ASSIGNMENT-4 Solution

1. We have $f(x) = x^3 - 5x + 1 = 0$

$$f(0) = 1$$
$$f(1) = -3 \Rightarrow f(0)f(1) < 0$$

iteration	n	a_n (-ve)	b_n (+ve)	$x_{n+1} = \frac{a_n+b_n}{2}$	$f(x_{n+1})$
1	0	1	0	$\frac{0+1}{2} = \frac{1}{2}$	-1.375
2	1	$\frac{1}{2}$	0	$\frac{0+\frac{1}{2}}{2} = \frac{1}{4}$	$-.234375$
3	2	$\frac{1}{4}$	0	$\frac{0+\frac{1}{4}}{2} = \frac{1}{8}$	$+.37695$
4	3	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{\frac{1}{4}+\frac{1}{8}}{2} = \frac{3}{16}$	$+.06909$
5	4	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{\frac{1}{4}+\frac{3}{16}}{2} = \frac{7}{32}$	$-.08328$

Hence the required root is $\frac{7}{32}$.

2. Let x be the cube root of a given positive number

$x = (N)^{1/3} \Rightarrow x^3 - N = 0$. Let $f(x) = x^3 - N = 0$

By Newton-Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$
$$= x_n - \frac{x_n^3 - N}{3x_n^2} = \frac{2x_n^3 + N}{3x_n^2} \text{ for } n = 0,1,2\ldots$$

*

n	x_n	x_{n+1}
0	3.5	3.6395
1	3.6395	3.6342
2	3.6342	3.6342

As $x_2 = x_3$, $(48)^{1/3} = 3.634$. Ans

3. Let $f(x) = e^x - 5x = 0 \Rightarrow f(0) = 1,\ f(1) = -2.2817$

and $f'(x) = e^x - 5$

The Newton-Raphson formulae becomes

$$x_{n+1} = x_n - \left(\frac{e^{x_n} - 5x_n}{e^{x_n} - 5}\right)$$

n	x_n	x_{n+1}
0	0	$\cdot 25$
1	$\cdot 25$	$\cdot 2591$
2	$\cdot 2591$	$\cdot 2591$

Hence the solution $x = \cdot 2591$.

4. Follow the solution on page 4 after solution of question 6.

5. Let $f(x) = x^3 + x^2 - 1 = 0$ ——— (1)

Equation (1) can be written as $x = \phi(x)$ in many ways

(i) $x^3 + x^2 - 1 = 0$

$\Rightarrow x = (1 - x^2)^{1/3} = \phi(x)$ (say)

$\Rightarrow \phi'(x) = \frac{1}{3}(1 - x^2)^{-2/3}(-2x)$

Now, $|\phi'(\cdot 8)| = 1.05 > 1$ which does not satisfies the convergency condition.

(ii) $x^3 + x^2 - 1 = 0$

$\Rightarrow x^2 = 1 - x^3$

$\Rightarrow x = (1 - x^3)^{1/2} = \phi(x)$ (say)

$\therefore \phi'(x) = \frac{1}{2}(1 - x^3)^{-1/2}(-3x^2)$

$\Rightarrow |\phi'(\cdot 8)| = 1.37 > 1$

Again does not satisfy convergency condi

iii) $x^3 + x^2 - 1 = 0$

$\Rightarrow x^2(x+1) = 1$

$\Rightarrow x = \frac{1}{\sqrt{1+x}} = \phi(x)$ (say)

$\phi'(x) = -\frac{1}{2}(1+x)^{-3/2}$

$\Rightarrow |\phi'(.8)| = .2 < 1$ which satisfies the condition.

n	x_n	$\phi(x_n)$
0	.8	.7454
1	.7454	.7569
2	.7569	.7544
3	.7544	.7550
4	.7550	.7549
5	.7549	.7549

Hence the required root $= .7549$.

6. Let $f(x) = x^2 + \ln x - 2 = 0$

$\therefore f(1) = -1 < 0$, $f(2) = 2.69 > 0$

Therefore one root lies between 1 and 2.

