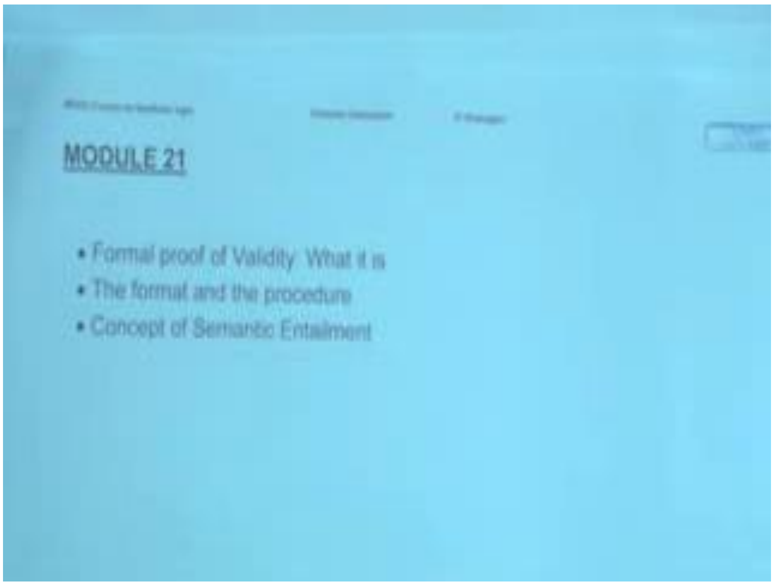
Symbolic Logic

Prof. Chhanda Chakraborti
Department of Humanities and Social Sciences
Indian Institute of Technology, Kharagpur

Lecture – 21
Formal Proof of Validity
What Is
The Format and the Procedure
Concept of Semantic Entailment

We are into module 21 of the Symbolic Logic course and today we are going to go forward. Remember, I told you that formal logical systems will going to have three basic components. One is syntax, semantics and the third one is proof theory.

(Refer Slide Time: 00:26)

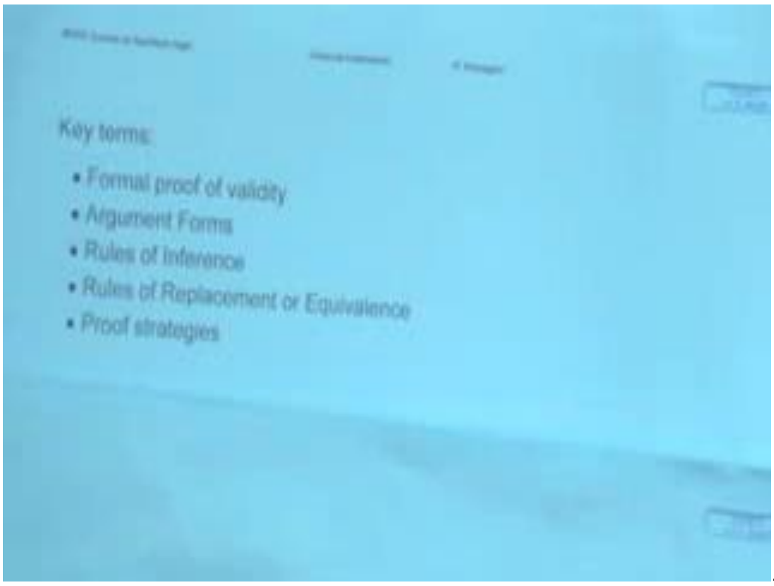


So, we have seen the syntax of propositional logic and we have also learnt the semantics of it; and we have been looking into the various procedures related to semantics, namely the truth trees and namely, the truth tables and so on. But now, we have come to talk about what is very important for a deductive logic system, namely proof, proof theory. How does it show, how does it prove its arguments to be valid?

So, we are going to learn the procedure called *formal proof of validity*. This is clearly a proof of validity. You cannot use this procedure to show invalidity. We will have a separate procedure for that. But this is the name of the procedure *formal proof of validity*, where you will see that we will employ the concept of logical forms, argument forms. So we have to learn what arguments forms are. And then it is pretty rigid procedure because any formal procedure is somewhat non-negotiable and rather strict. So, the format has to be learnt and the how the procedure goes. And we'll touch upon the concept of *semantic entailment* to understand how this… what is the underlying theoretical concept behind this proof procedure. So, our topic for today is formal proof of validity. We will continue on this also on a next segment next module. But let's learn this slowly.

