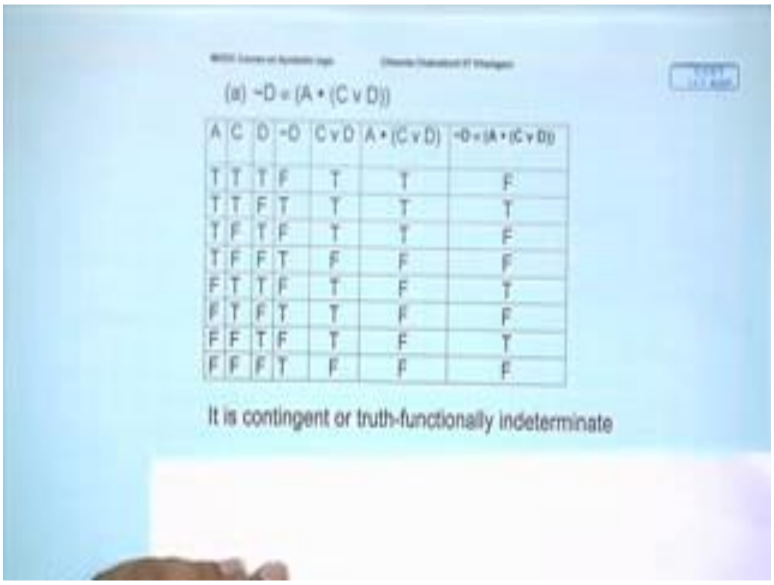

So we will start with the second one. Sorry, the very first one. This is our ~D ≡ (A ●(C ∨ D)). And this is what we are looking at we are trying to figure out whether it is a… what kind of proposition it is, in terms of truth values. Remember last module was about that. So the heads are going to be like this. We need columns; reference columns are A, C, D and then there has to be a separate column for ~D. Why? Because that is a component and remember it is a compound. So the way we are following it, there has to be a separate column assign to ~D also. Then this is C∨D. This is a sub-connective, and here (A ● C) ∨ D. That is a sub connective. The main connective is the triple bar (≡) and that we have saved for the last one.

How many rows? Well, as you see there are three discrete components, simple components; A, C, D. So, eight rows, and we remember how the truth values are distributed. So this is the way to distribute the truth values. Out of that we built the tilde ~D column. By looking at what we have assigned to D, that's how we get the tilde ~D. And then comes slowly we build it up like so, and final column if you have done it correctly looks like this –looks like this F T F F T F T F and so on and so forth. That kind of mixture is what makes this statement contingent, or truth functionally indeterminate. Alright?

So this is what the result shows us. The truth table, remember, doesn't tell you anything specific. It is a mindless procedure, but only when you pose a question to it, it can give you an answer. You need to interpret the result in a certain sort of way. So here we ask the question, can you show me in terms of truth values which category does this proposition belong to? Please note it's an actual proposition. It is not a propositional form, and you asked : Show me, truth table, which category does it belong to? And it has shown you that. You look into the final column and you find out that it's a smattering of Ts and Fs which make it qualify in to this category, contingent, truth functionally indeterminate. Is that what you found out in your answer too? If so, congratulate yourself. So you have learnt this task also.

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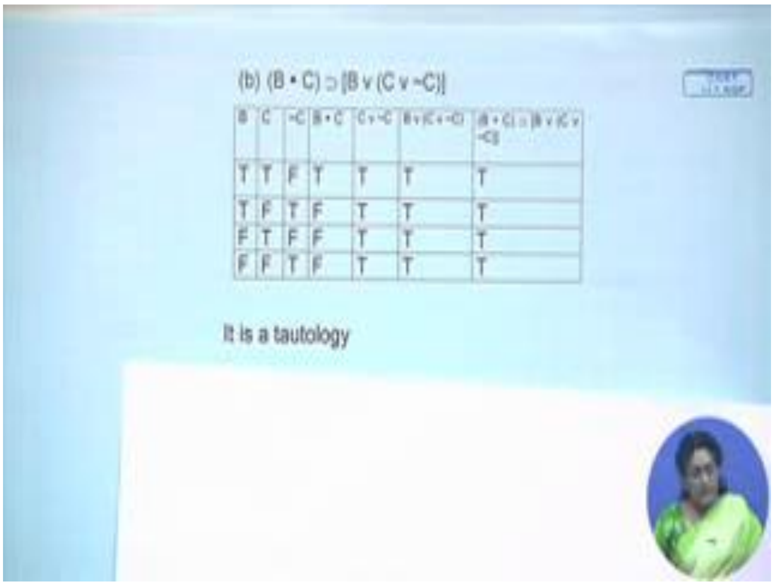

