Symbolic Logic

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Lecture – 23
How to Apply the Rules of Inference in a Proof
Introduction to the Equivalence Rules

So, we are back and this is module number 23, where we are going to now learn to apply these *Rules of Inference* that we have learnt in a proof. So that we know how to derive new lines and actually establish the validity of an argument using this procedure. I am also going to include the *Equivalence Rules.* So far we have only talked about the Rules of Inference. But you also need some equivalence rules. So, this module would introduce you to the equivalence rules also. Now what are these… just reminding ourselves that we have in hand the Rules of Inference and so far we have been exposed to nine of these.

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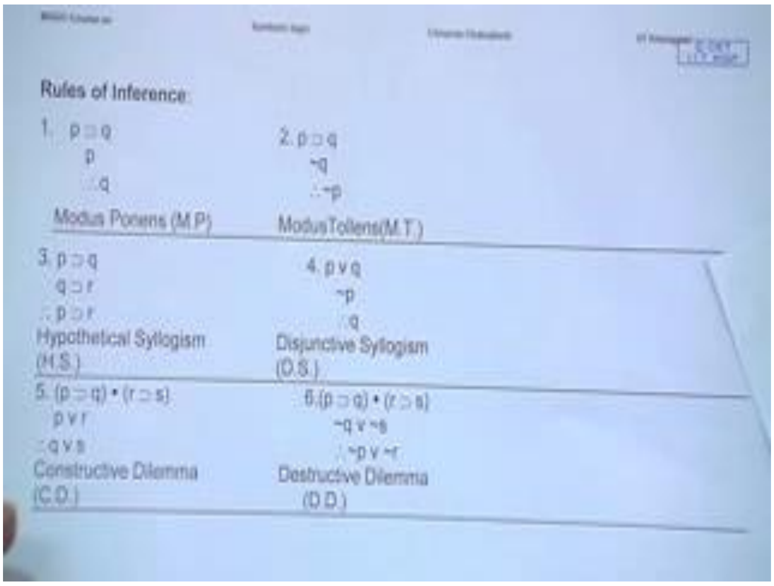

These 9 rules is going to see us through in… in a long way, in… in a major way they are going to see us through. But it is very important and you will see soon that you need to understand how these rules apply and where you can apply which one. Right? But slowly we are going to take it in. So, these are the nine rules we have in front of us.

We will keep this handy and will keep on referring to these rules. But let's start talking about the formal proof of validity.

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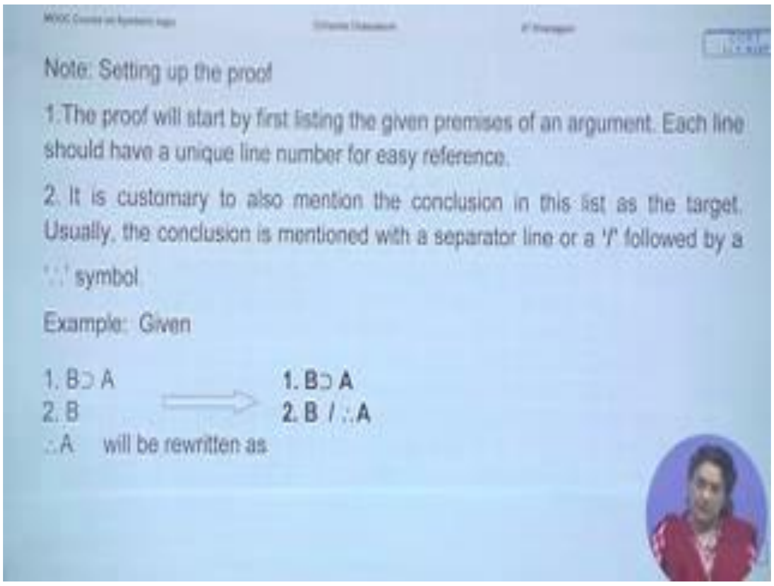

So, first how to set up the proof? The setting up means that you need to start the procedure. So, first of all there will be an argument that would be given to you, and you start by listing the given premises of those arguments. So, these would be given, right? And but each line is going to have a unique line number. So, you are going to start 1, 2, 3, etcetera. The conclusion also will be given because the whole point is that you don't have to find the conclusion. The conclusion will be given. All you have to do is to show why that conclusion can be claimed to be a valid consequence from the premises. How does it validly follow from the premises given?