There is going to be a little bit of theoretical talk before we actually start to take examples because I think very important that you grasp the idea first of this. Otherwise you are going to run into trouble to do this. You know before you start doing something it's better to understand it, make it your own, so that on the run you don't have any difficulties. Certainly you don't say what I am doing or what it is supposed to do? So, get yourself first of all clear on the concept.

(Refer Slide Time: 03:24)



.

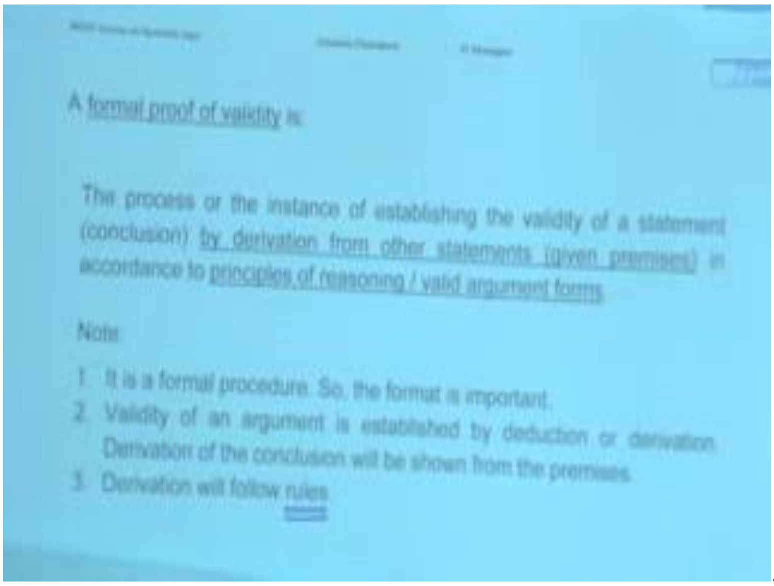
So, formal proof of validity, what it is , that's what we are going to talk about today. In this when we are going to learn formal proof of validity, somehow the things, the key terms are going to be like this, and you will you will ask yourself whether I have understood this or not. We are not going to cover all of them today, but some of the basic ones. That's why I said we have divided it up in so that the lecture remains interesting over the modules. And we are going to first look in to the formal proof of validity, then next module perhaps we'll be talking about argument forms under rules of inference, and then subsequent to that it will come to rules of replacement or rules of equivalence and then we'll talk about proof strategies, various kind of proof strategies.

As we go along, you need to come out of your truth table and truth tree mode, and take this formal proof of validity as something to learn for the first time. So, open your mind and sort of take it in. So, formal proof of validity, what it is and what can do and so on.

See, it's one thing when we say an argument is valid, *I know*. "How do you know that"? Alright? Is the question that somebody else can ask you. So, there is one kind of proof that you can do for yourself, right? Where *you* know that the argument is valid and it is seems like okay (Refer Time: 05:02) this is the procedure for that. The other one is what is known as public demonstration, where it's not enough just to make a claim that an argument is valid. Other people will say: How do you know that, you show us and so on.

So, the public demonstration or the logical demonstration of the validity of an argument is somewhat elaborate task. You will say that we have learnt the truth able method, we have learnt the truth tree method and that is how we can demonstrate it to the others. True, you can. But when you see the formal proof of validity, you will realize that this is *the* vehicle for proofing validity for any kind of argument, whether it has 18 variables or 18 constants or 200 constants. Doesn't matter. So, the formal proof of validity finally will survive as the proof of validity, no matter what kind of logic you are doing either in the propositional logic or in the predicate logic domain.

(Refer Slide Time: 06:11)



.

