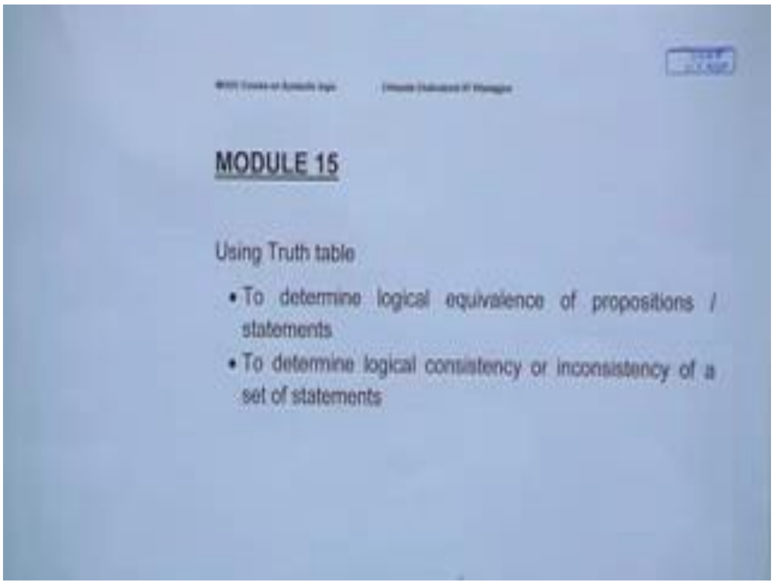
Symbolic Logic

Prof. Chhanda Chakraborti
Department of Humanities and Social Sciences
Indian Institute of Technology, Kharagpur

Lecture – 15
Using Truth Tables:
Testing a Set of Propositions for
Consistency and inconsistency, and for
Logical Equivalence

Ok, so our last module, this is our last module on the truth table task. So far we are progressing nicely with the truth table.

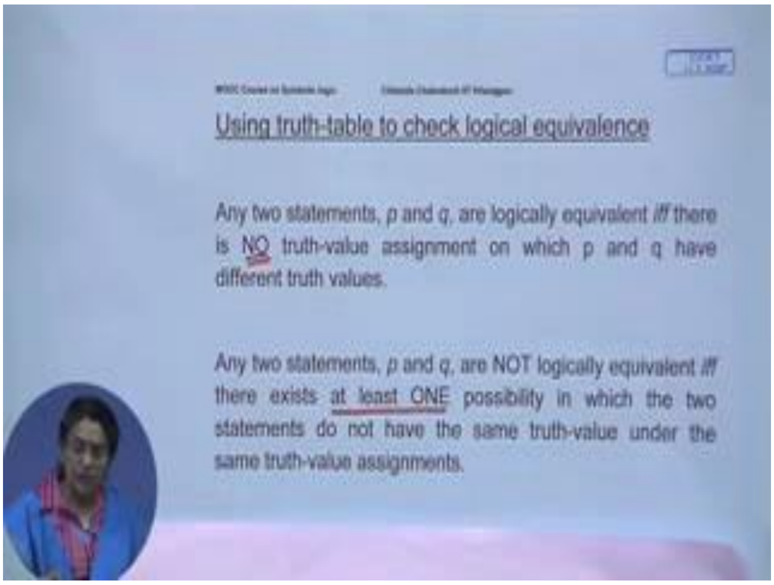
(Refer Slide Time: 00:30)



There is one more task that we need to show that the truth table can help us in this regard. We are still into the semantics of propositional logic and these are various properties. These are various characteristics that we are trying to sort of determine with the truth table technique.

The last one on our semantics portion is using the truth table to determine two different kinds of properties. One is, whether given two propositions we can tell that they are logically equivalent or not. So determining the logical equivalence of proposition, this is one. And then we have already learnt a little bit about what is consistency and inconsistency. So given a set of statements, can the truth table tell us whether the state is consistent or inconsistent? So this is our task at hand for today and we are going to use the truth table for doing this.

