Extra

Department of Mathematics

Indian Institute of Technology Kharagpur

Date: .... Time: 2 Hours Full Marks: 30 No of Students: 300

Spring Semester: 2013-14 Departments: AE+CY+NA+MA+CH+MA Sub. No. MA20102

Sub. Name: **Numerical Solution of Ordinary and Partial Differential Equations**

Instruction: *Answer all questions. All part of the same question should be done at one place.*

Question 1 [3+4]

a) Using the 2nd order **Taylor's series** method, solve

$$y' = 2t + 3y, \qquad y(0) = 1$$

for $y(0.2)$ using $h = 0.1$.

b) Use the classical **Runge-Kutta** method of fourth order to find the numerical solution at $t = 0.6$ for

$$y' = \sqrt{t + y}, \quad y(0.2) = 0.44.$$

Use the step length $h = 0.2$.

Question 2 [4+3]

a) Using the second order **implicit Runge-Kutta** method find the solution of the initial value problem

$$y' = -2ty^2, \quad y(0) = 1, \ 0 \le t \le 0.2 \text{ with } h = 0.2.$$

Use **Newton-Raphson** method to solve the non-linear algebraic equation.

b) Obtain the interval of absolute stability of the implicit method

$$u_{n+1} = u_n + \frac{h}{4}(K_1 + 3K_2), \quad K_1 = f(t_n, u_n), \quad K_2 = f\left(t_n + \frac{h}{3}, u_n + \frac{h}{3}(K_1 + K_2)\right)$$

when applied to the test equation $y' = \lambda y, \quad y(t_0) = y_0$.

Question 3 [4+3]

a) Consider the following third order differential equation

$$y''' + 2y'' + y' - y = \cos t,\ 0 \le t \le 1, \quad y(0) = 0,\ y'(0) = 1,\ y''(0) = 2.$$

Transform the above differential equation to a system of first order differential equations and solve the system to approximate $y(1)$, $y'(1)$ and $y''(1)$ using the **explicit Euler method** with the time step $h = 1$.

b) A single step method for solving $y' = f(y),\ y(0) = y_0$ is given by

$$u_{n+1} = u_n + hf\left(\frac{1}{2}(u_n + u_{n+1})\right).$$

Find the **local truncation error** and the **order** of the given implicit method.

Question 4 [2+4+3]

a) Define the (i) **order** and (ii) **root condition** of a linear multistep method.

b) Show that the **order** of the linear multistep method

$$u_{j+1} + (\alpha - 1)\, u_j - \alpha\, u_{j-1} = \frac{h}{4}\left[(\alpha + 3)\, u'_{j+1} + (3\alpha + 1)\, u'_{j-1}\right]$$ is TWO if $\alpha \neq -1$ and is THREE if $\alpha = -1$. Find the values of α for which the **root condition** is satisfied.

c) Find $u(0.4)$ correct to 4 decimal places from the IVP:

$$\frac{du}{dx} = -2u^3, u(0) = 1,\ h = 0.1$$ using the following **Predictor – Corrector method**:

$$P: u_{j+1} = u_{j-3} + \frac{4h}{3}(2f_j - f_{j-1} + 2f_{j-2}),$$

$$C: u_{j+1} = u_{j-1} + \frac{h}{3}(f_{j+1} + 4f_j + f_{j-1}).$$

Calculate the starting values using the 4th order **Taylor series** method. Find the error $x = 0.4$ if the exact solution is given by $u(x) = 1/(1 + 2x)$.

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