Symbolic Logic
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Lecture – 25
Proofs with All Rules

Hello! We are going to do today the module 25 with you. This is module 25 of the Symbolic Logic course where we are learning how to do the Formal Proof of Validity. And we have been exposed already to the nineteen rules so far, and we also were learning how to do the Formal Proof of Validity. By now I hope you have learnt your rules well enough, so that reference to the rules, if needed, we can always bring the rules to show you. But by now you should be familiar enough with the rules so that we can proceed without any further explanation about them.

Let's consider an example here. Here is the proof. Here is the argument, where you have this one. Premise is C ⊃ (A • ~B) and this is second, this is (~C ⊃ A) and this is (A • ~B) ∨ A. How to go about this? And how to construct the proof?

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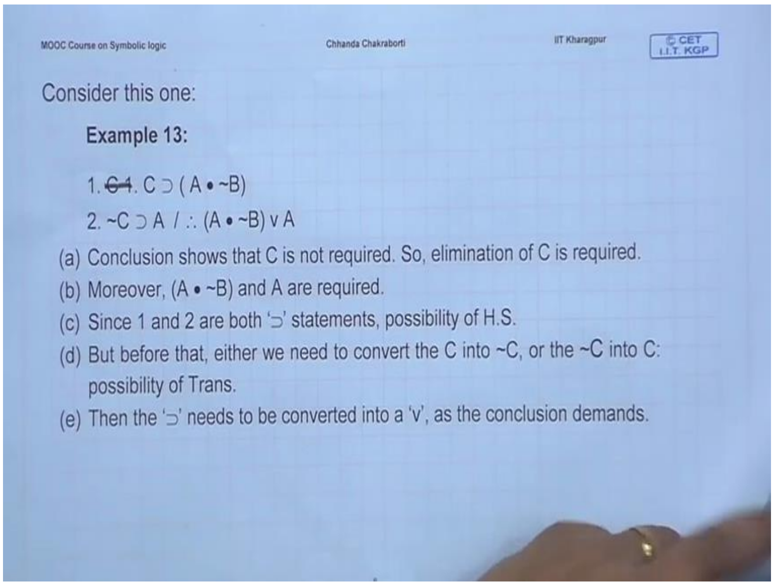

So, as you can see that there is no place of C in here. So somehow we have to eliminate the C, correct? So that we get this. Because we have A, and we also have (A • ~B) in the premises. Only thing is how do I get the C out? The elimination of C. And we want to keep the (A • ~B) and A in the place. Right. Since 1 and 2 both are horseshoe statements, this opens up certain possibility of horseshoe application rules. And it seems like probably there is a way to do the H.S here, so that we can keep only the A and (A • ~B). But, how? That you have to start thinking. Somewhere you remember H.S rules that we need to have an exact match in order to use the H.S rules. This is the H.S rule.

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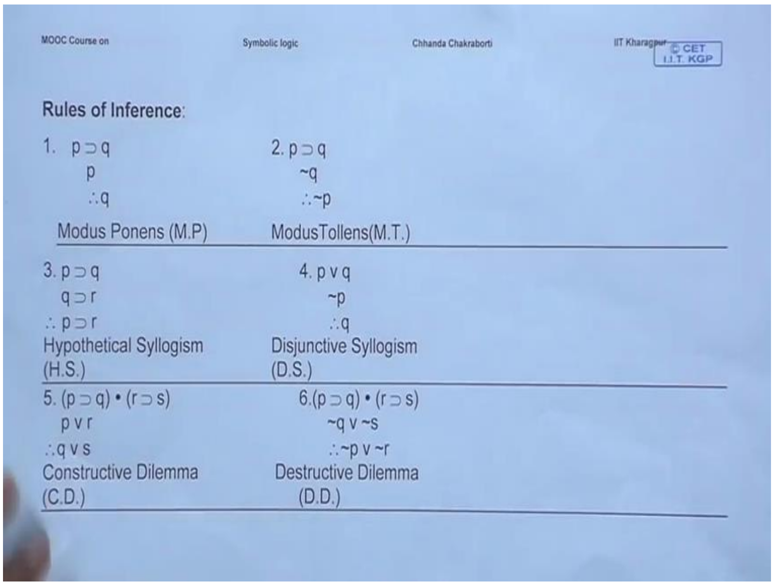

It shows that you have to have the q in the strategic position so that the q-s in these two premises should match. Meaning what? The consequent of the first conditional must exactly match with the antecedent of the second conditional. So that is required. Now if you look into these two statements here, 1 and 2, you have C here and you have ~C . And one of them is in the antecedent position, the other one is also in the antecedent position. So how can we use the H.S for example? So there is a… some brainstorming is required in order to change that.