The second one, the second problem was like this, (B ● C) ⊃ [(B ∨ (C ∨~ C)]. This was our previous from our previous (Refer Time: 04:13) module. So, again how many rows? The answer is very easy, we need only four rows. But how many heads, how many columns? Well, that is like this, that we need these two for the reference; the ~C because it appears here, so separate column. Then comes the sub-connectives, (B ● C), (C∨~ C), (B ∨ (C ∨~ C)) , and then the whole compound in itself. And then again if you have done it correctly, then this is what the truth table is like. Alright?

So in a way, this is your final column, which tells you what? That this proposition or statement is a tautology, always true. Ok? So simple. This is what we…this is our first task, and this is the procedure also I have shown, and you found out how to classify the proposition into these three categories.

If you have learnt that, then our next module, today's module, is about a second task that we can put the truth table to. Here we are trying to see whether an argument is valid or invalid, and we are dealing with deductive arguments. So it’s a very important property that also we need to remember. So, can the truth table tell us, given an argument ,whether it is valid or invalid? That’s the question we are posing to truth table.

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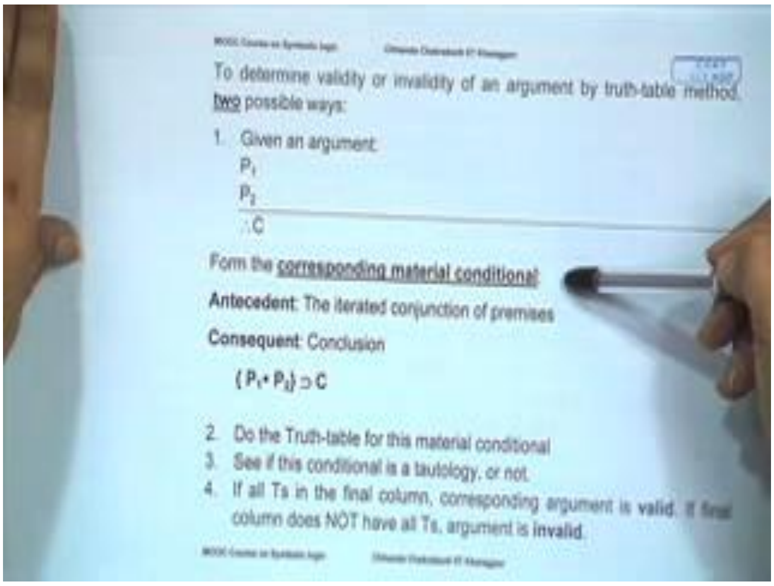

How to do this? Well, there are two possible ways in which you can go. One of them is that if you are given an argument, like say for example this, where there are two premises P1 and P2, and C is the conclusion. Then given this kind of an argument, what you can do… this is one of the ways, is to convert it into a *corresponding material conditional*. So, given an argument, you can form the corresponding ‘if then’ proposition. How? Where the antecedent of this conditional going to be the conjunction of the premises, the iterated conjunction of the premises. So here you have P1, P2. You put them in conjunction, and that becomes your antecedent, and the consequent is the conclusion. In this case C happens to your consequent. But the difference is this is an argument, and this is just a statement. So, given the argument, you can form the material conditional out of that. This is one of the ways. So here is, as I was saying, out of this argument, you can form this corresponding material conditional.

Then what? Well, now… because, remember, truth tables apply only to propositions or statement forms, but not to arguments per se. So directly on this argument you cannot apply the truth table. So you need to convert it into a statement of some kind, or statement form of some kind. And then the truth table becomes applicable. So that is the whole logic behind this conversion. Once you have converted into the material conditional of this kind, then you do the truth table as usual for this conditional.

Now the result. If I do the truth table, if I.. OK I do the truth table for the whole material conditional, then what? The idea is to see whether the conditional is a tautology, or not. If you have it all Ts, that is, if it turns out to be a tautology, you know the corresponding argument is valid. If the final column does not have all Ts, that is, there are some Fs there, then the argument is invalid. This is our one of the methods to apply the truth table to an argument.