We write the equation as

$$x = \sqrt{2 - \ln x} = \phi(x)$$

$$\Rightarrow \phi'(x) = -\frac{1}{2x\sqrt{2-\ln x}}$$

$\therefore |\phi'(x)| < 1$ as $\text{Max}\,[|\phi'(1)|, |\phi'(2)|]$

$= \text{Max}\,[.35, .21] = .35 < 1$

We take $x_0 = 1$

n	x_n	$\phi(x_n)$
0	1	1.4
1	1.4	1.29
2	1.29	1.32
3	1.32	1.312
4	1.312	1.3147
5	1.3147	1.3139
6	1.3139	1.31415
7	1.31415	1.31408
8	1.31408	1.31410
9	1.31410	1.31410

Hence the required root $= 1.3141$.

4. Here $f(x) = x^3 - 5x^2 + 7x - 3$

$f'(x) = 3x^2 - 10x + 7$

a) For first root with multiplicity 2 $(=m)$, if we use

$x_{n+1} = x_n - 2\dfrac{f(x_n)}{f'(x_n)}$. If we use $x_0 = 0$

n	x_n	x_{n+1}
0	0	.8571
1	.8571	.9953
2	.9953	.9999

b) If we use Newton Raphson Method with $x_0 = 0$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

n	x_n	x_{n+1}
0	0	.4285
1	.4285	.6856
2	.6856	.8328
3	.8328	.9132
4	.9132	.9557
5	.9557	.9887
6	.9887	.9943
7	.9943	.9971

See that (a) gives result .99 (correct upto 2 decimal) in 3 iterati[on] Whereas (b) gives this in 8 iteration.

* If we use $x_0 = 4$ in NR method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

n	x_n	x_{n+1}
0	4	3.4
1	3.4	3.1
2	3.1	3.008
	3.008	3.0000
	3.0000	3.0000

Third root $= 3.0000$ Ans

Here $|a_{11}| = 1$, $|a_{12}| = 1$, $|a_{13}| = 4$

$\therefore |a_{11}| < |a_{12}| + |a_{13}|$, the system is not diagonally dominant.

We re-arrange the system as

$$8x_1 - 3x_2 + 2x_3 = 20$$
$$4x_1 + 11x_2 - x_3 = 33$$
$$x_1 + x_2 + 4x_3 = 9$$

which is diagonally dominant.

Now we write the iteration formula for Gauss-Jacobi method as.

$$x_1^{(K+1)} = \frac{1}{8}\left[20 + 3x_2^{(K)} - 2x_3^{(K)}\right]$$

$$x_2^{(K+1)} = \frac{1}{11}\left[33 - 4x_1^{(K)} + x_3^{(K)}\right]$$

$$x_3^{(K+1)} = \frac{1}{4}\left[9 - x_1^{(K)} - x_2^{(K)}\right]$$

We now consider an initial arbitrary soln as:

$x_1^{(0)} = 0$, $x_2^{(0)} = 0$, $x_3^{(0)} = 0$

$$x_1^{(1)} = \frac{1}{8}\left[20 + 3\times 0 - 2\times 0\right] = 2.50$$

$$x_2^{(1)} = \frac{1}{11}\left[33 - 4\times 0 + 1\times 0\right] = 3.00$$

$$x_3^{(1)} = \frac{1}{4}\left[9 - 1\times 0 - 1\times 0\right] = 2.25$$

$x_1^{(2)} = 3.06$
$x_2^{(2)} = 2.30$
$x_3^{(3)} = 0.22$

$x_1^{(3)} = 3.142$
$x_2^{(3)} = 1.967$
$x_3^{(3)} = 0.910$

$x_1^{(4)} = 3.0101$
$x_2^{(4)} = 1.9402$
$x_3^{(4)} = 0.9728$

$x_1^{(5)} = 2.9844$
$x_2^{(5)} = 1.9938$
$x_3^{(5)} = 1.0124$

$x_1^{(6)} = 2.9946$ $\quad$ $x_1^{(7)} = 3.0012$

$x_2^{(6)} = 2.0068$ $\quad$ $x_2^{(7)} = 2.0024$

$x_3^{(6)} = 1.0054$ $\quad$ $x_3^{(7)} = 0.9996$

$\therefore x_1 = 3.00$, $x_2 = 2.00$, $x_3 = 1.00$ correct up to 2 decimal places.