And then we have this wedge situation we need to have a wedge in place. All these are clues for you to start the proof and sort of work on the constructed formal proof of validity. We have a solution already. But I suggest that you to try it on your own and not just copy the problem that is worked out. Because the whole idea is to get you started… get you started to think and to actually work on the proof.

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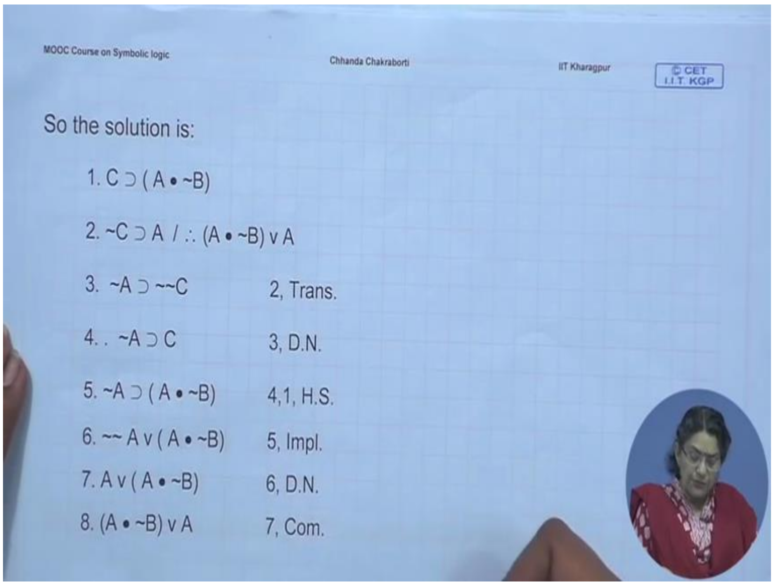

This is how I have done it. Not necessarily exactly how you have to do it. But this should set your thinking into motion. What I did is to do the Trans. on line number 2. Line number 2 is ~C ⊃ A, right? Now I have the… Trans. gives me ~A ⊃ ~~C. The question is why do I need to do the Trans on line number 2? That should be clear to you; otherwise there is… this is not a random move. I know why I needed to do that, because my plan is to somehow get this C as exactly matching with this C. So that I can then apply H.S to get this A and A • ~B somehow out. Alright? So this is what my motive is behind my using the Trans.

So now is we have done the double negation and this is what we have. When you have that, you can see that this is an exact match and we can apply the HS rule here. And that's what I did. So, 4 and 1, 4 and 1, right? Because I wanted to get rid of the C. So, 4, 1 gives me ~A ⊃ (A • ~B). Sounds very close to what we have here, right? Because the C is now eliminated. So, still some work is left.

So now, this horseshoe has to be changed into a disjunction. And you know which rule will give you that. That's your Impl. rule, which is applied to this horseshoe and we get this. Are we now at target? No, not quite. Because the positioning of these two disjuncts is not exactly like this. So we have to do this step also to get rid of the double negation.

Even in here you cannot stop the proof, because it's not exactly the conclusion that you want to derive. This is where you are. So, this is the complete proof of this argument.

This is how you show that from these 2 premises, this conclusion follows validly in this way. Get it? This is how….so we have now… as you can see there is a combination of Rules of Inference, along with the Rules of Equivalence, which is very very normal, very usual to do in a formal proof validity. But that's not the point. The point is to know which to apply and when. And with some already preplanned motive so that you know why you are doing what you doing.

