ASSIGNMENT-5 Solution

1. The difference table is

x	$y = \sin x$	Δy	$\Delta^2 y$	$\Delta^3 y$
30°	.5000			
		.0736		
			−.0044	
35°	.5736			
				−.0005
		.0692		
			−.0049	
40°	.6428			
				−.0005
		.0643		
			−.0054	
45°	.7071			
		.0589		−.0003
50°	.7660		−.0057	
		.0532		
55°	.8192			

Since $x = 32^\circ$ is near the begining of the table, we take Newton's Forward Formula

$$f(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{6}\Delta^3 y_0 + \cdots$$

Here $x = 32^\circ$, $x_0 = 30^\circ$ and $h = 5^\circ$ $\therefore$ $u = \frac{x - x_0}{h} = .4$

$$f(32^\circ) = .5000 + .4 \times .0736 + \frac{.4(.4-1)}{2} \times (-.0044) + \frac{.4(.4-1)(.4-2)}{6} \times (-.0005)$$

$$= .5000 + .02944 + .000528 - .000032$$

$$= .529936$$

$\therefore$ The difference table is

x	$y=f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1951	46				
		20			
1961	66		-5		
		15		2	
1971	81		-3		-3
		12		-1	
1981	93		-4		
		8			
1991	101				

Newton's forward difference formula

$$f(x) = f(x_0) + \frac{(x-x_0)}{h}\Delta y_0 + \frac{(x-x_0)(x-x_1)}{2!\,h^2}\Delta^2 y_0 + \frac{(x-x_0)(x-x_1)(x-x_2)}{3!\,h^3}\Delta^3 y_0$$
$$+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{4!\,h^4}\Delta^4 y_0$$

Now, $x = 1955$, $h = 10$

$$\Rightarrow f(1955) = 46 + 8 + \cdot 60 + \cdot 128 + \cdot 1248 = \boxed{54\cdot 8528}$$

Now, 1985 is near the end of table, we take backward form

$$f(x) = f(x_n) + \frac{x-x_n}{1!\,h}\Delta y_n + \frac{(x-x_n)(x-x_{n-1})}{2!\,h^2}\Delta^2 y_n$$
$$+ \frac{(x-x_n)(x-x_{n-1})(x-x_{n-2})}{3!\,h^3}\Delta^3 y_n + \frac{(x-x_n)(x-x_{n-1})(x-x_{n-2})(x-x_{n-3})}{4!\,h^4}\Delta^4 y_n$$

$x = 1985$, $h = 10$

$$\Rightarrow f(1985) = 101 + \frac{(-6)}{10}\times 8 + \frac{(-6)\,4\times(-4)}{2\times 100} + \frac{(-6)\times 4\times(14)}{6\times 1000}(-1)$$
$$+ \frac{(-6)\times(4)\times(14)\times(24)}{24\times 10000}\times(-3)$$
$$= \underline{96\cdot 8368}$$

1. (3) The difference table is

x	$y=f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1.0			
		.6		
5	1.6		1.6	
		2.2		.6
10	3.8		2.2	
		4.4		.6
15	8.2		2.8	
		7.2		
20	15.4			

Newton's Backward formula is

$$f(x) = y_n + u\,\Delta y_{n-1} + \frac{u(u+1)}{2!}\Delta^2 y_{n-2} + \frac{u(u+1)(u+2)}{3!}\Delta^3 y_{n-3} + \cdots$$

Here $x=21$, $x_n = 20$ and $h=5$

$$\therefore u = \frac{x-x_n}{h} = \frac{21-20}{5} = .2$$

$$\therefore f(21) = 15.4 + .2 \times 7.2 + \frac{.2 \times 1.2}{2} \times 2.8 + \frac{.2 \times 1.2 \times 2.2}{6} \times .6$$

$$= 15.4 + 1.44 + .336 + .0528$$

$$= 17.2288 \simeq 17.2$$

4. First we construct the divided difference table as follows

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
4	48	$\frac{100-48}{5-4} = 52$	$\frac{97-52}{7-4} = 15$	$\frac{21-15}{10-4} = 1$	0
5	100	$\frac{294-100}{7-5} = 97$	$\frac{202-97}{10-5} = 21$	$\frac{27-21}{11-5} = 1$	0
7	294	$\frac{900-294}{10-7} = 202$	$\frac{310-202}{11-7} = 27$	$\frac{33-27}{13-7} = 1$	
10	900	$\frac{1210-900}{11-10} = 310$	$\frac{409-310}{13-10} = 33$		
11	1210	$\frac{2028-1210}{13-11} = 409$			
13	2028				

