Answers to Selected Problems

Chapter 1

1.1	0.0173 yd; 0.104 yd (compared to a total of 5 yd)								
1.3	$\frac{5}{9}$	1.5	$\frac{7}{12}$	1.9	$\frac{6}{7}$	1.11	$\frac{19}{28}$	1.15	1
2.1	1	2.4	∞	2.7	e^2	2.9	1		

4.2 $a_n = \dfrac{1}{5^{n-1}} \to 0$; $S_n = \dfrac{5}{4}\left(1 - \dfrac{1}{5^n}\right) \to \dfrac{5}{4}$; $R_n = \dfrac{1}{4 \cdot 5^{n-1}} \to 0$

4.4 $a_n = \dfrac{1}{3^n} \to 0$; $S_n = \dfrac{1}{2}\left(1 - \dfrac{1}{3^n}\right) \to \dfrac{1}{2}$; $R_n = \dfrac{1}{2 \cdot 3^n} \to 0$

4.6 $a_n = \dfrac{1}{n(n+1)} \to 0$; $S_n = 1 - \dfrac{1}{n+1} \to 1$; $R_n = \dfrac{1}{n+1} \to 0$

5.2	Test further	5.4	D	5.5	D
5.6	Test further	5.8	Test further	5.9	D

6.5 b	D	6.7	D	6.9	C	6.10	C	6.14	D
6.18	D	6.20	C	6.22	C	6.23	D	6.24	D
6.26	C	6.29	D	6.31	D	6.32	D	6.35	C
6.36	D								
7.1	C	7.2	D	7.4	C	7.6	D	7.8	C
9.2	D	9.3	C	9.7	D	9.8	C	9.9	D
9.10	D	9.12	C	9.13	C	9.15	D	9.16	C
9.20	C	9.21	C	9.22	(b) D				

10.1	$\lvert x\rvert < 1$	10.3	$\lvert x\rvert \le 1$	10.4	$\lvert x\rvert \le \sqrt{2}$
10.5	All x	10.9	$\lvert x\rvert < 1$	10.10	$\lvert x\rvert \le 1$
10.11	$-5 \le x < 5$	10.13	$-1 < x \le 1$	10.15	$-1 < x < 5$
10.17	$-2 < x \le 0$	10.18	$-\frac{3}{4} \le x \le -\frac{1}{4}$	10.20	All x
10.21	$0 \le x \le 1$	10.22	No x	10.24	$\lvert x\rvert < \frac{1}{2}\sqrt{5}$
10.25	$n\pi - \frac{\pi}{6} < x < n\pi + \frac{\pi}{6}$				

13.4 $\binom{-1/2}{0} = 1$; $\binom{-1/2}{n} = \dfrac{(-1)^n(2n-1)!!}{(2n)!!}$

13.6 $\sum_0^\infty \binom{1/2}{n} x^{n+1}$ (see Example 2)

13.8 $\sum_0^\infty \binom{-1/2}{n} (-x^2)^n$ (see Problem 13.4)

13.11 $\displaystyle\sum_0^\infty \frac{(-1)^n x^n}{(2n+1)!}$ 13.14 $\displaystyle\sum_0^\infty \frac{x^{2n+1}}{2n+1}$

13.15 $\displaystyle\sum_0^\infty \binom{-1/2}{n} (-1)^n \frac{x^{2n+1}}{2n+1}$ 13.17 $\displaystyle 2\sum_{\text{odd } n} \frac{x^n}{n}$

