Lectione-1. Thursday - 7.1.16. Linear Algebra

F=
$$(F_x, F_y, F_z)$$
 $\vec{\alpha} = (\alpha_a, \alpha_y, \alpha_z)$
 $\vec{F} = m\vec{\alpha}$
 $\Rightarrow m\vec{\alpha} = \vec{F}$ $(\vec{m}, \vec{F} \rightarrow known)$
 $\Rightarrow (m \ o \ o \ m \ o)$
 $(a_x) = (F_x)$
 $(a_y) = (F_y)$
 $(a_z) = (F_y$

Grauss elimination method for solving system of canations

Ex1.
$$a_{11}=1$$
 $E_{1}: \chi + 2y - 3z = 4$
 $E_{2}: \chi + 3y + z = 11$
 $E_{3}: 2\chi + 5y - 4z = 13$
 $E_{4}: 2\chi + 6y + 2z = 22$
 $A = 1$
 A

Step-1: Eliminate a from E_2 , E_3 , E_4 , $E_i \rightarrow -\alpha_{ij} E_1 + \alpha_1 E_i$; i=2,3,4

E₁ unchanged
$$\chi + 2y - 3z = 4$$

E₂ $\xrightarrow{-1E_1+1.E_2}$ $q_{22}=1$ $+ 4z = 7$
E₃ $\xrightarrow{-2E_1+1.E_4}$ $q_{32}=1$ $+ 2z = 5$
E₄ $q_{42}=2$ $q_{4}=2$ $q_{4}=1$

Step-2. Keep 1st & 2nd earnations unchanged. Ei - - Ot2 Ez + O22 Ei ; i=3,4,

E₁ Unchanged
$$\chi$$
 + 2y - 3z = 4
E₂ Unchanged χ + 4z = 7
E₃ $\frac{-1 \cdot E_1 + E_3}{-2 \cdot E_2 + 1 \cdot E_3}$ $-2z = -2$
E₄ $0 = 0$

Step-3. Back substitution.

$$7=1$$
, $y=7-4z=3$, $\chi=1$
(1,3,1) \rightarrow unique solution.

2.
$$2x + y - 2z + 3w = 1$$

 $3x + 2y - z + 2w = 4$
 $3x + 3y + 3z - 3w = 5$

E₁ unchanged
$$2\pi + y - 2z + 3\omega = 1$$

E₂ $-\frac{\alpha_{21} \cdot E_1 + \Omega_{11} \cdot E_2}{-3E_1 + 2E_2}$ $y + 4z - 5\omega = 5$
E₃ $\frac{-\alpha_{31} E_1 + \Omega_{11} \cdot E_3}{-3E_1 + 2E_2}$ $y + 12z - 15\omega = 7$

$$E_{1} \xrightarrow{\text{unchanged}} 2x + y - 2z + 3w = 1$$

$$E_{2} \xrightarrow{\text{unchanged}} y + 4z - 5w = 5$$

$$E_{3} \xrightarrow{-\alpha_{32} E_{2} + \alpha_{22} E_{3}} 0 = -8$$

=> the system is inconsistent i.e. has no solution.

3.
$$2 + 2y - 2z + 3w = 2$$
 (E1)
 $2x + 4y - 3z + 4w = 5$ (E2)
 $5x + 10y - 8z + 11w = 12$ (E3),

Step-1 eliminate 2 from Ez & E3

E1 uncharged, $\chi + 2y - 2z + 3w = 2$ E2 $\frac{-2E_1+1.E_2}{}$ $\chi = 2w = 1$ E3 $\frac{-5E_1+1.E_3}{}$

Step 2. eliminate 7,

$$2+2y-2z+3w=2$$

 $z-2w=1$
0'=0.

y, w → free voviable,

(et w=1,
$$y=0$$
 $\Rightarrow z=3$
 $x=2+2z-3w=5$
(5,0,3,1) \rightarrow a solution of the system

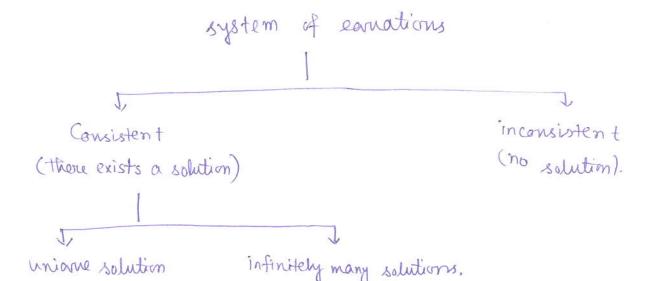
Let y=b, w=d Z=(+2w=1+2d) X=2-2y+2z-3w =2-2b+2(1+2d)-3d =4-2b+d (X,Y,Z,w)=(4-2b+d,b,1+2d,d).

$$2 + 2y - 2z + 3w = 2$$

 $-2w + z = 1$

n unknowns one there if there are rearrations in the reduced system. Then number of free variables = n-r.

In the above example n = 4 r = 2-. number of free variables = 4-2=2.



When there one more no. of unknowns than the earnations, then it is never possible to get a unique solution. The system will either have no solution or have infinitely many solutions.