(Refer Slide Time: 01:39)

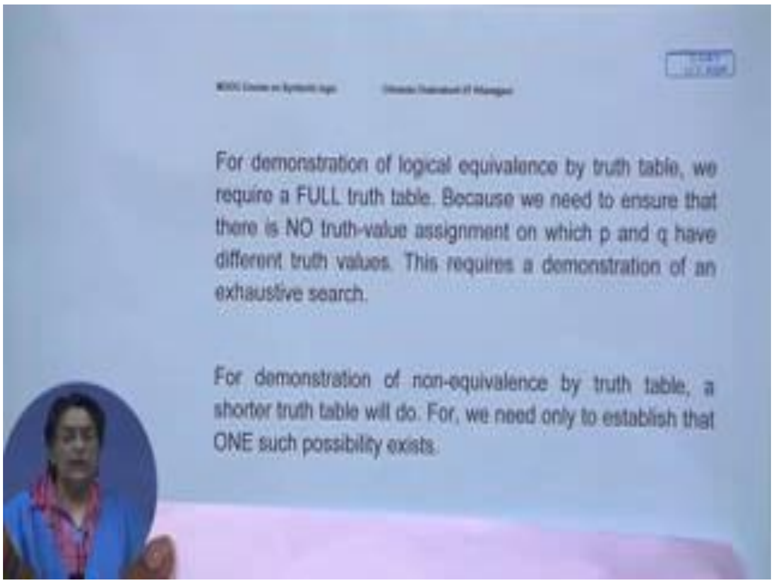


So, first task is finding out whether two given propositions or two given propositional forms, are they logically equivalent or not? Let us remind ourselves what logical equivalence means. Even if you remember the triple bar truth table, you recall that whenever the values match, the triple bar comes out to be true. Whenever there is a mismatch with the truth-values for the propositions, there is no equivalence. If you remember that, then given any two propositions, p and q, we will call them logically equivalent if and only if there is *no* truth-value assignment on which p and q differ, on which the truth values of p and q mismatch or are different. So I will repeat that. When can we call p and q logically equivalent? When there is *no* truth-value assignment, remember, this is no, none. You have to sort of show it exhaustively that there is no truth value assignment on which the values of p and q differ. Ok? So every single case, the values of p and q will match. That's when we call it, call them logically equivalent.

On the other hand when are they not logically equivalent? The answer is : For any given two statements p and q, they are *not* logically equivalent if and only if there exists at least one truth-value assignment on which the truth values of p and q do not match, or the truth values of p and q are different. How many such possibilities you need to show? At least one. Alright? How many such possibilities you need to establish here for logical equivalence? There is none. You understand? There is a very big difference in the way you are going to demonstrate this. So, this requires, logical equivalence demonstration requires that you need to exhaust all possibilities. Every single doubt has to be removed. On the other hand, for showing that they are not logically equivalent, you just need to demonstrate that there is at least one such possibility where the values are not going to be the same. Under the same truth value assignment to the components, the compounds are going to show that they have different truth values.

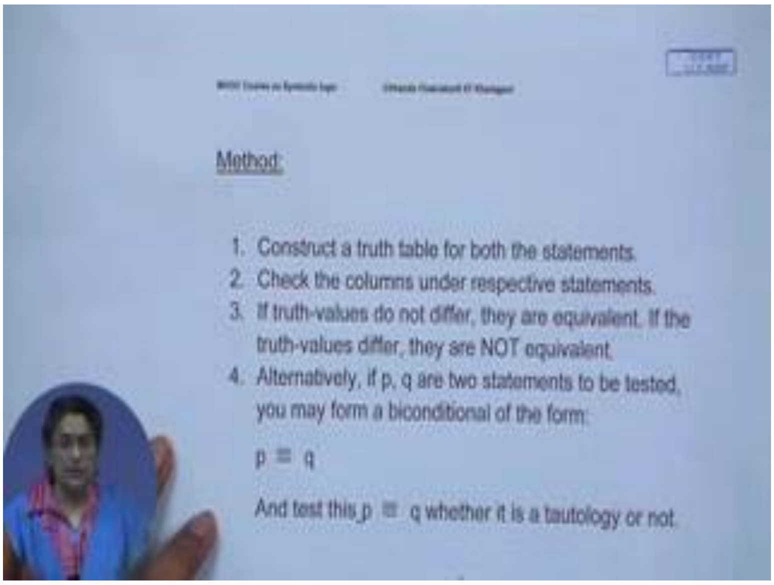
When I have said that, I think more or less you know in terms of truth table what would be your approaches. Right?