So, given premises, given conclusion; all you need to do is to sort of set it up. So, usually what we do is to use a separator line like so: A slant. Followed by the 'therefore' symbol. Right? So, suppose you have given an argument like this, then for the proof you can set it up like this, where the premises are all given numbers. And this is where when the premises end, that's where you put the slant line and you put the triple dot, and here is the conclusion that is claimed. Whatever you do now next, is not given. There would be line numbers to this and so on. But this is how the given is set up. So, conclusion is stated here, the premises are also stated here, with line numbers. That's your beginning point and then comes how to go further. So, the derivation will proceed by adding new lines to those new already given lines as you know.

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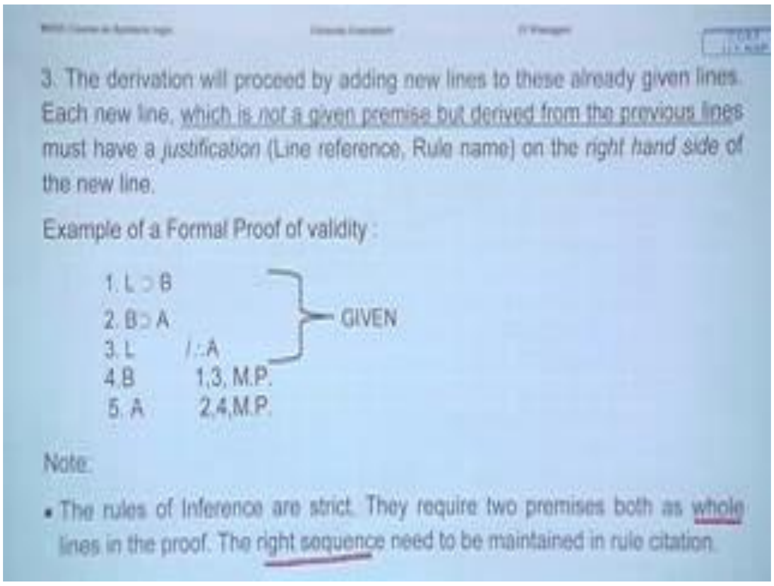

But every new line that you generate in a proof, you have to justify. Justify as what? Well, justify in terms of which line did you obtain it from, did you derive it from and which rule did you use? So, recall your knowledge of the truth tree and you will understand what we mean. That every new line that you add to the procedure, you are… you must be accountable. You are liable to refer back to the line that was… that you are using and the rule that you are using. Where would it show up? On the right hand side of the proof. Ok? So, we are going to now take a look in to actual examples.

So, suppose we have an argument like so. See? There are three premises lined up with line numbers. There is the slant line and here is the triple dot, followed by the conclusion of this. Whatever you now do is to show the derivation how from these, the conclusion A follows? That is what the formal proof of validity is going to be like. So, how do you go about it? Now you might think that… take a very good look at the premises. Automatically there should be some plan forming in your mind how you can get A. Right? Now, where is A in the premises? Right here, in the second premise which is a conditional statement. And you know, by now you should know, that there are rules that will help you to get that A out, provided you have the antecedent also.

In this case the antecedent is B, right? So, your target is if can I get B then I put it together with 2, line 2 and we can get A out. So, your now objective is how do I get B? Where is B here? Well, B is here and B is also here. So, once more, if you can have L ⊃

B, and L, you know you can get B out. Which rule helps that? That's the rule called *Modus Ponens*, right? And you happen to have L here given here in line number 3. So, if you have thought about it in this way, then it's a matter of just putting it all in lines. So, your line 4 would be B. How did you get that? From line 1 and 3. Notice the sequence, line 1 and line 3, because that is how the Modus Ponens works. You need to have the horseshoe statement first, and the assertion of the antecedent second.

So, 1, 3 and these commas are for separating the lines out. And here comes the rule name. Which rule have you used? The rule is Modus Ponens. That's how you got B. Am I done? No, look at the conclusion. This is where we need to go.