(8) Here $|a_{31}| = 4$, $|a_{32}| = 2$, $|a_{33}| = 1$

$$|a_{33}| < |a_{31}| + |a_{32}|$$

$\therefore$ The given system of equations is not diagonally dominant.

Now, we re-arrange the system to make it diagonally dominant as follows:

$$4x_1 - 2x_2 + x_3 = -8$$
$$3x_1 + 9x_2 - 2x_3 = 11$$
$$4x_1 + 2x_2 + 13x_3 = 24$$

Now, we write the iteration formula for Gauss-Seidel method as

$$x_1^{(K+1)} = \frac{1}{4}\left[-8 + 2x_2^{(K)} - x_3^{(K)}\right]$$

$$x_2^{(K+1)} = \frac{1}{9}\left[11 - 3x_1^{(K+1)} + 2x_3^{(K)}\right]$$

$$x_3^{(K+1)} = \frac{1}{13}\left[24 - 4x_1^{(K+1)} - 2x_2^{(K+1)}\right]$$

Starting with initial guess as $x_1^{(0)} = 0$, $x_2^{(0)} = 0$ and $x_3^{(0)} = 0$.

$$x_1^{(1)} = \frac{1}{4}\left[-8 + 2\times 0 - 0\right] = -2$$

$$x_2^{(1)} = \frac{1}{9}\left[11 - 3\times(-2) + 2\times 0\right] = 1.89$$

$$x_3^{(1)} = \frac{1}{13}\left[24 - 4\times(-2) - 2\times 1.89\right] = 2.17$$

$x_1^{(2)} = -1.60$ $\quad x_1^{(3)} = -1.38$ $\quad x_1^{(4)} = -1.425$ $\quad x_1^{(5)} = -1.4252$

$x_2^{(2)} = 2.24$ $\quad x_2^{(3)} = 2.12$ $\quad x_2^{(4)} = 2.128$ $\quad x_2^{(5)} = 2.1322$

$x_3^{(2)} = 1.99$ $\quad x_3^{(3)} = 1.94$ $\quad x_3^{(4)} = 1.957$ $\quad x_3^{(5)} = 1.9566$

$x_1^{(6)} = -1.4230$

$x_2^{(6)} = 2.1314$

$x_3^{(6)} = 1.9561$

Thus $x_1 = -1.42$, $x_2 = 2.13$, $x_3 = 1.96$ correct up to 3 significant digits.

(a) Gauss-seidel method:

The given system of equation is diagonally dominant.

Now, the iteration formula for Gauss-seidel method as

$$x_1^{(K+1)} = \frac{1}{2}\left[4 - x_2^{(K)} - x_3^{(K)}\right]$$

$$x_2^{(K+1)} = \frac{1}{2}\left[4 - x_1^{(K+1)} - x_3^{(K)}\right]$$

$$x_3^{(K+1)} = \frac{1}{2}\left[4 - x_1^{(K+1)} - x_2^{(K+1)}\right]$$

Starting with initial estimate as $x_1^{(0)} = 0$, $x_2^{(0)} = 0$, $x_3^{(0)} = 0$

