Symbolic Logic
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Lecture - 14
Using Truth Table:
Testing Arguments for Validity and invalidity

Hello, we are going to learn today a special technique to demonstrate invalidity specifically. Last time when we met, we were discussing the truth table technique to demonstrate validity and invalidity and that was what we would call a full truth table. So, constructing every single row and then demonstrating certain properties.

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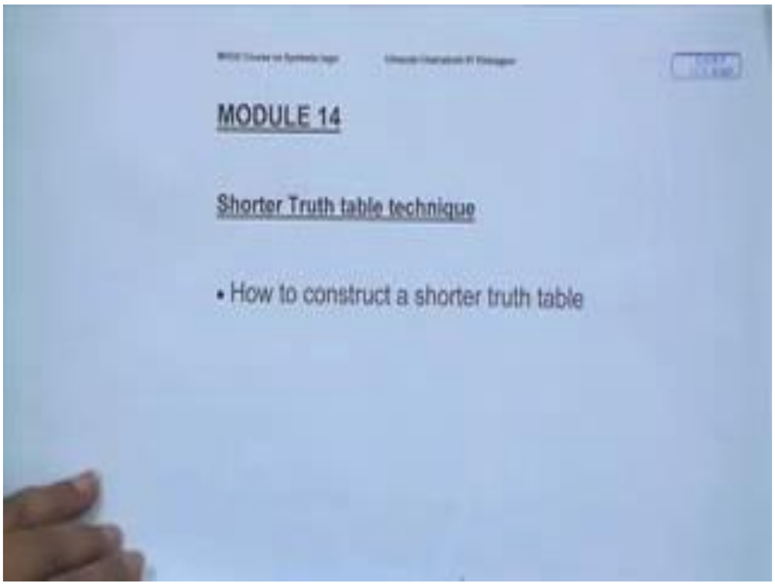

But today the task is to learn a special technique: That would be specifically, first of all, to *demonstrate invalidity*; second, it's not exactly what you have in mind when you have the full truth table. So, it's called the *shorter truth table*. And there is a reason that this is done in short, as if it is just a part of the truth table, but this is very effective for the task that is designated for. So, we are going to learn this technique; shorter truth table technique.

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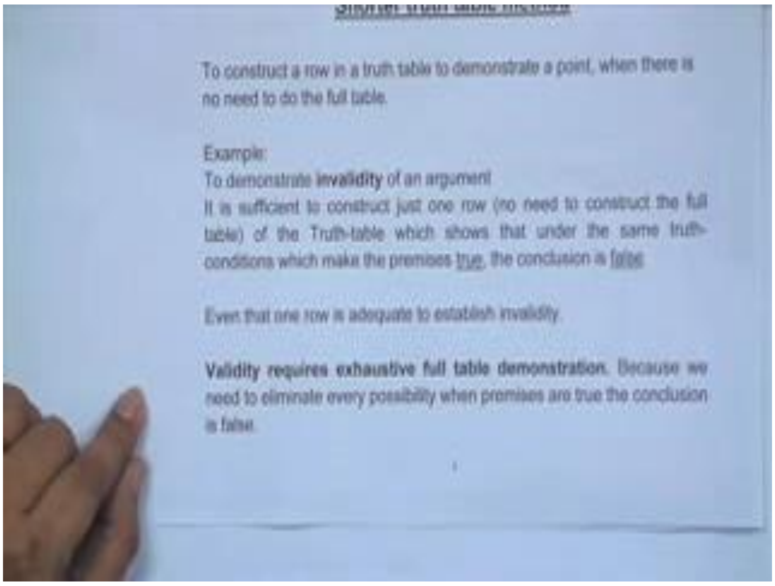

What, first of all, what does it do? How do you do it, is a separate question, but what does it do? The goal of this shorter truth table method is to construct *one particular row* in the truth table. See, when we learnt the full truth table,we were told that there is going to be rows and columns. Now, we are saying that the task of the shorter truth table is to construct a certain row. So, you are picking up one particular row, and you are constructing it with a certain objective. So, there is a point that you want to establish, and the goal of that row is to demonstrate that point.

