

2/01/20

Principles of Database Systems : JD Ullman

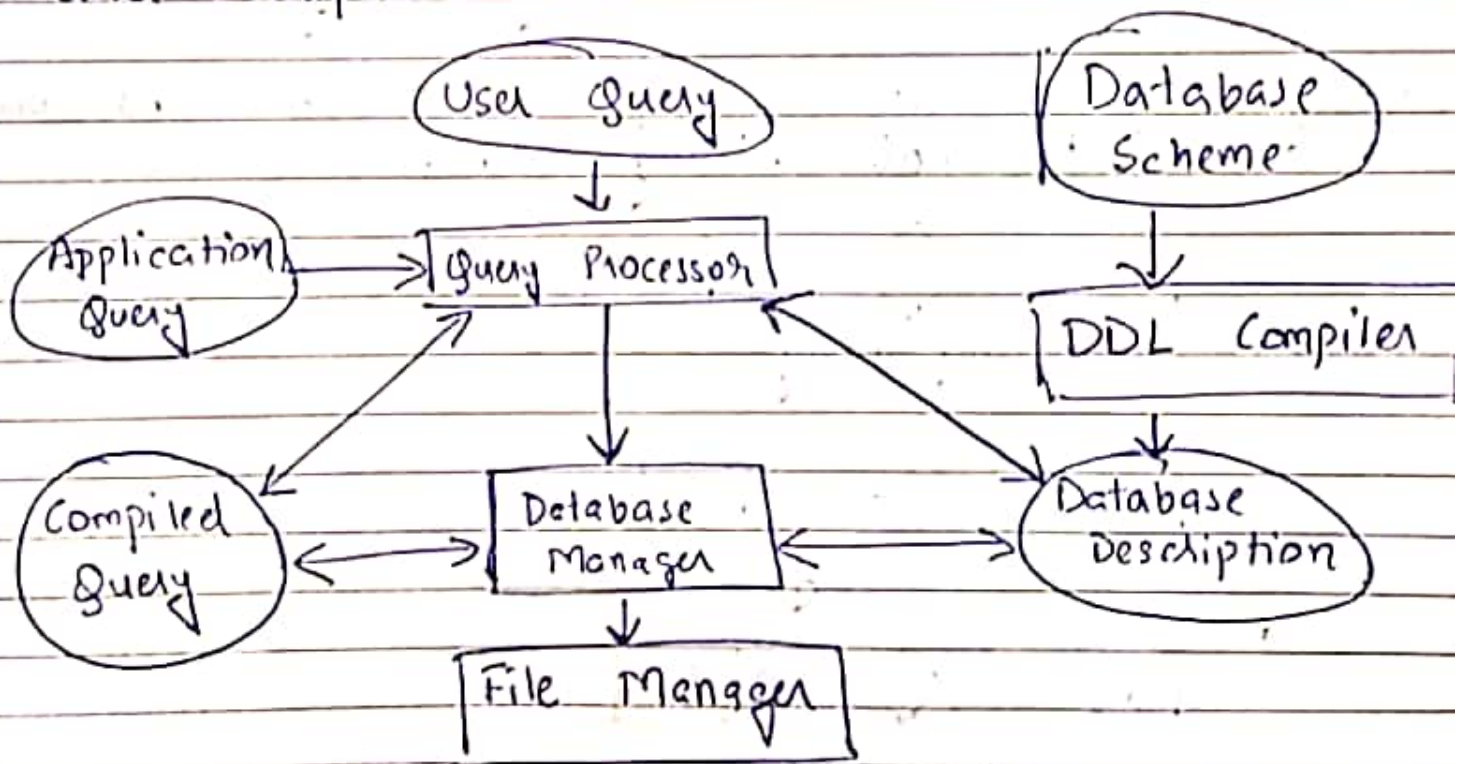
Database System Concepts : H.F. Korth & A Silberschatz

Database Management Systems : Raghu Ramakrishna

Raw Data \rightarrow Field(s) \rightarrow Records

Data File/Table \rightarrow Database

DBMS (Database Management System) : Software allows us to create database, datafiles, enter and manipulate data.



- ① Database Design
- ② Query Processing
- ③ Physical Database

Physical Database

- ① Remove data redundancy & inconsistency
- ② Multiple users (use same DB simultaneously)
- ③ Synchronization
- ④ Security Problems.

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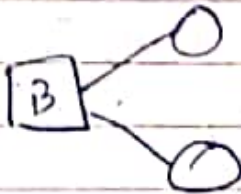
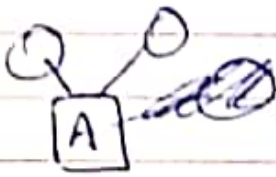
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Entity - Relationship Model

• Entity :- Entity is an object distinguishable from other objects. Eg. → Roll Number

• Entity Set :- { Roll No., Name, Dept., Hall }
Attributes.



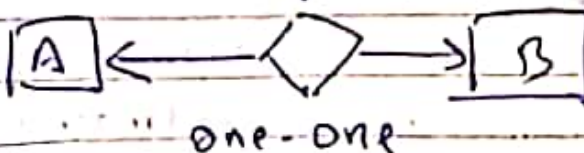
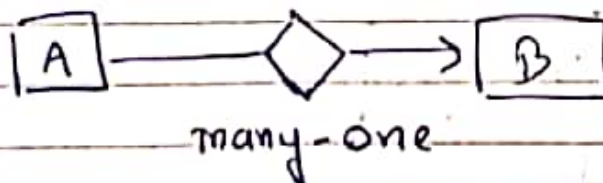
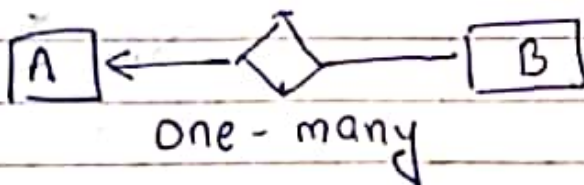
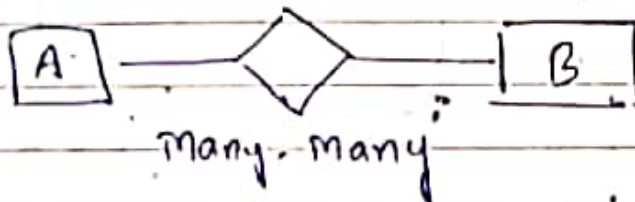
one - one