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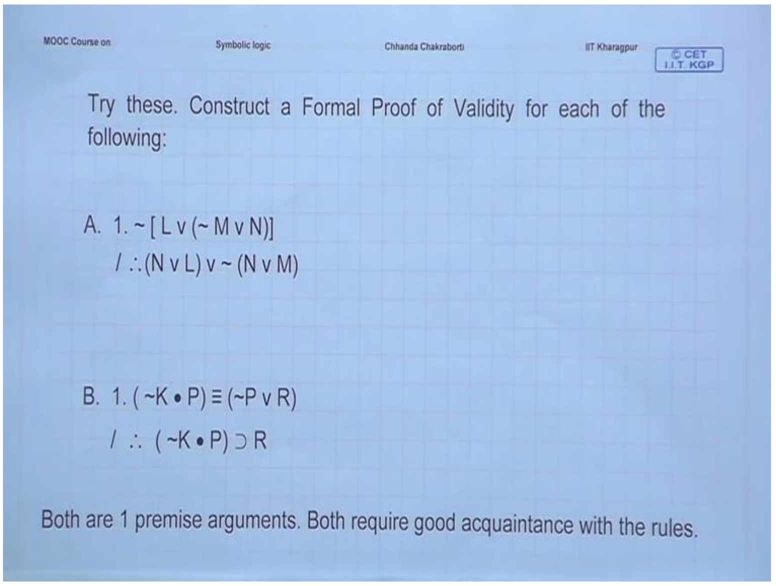

Let's see other problems and this I suggest that you don't…. even if I explain it here you try to do it on your own so that you have a feel of the problem. So here is one problem. Ok? I suggest that you take this down and before going any further into this module, where the solutions are presented, you try to work it. If you if you do not succeed in one attempt, no problem. Try it again, but do not give up. That is the point. So this is like a small puzzle that you have to be *at it* in order to solve it. This was my first problem and this is second problem. Ok? So these two, I suggest again that take it down on a piece of paper and try to solve them. You have all the rules at your disposal, you have all the time to think about it and have a plan before we go for it.

So, at this point I would suggest don't look into the module. Solve it first , and you come back to the module to see how it has been solved by me. Okay?

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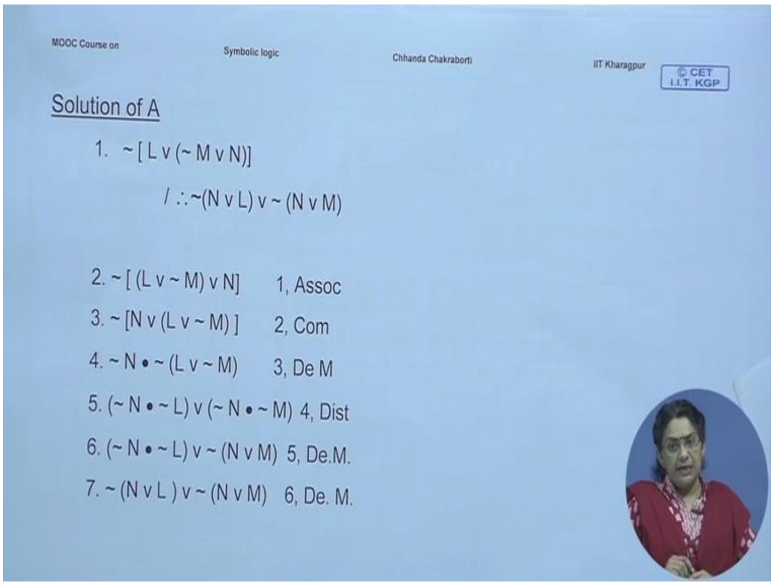

Both of these are one premise arguments. There is only one premise here and both require good grasp over the rules. The first one, the solution of the first problem. I have worked it in this way. See, this is the conclusion. It's $\sim(N \vee L) \vee \sim(N \vee M)$. If you look into the premise, all your components are there. All we need is somehow to approach it and they are all in as a wedge. Here also all your connectives are in wedge. So, somewhere we are talking about redistribution of these so that this becomes into this.

So, there is hope here. But you need to work with the statements a little. How I proceeded is that I regrouped them and there is a reason why I regrouped them in this way. So first let me explain that the regrouping is like so that first it was inside. Take a look. The negation sign is outside, but this L was the primary disjunct and $\sim M \vee N$ was the other one. What I did was to group them in this way, so $L \vee \sim M$ is one disjunct $\vee$ N is another disjunct. This is what Association allows, the regrouping, right? Provided you have all wedges or all dots, remember that? Question is Why? Why did I choose to use Association? The answer lies in the fact that there I see that N is distributed with L and M. And we need to have N in a position to distribute it over L and M. And this gives me that sort of possibility. It leaves open that possibility.