Now, using Newton's divided difference formula

$$f(x) = f(4) + (x-4)\,\Delta f(4) + (x-4)(x-5)\,\Delta^2 f(4) + (x-4)(x-5)(x-7)\,\Delta^3 f(4)$$

$$= 48 + (x-4)\times 52 + (x-4)(x-5)\times 15 + (x-4)(x-5)(x-7)\times 1$$

$$= 48 + 52(x-4) + 15(x^2-9x+20) + (x^3-16x^2+83x-$$

$$= x^3 + (15-16)x^2 + (52-(15\times 9)+83)x + (48 - 52\times 4 + 15\times 20$$

$$= x^3 - x^2 = x^2(x-1)$$

$$\therefore f(2) = 4\times 1 = 4$$

$$f(8) = 64\times 7 = 448$$

$$f(15) = 15^2(15-1) = 3150 \quad \text{Ans}$$

5. We know that velocity is the rate of change of displacement

$$v = \frac{ds}{dt} \Rightarrow ds = v\,dt$$

$$\Rightarrow s = \int v\,dt$$

So, the distance moved by particle in 12 sec

$$s = \int_0^{12} v\,dt$$

Since $n = 6 \Rightarrow h = \frac{12-0}{6} = 2$

So, by Simpson's $\frac{1}{3}$rd Rule

$$s = \frac{h}{3}\left[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$$

$$= \frac{2}{3}\left[140 + 4 \times 134 + 2 \times 76\right]$$

$$= \frac{2}{3} \times 828$$

$$= 552 \text{ m.}$$

Here $f(x) = \dfrac{x}{1+x}$, $a = 0$, $b = 1$, $n = 6$ $\therefore h = \dfrac{b-a}{n} = \dfrac{1}{6}$

using Trapezoidal rule, we get

$x_0 \quad x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6$

$$\int_0^1 f(x)\,dx \approx \frac{h}{2}\Big[f(x_0) + f(x_6) + 2\big(f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)\big)\Big]$$

$f(x_0) = f(0) = 0$

$f(x_1) = f(1/6) = \dfrac{1/6}{1 + 1/6} = 0.14286$

$f(x_2) = f(\cancel{x}2/6) = 0.25$

$f(x_3) = f(3/6) = 0.33333$

$f(x_4) = f\left(\dfrac{4}{6}\right) = 0.4$

$f(x_5) = f\left(\dfrac{5}{6}\right) = 0.45454$

$f(x_6) = f(1) = 0.5$

so, we have

$$\int_0^1 f(x)\,dx \approx \frac{1}{12}\Big[0 + 0.5 + 2\big(0.14286 + 0.25 + 0.33333 + 0.4 + 0.45454\big)\Big]$$

≈ 0.305, correct up to 3-significant digits.

7.

Here $f(x) = \sqrt{\sin(x)}$, $a = 0$, $b = \frac{\pi}{2}$, $n = 6$ $\therefore h = \frac{\pi/2 - 0}{6} = \frac{\pi}{12}$.

using Simpson's $\frac{1}{3}$rd rule, we get

$$\int_0^{\pi/2} f(x)\,dx \approx \frac{h}{3}\left[f(x_0) + f(x_6) + 4\left(f(x_1) + f(x_3) + f(x_5)\right) + 2\left(f(x_2) + f(x_4)\right)\right]$$

$f(x_0) = f(0) = 0$

$f(x_1) = f\left(\frac{\pi}{12}\right) = 0.5087$

$f(x_2) = f\left(\frac{2\pi}{12}\right) = 0.7071$

$f(x_3) = f\left(\frac{3\pi}{12}\right) = 0.8409$

$f(x_4) = f\left(\frac{4\pi}{12}\right) = 0.9306$

$f(x_5) = f\left(\frac{5\pi}{12}\right) = 0.9828$

$f(x_6) = f\left(\frac{\pi}{2}\right) = 1.0$

$$\therefore \int_0^{\pi/2} f(x)\,dx \approx \frac{\pi}{36}\left[0 + 1.0 + 4\left(0.5087 + 0.8409 + 0.9828\right) + 2\left(0.7071 + 0.9306\right)\right]$$

≈ 1.187, correct up to 4 significant digits.