So, here is next line A. how did you obtain it? From line 2 and 4 you derived by using Modus Ponens. So, line 2 and 4. 4 is not a given line, 4 is something that you have derived. But that is perfectly alright. Anything that you legitimately derive becomes the automatically part of the proof. So, here is a formal proof of validity that shows.

If you just count how many… how many constants you have? 1, 2, 3. So, if you are doing by the truth table, how many rows would you be requiring? 8 rows minimum, right? 8 rows you need… you need to do the 8 rows truth table. Here you have done in two lines. And that should be an advantage that you should remember.

How do I know each line is correct or each line is true? The answer is because you are using valid argument forms: Modus Ponens. So, all these nine rules will become the basic guarantee that you are doing something where your truth is preserved and that is very important.

I will remind you once more what I have already said is that you are dealing with rules of inference which are rather rigid and formal in nature. Now they require that you pay attention to them. If they say that we need two premises, then you need to have two premises as standalone statements in your proof like here we did. $L \supset B$ was required and L is also required before you could do the B, derivation of B. So, just from $L \supset B$, you cannot have B out by this rule, right? And notice this, that L is a standalone line. It's not a part of another statement. So, that is something to also recognize and to remember when you are using the rules.

So, let's revisit the rule once more. Which rule did we use? We used this Modus Ponens which looks like this. That you have p ⊃ q and p, then you can pull q out. This is what we have used in this proof.

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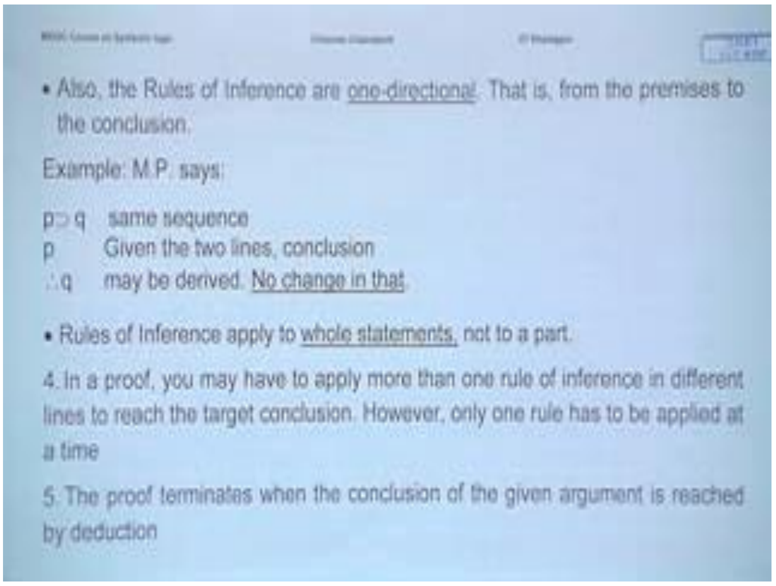

Let me now take you to this further that rules of inferences, remember, are one-directional. That is, from premises we go to the conclusion and that direction is not to be changed. That is, you cannot come from conclusion back to the premises. Ok? That's why we call them the Rules of Inference. You will soon find in… in as we introduce you the rules of equivalence that the Rules of Equivalence would be bi-directional. It does not matter of which side you are coming from. But Rules of Inferences are rather strict sequences. From the… given the premise you can go to the conclusion, not the other way around. Now here for example, Modus Ponens says that you have to have p ⊃ q and p. Given these two lines, the conclusion may be derived. There is no negotiation, no change is possible.

The second point which we will try to understand it also with example, is that rules of inference apply only to whole statements, not to a part. So, you have to have them as whole statement before you can apply. Which is what I have tried to explain to you. But we'll take this point again with an example to make this point more clear to you.

And the other thing is that it's not necessary that in a proof you can use only one rule. In fact, there are different lines as you gain experience you will see that there will be many lines in a proof and you can have a combination of various rules to reach your target as long as those are in your rule base. You cannot create a new rule or import a new rule. But it has to be within these nine rules that you have just learnt. But when you are applying, remember, in one step one rule at a time. On the whole in the proof you can use many, many rules. But in one step only one rule is permitted.

And then, since we defined what would be your starting point, namely, the given premises is our starting point, then when does the proof end? What is the definite end of the proof? The answer is when you have reached the conclusion of the given argument. Which is also given. The conclusion is given, right? So, that is where you stop.

