Problem Set - 11

AUTUMN 2016

ANSWER/HINTS

MATHEMATICS-I (MA10001)

1. (a) The limit exists and equal to 0 (use $\epsilon - \delta$ definition, take $\delta = \epsilon$)
   (b) The limit exists and equal to i (use limit theorem)
   (c) The limit exists and equal to $4i$ (use limit theorem)
   (d) The limit exists and equal to i (take $x = \frac{1}{w}$)
   (e) The limit does not exist (substitute $y = mx$)
   (f) The limit does not exist (substitute $y = mx$)
   (g) The limit does not exist (substitute $y = mx$)
   (h) $\lim_{z \to 1} \dfrac{z^2 + 1}{z^2 - 3z + 2} = \infty$
   (i) The limit does not exist (substitute $y = mx$).
   (j) The limit exist and equal to 1 (take $z = \frac{1}{w}$.
   (k)The limit doesn't exists (evaluate along $x = 0$ and $y = 0$)

2. (a) f is continuous at $z = 0$ (use $\epsilon - \delta$ definition, take $\delta = \epsilon$)
   (b) f is continuous at $z = 0$
   (c) f is continuous at $z = 0$ (use $\epsilon - \delta$ definition, take $\delta = \epsilon$)
   (d) f is continuous at $z = 0$ (use $\epsilon - \delta$ definition, take $\delta = \epsilon$)
   (e) f is not continuous at $z = 0$ (substitute $z = re^{i\theta}$)
   (f) f is continuous at $z = 0$
   (g) f is not continuous at $z = 0$ because the limit does not exist (substitute $z = re^{i\theta}$)
   (h) f is continuous at $z = 0$.

3. (a) Take $\delta = \min\{1, \frac{\epsilon}{28}\}$
   (b) Take $\delta = \min\{1, \frac{\epsilon}{4}\}$
   (c) Take $\delta = \min\{1, \frac{\epsilon}{3}\}$
   (d) (i) For $\epsilon > 0$ $\exists \delta > 0$ s.t for $|z - z_0| < \delta$, $|f(z)| > \frac{1}{\epsilon}$.
   (ii) For $\epsilon > 0$ $\exists \delta > 0$ s.t $|f(z) - w_0| < \epsilon$ whenever $|z| > \frac{1}{\delta}$. Replace z by $\frac{1}{z}$.
   (e) Take $\delta = \min\{\sqrt{5}, \frac{\epsilon}{3\sqrt{5}}\}$
   (f) Take $\delta = \frac{\epsilon}{1+\epsilon}$
   (g) $f''(x) = 2$ for $x \geq 0$ and $f''(x) = -2$ for $x < 0$.

4. Take $\delta > 0$ such that $\delta^2 + 2|z_0|\delta = \epsilon$.

5. $|\overline{f(z)} - \overline{f(a)}| = |\overline{f(z) - f(a)}| = |f(z) - f(a)|$.

6. (a) $f'(a) = 3a^2 - 2$ (b) $f'(-1) = 1$.

7. Use that $\lim_{z \to 0} \dfrac{\bar{z}}{z}$ does not exist.

8. $\frac{\partial F}{\partial x} = \frac{\partial F}{\partial z}\frac{\partial z}{\partial x} + \frac{\partial F}{\partial \bar{z}}\frac{\partial \bar{z}}{\partial x}$. Follow similarly.

9. Take $x = r\cos\theta, y = r\sin\theta$ and use the chain rule.

10. $f'(z) = \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$ and put $y = 0$.

11. (a) Verify that $u_{xx} + u_{yy} = 0$
(b) Use CR-equations find v is $2y - y^2 + x^2 + c$
(c) $f(z) = (2 + iz) + k$.

12. Note that the CR-equations are true only for $z = 0$.

13. (a) $|z + 3| = 2$ (b) the real axis (that is x-axis).

14. f is nowhere differentible. f satisfying the CR-equations at the origin.

15. Apply CR-equations to show all partial derivatives of u and v are zero and therefore the function is constant.

16. Apply CR-equations.

17. Use CR-equations to find v and $f(z) = -z^3 - 2iz^2 + ik$, where k is a real constant.

18. Apply CR-equations.

19. $f(z) = z^3 + 2iz^2 + 6 - 2i$ (Use CR-equations).