Assignment 3

Transform Calculus (MA 20101)

Section 4

To be submitted on or before 23rd October 2017 (Monday).

Q1) Determine the Fourier series of the function

$$f(x) = |x|, \quad -3 \le x \le 3,$$
$$f(x) = f(x+6).$$

Q2) Determine the Fourier series for the function $f(x) = x^2$,

$$f(x) = f(x+2\pi), \quad -\pi \le x \le \pi.$$

Hence, obtain the value of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

** Q3) Determine the Fourier series for $f(x) = H(x)$, the Heaviside Unit step fn, in the range $[-\pi, \pi]$, $f(x) = f(x+2\pi)$.

Hence, find the value of the series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$

Q4) Given the half-range sine series

$$t(\pi - t) = \frac{8}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2n-1)t}{(2n-1)^3}, \quad 0 \le t \le \pi.$$

Use Parseval's theorem to deduce the value of the series $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^6}$. Hence, find $\sum_{n=1}^{\infty} \frac{1}{n^6}$.

Q5) Find the complex Fourier series for the f^n

$$f(t) = t^2 + t, \quad -\pi \le t \le \pi,$$

$$f(t) = f(t+2\pi).$$

Hence, obtain the usual Fourier series from it.

Q6) Let $f(t)$ be defined as

$$f(t) = \begin{cases} \pi^2, & -\pi < t < 0 \\ (t-\pi)^2, & 0 \le t < \pi. \end{cases}$$

& $f(t) = f(t+2\pi)$.

Determine the two Fourier half-range series for the f^n $f(t)$, & sketch the graphs of the f^n in both cases over the range $[-2\pi \le t \le 2\pi]$.

Q7) Show that $\int_0^\infty \frac{\cos \lambda x}{\lambda^2+1} d\lambda = \pi/2\, e^{-x}, \quad x \ge 0.$

(Hint: Use Fourier Integral Theorem)

Q8) Solve the integral equation

$$\int_0^\infty f(x) \sin(\alpha x)\, dx = \begin{cases} 1-\alpha, & 0 \le \alpha \le 1 \\ 0, & \alpha > 1. \end{cases}$$

Q9) Find the Fourier integral representation of the piece-wise continuous function

$$f(x) = \begin{cases} 0, & x < 0 \\ 1, & 0 < x < 2 \\ 0, & x > 2. \end{cases}$$

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