So, this is our elementary understanding of how the proof works. We have a definite starting point. We have a definite end point. In between, I have tried to explain how to derive the new lines, right? Which would be with the valid argument forms or the Rules of Inference so far.

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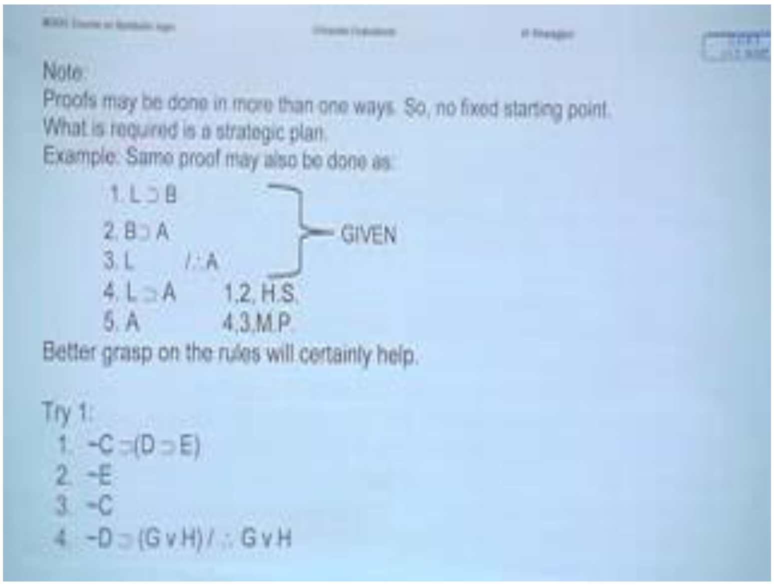

Nowcomes, what I would say, is additional, more information to process. A proof may be done in more than one ways. Different people think in different sort of way. So, the more advanced proofs, you will see that there can be multiple ways of solving the same problem. So, there is no fixed starting point. If you are lucky and the problem is very simple, then maybe there is a fixed starting point where you can start. But it does not have to be that way at all. So, what is required whenever you are doing a proof, I suggest that you first form a *strategy*. Sort of like the tree. You try to think ahead and think a little in steps before doing the thing. May be a rough planning, may be a rough work would be needed: What is it that you want to do in this proof? Now let me justshow you that first of all that the same proof may be done in more than one ways. So, let's take our earlier argument. This is what we encountered just now and this is how given.

Now, in… in what we saw the example we used the Modus Ponens twice. Right? But you can do it also like this. If you now see 1 and 2, you might find that there is an opportunity to apply the chain rule or the Hypothetical Syllogism. You have them in the right order. Let's visit the hypothetical syllogism. What does it say? That if you have p then q, you also have q and then r. Notice, notice the nice formation of this sort of zee or z that you have it like so, then you can derive $p \supset r$. So, let's take a look. Is this what we have in the argument? If so, here is a prime opportunity to apply the HS rule and it will give you $L \supset A$. But you need to know why you are applying this. What will you do with $(L \supset A)$? The answer is because I already have L and I can pull A out by Modus Ponens. So, it might work like this also.

So, this is another way of doing the same proof. We are using another rule. The more rules that you are… you have command over, the more rules that you have grasp over, the better. Because sometimes the rules can save you steps, sometime rules can give you more. Instead of using one rule if you use the other one you might have a more efficient proof. So, get acquainted very, very well with the rule base, the nine rules that we have learnt. Try to get it.

Shall we try a new problem together? So, here is a given argument. Can you construct a formal proof of validity? This one is, four premise and here is the conclusion $G \vee H$. What is the first point? This is already set up like so. You are going to add new lines here. But before that what is required , absolutely essential, is a plan, a strategy plan. How are you going to go so that this would be the last line of your proof? How can you derive $G \vee H$ from all of this? Take a good look at the premises and try to form a plan. Then, those of you who are already keen or, you know, have a hobby of puzzle solving, you may have already noticed something. Namely, that this $G \vee H$ is right here. So, if you can have $\sim D$, then pulling $G \vee H$ is no problem. Because, you know, Modus