So you need to know Distribution rule well enough to see this possibility clear, and then Association is just one step to prepare the statement to the application of Distribution. Let me show you. Then I got the position of N as the first disjunct, right? But there is still a tilde in front, fine? So, De Morgan will give me a conjunction. So ~N • ~(L ∨ ~M), fine? This is what we have so far, and then we bring in Distribution. We have here. What am I doing? I am distributing this ~N with L and inside ~M and with distributed it looks like this. Then De Morgan on that, only on this part, gives me this. This is untouched. One more De Morgan will give me this, and this is your target conclusion. Take a look. I leave it in front of you for you to absorb, alright?

The point is that you need this. But in order to get here, you need to anticipate these moves a little bit. So the whole thing is to prepare the proposition in such a way that legitimately you can derive this line. It's a little bit like playing chess where you have to anticipate the moves. At least 4 or 5 steps ahead, you need to think a little, you need to arrange your pawns in such a way that you get that move open.

So similarly in the proof of validity you need to think a little bit. The plan has to be done so that the steps come through one by one, when you land at the target line. This was our first problem.

The second problem, the solution is somewhat like this. You see, this is the premise and this is the conclusion. Somewhere, again, all your components are there, but not in the way you want. So we have to process, we have to somehow replace, to get this thing out of here.

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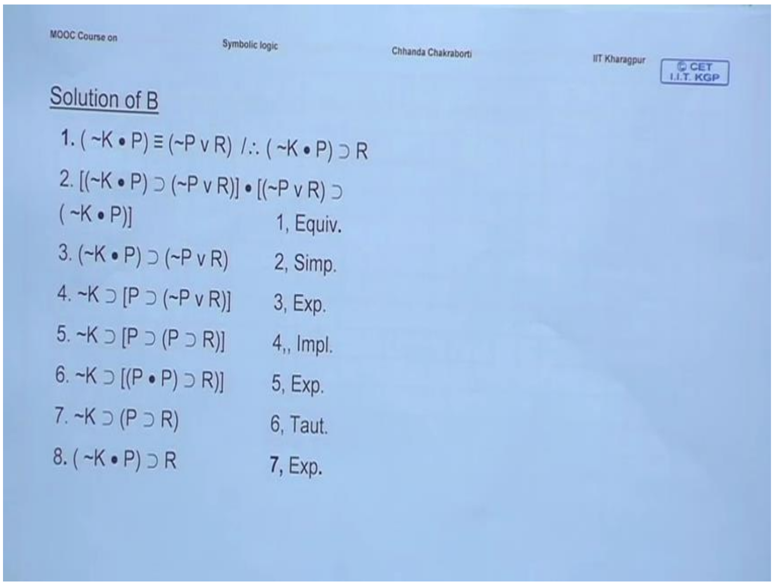

First of all, this is a triple bar and we don't have place for the triple bar anywhere here. What we need is a horseshoe, right? That should give you some clue. That we are going to break open this triple bar into the horseshoes. And then we may have to work on the statements to get rid of the redundant, for example, there is P here, there is ~P we have to somehow get rid of all that to get them into this kind of primed form. So here is the way I have approach the problem.

This is my application of material equivalence rule on line number 1. There is only one premise. So, the we have to start here and what you get is a conjunction of 2 horseshoes. If you look into this, the horseshoe here is (~K • P) ⊃ (~P ∨ R) and then this is (~P ∨ R) ⊃ (~K • P) right. This is what (p ⊃ q) • (q ⊃ p). The point is this looks long and are these all relevant for our proof? The answer is: No. We can do very well with only with the first conjunct, the first horseshoe. That itself is enough. So there is a possibility of using the Simplification. What you have done is to reduce this triple bar into a horseshoe, first of all. So lot of extra, extraneous material has been simply chopped out. Then we do this and here again I have to make sure that you know this rule, the Exportation rule, which is a Rule of Equivalence. The Rule of Equivalence; among this exportation is… allows you to work with the horseshoe statement in a certain sort of a way.

