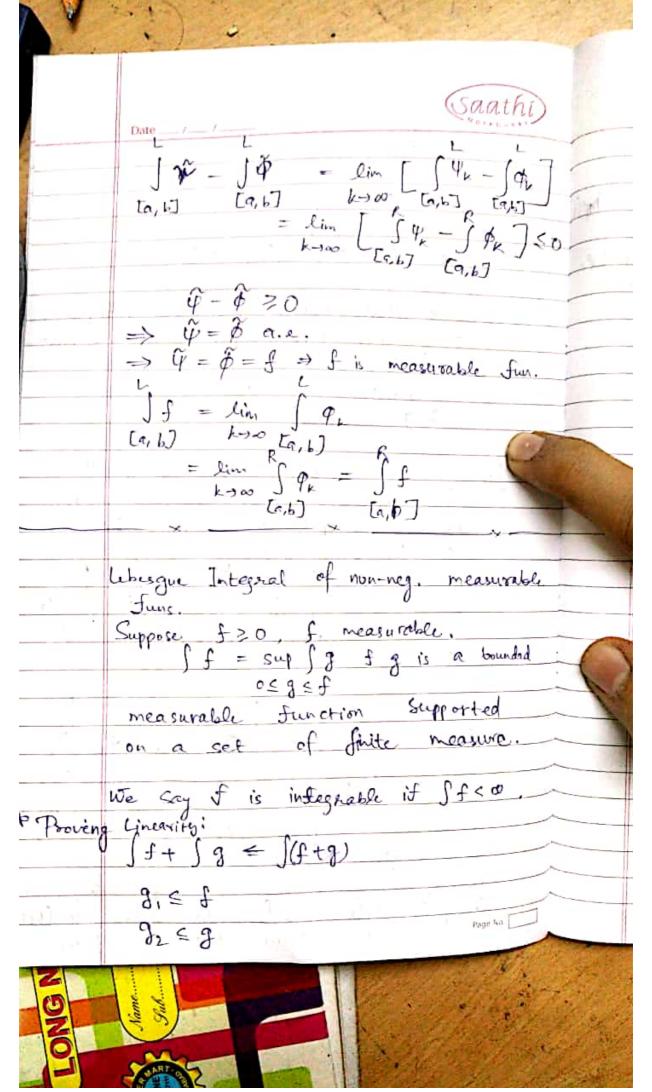
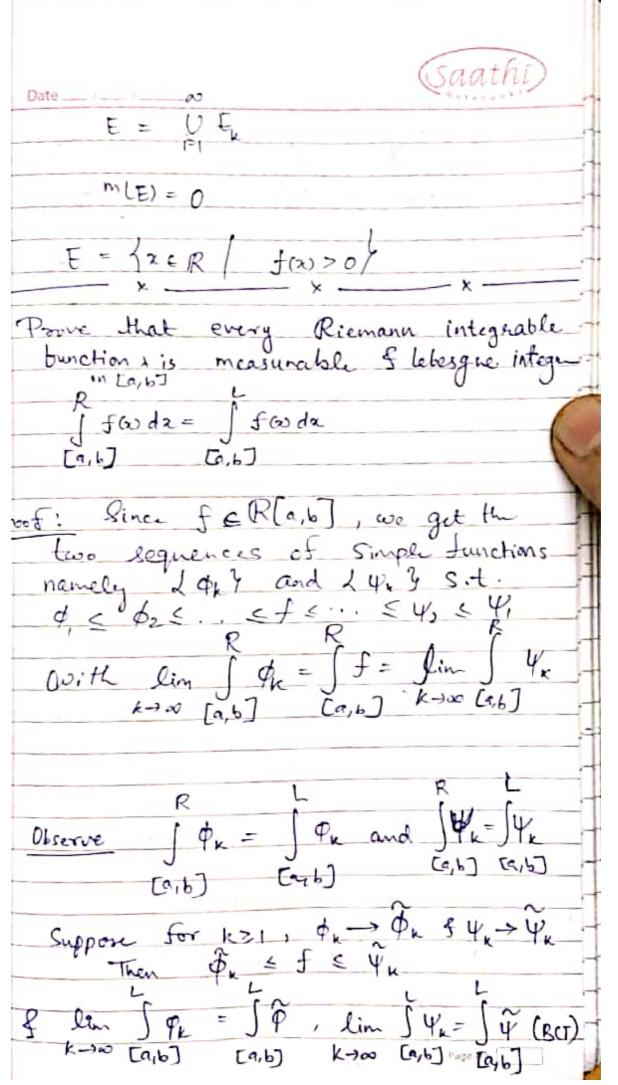
(Suath) Fatou's Lemma Suppose (for Be & Sequence of non-nog. measurable when St. In- of ptwin . The Stability . First sit g in a bounded measurable duration basing impost of finite measure st. 05g f. Let of = min of J. J. J. In's some Procesurable Simption having Sinte measure. 30-3 of none (persone) -, e2 e-, e2 e-5 2n = f. = 12" = 2t => Sq = le-inf So Jef in us. => \f \le \lim \hf \f \f_n

(Saathi) $\int_{g_1}^{f_2} + \int_{g_2}^{f_2} = \int_{g_1 + g_2}^{f_2} = \int_{g_1 + g_2}^{f_2} (f + g)$ Take supremum in the LHS Then, we have $ff+fg \in \int (4+g)-0$ Conversely, let 1 = f+g, n is non-n.