Formal proof of validity. What is it? Let me slowly take you in first of all what it is, is a *detailed demonstration*. Remember, in the truth table when an argument was valid what you had, is you had several rows in the table and each row showed you that whenever the premises are true, the conclusion is also true. Right? This is what you showed. But what you didn't know, what the truth table did not tell you, is *how* is it that given the premises' truth, the conclusion's truth is guaranteed. All you were shown is that under the premises you had… whenever you had 'true' truth value under the premises, you also had under the conclusion truth value 'true'. What you did not have is the logical explanation: *Why* or *how* that if you have the premises as true, how does it show, how does it entail that the conclusion has to be true, right? Same thing also can be said about the truth tree. All you showed is that it cannot be that whenever the premises are true, the conclusion cannot be false. If it is false then the result would be a closed tree. That's all. But why? In what way the truth of the premises entailed the truth of the conclusion? That was not shown.

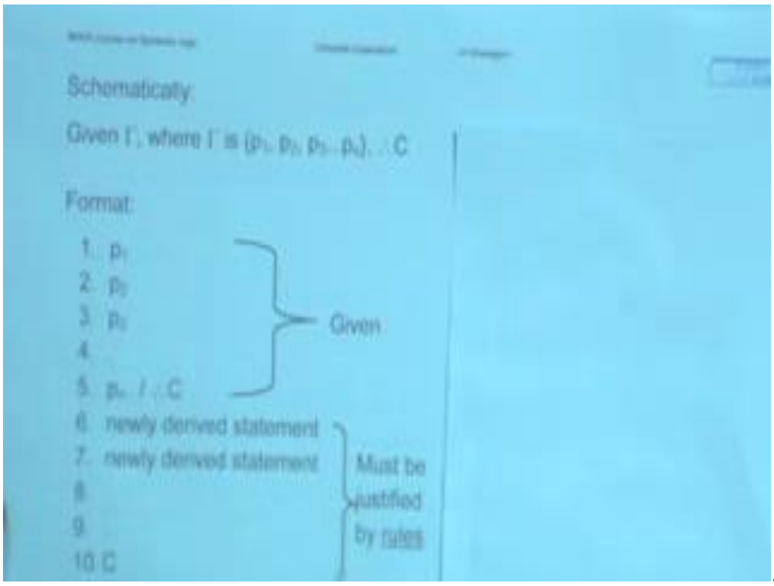
Now, here in the formal proof of validity, this is what you are going to learn for the first time. That it is not enough just to claim that whenever the premises are true, the conclusion is going to be true. But you need to *derive, step by step* through a proper formal process and you can demonstrate that given, or if the premises are true, these are the reasons why the conclusion has to be true. Ok? So, step by step you sort of derive in a sequential manner, *how* from the premises the conclusion can be derived or deduced. Actual analysis of the process. That's what is captured in this formal proof of validity. So, we will try to give a definition like this - first of all, note, that I have said it is a process. It is a procedure, or it is an instance where your goal is to establish the validity of an argument. How do you show that? By showing how the conclusion can be derived from other statements which happen to be the given premises.

And this derivation is not random, it is not your opinion, it's not my opinion either, but it is a legitimate logical derivation, the guarantee of which comes from certain principles of reasoning. Now you will say: What? Suddenly where this principles of reasoning are coming from? Well that's why we will try to bring in the idea of *valid argument forms*. Ok? Right now you will not understand what the valid argument forms are. I am going to save that for the next module. Right now what did we say? That take the formal proof of validity as a detailed, step by step sequence to show how the truth of the conclusion can be derived from the assumed truth of the premises given. This is what we will try to show. Few things that we will… we have already covered, but let us put it all in black and white in nonetheless.

First of all we will utilize the idea of form, as you can see you are going to talk about valid argument forms and it's a formal procedure. And all formal procedures are rather precise, rather rigid. So, the format there is going to be a, say, prescribed format and we *have* to follow that prescribed format for the proof. We cannot do it any which way we want.

Second, that there will be a deduction or a derivation that will be shown step by step. And we already said that this derivation is not going to be random. It's going to follow certain rules. So, its time that we also note this that we need to learn these rules of reasoning, so that will be a part of the training of this for formal proof of validity. Now schematically, you know, some of us said: Enough words, show us what it looks like. I am not going to show you how to do the proof immediately because you still have to learn quite a bit, but we can show you the schematic form for example. Ok?

(Refer Slide Time: 11:31)



.

So, here is the schematic idea, the skeletal form. Given Gamma (Γ), where Gamma happens to be a set of propositions, various propositions, say P_1, P_2, P_3, up to P_n. This is the set gamma and here is the claim that from this set gamma, C follows. Therefore, C, where C is your conclusion. Get it?