13.21 $x^2 + 2x^4/3 + 17x^6/45 \cdots$

13.22 $1 + 2x + 5x^2/2 + 8x^3/3 + 65x^4/24 \cdots$

13.25 $1 - x + x^2/3 - x^4/45 \cdots$

13.27 $1 + x + x^2/2 - x^4/8 - x^5/15 \cdots$

13.28 $x - x^2/2 + x^3/6 - x^5/12 \cdots$

13.29 $1 + x/2 - 3x^2/8 + 17x^3/48 \cdots$

13.34 $x - x^2 + x^3 - 13x^4/12 + 5x^5/4 \cdots$

13.35 $1 + x^2/3! + 7x^4/(3\cdot 5!) + 31x^6/(3\cdot 7!) \cdots$

13.41 $e^3[1 + (x-3) + (x-3)^2/2! + (x-3)^3/3! \cdots]$

13.44 $5 + (x-25)/10 - (x-25)^2/10^3 + (x-25)^3/(5\times 10^4) \cdots$

14.8 For $x < 0$, error < 0.001; for $x > 0$, error < 0.002.

15.1 $-x^4/24 - x^5/30 \cdots \cong -3.376 \times 10^{-16}$

15.3 $x^5/15 - 2x^7/45 \cdots \cong 6.667 \times 10^{-17}$

15.6 12 15.8 1/2 15.10 -1 15.12 1/3

15.14 $t - \frac{t^3}{3}$, error $< 10^{-6}$ 15.17 $\cos(\pi/2) = 0$

15.19 $\sqrt{2}$ 15.20 (b) $5e$ 15.21 (b) 0.937548

15.22 (b) 1.202057 15.23 (a) 1/2 (c) 1/3

15.24 (a) $-\pi$ (d) 0 (f) 0

15.27 (a) $1 - \frac{v}{c} = 1.3 \times 10^{-5}$, or $v = 0.999987c$

(d) $1 - \frac{v}{c} = 1.3 \times 10^{-11}$

15.28 $mc^2 + \frac{1}{2}mv^2$

15.29 (b) $\dfrac{F}{W} = \dfrac{x}{l} + \dfrac{x^3}{2l^3} + \dfrac{3x^5}{8l^5} \cdots$

15.30 (b) $T = \dfrac{1}{2}\dfrac{F}{\theta}\left(1 + \dfrac{\theta^2}{6} + 7\dfrac{\theta^4}{360} \cdots\right)$

15.31 (a) finite (b) infinite

16.6 C 16.7 D 16.9 $-1 \le x < 1$

16.10 $-4 < x < 4$ 16.13 $-5 < x \le 1$

16.15 $-x^2/6 - x^4/180 - x^6/2835 \cdots$

16.16 $1 - x/2 + 3x^2/8 - 11x^3/48 + 19x^4/128 \cdots$

16.19 $-(x-\pi) + (x-\pi)^3/3! - (x-\pi)^5/5! \cdots$

16.20 $2 + \dfrac{x-8}{12} - \dfrac{(x-8)^2}{2^5\cdot 3^2} + \dfrac{5(x-8)^3}{2^8\cdot 3^4} \cdots$

16.26 $-1/3$ 16.28 1

16.31 (b) 2.66×10^{86} terms. For $N = 15$, $1.6905 < S < 1.6952$

Chapter 2

	x	y	r	θ	
4.1	1	1	$\sqrt{2}$	$\pi/4$	See Fig. 5.1
4.2	-1	1	$\sqrt{2}$	$3\pi/4$	See Fig. 9.6
4.3	1	$-\sqrt{3}$	2	$-\pi/3$	
4.5	0	2	2	$\pi/2$	See Fig. 5.2
4.7	-1	0	1	π	See Fig. 9.2
4.9	-2	2	$2\sqrt{2}$	$3\pi/4$	
4.11	$\sqrt{3}$	1	2	$\pi/6$	See Fig. 9.1
4.14	$\sqrt{2}$	$\sqrt{2}$	2	$\pi/4$	
4.15	-1	0	1	$-\pi$ or π	See Fig. 9.2
4.17	1	-1	$\sqrt{2}$	$-\pi/4$	See Fig. 9.5
4.20	-2.39	-6.58	7	$-110°$ $= -1.92$ radians	
5.2	$-1/2$	$-1/2$	$1/\sqrt{2}$	$-3\pi/4$ or $5\pi/4$	
5.4	0	2	2	$\pi/2$	
5.6	-1	0	1	π	
5.8	1.6	-2.7	3.14	$-59.3°$	
5.10	$-25/17$	$19/17$	$\sqrt{58/17}$	$142.8°$	
5.12	2.65	1.41	3	$28°$	
5.14	1.27	-2.5	2.8	-1.1 radians $= -63°$	
5.16	1.53	-1.29	2	$-40°$	
5.17	-7.35	-10.9	13.1	$-124°$	
5.18	-0.94	-0.36	1	$201°$ or $-159°$	

