**Symbolic Logic**
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**Lecture - 12**
**Using Truth Table**
**Tautology**
**Contradiction**
**Contingent Propositions**

Hello. We have looked into the truth-tables and today we will talk about applications of this truth table to find out some interesting logical properties.

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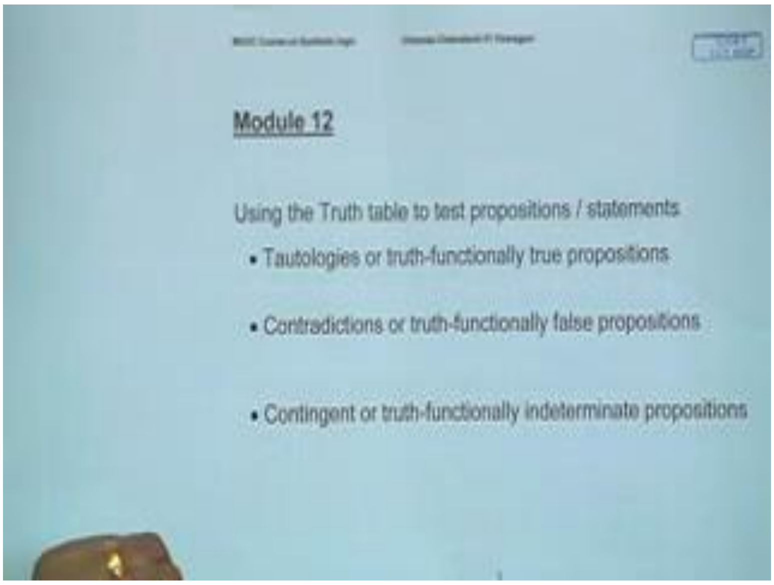


So, if you have by now mastered, how to do the truth-table, that would help you to understand the procedure better. What we are going to do today is to look into proposition or statement forms, and then classify them in a certain sort of order. These are some of the classifications that we will be doing. But before we do that, I have used the word ‘propositional form’ or a ‘statement form’. So earlier we have talked about propositions, statements, and suddenly I am bringing in the idea about a *statement form* or a *propositional form*.

So first thing is to learn that. Why? What is the advantage of learning the form? I will explain that, but remember in formal logic the *forms* are going to play a crucial role. So we will start by learning what is this statement form or a propositional form to start with.

To remind you that when we were doing the syntax, we decided that there is a certain way to represent the variables and the constants.

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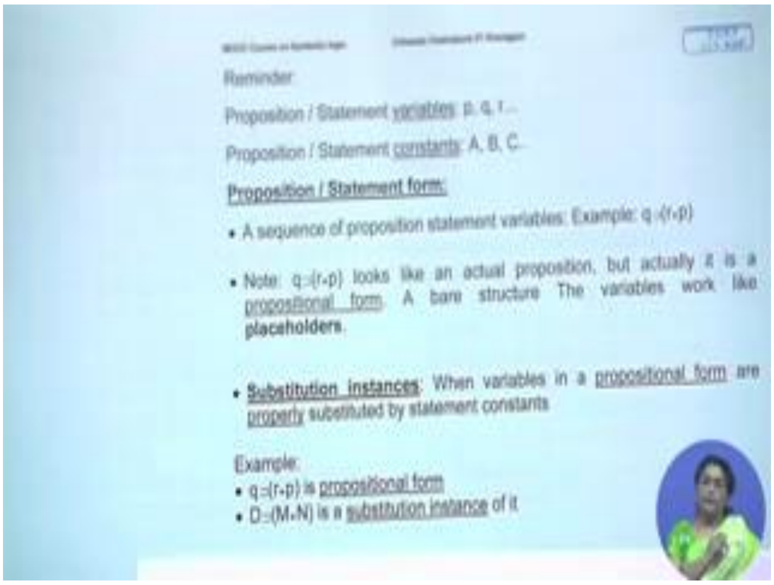


The propositional variables, or the statement variables, we said are going to be the lowercase letters, such as p, q, r, and so on and so forth. And then we said, the proposition or the statement constants, which stand for the actual propositions, which will pick up *one specific proposition* from your domain, and we reserve the capitals letters A, B, C as appropriate for them. So there is a case sensitivity.

