# The Transportation Model –

Formulations

## The Transportation Model

The transportation model is a special class of LPPs that deals with transporting(=shipping) a commodity from sources (e.g. factories) to destinations (e.g. warehouses). The objective is to determine the shipping schedule that minimizes the total shipping cost while satisfying supply and demand limits. We assume that the shipping cost is proportional to the number of units shipped on a given route.

We assume that there are m sources 1,2, ..., m and n destinations 1, 2, ..., n. The cost of shipping one unit from Source i to Destination j is  $c_{ij}$ .

We assume that the availability at source i is  $a_i$  (i=1, 2, ..., m) and the demand at the destination j is  $b_j$  (j=1, 2, ..., n). We make an important assumption: the problem is a **balanced** one. That is

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$$

That is, total availability equals total demand.

We can always meet this condition by introducing a dummy source (if the total demand is more than the total supply) or a dummy destination (if the total supply is more than the total demand).

Let  $x_{ij}$  be the amount of commodity to be shipped from the source i to the destination j.

### Thus the problem becomes the LPP

Minimize

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to

$$\sum_{i=1}^{n} x_{ij} = a_i \quad (i = 1, 2, ..., m)$$

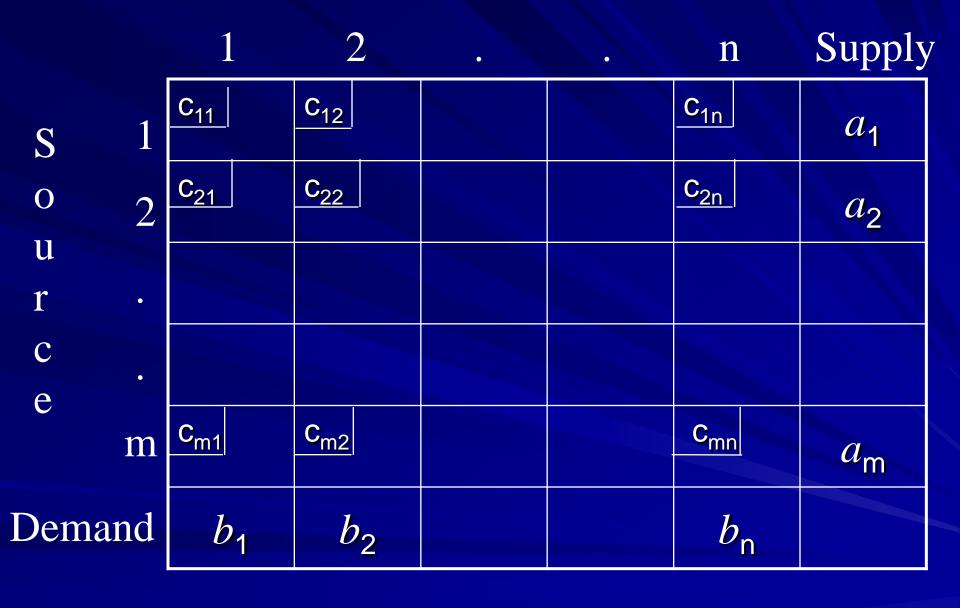
$$\sum_{i=1}^{m} x_{ij} = b_j \quad (j = 1, 2, ..., n)$$

$$x_{ij} \ge 0$$

Thus there are m×n decision variables xij and m+n constraints. Since the sum of the first m constraints equals the sum of the last n constraints (because the problem is a balanced one), one of the constraints is redundant and we can show that the other m+n-1 constraints are LI. Thus any BFS will have only m+n-1 nonzero variables.

Though we can solve the above LPP by Simplex method, we solve it by a special algorithm called the transportation algorithm. We present the data in an m×n tableau as explained below.

## Destination



#### Destination

		Denver	Miami	Supply
S	Los Angeles	80	215	1000
0	Detroit	100	108	1300
u r	New Orleans	102	68	1200
c e	Dummy	0	0	200
	Demand	2300	1400	

We write inside the (i,j) cell the amount to be shipped from source i to destination j. A blank inside a cell indicates no amount was shipped.

