1. Structure Theorem for finitely generated abelian groups

(1) How many number of non isomorphic abelian groups are there of order 320. Write down the invariant decomposition and also the elementary decomposition for each of the groups.

(2) Let G and H be finite groups. Let $|g| = m$ for $g \in G$ and $|h| = n$ for $h \in H$. Then prove that $|(g,h)| = l.c.m(m,n)$ for $(g,h) \in G \times H$.

(3) Which pair of abelian groups are isomorphic from the list below, where the expression $\{a_1, \ldots, a_k\}$ denote the abelian group $\mathbb{Z}_{a_1} \times \ldots \times Z_{a_k}$.

$$\{4, 18\}, \{12, 6\}, \{72\}, \{36, 2\}$$

(4) Let G be a finite abelian group with invariant factor type $(n_1, \ldots, n_t)$. Prove that G contains an element of order m if $m|n_1$.

(5) Suppose that G is a finite abelian group that has exactly one subgroup for each divisor of $|G|$. Show that G is cyclic.