5.19 $(2+3i)/13$; $(x-yi)/(x^2+y^2)$
5.21 $(1+i)/6$; $(x+1-yi)/[(x+1)^2+y^2]$
5.23 $(-6-3i)/5$; $(1-x^2-y^2+2yi)/[(1-x)^2+y^2]$
5.26 1 5.30 3/2 5.31 1
5.32 169 5.34 1 5.35 $x=-4,\ y=3$
5.36 $x=-1/2,\ y=3$ 5.39 $x=y=$ any real number
5.42 $x=-1/7,\ y=-10/7$ 5.43 $(x,y)=(0,0)$, or $(1,1)$, or $(-1,1)$
5.45 $x=0$, any real y; or $y=0$, any real x
5.46 $y=-x$ 5.48 $x=36/13,\ y=2/13$
5.49 $y=0,\ x=1/2$
5.53 Circle (Find center and radius)
5.55 Straight line (What is its equation?)
5.56 Part of a straight line (Describe it.)
5.57 Hyperbola (What is its equation?)
5.60 Circle (Find center and radius)
5.62 Ellipse (Find its equation; where are the foci?)
5.63 Two straight lines (What lines?)
5.68 $v=2,\ a=4$

6.2 D 6.3 C 6.4 D
6.5 D 6.10 C 6.12 C

7.1	All z	7.3	All z	7.6	$\lvert z\rvert < 1/3$
7.7	All z	7.10	$\lvert z\rvert < 1$	7.12	$\lvert z\rvert < 4$
7.14	$\lvert z-2i\rvert < 1$	7.16	$\lvert z+(i-3)\rvert < 1/\sqrt{2}$		

8.3 See Problem 17.30

9.3	$-9i$	9.4	$-e(1+i\sqrt{3})/2$	9.6	1
9.7	$3e^2$	9.8	$-\sqrt{3}+i$	9.10	-2
9.11	$-1-i$	9.13	$-4+4i$	9.14	64
9.17	$-(1+i)/4$	9.19	16	9.20	i
9.21	1	9.24	$4i$	9.26	$(1+i\sqrt{3})/2$
9.29	1	9.32	$3e^2$	9.34	$4/e$
9.35	21	9.38	$1/\sqrt{2}$		

10.3 $\pm 1,\ \pm i$ 10.4 $\pm 2,\ \pm 2i$
10.7 $\pm\sqrt{2},\ \pm i\sqrt{2},\ \pm 1 \pm i$
10.9 $1,\ 0.309 \pm 0.951i,\ -0.809 \pm 0.588i$
10.16 $\pm i,\ (\pm\sqrt{3} \pm i)/2$
10.17 $-1,\ 0.809 \pm 0.588i,\ -0.309 \pm 0.951i$
10.18 $\pm(1+i)/\sqrt{2}$ 10.21 $\pm(\sqrt{3}+i)$
10.22 $r=\sqrt{2}$, $\theta = 45^\circ + 120^\circ n$: $1+i,\ -1.366+0.366i,\ 0.366-1.366i$
10.24 $\pm(\sqrt{3}+i)/2,\ \pm(1-i\sqrt{3})/2,\ \pm(0.259+0966i),\ \pm(0.966-0.259i)$
10.25 $0.758(1+i),\ -0.487+0.955i,\ -1.059-0.168i,\ -0.168-1.059i,$
$0.955-0.487i$