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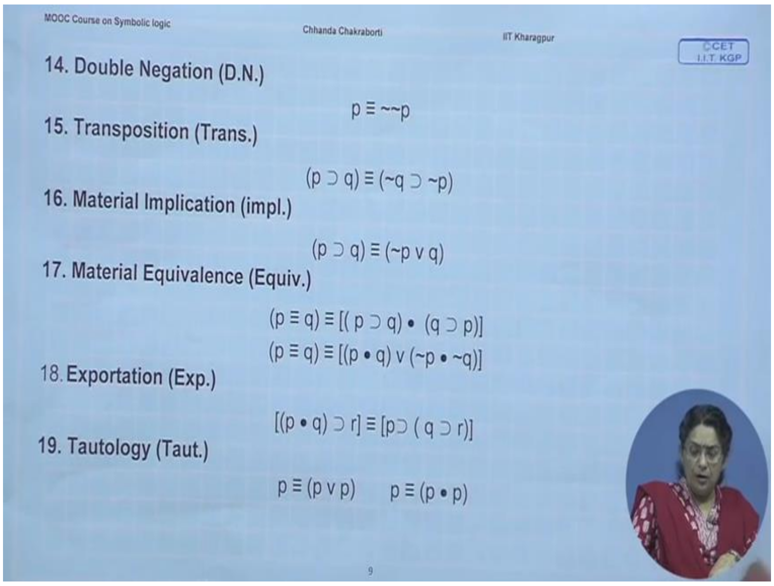

So let's remind ourselves. What does it say? The exportation rule says that if you have (p • q) ⊃ r, right? Then you can expand it as p ⊃ (q ⊃ r). Or, if you have a p ⊃ (q ⊃ r) situation like this, you can rewrite it as $(p \bullet q) \supset r$. Ok? You can compact, make it into compact.

So if that is that, then look at this sentence very well and you need to you understand that what we have here is $(p \bullet q) \supset r$. This whole thing is your r. So what we can do now is to rewrite it like so, by Exportation. This is p, $\sim K \supset [P \supset$, this is your r. This vision will come, provided you understand the rule Exportation very well, right? So that flexibility is something that you need to work on and understand. So this is what Exportation will allow you. This is expansion of what is being said here.

But there is also an agenda behind it. Why are we applying exportation here? That should become clearer to you as we go on. See the reason is that in $\sim P \vee R$, I have seen that this is nothing but $P \supset R$. So if you can get another P here, I say the possibility of collapsing these two Ps into one by using the rule called Tautology. Let's see. So let me just … I am getting ahead of that. So let's see.

First of all I have done Exportation, then this is your Implication. And this is what we can now do by Exportation on this part. This is your p, this is your q, this is your r. So $P \supset (P \supset R)$ by Exportation would become $(P \bullet P) \supset R$, alright? . So I am applying the Rule of Equivalence only into the part where I want to. And, this is the Exportation. After applying Exportation, this is what you are going to get. Now you see what I was trying to get at. The $\sim K$ is right where we want it to. The P was too many times. So, we needed to work on that. The R is also in the right position. So this would give you an opportunity to apply which rule? The Tautology which says $(p \bullet p)$ is equivalent to p, right? This is the rule that we are referring to. This is Tautology. And this rule is what I am applying here. So once we have that $\sim K \supset (P \supset R)$, you can see that its a matter of just one step to reach that. By another application of Exportation. So here you are $(\sim K \bullet P) \supset R$.

The lesson from this is that no rule is unimportant. So you… the lesser known, the unfamiliar rules are the ones that you may have to use a lot to sort of get a grasp on this. And Rules of Inference have their place, but also the Rules of Equivalence are also equally important. This was our way to do the Formal Proof of Validity, but let's check whether you have enough command over the rule base by now.

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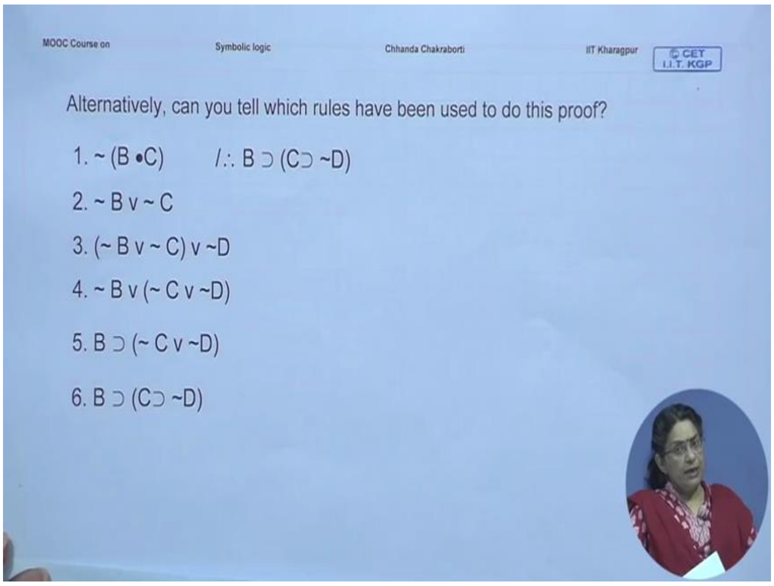