So when you are using the variables group, you are at a different plane, which is more abstract than the level of the constants. because the variables have no specific reference. It means *any* proposition. If you keep that in mind then you will soon start to see what we talk about the proposition of the statement form is like a structure, a bare structure. You know, we say like if you compare the human body with, then there is a skeleton and there is the flesh. So when you put this flesh over the skeleton, you get a whole human being. But when you have a bare structure, namely, the skeleton, that’s what this proposition statement form is going to be looking like.

So if you say how do, how do they look like? The answer is it is a sequence of propositional variables, or statement variables; such as for example, q horseshoe r dot p; $q \supset (r \bullet p)$.  This is a proposition form, or a statement form. Now the point to note is that it may look to you like as if it is a real proposition, because we are using the real connectives. But please note that we are using these strange symbols, small q, small r, small p, and they are actually like holes. You know how people play with playdough (Refer Time: 03:56), or how people do die casting and so on. So, what you have is a mould, in which you pour some liquid, and the liquid takes that shape. Right? So, if you remember that, what we are talking about is a mould, then this is what these moulds of propositions look like. They are like little holes, where if you put the actual proposition then you understand what this sentence means. Otherwise they are just propositions shaped holes, place holders. That's what we call. So first thing to notice though they look like actual propositions, they are not really propositions. They are structures or forms which will be exemplified, which will be instantiated in actual propositions. But that's coming up soon.

Now, formal logic, and specially the contribution of Aristotle, or formal logicians like Aristotle, is this key understanding that the form is the important thing. The form is the bare structure; the underlying logical structure which gets instantiated by actual propositions. So it's not like we take actual propositions and somehow we eliminate the content, and we get the form. That's not at all what they are saying. Rather, what they are saying is the starting point is the structure like this, which when properly substituted; which when properly instantiated, that's when we get actual form, actual propositions of the same form and we will show you examples.

So remember, when you are going into the formal level, the first thing to note that substitution instances - instances which substitute the variables by constants. So this is what, how the process of obtaining an actual proposition is in formal logic, when the variables in the propositional form or the statement form are properly substituted. This is certain way to substitute it and by the statement constant. That's when you get substitution instances. Now why we are saying this. you have to understand that as if the statement form is the fountainhead; and when you instantiate it, when you try to exemplify it, then you get a bunch of many substitution instances which are your actual propositions.

So let's take a look. For example, this is what we started with. This is a structure: the q horseshoe r dot p, $q \supset (r \bullet p)$. Now what will happen if we substitute it properly? Meaning, look, there are these variables. Each of them is a simple propositional variable. So, each one will have to be substituted by a simple actual proposition. Now symbolically, when we represent it, there's going to be a propositional constant for each of these variable occurrences. So if we do that, that substitution instance~~, it~~ will look like this. So for example, we have chosen arbitrarily D for q,  M for r, N for p and this is an actual proposition. Each of this is a constant, which refers to a specific proposition in your domain, and the whole thing is a compound which is a *substitution instance* of this *form*. We will continue to talk about these kind of forms as we go along, but it is very important that you see the angle from which the formal logic sees these propositions, the generation of actual propositions as. And the reasons why we are talking about this also will be explained in a minute, but let's take the idea through.

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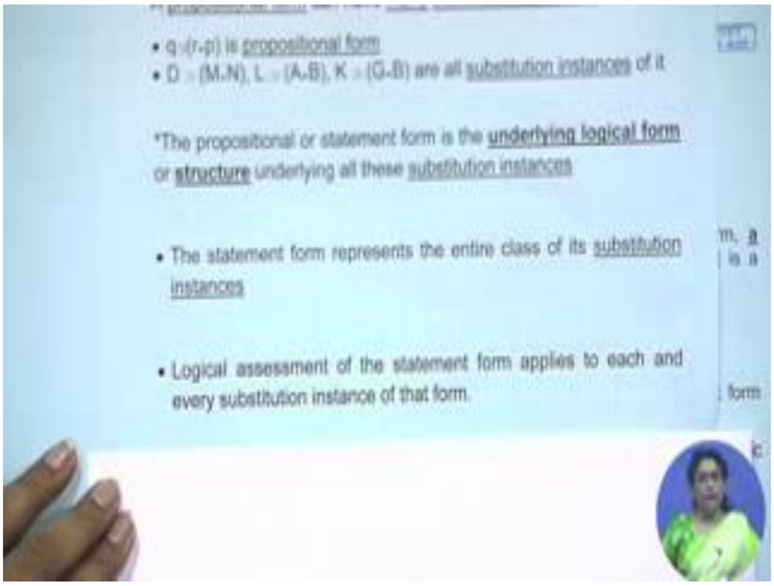


