Symbolic Logic (HS 30068)

Dept of HSS, IIT Kharagpur

Date: 18.10.2020 Total Marks: 60 Duration: 90 minutes

Section I. T or F, ¼ mark off for each wrong answer. 20×1=20

1. All truth-functional statements of propositional logic are compound statements.
   Ans: True
   Detailed Solution: Compound statements are statements which have at least one connective in a combination of simple statements. In propositional truth-functional statements are associated with 5 basic connectives. These connectives are used only in compound statements. Hence, the given statement is true.

2. The statement, 'The lightning struck somewhere nearby and he cried out for help' is a truth-functional statement.
   Ans: False
   Detailed Solution: The 'and' in this statement is not a truth functional 'and' or '•'. For, the 'and' here is not commutable. 'He cried for help and the lightning struck somewhere nearby' is not the same as the given statement. In order to know its truth-value, one has also to know the right sequence of the events.

3. The tree for '[A • (B v C)] ⊃ ~ (A v C)' does not show any closed branch, but the tree for '~[[A • (B v C)] ⊃ ~(A v C)]' shows one closed branch.
   Ans: False.
   Detailed Solution: The given statement is False because if we done the tree correctly then we found that both trees will have all open branches.

4. A completed tree must be an open tree.
   Ans: False
   Detailed Solution: The given statement is false because a completed tree need not be an open tree. A completed tree, on which all decompositions are done, may turn out to be a close tree.

5. The Megarian School of logic was founded by Aristotle.
   Ans: False
   Detailed Solution: The given statement is false because the Megarian School of logic was not founded by Aristotle.

6. The discovery of Aristotle's logic was a reason for decline in interest among the European mathematicians in symbolic algebra.
   Ans: False

Detailed Solution: It was just the opposite. The interest in symbolic algebra was a reason for decline in interest in Aristotle's logic among the European mathematicians.

7. Before the Moors brought the knowledge to Europe, European mathematicians did not know algebra.
   Ans: True
   Detailed Solution: The Islamic mathematicians created Algebra. Before they were exposed to books on Algebra by Islamic mathematicians, the Europeans knew only Arithmetic and Geometry.

8. Russell's paradox shows that mathematics is derivable from logic.
   Ans: False
   Detailed Solution: False. Because the paradox raised a fundamental problem in the very idea of Logicism: That mathematics is derivable from logic.

9. The translation for "Both India and Pakistan will play Cricket only if it is a friendship series, unless there are security reasons for the players" is $S \supset (F \supset (B \bullet P))$ [I: India will play Cricket, P: Pakistan will pay Cricket, F: It is a Friendship series, S: There are security reasons for the players]
   Ans: False.
   Detailed Solution: The given translation is False. The correct Translation will be: $\sim S \supset [(B \bullet P) \supset F]$.

10. 'Overuse of antibiotics does not cause harm to people' is a structurally compound statement.
    Ans: True
    Detailed Solution: By definition structurally compound statements are a combination of simple statements with at least one connective. The given statement is uses the negation connective with a statement. Hence, it is a structurally compound statement and the answer is true.

11. The argument:
    P1: $((L \supset M) \supset N) \supset P$
    P2: $(P \supset (M \bullet N)) \supset L$
    P3: $M \vee N \ / \therefore L \equiv P$

    Will be invalid when L: T, P: F, M: T, N: F
    Ans: True
    Detailed Solution: A shorter truth-table will establish that with these truth-values the argument will be invalid

12. If '(P ⊃ Q) v (Q ⊃R)' is a tautology, its negation may or may not lead to a contradiction.
    Ans: False.
    Detailed Solution: A tautology is always true in every truth value assignment and a contradiction is false on every possible truth value assignment. If '(P ⊃ Q) v (Q ⊃R)' is a tautology, then its negation will definitely be a contradiction. Hence the given statement is false.

13. Any two statements, *p*, *q*, are logically equivalent, the tree for p ≡ ~q must remain open.
    Ans: False
    Detailed Solution: Logical equivalence means having identical truth conditions. If *p*, *q*, are logically equivalent then the tree for p ≡ ~q will be a closed tree. Hence, the given statement is false.

14. The set { (G ⊃ E), (G v (A • C), ~ (A v (G •E)} is inconsistent.
    Ans: True.
    Detailed Solution: The given set is inconsistent. This can be shown by Truth table or Truth-tree.