11.3 $3(1-i)/\sqrt{2}$ 11.5 $1+i$ 11.8 $-41/9$ 11.9 $4i/3$

12.25 $\sin x \cosh y - i \cos x \sinh y,\ \sqrt{\sin^2 x + \sinh^2 y}$
12.26 $\cosh 2 \cos 3 - i \sinh 2 \sin 3 = -3.72 - 0.51i,\ 3.76$
12.28 $\tanh 1 = 0.762$
12.30 $-i$ 12.32 $-4i/3$
12.33 $i \tanh 1 = 0.762i$ 12.35 $-\cosh 2 = -3.76$

14.2 $-i\pi/2$ or $3i\pi/2$ 14.3 $\mathrm{Ln}\,2 + i\pi/6$
14.5 $\mathrm{Ln}\,2 + 5i\pi/4$ 14.6 $-i\pi/4$ or $7i\pi/4$
14.8 $-1,\ (1 \pm i\sqrt{3})/2$ 14.10 $e^{-\pi^2/4}$
14.11 $\cos(\mathrm{Ln}\,2) + i \sin(\mathrm{Ln}\,2) = 0.769 + 0.639i$
14.14 $0.3198i - 0.2657$ 14.15 $e^{-\pi \sinh 1} = 0.0249$
14.18 -1 14.20 1
14.23 $e^{\pi/2} = 4.81$

15.2 $\pi/2 + n\pi + (i\,\mathrm{Ln}\,3)/2$ 15.3 $i(\pm\pi/3 + 2n\pi)$
15.4 $i(2n\pi + \pi/6),\ i(2n\pi + 5\pi/6)$ 15.5 $\pm[\pi/2 + 2n\pi - i\,\mathrm{Ln}(3+\sqrt{8})]$
15.8 $\pi/2 + 2n\pi \pm i\,\mathrm{Ln}\,3$ 15.9 $i(\pi/3 + n\pi)$
15.12 $i(2n\pi \pm \pi/6)$ 15.14 $2n\pi + i\,\mathrm{Ln}\,2,\ (2n+1)\pi - i\,\mathrm{Ln}\,2$
15.15 $n\pi + 3\pi/8 + (i/4)\,\mathrm{Ln}\,2$

16.3 $\lvert z\rvert = \sqrt{2}$; motion around a circle of radius $\sqrt{2}$, at constant speed $v=\sqrt{2}$, constant acceleration $a=\sqrt{2}$.

16.5 $v = |z_1 - z_2|$; $a = 0$
16.6 (a) Series: $3 - 2i$; parallel: $5 + i$
(b) Series: $2(1 + i\sqrt{3})$; parallel: $i\sqrt{3}$
16.8 $[R - i(\omega CR^2 + \omega^3 L^2 C - \omega L)]/[(\omega CR)^2 + (\omega^2 LC - 1)^2]$; this simplifies to $L/(RC)$ at resonance.
16.9 (b) $\omega = 1/\sqrt{LC}$
16.12 $(1 + r^4 - 2r^2 \cos\theta)^{-1}$

17.2 $(\sqrt{3} + i)/2$
17.4 $i \cosh 1 = 1.54i$
17.6 $-e^{-\pi^2} = -5.17 \times 10^{-5}$
17.7 $e^{\pi/2} = 4.81$
17.9 $\pi/2 \pm 2n\pi$
17.11 i
17.13 $x = 0,\ y = 4$
17.15 $|z| < 1/e$
17.26 1
17.27 (c) $e^{-2(x-t)^2}$
17.28 $1 + (a^2 + b^2)^2 (2ab)^{-2} \sinh^2 b$
17.30 $e^x \cos x = \sum_{n=0}^{\infty} \left(2^{n/2} x^n/n!\right) \cos n\pi/4$
$e^x \sin x = \sum_{n=0}^{\infty} \left(2^{n/2} x^n/n!\right) \sin n\pi/4$