Now notice that the propositional form, a single propositional form, as you can guess, may have many *substitution instances*. For example, the same propositional form, you saw one substitution instance, but you can imagine that there can be many that exhibit the same form. This one does, this one also does, this one also does. So are they all substitution instances? Yes, of the same form. What we say, is that they exemplify the *same propositional form*. And there can many more.  Right? There can be many more, depending upon what you are substituting the variable with, the constant domain. If you have understood this idea, then you can also understand that the *same form* may represent a whole class of substitution instances. A whole set of substitution instances. And that is the idea that we are going to catch on too.

So, one more time, what we are talking about is the bare skeletal form; the underlying logical form. At the surface, you may see ~~the~~ language, at the surface you may see a lot of content, which are which we seem to defer, but underlying the bare structure is what we called the *logical form* of a statement. And as I said, it represents the entire class of substitution instances. Ok? So that is something to remember by.

Then, the advantage of doing this is that practically when you take the whole discussion to the form level, then if you do any logical assessment of the form, the underlying form, then you can make a comment about the entire class, which is a result of substitution of this form. That is the game and we'll show you with actual examples.

But first of all, different types of statement forms. So if you have understood the *form* in statement form, then there can be two types, and these are *generic* and *specific*. So the specific statement forms are what is more desirable; because they capture the structure, the logical structure with greater details.  And sometimes that could be logically more important and informative. What do we mean by a specific statement form? We will try to explain that. What we mean is that the structure should be exactly replicated, and one way to say that that whenever there is a distinct statement variable occurring, it has to be consistently substituted by a unique, simple, actual statement constant.

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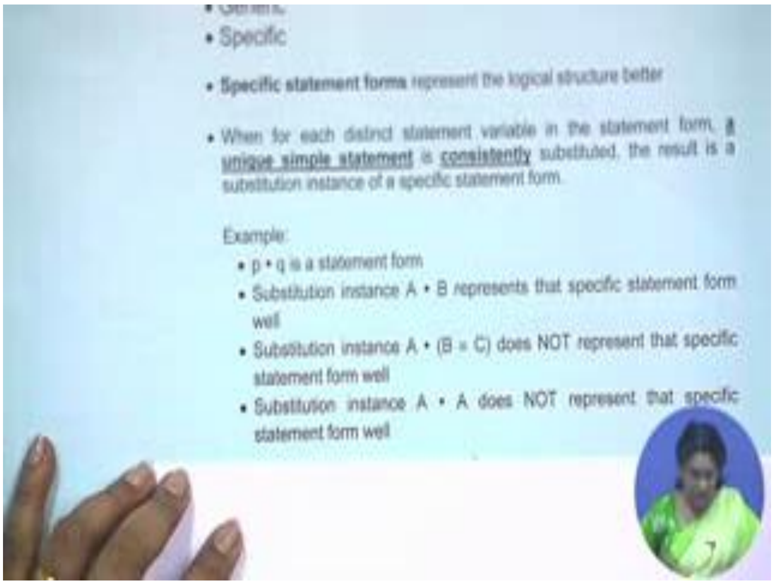


So two things.  One, whenever you find that the variable is a distinct one; it's not the same, it's a different one, you need to pick a different statement constant to substitute it.

~~Second point is~~ So that is a point of saying unique, and if the distinct statement variable is *simple*, you need to preserve the structure by picking up a *simple* constant. Second point, *consistent substitution* which means that it should not be random, and it should not be also out of sync. So if you at one place if you have substituted small p with big or capital A, next time it should not happen in the same form, that you, the occurrence of p is now substituted by a capital B, because then you are changing the meaning. So whereever small p occurs, if you have decided to substitute it by capital A, that should be done *thoroughly* and *consistently throughout the sentence*. This is something to remember. So the specific statement form.

