Assignment for submission - Last date 16-04-2019

Numerical Solutions of Ordinary and Partial Differential Equations

1. Find the solution of the BVP

$$x^2y'' - 2y + x = 0, \ y(2) = 0, \ y(3) = 0$$

using the shooting method. Use the Taylor series method of order 2 with $h = \frac{1}{4}$ to solve the resulting initial value problems (IVPs).

2. Use shooting method to solve the BVP

$$y'' = 6y^2, \ y(0) = 1, \ y\left(\frac{3}{10}\right) = \frac{100}{169}.$$

Use Taylor series of order 3 with $h = \frac{1}{10}$ to solve the resulting IVPs and the secant method for iteration. Take $s^{(0)} = -\frac{9}{5}$, $s^{(1)} = -\frac{19}{10}$ and perform two iterations of the secant method. Compare the numerical results with the exact solution $y(x) = \frac{1}{(1+x)^2}$.

3. Use shooting method to solve the BVP

$$y'' = (3/2)y^2, \ y(0) = 1, \ y(1) = 4.$$

Use Runge-Kutta method of order 2 with $h = \frac{1}{4}$ to solve the resulting IVPs and the Newton's method for iteration. Take $s^{(0)} = 0.9$, and perform one iteration.

4. Use second order finite difference method to solve the following boundary value problems:

   i. $y'' - 3y' + 2y = 0$, with $2y(0) - y'(0) = 1$, $y(1) + y'(1) = 2e + 3e^2$

   ii. $y'' = 2yy'$, with $y(0) = \frac{1}{2}$, $y(1)=1$

   taking $h = \frac{1}{2}$.

5. Using the standard five point formula to solve

$$u_{xx} + u_{yy} + 10xy = 0, \quad (x, y) \in (-3, 3) \times (-3, 3)$$

with $u = 10$ on the boundary of this square. Take step length $h = 1.0$ along x and y axes.

6. Use the Crank Nicolson method to solve the partial differential equation

$$\mathbf{u_t = 12u_{xx}},\ \mathbf{0 < x < 1},\ \mathbf{t > 0}$$

with the initial condition $u(x,\ 0) = x+1$ and the boundary conditions $u_x(0,\ t) = 0$ and $u_x(1,\ t) = 1$. Use the central difference approximation for discretizing the spatial derivatives in the boundary conditions. Take the step lengths $\Delta x = 1/4$, $\Delta t = 1/36$ and integrate up to two time levels.

7. Use the explicit and implicit (with $\theta = 1/2$) schemes to solve the hyperbolic equation

$$u_{tt} = 9u_{xx}$$

with the initial conditions

$$u(x,\ 0) = \sin(x) \quad \text{and} \quad u_t(x,\ 0) = -\cos(x)$$

and the boundary conditions

$$u(0,\ t) = -\sin\left(\frac{t}{5}\right) \quad \text{and} \quad u(1,\ t) = \sin\left(1 - \frac{t}{5}\right).$$

Take the step lengths $\Delta x = \Delta t = 0.25$. Solve for TWO time levels. Compare your results.

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