Ex1.
$$A = \begin{pmatrix} 1 & 2 & -3 \\ 1 & 3 & 1 \\ 2 & 5 & -4 \\ 2 & 6 & 2 \end{pmatrix} \begin{pmatrix} \gamma \\ \gamma \\ \overline{\gamma} \end{pmatrix} = \begin{pmatrix} 4 \\ 11 \\ 13 \\ 22 \end{pmatrix}$$

After Gauss elimination

$$2 + 2y - 3z = 4$$

$$1 + 4z = 7$$

$$-2z = -2$$

$$1 + 4 = 7$$

$$0 + 4 = 7$$

$$0 - 2 = 7$$

$$7 = 7$$

$$7 = 7$$

Row operations

2.
$$R_i \rightarrow kR_i$$

3. $R_j \rightarrow k'R_j + R_i \leftarrow R_j \rightarrow k'R_j + kR_i$

$$\begin{pmatrix} 1 & 2 & -3 \\ 1 & 3 & 1 \\ 2 & 5 & -4 \\ 2 & 6 & 2 \end{pmatrix} \xrightarrow{R_2 \to -R_1 + R_2} \begin{pmatrix} 1 & 2 & -3 \\ R_3 \to -2R_1 + R_3 \\ R_4 \to -2R_1 + R_4 \end{pmatrix} \begin{pmatrix} 0 & 1 & 4 \\ 0 & 1 & 2 \\ 0 & 2 & 8 \end{pmatrix}$$

$$\begin{array}{c} R_3 \longrightarrow -R_2 + R_3 \\ R_4 \longrightarrow -2R_2 + R_4 \end{array} \left(\begin{array}{c} \boxed{1} & 2 & -3 \\ 0 & \boxed{1} & 4 \\ 0 & 0 & \boxed{-2} \\ 0 & 0 & 0 \end{array} \right) \quad \text{echelon modriz.}$$

Echelon moutrix: A matrix is an echelon moutrix of no. of Zeros proceeding the 1st nonzero element of each row increases row by row, until we arrive at the zero row, if there be any.

Distinguished element: The first non zero element in each row is the distinguished element of their row. In the previous example 1,1,-2 are the distinguished elements.

An echelon matrix is said to be row reduced

- (i) it all the distinguished elements are 1
- (ii) the distinguished elements in its nespective columns is the only non-zero elements.

$$\begin{pmatrix}
0 & 11 & 3 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 11 & 0 & -3 & 0 \\
0 & 0 & 0 & 0 & 11 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$
The reduce d eathelon matrix.

Rank of a mostrix.

Rank of a matrix A = no. of non-zero rows in echelon form.

Ext.

$$A = \begin{pmatrix} 1 & 2 & -3 \\ 0 & 1 & 4 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix}$$
 $A = \begin{pmatrix} 2 & 1 & -2 & 3 \\ 0 & 1 & 4 & -5 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
 $A = \begin{pmatrix} 1 & 2 & -2 & 3 \\ 0 & 1 & 4 & -5 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
 $A = \begin{pmatrix} 1 & 2 & -2 & 3 \\ 0 & 1 & 4 & -5 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
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 $A = \begin{pmatrix} 1 & 2 & -2 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
 $A = \begin{pmatrix} 1 & 2 & -2 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

Augmented moutrix.

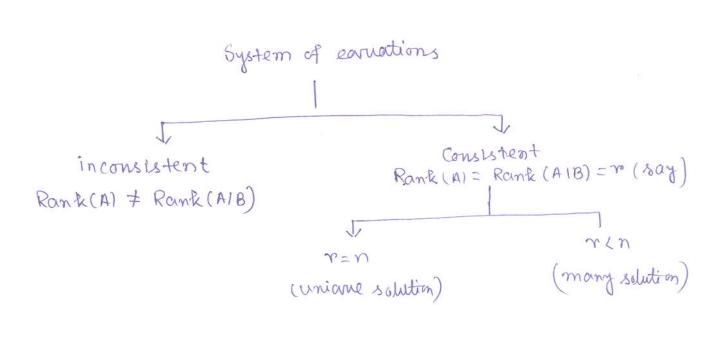
$$\frac{\text{Ex1.}}{(\text{A18})} = \begin{pmatrix} 1 & 2 & -3 & | & 4 \\ 0 & 1 & 4 & | & 7 \\ 0 & 0 & -2 & | & 2 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$$\frac{\text{Cank}}{(\text{A18})} = 3$$

$$(\text{A18}) = \begin{pmatrix} 2 & 1 & -2 & -3 & | \\ 0 & 1 & 4 & -5 & 5 \\ 0 & 0 & 0 & -8 \end{pmatrix}$$

$$\text{Pank}(\text{A18}) = 3$$

$$\begin{pmatrix} 1 & 2 & -2 & 3 & 2 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$
 $\text{ rank (AIB)} = 2.$



Homogeneous system of earnations.

$$x + 2y - 3z = 0$$
A homogeneous system is
$$x + 4y - 6z = 0$$

$$3x - 2y + 5z = 0$$
Since $(0,0,0)$ is a solution.

Either it will have unione solution i.e. the zero solution, or it will have many solutions.

Theorem In a homogeneous system of linear earnations, then the 18 there are more unknowns than earnations, then the system will has a non Zero solution.