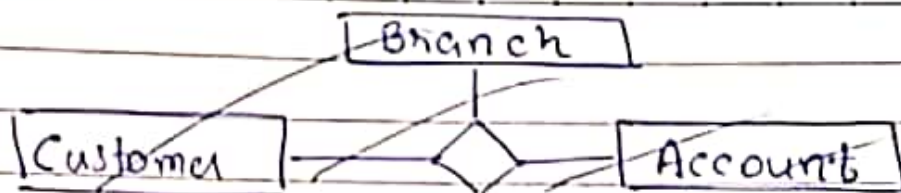
one - many

many - one

many - many

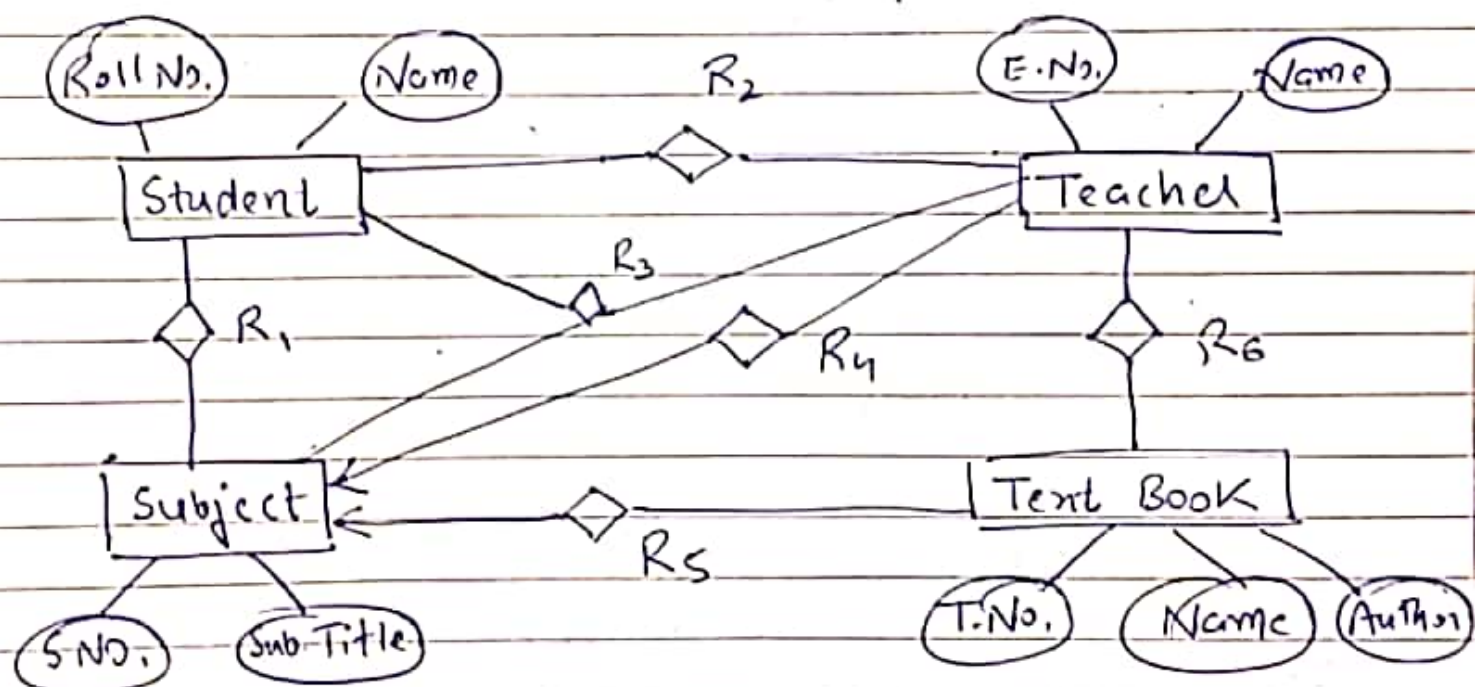
• Key :- It is a set of one or more than one attribute that uniquely identifies a row





Draw an ER Diagram with entities — Text book, subjects, students, teachers.

- 1) For each subject each student is taught by single teacher.
- 2) Each teacher teaches only one subject.
- 3) Each subject is taught by several teachers.
- 4) Teachers choose their resp. text book.



Student	
Roll No.	...

Teacher	
E.No.	—

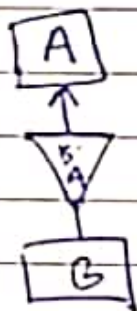
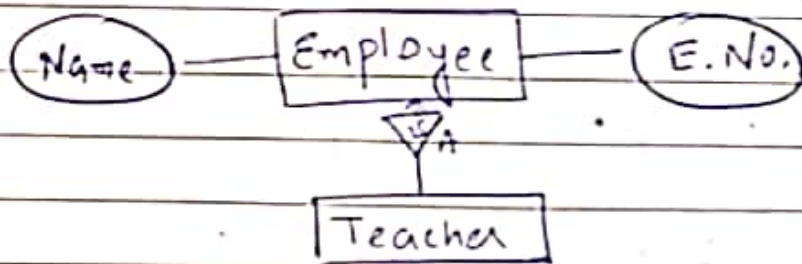
Subject	
S.No.	---

Text Book	
T.No.	—

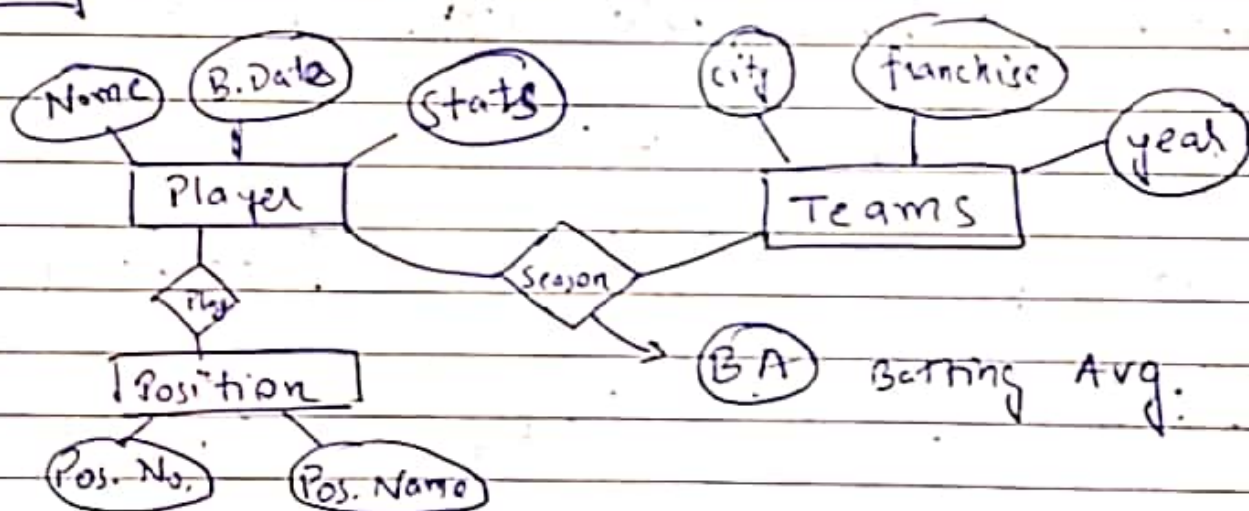
R1	
Roll No.	S.No.