(Refer Slide Time: 04:49)



So, let me make that point clearer or more explicit, because I think you have already guessed what we are trying to talk about here. But this is where we are. That the demonstration of logical equivalence by truth table, we are going to require a full truth table, right? This is already, should be already evident from what we have discussed. Because we need to ensure that there is *no* truth-value assignment on which p and q start to exhibit different truth-values. So, this elimination of any possibility, or elimination of a single possibility is a major exhaustive task, for which you are going to require the full truth table. Correct? Whereas, for the demonstration of not logically equivalent, you are going to require the shorter truth table technique. Remember the shorter truth table technique which we have covered in the previous module. It demonstrates that there is at least one such possibility, and that is exactly what you need to show that any two given proposition p and q; they are not logically equivalent. So, here we are, and this is, again , when you understand it is pretty simple. The method, how to do this, etcetera… I am going to explain in a moment. But it's… it's much more important to understand and grasp the idea behind what it is that you doing. I hope that difference has been clarified in this.

(Refer Slide Time: 06:24)

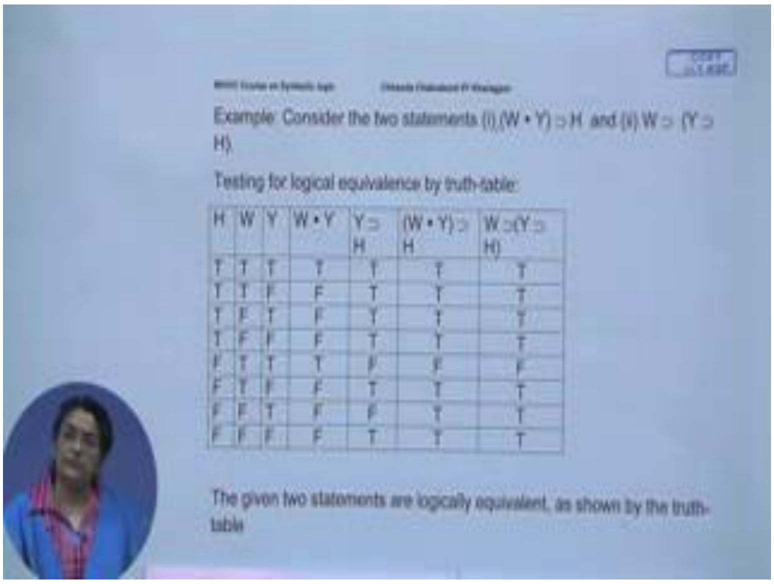


So, logical equivalence or not logically equivalent, what is the method? Given any two propositions p and q, what do we have to do? What we do is that we construct a truth table for both the statements. This is one approach. How do we do that? We do it like this. That we do regular truth table for them. If they have a component that they share, well and good. So reference columns, and then for each statement we need to have a column. Under these respective statements what do we look for? We look for every single row whether under different truth value assignments to the component, there are truth values of this given two propositions: Whether they differ or they do not differ? If the values match exactly, that is they do not differ, then you have equivalence, logical equivalence. On the other hand, if their truth values differ, right? There is a mismatch, sometimes this one is true when the other one is false, etcetera, then obviously, the two propositions are not equivalent.

So, this is one approach to show or to try out the truth table technique to show logical equivalence or the lack of it. The alternative way it do this would be that you take suppose p and q are two any two given propositions, and they are to be tested for logical equivalence. What you can do is to form a biconditional out of them, and the form will be like this. Remember p and q are any two given propositions. What you do is to form a biconditional of this kind. Just remember that the p and q themselves might be rather complex compound propositions, as we will see in some of the examples. But that should not deter you. Treat the whole complex compound statement as your p, one of them; and the other one, you treated as q and try to form this p triple bar q, $p \equiv q$ biconditional out of that. Once you form that, then by truth table just do the regular truth table on $p \equiv q$. And if $p \equiv q$ comes out to be a tautology, you know p and q must be equivalent. If it does not turn out to be a tautology, you also know the answer that p and q cannot be logically equivalent. Both are accepted approaches.

So, here you have one, where you form the respective propositions, you give them columns, and you sort of see under the same truth value assignment whether their values match or differ. Here you convert, you take those two propositions and form a biconditional and test it for whether it is a tautology or not by truth table. So these two are the methods and we will try to see how it goes along here.