Now the format of the formal proof of validity is going to be like this that you have your given set here. This is Gamma, represented each member is given a line. What are you claiming? That from all of this, C follows. So, you already have a claim that C follows validly from this. What is the job of the proof then? Is to show *how* C follows from this premises. How? Ok?

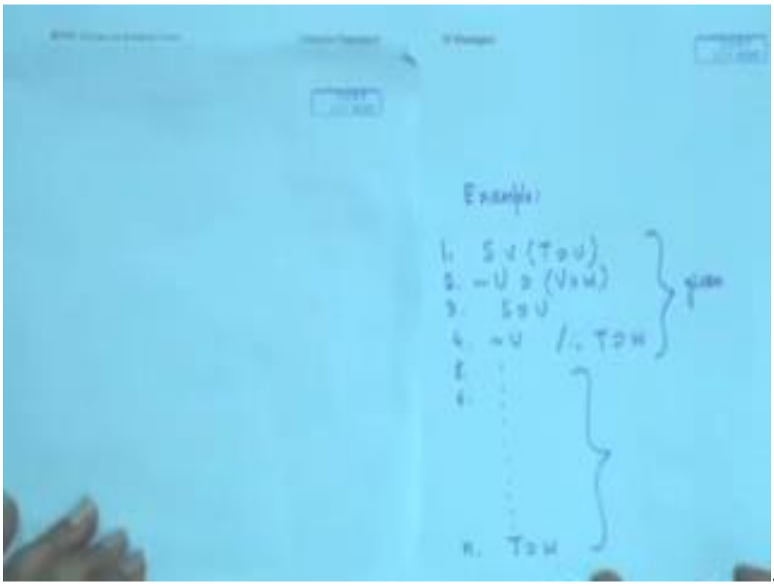
First the format and we will talk about the format more, but take a note.

What are these? These are line members. So there is going to be unique line numbers for every line. Both for the given, the given set of premises and also for lines, any line that you would be adding to the proof in terms of newly derived lines. So, every line is going to have a line number for your reference's sake. Is that correct? Notice that the conclusion is already mentioned here, but there is a slant and there is a triple dot. This is just an indication. These are the premises; this is conclusion, as we learned long ago about the format. Notice that this is going to be, the C is going to appear in the last line of your proof. Why? Because that's what I said. I mean the whole proof is to show that how C follows from these premises.

So, with these premises, you are going to derive new statements, new statements and new statements until you reach C, and when you reach C that is the end of proof. Now for the given set, there is no need for giving any justification. But for every line that you add, there must be justification in terms of which line, and which rule did you apply to get this new line in the proof. So that it is known as not a spurious line or ad hoc (Refer Time: 14:22) line. But something that legitimately follows from previously, what has been previously claimed to be true. Get me? So, there is lot to learn, as we can see. That first of all we are referring again and again to rules which we'll come to do. But it is also very important at this point to understand the layout or the format. As I was saying, the format is absolutely non-negotiable. You have to follow this procedure in order to do the formal proof of validity.

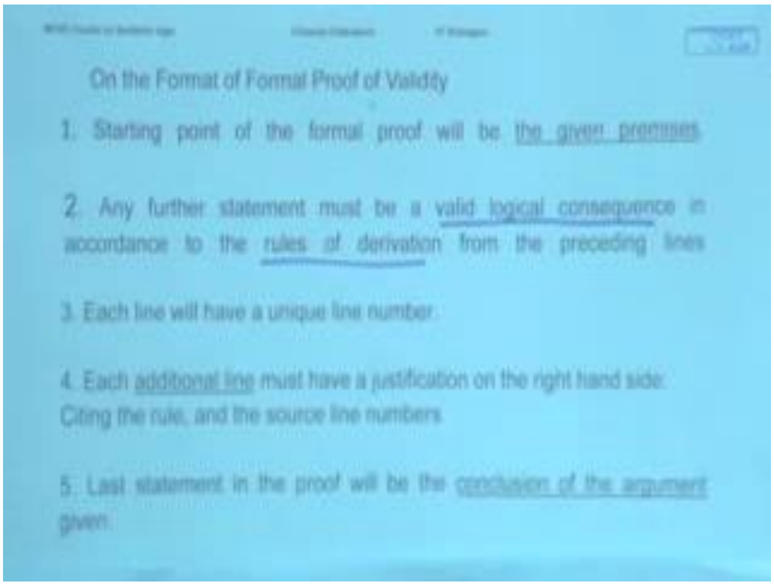
If this was a skeletal representation of the format, let me show you what would be the actual example like.