When do we do this? Construct just one row? Obviously, when there is no need to have a full truth table in front. That is, this row in itself should be sufficient to demonstrate the point that you want to make. So, it depends very much on the problem at hand. What is it that you are trying to establish? And if the nature of that objective is such that by constructing one row; remember, one row means one possibility, or one situation. So, if that showing that one possibility or one situation demonstrates your point then shorter truth table is the method to follow instead of the whole complete truth table. There will be occasions and there will be required situations when you are bound to, you *must* construct a full truth table. But there are also other situations where the problem sort of deserves that you construct just one row, and the point is taken care of. And that latter kind of situation is where you are going to use the shorter truth table method.

So, one such point where the shorter truth table method can be extremely effective, if done correctly, is to demonstrate invalidity of an argument; invalidity of an argument. We have already been through this topic how you can demonstrate the validity and invalidity, as I said earlier. But specifically for invalidity of an argument, the shorter truth table method is extremely effective.

Let us remind ourselves what this invalidity is about. A deductive argument is invalid when you have all the premises as true, but the conclusion is still false. That we know. The point is: How many such cases do I need to demonstrate for the argument to be invalid ? And the answer is that it's sufficient to show even one case. Even one case, if you can establish correctly, that this is where the premises are all true, but the conclusion turns out to be false, that is enough to take care of invalidity of the argument. So instead of the whole truth table, it's sufficient if you construct just one row, where you show that under the same truth-value assignment to the components, the premises are coming out to be true and the conclusion is turning out to be false. Just that one row, remember, is sufficient to establish invalidity. So this is one situation where the shorter truth-table technique is going to be effective and also time wise I mean it's going to be efficient.

On the other hand, this is a way to remind yourself, remember, that validity, on the other hand, requires exhaustive possibility search. Validity means there is no possibility that you have the premises all true, but the conclusion is false. There is no such possibility. When you say there is no such possibility, or it is impossible, what you need to do is to eliminate all such possibilities. So, you need to show every single possible situation that there is no such situation available. This is the reason why validity establishment requires exhaustive truth-table or the full truth table. You can't establish validity by showing "here is one case where the premises are true and the conclusion is not false". Because you need to argue that there is no such case where this is going to happen. Ok? So, this is why, the elimination of every possibility requires that you show it exhaustively by a complete or full truth table. So, invalidity is possible, but validity you cannot show by shorter truth table technique. So, that is something to start with and then we move forward with examples to see how the shorter truth table works. But I hope the point has gotten through that this is for a task at hand, and this is your invalidity. Validity we will not be able to demonstrate with shorter truth table method.

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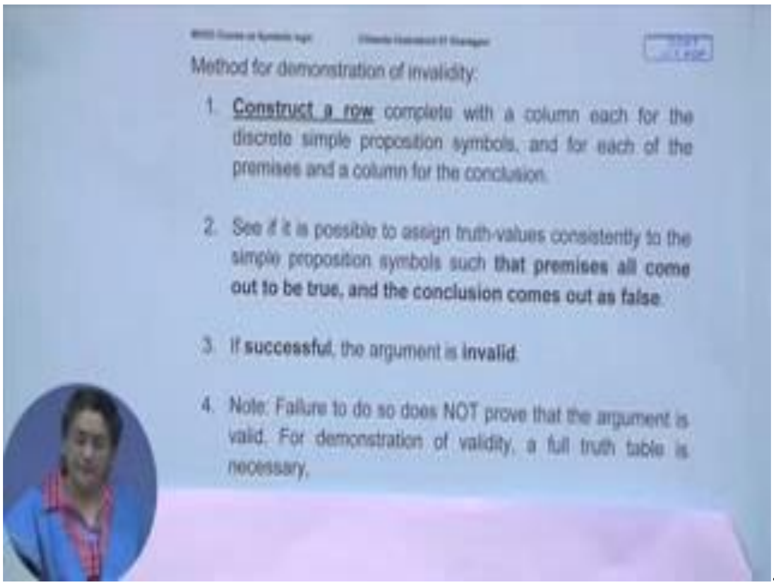

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Next is how do we do this? So the purpose of the shorter truth table technique that we are learning is clear, that we are going to do it for invalidity, at least in this module. The question is then how? How do I do construct this row?

