Answer and hints of tutorial Sheet - 12

SPRING 2017

MATHEMATICS-II (MA10002) *January 2, 2017*

1. Answer $\frac{7}{12}$. Hint: Put $y = x^2$ and convert to integration of one variable. .
2. (a) Answer $-\pi$. Hint: Convert to parametric form $x = \cos t, y = \sin t, z = 0$. .
   (b) Answer -1. Hint: Draw the picture the picture of the triangle and do line integral for each side.
   (c) Answer 1. Hint: Break the problem in three parts e.g. from $(0,0,0)$ to $(1,0,0)$, form $(1,0,0)$ to $(1,1,0)$, form $(1,1,0)$ to $(1,1,1)$ etc. and do the surface integration on those parts and add.
3. (a) Hint: Show that $\nabla \times \vec{F} = 0$.
   (b) Answer $\phi = x^3y + xz^2 + c$ (where c is an arbitrary constant). Hint: Solve $\phi_x = 2xy + z^3, \phi_y = x^2, \phi_z = 3xz^2$.
   (c) Answer 202. Hint: $\phi(3,1,4) - \phi(1,-2,1)$.
4. Ans $-\arctan 3 + \arctan(-\frac{1}{2}) + \frac{1}{3}\arctan(\frac{1}{3}) - \arctan(-2)$. Hint: Show that $\nabla \times \vec{F} \neq 0$ and then calculate the integral.
5. Answer 90. Hint: Calculate $\hat{n} = \frac{\nabla(x^2+y^2)}{|\nabla(x^2+y^2)|}$ then $\vec{F} \cdot \hat{n}$. $dS = \frac{dxdz}{y/4}$ and do the surface integral over $x = 0$ to $x = 4$ and $z = 0$ to $z = 5$.
6. Answer 0. Hint: Do the surface integration on the disc $x^2 + y^2 = a^2$.
7. Answer $\frac{3}{2}$. Hint: Calculate surface integration for each of 6 sides and sum them up.
8. Hint: Calculate both the integration (first normally then using Green's theorem) and show they are equal. In the both cases the answer is $\frac{1}{20}$.
9. Answer 0. Hint: Calculate the integration by transforming it to surface integral using Green's theorem.
10. Hint: Use Gauss's divergence theorem to calculate the integral then compare with question 7.
11. Answer 60π.
12. Hint: Use Gauss's divergence theorem and the identity $\nabla \cdot (\nabla \times \vec{F}) = 0$.
13. Hint: Do the line integral in the boundary $x^2 + y^2 = 1, z = 0$ and then surface integral (using Gauss's divergence theorem) on the surface of the sphere. Finally compare those two values.
14. Answer 0.
15. Hint: Show $\nabla \times \vec{F} = 0$ then use Stokes' theorem.
16. Hint: For necessary part assume $\nabla \times \vec{F} \neq 0$ at some point and show the line integral is non zero by Stokes' theorem.