R3		
Roll No.	S.No.	E.No.

Relational Model



\Rightarrow B is a subset of A



Relational Table

Players (Name, B.date, state)

Position (Pos. No., Pos. Name)

Teams (City, Franchise, Year)

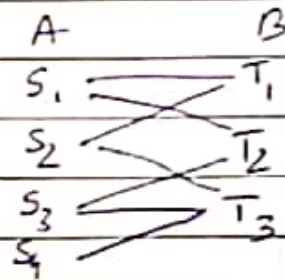
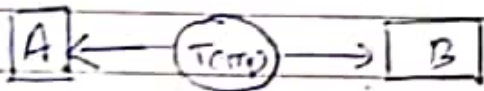
Play (Name, Pos. No.)

Season (Name, city, franchise, B.A.)

Network

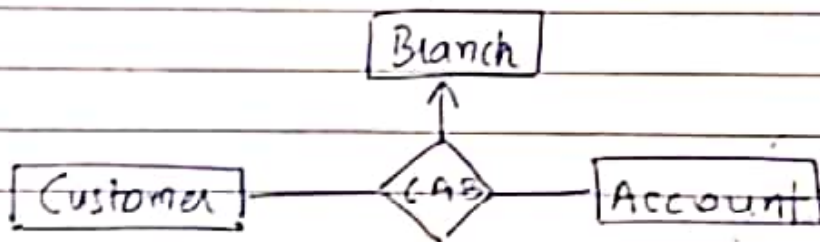
Network Model :- It is ER-Model restricted to binary & many-one relations only.

Drawing Network Model from ER model →



• Many-many relationship is converted to binary many-one relationships.

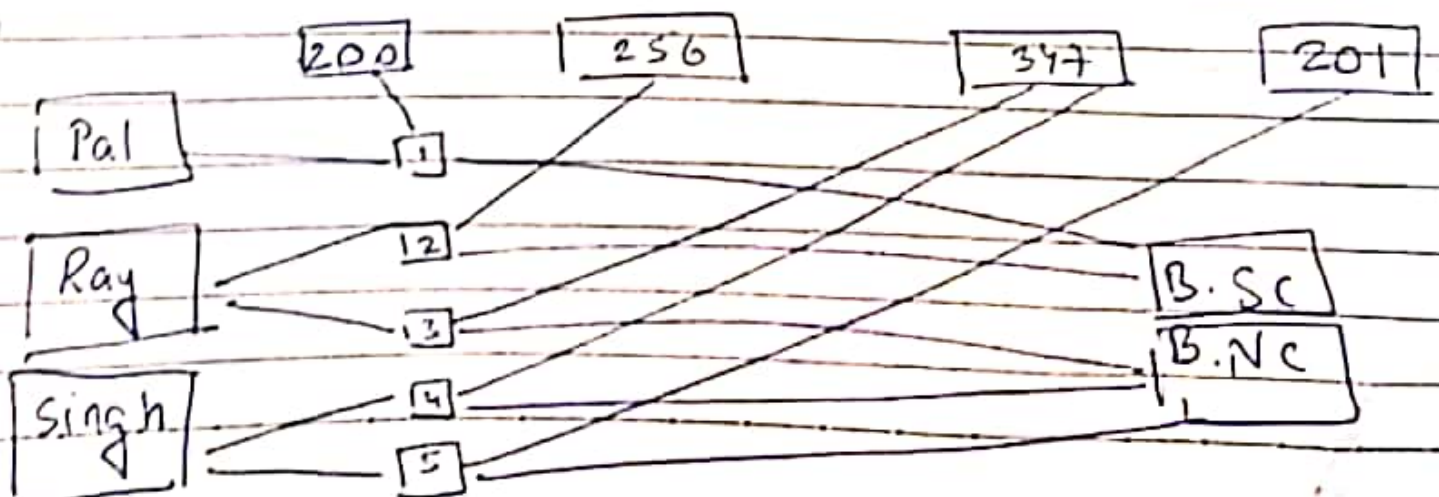
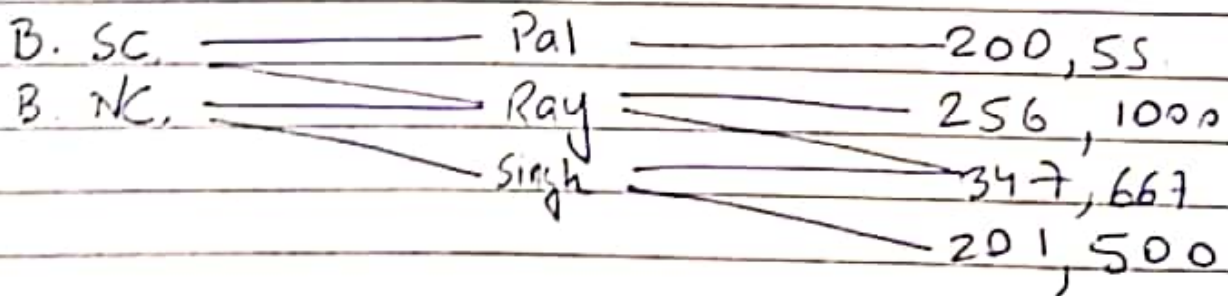
Eg :-