15. The preliminary version of Conditional Proof is more limited in its scope than the strengthened version of Conditional Proof.
    Ans: True
    Detailed Solution: The given statement is true. The preliminary version of Conditional Proof is more limited in its scope because it is only applied in those arguments that have conditional statements as their conclusion. On the other hand we can apply strengthened version of Conditional Proof to all kinds of arguments.

16. It is logically impossible for a contradiction to have a truth-value.
    Ans: False.
    Detailed Solution: The given statement is false. A contradiction is false on every truth-value assignments. Hence, a contradiction carries the 'Falsity' as its truth value.

17. We can successfully use a shorter truth-table to demonstrate the logical inconsistency of a set of statements.
    Ans: False
    Detailed Solution: Shorter truth table contains only one single row; whereas for demonstration of inconsistency we need to establish that there is no possibility of the members of the set being true together. Hence, the given statement is false.

18. Only truth-functional statements can be compound statements.

Ans: False

Detailed Solution: The given statement is false because non-truth-functional statement can also be compound statement. A compound statement is that state which contains another simple statement as its component. There is no necessity that the connective in it must be a truth-functional one.

19. Given that X is False, Y is true, and Z is unknown, the truth-value of 'Z ⊃ {(X v Y) ≡ X} is False.

Ans: False

Detailed Solution: 'Xv Y' will be T, hence (X v Y) ≡ X will be F, hence the whole statement will be F when Z is T. But when Z is F, the whole statement will be T.

20. In Truth-tree to enter a statement in the root or branch of a truth tree is called decomposition.

Ans: False

Detailed Solution: The given statement is false because to enter a statement in root or branch is called Listing. Decomposition is the process of breaking a string of symbols or a compound statement down to literals, following certain rules.

Section-II: MCQ **2x20=40 Marks**

21. Which statement of the following is/are true?

The difference between a statement variable and a statement constant in propositional logic is:

(a) That the statement constant is a substitution instance of the variable, but not vice versa.

(b) That the constant represents a specific statement form, but the variable represents a generic statement form.

(c)That the statement variable is the substitution instance of the statement constant, but not vice versa.

(d)That the statement constant syntactically must be represented by a suitable lower case alphabet, but the statement variable must be represented by an alphabet from the lower end of the alphabet series.

Ans: (a)

Detailed Solution: In propositional logic statement variables are used to stand for arbitrary and unspecified statements by lowercase letters. On the other hand statement constants are symbolically represented actual statements in a given context. The main difference between these two is that the statement constant is a substitution instance of the variable, but not vice versa. Hence, the right choice is option (a).

22. Which among the following is /are true?

An 'incomplete' formal logical system:

(a) Cannot guarantee the validity of valid deductive arguments.
(b) Can provide a proof for every logical truth in the system.
(c) Can demonstrate some semantic consequences as syntactic consequences.
(d) Must be non-monotonic in character.

Ans: (c)
Detailed Solution: Incompleteness of a formal logic system means in the system in it not all semantic consequences can be provided a proof. That is, for every semantic consequence, it cannot claim that it is also a syntactic consequence. So, (c) is the only right choice. Rest are wrong.

23. Which of the following is/are false?

(a) The difference between a unary and a binary connective is in the number of components it can connect.
(b) The difference between an inclusive OR and an exclusive OR is not in the falsity condition, but in the conditions which make them true.
(c) The Rules of Replacement or Equivalence Rules apply to the whole statement, and not to a part.
(d) The Addition Rule is: p; q / ∴ p • q

Ans: (c) and (d)
Detailed Solution: In the given question option (a) is true because unary connective can connect only one component and binary connectives can connect two components at a time. Option (b) is also true because inclusive *or* is true when either of the disjuncts is true and also when both the disjuncts are true. But exclusive *or* is false when both the disjuncts are true. Option (c) is false because we can apply Replacement or Equivalence Rules to a part of the statement, as well as to the whole statement. Option (d) is false because the Addition Rule is: p / ∴ p v q. Hence, the right choice would be option (c) and (d).