So alternatively we can ask you even this, that here is a worked out Formal Proof of Validity. Take a look at this and try to tell by which rule these lines have been generated. Can you do that? So this is your original argument. Here is the worked out proof. Just try to fill up the gaps and say by which rule did we get these lines, for example this, this, this. Ok? That would be an exercise for you to also check back whether how much of the rules you have grasped and whether you know what they do.

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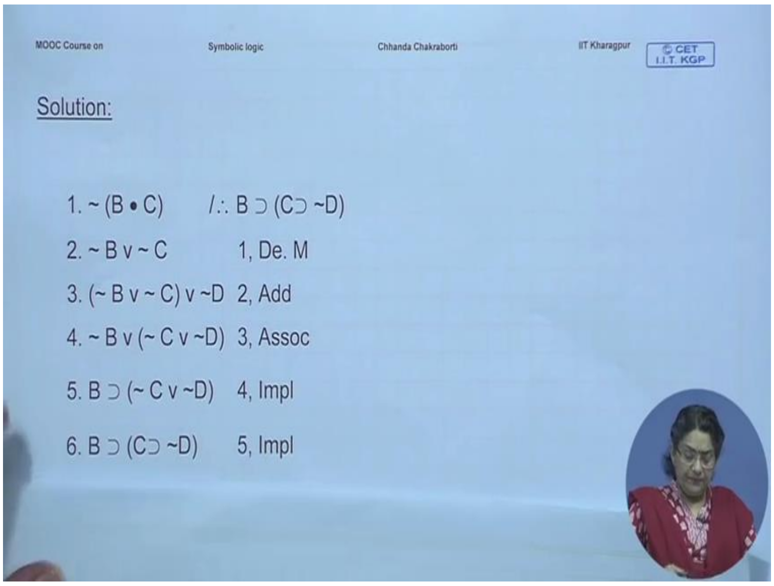

And this again I suggest that you instead of looking into the remaining part of the module, you stop and work it out and then check the result with the worked out solution. So here is the solution which comes next. See this is..this line, this was given and from this the next line followed by the rule De Morgan. De Morgan does that. This tilde and then within bracket the dot becomes ~B ∨ ~C, De Morgan.

The next one, where the tilde D appears is nothing but adding the tilde D onto this line. So 2, Addition will give you this. Right? Here comes the application of Association. This is how we got that. And then this is your Implication, by which we have changed this wedge into the horseshoe. Still not done. There is a part that we want to work on. So this you obtained by applying Implication again on that part. The whole point of this exercise is to see whether you can recognize the results of rule applications in each case and which rule is being applied; so that it opens your eye and also you see, you find out, realize that this is how the rules can work out. And may be it will give you some idea about how to do the proofs is also.

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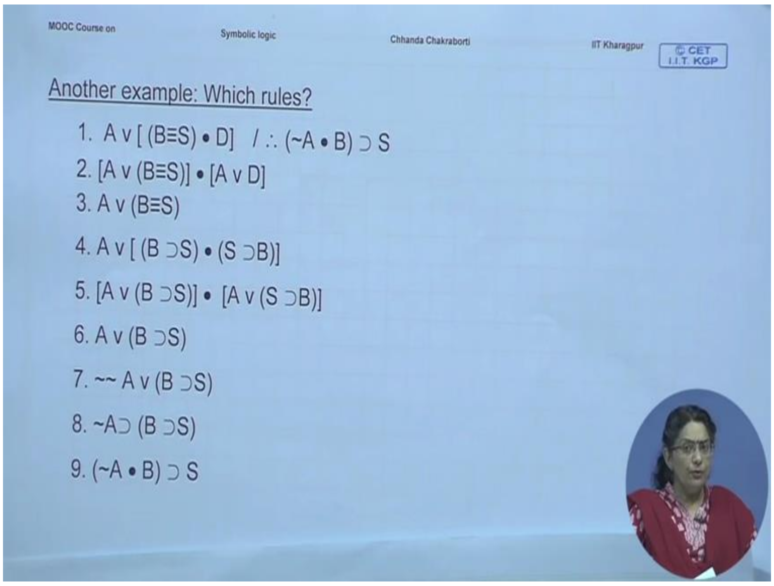