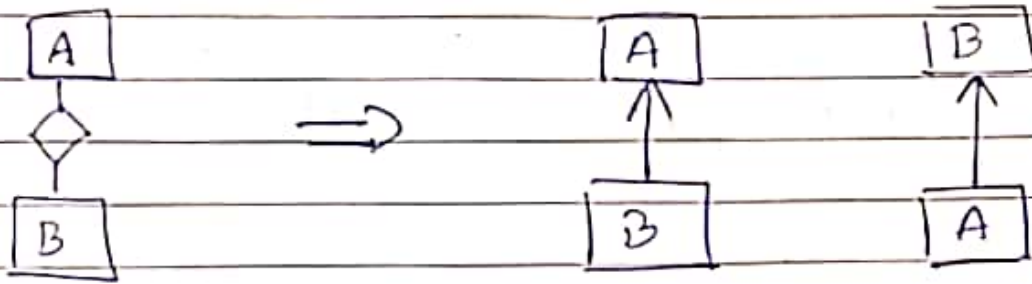
Branch

Customer

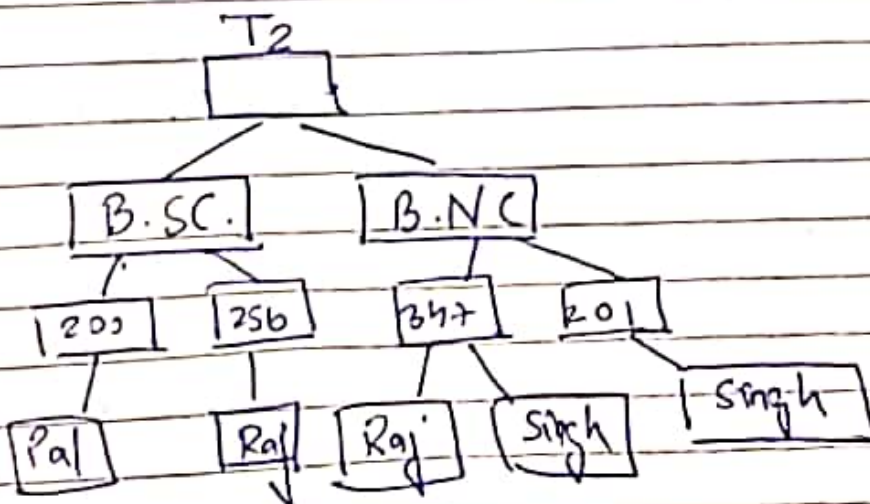
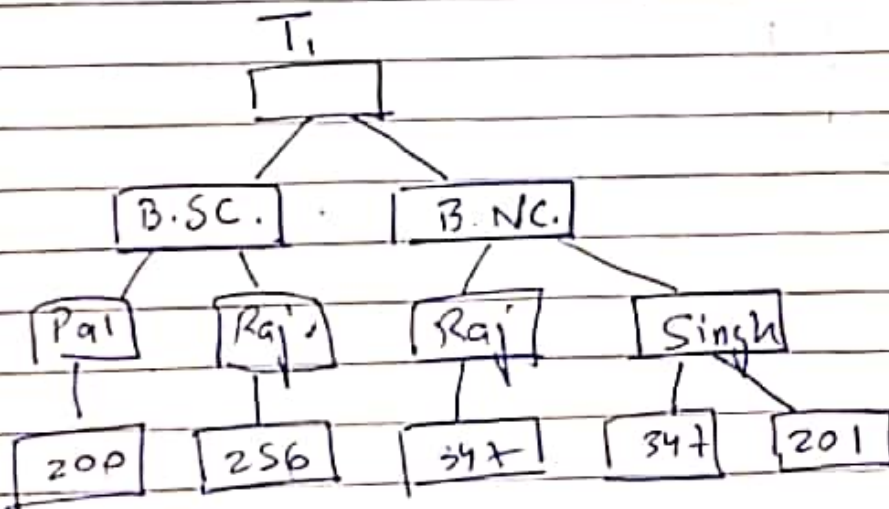
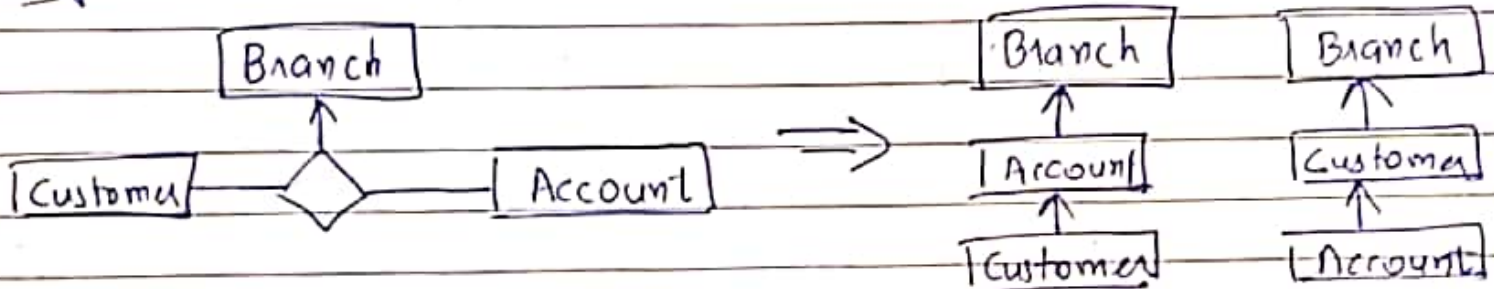
Account



Hierarchical Model



Eg.



Relational Database Design

Redundancy	Anomaly
Updation	Anomaly
Insertion	Anomaly
Deletion	Anomaly

SGP (RN, S NAME, HALL, GAME, FEE)
↑
Student
Game
participation

~~SGP~~ Break into different tables to avoid the anomalies.

T_1 (RN, S NAME, HALL)
 T_2 (GAME, FEE)
 T_3 (R.N., GAMA)

Functional Dependency

$U \rightarrow$ Universal set of attributes.

$U_i \subseteq U$ is a subset of attributes

$X \rightarrow Y$; $\pi(U_i)$ M_1 & M_2 are two
tuples
(records)

$$M_1[X] = M_2[X] \Rightarrow M_1[Y] = M_2[Y]$$

$R(A, B, C, D)$

A	B	C	D
a_1	b_1	c_1	d_1
a_1	b_2	c_1	d_2
a_2	b_2	c_2	d_2
a_2	b_3	c_2	d_3
a_3	b_3	c_2	d_4

$A \rightarrow C$

$D \rightarrow B$

ASA

Trivial
F.D.