24. Which among these is/are the correct translation of the given argument into Propositional Logic language:

Provided that a vaccine arrives next year, not both India and China can be benefited. Though it is not that neither India nor China can be benefited, the arrival of the vaccine next year is the necessary condition for the end of the pandemic. Hence, India can be benefited. [V: Vaccine arrives next year, I: India can be benefited, C: China can be benefited, P: The pandemic ends]

(a) V ⊃ ~ (I • C); ~ (~I • ~C) • (V ⊃ P) / ∴ I
(b) V ⊃ ~ (I • C); ~ (~I • ~C) • (P ⊃ V)/ ∴ I

(c) $V \supset (\sim I \bullet \sim C); \sim (I \vee C) \bullet (P \supset V) / \therefore I$

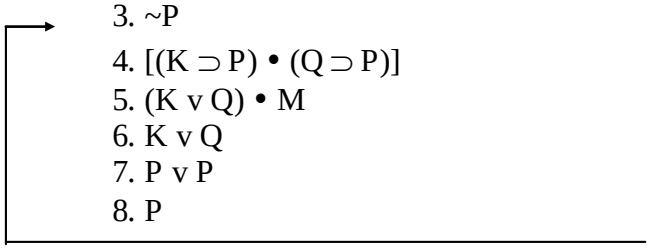
(d) $V \supset \sim (I \bullet C); \sim (\sim I \vee \sim C) \bullet (V \supset P) / \therefore I$

Ans: (b)

Detailed Solution: Choice (a) cannot be the right translation because V is the necessary condition for P. It cannot be (V ⊃ P). Choice (c) cannot be the right translation because it is not the case either I or C. It cannot be (I v C). Choice (d) cannot be the right translation because again it cannot be (V ⊃ P). Hence, the right answer would be option (b) as it contains the right translation.

25. Which of the following claims about the given proof by strengthened version of Conditional Proof is / are false?

1. $P \vee [(K \supset P) \bullet (Q \supset P)]$
2. $M \bullet (K \vee Q) \quad / \therefore \sim P \supset P$



3. $\sim P$
4. $[(K \supset P) \bullet (Q \supset P)]$
5. $(K \vee Q) \bullet M$
6. $K \vee Q$
7. $P \vee P$
8. P

9. $\sim P \supset P$ 3-8, CP

(a) The bent arrow closing line should be after line 7 and before Line 8.
(b) On Line 7, the justification is 4,6, by HS
(c) Justification for Line 4 is 1,3, DS
(d) Line 8 is obtained from Line 7 by Tautology

Ans: (a) and (b)

Detailed Solution: Option (a) is false, because the closing line should be after Line 8. Option (b) is false because line 7 is by 4, 6, by C.D. Rest are correct.

26. Choose the correct option from the following:

$(K \bullet (L \vee C)) \supset \sim (K \vee C)$ is:

(a) A contradiction
(b) A contingent
(c) A tautology
(d) Not a syntactically correct statement of Propositional Logic

Ans : (b)

Detailed Solution: The given statement is a contingent because if we prepare a truth table for the given statement correctly we can find that there is at least one T and at least one F in the final column. Hence, the right choice is option (b).

27. Suppose that the completed truth-tree of premises and the conclusion of an argument is open. This shows that:

(a) The argument is valid
(b) The argument is invalid.
(c) Nothing conclusive follows about the validity of the argument.
(d) The validity of the argument cannot be demonstrated by the truth-tree method.

Ans: (c)

Detailed Solution: The validity of the argument cannot be directly demonstrated by the truth-tree method. The argument will be valid if the premises and the negation of the conclusion have a closed tree. If, the completed truth tree of premises and the conclusion of an argument is open then we know that premises are compatible with conclusion. But we do not know that there is no possibility for the conclusion to be false when the premises are true. Hence the right choice would be option (c).

28. Which of the statements is / are false?

a. The ~•D rule bifurcates into two branches
b. All closed branches are branches in which all decompositions are completed.
c. The ~⊃D rule does not bifurcate into two branches.
d. A completed open tree must have at least one completed open branch.

Ans: (b)

Detailed Solution: Statement (a) is true. Statement (b) is false because closed branch is a branch on which a simple statement and its negation both occur. There is no necessity that all decompositions must be finished before the branch is closed. Statement (c) and (d) is true. Hence, option (b) is the only false statement in this question.

29. Which statement of the following is correct?

A difference between a limited scope assumption proof procedure and formal proof of validity in propositional logic is:

a. That the formal proof of validity uses the rules of inferences and replacement, but a limited scope assumption proof procedure cannot use those rules.
b. A limited scope assumption proof is necessarily shorter than a formal proof of validity.
c. That the formal proof of validity allows the bent arrow format, but a limited scope assumption proof does not allow that.
d. That the limited scope assumption allows extra but temporary premise(s), which is not allowed in the formal proof of validity.

