# Problem Set - 8

**AUTUMN 2016**

**ANSWER/HINTS** MATHEMATICS-I (MA10001)

1.

(i) $M = y + \dfrac{y^3}{3} + \dfrac{x^2}{2}, \quad N = \dfrac{x}{4}(1+y^2)$

$\dfrac{\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}}{N} = \dfrac{3}{x}$, IF$= x^3$,

$f(x,y) = \dfrac{x^4}{4}\left(y + \dfrac{y^3}{3}\right) + \dfrac{x^6}{12} = C.$

(ii) $\dfrac{\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}}{N} = -1$, IF$=e^{-x}$,

$f(x,y) = e^{-x} + e^{-y} = C.$

(iii) $\dfrac{\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}}{N} = \dfrac{2}{x}$, IF$= x^2$,

$f(x,y) = \dfrac{x^3y^3}{3} + \dfrac{x^6}{6} + \dfrac{x^3}{3} = C$

(iv) $\dfrac{\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}}{M} = \dfrac{3}{y}$, IF$= \dfrac{1}{y^3}$,

$f(x,y) = x^2e^y + \dfrac{x}{y} + \dfrac{x}{y^2} = C.$

(v) $\dfrac{\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}}{-M} = \dfrac{1}{y}$, IF$= y$,

$f(x,y) = xy^2 + \dfrac{x^2y^4}{2} + \dfrac{y^6}{3} = C.$

(vi) $Mx + Ny \neq 0$, IF$= \dfrac{1}{x(x^2-y^2)}$,

$f(x,y) = \dfrac{x^2-y^2}{x} = C.$

(vii) $Mx + Ny = x^2y$, IF$= \dfrac{1}{x^2y}$,

$f(x,y) = \dfrac{-y}{x} + \ln y = C.$

(viii) $Mx + Ny = 0$,

$\dfrac{ydx - xdy}{y^2} = 0 \Longrightarrow d\left(\dfrac{x}{y}\right) = 0,$

So $f(x,y) = \dfrac{x}{y} = C$

2. Given DE $Mdx + Ndy = 0$. If $Mdx + Ndy = 0$ is exact, then $F = 1$. If $Mdx + Ndy = 0$ is not exact, then $F(x,y)(Mdx + Ndy) = 0$ is exact iff

$$\frac{\partial}{\partial y}(MF) = \frac{\partial}{\partial x}(NF) \Longleftrightarrow M\frac{\partial F}{\partial y} - N\frac{\partial F}{\partial x} + F\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = 0.$$

3.

$$M = e^{\int p(x)dx}\left[p(x)y - q(x)\right], N = e^{\int p(x)dx},$$

$$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}.$$

$$f(x,y) = e^{\int p(x)dx}y - \int q(x)e^{\int p(x)dx}dx = C.$$

4.

(i) IF $= e^{\frac{1}{x}}$, $y = -2\left(\sin\frac{1}{x} - \cos\frac{1}{x}\right) + Ce^{\frac{-1}{x}}$.

(ii) IF$= e^{x^2}$, $y = (x+1)e^{-x^2}$.

(iii) IF $= 1 + x^2$, $y = \dfrac{C + \sin x - x\cos x}{1+x^2}$.

(iv) IF $= \dfrac{1}{x^3}$, $y = x^3\left(e^x + \sin x\right) + 2x^2$

5.

(i) Set $z = \dfrac{1}{y} \Longrightarrow \dfrac{dz}{dx} + \dfrac{z}{x} = 1$, then $y = 1/(\dfrac{x}{2} + \dfrac{C}{x})$

(ii) Set $z = y^{-3} \Longrightarrow \dfrac{dz}{dx} + \dfrac{3z}{x} = \dfrac{3\cos x}{x^3}$, then $\dfrac{x^3}{y^3} = 3\sin x + C$.

(iii) Set $z = \dfrac{1}{y^{2/3}} \Longrightarrow \dfrac{dz}{dx} - \dfrac{2z}{3} = \dfrac{-2x}{3}$ then $y^{-2/3} = \frac{1}{2}(2x+3) + Ce^{\frac{2x}{3}}$