PRACTICE PROBLEMS (Euler & Taylor's Series)
Numerical Solutions of ODE and PDE

1. Compute the first few terms using Picard's iteration with $y_0(x) \equiv 0$ for the initial value problem $y' = xy + 2x - x^3$ with $y(0) = 0$, and show that they converge to the solution $y(x) = x^2$ for all x.

2. Solve $y' = 3x + y^2$, y=1, when x = 0, numerically for x = 0.1 by Taylor's series method of order 2.

3. Use Taylor's series to find a numerical solution at $x = 0.5$ of the differential equation $y'' = xy$ given that $y' = 1$ and $y = 1$ when $x = 0$ using $h=0.5$.

4. Solve the differential equation $y' = 2y + 3e^x$ with $x_0 = 0, y_0 = 0,$ using Taylor's series method of order 2 to approximate y for x = 0.1, 0.2.

5. Solve $y' = x - y^2$, $y(0) = 1$ by Euler's method for x =0.2 to 0.6 with h = 0.2.

6. Given $y' = y - x$, where $y(0)$ = 2. Find $y(0.1)$ and $y(0.2)$ by Euler's method taking step size $h = 0.1$.

7. Given that $\frac{dy}{dx} = x + y^2$, $y(0)$=1. Find $y(0.2)$, by backward Euler's method in one step.

8. Use Euler's method to find a numerical solution at $x = 0.1$ of the differential equation $y'' = 4y - 2xy'$ if $y' = 0.5$ and $y = 0.2$ when $x = 0$. Use $h = 0.05$.

9. Given $\frac{dy}{dx} = \frac{1}{x^2 + y}$, $y(4)$ = 4, find $y(4.4)$ by Taylor's series method of order 2, taking h=0.1.

10. Find $y(1)$ by Euler's method from the differential equation $\frac{dy}{dx} = \frac{-y}{1+x}$ when $y(0.3)$ = 2. Use step length h = 0.1.

11. Given $\frac{dy}{dx} = -\frac{y-x}{1+x}$, with initial condition $y(0)$ = 1, find approximately y for x = 0.1, by backward Euler's method (two steps).

12. Use Euler's method to approximate a set of particular solutions of the system of differential equations $y' = x + z^2$; $z' = y - x$ over the interval $0 \le x \le 1.5$ given that $(x_0, y_0, z_0) = (0,0,1)$. Use $h=0.5$.