Ans: (d)

Detailed Solution: Statement (a) is incorrect because both formal proof of validity and limited scope assumption proof procedure can use the rules of replacement. Statement

(b) is incorrect false because in both the cases the length of the proof is depends on the appropriate use of the rules. Statement (c) is also incorrect because it is the limited scope assumption proof which allows the bent arrow format. In this question only option (d) is correct because inclusion of extra but temporary premise(s), are allowed only in the limited scope assumption and not in the formal proof of validity.

30. Consider the following argument:

1. $P \supset Q$
2. $R \supset P$
3. $R \vee (Q \bullet S)$ / $\therefore Q$

Which of the following claims is / are true?

(a) Q is not derivable from the given premises using the 19 Rules and Formal Proof of validity.
(b) Q is derivable, by insertion of R v ~R as a permissible premise in Formal Proof of Validity.
(c) Q is derivable by Formal Proof of Validity, without the insertion of R v ~R.
(d) Q is not derivable, the given argument is invalid.

Ans: (c)

Detailed Solution: Claim (a) is not true because Q is derivable from the given premises by Formal Proof. Claim (b) is not true because R v ~R is not needed as a premise to derive Q. Claim (d) is also not true because the given argument is valid. Hence, only claim (c) is true because Q is derivable by Formal Proof of Validity with the help of the given premises without the insertion of R v ~R.

31. Select the correct option from the following. If correctly done, the final column in the truth table for $J \bullet (K \equiv M)$ will have:

(a) All Ts.
(b) Three Ts, and five Fs.
(c) Only one T, followed by all Fs.
(d) Only 2 Ts, and six Fs

Ans: (d)

Detailed Solution: Check the final column of the below truth table. It has only 2 Ts, and 6 Fs. Hence, the right choice is option (d).

J	K	M	(K ≡ M)	J • (K ≡ M)
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	T	T

F	**T**	**T**	**T**	**F**
F	**T**	**F**	**F**	**F**
F	**F**	**T**	**F**	**F**
F	**F**	**F**	**T**	**F**

32. The Formal Proof of validity for the following argument:

If either the map (**M**) or the address (**A**) is available, then the person can be located (**L**). Therefore, if the map is available, then the person can be located.

(a) Will start by assuming M v A.
(b) May start by applying the rule Implication (Impl.) on the symbolized premise.
(c) Will start by assuming the negation of the conclusion.
(d) May start by assuming M.

Ans: (b)
Detailed Solution: A formal proof of validity cannot assume anything besides what is given as premises. Hence, a, c, d all are incorrect. Only option (b) is the right choice.

33. In the truth-tree method, the following pair will <u>not</u> be considered as logically equivalent:

{(J• L), (J v L)}

(a) If the tree for ~ [(J • L) ≡ (J v L)] is a not closed tree.
(b) If the tree for ~[(J • L) ≡ (J v L)] is a closed tree
(c) If the tree for [(J • L) ≡ (J v L)] has at least one closed branch.
(d) If the tree for [(J • L) ≡ (J v L)] has both a literal and its negation in any branch.

Ans: (a)
Detailed Solution: In the truth tree method two statements (J• L) and (J v L) are not logically equivalent if the set ~ [(J • L) ≡ (J v L)] is not a closed tree. Hence, the right answer is option (a).

34. Proof by Indirect Proof of the following:

(D• J) ⊃ [L⊃ (D v J)]

(a) Will end in deriving [L⊃ (D v J)]
(b) Will start by assuming (D ● J)
(c) Will end in deriving J v ~J
(d) Will end by deriving D • ~D

Ans: (d)

Detailed Solution: In Indirect proof we need to derive an explicit or obvious contradiction in the form of p• ~p by the use of the inference and replacement rules. Hence, in the given proof also we have to end it by deriving D • ~D. Thus, (d) is the right choice here.

35. Which of these is / are not true?

In *Reductio ad absurdum* proof, an 'absurdity' may mean:

(a) A self-contradiction
(b) A shockingly inappropriate statement.
(c) A patently false statement.
(d) An implausible statement.

Ans: (b)

Detailed Solution: The Indirect Proof (I.P) is also known as *Reductio ad absurdum* proof. In I.P. an 'absurdity' does not mean a shockingly inappropriate statement. Hence, the right choice is option (b).