Chapter 3

2.4 $\begin{pmatrix} 1 & 0 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & 1 \end{pmatrix}$, $x = \frac{1}{2}(z + 1)$, $y = 1$

2.8 $\begin{pmatrix} 1 & -1 & 0 & -11 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, $x = y - 11$, $z = 7$

2.9 $\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$, inconsistent, no solution

2.12 $\begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$, $x = -2$, $y = 1$, $z = 1$

2.17 $R = 2$

3.1 -11
3.5 -544
3.12 16
3.16 $A = -(K + ik)/(K - ik)$, $|A| = 1$

4.12 $\arccos(-1/\sqrt{2}) = 3\pi/4$
4.14 (a) $\arccos(1/3) = 70.5°$
4.14 (c) $\arccos\sqrt{2/3} = 35.3°$
4.15 (b) $8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}$
4.18 $2\mathbf{i} - 8\mathbf{j} - 3\mathbf{k}$
4.19 $\mathbf{i} + \mathbf{j} + \mathbf{k}$
4.22 Law of cosines
4.24 $A^2 B^2$

5.1 $\mathbf{r} = (2\mathbf{i} - 3\mathbf{j}) + (4\mathbf{i} + 3\mathbf{j})t$ [Note that $2\mathbf{i} - 3\mathbf{j}$ may be replaced by *any* point on the line; $4\mathbf{i} + 3\mathbf{j}$ may be replaced by *any* vector along the line. Thus, for example, $\mathbf{r} = 6\mathbf{i} - (8\mathbf{i} + 6\mathbf{j})t$ is just as good an answer, and similarly for all such problems.]
5.4 $\mathbf{r} = \mathbf{i} + (2\mathbf{i} + \mathbf{j})t$
5.6 $(x - 1)/1 = (y + 1)/(-2) = (z + 5)/2$, or $\mathbf{r} = \mathbf{i} - \mathbf{j} - 5\mathbf{k} + (\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})t$
5.8 $x/3 = (z - 4)/(-5)$, $y = -2$; or $\mathbf{r} = -2\mathbf{j} + 4\mathbf{k} + (3\mathbf{i} - 5\mathbf{k})t$
5.9 $x = -1$, $z = 7$; or $\mathbf{r} = -\mathbf{i} + 7\,\mathbf{k} + \mathbf{j}\,t$

5.11 $(x-4)/1=(z-3)/(-2),\ y=-1$; or $\mathbf{r}=4\mathbf{i}-\mathbf{j}+3\mathbf{k}+(\mathbf{i}-2\mathbf{k})t$
5.12 $(x-5)/5=(y+4)/(-2)=(z-2)/1$; or $\mathbf{r}=5\mathbf{i}-4\mathbf{j}+2\mathbf{k}+(5\mathbf{i}-2\mathbf{j}+\mathbf{k})t$
5.14 $36x-3y-22z=23$ 5.16 $5x-2y+z=35$
5.18 $x+6y+7z+5=0$ 5.20 $x-4y-z+5=0$
5.21 $\cos\theta=25/(7\sqrt{30})=0.652,\ \theta=49.3°$
5.22 $\cos\theta=2/\sqrt{6},\ \theta=35.3°$
5.24 $\mathbf{r}=2\mathbf{i}+\mathbf{j}+(\mathbf{j}+2\mathbf{k})t,\ d=2\sqrt{6/5}$
5.25 $\mathbf{r}=\mathbf{i}-2\mathbf{j}+(4\mathbf{i}+9\mathbf{j}-\mathbf{k})t,\ d=\big(3\sqrt{3}\big)/7$
5.29 $2/\sqrt{6}$ 5.31 $5/7$ 5.33 $\sqrt{43/15}$
5.34 $\sqrt{11/10}$ 5.36 3 5.38 $\arccos\sqrt{21/22}=12.3°$
5.39 Intersect at $(3,2,0)$; $\cos\theta=5/\sqrt{60},\ \theta=49.8°$
5.42 $1/\sqrt{5}$ 5.43 $20/\sqrt{21}$ 5.45 $d=\sqrt{2},\ t=-1$