$$x_1^{(1)} = \frac{1}{2}\left[4 - 0 - 0\right] = 2.0$$

$$x_2^{(1)} = \frac{1}{2}\left[4 - 2 - 0\right] = 1.0$$

$$x_3^{(1)} = \frac{1}{2}\left[4 - 2 - 1\right] = 0.5$$

$x_1^{(2)} = 1.25$ $\quad x_1^{(3)} = 1.03125$ $\quad x_1^{(4)} = 0.98828$

$x_2^{(2)} = 1.125$ $\quad x_2^{(3)} = 1.01813$ $\quad x_2^{(4)} = 1.0332$

$x_3^{(2)} = 0.8125$ $\quad x_3^{(3)} = 0.94531$ $\quad x_3^{(4)} = 0.98926$

$x_1^{(5)} = 0.98877$ $\quad x_1^{(6)} = 0.99446$ $\quad x_1^{(7)} = 0.99794$

$x_2^{(5)} = 1.010985$ $\quad x_2^{(6)} = 1.00272$ $\quad x_2^{(7)} = 1.0003$

$x_3^{(5)} = 1.0001$ $\quad x_3^{(6)} = 1.00141$ $\quad x_3^{(7)} = 1.00088$

Thus $x_1 = 1.00$, $x_2 = 1.00$, $x_3 = 1.00$ correct up to 2-decim places.

(b) Gauss - Jacobi method:

The given system of equ^n is diagonally dominant.

The iteration formula for Gauss. Jacobi method as

$$x_1^{(K+1)} = \frac{1}{2}\left[4 - x_2^{(K)} - x_3^{(K)}\right]$$

$$x_2^{(K+1)} = \frac{1}{2}\left[4 - x_1^{(K)} - x_3^{(K)}\right]$$

$$x_3^{(K+1)} = \frac{1}{2}\left[4 - x_1^{(K)} - x_2^{(K)}\right]$$

The initial guess solution as $x_1^{(0)} = 0$, $x_2^{(0)} = 0$, $x_3^{(0)} = 0$

$x_1^{(1)} = \frac{1}{2}\left[4-0-0\right] = 2.0$ $\qquad x_1^{(2)} = \frac{1}{2}\left[4-2-2\right] = 0$

$x_2^{(1)} = \frac{1}{2}\left[4-0-0\right] = 2.0$ $\qquad x_2^{(2)} = \frac{1}{2}\left[4-2-2\right] = 0$

$x_3^{(1)} = \frac{1}{2}\left[4-0-0\right] = 2.0$ $\qquad x_3^{(2)} = \frac{1}{2}\left[4-2-2\right] = 0$

$x_1^{(3)} = \frac{1}{2}\left[4-0-0\right] = 2.0$

$x_2^{(3)} = \frac{1}{2}\left[4-0-0\right] = 2.0$

$x_3^{(3)} = \frac{1}{2}\left[4-0-0\right] = 2.0$

Repeated this process.

Thus, the system of equations is not convergent for Gauss. Jacobi method.

Gauss - Jacobi method:

$$x_1^{(K+1)} = \frac{1}{2}\left[9 - x_2^{(K)} - 6x_3^{(K)}\right]$$

$$x_2^{(K+1)} = \frac{1}{3}\left[13 - 8x_1^{(K)} - 2x_3^{(K)}\right]$$

$$x_3^{(K+1)} = 7 - x_1^{(K)} - 5x_2^{(K)}$$

Starting with initial guess as $\left(x_1^{(0)}, x_2^{(0)}, x_3^{(0)}\right) = (0,0,0)$

$x_1^{(1)} = 4.5$
$x_2^{(1)} = 4.333$
$x_3^{(1)} = 7$

$x_1^{(2)} = -18.666$
$x_2^{(2)} = -12.333$
$x_3^{(2)} = -19.165$

$x_1^{(3)} = 47.665$
$x_2^{(3)} = 66.886$
$x_3^{(3)} = 87.331$

$x_1^{(4)} = -290.936$
$x_2^{(4)} = -180.995$
$x_3^{(4)} = -375.095$

$x_1^{(5)} = 1220.284$
$x_2^{(5)} = 1030.226$
$x_3^{(5)} = 1202.911$

We can see in above results the Jacobi method become progressively worse instead of better.

Thus we can conclude that the method diverges.