36. By the shorter truth table method the following argument may be shown as invalid when:

1. B ⊃ (B v H)
2. (B v H)• (Q v H) / ∴ B • Q

(a) B is F, H is T, Q is T
(b) B is T, H is T, Q is F
(c) B is F, H is T, Q is F
(d) B is T, H is F, Q is T

Ans: (a), (b), (c)

Detailed Solution: If we formulate the shorter truth table by the given truth values in the options then we will find that only for option (d) the premises and the conclusion are true hence, the argument may be valid. In case of option (a), (b), and (c) though the premises are true but the conclusion is false hence, invalid.

Option	B	H	Q	B v H	Q v H	B ⊃ (B v H) (1)	(B v H)•(Q v H) (2)	B • Q(3)
(a)	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**F**
(b)	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**F**
(c)	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**F**
(d)	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**

37. Which of these statements is /are incorrect?

(a) Valid argument forms are truth-preserving by nature.
(b) In 'q ⊃ p', q is the necessary condition.
(c) In Propositional logic, not all connectives are binary.

(d) The formula for determining the number of rows in a Truth-table is n^2,where n stands for the number of simple statements in the given statement

Ans: (b) and (d)

Detailed Solution: statement (a) is correct because if an argument is valid, then, if all its premises are true, then its conclusion is also true. Statement (b) is incorrect because in material condition the consequent is also known as the necessary condition and antecedent is sufficient condition. Then in 'q ⊃ p' p is the necessary condition and not q. Statement (c) is also correct because ~ is a unary connective. Statement (d) is incorrect because the formula for determining the number of rows in a Truth-table is 2^n, where 2 is constant for the number of truth values and n is the number of discrete atomic components.

38. Which of these statements is/are true?

(a) The inclusive OR and exclusive OR are both false when both disjuncts are true.
(b) In Formal Logic, invalidity of an argument is the result of its form.
(c) A • ~A is a specific statement form.
(d) Logical connectives are the truth-value bearers.

Ans: (b)

Detailed Solution: Statement (a) is not true because when both the disjuncts are true then, inclusive OR will be true and exclusive OR will be false. Statement (b) alone is true because in formal logic invalidity of argument considered to be in virtue of its form. Statement (c) is not true because A • ~A is not a specific statement form. Statement (d) is not true because propositions are the truth value bearers.

39. For the 'zero-premise' proof of tautology by C.P. of :

(X v Y) ⊃ (Z ⊃ (X v Y)

Which among these is the correct choice?

(a) We may first assume (X v Y), then Z, then ~X.
(b) We may first assume Z, and then (X v Y)
(c) We may first assume X, then Y, then Z
(d) We may first assume Z ⊃ (X v Y), then X v Y.

Ans: (a)

Detailed Solution: If we derive the tautology then it will looks like

$(X \vee Y) \supset (Z \supset (X \vee Y))$

1.	$(X \vee Y)$	
2.	Z	
3.	$\sim X$	
4.	Y	1,3, D.S.
5.	$(Y \vee X)$	4, Add.
6.	$(X \vee Y)$	5, Com.
7.	$Z \supset (X \vee Y)$	2-6, C.P.
8.	$(X \vee Y) \supset (Z \supset (X \vee Y))$	1-7, C.P

Hence, the right answer is option (a).

40. Consider the Truth-table for $(W \vee (R \bullet U)) \supset (U \equiv (R \vee \sim W))$

1’	2’	3’	4’	5’	6’	7’	8’
R	W	U	R •U	R v ~W	W⊃(R• U) (1)	U ≡ (R v ~W) (2)	(1) ⊃(2)
T	T	T	T	T	T	T	T
T	T	F	F	T	T	F	F
T	F	T	T	T	T	T	T
T	F	F	F	T	F	F	F
F	T	T	F	T	F	T	T
F	T	F	F	T	F	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	F	F

Which statement is/are false about this Truth-table?

(a) There is no separate column for the connective with the least scope.
(b) There are mistakes in columns 5’, 6’, 7’, 8’.
(c) There are mistakes in columns 4’ and 5’.
(d) There is problem in the sequence of the reference columns.

Ans: (c)

Detailed Solution: Statement (a) is true because there is no separate column for ~W. Statement (b) is true. In column 5’ there will be 2 Fs and 6 Ts instead of 8 Ts. In column 6’ the T/F values are not in right place. In column 7’ there will be 4 Ts and 4 Fs instead of 5 Ts and 3 Fs. In column 8’ there will be 2 Fs and 6 Ts instead of 5Ts and 3Fs. Statement (c) is false because there is no mistake in column 4’ and 5’. Statement (d) is also true. Hence, option (c) is the right answer.