



Indian Institute of Technology Kharagpur

QUESTION-CUM-ANSWERSCRIPT

Stamp / Signature of the Invigilator

END-SEMESTER EXAMINATION

SEMESTER (SPRING-2018)

Roll Number											Section		Name	
Subject Number	M	A	2	0	1	0	4				Subject Name	Probability and Statistics		
Department / Centre / School														

Important Instructions and Guidelines for Students

1. You must occupy your seat as per the Examination Schedule/Sitting Plan.
2. Do not keep mobile phone or any similar electronic gadgets with you even in switched off mode.
3. Loose papers, class notes, books or any such materials must not be in your possession; even if they are irrelevant to the subject you are taking examination.
4. Data book, codes, graph papers, relevant standard tables/charts or any other materials are allowed only when instructed by the paper-setter.
5. Use of instrument box, pencil box and non-programmable calculator is allowed during the examination. However, the exchange of these items or any other papers (including question papers) is not permitted.
6. Write on both sides of the answer-script and do not tear off any page. Use last page(s) of the answer-script for rough work. Report to the invigilator if the answer-script has torn or distorted page(s).
7. It is your responsibility to ensure that you have signed the Attendance Sheet. Keep your Admit Card/Identity Card on the desk for checking by the invigilator
8. You may leave the Examination Hall for wash room or for drinking water for a very short period. Record your absence from the Examination Hall in the register provided. Smoking and consumption of any kind of beverages is strictly prohibited inside the Examination Hall.
9. Do not leave the Examination Hall without submitting your answer-script to the invigilator. **In any case, you are not allowed to take away the answer-script with you.** After the completion of the examination, do not leave your seat until invigilators collect the answer scripts.
10. During the examination, either inside or outside the Examination Hall, gathering information from any kind of sources or exchanging information with others or any such attempt will be treated as '**unfair means**'. Don't adopt unfair means and don't indulge in unseemly behaviour.
11. Please see overleaf for more instructions.

**Violation of any of the above instructions may lead to severe punishment.**

To be filled by the examiner

Question	1	2	3	4	5	6	7	8			Total
Marks Obtained											
Marks obtained (in words)				Signature of Examiner				Signature of Scrutinizer			

# Special Instructions

1. There are total **13** questions in this paper.
2. There are total **24** number of pages.
3. Final answer to each question or sub-question must be clearly written in the box provided.
4. Statistical tables may be used whenever appropriate.
5. Total marks: 50

**Rough work**

1. Let  $X, Y$  be independent and identically distributed  $N(0, 1)$  random variables.

(a)  $E(|X|) =$   [3 marks]

(b)  $E(\max\{X, Y\}) =$   [3 marks]

4. Let  $X$  and  $Y$  be independent and identically distributed random variables following  $\text{geometric}(0.6)$  distribution. Find the value of  $E(Y|X + Y = 5)$ . [3 marks]

Answer:

5. Suppose the joint density  $f(x, y)$  of the random vector  $[X, Y]$  is given by

$$f(x, y) = \begin{cases} x^2 + \frac{xy}{3} & \text{for } 0 < x < 1, 0 \leq y \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Compute

(a)  $E(Y) =$

[2 marks]

(b)  $E(Y \mid X = 0.5) =$

[2 marks]

7. Marks (out of 100) in a course on Statistics are normally distributed with mean  $\mu$  and standard deviation  $\sigma$ . Ten percent of students get more than 80 marks and twenty percent of students get less than 30 marks. Then

(a)  $\mu =$

[2 marks]

(b)  $\sigma =$

[2 marks]

(c) What is the probability that a student gets marks between 60 to 70?

Answer:

[1 mark]

8. A manufacturing plant has two machines for production. The machines lives (in months) have independent exponential distributions with mean 6 months.

- (a) The probability that a machine will work for at least a year is

[1 mark]

- (b) If both the machines are used in the production simultaneously, then the probability that the production will not stop at least a year is

[1 mark]

- (c) If the two machines form a series system, then the probability that the production will stop before a year is

[1 mark]

- (d) Suppose the machines are used serially (one after the other). That is, first machine 1 is used. When it stops working, then the machine 2 is used and when that fails, the production stops. Then the probability that the production will not stop for at least two years is

[2 marks]

9. A delivery truck is to be loaded with the refrigerators which are to be carried from the warehouse to the stores. The weight of each packed refrigerator is not fixed and follows distribution with mean 120 kilograms and a standard deviation of 500 grams. The delivery truck can accommodate 50 refrigerators.

(a) What is the approximate distribution of weight load in kilograms in the truck?

Answer:

[1 mark]

- (b) Suppose that the capacity of the delivery truck is 6025 kilograms and it is carrying 50 refrigerators, then what is the probability that the truck will be carrying load which exceeds its capacity?

Answer:

[2 marks]



10. Find a 95% confidence interval for the mean of a normal population with  $\sigma = 3$  for the given sample  $(2.3, -0.2, -0.4, -0.9, 1.3, 2.5, 2.1, -0.15)$ .

Answer:

[3 marks]

11. The number of marks scored by 10 randomly selected students in a very large class are 21, 37, 48, 6, 19, 31, 27, 32, 25, 39. Under the normality assumption, find the 90% confidence interval for the variance of marks scored by the students in the class.

Answer:

[3 marks]

12. Let  $X$  and  $Y$  be two random variables associated with a probability space such that  $Var(X + 2Y) = 40$  and  $Var(X - 2Y) = 20$ .

(a) Compute  $Covariance(X, Y)$ .

Answer:

[2 marks]

(b) If  $Var(X) = 2 Var(Y)$  then compute the correlation coefficient  $\rho(X, Y)$ .

Answer:

[1 mark]

13. Let  $X$  be a discrete random variable with the p.m.f

$$\begin{aligned}P(X = 0) &= \frac{2}{3}\theta, & P(X = 1) &= \frac{1}{3}\theta \\P(X = 2) &= \frac{2}{3}(1 - \theta), & P(X = 3) &= \frac{1}{3}(1 - \theta).\end{aligned}$$

where  $0 \leq \theta \leq 1$  is a parameter. Consider the following 10 independent observations that are taken from the above distribution

$$(3, 0, 2, 1, 3, 2, 1, 0, 2, 1).$$

- (a) Compute the estimate of  $\theta$  using the method of moments.

Answer:

[2 marks]

- (b) Compute the maximum likelihood estimate of  $\theta$ .

Answer:

[3 marks]