Idd mean having finite Supp. Set 1, = min (5, 79 n = n-n, Three 0 = 1, < f , 0 = 1, = 9 for non-neg functions. fn(x) =) n, ocx = 1/n 0 1 1/2 5 2 6 1 If = 1 for each n fr -> 0 ptwise. 1= Sf -> So=0 (f & lin inf Ifn





->
→ In must converge i.e. lim & exists.
> 6102 J=0 a.r. given
(ii) I, & (5) = 0 a.e. given
=) \f = 0
648
O. P. T. f is a non new measurable
Def : If f is a non-neg. I measurable function supported on a set of
Finite measure, then
$\int f = \lim_{n \to \infty} \int \phi_n$
7-100
where on's are Simple, Supp (Pm)
c c cc) e d of stuise.
Supp (f) & qn → f ptwise.
Proposition
() (af + bg) = aff + bfg (dinearity)
$(1) f \leq g \Rightarrow f \neq f \leq fg.$
TO (f - (f+)f, En P= q.
FUE E F
(IV) J J = J 1 1 1 1 1 1 1 1 1
E
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f>0 measureable, If(n) 1 < M ∀x∈E, Supp (f) ≤ E. We can find a sequence of simple functions on Such that supp (op) ∈ E ∀n & op→ f phosise (op) (or)

Lemna

Let f be a nonneg bounded measurable function supported on a set E of finite measure.

Let In -> f (9n's are as above).

Then () lim I 9n exists.

(i) If f=0 a.e., then lim fox =0.

Proof By Egorov's theorem. We can find a closed set AcE s.t. In - if uniformly on Az & m(E) Az) < E.

Define In = I on

In-Im | = [] (\$\frac{1}{2}n - \phi_m) |

\[
\leq \int \left[\frac{1}{2}n - \phi_m] + \int \left[\frac{1}{2}n - \phi_m] \]

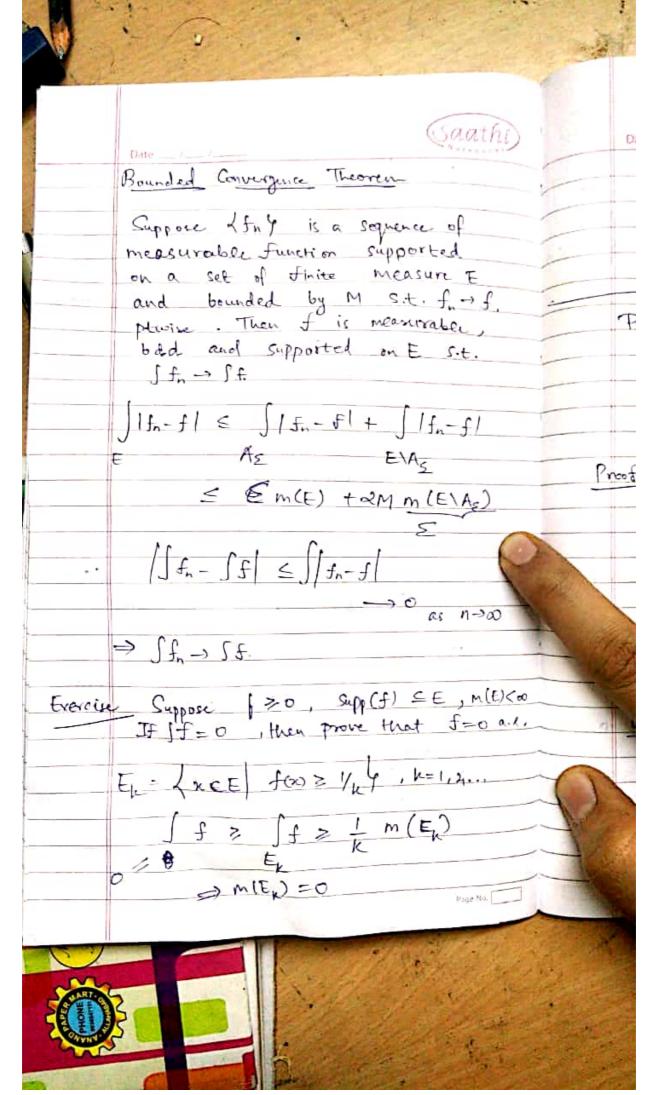
\[
\text{A}_E \quad \text{E\A}_E \quad \text{A}_E \quad \text{E\A}_E \quad \text{A}_E \quad \text{E\A}_E \quad \text{A}_E \quad \quad \text{A}_E \quad \quad \text{A}_E \quad \quad \text{A}_E \quad \text{A}_E \quad \quad \text{A}_E \quad \quad \quad \quad \quad \quad \quad \quad \quad \qu

< \(\int \left[\phi_n - \phi_m \right] + 2 ME
\(\Lambda \)

Choose Min large enough s.t.

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P = 5 CK XEK, CK + 0, EKN Ej = 6 lebesque integral. Jodx = : E Chm(E) -> Integral of bounded measurable functions supported on a set of finite measure Def ": (Support of f) = $\sqrt{x \in E} |f(x) \neq 0|^6$ * If f is measurable, then the Supp (f) is also So. Page No.



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