Gauss - Seidel method:

$$x_1^{(k+1)} = \frac{1}{2}\left[9 - x_2^{(k)} - 6x_3^{(k)}\right]$$

$$x_2^{(k+1)} = \frac{1}{3}\left[13 - 8x_1^{(k+1)} - 2x_3^{(k)}\right]$$

$$x_3^{(k+1)} = 7 - x_1^{(k+1)} - 5x_2^{(k+1)}$$

Starting with initial estimate as $\left(x_1^{(0)}, x_2^{(0)}, x_3^{(0)}\right) = (0, 0, 0)$

$x_1^{(1)} = 4.5$

$x_2^{(1)} = -7.666$

$x_3^{(1)} = 40.833$

$x_1^{(2)} = -114.166$

$x_2^{(2)} = 281.554$

$x_3^{(2)} = -1286.604$

$x_1^{(3)} = 3723.535$

$x_2^{(3)} = -9067.357$

$x_3^{(3)} = 41620.231$

Therefore neither the Jacobi method nor the Gauss - Seidel method converges to the solution of the system of linear equations.

(11)

The given system of equations can be written as $Ax = b$

where $A = \begin{pmatrix} 4 & 1 & 1 \\ 1 & 5 & 2 \\ 1 & 2 & 3 \end{pmatrix}$, $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$, $b = \begin{pmatrix} 2 \\ -6 \\ -4 \end{pmatrix}$

The matrix A can be written as $A = L + D + U$

where $L = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 2 & 0 \end{pmatrix}$, $D = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{pmatrix}$, $U = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$

using Gauss-Jacobi method we get

$$x^{(K+1)} = D^{-1}\left(b - (L+U)x^{(K)}\right)$$

$$= D^{-1}b - D^{-1}(L+U)x^{K}$$

let $T = -D^{-1}(L+U)$

$$= -\begin{pmatrix} 1/4 & 0 & 0 \\ 0 & 1/5 & 0 \\ 0 & 0 & 1/3 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1/4 & -1/4 \\ -1/5 & 0 & -2/5 \\ -1/3 & -2/3 & 0 \end{pmatrix}$$

let $c = D^{-1}b = \begin{pmatrix} 1/4 & 0 & 0 \\ 0 & 1/5 & 0 \\ 0 & 0 & 1/3 \end{pmatrix}\begin{pmatrix} 2 \\ -6 \\ -4 \end{pmatrix} = \begin{pmatrix} 1/2 \\ -6/5 \\ -4/3 \end{pmatrix}$

Therefore,

$$x^{(K+1)} = \begin{pmatrix} 0 & -1/4 & -1/4 \\ -1/5 & 0 & -2/5 \\ -1/3 & -2/3 & 0 \end{pmatrix}x^{(K)} + \begin{pmatrix} 1/2 \\ -6/5 \\ -4/3 \end{pmatrix} \quad K = 0, 1, 2, \ldots$$

Starting with $x^{(0)} = \begin{pmatrix} 0.5 \\ -0.5 \\ -0.5 \end{pmatrix}$, we obtain

$$x^{(1)} = \begin{pmatrix} 0 & -1/4 & -1/4 \\ -1/5 & 0 & -2/5 \\ -1/3 & -2/3 & 0 \end{pmatrix} \begin{pmatrix} 0.5 \\ -0.5 \\ -0.5 \end{pmatrix} + \begin{pmatrix} 1/2 \\ -6/5 \\ -4/3 \end{pmatrix}$$

$$= \begin{pmatrix} 0.75 \\ -1.1 \\ -1.1667 \end{pmatrix}$$

$$x^{(2)} = \begin{pmatrix} 0 & -1/4 & -1/4 \\ -1/5 & 0 & -2/5 \\ -1/3 & -2/3 & 0 \end{pmatrix} \begin{pmatrix} 0.75 \\ -1.1 \\ -1.1667 \end{pmatrix} + \begin{pmatrix} 1/2 \\ -6/5 \\ -4/3 \end{pmatrix} = \begin{pmatrix} 1.0667 \\ -0.8833 \\ -0.8500 \end{pmatrix}$$

