Indian Institute of Technology Kharagpur

QUESTION-CUM-ANSWERSCRIPT

Stamp/Signature of the Invigilator

MID-SEMESTER EXAMINATION 2016	SEMESTER (Autumn)

Roll Number											Section		Name	
Subject Number	M	A	2	0	1	0	1		Subject Name		**Transform Calculus**			
Department/Centre/School											Additional Sheets			

Important Instructions and Guidelines for Students

1. You must occupy your seat as per the Examination Schedule/Sitting Plan.
2. Do not keep mobile phones or any similar electronic gadgets with you even in the switched off mode.
3. Loose papers, class notes, books or any such materials must not be in your possession; even if they are irrelevant to the subject you are taking examination.
4. Data book, codes, graph papers, relevant standard tables/charts or any other materials are allowed only when instructed by the paper-setter.
5. Use of instrument box, pencil box and non-programmable calculator is allowed during the examination. However, the exchange of these items or any other papers (including question papers) is not permitted.
6. Write on both sides of the answer-script and do not tear off any page. **Use last page(s) of the answer-script for rough work**. Report to the invigilator if the answer-script has torn or distorted page(s).
7. It is your responsibility to ensure that you have signed the Attendance Sheet. Keep your Admit Card/Identity Card on the desk for checking by the invigilator.
8. You may leave the Examination Hall for wash room or for drinking water for a very short period. Record your absence from the Examination Hall in the register provided. Smoking and the consumption of any kind of beverages are strictly prohibited inside the Examination Hall.
9. Do not leave the Examination Hall without submitting your answer-script to the invigilator. **In any case, you are not allowed to take away the answer-script with you**. After the completion of the examination, do not leave your seat until the invigilators collect all the answer-scripts.
10. During the examination, either inside or outside the Examination Hall, gathering information from any kind of sources or exchanging information with others or any such attempt will be treated as '**unfair means**'. Don't adopt unfair means and also don't indulge in unseemly behavior.

Violation of any of the above instructions may lead to severe punishment.

Signature of the Student

To be Filled by the Examiner

Question Number	1	2	3	4	5	6	7	8	9	10	Total
Marks Obtained											

Marks Obtained (in words)	Signature of the Examiner	Signature of the Scrutineer

Instructions:
(i) Answer all questions.
(ii) All the parts of the same question should be done at one place.
(iii) $\mathcal{L}$ and $\mathcal{L}^{-1}$ denote the Laplace and Inverse Laplace operators, respectively.

Instruction to the Invigilators: No additional answer sheet to be supplied.

Space for Rough work

Question 1. [2+2+4]

a) If

$$f(t) = \begin{cases} t/T & \text{for } 0 < t < T, \\ 0 & \text{for } T < t < 2T \end{cases}$$

with period $2T$, find $\mathcal{L}(f(t))$.

b) Evaluate $\int_0^\infty te^{-2t} \sin t \, dt$ using the Laplace transform technique.

c) Express the following function

$$g_(t) = \begin{cases} t-1 & \text{for } 1 \leq t < 2, \\ 3-t & \text{for } 2 < t \leq 3, \\ 0 & \text{otherwise} \end{cases}$$

in terms of an unit step function and hence find $\mathcal{L}(g(t))$. Does $g(t)$ satisfy the sufficient conditions for the existence of Laplace transformation? Justify your answer.

Question 2. [3+2+3]

a) Find $\mathcal{L}^{-1}\left(\dfrac{1}{s^3(s^2+1)}\right)$.

b) Find $\mathcal{L}^{-1}\left(\dfrac{1}{\sqrt{s}+a}\right)$.

c) Using the convolution property of Laplace transformation, find $\mathcal{L}^{-1}\left(\frac{1}{s(s^2+4)^2}\right)$.

Question 3. [3+4]

a) Using the Laplace transform technique, solve the integral equation

$$\int_0^t f(x)\frac{1}{\sqrt{t-x}}\,dx = 1 + 2t - t^2.$$

b) Solve the following differential equation

$$t\frac{d^2y(t)}{dt^2} + (t+1)\frac{dy(t)}{dt} + 2y(t) = e^{-t}$$

subject to $y(0) = 0$ by using the technique of Laplace transformation.

Question 4. [2+5]

a) Find the Fourier Cosine coefficients of a periodic square wave function $f(x)$ defined by

$$f(x) = \begin{cases} 0 & \text{for } -2 < x < -1, \\ k & \text{for } -1 < x < 1, \\ 0 & \text{for } 1 < x < 2. \end{cases}$$

b) Find the Fourier series expansion for the periodic function

$$f(x) = \begin{cases} 0 & \text{for } -\pi < x < 0, \\ \sin x & \text{for } 0 < x < \pi. \end{cases}$$

Hence find the sum of the series

$$\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \cdots .$$

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