6.2 $\mathrm{AB}=\begin{pmatrix}-2&-2\\1&2\end{pmatrix}$, $\mathrm{BA}=\begin{pmatrix}-6&17\\-2&6\end{pmatrix}$, $\mathrm{A}+\mathrm{B}=\begin{pmatrix}1&-1\\-1&5\end{pmatrix}$, $\mathrm{A}-\mathrm{B}=\begin{pmatrix}3&-9\\-1&1\end{pmatrix}$, $\mathrm{A}^2=\begin{pmatrix}9&-25\\-5&14\end{pmatrix}$, $\mathrm{B}^2=\begin{pmatrix}1&4\\0&4\end{pmatrix}$, $5\mathrm{A}=\begin{pmatrix}10&-25\\-5&15\end{pmatrix}$, $3\mathrm{B}=\begin{pmatrix}-3&12\\0&6\end{pmatrix}$, $\det(5\mathrm{A})=5^2\det\mathrm{A}$ for a 2×2 matrix
6.4 You should have found BA, C^2, CB, C^3, $\mathrm{C}^2\mathrm{B}$, and CBA; all others are meaningless. $\mathrm{C}^2\mathrm{B}=\begin{pmatrix}32&12\\53&7\\-13&-9\end{pmatrix}$, $\mathrm{CBA}=\begin{pmatrix}36&46&14&-36\\40&22&1&91\\-8&-2&1&-29\end{pmatrix}$
6.13 $\begin{pmatrix}5/3&-3\\-1&2\end{pmatrix}$ 6.15 $-\dfrac{1}{2}\begin{pmatrix}4&5&8\\-2&-2&-2\\2&3&4\end{pmatrix}$
6.19 $\mathrm{A}^{-1}=\dfrac{1}{7}\begin{pmatrix}1&2\\-3&1\end{pmatrix}$, $(x,y)=(5,0)$
6.22 $\mathrm{A}^{-1}=\dfrac{1}{12}\begin{pmatrix}4&4&0\\-7&-1&3\\1&-5&3\end{pmatrix}$, $(x,y,z)=(1,-1,2)$
6.30 $\sin k\mathrm{A}=\mathrm{A}\sin k=\begin{pmatrix}0&\sin k\\\sin k&0\end{pmatrix}$, $\cos k\mathrm{A}=\mathrm{I}\cos k=\begin{pmatrix}\cos k&0\\0&\cos k\end{pmatrix}$, $e^{k\mathrm{A}}=\begin{pmatrix}\cosh k&\sinh k\\\sinh k&\cosh k\end{pmatrix}$, $e^{ik\mathrm{A}}=\begin{pmatrix}\cos k&i\sin k\\i\sin k&\cos k\end{pmatrix}$

7.1 Not linear 7.4 Linear 7.6 Not linear
7.8 Not linear 7.11 Not linear 7.12 Linear
7.14 Not linear 7.15 Linear
7.22 $D=1$, rotation $\theta=-45°$ 7.24 $D=-1$, reflection line $x+y=0$
7.26 $D=-1$, reflection line $x=2y$
7.30 $\mathrm{R}=\begin{pmatrix}0&-1&0\\1&0&0\\0&0&1\end{pmatrix}$, $\mathrm{S}=\begin{pmatrix}1&0&0\\0&0&-1\\0&1&0\end{pmatrix}$,
R is a 90° rotation about the z axis, S is a 90° rotation about the x axis.
7.32 180° rotation about $\mathbf{i}-\mathbf{k}$
7.35 Reflection through the (x,y) plane and 90° rotation about the z axis.

