**Solution of Tutorial Problems set-III**

**Note:** All these problems can be solved using the results of Chapter-3.

**[0.0.1]** ***Exercise*** Can you construct a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^4$ such that $R(T) = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1 + x_2 + x_3 + x_4 = 0\}$ ?

**[0.0.2]** ***Exercise*** Can you construct a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ such that $R(T) = \{(x, y, z) \in \mathbb{R}^3 : x + y + z = 0\}$ ?

**[0.0.3]** ***Exercise*** Let $\mathbb{V} = \mathbb{R}^n$ and $A$ be a $n \times n$ matrix. If $Ax = 0$ has a unique solution $Ax = b$ has a unique solution for every $b \in \mathbb{R}^n$.

**[0.0.4]** ***Exercise*** Let $T : \mathbb{V} \to \mathbb{V}$ be a linear map such that $R(T) = Ker(T)$. What can you say about $T^2$?

**[0.0.5]** ***Exercise*** Let $\mathbb{V}$ and $\mathbb{W}$ be two vector spaces over the field $\mathbb{Q}$. $f$ is a map from $\mathbb{V}$ to $\mathbb{W}$ such that $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{V}$. Sow that $f$ is a linear transformation.

**[0.0.6]** ***Exercise*** Let $\mathbb{V}$ and $\mathbb{W}$ be two vector spaces over the field $\mathbb{R}$. $f$ is a map from $\mathbb{V}$ to $\mathbb{W}$ such that $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{V}$. Is $f$ a linear transformation.

**[0.0.7]** ***Exercise*** Let $f$ be a linear transformation from $\mathbb{V}$ to $\mathbb{W}$. If $S$ is a subspace of $\mathbb{V}$ then $f(S)$ is a subspace of $\mathbb{W}$. Moreover, if $x_1, \ldots, x_k$ generates $S$ then $f(x_1), \ldots, f(x_k)$ generates $f(S)$.

**[0.0.8]** ***Exercise*** Check that following are linear transformations.

1. Let $\mathbb{V}$ be the vector space of all convergent real sequence. $T : \mathbb{V} \to \mathbb{R}$ be defined by $T(x_n) = \lim\limits_{n\to\infty} x_n$
2. Let $\mathbb{D}[0, 1]$ be the set of set of all continuously differentiable function on $[0, 1]$ and let $T : \mathbb{D}[0, 1] \to \mathbb{C}[0, 1]$ defined by $T(f) = f'$.
3. Let $\mathbb{V} = \mathbb{R}^n$ and $\mathbb{W}$ be the subspace given by $\mathbb{W} = \{(x_1, \ldots, x_n) \in \mathbb{R}^n : x_n = 0\}$. Consider $P : \mathbb{V} \to \mathbb{W}$ given by $P(x_1, \ldots, x_n) = (x_1, \ldots, x_{n-1}, 0)$.