Ponens will help you there. But not-D is not given anywhere, which means that not-D has to be somehow generated from all of this. So, now, our next question is how can we get not-D out? Where is D? D is right here. D is here, but what we need is not-D. Can you see any opportunity of getting a not-D? Well, if you recall your rules very well, this is why we say that you need to know your rules really, really well, you know that if you have… you… you have D ⊃ E that can be somehow separated out, pulled out and if you also have not-E , then there is a rule that will help you to get not-D. Which rule is that? Let's take a look into the rule base, and we find that the rule is called Modus Tollens, Modus Tollens or MT. What does it say? That if you have p ⊃ q and ~q then you can pull the negation of the antecedent out. So, in our case, this Modus Tollens application here would be possible provided we have ~E.

Do we have ~E? Yes, we have here. But can we apply it here? Because we said the rules of inference are all applied to whole statement. So, it cannot apply just to this part. So, therefore, we need to think about what to do about this ~C. Well, we find that 1 and 3 combined, ~C is already given. So, Modus Ponens will help you to get the D ⊃ E out as a stand-alone statement. Correct? Once you have that, you can plug in ~E, use Modus Tollens to get ~D. The moment you get ~D, you can get G ∨ H. Am I making sense here? If I am, that's very good; if not, listen to me slowly. But what is required is a plan. That was my plan : How am I going to do this. But you work with me so that you have your plan how to work on problems such as this. Now let's plug it all together and construct the formal proof of validity.

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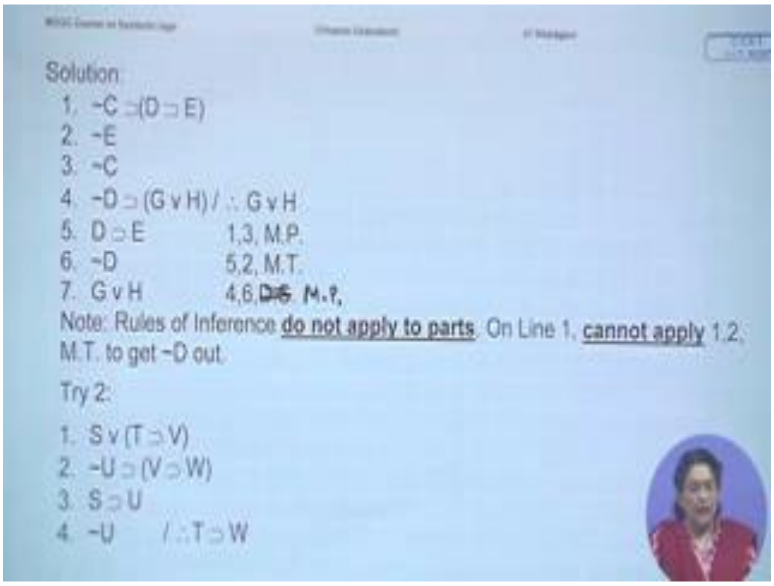

And your answer is going to sort of look like this. So, we are now putting everything that we have learnt together, putting it together. So, where do we start? We start where we wanted; namely, we are going to use 1 and 3 Modus Ponens and pull out D ⊃ E. Why D ⊃ E? So that we can use the ~E in line 2 get not-D out. So that's our next step. We are pulling not-D out by this rule called Modus Tollens.

This is Modus Ponens, this is Modus Tollens. And here comes then the simple idea of G ∨ H and by the rule that is you are using 4 and 6 and this is your classic Modus Ponens. Right? So, that's what will give you this result. Fine? Simple.

Again how many constants can we find? 1, 2, 3, 4, 5. So, it would have been a 2^5 rowed truth table for you to establish the validity. But here you have done it only by adding three lines, right? That is the formal proof of validity. And each of them is a valid argument form which ensures that in every step the truth has been preserved. If you started with truth, then you landed in truth, right? So, that's how it goes.

So, here probably is a chance to also appreciate when we said that the rule of inference does not apply to parts, so, on line 1 for example, you cannot apply line 2 directly onto this part D ⊃ E. Why not? Because, D ⊃E is a part of our whole sentence. So, until you have sort of operated on this horse shoe, this horse shoe is not accessible to you. Alright? So, this is why we have to do the first get the D ⊃ E as a standalone line, before we can do the Modus Tollens on line number 5 and get ~D out.