$A \rightarrow B$, $B \rightarrow C$ then $A \rightarrow C$

Closure of a set of FDs

$$F^+ = \{ X \rightarrow Y \mid F \models X \rightarrow Y \}$$

$$R(A, B, C) \quad F = \{ A \rightarrow B, B \rightarrow C \}$$

$$F^+ = \{ A \rightarrow C, AB \rightarrow ABC, B \rightarrow BC, \dots \}$$

Armstrong's Axioms

1) If $Y \subseteq X \subseteq U$ then $X \rightarrow Y$

2) If $X \rightarrow Y$ and $Z \subseteq U$

$$XZ \rightarrow YZ, \quad XZ = X \cup Z$$

3) If $X \rightarrow Y$ and $Y \rightarrow Z$ then $X \rightarrow Z$

Eg: $R(\text{CITY}, \text{ST}, \text{PIN})$; $\text{CITY ST} \rightarrow \text{PIN}$
 $\text{PIN} \rightarrow \text{CITY}$

$$\rightarrow (\text{CITY ST}) \cup (\text{CITY ST}) \rightarrow \text{PIN} \cup (\text{CITY ST})$$

$$\Rightarrow \boxed{\text{CITY ST} \rightarrow \text{PIN CITY ST}}$$

↑ PR
Key

$$\text{PIN} \cup (\text{PIN ST}) = \text{CITY} \cup (\text{PIN ST})$$

$$\Rightarrow (\text{PIN ST}) = \text{CITY} \cup \text{PIN ST}$$

↑
Key

Inference Rule

$$1) \{ X \rightarrow Y, X \rightarrow Z \} \Rightarrow X \rightarrow YZ$$

$$2) \{ X \rightarrow Y, WY \rightarrow Z \} \Rightarrow XW \rightarrow Z$$

← Pseudo Transitivity Rule

$$3) \text{ If } X \rightarrow Y \text{ and } Z \subseteq Y \text{ then } X \rightarrow Z$$

$$4) \{ X \rightarrow YZ \} \Rightarrow X \rightarrow Y, X \rightarrow Z$$

$$R(A, B, C, G, H, I)$$

$$F = \{ A \rightarrow B, A \rightarrow C, CG \rightarrow H, CG \rightarrow I, B \rightarrow H \}$$

$$F^+ = \{ A \rightarrow BC, A \rightarrow H, CG \rightarrow HI, AG \rightarrow HI \dots \}$$

Closure of a set of Attributes

Let F be a set of FDs, The closure of attribute X denoted by X^+ is a set of attributes A , s.t. $X \rightarrow A$ can be derived from F using Armstrong's Axioms

Lemma: $X \rightarrow Y$ follows from Armstrong's Axiom, if and only if $Y \subseteq X^+$

Th: Armstrong's Axioms are sound and complete
→ Sound: If $X \rightarrow Y$ from F , then it's true in all cases.

Complete: The set of FDs obtained by Armstrong's axioms are all the possible FDs.

X^+ algorithm of Bernstein

1) Let $N=0$ & $X(N) = X$

2) If \exists a dependency $AB \rightarrow C$ in F then left-hand side A is contained in $X(N)$, while right-hand side C is not then,

$$X(N+1) = X(N) \cup B$$

3) $N = N+1$, go to step 2

eg: $R(A, B, C, D, E, G)$

$$F = \{ AB \rightarrow C, C \rightarrow A, BC \rightarrow D, ACD \rightarrow B, D \rightarrow EG, BE \rightarrow C, CG \rightarrow BD, CE \rightarrow AG \}$$

$$(BD)^+ = ?$$

→

$$X(0) = BD$$

$$D \rightarrow EG, \quad X(1) = BD \cup EG = BDEG$$

$$BE \rightarrow C, \quad X(2) = BDEGC$$

$$A \rightarrow C, \quad X(3) = BDIGCA$$

$$\boxed{(BD)^+ = BDEGCA}$$

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$R \{$
↑ family name
↑ middle name
↑ salary
↑ spouse's salary
↑ city of residence
 $\{$
 $FI, MOC, SAL, SS, ~~FI~~, JOB, COR, LUCK, ED\}$

$effort \{$
↑ sincerity
 $EFF, SEN\}$

$\{ \text{FI} \rightarrow \text{MOC, SAL}, \text{SS} \rightarrow \text{FI}, \text{JOB FE} \rightarrow \text{COR},$
 $\text{LUCK ED} \rightarrow \text{JOB}, \text{JOB} \rightarrow \text{ED},$
 $\text{EFF SEN JOB} \rightarrow \text{SAL} \}$

- To find Key, we have to find min set of attributes whose closure, if it gives you all the attributes we say that it is a key.

$\{ \text{JOB}, \text{EF}, \text{SEN}, \text{LUCK} \}$ etc. are not appearing on the RHS at all, so they must be a part of the key. So, find the closure of these.

Simplification of FDs:

- $X \rightarrow Y$ is a non trivial FD if $Y \neq X$
- If \nexists no $X' \subset X$ where $X' \rightarrow Y$ then $X \rightarrow Y$ is called an elementary FD.

Eg $\rightarrow AB \rightarrow C, B \nrightarrow C, A \nrightarrow C$

i.e. If I supply A & B, I'll get C.

- If \exists some $X' \subset X$ and $X' \rightarrow Y$, then \exists partial dependency on that FD.

Eg $\rightarrow AB \rightarrow C, A \rightarrow C$, here supplying B was not necessary so it's called as extraneous attribute while $AB \rightarrow C$ is called as partial dependency.

- Let $X \rightarrow A, B \in X$, if $\{X - B\} \rightarrow A$ is in F^+ , then we can say that B is an extraneous attribute A and is not required.

• Let $F = \{AB \rightarrow DEF, AC \rightarrow G, A \rightarrow C\}$

$$AC^+ = ACG$$

$$A^+ = ACG$$

So in the FD $AC \rightarrow G$, C is extraneous attr., so you can write it down as

$$F = \{AB \rightarrow DEF, A \rightarrow G, A \rightarrow C\}$$

• Each FD is represented by a table, and if we remove an attr. without losing the properties, then we are reducing the size of the table and hence, processing time.

• In the prev. case, we tried to find if an attr. is redundant, now we try to ^{find out} ~~prove~~ if an FD is redundant.

Let an FD f is redundant on a set of FDs ' F '

$$\Rightarrow (F - f)^+ = F^+$$

i.e. by removing f^+ , we should be able to obtain F^+ .

• A set of FDs ' F ' is 'minimum' if \exists no set G with less no. of FDs than F s.t. $G^+ = F^+$
 $\{A \rightarrow B, A \rightarrow C\}$ and $\{A \rightarrow BC\}$

• A set of FDs ' F ' is 'L-minimum' if

1) F is min.

