Indian Institute of Technology Kharagpur
Department of Mathematics

End-Autumn Semester Examination

November 2011

Sub No: MA20103	No of Students:470	Partial Differential Equations
Full Marks: 50	BT/AG/EX/ME/MA/HS/MT	Time: 3 hrs

Notes:

- *Start each group on a new page. Answer each group on continuous pages. Clearly mention the group at the top of the page along with the serial number of questions.*
- *Attempt all the questions. Marks are shown against each question. Show the relevant steps.*
- $p \equiv \frac{\partial z}{\partial x}, q \equiv \frac{\partial z}{\partial y}$.

Group I

1. Obtain general solution of the following PDE 4

$$(y + zx)p - (x + yz)q = x^2 - y^2 .$$

2. Reduce the following equation to its canonical form 4

$$\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0 .$$

3. Using Laplace transform, solve 4

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad 0 \le x \le 1, t \ge 0,$$

subject to

$$\begin{aligned} u(0,t) &= u(1,t) = 0 \\ u(x,0) &= \sin \pi x, \ u_t(x,0) = -\sin \pi x . \end{aligned}$$

4. Use the method of separation of variables to solve 4

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad 0 \le x \le 1, \ t \ge 0,$$

such that

$$\begin{aligned} u(0,t) &= u(1,t) = 0, \\ u(x,0) &= x(1-x), \ 0 \le x \le 1, \\ u_t(x,0) &= 0, \ 0 \le x \le 1 . \end{aligned}$$

Group II

5. Consider a hemispherical surface whose surface is kept at a constant temperature. There is no source or sink inside the hemisphere. Using the method of separation, obtain an axisymmetric solution for steady-state temperature inside the hemisphere. 4

6. Solve the following PDE, using the method of Laplace transform 4

$$\frac{\partial^2 u}{\partial x^2} = 4\frac{\partial u}{\partial t}, \quad 0 \le x \le 1/2\,,\ t \ge 0\,,$$

with the initial conditions $u(x,0) = (1/4) + \sin 2\pi x$ and the boundary conditions $u(0,t) = u(1/2,t) = 1/4$.

7. (a) Find the particular integral of the PDE 1

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 2\frac{\partial^2 z}{\partial y^2} = e^x$$

(b) Obtain general solution of the PDE 3

$$2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} - 3\frac{\partial z}{\partial y} = \sin(x-y)$$

8. Consider the following second order PDE

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = 0\,.$$

Reduce this equation to a linear PDE with constant coefficients by the change of variables $x = e^u, y = e^v$. Hence obtain the general solution. 4

Group III

9. Consider a metal rod of uniform thickness which is of unit length fixed along x-axis. One end of the rod is fixed at $x = 0$ and the other end at $x = 1$ which are kept at a constant temperature of 1 and 0 respectively. If the initial temperature throughout the rod is given by $\frac{7}{2}$, formulate the problem in order to determine the heat conduction inside the rod at any given time t. The thermal conductivity of the rod is assumed to be unity. Solve using separation of variables. 6

10. Solve, using the method of separation of variables, $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$ subject to the boundary conditions $f(x,0) = 0 = f(x,\pi)$, $f(0,y) = 0, f(\pi,y) = \cos^2 y$. 4

11. Classify the second order PDE

$$\frac{3}{4}u_{xx} - 2yu_{xy} + y^2 u_{yy} + \frac{1}{2}u_x = 0$$

depending on the domain. Reduce it to canonical form and integrate to obtain the general solution. 5

12. Solve the non-linear PDE using Charpit's method $z^2 - pq + 25 = 0$. 3

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