Answers-Hints Tutorial Sheet - 8 SPRING 2017

MATHEMATICS-II (MA10002) *January 13, 2017*

1.
   (a) Use Gamma function
   (b) Use Gamma function
   (c) put, $x = \sin\theta$, then use Beta function
   (d) use Beta function
   (e) put, $x^4 = u$, then use Beta function
   (f) use Beta function
   (g) integration by parts
   (h) put, $x = a\cos^2\theta + b\sin^2\theta$
   (i) put $x = \sin\theta$ use Beta function
   (j) put, $x^r = z$ use Beta function
   (k) put, $\ln\frac{1}{x} = z$,Use Gamma function
   (l) put, $x^n = z$ use Beta function

2. Given $\beta(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt,\ x>0,\ y>0,$
   (a) put $t = \sin^2\theta$
   (b) put $t = \frac{u}{u+1}$.
   (c) put $t = m^2$ in $\Gamma(x)\Gamma(y)$ and then $x = r\cos\theta,\ y = r\sin\theta$
   (d) $\beta(\frac{1}{2},\frac{1}{2}) = \pi$

3.
   (a) put, $ax^n = z$
   (b) put, $\log\frac{1}{x} = t$.
   (c) $n^{-x} = e^{-x\log n}$, put $x\log n = z$

4. expand $\Gamma(m+\frac{1}{2})$

5. $\int_0^1 \frac{x^n + x^{-n}}{1+x^2}dx = \int_0^\infty \frac{x^n}{1+x^2}dx$, put $x^2 = u$.

6. put $x^n = a\tan^2\theta$

7.
   (a) take 2 common
   (b) multiply 2. 4. 6, ..., $(2m-2)$ and divide

8. $\log(1+\alpha)$, (Hint: Define a function $F(\alpha)$ differentiate w.r.to 'α' and then integrate $F'(\alpha)$ w.r.to 'α')

9. (i) Define a function $F(b)$ differentiate w.r.to 'b' and then integrate $F'(b)$ w.r.to 'b'

   (ii) Define a function $F(\alpha, \beta)$ differentiate partially w.r.t 'α' and 'β' and then integrate $\frac{\partial^2 F(\alpha,\beta)}{\partial\beta\partial\alpha}$ w.r.t. β and α

   (iii) Define a function $F(\alpha, \beta)$ differentiate partially w.r.t 'α' and then integrate $\frac{\partial F(\alpha,\beta)}{\partial\alpha}$ w.r.t. α

10. (i) $t^6 + 2t^3 + 4/3$

11. (i) $F'(x) = \begin{cases} \frac{\sin\frac{\pi}{2}x}{x}, & x \neq 0 \\ \pi/2, & x = 0 \end{cases}$, (ii) $f'(x) = x(\pi/2 - \log 2)$

12. No. Since $\frac{\partial f(x,t)}{\partial t}$ is not continuous function.

13. $\tan^{-1}(a/b)$ and $\pi/2$

14. (i) $\frac{1}{2}\log(1 + \frac{a^2}{b^2})$ (ii) $\frac{1}{2}\log\left(\frac{a^2+b^2}{p^2+q^2}\right)$ (iii) $\frac{\sqrt{\pi}}{2}e^{-a^2}$