2) \exists no partial dependency in each FD.

• A set of FDs ' F ' is 'LR-minimum' if

1) F is L-minimum

2) \exists no redundancy on RHS

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g) $\{AB \rightarrow C, C \rightarrow A, BC \rightarrow D, ACD \rightarrow B, D \rightarrow EG, BE \rightarrow C, CG \rightarrow BD, CE \rightarrow AG\}$ simplify.

$AB \rightarrow C$	N.R.	$BE \rightarrow C$	N.R.	R: Redundant NR: Non-Redundant
$AC \rightarrow A$	N.R.	$CG \rightarrow B$	N.R.	
$BC \rightarrow D$	N.R.	$\times CG \rightarrow D$	R	
$\times ACD \rightarrow B$	R	$\times CE \rightarrow A$	R	
$D \rightarrow E$	N.R.	$CE \rightarrow G$	N.R.	
$D \rightarrow G$	N.R.			

$(ACD)^+ = ACDEGB$ (All elements)

$D^+ = DE$; $(CG)^+ = CGADE$

Allowed minimum:

$\{AB \rightarrow C, C \rightarrow A, BC \rightarrow D, D \rightarrow EG, BE \rightarrow C, CG \rightarrow B, CE \rightarrow G\}$

Decomposition of a Relational scheme

$\rho: R(A_1, A_2, \dots, A_n)$

Obtain, $P = \{R_1, R_2, \dots, R_n\}$ s.t.

$$R = R_1 \cup R_2 \cup \dots \cup R_n$$

Lossless Join

$R \quad R_1, R_2, \dots, R_K \quad D$

$$r = \pi_{R_1}(r) \bowtie \pi_{R_2}(r) \bowtie \pi_{R_3}(r) \bowtie \pi_{R_4}(r) \bowtie \dots \bowtie \pi_{R_K}(r)$$

If $\rho = (R_1, R_2)$ is a decomposition of R then ρ is a lossless join decomposition w.r.t FDs iff

$$(R_1 \cap R_2) \rightarrow (R_1 - R_2)$$

$$(R_1 \cap R_2) \xrightarrow{\text{or}} (R_2 - R_1)$$

eg: $R(A, B, C) \quad F = \{A \rightarrow B\}$
 (i) $R_1(A, B), R_2(A, C)$

$$R_1 \cap R_2 = A \quad ; \quad R_1 - R_2 = B$$

$$\therefore \rho(R_1, R_2) \text{ is lossless}$$

(ii) $R_1(A, B), R_2(B, C)$
 NOT lossless.

eg: $\eta = \{a_1 b_1 c_1, a_2 b_1 c_2\}$

$$\text{Exp } R_1 = \pi_{AB}(R) = \{a_1 b_1, a_2 b_1\}$$

$$R_2 = \pi_{BC}(R) = \{b_1 c_1, b_1 c_2\}$$

$$R \neq R_1 \cup R_2 = \{a_1 b_1 c_1, a_1 b_1 c_2, a_2 b_1 c_1, a_2 b_1 c_2\}$$

Preservation of Dependencies

$$\begin{array}{ccccccc} R & & R_1 & , & R_2 & , & \dots & , & R_K \\ \downarrow & & \downarrow & & \downarrow & & & & \downarrow \\ F & & F_1 & & F_2 & & & & F_K \end{array}$$

$$F = F_1 \cup F_2 \cup \dots \cup F_K$$

Note:

- Lossless join property must be satisfied.

$$R = (C, S, P)$$

~~$$R_1 (S, P)$$~~

$$\{ CS \rightarrow P, P \rightarrow C \}$$

$$R_1 (S, P), R_2 (C, P)$$

$$F_1 = \phi, F_2 = \{ P \rightarrow C \}$$

- Super Key :- Super key is the combination of fields by which row is uniquely identified. If you add any column or attribute to a primary key, then it becomes a super key.

- Candidate Key :- Candidate key is the minimal super key. These are individual columns in a table that qualifies for uniqueness of all rows.

- Foreign Key :- Foreign key is a collection of field or fields in one table, that uniquely identifies a row of another table. Thus foreign key is defined in a second table, but it refers to the primary key in the first table.

Eg Employee { E-id, Fullname, SSN, Dept-ID }

{ E-id, Fullname }

{ E-id, Dept ID }

Normal Forms

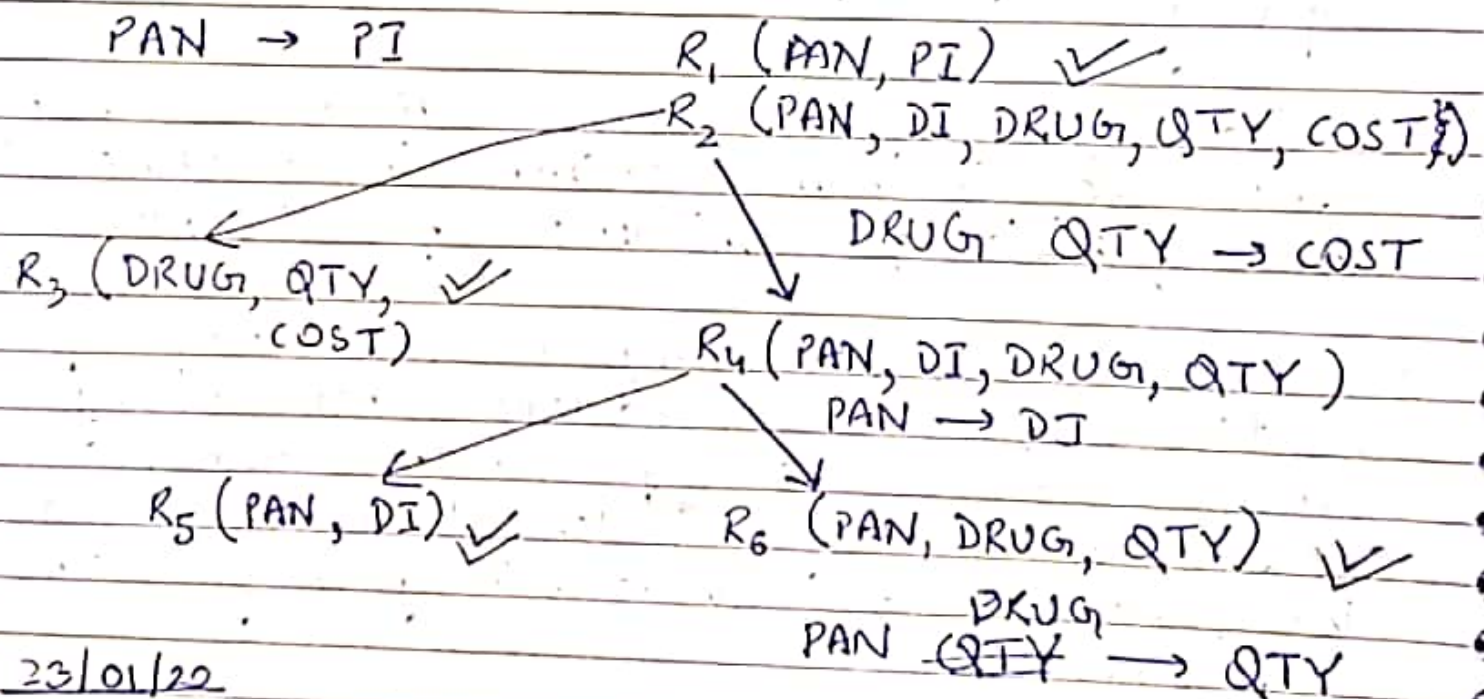
Boyce Codd Normal Form (BCNF)