Well, first that get used to the idea that what you are doing is almost like picking up one row in the truth table, but you are doing it; it's not blindly done. In full truth table it is almost blindly done. You know how many rows are going to be there, you distribute the truth values, you then compute the value of the sub connectives, the main connectives. Automatically the rows are formed and rows are filled out. That's not how it is going to happen here. You are constructing a row. The row is going to have first of all the reference columns. So every discrete simple proposition symbols whether it is variable or whether it is a constant, you are going to assign a column each, a reference column each for these symbols, and then slowly you pick up the premises and assign them each a column and a column for the conclusion. Alright?

So, this is the first thing and these are going to be the heading of your rows. We will show you actual examples. But try to get used to seeing it conceptually first. So, that's your first task. Reference columns and then assigning one column each for the premises and a column for the conclusion. Fine? You do not need to break them up into further shorter or the smaller components.

Then comes the major thing; the major thing, the major task. Once you have laid out the row like this, your job is to see: How can I assign truth values in a consistent manner to the propositional symbols; that is, to the variables and constants, so that the premises all come out to be true and the conclusion turns out to be false?

Your goal is to make the premises true and the conclusion false. But remember these are going to be your compound statements. So, the idea is that you assign truth values to the simple propositional symbols in such a way that the premises all come out to be true and the conclusion turns out to be false. The whole task is, the challenge is whether you can do this or not. If you can do it, then the argument is *invalid*. Alright? So, if you are successful in turning the premises true and the conclusion false by some consistent value assignment to the symbols, then the argument is invalid.

But what if you fail? Does that mean the argument is valid? Well, that is a further question that you need to sort of understand. If you are successful, the argument is invalid. But if you fail to do that, does that automatically show that the argument is valid? The answer is: Validity requires something more than that. Validity is not just a failure of invalidity. In order to demonstrate validity, you are going to need to develop the complete truth table. As I said earlier, you have to have an exhaustive possibility elimination in case of validity. So, for that you need to have to develop a full truth table. Alright?

So, once more, then what is it that we are doing? One, shorter truth table means that you construct a row. A row in a truth table which shows what? That there is at least one truth value assignment possible to the simple propositional symbols, which will make the premises true and the conclusion false. And that *one* possibility; that there exist one such possibility, is good enough to establish invalidity of the argument. So, if you are successful in showing this, then you have shown the argument to be invalid. But demonstration of validity, as I said, is not going to be proven by the shorter truth table. You are going to need the full truth table. Is that clear?

So, if that is clear then we can go into the problems, the actual problems and we will try to do it together.

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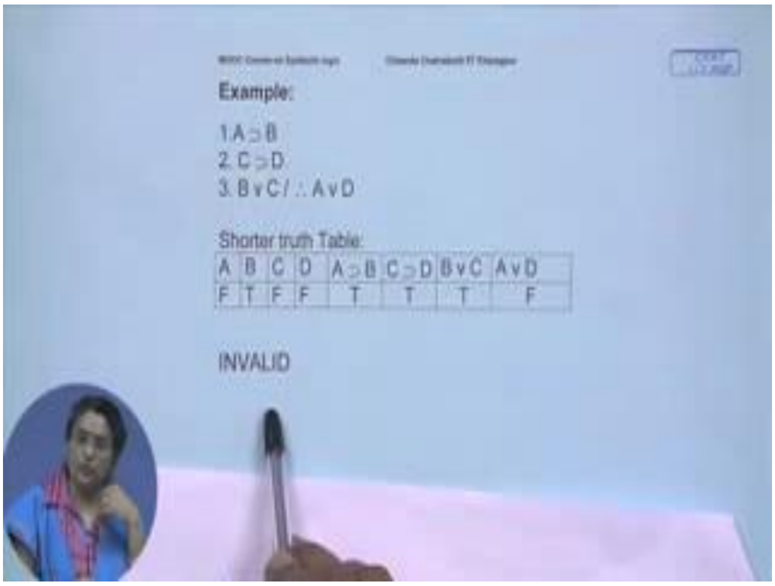