Now what is the insight behind this? See, when you are saying this, what is it that you are saying? You converted the argument into this material conditional. What are you saying here? If P1 and if P2, then C. Right? So, if the argument is valid, then whenever P1 and P2 are true, C must be true. Correct? So in a way, what you are saying that if this is always true, the argument has to be valid. This is what we said. If you find this is to be tautology, the argument has to be valid.

Now when can this be false? When you have P1 true, P2 true, but C false. That is when we say it is an invalid argument. Right? When premises all true, but conclusion is false. If there are such situations, then you are not going to have this material conditional as a tautology, there will be some Fs there. When that happens, you know the argument is invalid. Did you follow that?

So, I will repeat what I just said. That one of the ways, there are two ways to do this with the truth table method, one of them is to take the argument, form the material conditional with antecedent being formed by the conjunction of the premises given, and the consequent becomes the conclusion, and then do the regular truth table on this proposition, or propositional form. And what you see, whether this term out to be a tautology or not. If it is a tautology argument is valid, if not, the argument is invalid. Right? So that is our first method.

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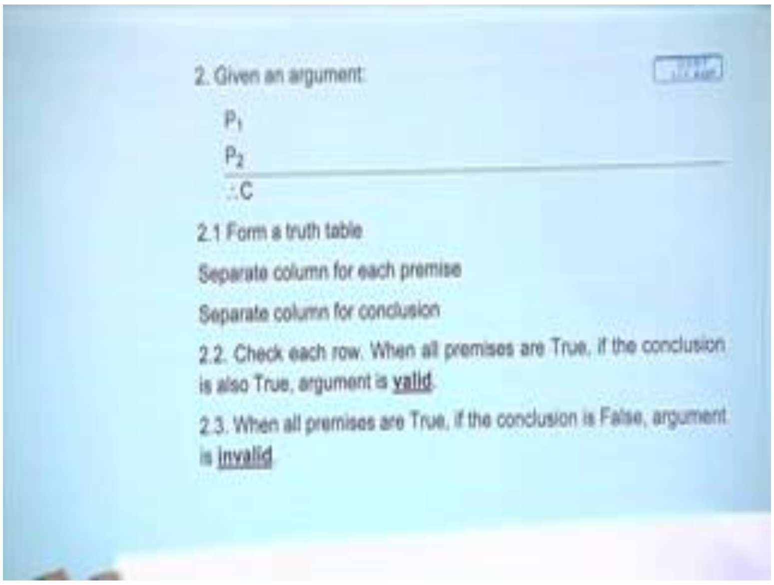

What is the other one to do? Well, similar, something similar, but not… the logic remains same, but just few steps we don't have to do. For example, given this kind of argument, that you have P1, P2 and you have *therefore*, C. What we do is we do the truth table. We do not convert it into a material conditional. We do the formal truth table on what? On…just take the premises, give them separate column, one separate column for the conclusion. So, we do not form any material conditional, but we have the reference columns, and we directly bring in each premise and assign it a column, each and then one column for the conclusion.

So, then still the logic remains the same. We are going to look into every row and check what is happening when all premises are true. Remember, if the argument is valid, whenever all the premises are true, the conclusion is also going to be true. And, when all premises are true, but conclusion can be still false, you have invalidity at hand. So I will repeat the second method also, whichever suits you. You don't have to do both the things, but either of this will do. The second method simply says that given an argument like this, do not convert it into a material conditional; instead you form a truth table. As is usual for truth table, you have the reference columns for the components. Then, one column for each premise and a separate column for the conclusion. We will show you example and then we check, we do the truth table as usual and then we check every row and check specifically when every premise is true, what is happening in the conclusion.

When if all the premises are true, then also the conclusion is coming out to be true because you can see, you have the rows. Right? So that rows represent possibilities. So you check what is happening when the premises are true, and if you find the conclusion also true, every case, then argument is valid. But if you find *even one case* where the premises are all true, but the conclusion is false, call the argument invalid. Did you understand? So these are the two methods.