- Whenever $X \rightarrow Y$, and Y is not contained in X and X is a key.

eg:- $R \{PAN, PI, DI, DRUG, QTY, COST\}$

$F = \{ PAN \rightarrow PI, PI \rightarrow DI, PI \ DRUG \rightarrow QTY, DRUG \ QTY \rightarrow COST \}$

→ Key: $\{ (PAN, DRUG) \}$



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- Th: BCNF decomposition always produces lossless joint decomposition.
- After BCNF we might not be able to answer certain queries, while those which we can will have correct information. i.e. it may or may not preserve dependencies.

9 R(A, B, C, D, E, F)
 $F = \{ C \rightarrow A, AE \rightarrow B, BF \rightarrow C, CD \rightarrow EF, EF \rightarrow AD \}$
 \rightarrow
 Key : $\{ CD \text{ or } EF \}$

$R_1(A, C), R_3(B, F, C), R_5(B, D, E, F)$

Prime Attribute

- Any attribute A is called as a prime attribute if its a member or subset of a ~~the~~ key. Otherwise its non-prime attribute.
- Key : $\{ AB, BC \}$. A, B, C are prime attributes.

Third NF (3NF)

- $X \rightarrow A$ A is not a subset of X, then X is a key of the relation or A is prime attribute.
- If a relation is in BCNF, it ~~defin~~ certainly is 3NF.
- It removes two things — Partial Dependency and Transitive Dependency.

Bernstein's 3NF Algorithm

- ① Rewrite the FDs s.t. each FD contains only one attribute on the RHS.
- ② Rewrite the list of FDs s.t. \exists no redundant FDs.
- ③ Rewrite FDs s.t. no proper subset of LHS functionally determines RHS. This ensures no partial dependencies.

- ④ Combine dependencies with same LHS using union rule. Then if $X \rightarrow Y$ is a FD make a decomposition $\{X, Y\}$.
- ⑤ If key is not present in any of the subrelation then create a new subrelation with key only.
- ⑥ If any subrelation is subset of other subrelation remove it.

eg $R_1(PAN, PI), R_2(PI, DI), R_3(PI, DRUG, QTY)$
 $R_4(DRUG, QTY, COST), R_5(PAN, EDROG)$

Note :- In 3NF The result is unique as it is structured.

• 3NF ensures lossless join as well as preserves dependency

eg: $R(A, B, C, D, E, F, G, H, I)$

$\{A \rightarrow BCD, AE \rightarrow FG, F \rightarrow AEG, C \rightarrow HI\}$

→ Key: AE, F

$R_1(A, B, C, D), R_2(C, H, I), R_3(AE, F, G)$

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Query LanguagesRelational Algebra

① Union (\cup) :- $R \cup S$, The set of tuples which are in either in R , or in S , or in both of them.

R		
A	B	C
a	b	c
d	a	f
c	b	d

S		
D	E	F
b	g	a
d	a	f

$R \cup S$		
A	B	C
a	b	c
d	a	f
c	b	d
b	g	a

- Checks the tuple & not the attribute name.
- R & S must have same number of attributes.

② Set Difference :- $(-)$, $R - S$.

- The set of tuples which are in R but not in S .

$R - S$ (R & S same as prev.)

a	b	c
c	b	d

- R & S must have same no. of attributes.

③ Cartesian Product :- (\times) , $R \times S$ (R & S same as prev.)

- Number of attributes is known as arity.
- Let R & S be tables of arity K_1 & K_2 . In $R \times S$, The first K_1 components come from a tuple of R & last K_2 from a tuple of S .

- Here column names will appear,

A	B	C	D	E	F
a	b	c	b	g	a
a	b	c	d	a	f
d	a	f	b	g	a
d	a	f	d	a	f
c	b	d	b	g	a
c	b	d	d	a	f

④ Projection (π) :- $\pi_{i_1, i_2, \dots, i_k}(R)$

- This will take out k attribute values from each tuple of R .

• Eg:- $\pi_{C,A}(R) =$

C	A
c	a
f	d
d	c

$\pi_{2,3}(S) =$

E	F
g	a
a	f

⑤ Selection (σ) :- Suppose F is a formula.

• $\sigma_F(R)$ means from the relation R we will take only those tuples which will satisfy the formula F .

• Eg :- $\sigma_{B=c_1}(R) \equiv$

A	B	C
a	b	c
c	b	d

 ; $\sigma_{B \neq c_1}(R) \equiv$

A	B	C
Empty		

$\sigma_{(I=c_1) \vee (I=c_2)}(R) \equiv$

A	B	C
a	b	c

 Either a or b

Additional Operators

① Intersection (\cap) :- $R \cap S$:-

• Set of tuples that are both in R and S .
 $R \cap S = R - (R - S)$

② Quotient (\div) :- $R \div S$. Consider arity of R and S as n & s resp. s.t. $n > s$.

$R \div S$ is the tuple set of tuples t with arity $n-s$ s.t. \forall tuples u is S , the tuple tu is in R .

R			
a	b	c	d
a	b	e	f
b	c	e	f
e	d	c	d
e	d	e	f
a	b	d	e

S	
c	d
e	f

$R \div S$	
a	b
eb	d

$$R \div S = \pi_{1,2,\dots,n-s}(R)$$

$$= \pi_{1,2,\dots,n-s}((\pi_{1,2,\dots,n-s}(R) \times S) - R)$$

③ θ -Join :- $R \bowtie_{i\theta j} S$, here i & j are attributes of R & S resp. We first find the cartesian product of R and S , and then takes out those tuples which satisfy $i\theta j$ formula.