3.
Here $f(x) = 4x - 3x^2$, $a = 0$, $b = 1$, $n = 10$ $h = \frac{b-a}{n} = 0.1$

$f(x_0) = f(0) = 0$
$f(x_1) = f(0.1) = 0.37$
$f(x_2) = f(0.2) = 0.68$
$f(x_3) = f(0.3) = 0.93$
$f(x_4) = f(0.4) = 1.12$
$f(x_5) = f(0.5) = 1.25$

$f(x_6) = f(0.6) = 1.32$
$f(x_7) = f(0.7) = 1.33$
$f(x_8) = f(0.8) = 1.28$
$f(x_9) = f(0.9) = 1.17$
$f(x_{10}) = f(1) = 1.0$

using Trapezoidal rule, we get

$$\int_0^1 f(x)\,dx \approx \frac{h}{2}\left[f(x_0) + f(x_{10}) + 2\left(f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6) + f(x_7) + f(x_8) + f(x_9) + f(x_{10})\right)\right]$$

$$\approx \frac{0.1}{2}\left[0 + 1.0 + 2\left(0.37 + 0.68 + 0.93 + 1.12 + 1.25 + 1.32 + 1.33 + 1.28 + 1.17\right)\right]$$

≈ 0.995, correct up to 3-decimal places.

using simpson's one-third rule

$$\int_0^1 f(x)\,dx \approx \frac{h}{3}\left[f(x_0) + f(x_{10}) + 4\left(f(x_1) + f(x_3) + f(x_5) + f(x_7) + f(x_9)\right) + 2\left(f(x_2) + f(x_4) + f(x_6) + f(x_8)\right)\right]$$

$$\approx \frac{0.1}{3}\left[0 + 1.0 + 4\left(0.37 + 0.93 + 1.25 + 1.33 + 1.17\right) + 2\left(0.68 + 1.12 + 1.32 + 1.28\right)\right]$$

≈ 1.000, correct up to 3-decimal places.

(9.)

Here $f(x) = e^{x} + 2x$ $a = 0.1,\ b = 0.7,\ h = 0.1,\ \therefore n = \frac{b-a}{h} = 6$

$f(x_0) = f(0.1) = 1.30517$

$f(x_1) = f(0.2) = 1.62140$

$f(x_2) = f(0.3) = 1.94986$

$f(x_3) = f(0.4) = 2.29182$

$f(x_4) = f(0.5) = 2.64872$

$f(x_5) = f(0.6) = 3.02212.$

$f(x_6) = f(0.7) = 3.41375$

Trapezoidal rule:

$$\int_{0.1}^{0.7} f(x)\,dx \approx \frac{h}{2}\left[f(x_0) + f(x_6) + 2\left(f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)\right)\right]$$

$$\approx \frac{0.1}{2}\left[1.30517 + 3.41375 + 2\left(1.62140 + 1.94986 + 2.29182 + 2.64872 + 3.02212\right)\right]$$

≈ 1.389338

≈ 1.3293 , correct up to 4-decimal places.

Simpson's one-third rule:

$$\int_{0.1}^{0.7} f(x)\,dx \approx \frac{h}{3}\left[f(x_0) + f(x_6) + 4\left(f(x_1) + f(x_3) + f(x_5)\right) + 2\left(f(x_2) + f(x_4)\right)\right]$$

$$\approx \frac{0.1}{3}\left[1.30517 + 3.41375 + 4\left(1.62140 + 2.29182 + 3.02212\right) + 2\left(1.94986 + 2.64872\right)\right]$$

≈ 1.3885813

≈ 1.3886 , correct up to 4-decimal places.