Department of Mathematics
Indian Institute of Technology Kharagpur
Mid Semester Examination

Date of Exam.: .09.15 (FN/AN) Time: 2 Hrs . Full Marks: 30 No. of Students: 60
Subject Name: Probability and Statistics Subject No.: MA 41009
1^{st} yr M.Sc. (Maths.) & others

Instructions: Answer all questions. Marks are indicated at the end of each question. All parts of a question must be answered at one place.

1. (a) A random variable X has MGF $M_X(t) = \left(\frac{1+5e^t}{6}\right)^6$. Find $E(X)$, $Var(X)$, $P(X = 3)$.
   (b) A die is thrown until 5 appears. What is the probability that it must be thrown more than 5 times.
   (c) Suppose a survey of 100 computer installations in a certain city shows that 75 of them have atleast one brand Z computer. If four of these installations are chosen at random without replacement, what is the probability that each of them has atleast one brand Z computer.
   (d) Let $X \sim N(10, 100)$. Find $P(Y \leq 101)$, where $Y = X^2 + 1$. Here $\Phi(2) = 0.9772$, $\Phi(1) = 0.8413$.
   (e) If X is a random variable such that $E(X) = 3$ and $E(X^2) = 13$, use Chebyshev's inequality to determine a lower bound for the probability $P(-2 < X < 8)$.
   (f) Let $A_k = \{(x, y) : 0 \leq x^2 + y^2 \leq \frac{1}{k},\ k = 1, 2, 3, \ldots\}$, find $\lim_{k\to\infty} A_k$.
   (g) In a league tournament with 10 teams, every team plays every other team once. The winner of a game gets two points, the loser gets no points and in the case of a drawn game, each team gets one point. At the end of the tournament, the aggregate points by one team is 17. What is the maximum possible number of aggregate points received by another team in the tournament?
   [1×7 M]

2. (a) A random number N of the balanced dice are tossed, where $P(N = n) = p(1 - p)^{n-1}$, $n = 1, 2, \ldots, \infty$, and $0 < p < 1$. Find the probability q_k that the largest number shown by any of the dice does not exceed k, where $k = 1, 2, \ldots, 6$.
   (b) A blood test is 95% effective in detecting a certain disease, when it is, in fact present. The test also yields a false positive result for 1% of the healthy persons tested. If 0.5% of the population actually has the disease, what is the probability that a person has a disease given that his test result is positive?
   [4+3 M]

3. (a) Consider X be a random variable with PDF

$$f(x) = \begin{cases} 1/3 & , \quad -1 < x < 2 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

   Find the CDF and PDF of $Y = X^2$.
   (b) Let $X \sim U(0, 1)$. If $Y = -2\log X$, determine $E(Y)$, $Var(Y)$ and median of Y. [4+4 M]

4. (a) The number of failures which occur in a computer network over the time interval can be described by a Poisson process. On an average, there is a failure after every 4 hours, i.e. the intensity of the process is equal to 0.25.
      i. What is the probability of at most 1 failure in [0, 8); at least 2 failures in [8, 16), and at most 1 failure in [16,24).
      ii. What is the probability that the third failure occurs after 8 hours?
   (b) Let A and B shoot independently until each has his own target. The probability of their hitting at each shot is 3/4 and 4/6, respectively. Find the probability that B will require more shot than A.
   [4+4 M]

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