Eg $\rightarrow R$

A	B	C	D	E
1	1	3	3	1
4	5	6	6	2
7	8	9		

$R \times S$

$B < D$

A	B	C	D	E
1	1	3	3	1
1	1	3	6	2
4	5	6	6	2

④ Natural Join (\bowtie) :- $R \bowtie S$

* Here R and S must have column/attr. names.
 * If the attr. name of two tuples ^{columns} are same it will merge those two and output only those tuples which have same values in those two attributes.

Eg:-

R

A	B	C
a	b	c
d	b	c
b	b	f
c	a	d

S

B	C	D
b	c	d
b	c	e
a	d	b

$R \bowtie S$

A	B	C	D
a	b	c	d
d	b	c	e
a	b	c	e
d	b	c	e
c	a	d	b

* Examples :- (Tables to be used in this course)

Customer (c-name, street, c-city)

Branch Deposit (b-name, b-city, asset)

Borrow (b-name, loan-no, c-name, amount)

Client (c-name, emp-name)

Deposit (b-name, ac-no, c-name, balance)

Q1: Find the customer name, where customer name and the employee name who handle the customer are same.

$$\rightarrow \pi_{c_name} \left(\sigma_{c_name = emp_name} (Client) \right)$$

Q2: Find all clients of employee XYZ, and corresponding cities of those clients

$$\rightarrow \pi_{client.c_name, c_city} \left(\sigma_{emp_name = 'xyz' \wedge client.c_name = customer.c_name} (client \times customer) \right)$$

Q3: Find all customers who have both account & loan at KGP branch.

$$\rightarrow \pi_{c_name} \left(\sigma_{b_name = 'KGP'} (Deposit \times Borrow) \right)$$

Q4: Find all customers who have account at all branches in the city Kolkata.

$$\rightarrow \pi_{c_name, b_name} (Deposit)$$

$$\div \pi_{b_name} \left(\sigma_{b_city = 'Kolkata'} (Branch) \right)$$

30/01/20

- Employee (Emp-id, Name, Salary, manager-id)
- If manager-id = 1 \Rightarrow employee is a manager otherwise not a manager.

Q Display the name of employees (not manager) whose salary is more than any manager.

$$\rightarrow \pi_2 \left(\sigma_{4 \neq 1 \wedge 5 > 1 \wedge 3 > 2} (\text{Employee} \times \text{Employee}) \right)$$

Here $4 \rightarrow \text{Employee A. manager_id}$
 $2 \rightarrow \text{Employee B. manager_id}$

Lives (p-name, street, city)
 Works (p-name, c-name, salary)
 Located-in (c-name, c-city)
 Manages (p-name, m-name)

Q Find the name and city of all persons who work for "XYZ" company.

$$\rightarrow \pi_{p\text{-name}} \left(\sigma_{\substack{c\text{-name} \\ = 'XYZ'}} (\text{Lives} \bowtie \text{Works}) \right)$$

or.

$$\pi_{\text{Lives.p-name}} \left(\sigma_{\substack{c\text{-name} \\ = 'XYZ'}} (\text{Lives} \times \text{Works}) \right)$$

$\pi \wedge \text{Lives.} \cancel{\text{work}}_{p\text{-name}} = \text{Works.p-name}$

Q Find all persons who live in the same city and the company they work for.

$$\rightarrow \pi_1 \left(\sigma_{3=8 \wedge 4=9} \left((\text{Lives} \bowtie \text{Works}) \times (\text{Lives} \bowtie \text{Works}) \right) \right)$$

Q Find all persons who live in the same city & street, as their manager.

$$\rightarrow \pi_1 \left(\sigma_{1=5 \wedge 2=6 \wedge 4=7 \wedge 5=8} (\text{Manages} \times \text{Lives} \times (\pi_2 (\text{Manages}) \bowtie \text{Lives})) \right)$$

3/02/20

Q Find all persons whose salary is less than every employee of 'xyz' company.

$$\pi_1 (\sigma_{1 > 2} (P \times P)) = Q$$

$$\boxed{\text{min} = P - Q}$$

Propositional Logic

\neg (not), \wedge (and), \vee (or), \Rightarrow (implication),
 \equiv (equivalence)

- Proposition \rightarrow is a declarative statement that's either true or false, made up of symbols, known as atoms.
- Any expression representing a proposition using atoms is called a formula

Tuple Relational Calculus

- An expression in T.R.C. is written as $\{t \mid \psi(t)\}$ where t is a tuple variable, $\psi(t)$ is a formula
- $R(s) \leftarrow s$ is a tuple of relation R
e.g. $\rightarrow n[i] \theta m[j]$

Universal Quantifier $(\forall x)$

- $(\forall x) G(x)$ means that $G(x)$ is true for all x .

Existential Quantifier $(\exists x)$

- $(\exists x) G(x)$ then means there exist at least one value of x s.t. $G(x)$ holds.

eg. $\rightarrow (\exists s) (R(s))$

• $R \cup S$ is T.R.C. $\rightarrow \{t \mid R(t) \cup S(t)\}$

• $R - S$ is $\{t \mid R(t) \wedge \neg S(t)\}$

• $R \times S$ is $\{t^{(1+s)} \mid \{(\exists u)(\exists v)(R(u) \wedge S(v) \wedge$
 $t[1] = u[1] \wedge \dots \wedge t[\lambda] = u[\lambda] \wedge$
 $t[\lambda+1] = v[1] \wedge \dots \wedge t[\lambda+s] = v[s]\}\}$

• $\pi_{i_1, i_2, \dots, i_n}(R)$ is $\{t^{(n)} \mid (\exists x)(R(x) \wedge t[i_1] = x[i_1]$
 $\wedge \dots \wedge t[i_n] = x[i_n])\}$

• $\nabla_F(R)$ is $\{t \mid R(t) \wedge F'\}$
 \uparrow tuple version of F

Th: Every F is relational algebra can be written in tuple algebra.

eg. $\rightarrow R^{(2)}, S^{(2)} \quad \pi_{1,4}(\nabla_{2=3}(R \times S))$

$\{t \mid (\exists u)(\exists v)(R(u) \wedge S(v) \wedge u[2] = v[1] \wedge t[1] = u[1]$
 $\wedge t[2] = u[2])\}$