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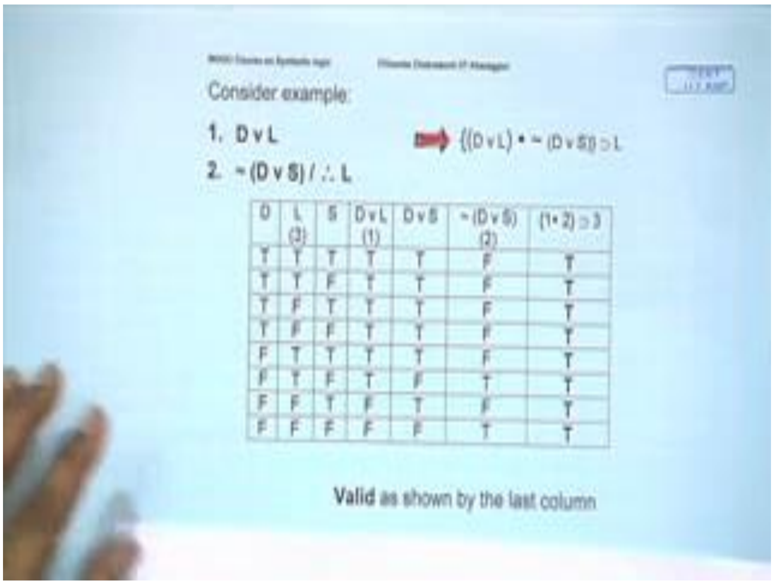

And now the time comes to see some actual examples, and I suggest that you also try to do this. As I explain it to you, as I show you the example, you try to do this. So here is an argument. The first premises D ∨ L. The second premise is: It is not the case that D wedge S, or ~(D ∨ S). This slant line is a separator line between the premises and the conclusion. The conclusion is L, preceded by the triple dot. The triple dot stands for therefore. Got it? This slant line is a separator line that separates the conclusion from the premises.

So, we are following… suppose you say I am going to follow the first method. Right? To check by truth table we'll follow the first method. What does the first method say? Convert it into the corresponding material conditional. And then the sentence becomes like this. This is an argument; this is a compound proposition. What have you done? You made… you took the two premises, formed a conjunction; and the consequent is the conclusion. Get it? And then we do truth table for this one.

So how many rows we have one, two, three. Three discrete constants. There is going to be 8 rows, and we will do the truth table as we know it is. So here is our table heads, or the column heads. These three are your reference columns. Please note these are alphabetically ordered.

Then we take the sub connectives. $(D \vee L)$, then $(D \vee S)$ and this is a $\sim(D \vee S)$. So first we need to compute this, and then we have $\sim(D \vee S)$ and then we have the whole proposition, namely, the whole material conditional here. But we are using this numbers and numericals. Why? Well, into for abbreviation. Remember this is our premise number 1. So we call it 1. This is our premise number 2. So we gave it a number 2, and this L which happens to be now part of the reference column is our 3. So these 3 columns are what we need to check. When this is true, this is true, what is happening with L, that is what we are going to check.

But one way to do this is the first method says we are going to put the whole conditional statement as the last column, and see whether it comes out to be a tautology or not.

If you have done the truth table correctly and assigned the truth values properly and computed the sub connectives properly, then this is what you are going to find. What we are looking for is, when this is true and this is true, what is happening. Please check that this is not the case where you have both the true, but this is true anyway. So you have T. Ok? You don't have a situation where the premises are all true, but the conclusion is false. You do not have any situation like that. Please look through into the rows. This is T, this is F and this is T. Ok? The combination that you are looking for to rule out is that when this is true and this is true, but this is false. You will not find any of, any possibility like that. And this is all going to be true. So what you found out? You found out that the given argument is *valid*. By looking at the truth table of this statement, you are commenting on the argument, and argument is valid as shown by the last column.

Let's take the same problem, but we are now using the second method. So I will remind you, this is the argument. If we do it by second method, this was our first method, the result is not going to change, but look at the way the second method is applied.

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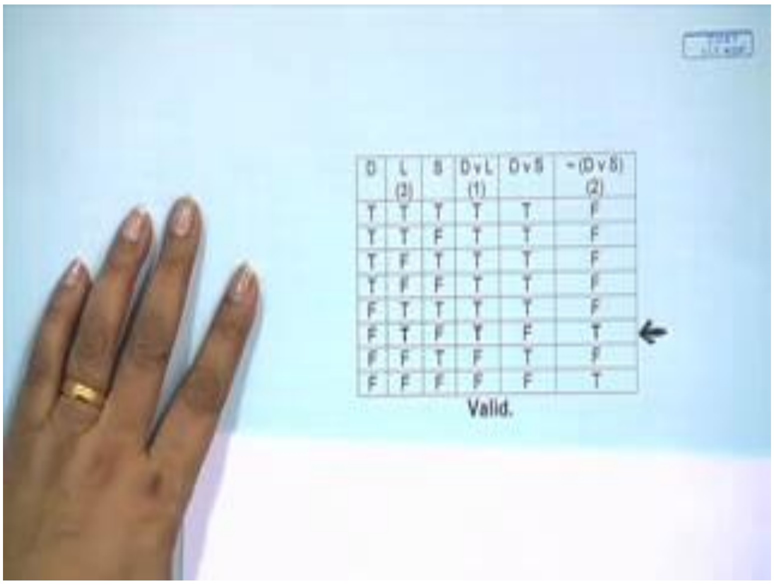