8.1 In terms of basis $\mathbf{u} = \frac{1}{9}(9,0,7)$, $\mathbf{v} = \frac{1}{9}(0,-9,13)$, the vectors are: $\mathbf{u}-4\mathbf{v}$, $5\mathbf{u}-2\mathbf{v}$, $2\mathbf{u}+\mathbf{v}$, $3\mathbf{u}+6\mathbf{v}$.

8.3 Basis $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$

8.6 $\mathbf{V} = 3\mathbf{A} - \mathbf{B}$

8.19 $x = y = z = w = 0$

8.20 $x = -z$, $y = z$

8.23 For $\lambda = 3$, $x = 2y$; for $\lambda = 8$, $x = -2y$

8.25 For $\lambda = 2$: $x = 0$, $y = -3z$; for $\lambda = -3$: $x = -5y$, $z = 3y$; for $\lambda = 4$: $z = 3y$, $x = 2y$

8.26 $\mathbf{r} = (3,1,0) + (-1,1,1)z$

9.4 $\mathrm{A}^\dagger = \begin{pmatrix} 0 & i & 3 \\ -2i & 2 & 0 \\ -1 & 0 & 0 \end{pmatrix}$, $\quad \mathrm{A}^{-1} = \dfrac{1}{6}\begin{pmatrix} 0 & 0 & 2 \\ 0 & 3 & i \\ -6 & 6i & -2 \end{pmatrix}$

9.14 $\mathrm{C^TBA^T}$, $\quad \mathrm{C^{-1}M^{-1}C}$, $\quad \mathrm{H}$

10.1 (b) $d = 8$

10.2 The number of basis vectors given is the dimension of the space. We list one possible basis; other bases consist of the same number of independent linear combinations of the vectors given.
(b) $(1,0,0,5,0,1)$, $(0,1,0,0,6,4)$, $(0,0,1,0,-3,0)$

10.3 (a) Label the vectors $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$. Then $\cos(\mathbf{A},\mathbf{B}) = 1/\sqrt{15}$, $\cos(\mathbf{A},\mathbf{C}) = \sqrt{2}/3$, $\cos(\mathbf{B},\mathbf{D}) = \sqrt{17/690}$.

10.4 (b) $\mathbf{e}_1 = (0,0,0,1)$, $\mathbf{e}_2 = (1,0,0,0)$, $\mathbf{e}_3 = (0,1,1,0)/\sqrt{2}$

10.5 (b) $\|\mathbf{A}\| = 7$, $\|\mathbf{B}\| = \sqrt{60}$, $\quad$|Inner product of $\mathbf{A}$ and $\mathbf{B}$| $= \sqrt{5}$

11.5 $\theta = 1.1 = 63.4°$

11.11 $\begin{pmatrix} x \\ y \end{pmatrix} = \dfrac{1}{5}\begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} x' \\ y' \end{pmatrix}$, not orthogonal

In the following answers, for each eigenvalue, the components of a corresponding eigenvector are listed in parentheses.

11.12	4	$(1,1)$
	-1	$(3,-2)$
11.15	1	$(0,0,1)$
	-1	$(1,-1,0)$
	5	$(1,1,0)$
11.18	4	$(2,1,3)$
	2	$(0,-3,1)$
	-3	$(5,-1,-3)$
11.20	3	$(0,-1,2)$
	4	$(1,2,1)$
	-2	$(-5,2,1)$
11.22	-4	$(-4,1,1)$
	5	$(1,2,2)$
	-2	$(0,-1,1)$

11.23	18	$(2,2,-1)$	The two eigenvectors corresponding to the eigenvalue 9
	9	$(1,-1,0)$	may be any two vectors orthogonal to $(2,2,-1)$ and or-
	9	$(1,1,4)$	thogonal to each other.