May be the example will tell you better. See here for example, p dot q; p ● q; that is a statement form. p and q if you notice are simple statement variables. There is no compoundness here. The whole sentence is a compound, but p, q alone they are actually simple statement variable. So if you if you want to preserve that structure in your substitution instance, then say A dot B, (A●B) , A dot B. Ok? (A ● B) would be good example of specific statement form representation. Whatever was the form, said that is what you have tried to capture in this substitution instance. But A dot B triple bar C; A● (B≡C) will not represent the specific statement form. Why not? Because you know you have replaced p with capital A, but what you have done is to replace q with a compound statement. What have you done? You have changed, in a way, you have tampered the simple structure, what you saw in the variable somehow has not been preserved by replacing it with a compound. Ok? So, that simplicity structure needs to be retained if you are looking for specific statement form substitution.

Similarly, A dot A, (A ● A) is not a proper specific form substitution. Why not? Because the variety that was present in p ● q, p and q, these are discrete different statement variables, but when you replace them with the same constant, in a way you are playing with the form. In a way you are distorting what this says. This says something else and this is quite different. So in a way you have changed the structure. How? By choosing to replace the distinct variable by the same constant, and that is not the nature of specific statement form. Hope this is going through well with you. So, in a way specific statement form keeps us more about the, more attuned to the specificity of the statement structure and they are desirable for representing many things.

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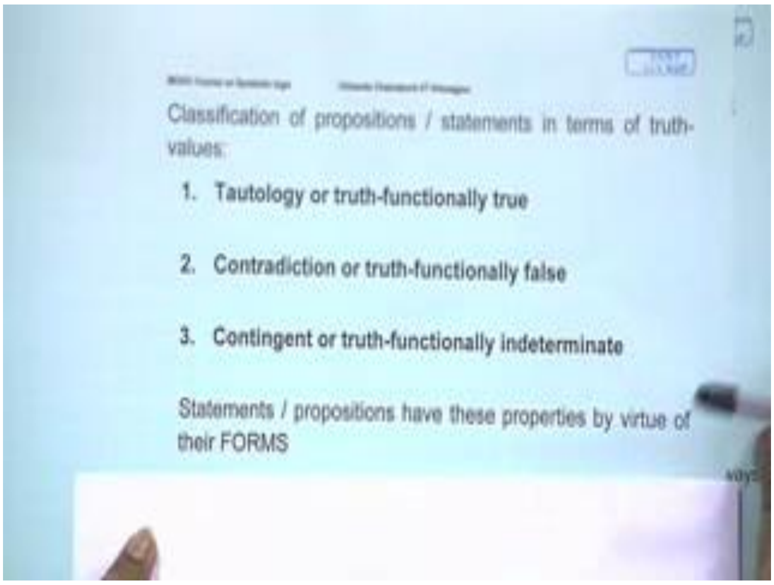


One of the tasks that we are going to take upon today by use of truth-table is this classification. We are going to classify proposition in terms of their truth-values and the truth-table, of course, is going to help us in doing that.  It's the broad classifications I will show you in a second is like this.

There are propositions that are called tautology or truth functionally true. What is there nature we will talk about that. The second grouping is contradiction or truth functionally false, and the third category is contingent or truth functionally indeterminate. This classification is not arbitrary, but in the terms of their truth values.  And this is where we will try to utilize the truth table technique to see whether the table can help us to categorize unknown propositions into this kind of categories, or unknown statements into this kind of categories. But the point to note is that we as being for at the formal logic level, we are going to say that the propositions or the statements have this kind of properties, this classificational properties by *virtue of the forms*, by virtue of what logical form they exhibit, by virtue of the form that is underlying them. So the… ultimately the comment is about the form. So in a way what we will learn here is that the tautologous propositional forms will always yield tautologies as substitution instances. Similarly the forms, the statement forms which are of this kind of nature, contradictions will automatically and always yield substitution instances that are of this type.  And similarly for the contingent. So if the logical form is contingent, no matter how you do it, the substitution instances are going to be always of this category and so on.  Let's try that.

