ASSIGNMENT – 2

Numerical Solutions of ODEs & PDEs

1. Use Runge-Kutta method of order 2 to solve $y' = xy$ for x = 1.4. Take initial value as *y(1)* = 2 and step-length h = 0.2.

2. Use implicit Runge-Kutta method with 2 slopes to calculate the value of y at x = 0.1, to five decimal places after a single step of 0.1, if $\frac{dy}{dx} = 0.31 + 0.25y + 0.3x^2$ and y = 0.72 when x = 0.

3. Find by implicit Runge-Kutta method with 2 slopes, an approximate value of y for x=0.8, given that y=0.41 when x=0.4 and $\frac{dy}{dx} = \sqrt{x+y}$. Take h=0.4.

4. Consider $\frac{dy}{dx} = -\frac{y^2 - 2x}{y^2 + x}$ and use the classical fourth-order Runge-Kutta method to find y at 0.1, 0.2, given that y=1 when x=0.

5. Solve the differential equation $\frac{dy}{dx} = \frac{1}{x+y}$ for x = 1.0 by the classical fourth-order Runge-Kutta method, given that y(0)=1, interval length h = 0.5.

6. Solve $y'' = y + ty', \quad y(0) = 1, \; y'(0) = 0$, to find *y(0.2)* and *y'(0.2)* using the 4th order R-K method. Take $h = 0.1$.

7. Use the classical fourth order Runge-Kutta method to find a numerical solution at $x = 0.1$ of the differential equation $y'' = 4y - 2xy'$ if $y' = 0.5$ and $y = 0.2$ when $x = 0$. Take h=0.1.

8. Solve the system equations $u' = -3u + 2v, \; u(0) = 0$ and $v' = 3u - 4v, \; v(0) = 1/2$ using

   (i) Forward Euler method and

(ii) 2nd order Taylor Series method by taking h = 0.2 on the interval [0, 0.6].

Programming Exercises

9. Consider the IVP $y' = 2ty,\ y(1) = 1$. Approximate y(1.5) using Runge-Kutta method of order 4 and compare by plotting the approximate solution with the exact one $y = e^{t^2-1}$.

10. Using R-K method of order 2, solve the IVP $y' = 2 + \sqrt{y - 2t + 3},\ y(0) = 1$ in $[0,\ 1.5]$. Compare numerical solution with the exact solution $y(t) = 1 + 4t + \frac{1}{4}t^2$.

11. Using the implicit R-K method of order 2 solve the IVP $y' = \frac{y^2 + ty - t^2}{t^2},\ y(1) = 2$ in *[1, 1.5]*. Plot your numerical solution with the exact solution $y(t) = \frac{t(1+t^2/3)}{1-t^2/3}$.

12. Solve the following system of differential equations using the RK method of order 2.

$$x'(t) = -3x + 4y, \quad x(0) = 1$$

$$y'(t) = -2x + 3y, \quad y(0) = 2$$

Compare you results with the values of the exact solution

$$x(t) = 3e^t - 2e^{-t}$$

$$y(t) = 3e^t - e^{-t}$$