(Refer Slide Time: 15:06)



So, here is a small example where you have four premises, given premises. So, this entire thing is your given. Not only the premises are given, also the conclusion is also given. So, there is a claim that this follows from these premises validly. So, the entire argument is valid that is the claim, but claim is not enough you need to show how it follows. So, what the formal proof will do is with this given premises it will now derive line after line after line, until it shows how T ⊃ W or the set conclusion can be derived from this whole thing. Ok? As I said, every line that you generate will have numbers, as you can see, sequential numbers and every new line that you generate must have justification in terms of the rules of reasoning and the new line number that you have tried to add. So, this is what the formal proof of validity is going to look like. Ok?

(Refer Slide Time: 16:36)



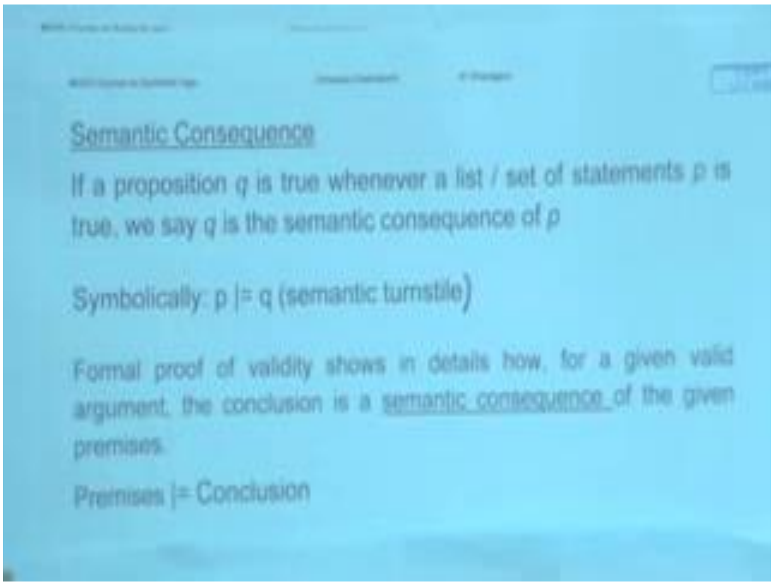
The rest is as I am going to further and further add is just to sort of elaboration on these points. But these are all important to remember, especially because you are learning it for the first time. So, just like any other procedure, there will be a starting point and there will be an end point. What is the starting point? The starting point has always, as you have seen, in the case of the trees also. But in here, in the formal proof your starting point are the given premises, given specifications, which you are claiming to be true. Suppose these are true. Remember the conclusion is also given, but the conclusion is not at this point to be entered in the proof. The conclusion is something to be derived from the premises. So, your first point, starting point, are the premises.

And then, as we just saw, that you can add lines in the proof. This is given, then anything that you add to the proof is a new line. But these are not random insertions or random additions. Any further statement that you add must be a valid logical consequence in accordance to the rules of derivation, ok? From the proceeding lines. So, these are not just any which conclusion that you can draw; rather, these are derived conclusions from using certain rules. This we have already established that every line in the proof is going to have a unique and sequential line number.

We have also established that every new line that you add to the proof must have justification. As I said, you need to cite which rule of inference or which rules of reasoning you are following, and the source line number, on which line you are doing the operation. That has to be part of your justification for every new line. And this is the termination point, your proof ends when you have derived the conclusion. Which conclusion? The conclusion that was announced as the valid conclusion of the premises right at the beginning, ok? So this is what is going to be the format of the formal proof validity.

