Indian Institute of Technology, Kharagpur

Date..................... FN/AN Time: 2 Hrs Full Marks: 30 No. of Students: 100
Mid (Spring) Semester 2013-14 Subject Name: Switching and Finite Automata
Sub. No. MA 30006 Deptt: MA/AG/IM/ME/MF/SI/EC/CE/EE/HS/IE/NA/MI
Instruction: Answer **all** questions.

Question 1 [$6 \times 1 = 6$ marks]

a) Show by giving an example that, if M is an NFA that recognizes language L, swapping the accept and non-accept states in M does not necessarily yield a new NFA that recognizes $\overline{L}$, the complement of L.

b) Is the class of languages recognized by NFAs closed under complement? Explain your answer.

c) Suppose M_1 and M_2 are two DFA's defined over the same input alphabet Σ. Design a DFA D such that $L(D) = L(M_1) \cup L(M_2)$.

d) What does it mean for two regular expressions over an alphabet Σ to be *equivalent*? Describe an algorithm for deciding equivalence of regular expressions. Any standard results you use should be clearly stated, but need not be proved.

e) Prove that $(r^*)^R = (r^R)^*$ for any regular expression r where r^R stands for the reversal of r.

f) Show that if a DFA M accepts any string at all, then it accepts one whose length is less than the number of states in M.

Question 2 [$2 + 3 = 5$ marks]

a) Given any ϵ-NFA M, describe how to construct a regular expression r whose language of matching strings $L(r)$ is equal to the language $L(M)$ accepted by M.

b) Find a regular expression r with $L(r) = L(M)$ when M is the following ϵ-NFA.

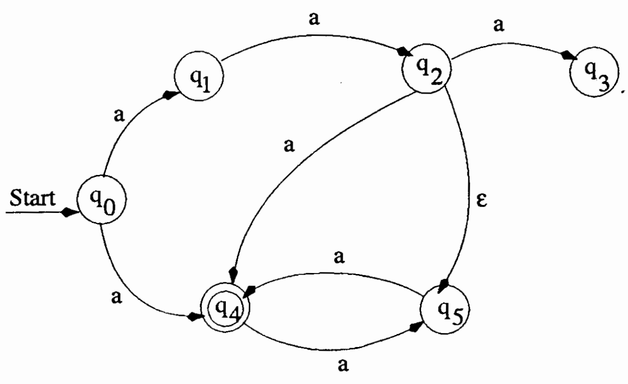

Figure 1: ϵ-NFA

——-P.T.O.———

Question 3 [$2+3+2+2=9$ marks]

a) Design a DFA which accepts the language $L = \{w \in \{0,1\}^* |$ the number of 1's is even and the number of 0's is a multiple of 3 and w contains at least one 0 and 1$\}$.

b) Construct a DFA with reduced states equivalent to the regular expression $\mathbf{10 + (0 + 11)0^*1}$.

c) Define 2-DFA and the language accepted by it.

d) Define equivalence of Moore and Mealy machines. Given a Mealy machine M, describe how to construct a Moore machine M' equivalent to M.

Question 4 [$2+4+2+2=10$ marks]

a) State the Pumping Lemma and explain how it is used to prove that languages are not regular.

b) Are the following languages regular? Justify your answer in each case.

(i) $L = \{x^p x^q \in \{x,y\}^* | p, q$ are integers with $p > q\}$ and

(ii) $L = \{x^n y^l x^k \in \{x,y\}^* | n, l, k$ are integers with $k \geq n + l\}$.

c) Construct a grammar accepting $L = \{w \in \{a,b\}^* |$ the number of a's in w is divisible by 3$\}$.

d) Show that the language generated by the grammar $G = (V, T, P, S)$, where $V = \{S\}$, $T = \{a, b\}$ and $P = \{S \to aSbS, S \to, S \to bSaS, S \to \epsilon\}$ is the set of all strings with an equal number of a's and b's.

——-The End———