Ends

Indian Institute of Technology Kharagpur QUESTION-CUM-ANSWERSCRIPT	Stamp / Signature of the Invigilator

MID-SEMESTER EXAMINATION	SEMESTER (SPRING-2018)

Roll Number											Section		Name	
Subject Number	M	A	2	0	1	0	4		Subject Name	Probability and Statistics				
Department / Centre / School														

Important Instructions and Guidelines for Students

1. You must occupy your seat as per the Examination Schedule/Sitting Plan.
2. Do not keep mobile phone or any similar electronic gadgets with you even in switched off mode.
3. Loose papers, class notes, books or any such materials must not be in your possession; even if they are irrelevant to the subject you are taking examination.
4. Data book, codes, graph papers, relevant standard tables/charts or any other materials are allowed only when instructed by the paper-setter.
5. Use of instrument box, pencil box and non-programmable calculator is allowed during the examination. However, the exchange of these items or any other papers (including question papers) is not permitted.
6. Write on both sides of the answer-sceipt and do not tear off any page. Use last page(s) of the answer-script for rough work. Report to the invigilator if the answer-script has torn or distorted page(s).
7. It is your responsibility to ensure that you have signed the Attendance Sheet. Keep your Admit Card/Identity Card on the desk for checking by the invigilator
8. You may leave the Examination Hall for wash room or for drinking water for a very short period. Record your absence from the Examination Hall in the register provided. Smoking and consumption of any kind of beverages is strictly prohibited inside the Examination Hall.
9. Do not leave the Examination Hall without subitting your answer-script to the invigilator. **In any case, you are not allowed to take away the answer-script with you**. After the completion of the examination, do not leave your seat until invigilators collect the answer scripts.
10. During the examination, either inside or outside the Examination Hall, gathering information from any kind of sources or exchanging information with others or any such attempt will be treated as **'unfair means'**. Don't adopt unfair means and don't indulge in unseemly behaviour.
11. Please see overleaf for more instructions.

Violation of any of the above instructions may lead to severe punishment.

To be filled by the examiner

Question	1	2	3	4	5	6	7	8			**Total**
Marks Obtained											

Marks obtained (in words)	Signature of Examiner	Signature of Scrutinizer

Special Instructions

1. There are total **8** questions in this paper.
2. There are total **20** number of pages.
3. Final answer to each question or sub-question must be clearly written in the box provided.
4. Total marks: 30

Rough work

1. Three distinct numbers are selected from $\{1, 2, 3, \ldots, 20, 21\}$ randomly without replacement. What is the probability that the sum of these numbers is divisible by 3? [5 marks]

Answer:

2. A batch of 580 students taking the course Probability and Statistics is divided into three sections A,B,C. Section A has 180 students while Sections B and C have 200 students each. After the examination, it is observed that 11 students from Section A scored EX grade while 7 students failed the course. In Section B, 9 students scored EX while 12 students failed the course. In Section C, 13 students scored EX while 12 students failed the course.

   (a) If a randomly selected student from the batch has scored EX grade, what is the probability that the student is from Section A? [1 mark]

   Answer:

   (b) If a randomly selected student from the batch has failed the course, what is the probability that the student belongs to either Section B or Section C? [2 marks]

   Answer:

   (c) If two students are selected randomly from the batch and it is noted that one of them has scored EX while the other has failed course. What is the probability that both the students belong to the same section? [2 marks]

   Answer:

3. Suppose n balls are distributed at random into r boxes. Find the probability that there are exactly k balls in the first $r_1 (< r)$ boxes. [2 marks]

Answer:

4. Tattoo bubble gums are on sale for Rs. 5 each. Each bubble gum contains exactly one tattoo, which can be one of five types with equal probability. Suppose you keep on buying bubble gums and stop when you collect all the five types of tattoos. What will be your expected expenditure? [5 marks]

Answer:

5. Let X be a discrete random variable with the properties $E(X) = 0$, $E(X^2) = 2$ & $E(X^4) = 4$.

(a) Find the moment generating function of X. [3 marks]

Answer:

(b) Compute $E(X+1)^3$. [2 marks]

Answer:

6. Let $\begin{pmatrix} X \\ Y \end{pmatrix}$ be a discrete random vector with the joint probability mass function given as follows:

$$f_{X,Y}(x,y) = \begin{cases} \frac{(1-p)^x p}{s} & \text{for } x = 0, 1, \ldots; \quad y = 1, 2, \ldots, s \\ 0 & \text{otherwise} \end{cases}$$

for some $0 < p < 1$.

(a) Compute $E(X)$. [1 mark]

Answer:

(b) Compute $Var(Y|X)$. [1 mark]

Answer:

(c) Compute $Var(X)$. [1 mark]

Answer:

7. Let X be a discrete random variable which follows geometric distribution with parameter $p = \frac{1}{3}$. Define a new random variable

$$Y = \frac{2^X}{X!}$$

(a) Compute $P(Y \leqslant 2)$. [1 mark]

Answer:

(b) Compute $E(Y)$. [2 marks]

Answer: