INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Date: , FN/AN, Time: 2 Hrs, Full Marks: 30, Deptt: MA/ AE/ AG
No. of Students: 54, Mid-Autumn Semester Examination, 2015
Sub. No. - MA40011, Sub. Name - Fluid Mechanics

Instruction*: *Answer all the questions. Show all the intermediate steps of your calculations. No marks will be awarded for incorrect procedure, even if your answer is correct.

1. Evaluate the constants a, b and c in order that the velocity

$$\vec{q}(x,y,z) = \frac{(x+ar)\hat{i}+(y+br)\hat{j}+(z+cr)\hat{k}}{r(x+r)}; \; r = \sqrt{x^2+y^2+z^2}$$

may satisfy the equation of continuity for a liquid. (5)

2. For the velocity field $\vec{q}(x,y,z,t) = \left(\frac{x}{1+t}, \frac{y}{1+2t}, 0\right)$, find the pathlines, streamlines and the streaklines which pass through (x_0, y_0, z_0) at $t = 0$. (6)

3. A small thin plate is pulled through a gap between the two parallel horizontal planes filled with a fluid of viscosity μ.

(a) If the distance between the horizontal planes is h, find at what height from the lower plate the shear stress on the thin plate will be minimum.

(b) If the plate moves with a velocity 5m/s and viscosity is $\mu = 0.014\text{NS/m}^2$, then what will be the shear stress on the plate at a height 2 mm from the lower plane. (4)

4. Find the relation between $f(t), \varphi(t)$ and $\psi(t)$ (t = time) such that

$$\frac{x^2}{a^2}f(t) + \frac{y^2}{b^2}\varphi(t) + \frac{z^2}{c^2}\psi(t) = 1; \; a, b, c \text{ being constants,}$$

is a possible form of the boundary surface. (5)

5. Define circulation around a closed curve moving with a fluid. State and prove Kelvin's circulation theorem. (5)

6. A complex potential $W(z)$ is given by

$$W(z) = Uz + \frac{Ua^2}{z} + ik\ln z, \;\; U, k, a\,(>0) \text{ being constants}$$

and represents the two-dimensional flow of an incompressible fluid of density ρ. If the cylinder experiences a force $\vec{F} = (F_x, F_y)$ per unit length where $F_x = -\int p\,dy, F_y = \int p\,dx$ (p = pressure), find $|\vec{F}|$. (5)

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