#### Destination

		Denver	Miami	Supply
S	Los Angeles	80	M	1000
O	Detroit	100	108	1300
u r	New Orleans	102	68	1200
c	Dummy	200	300	200
e	Demand	2300	1400	

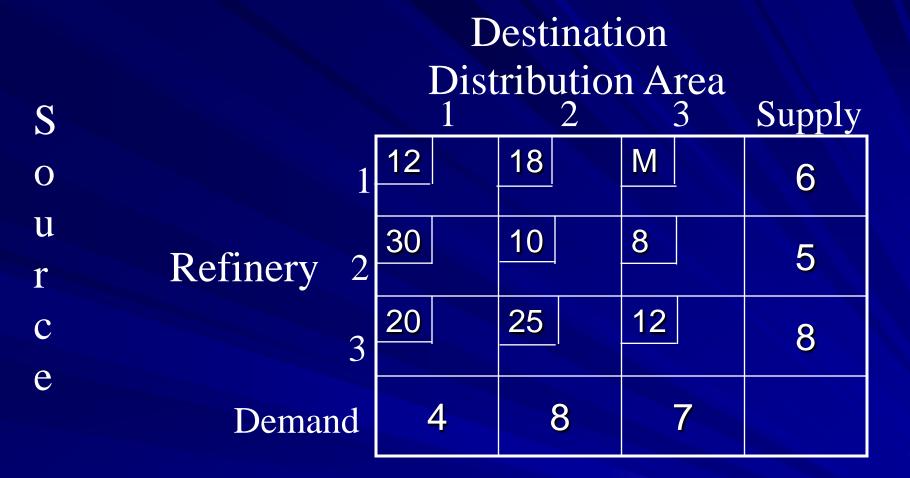
Note: M indicates a very "big" positive number. "infinity".

Distribution Area
1 2 3

1 120 180 Refinery 2 300 100 80
3 200 250 120

Construct the associated transportation model.

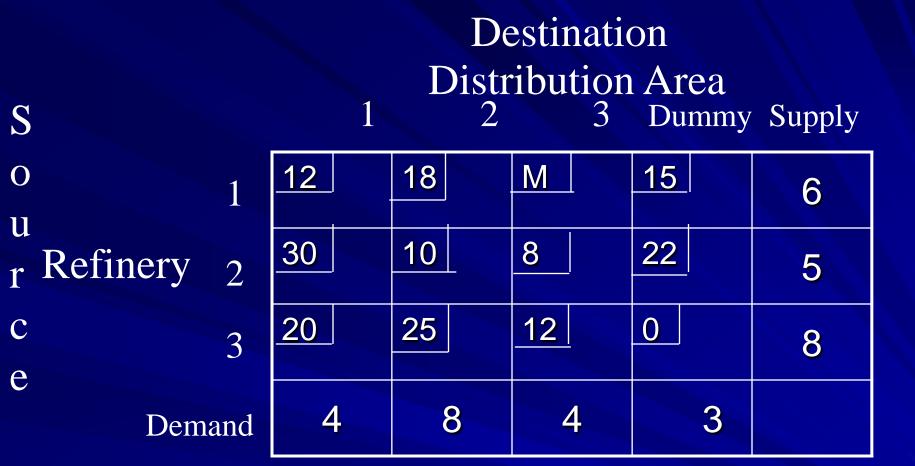
(Solution in the next slide)



The problem is a balanced one. M indicates a very "big" positive number.

The total cost will be 1000\*

$$\sum_{i=1}^{3} \sum_{j=1}^{3} c_{ij} x_{ij}$$



M indicates a very "big" positive number.

The total cost will be 1000\*

$$\sum_{i=1}^{3} \sum_{j=1}^{3} c_{ij} x_{ij}$$

Destination Retailer

		Retailer					
S	O	1	2	3	4	Dummy	Supply
Ο	r 1	1	2	3	2	0	350
u r	h 2	2	4	1	2	0	400
C	a r 3	1	3	5	3	M	250
e	d Demand	150	150	400	100	200	