So, here comes an argument. We have all the constants here. So they mean certain actual propositions in the world. These are the three premises and here is your conclusion. What is the task at hand? To construct a row. See, normally if you are doing in full truth table, how many rows you are going to need? One, two, three, four. So, 2 to the power 4 is 16 rows truth table. In shorter truth table, what is it that you are doing? You are constructing only one row. And the heading of that row is going to be like this, that we have the reference columns like this. So, a, b, c, d alphabetically arranged, then we pick up the premises one by one $A \supset B$, $C \supset D$, $B \vee C$. And we assign a column each, and here is your conclusion. And let us remind ourselves, Ok that is laying it out, the row, but what is the objective? The objective is to assign truth values to A B C D in such a way that each of these turns out to be true; $A \vee D$ comes out to be false. Get me? (Refer Time: 13:49).

So if that is the case, then we are going to figure out what values we are going to do. So, let's start. Where do you think we should start? Well, you know that $A \vee D$ has to be false, because that is the conclusion. We want to make it false. But $\vee$ or disjunction is false when both the disjuncts are false. When that happens, you know what value A and D are going to have. A is going to be false, D is going to be false. Get me? So, you can copy those values under A and D. You know now what value A and D are going to have. Let's copy that there. Once that is done, you know that this A is going to be false, and this D is going to be false. Right?

Now, you might argue that if this D is false; C has to be false. Why? Because, our goal is to make each of the premises true. If D is false and C becomes true, then the premise will become false, because horseshoe is false when antecedent is true, consequent is false. So, C has no other choice but to be false. We know that C is false, so we can copy it here. These values that you are picking up or finding out here, you should copy it here in the reference columns. So, now, we have fixed that the value of C is going to be false. D is already false, A is already false. Right? So now let's plug that in. We have B $\vee$ C and C is already false. Now if B becomes false, then F $\vee$ F, B is false, C is false, then the disjunction will become false. So B has to be true. Right? So B has to be true, because C is already false here. So we will copy that here that B has to be true.

So, let's now plug everything in and let us see whether we have accomplished this. Again, remind yourself, we need to make premise one, premise two, premise three true; conclusion false. So, let us see. A is false, we knew that; and B is true which we found out. Is A $\supset$ B true? The answer is: Yes! Right? And then C is already false, we knew that. So, there you are. This is true and B $\vee$ C we already computed that B is true then B $\vee$ C is going to be true, and this one is false. So, let us see. This is how the row is going to look like. How we are doing this? I just showed you some arguments or reasoning process the way you can go back and forth to see what values the simple propositions will have. You can do it in some other way, but the point is that the goal is accomplished. We have shown that there is at least one truth value assignment on which the premises are all true, conclusion is false. What we did, you might call it a backward calculation, we go from what we need here to what values the components must have and that is perfectly legitimate. That's perfectly logical, because your need is what defines what values are going to show up here. Is this process clear?

Now, what we accomplished, is to show by the single row that this given argument has to be invalid. Under what truth value assignment? Under this truth value assignment. Is this enough to establish that this is invalid? Yes, even that *one* possibility shows that the argument has to be invalid. Right? So, this is what shorter truth table looks like, and I remind you that the same result you would have obtained by doing the whole sixteen row truth table. And there could have been some other cases also, but how many cases do you need to establish the point? The answer is: One. Even one will do, so here is this one, and we have shown the point to be taken. Ok?

So, this was our first example. I hope there are no queries, because we will do more of this. The more you practice, the more you sort of work this kind of problems, the clearer are the conceptions are bound to be. So, let us see. This is our result of the first problem done, that this is shown to be invalid by shorter truth table technique. Right?