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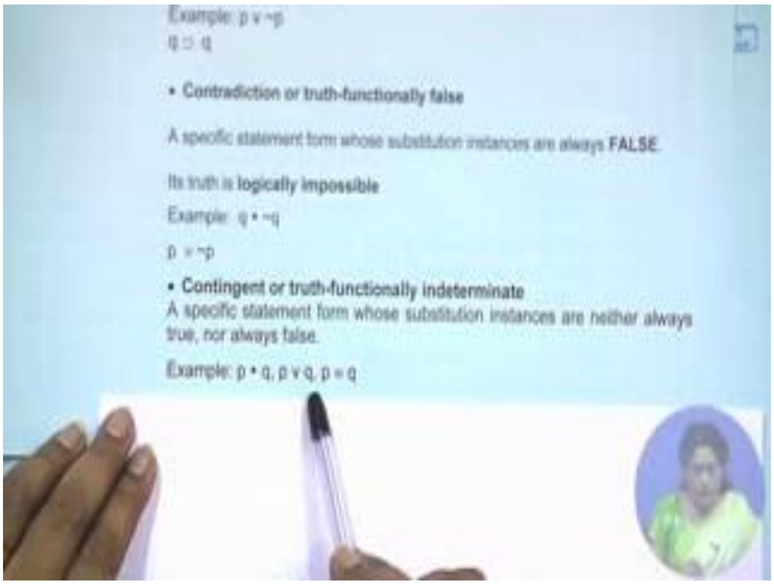


First, our acquaintance is with this classification - what are tautology or truth functionally true propositions and the answer to that is like this: That it is a specific statement form whose substitution instances are always true. Ok? So that’s the first thing to remember that it is a statement form whose substitution instances are always going to true. That’s what we call the tautology or the truth functionally true. So its falsity is logically impossible. Right? If you want to know what are they, what which forms we are talking about? Well one of the very well known form that you can probably identify with immediately is $p \vee \sim p$.

If you understand how the ‘vel’ or the ‘$\vee$’ works, then you know one of them will be true and one of them when they are true, it makes the whole compound, that disjunction, true always. Now if you replace this p with any constant of your choice and if you keep the specific logical form intact, then there is no way that you are going to have any substitution instance that is going to be anything but true. That is the beauty. So this form is going to generate always true substitution instances.

There are many more tautologies by the way, I mean; this is not the only one, though it may be the most well known one. But there are many.  For example, here there is another example, q horseshoe q, $q \supset q$, and that again if you replace or substitute properly, you will see that this form is going to only yield true substitution instances. So this is what tautologies are.

Next is contradiction or truth functionally false. What are they? Again they are also specific statement form. But it is a statement from, specific statement form, whose substitution instances are always, always false.

So, its truth is logically impossible; and so they are just the opposite of tautologies. Alright? ~~all right~~. So examples are many, but then again we will start with the most well-known example perhaps, is this one for example, q dot not q, ($q \bullet \sim q$). So in a way you're saying a proposition, and then you are also negating it and joining it together with the conjunction. This structure, this propositional form can only yield substitution instances that are going to be always false. That is what makes them contradictions. Are there other examples? There are many, and you will soon find out many more. Say, p triple bar tilde p, ($p \equiv \sim p$), that statement form again is a contradiction because it is going to yield only false substitution instances.

So here is ~~the~~ tautology, here is contradiction; this is always true, and this always false. And then there is a third category that we call contingent or truth functionally indeterminate ones. What is the situation here? It's a statement form, specific statement form, whose substitution instances are neither always true nor always false. So they yield substitution instances which are sometimes true and sometimes false. Ok? Being always true makes you a tautology. So if you are neither always true then you're not a tautology. And being always false makes you a contradiction; so being not always false makes you different from the contradiction. So it's a third category by itself, and it is an important category because there are so many examples possible of this.