(Refer Slide Time: 09:39)



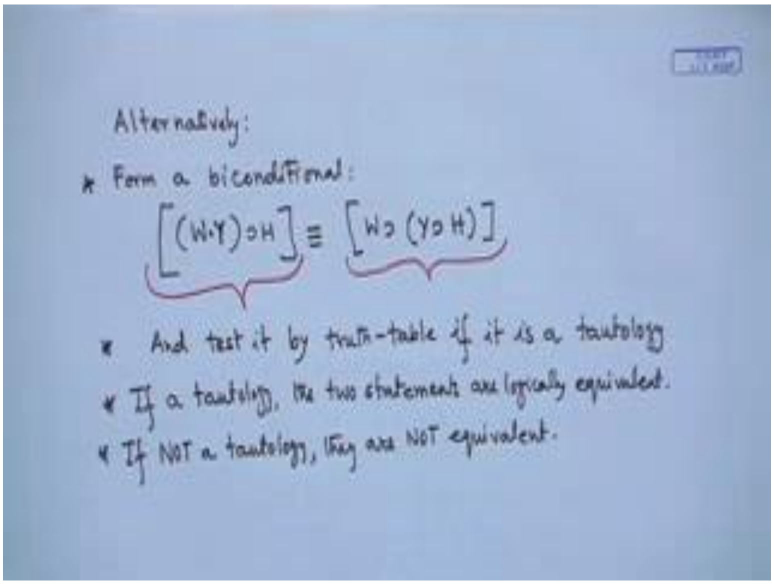
Let us consider these two statements. One of them is $(W \bullet Y) \supset H$. and here is the second one $W \supset (Y \supset H)$. Look at the grouping and look at the separate it out. Are they equivalent? This is what we are asking.

Now, suppose you are following the first method, right? Where you just do the truth table for both of them. In a single truth table you try to fit in, and here we have the advantage also that their components are same. So we can do the truth table like this. We have $(W \bullet Y) \supset H$ is one of the statements, and $W \supset (Y \supset H)$ is another statement. So, the heading of your truth table is going to look like this. See, these are your reference column. Please note they are alphabetically arranged; not by the order of their appearance, but alphabetically. So, H, W, Y and then we are doing the regular truth tables. So we have done, these sub-connectives have shown up. But finally, this is your one statement p, this is your second statement, that is your q. You can skip these two if you want to, and you can go directly here also to see this. But you need to compute this in separate form

So, I have included them as columns inside here. How many rows? Answer is 8 rows. Distribution of truth values, you know that already. So here is how this is going to be distributed and then we just compute the value of $W \bullet Y$, and here $Y \supset H$ and then we come here and we come here, right? If you have done it correctly, then this is the result. This is the result you should have. Now what are we looking at? We are specifically interested in this column, and this one. What we are looking for? What we are looking for is under the same truth value assignments whether there is even one case, when the truth value under this and this match, or it doesn't match. If you are looking for that one possibility then you are looking for a difference in truth values.

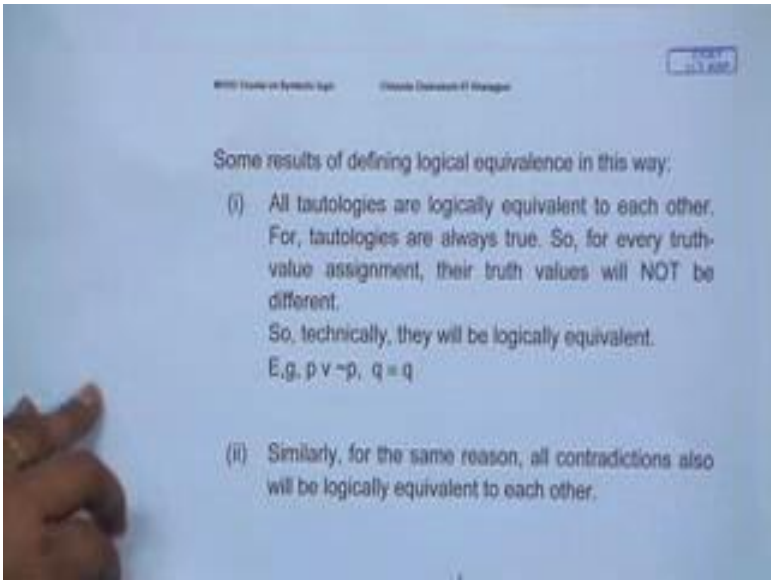
So we check every row, and we find that there is not even one possibility, not even one row, in which the truth values of this two propositions differ. Right? Because it's an exact match, as you can see. Therefore your result is that the given two propositions are logically equivalent as is shown by the truth table. Get it? So this is how you show, demonstrate whether two propositions are logically equivalent or not.