So, that's how the Rules of Inference work. Are we ready to try another example? So, let us see. Here, this is an argument and where you need to construct the formal proof of validity. Again what is required is some sort of planning. Don't go into proofs haphazardly, or with random steps, because then you are…it is likely that you will be laid astray or not finding your way and so on. Let's have a definite plan before you actually start the proof.

So, what is required is $T \supset W$. This is what we have to reach. Where is $T \supset W$? Anywhere? Nowhere it is given. Is there a chance to get $T \supset W$ anywhere? Can you see that? Can you spot that? Look at your premises. Those are your friends. So, look at your premises and try to locate the possibility of having $T \supset W$. Ok? And if you are keen and closely watching the premises, you might see that you don't have $T \supset W$. But what you have is close enough possibility in line number 1 and 2. Where? Here you have $T \supset V$ and here you have $V \supset W$, and that should ring a bell in your mind that here is a chance. If we can get $T \supset V$ and $V \supset W$ out, then the chain rule of the hypothetical syllogism will give us $T \supset W$. So, the point is how do I get $V \supset W$ separate standalone? How do I get $T \supset V$ separately out?

Now, for $V \supset W$ the problem is easy. Why? Because look at your premise, line number 2 and 4 combined application of Modus Ponens will give you $V \supset W$ separate. Got it? But $T \supset V$ is still not very clear. How do I get $T \supset V$ out? Well, you remember this $S \vee (T \supset V)$. That's a wedge right? If you somehow can get not-S, then there is a rule that will allow you to get $T \supset V$. That rule is called the Disjunctive Syllogism. This is why you need to know your rules thoroughly. This is the rule that we are referring to. If you have a wedge, and you also have the negation of the first disjunct, then this rule will allow you to get the second disjunct out. $p \vee q$ and $\sim p$, together you can derive q, right? So, this is the rule which we are plugging in here. $S \vee (T \supset V)$ and if we can get not-S then we can use the DS rule to get the $T \supset V$ out.

So, how do I get not-S? Well, that should be easy. Isn't it? 3 and 4, line 3 and 4. Just take a good look. Do you see how you can get not-S out? You are right! So, this is going to be a Modus Tollens opportunity to get the not-S out. Modus Tollens is this rule. Once more, $p \supset q$ and $\sim q$ therefore, $\sim p$, right? So, this is what we do. Alright. Now that sounds like a good plan to work on. Let’s do that. So, are you going to do it or shall I do it?

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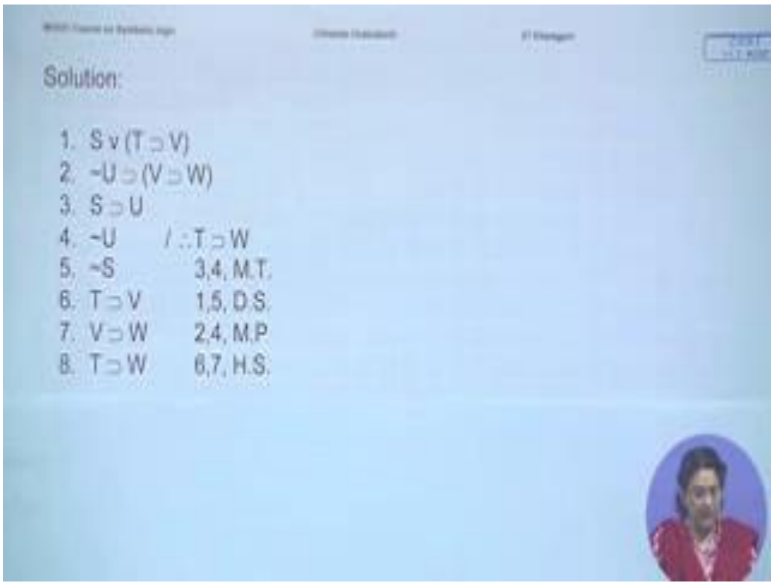

So, let’s see what it looks like when we have done it. So, here is the proof. That’s your beginning point and then you start wherever you want to start. I started out with getting the not-S out. How? From line 3 and 4 by Modus Tollens.