So here we don't convert into a material conditional, instead what we have done? We have taken the discrete constants, and these are your part of the reference column. Here is first premise, one column given, this is D ∨ S, but you can directly also go into this column and call it the second column. What are we comparing? This column, this column and this column, as before. And what are we checking? When this is true, this is true, what is happening with the conclusion or the consequent. And again you will find that there is not a single case where you will find that when this is true and this is true, this is false.

Now you please go through the rows by yourself to see when is 1 and 2 both true, and you will find that this is the *only* row, this is the *only* time when that has happened. First premises is true here, second premises is true here. So true and true, and this is the crucial row, which will tell you whether the argument is, what fate the argument has. And you find that is when the conclusion is also true. Which makes it valid, but you have to check every single row. Validity is something where you need to eliminate all possibilities. So every case there is, first of all there is in here, there is not even one situation when you have both the premises true. This is the only time both the premises are true and luckily, fortunately that's when your conclusion is also true. Which makes the argument valid.

The result does not change, obviously; the method changes, but the result does not change. So this was our first way of doing it, converting into a material conditional, looking for tautology, or just assign columns under premises and the conclusion. Check, what is happening when both of the premises are true what is happening with the conclusion, and the result or the verdict remains the same. Right?

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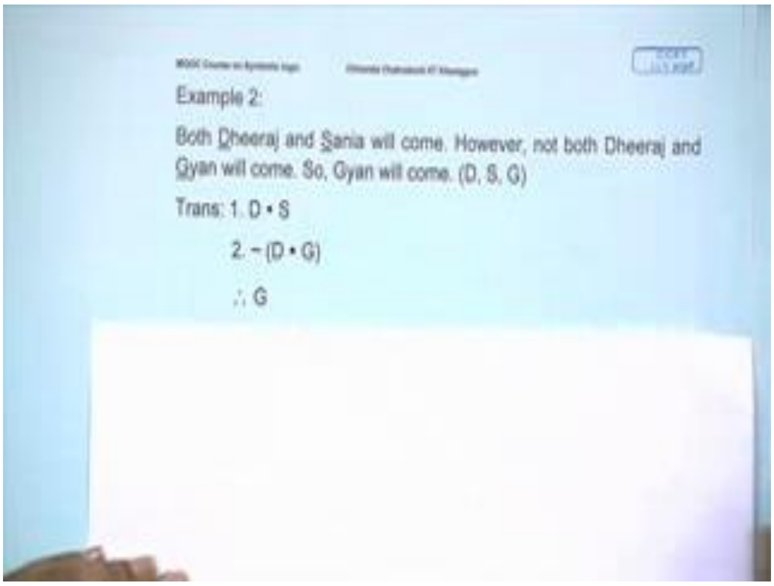

So this is how we go. If you gained a little bit of confidence in this procedure, then let's try this kind of word problem. See, so far we have gone into the symbolic level, but here is an argument. 'Both Dheeraj and Sania will come. However, not both Dheeraj and Gyan will come. Therefore, or, so, Gyan will come'. This is a problem, the argument is given in word. So, the translation skill also shows up, and then you do the truth table technique to find out whether this is valid or invalid argument.

So, first thing is to do is to translate it. We have done translations already. So that should not be a problem, if you still remember the translation skills… have to be practiced and have to be learnt. So this is first sentence premise number 1, this is premise number 2, and this is your conclusion. And these are the given keys, D, S, G stands for Dheeraj, Sania, Gyan, sentences involving them. What is the translation of this argument? Let's try.

'Both Dheeraj and Sania will come', that is clearly (D ● S). 'Both Dheeraj and Sania will come'. Next sentence is 'however, not both Dheeraj and Gyan will come', *not both*.

Ok? So the translation will be like *this: 'not both'. It is not the case both Dheeraj and Gyan will come.* ~(D ● G). And then the conclusion is very simple it is just G.

