ASSIGNMENT - 4

Numerical Solutions of Ordinary and Partial Differential Equations

1. Solve the differential equation $y' = x^2 + y^2 - 2$ using the Modified Euler predictor-corrector method (Euler as predictor and Euler-Cauchy as corrector) for $x = 0.1$ given the initial value $x = 0, y = 1$.

2. Using the Adams-Bashforth-Moulton predictor-corrector formulas, evaluate $y(1.4)$, if y satisfies

$$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2} \text{ and } y(1) = 1, y(1.1) = 0.996, y(1.2) = 0.986, y(1.3) = 0.972.$$

3. Consider the linear multistep method

$$u_{n+2} + 28u_{n+1} - 28u_{n-1} - u_{n-2} = h(12f_{n+1} + 36f_n + 12f_{n-1})$$

used to solve the test equation $y' = \lambda y, y(t_0) = y_0, t \in [t_0, b]$;
(i) Is this method convergent ?
(ii) Obtain all the growth factors

4. Given $\rho(\xi) = \xi^2 - 1$, find the corresponding implicit linear multistep method by finding $\sigma(\xi)$. Discuss the numerical stability of this method.

5. Given the initial value problem, $u' = t + u, u(0) = 1$; if $u(0.1) = \alpha$ and $u(0.2) = \beta; u(0.3) = \gamma$; find α using the 3^{rd} order Taylor series method , β and γ using the implicit linear multistep method derived above.

6. Find the growth parameters corresponding to the roots of the reduced characteristic equation $\rho(\xi) = 0$ for the linear multistep method

$$u_{n+3} = u_n + \frac{3h}{8}[u'_n + 3u'_{n+1} + 3u'_{n+2} + u'_{n+3}].$$

7. Find the interval of absolute stability for the linear multistep method

$$u_{n+1} = u_n + \frac{h}{12}[5u'_{n+1} + 8u'_n - u'_{n-1}]$$

used to solve the IVP $y' = -y, y(x_0) = y_0$.

8. Find the general solution of the difference equations

$$(i)\Delta^2 u_n - 3\Delta u_n + 2u_n = 0$$

$$(ii)\Delta^2 u_n + \Delta u_n + \frac{1}{4}u_n = 0$$

$$(iii)\Delta^2 u_n - 2\Delta u_n + 2u_n = 0$$

$$(iv)\Delta^2 u_{n+1} - \frac{1}{3}\Delta^2 u_n = 0$$

9. Find the range for α so that the roots of the characteristic equation of the difference equations

$$(i)(1-5\alpha)y_{n+2} - (1+8\alpha)y_{n+1} + \alpha y_n = 0$$

$$(ii)(1-9\alpha)y_{n+3} - (1+19\alpha)y_{n+2} + 5\alpha y_{n-1} - \alpha y_n = 0$$

are less than one in magnitude.

10. Show that all solutions of the difference equation

$$y_{j+1} - 2\lambda y_j + y_{j-1} = 0$$

are bounded when $j \to \infty$ if $-1 < \lambda < 1$, while for all other complex values of λ there is atleast one unbounded solution.

11. Show that the two step method

$$u_{j+1} = \frac{4}{3}u_j - \frac{1}{3}u_{j-1} + \frac{2}{3}hu'_{j+1}$$

when applied to

$$u' = \lambda u, \lambda < 0$$

is A-stable.

12. Find the condition on $'a'$ and $'b'$ for which the linear multistep method

$$u_{j+1} - (1+a)u_j + au_{j-1} = h[\{\frac{1}{2}(1+a) + b\}u'_{j+1} + \{\frac{1}{2}(1-3a) - 2b\}u'_j + bu'_{j-1}]$$

when applied to the test equation

$$u' = \lambda u, \lambda < 0$$

is A-stable.