So, I already have the not-S out and that will give me T ⊃ V. On line 1 and 5 by DS and then V ⊃ W we got from 2 and 4 and this is my Modus Ponens, right? So, we have it now what we wanted, and line number 8 should be immediately clear to you. T ⊃ W from 6 and 7. The order is important. So, 6 and 7 and HS. The order, by order I mean the order in the rule. So, the rule has a certain way of the sequences given and you follow that also in your citation of the lines. So, this is a proof that we have ready. This is how the rules of inferences are applied.

Time has come to join or bring more rules into our rule base.

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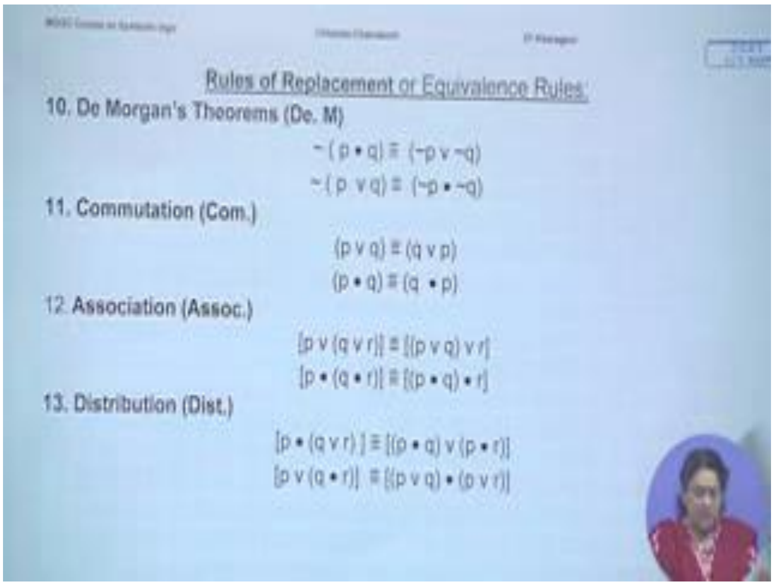

So, this is our Rules of Replacement, equivalence rules. First we will introduce you the rules. What they are. Some of them are already known to you. I don’t have to explain. But still we are all beginners, so, we’ ll go like this.

First set is De Morgan's theorems and we have it like this. If you have a situation with negation of the dot, you can rewrite it as a wedge with negation attached to each of these. So, what you have done? You have a negation of a conjunction; you can rewrite it as a disjunction. But there will be tilde attached to each of these components. Remember this is a triple bar, this is an equivalent rule. So, you can come from the left hand side to the right hand side, or from right hand side to the left hand side also, either way. So, equivalence rules are going to be bi-directional.

So, this is corollary of that. If you have negation of wedge, then you can convert it into a conjunction. But negations will attach to the components. Once more, from left to right, from right to left.

This is Commutation. This is the rule that you needed to change the position. So, but if you have wedge then you can write $p \vee q$, in place of it, you can replace it by $q \vee p$. Doesn’t matter. Similarly, if you have $p \bullet q$, you can rewrite it as $q \bullet p$. So, what you are doing? You are switching the position without changing the actual connective.

This is the equivalence rule. That's known as Association, this is Commutation. We' ll refer to is as Comm. This is Association, we' ll refer to it as Assoc. What does it do? If you have throughout wedges, then what it allows you is to regroup it. Take a look. This is $p \vee (q \vee r)$. This is still all the wedges are in place, but the grouping has changed. Now $(p \vee q) \vee r$. The emphasis has changed. Same goes for the dots. If you have throughout dot, you can change the grouping like so by the Association rule. We are going to revisit these rules more in our next modules. But let's go through this.

This is Distribution rule. This is known as: if you have $(p \bullet q)$, it sort of works like a multiplication, so, if you have $p \bullet (q \vee r)$, distributed it becomes $(p \bullet q) \vee (p \bullet r)$. If you want to understand mechanically, the inside connective become outside, the outside connective becomes inside, and you just multiply it through. Here is $p \vee (q \bullet r)$. It will become $(p \vee q) \bullet (p \vee r)$ and you can come back from both directions. So, these are just 4 of these rules of replacement. More, there are more rules of replacement. But, we will visit them in our next module and we will talk a little bit more about the equivalence rules how to apply them and so on. But this is where we will end this module here.

Thank you very much.