Indian Institute of Technology, Kharagpur

Date of Exam.: .02.14 (FN/AN) Time: 2 Hrs. Full Marks: 30 No. of Students: 400
Mid (Spring) Semester Examination (2013-14) Department: Mathematics
Subject No. MA20106 Subject Name : Probability & Stochastic Processes
II yr B.Tech. EE/E&ECE/IE/MT

Instructions:
(i) Use of calculator and Statistical tables is allowed. All the notations are standard and **no query or doubts will be entertained**. If any data/statement is missing, identify it in your answer script.
(ii) Answer **All** questions.
(iii) All parts of a question **Must Be** answered at **One Place**.

1. Examine each part of this question and carry out your solution as usual. Write **ONLY THE ANSWER** for each part of this question **on the first page** of your answerscript. [1 × 7]

   (a) Suppose the metal parts produced by a machine contains 5% defective parts. How many parts should be produced in order that the probability of at least one defective is 50% or more?

   (b) A pressure control apparatus contains 3 electronic tubes. The apparatus will not work unless all the tubes are operative. If the probability of failure of each tube during some interval of time is 0.04, find the corresponding probability of failure of the apparatus.

   (c) A hunter's chance of shooting an animal at a distance $r(> a)$ is $\frac{a^2}{r^2}$. He fires when $r = 2a$ and if he misses, he reloads and fires again when $r = 3a, 4a, \ldots, na$. If he misses at a distance na the animal escapes. Find the probability that the animal escapes.

   (d) If random variable X has probability mass function (pmf) $p(x) = \frac{k}{x!}$, $x = 0, 1, 2, \ldots$; where k is a constant. Find the value of k and $P(X \geq 3)$.

   (e) Let random variable X has probability density function (pdf) $f(x) = 3x^2$, $-1 \leq x \leq 0$. If a is a number satisfying $-1 < a < 0$, compute $P\left(X > a | X < \frac{a}{2}\right)$

   (f) If X is a poisson variate such that $2P(X = 0) + P(X = 2) = 2P(X = 1)$, what is $E(X)$?

   (g) Let the r.v. X be exponentially distributed, and $E(X) = 2$. Find $P(3 \leq X \leq 4 | X > 1)$.

2. (a) Suppose that we have a test for the HIV virus. The probability that a person who really has the virus will test positive is .90 while the probability that a person who does not have it will test positive is .20. Finally, suppose that the probability that a person in the general population has the virus is .01. Tom is tested positive. What is the chance that Tom has HIV ?

   (b) A hat-check staff has had a long day, and at the end of the party they decide to return people's hats at random. Suppose that n people have their hats returned at random. Calculate the expectation and variance in the number of people who get their hat back.

   (c) A couple decided to have children until they have both a boy and a girl. What is the expected number of children that they'll end up with? Assume that each child is equally likely to be a boy or a girl and genders are mutually independent. [2+2+2]

——-P.T.O——

(d) For each of the following transition probability matrices, state whether the chain is (i) irreducible, (ii) a periodic, (iii) positive recurrent.

$$(a)\begin{pmatrix} 1/4 & 1/4 & 1/2 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad (b)\begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad (c)\begin{pmatrix} 1/3 & 0 & 2/3 \\ 1/4 & 3/4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3. (a) Consider a drunkard who is walking on a circle. He can only stand on one of five positions, equally spaced on the circle, labeled 1, 2, 3, 4, 5. At any step, the probability that he makes one step in counterclockwise direction is p and one step in clockwise direction $1-p$. Find the limiting distribution of his location.

(b) Let $\{\alpha_i : i = 1, 2, ...\}$ be the probability distribution, and consider the Markov chain whose TPM is

$$M = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 & \cdots \end{array} \\ \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & \alpha_6 & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 1 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 1 & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots \end{pmatrix} \end{array}$$

What condition on $\{\alpha_i : i = 1, 2, ...\}$ is necessary and sufficient in order that limiting distribution exist, and what is this limiting distribution? Assume $\alpha_1 > 0$ and $\alpha_2 > 0$.

4. (a) A rat is put into the linear maze as shown below

0	1	2	3	4	5
Shock			*Rat*		Food

(i) Assume that the rat is equally likely to move right or left at each step. What is the probability that the rat finds food before getting shocked?
(ii)As a result of learning at each step the rat moves to the right with probability $p > 1/2$ and to the left $q = 1-p$. What is the probability that the rat find the food before getting shocked?

5. (a) Describe the Gambler's ruin problem where the player A plays against the adversary B with total fortune of the game being N. Derive the expression for A's ruin when A starts the play with the fortune k ($< N$). Also find the mean duration of the game. Assume p and q as the probabilities of winning and loosing one unit on each play for the player A, respectively.

(b) For N= \$100, k=\$ 50 and p=0.60, derive the mean duration of the game if the player A wins on a given bet (assuming each player bets \$1 at each play).