(Refer Slide Time: 12:49)



Alternatively, you can do this. You can approach this problem like this that you take these two propositions and form a biconditional, which is going to look like this. This is your proposition p, this is your proposition q. Both are complex compounds. That doesn't matter. But this is your p, this is your q ,and this is the whole biconditional and you do the truth table on this one. Ok? When you do the truth table, what is it that you are looking for? You are looking whether it is a tautology or not. If it turns out to be a tautology, if this triple bar (≡) turns out to be tautology, the two statements must be logically equivalent. If this triple bar (≡) is not a tautology then these two are not equivalent. Get me?

(Refer Slide Time: 13:43)



So this is how we do the logical equivalence test.

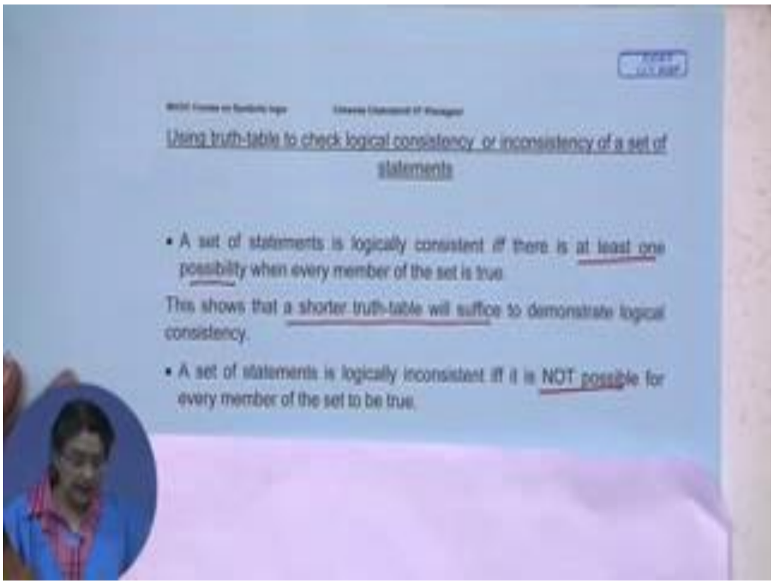
There is a strange result of defining logical equivalence in this way and I must share you. You have to sort of admit or accept the fact that, because the way we have defined logical equivalence in terms of truth table, all tautologies are going to be logically equivalent to each other. Because it doesn't matter what truth-values. Tautologies are always true. So there is not going to even a single possibility, when they are going to show a different truth values. Take any two tautologies of your choice, any two tautologies which work as your p and q, and there will not be not even be a single situation when they are going to result into different truth values because tautologies are always true.

So, technically speaking they are going to turn out as logically equivalent. For example, take $p \vee \sim p$ and take $q \equiv q$, and you will say: But they don't mean the same. That is precisely the point, that technically speaking they will turn out to be logically equivalent; but not if you are looking into *meaning* of this. That's why I called it rather strange result.

Same reason you will see that all contradictions are also going to be logically equivalent. For the same reason, because contradictions are always false. So take any two contradictions, under every single truth value assignment what will happen? The values will never ever differ for those two contradictions. So technically speaking, you will have to say all contradictions are logically equivalent to each other.

Now why it will I share this, just to show that sometimes there are unwanted cases also which passes through the definition, but the formal definitions sometimes bring this kind

(Refer Slide Time: 15:53)



of strange results, but this was one thing is to notice also.

And then we come to…so logical equivalence we have seen and we have learnt how to do. Now the only thing that is remaining is whether the truth table can guide us in finding out whether a set of statements is logically consistent or inconsistent. So I will remind you again when you are judging, determining logical equivalence, you have only two propositions or propositional forms.

