Mab

INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Date——FN/AN 2 Hrs. Full Marks: 30 No. of Students 90

Mid Autumn Semester 2015-2016 Deptt: MATHEMATICS Sub No: MA 40001/41007

——Yr. B.Tech.(H)/B.Arch.(H)/M.Sc. Sub. Name: Functional Analysis

Instruction: Answer all questions, which are of equal values

1. (a) Show that if X and Y are isometric, then they are homeomorphic. Give an example that a complete and incomplete metric space may be homeomorphic.

   (b) Show that the image of an open set under a continuous mapping need not be open.

2. (a) Show that l^∞ is not separable.

   (b) Lets X be the set of all continuous real-valued functions on $J = [0, 1]$, and let $d(x, y) = \int_0^1 |x(t) - y(t)|\, dt$. Then show that the metric space (X, d) is not complete.

3. (a) If X is a finite dimensional normed linear space and Y be any normed linear space, then show that every linear map from X to Y is continuous.

   (b) Let $T : D(T) \subset X \mapsto Y$ be a linear operator, where X and Y are normed linear spaces. Then show that T is continuous iff it is bounded.

4. (a) For $x = (\xi_j) \in l^p$, $y = (\eta_j) \in l^q$ where $p > 1$ and $\frac{1}{p} + \frac{1}{q} = 1$, then show that

$$(\sum_{j=1}^{\infty} |\xi_j \eta_j|) \leq (\sum_{k=1}^{\infty} |\xi_l^p)^{\frac{1}{p}} (\sum_{m=1}^{\infty} |\eta_m|^q)^{\frac{1}{q}}.$$

   (b) Show that a normed linear space X is a Banach space iff every absolutely convergent series of elements in X is convergent in X.

5. (a) Let $T : X \mapsto K$ be a linear operator (where X is a normd linear space over the scalar field K). Then show that T is continuous iff $Z(T)$ is closed.

6. State and prove the Riesz lemma.

The END