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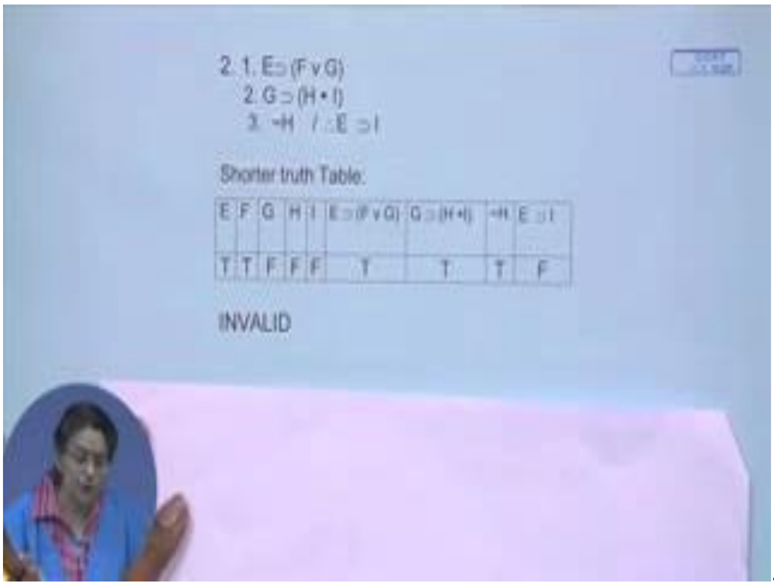

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Let us now go into another one; second problem. Second problem . And I expect now that since you have seen how the first one was done, that you start doing it on your own. So, once more you have three premises. One of the premises is ~H, and here is your conclusion E ⊃ I. What is your objective? To assign truth values to this simple components in such a way that the premises come out be true, conclusion turns out to be false. That’s all.

Normally if you would be doing the full truth table, this is going to be how many rows? 2 to the power 5; 1, 2, 3, 4, 5; 5 components. So, 32 rows of truth table. Instead, if you are, if you are lucky or successful then you can save a lot of work by constructing that one definitive row, and that’s what you are trying to do. So, let us set it up. You know what to do. First the reference columns, and alphabetically ordered propositional symbols, and then a column each for the premises and then one column for the conclusion. Set it up, and let me show you the result so that you can compare with your work.

Alright, let’s see whether you have done like this. So here is the shorter truth table for this problem and the headings are going to look like this. We have E F G H I lined up as your reference columns, and then each of the premises and here comes the conclusion. You know the task at hand. You have to assign values in such a way this is true, this is true, this is true, but this is false. Ok? You can start wherever you want to, as you wish. But find out at least one truth value assignment which makes this argument invalid. That's your task. Right?

So, by now I am sure you have figured out the strategy; but may be this is one of the ways to do it. So, you know just by looking at it what value H must have. What is it? ~H has to be true, because it is one of premises. Therefore, the value of H is going to be what? False. So, you know H is going to be false, wherever H is occurring H is bound to be false. Otherwise ~H will not be true. So that much you know. You also know what value E and I are going to have. Why? Because, horseshoe (⊃) is false only in one condition when antecedent is true and consequent is false. So, E has to be true. Wherever E occurs you know that E has to be true and I has to be false. So wherever it occurs, you know I has to be false. E has to be true, I has to be false, H has to be false. This much you know. And I will give you one moment to find out what value G and F should have. Plug it in and I will show you the result in just one moment.

You know that H has to be false. So this truth value of (H • I) is going to be false no matter what. Right? So, G has to be false; otherwise this ⊃ will become false. You do not want that, you want this premise to be true. Ok? So, now, you can plug in the value G here. Remember E is true, right? And G is, already you know, it has to be false. So in order to save this horseshoe ⊃ from being false, because E already true, F has to be true so that T ∨ F becomes true and your horseshoe is saved. So, if you, remember that's why I said, you know, the connectives truth table should be completely at the finger tips. You should remember this, because these are all applications of that old knowledge about the connectives, and unless you know them really pretty well, then the application problems will show up even at this stage.