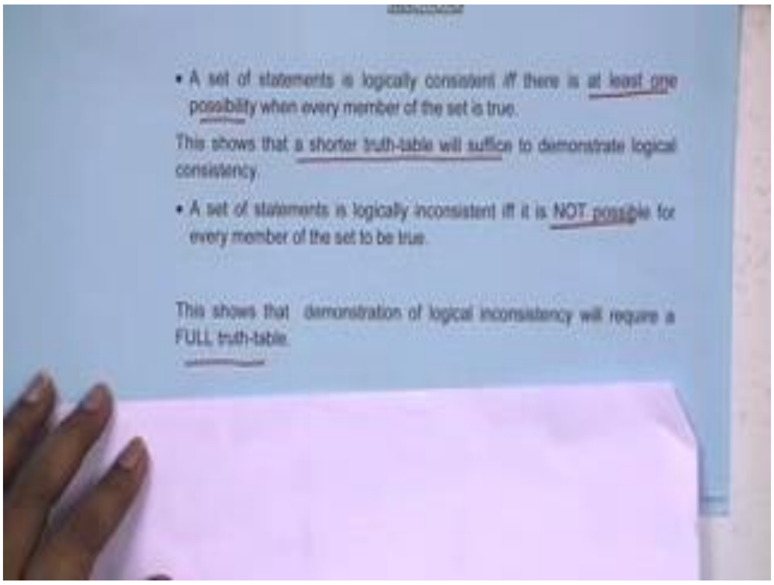
Consistency or inconsistency, on the other hand, is the property of a set of statements. Now a set of statements may have only two statements, right? Or, it can have more than two. But, the idea to understand that the property of consistency / inconsistency belongs to a set of statements, and not to individual statements. And it does not belong to an argument. It is a property of the set of statements. I said this earlier and I am reminding you again, once more, that you treat logical consistency / inconsistency in a slightly different manner.

What is logically consistent? I will remind ourselves that a set of statements is known to be logically consistent, when there is at least one possibility when every member of the set is true, at the same time; simultaneously, right? Or, in a sense that it is compatible. They don’t cancel the truth of each other.

How many such possibilities you need? You need at least one such possibility. That should immediately tell you what kind of truth table you need to construct. So for this task, even a shorter truth table will do, because all we need is to show one such possibility.

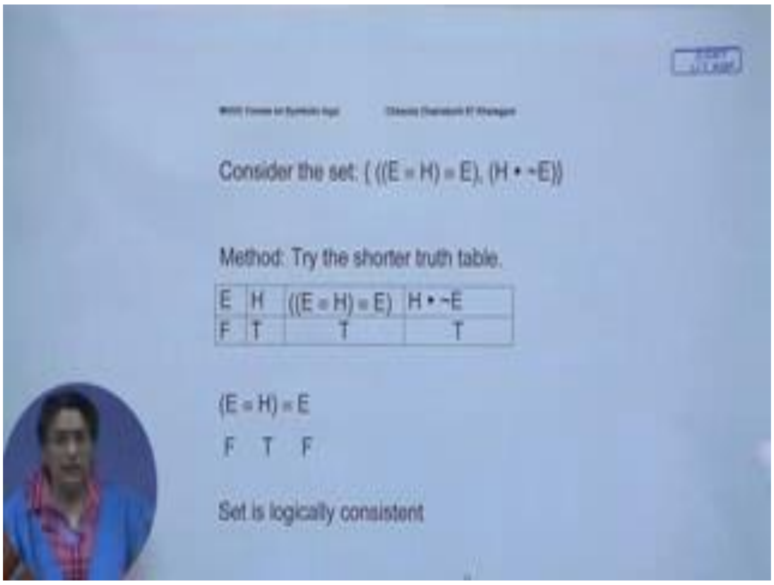
What is inconsistency? Inconsistency, on the other hand, where you need to show that it’s not possible for every member of the set to be true. It is *impossible*, in fact, for every member of that set to come out as true at the same time. That is when we call the set inconsistent. This impossibility demonstration, meaning elimination of even a single possibility, which means that you have to really go through a careful examination of every single possibility to rule out the fact that there is no such possibility, right? The moment you say that, you know by now that you are going to require a full truth table demonstration to show logical inconsistency, alright?

(Refer Slide Time: 18:23)



So, keep that in mind for consistency we need shorter truth table; for inconsistency however we are going to need a full truth table demonstration.

