INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR
End - Autumn Semester 2018-19

Date of Examination: **22-04-2019** Session: **AN** Duration: **3 hrs.**
Subject No.: **MA20013;** Subject: **Discrete Mathematics** Total Marks : **50**
Department: **Mathematics**
No specific charts, graphs, log book etc. required
Special Instructions:
Step marking depends on the procedure and the final answer. Hence, it is essential to show the detailed working. Answer all the questions.

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1. [9 x 2 = 18 marks] True or false: Justify your answer:
   (a) $2^{4n} + 3n - 1$ is divisible by 9, for all positive integers n.
   (b) The number of terms in the expansion the expression $(b+c)(d+e+f)(x+y+z)$ is 16.
   (c) The number of solutions to the equation $x_1 + x_2 + x_3 + x_4 = 7$ is 120, where the variables x_1, x_2, x_3 and x_4 are nonnegative integers.
   (d) In a group of 5 people, there are three mutually known people or there are three mutually unknown people.
   (e) Every graph on more than 1 vertex has two vertices of same degree.
   (f) Let G be a graph of order $n \geq 2$, and suppose that for every vertex v of G, $deg(v) \geq \frac{(n-1)}{2}$. Then G is connected.
   (g) Let G be a connected graph such that degree of every vertex of G is even. Then G can not contain a cut-vertex.
   (h) The sequence $(6, 5, 4, 4, 4, 4, 3, 2)$ is a Prufer sequence of a tree.
   (i) If G is a connected graph such that degree of every vertex v of G is greater than or equal to 2, then G is Hamiltonian.

2. [3 + 2 = 5 marks] Evaluate the following sums using combinatorial arguments (n is a positive integer):
   (a) $1\binom{n}{1} + 2\binom{n}{2} + \cdots + n\binom{n}{n}$.
   (b) $1\binom{n}{1} + 3\binom{n}{3} + \cdots + (2k+1)\binom{n}{2k+1} + \cdots$.

3. [5 marks] Solve the recurrence relation: $a_n = 5a_{n-1} - 6a_{n-2} + 3.5^n$, where $a_0 = 4$ and $a_1 = 7$.

4. [5 marks] Using generating functions, solve the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2}$, where $a_0 = 2$ and $a_1 = 3$.

(please turn over)

5. [6 marks] Define a minimal spanning tree of a connected graph. Explain the Kruskal's algorithm for computing a minimal spanning tree of a connected graph.

6. [3 marks] Let G be the graph obtained from K_4, the complete graph on 4 vertices, by removing one edge. Compute the number of spanning trees of G.

7. [5 marks] State and prove Ore's theorem for Hamiltonian graphs.

8. [3 marks] Let G be a plane drawing of a connected planer graph, and let n, m and f denote respectively the number of vertices, edges and faces of G. Then, prove that $n - m + f = 2$.

***** All the best *****