Now you have a choice. So translation is done. Now you have a choice, how to apply the truth table. You can either form a material conditional, or you can do the straightway column assignment to each of the premise and conclusion and do this. The question is what is the result of this? So if you can, you quickly do the truth table and will compare the result right now. How many rows? You know already; there is going to be 8 rows and it's a matter of just lining up. One thing I must tell you that at the alphabetical order. Try to follow the alphabetical order. So it is going to be D, G, S and not D, S, G. So, not by the appearance order, but by alphabetical order.

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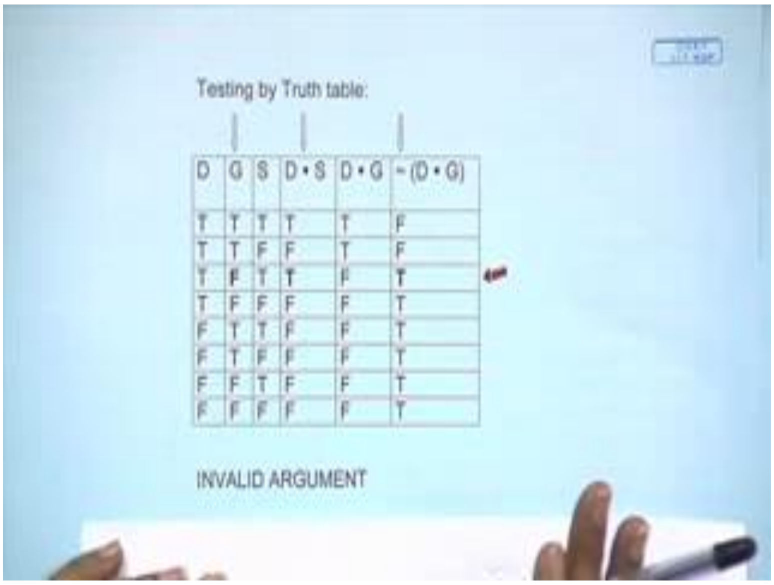

So if you are fast, you can do the truth table as I said right away. But let me at least show you the column heads. If you are following the second method, that is not converting into material conditional, then these are your reference columns. This is the first premise, this is the second premise. You can skip this one also. If you want you can skip this column and go directly here. But since we are beginners, so I thought about putting that in. And we are comparing these two columns with G, which happens to be conclusion.

Once more, be very careful to see each row, when this is true, when this is true, what is happening with G. Ok? If you find that one of them is false, obviously that is not a point of worry, but it must not happen when this is true and this is true, this is false. That must not happen. If that happens then the argument is invalid. Right? That is our modus operandi.

So we check how we have done it. If you have done it correctly, then this is the kind of table that you are going to generate. 8 rows this is the way to distribute the values and these are the results for each column. Ok? We are checking now what is happening here, this is true this is false and this is true. So we are safe. This is false, this is false and this is true, no problem. This is true, this is true and then here we find conclusion is false. Once more. This one is true, premise number 1 is true, premise 2 is true and the conclusion is false. And that row is crucial to settle what? That the argument is invalid. Invalid. Alright?

So whether you look into this, whether there is a second row or third row displaying this, is not the issue. Having even one such row can demonstrate that the argument is invalid. That’s what we earlier established. Validity is an exhaustive notion, all or none, either always valid or it is invalid. So it does not matter, how many times it happens that your premises true and the conclusion false. Here we have found one clear decisive situation when you have the premises all true, but the conclusion is false. That is what is sufficient make the argument invalid.

So this was our module on testing arguments with truth table and determining whether they are valid or invalid. You have learnt earlier that validity and invalidity are very important notions. They differentiate between what we call good deductive arguments from bad deductive arguments. So you just saw that the truth table technique is quite adequate and competent to perform this task. Earlier we have learnt how to classify say propositions into pigeon's holes like tautologies, contradiction, and contingent. This was with the arguments which are not statements. Right? We have learnt that. But it’s important to know that even with this truth table technique we can compute values in such a way that the arguments can be classified into valid and invalid. The invalid arguments are not desirable ones, the valid ones are. Right? So this was our module on that. In the modules to come, we will put the truth table to other tasks. But right now this is where we were, and let me remind you that we have learnt two techniques; two ways to demonstrate validity or invalidity; namely, the conversion into material conditional or not.

So with that I am going to end this module here.

Thank you very much. Hope you have learnt something.