So, let us see. Is this what you have? Is this how you computed the shorter truth table? So, here is a set of truth-values given to the propositional symbols, which make each of this premises true, but the conclusion turns out to be false. That is a definitive one row that proves what? That the argument has to be invalid. Remember to write the result.

The truth tables, whether it is short, whether it is full, they are all decision procedures and they are supposed to show something. And the result should be clearly claimed.

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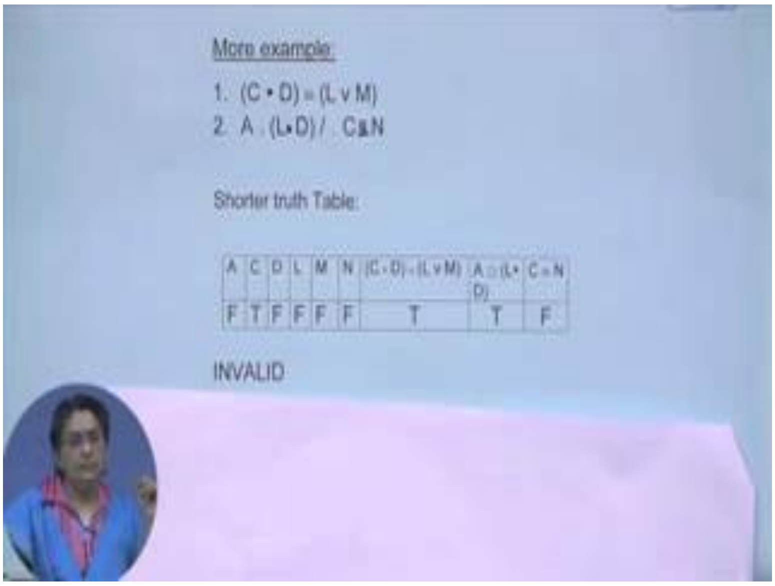

So here is it, we are saying that what it shows is that the argument given is invalid. Ok?

Finally, the last example here, and this time I think you should take the lead and try to set it up as soon as possible. This one has two premises and there is this conclusion. I am sorry; this is the conclusion that says C ≡ N, C ≡ N. So, here you are, and this is a dot. Fine? So, this is the problem. But the reason I bring that in, in addition to the previous two, is that otherwise you will have the wrong idea that it is pretty easy for shorter truth table. See, here the conclusion is false under two conditions; more than one condition. Either C can be true or N can be false. That's one. Or, C is false or N is true.

So, there are more than one way probably to work this truth table. Do I need to show all of those ways? The answer is no. Your goal is to show at least *one* truth-value assignment. So, just showing that one is good enough; and that is what we will try to do. So, I will give you half a moment to set it up as we have done earlier. This is shorter truth table for this problem, and here is your heading of the shorter truth table. Again you have one two three four five six, right? It's pretty …. if you are asked to do the full truth table, it would have been really long one. And here is premise number one, premise number two and here is your conclusion. You know that you have to make the conclusion false.

Please try it out. Which of these conditions would fit in? Which would make it all nicely true and this one false? Which conditions? You have two conditions where C ≡ N can be false. C can be true, N can be false. Will it work for the rest of it? You need to try that out. C is false, N is true. Again will that workout? So work it out. But ultimately you need to produce just one row, one row like that. So, a little bit of a decision is needed here which one is more effective in this case. But finally you will come up with only one.

So, here you are and remember whatever value you are picking up here, you are going to repeat here, and wherever they occur. For example, here you have C, and N does not appear anywhere here so that is rather free for you to do this. And if you try a little bit then you will see, this is the group of truth-values that will work out for you. Clearly this is going to make the premises true and conclusion false. The one value said that this one uses is where C is true and N is F. Ok? So this is how we construct shorter truth table. We are going to utilize this knowledge in our next module also for other task. Here we just learnt how to demonstrate invalidity of an argument by shorter truth table technique.

Thank you very much. This is where we will end.