So here is another example, again I will ask you which rules. So this is a problem that is worked out. Ok? Here is the premise, one premise and here is the conclusion. Every line there has been derived. The question is: How is it derived? So take a look into this worked out example and try to fill out the justification part on your own. The rule and the line number, in each case, try to fill this out. And that would be another way to know what is it that the rules do and how can you generate a proof like this.

So this is the problem for you to work on, but I will finish the module by showing you the worked out result, which you can compare with once you finish your own work.

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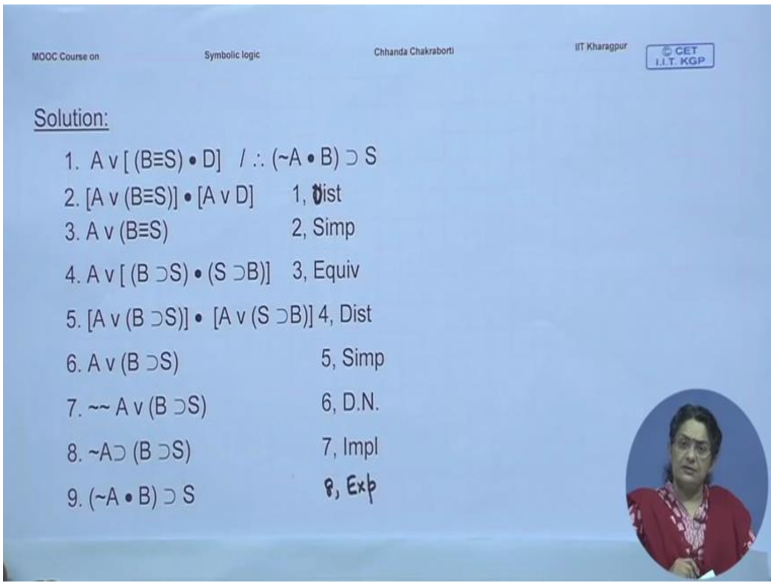

This was the original one, given, the argument itself. This is the line that we obtained from there. How? The answer is what happened is that the A has been distributed. So A ∨ (B ≡ S) is one, then A ∨ D is the other conjunct. What was inside became the outside connective. So 1 Distribution. Then comes this, where A ∨ D has been chopped out. What you derived by Simplification rule is A ∨ (B ≡ S). This is line 3 and application of Equivalence, where this triple bar has been broken down into the conjunction of two conditionals. This is 4, Distribution. Where? What are you distributing? Again A. Over what? First B ⊃ S and then S ⊃ B. So look at the result, look at the result and the dot becomes the main connective. Which was inside will become the outside connective, that is how Distribution works.

Once you have obtained that, this part is being removed, eliminated. By what? Again the rule is Simplification. So now we have only this. This is your final target. But we are getting close. This is Double Negation of course, requires no explanation and this is Implication. Is this step necessary to come here from for this line? Yes, at this point yes. From here we need to have a Double Negation to get the Implication of this for. Once you have that, you can probably see now that we are absolutely there except this one step which is your Exportation. Ok? By Exportation we are joining these two antecedents and this is your whole antecedent. And here is the S. Alright?. So this is where we can see how the proof can be worked out, what rules we are applying and so on and so forth.

Now, this is where I am going to end this module, because I think all that you needed has been already touched upon. The remaining part is your practice and further practice to see how you can get a proof going. Without a plan and without the knowledge of the rule base its going to be rather problematic. But I know that the rules are like that and probably by now you have understood and mastered them a lot. So I don’t think there is going to be any problem for doing the Formal Proof of Validity.

So with that good luck and all the best for doing this Formal Proof of Validity or derivations, and this is where I am going to end this module.

Thank you very much.