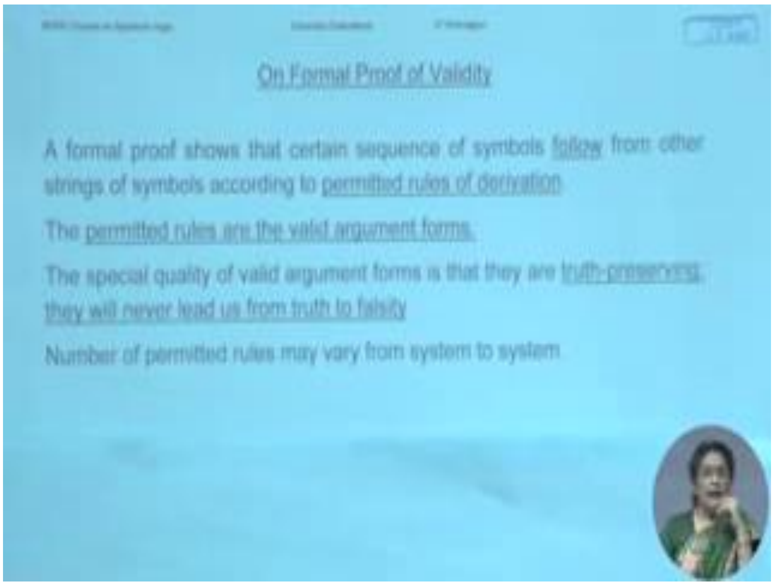
Why we are doing it like this? And we'll try to just I said touch upon this concept of semantic consequence.

(Refer Slide Time: 19:02)



See, if a proposition q is true whenever a list of statements or set of statements p is true, ok? we say then that q is the semantic consequence of p. Alright? So, this is the symbolic representation of this idea. This is called turnstile, double turnstile. So, p double turnstile q. What you are claiming is that q is true whenever p is true, where p is a set of statements or list of statements. You can put it now in the argument situation like this. That in a valid argument what we are claiming is that the conclusion is a semantic consequence of the set of premises. Thus the conclusion q is true whenever the set of premises is true. So, we can put this, in this way that the premises double turnstile conclusion, or the conclusion is the semantic consequence of the set of premises. That is the underlying thought behind this formal proof of validity and this is what we build upon… our proof procedure on.

(Refer Slide Time: 20:49)



Now, basically, syntactically speaking what you are doing is that you are you are generating a certain array of symbols from earlier given arrays of symbols. But in between these two steps, there are certain rules of derivation. What they are, we are going to learn soon. So, just keep it alive, but I have used the word ‘permitted’. Why? Because you know the ‘permitted’ as in ‘permitted in the system’. Every logical system is rather independent. Their actions are not the same and the rules of derivation that they allow are also may not be the same. So, they can pick and choose and decide that we are going to only allow these rules to be our rules of (Refer Time: 21:26) derivation. This is the reason I have use the word ‘permitted’.

So, the system that we are going to follow will have very specified set of rules of inference or derivation and we are going to the learn those. If you are in another logical system, you may have to use some other rules of derivation. What you need to note is that what will function or act as the rules of derivation are actually valid argument forms. I know I have used this phrase earlier also, valid argument forms, and I have not given you any definition. Don’t worry; I will not give the definition yet also. I will save it for the next module. So, just listen to this that what these rules are going to be, are going to be valid argument forms, and that is, that is done consciously. The reason behind is that what we call valid argument form has a very, very special quality. What is that special quality? That they are truth-preserving in nature, they preserve truth. Meaning, that if you use them and if you start with truth, they will never ever land you into falsity. They preserve truth, if you begin with truth, they will ensure that you land also in truth. This is the very special quality of these things called valid argument forms.

So, this is going to be the crux (Refer Time: 23:07) of the matter in formal proof of validity that we will use certain valid argument forms as rules of derivation. Because they will ensure that when we are starting with truth, that we are also deriving something that is true. And when you do it sequentially with the valid argument forms, what you are doing is a tightly linked proof, each step of which is ensured by valid argument form so that nowhere are you missing out on truth. You started with truth, your truth is ensured next step, next step, next step, until you have derived the truth of the conclusion. Right?

So, this is the whole idea behind it, and as I said the number of permitted rules the nature of permitted rules vary from logical system to system. But let us keep it, keep that point right now slightly away from our main focus. The main focus in this module was to sort of get you acquainted with the idea of formal proof of validity, the format and more or less where do you start and where you end. What we now going to look into in the next module and so on is how to actually construct the proof. But with that I will end this module.

Thank you very much.