Tutorial Sheet - 3 (Hints and Answer) **SPRING 2017**

MATHEMATICS-II (MA10002) *January 10, 2017*

1. (a) Rank A=2.
   $\det A = 0$. Show atleast one second order minor is nonzero.
   (b) Rank A=2.
   Every minor of order 3 is zero. Show atleast one second order minor is non zero.

2. Use elementary row operation to reduce to the row-echelon form.
   (a) Row-echelon form will be $\begin{pmatrix} 1 & 3 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$. Number of non zero row is 3. So rank is 3.
   (b) Row-echelon form will be $\begin{pmatrix} 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$. Rank is 2.
   (c) Row-echelon form will be $\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$. Rank is 3.

3. Since rank is less than 3, $det = 0$. Then solve for x. $x = -\frac{1}{2}, 1, 1$.

4. Since coefficient of x^2 is 0, sum of the roots will be 0.Using this fact, show $det = 0$ and atleast one second order minor is non-zero.

5. (a) After row operations, the coefficient matrix will be $\begin{pmatrix} 1 & 1 & -1 \\ 0 & -5 & 3 \\ 0 & 0 & 0 \end{pmatrix}$. Solution is $(\frac{2}{5}c, \frac{3}{5}c, c)$, c is an arbitrary constant.
   (b) After row operations, the coefficient matrix will be $\begin{pmatrix} 1 & 1 & -1 \\ 0 & 2 & 1 \\ 0 & 0 & \frac{11}{2} \end{pmatrix}$. Solution is $(\frac{3}{2}c, -\frac{c}{2}, c)$.
   (c) After row operations, the system of equations reduces to

$$x_1 - 3x_2 + 2x_3 - x_4 + 2x_5 = 2$$
$$x_3 + 2x_4 - x_5 = 1$$

6. (i) Augmented matrix $\overline{A} = \begin{pmatrix} 1 & 1 & 0 & 4 \\ 0 & 1 & -1 & 1 \\ 2 & 1 & 4 & 7 \end{pmatrix}$ Apply row operations on $\overline{A}$. Solution $(3, 1, 0)$.
   (ii) inconsistent

(iii) $\overline{A} = \begin{pmatrix} 1 & 3 & 1 & 0 \\ 2 & -1 & 1 & 0 \end{pmatrix}$. After row operation equivalent system is given by

$$x_1 + \frac{4}{7}x_3 = 0$$
$$x_2 + \frac{1}{7}x_2 = 0$$

Solution $k(4, 1, -7)$, $k \in \mathbb{R}$.

7. For non-trivial solution det(coefficient matrix)$= 0$, $rank < 3$.
   (i) $k = 1$. (ii) $k = 2, \frac{11}{3}, \frac{11}{3}$.

8. (i) For $a \neq 1$, the system has unique solution.
   For (ii) $a = 1, b \neq -1$, the system has no solution.
   For (iii) $a = 1, b = -1$, the system has many solutions.
   (b) (i) $a \neq 1$, the system has unique solution.
   (ii) $a = 1, b \neq 1, -3$, the system has no solution.
   (iii) $a = 1, b = 1$ or $a = 1, b = -3$, system has many solutions.
   (c) (i) $detA = 0$, the system cannot have unique solution.
   (ii) For $a \neq 1, \frac{2}{3}$, the system has no solution.
   (iii) For $a = 1, \frac{2}{3}$, the system has infinitely many solution.

9. $det(A) = 0$. Applying elementary row/column operations $a + b + c = 0$ or $a = b = c$.

10. $A = \frac{1}{2}[A + A^*] + \frac{1}{2}[A - A^*]$ where $A^* = (\bar{A})^T$ and $\frac{1}{2}[A + A^*]$ is Hermitian part and $\frac{1}{2}[A - A^*]$ is Skew-Hermitian part.

11. Find AA^* and show $(AA^*)^* = AA^*$.

12. Considering any third order arbitrary matrix A, show $(A - A^T)$ is skew symmetric matrix of odd order. Hence the determinant is zero. Considering the second order minors, prove that $rank(A - A^T) = 2$.

13. Prove from the condition of unitary matrix $A\overline{A}^T = I$.

14. $\overline{a} = e^{-\frac{2i\pi}{3}}$, $\overline{a}^2 = e^{-\frac{4i\pi}{3}}$,$a^3 = 1$ and $\overline{a}^3 = 1$. Therefore a is the cube root of unity. Hence show $M\overline{M} = 3I$.