For example take any of your choice p dot q; ($p \bullet q$) for example. If you recall its ~~is~~ truth table, you will see that it has a mixture of truth and falsity as its value. $p \bullet q$ is true only when both p and q are true; otherwise it is false. So there is a mingling, there is a mixture of truth and falsity truth-values. Same goes to for $p \vee q$ or p vel q. It's true when both are true, or one of them is true; and it is false when both disjuncts are false. So then again there is a mixture of T-s and F-s. Same goes for your $p \equiv q$ or $p \supset q$ either way. So this is a very large category and very important also, but it's good to know that we don't have just one set of truth-values coming up. There is, there can be also mixtures.

But these are the three basic categories and now comes our truth table knowledge and how we are going to use the truth table to classify propositions into, so that they fit into this kind of categories.

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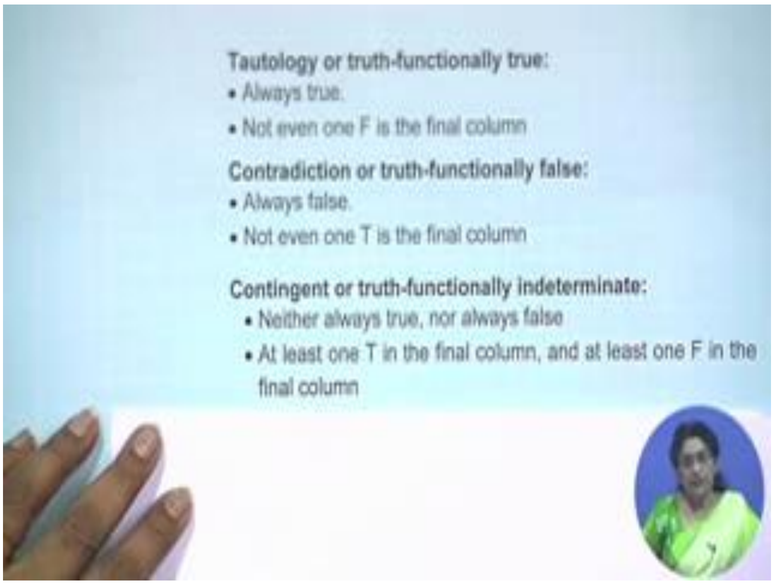


So if you are testing by truth table, then it's easy to figure out even common sense is that in the case of the propositional form, or the proposition, is a tautology or truth functionally true, what will happen? What will happen is that if you look at the final column, you will not find even one F. It's going to be all throughout and without any exception T-s, true-s because it is a tautology. If you that in the final column, then you know it's a tautology. Alright? Similarly, the contradiction. What will happen is that, if you look at the final column of the truth table, you will not even find a single T in that final column. It will be throughout false and only false. When that happens, your truth table is telling you that what you are dealing with is a contradiction or truth functionally false proposition.

On other hand, in case of contingent or truth functionally indeterminate propositions what will happen? Well, as you know that it is neither always true or nor always false. But what does that mean logically? How many Ts, how many Fs are going to be there? What you say? And the answer, as you probably thinking, that they should be at least one T in the final column to make it, to distinguish it from being a contradiction, and at least one F in the final column so that it is distinguished from being a tautology. So that's our answer. There can be many more various kind of combinations possible, but at least one T and at least one F in the final column. That's what is going to happen in the case of contingent.

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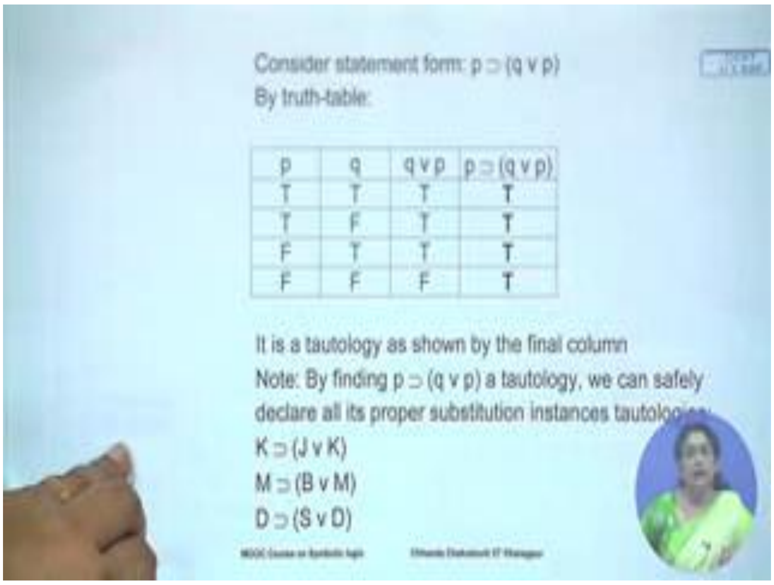