$$x^{(3)} = \begin{pmatrix} 0 & -1/4 & -1/4 \\ -1/5 & 0 & -2/5 \\ -1/3 & -2/3 & 0 \end{pmatrix} \begin{pmatrix} 1.0667 \\ -0.8833 \\ -0.8500 \end{pmatrix} + \begin{pmatrix} 1/2 \\ -6/5 \\ -4/3 \end{pmatrix} = \begin{pmatrix} 0.9333 \\ -1.0733 \\ -1.1000 \end{pmatrix}$$

(12) Solve the following system of linear equations

$2x_1 - x_2 = 7$

$-x_1 + 2x_2 - x_3 = 1$

$-x_2 + 2x_3 = 1$

using Gauss Seidel method with the initial approximation as $x^{(0)} = (0, 0, 0)^T$ and perform three iteration. (using matrix form)

Ans: The above system of equations can be written as

$$Ax = b$$

where $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$, $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ and $b = \begin{pmatrix} 7 \\ 1 \\ 1 \end{pmatrix}$

matrix A can be written as.

$$A = L + D + U$$

re $L = \begin{pmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$, $D = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ and $U = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}$

Gauss-Seidel method gives

$$x^{(k+1)} = (D+L)^{-1}\left(b - U x^{(k)}\right)$$

$$= -(D+L)^{-1} U x^{(k)} + (D+L)^{-1} b$$

t $T = -(D+L)^{-1} U = -\begin{pmatrix} 1/2 & 0 & 0 \\ 1/4 & 1/2 & 0 \\ 1/8 & 1/4 & 1/2 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1/2 & 0 \\ 0 & 1/4 & 1/2 \\ 0 & 1/8 & 1/4 \end{pmatrix}$

t $c = (D+L)^{-1} b = \begin{pmatrix} 1/2 & 0 & 0 \\ 1/4 & 1/2 & 0 \\ 1/8 & 1/4 & 1/2 \end{pmatrix} \begin{pmatrix} 7 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 7/2 \\ 9/4 \\ 13/8 \end{pmatrix}$

erefore, we obtain the iteration scheme.

$$x^{(k+1)} = \begin{pmatrix} 0 & 1/2 & 0 \\ 0 & 1/4 & 1/2 \\ 0 & 1/8 & 1/4 \end{pmatrix} x^{(k)} + \begin{pmatrix} 7/2 \\ 9/4 \\ 13/8 \end{pmatrix}$$

tarting with initial vector $x^{(0)} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, we get

$$x^{(1)} = \begin{pmatrix} 0 & 1/2 & 0 \\ 0 & 1/4 & 1/2 \\ 0 & 1/8 & 1/4 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 7/2 \\ 9/4 \\ 13/8 \end{pmatrix} = \begin{pmatrix} 3.5 \\ 2.25 \\ 1.625 \end{pmatrix}$$

$$x^{(2)} = \begin{pmatrix} 0 & 1/2 & 0 \\ 0 & 1/4 & 1/2 \\ 0 & 1/8 & 1/4 \end{pmatrix} \begin{pmatrix} 3.5 \\ 2.25 \\ 1.625 \end{pmatrix} + \begin{pmatrix} 3.5 \\ 2.25 \\ 1.625 \end{pmatrix} = \begin{pmatrix} 4.625 \\ 3.625 \\ 2.3125 \end{pmatrix}$$

$$x^{(3)} = \begin{pmatrix} 0 & 1/2 & 0 \\ 0 & 1/4 & 1/2 \\ 0 & 1/8 & 1/4 \end{pmatrix} \begin{pmatrix} 4.[\text{illegible}] \\ 3.625 \\ 2.3125 \end{pmatrix} + \begin{pmatrix} [\text{illegible}] \\ 2.25 \\ 1.625 \end{pmatrix} = \begin{pmatrix} 4.31[\text{illegible}] \\ 2.6563 \end{pmatrix}$$