(Refer Slide Time: 18:40)



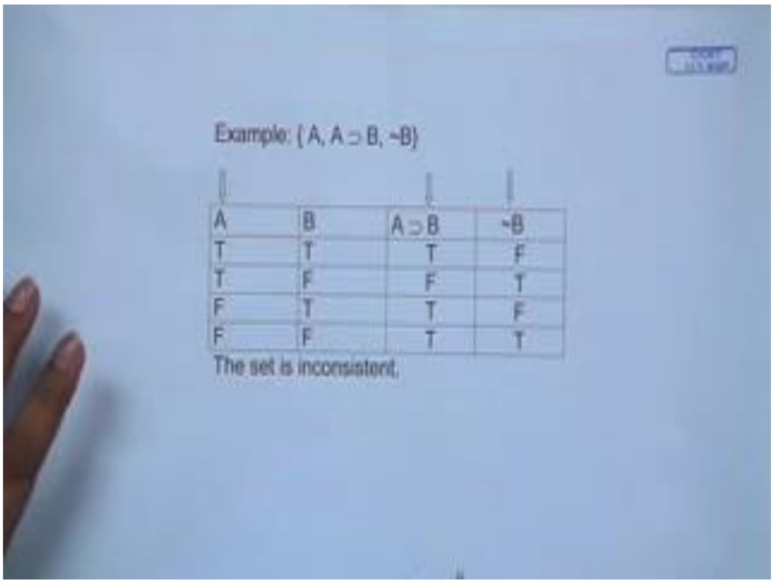
Let us go now to actual examples. Here is a set, which has this as a member, which has this as a member, right? So here we are, what do we do? Well, it is up to you. You can do the full truth table. This is going to be a four row one. Or, you can first try the shorter truth table to see whether it comes out to be consistent or not. If you are doing the shorter truth table, then here is the column heads for the shorter truth table. You are going to have the reference column; you are going to have one each for the members. So here is first member, here is second. And by now if you know how to do the shorter truth table, you can do it very quickly and easily. For example, you know that the dot (•) is going to be true , you are trying to make. What is your goal? To see assignment of truth values here in such a way such that each one of them comes out to be true. If you are successful in doing that, you have shown the set to be consistent. You have shown that one possibility when every member of that set comes out to be true. That’s all.

So, here you are. H • ~E has to be true, and conjunction requires there is exactly one condition under which a conjunction or a dot is true. Which is what? When H is true and not-E is true. When not-E is true, what is the value of E? E has to be false. So, you know that wherever E occurs, you need to repeat that. You also know what value of H is by now. It has to be true. So here is your clear answer: That E has to be false, H has to be true, and the overall result of this is going to be true, overall result of this is going to be true. So, you have shown that one row where the every member of that set is true.

Those of you who are confused about this, how this turned out to be true, let me just show you. This is triple bar (≡), remember? Triple bar is true only when every member has… every component shows the same value. So here you have E is F, E is F, right? Here you have E as F, H as true. The value here is going to be what? A triple bar is going to be false. When this is false, and E is also false, what happens to this triple bar? This will be true. Alright? This is what you have shown here, and write the results the set is logically consistent, right?

Let's take another example so that we are our understanding is more or less clear and this is where we stop the discussion also.

(Refer Slide Time: 21:34)



So, here is the example, and it has three members, right? Now you have the choice you can go with the four row truth table to see whether it is consistent or inconsistent, or you can do that one shorter truth table method to see whether it comes out to be consistent even in that one possibility.

So here you have A, B and…. sorry A , ~B and you have the A ⊃ B . Your truth table column heads are going to be like this. These two are the reference columns. But this is also a member of the set, and you have the ~B. That is a member of the set also, and this is your A ⊃ B, right? And what is it that you are looking for? Whether you can make each of this members true at least once. Ok? Whether there is any truth-value assignment under which each of these members come out be true. That's what you are trying to find out. If you do this, If you do this full truth table, take a look in the way it progresses. The value distributions are like this, and this is automatic and you also know that A ⊃ B has to follow that truth table.

Now, you go by true every single row. Do you have a situation where A is true, A ⊃ B is true and ~B is true? Take a look. So T, T, but this one is F, this is T, this is T, but this one is F, this is already F. So you do not have a situation where you have all the members to be true. Again you have A as false in the last row; you do not have a situation where all the members are true at the same time. You understand? So there is not even one row in which all the members have come out to be true simultaneously. What does that make the set? The set becomes inconsistent. Ok? So, this is where we have learnt that the truth table technique helps us to determine this kind of properties and this concludes our discussion on the truth tables and specifically on this kind of properties.

Next module onwards we are going to learn some other technique. We are still in the semantics, but the truth table discussion ends here. I suggest that you try this on your own with problems so that your skill is developed for truth tables: how to apply the truth tables, how to figure out these properties, logical properties through truth table technique. Thank you. That's all. Thank you very much.

Thank you very much.