So now comes the point where we are going to try this out. Alright? And we are going to start out with a statement form of this kind, $p \supset (q \vee p)$, and we are going to do truth table as we have learnt to do truth table on this. So what will happen is that… why do not you do it along with me, so that we all learn at the same time… is that there is going to how many rows? Four, because there are two statement variables. So, *that* we know, and then the top heads of the table is going to be like this. You have two reference columns, here is $q \vee p$ because we need to have a column for that, and here is the column for the whole compound statements. And if you remember how to distribute the truth values and all, then this how to distribute the truth values. This is how we do it and then the third column looks like this. Right? Because this disjunction is false only when both the disjuncts are false; otherwise it's always true.  And here comes the truth value of the whole sentence, the compound. All of them is going to be true. Remember the horseshoe truth table. That it is false when the antecedent is true, but the consequent is false; and if you now check p is here. Right? So this is what we are comparing with this column, which is your consequent. So T T that is T. T T that is T. This is F T. That's not the same as p antecedent being true consequent being false. What you have is false antecedent and true consequent, and that is when horseshoe takes the value true.  And this is when both of them are false. That is also when the horseshoe takes the value true. But ultimately what you have gained or what you have found out? That this sentence is what you would call always true. Which means it is a tautology or a truth functionally true proposition.

So this is what we did. You found out that this is a tautology as shown in the final column. But notice, that this is not an actual proposition, but this is a statement form. If you have shown that this statement form is a tautology, what have you gained? Now you are in a position to say that any substitution instance that exemplify this form is also going to be a tautology. So if I give you this, if I give you this, if I give you this, you do not have to do separate truth tables for these anymore. This alone settles that each of this *must* be a tautology by virtue of their form. The form, common form that exhibit is this one and we have established it is a tautology. That is what you gain by doing logic at formal level.

So this one task we have accomplished. We just figured out that this statement form is a tautology. You may try this. These are the some of the examples that you may try right now or we come back in the next module and you can do that.

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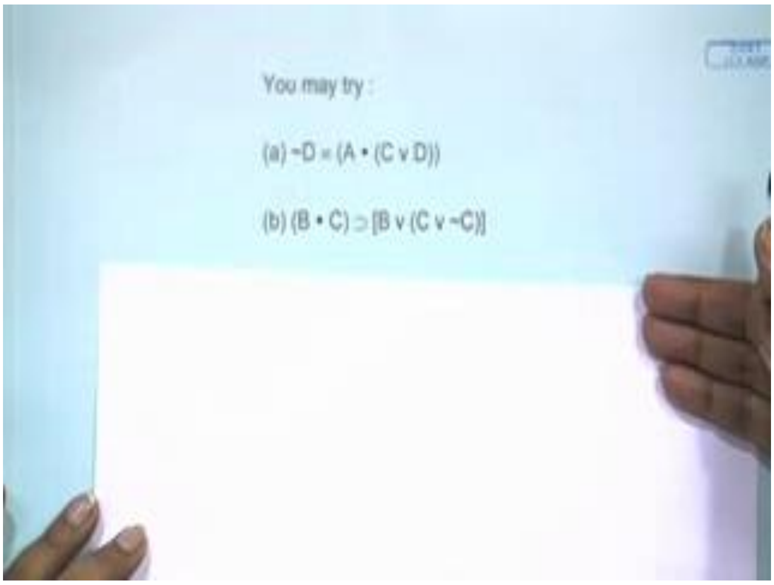


This is a simple ones take a look. So here is this, $\sim D \equiv (A \bullet (C \vee D))$ . This is $(B \bullet C) \supset [B \vee (C \vee \sim C)]$. Why do not you try this? And we compare the result in our next module. Ok?

So let us close this module. We have learnt a task and we will do this and we will continue with this in the next module. Thank you very much.

Thank you very much.