INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR
Mid-Spring Semester 2017-18

Date of Exam: .02.18 Session(FN/AN) Duration: 2 hrs
Subject Number MA41017/MA60067 Max. Marks: 30 Department: Mathematics
Subject Name: Stochastic Processes/ Stochastic Process and Simulation No. of students: 72

Instructions:

(i) Use of calculator and Statistical tables is allowed.
(ii) All answers of numerical questions must be in **at least two decimal places**.
(iii) All the notations are standard and no query or doubts will be entertained. If any data/statement is missing, identify it on your answer script.
(iv) Answer **All** questions.
(v) All parts of a question **Must Be** answered at **One Place**.

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1. (a) Events occur according to a Poisson process with rate $\lambda = 2$ per hour.

   i. What is the probability that two or more events occur between 7 pm and 9 pm?

   ii. Starting at noon, what is the expected time at which the fourth event occurs?

   (b) Consider a taxi station where taxis and customers arrive in accordance with Poisson processes with respective rates of one and two per minute. A taxi will wait no matter how many other taxis are present. However, an arriving customer that does not find a taxi waiting leaves. Find

   i. the average number of taxis waiting.

   ii. the proportion of arriving customers that get taxis.

   (c) A communication link transmits binary characters, $\{0, 1\}$. There is a probability p that a transmitted character will be received correctly by a receiver, which then transmits to another link, etc. If X_0 is the initial character and X_1 is the character received after the first transmission, X_2 after the second transmission, etc., If we assume the independence of each transmission, then find one-step TPM of the Markov chain $\{X_n, n = 0, 1, 2, \ldots\}$ and its stationary distribution (π_0, π_1).

   [2+3+1]

2. (a) Consider a two-state Markov chain $\{X_n\}$, having state space $E = \{0, 1\}$, with TPM

$$\begin{bmatrix} 1/2 & 1/2 \\ 1/3 & 2/3 \end{bmatrix}.$$

   Then whether $Z_n = (X_{n-1}, X_n)$ is a Markov Chain. If so then determine state space and TPM of $\{Z_n\}$.

   (b) A Markov chain $\{X_n\}$, having state space $E = \{0, 1, 2\}$, with TPM

$$\begin{bmatrix} 0.1 & 0.2 & 0.7 \\ 0.2 & 0.2 & 0.6 \\ 0.6 & 0.1 & 0.3 \end{bmatrix}.$$

   Let the initial state probability distribution is $\mathbf{p} = (1/3, 1/3, 1/3)$.

   i. Find the stationary state probability distribution.

   ii. Whether states are transient or recurrent. Why?

   iii. Find $P(X_3 = 1|X_1 = 0)$, $P(X_0 = 1, X_1 = 0, X_3 = 1)$.

   [3+3]

3. (a) The probability of the thrower winning in the dice game called "craps" is $p = 0.49$. Suppose Player A is the thrower and begins the game with \$ 5, and Player B, his opponent, begins with \$ 10. What is the probability that Player A goes bankrupt before Player B? Assume that the bet is \$ 1 per round.

   (b) A small flightless insect moves around the vertices of a triangle in the following manner: Whenever it is at vertex i it moves to its clockwise neighbor vertex with probability p_i and to the counterclockwise neighbor with probability $q_i = 1 - p_i$, $i = 1, 2, 3$. Let $p_1 = 1/2$, $p_2 = 1/3$, $p_3 = 1/4$.

      i. Find the proportion of time that the insect is at each of the vertices.

      ii. How often does the insect make a counterclockwise move that is then followed by five consecutive clockwise moves?

[2+4]

4. (a) The first generation of particles is the collection of offsprings of a given particle. The next generation is formed by the offsprings of these members. If the probability that a particle has k offsprings is p_k, where $p_0 = 1/7$; $p_1 = 3/7$ and $p_2 = 3/7$. Assume that particles act independently and identically irrespectively of the generation. Then find the probability of extinction.

   (b) Consider a post office with two clerks. Three people, A, B, and C, enter simultaneously. A and B go directly to the clerks, and C waits until either A or B leaves before he begins service. What is the probability that A is still in the post office after the other two have left when the service times are exponential with mean 1/2?

[3+3]

5. (a) Suppose that the customers flow to a shop is described by a Poisson process with rate 10 per hour. We know that each of the customers is a female with probability 0.4. What is the probability that

      i. no male customer will enter the shop between 10:00 am to 10:30 am?

      ii. time for 4 male customers that enters the shop exceeds 120 minutes?

   (b) Ships pass a bird sanctuary according to a Poisson Process with rate of one per hour. 40% of the ships are oil tankers.

      i. What is the probability that at least one oil tanker will pass the bird sanctuary during a (24 hrs) day?

      ii. If 50 ships have passed by in a day, what is the probability that 5 of them were oil tankers?

   (c) Customers arrive at an automatic teller machine (ATM) in accordance with a Poisson process with rate 12 per hour. The amount of money withdrawn on each transaction is a random variable with mean Rs. 30 and standard deviation Rs. 50. (A negative withdrawal means that money was deposited). Suppose that the machine is in use 15 hours per day. Find the approximate probability that the total daily withdraw is less than Rs. 6000.

[2+2+2]

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