11.26	4	$(1,1,1)$
	1	$(1,-1,0)$
	1	$(1,1,-2)$

11.27 $\mathrm{D} = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$, $\mathrm{C} = \dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

11.29 $\mathrm{D} = \begin{pmatrix} 11 & 0 \\ 0 & 1 \end{pmatrix}$, $\mathrm{C} = \dfrac{1}{\sqrt{5}}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$

11.31 $\mathrm{D} = \begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix}$, $\mathrm{C} = \dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

11.41 $\lambda = 1,\ 3$; $\quad \mathrm{U} = \dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$

11.44 $\lambda = 3,\ -7;\quad \mathrm{U} = \dfrac{1}{5\sqrt{2}}\begin{pmatrix} 5 & -3-4i \\ 3-4i & 5 \end{pmatrix}$

11.52 60° rotation about $-\mathbf{i}\sqrt{2}+\mathbf{k}$ and reflection through the plane $z = x\sqrt{2}$

11.53 180° rotation about $\mathbf{i}+\mathbf{j}+\mathbf{k}$

11.56 45° rotation about $\mathbf{j}-\mathbf{k}$

11.58 $\mathrm{M}^{10} = \dfrac{1}{5}\begin{pmatrix} 1+4\cdot 6^{10} & 2-2\cdot 6^{10} \\ 2-2\cdot 6^{10} & 4+6^{10} \end{pmatrix}$

11.59 $e^{\mathrm{M}} = e^3\begin{pmatrix} \cosh 1 & -\sinh 1 \\ -\sinh 1 & \cosh 1 \end{pmatrix}$

12.2 $3x'^2 - 2y'^2 = 24$

12.3 $10x'^2 = 35$

12.6 $3x'^2 + \sqrt{3}\,y'^2 - \sqrt{3}\,z'^2 = 12$

12.15 $y = 2x$ with $\omega = \sqrt{3k/m}$; $x = -2y$ with $\omega = \sqrt{8k/m}$

12.17 $x = -2y$ with $\omega = \sqrt{2k/m}$; $3x = 2y$ with $\omega = \sqrt{2k/(3m)}$

12.19 $y = -x$ with $\omega = \sqrt{3k/m}$; $y = 2x$ with $\omega = \sqrt{3k/(2m)}$

12.22 $y = -x$ with $\omega = \sqrt{2k/m}$; $y = 3x$ with $\omega = \sqrt{2k/(3m)}$

13.6 The cyclic group

13.11 The four matrices of the symmetry group of the rectangle are:
$\mathrm{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\ \mathrm{P} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix},\ \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = -\mathrm{P},\ \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -\mathrm{I}$
This group is isomorphic to the 4's group.

13.21 SO(2) is Abelian; SO(3) is not Abelian.

14.3 $x,\ \cos x,\ x\cos x,\ e^x\cos x$

14.5 $1,\ x+x^3,\ x^2,\ x^4,\ x^5$

14.6 Not a vector space

14.8 $1,\ x^2,\ x^4,\ x^6$

15.3 (a) $(x-4)/1 = (y+1)/(-2) = (z-2)/(-2)$; or $\mathbf{r} = (4,-1,2) + (1,-2,-2)t$
(b) $x - 5y + 3z = 0$
(c) $5/7$
(d) $5\sqrt{2}/3 = 2.36$
(e) $\arcsin 19/21 = 64.8^\circ$

15.5 (a) $y = 7$, $(x-2)/3 = (z+1)/4$; or $\mathbf{r} = (2,7,-1) + (3,0,4)t$
(b) $x - 4y - 9z = 0$
(c) $\arcsin(\frac{33}{70}\sqrt{2}) = 41.8^\circ$
(d) $12/\sqrt{98} = 1.21$
(e) $\sqrt{29}/5 = 1.08$

15.7 You should have found all except $\mathrm{A^TB^T}$, $\mathrm{BA^T}$, ABC, $\mathrm{AB^TC}$, $\mathrm{B^{-1}C}$, and $\mathrm{CB^T}$, which are meaningless.
$\mathrm{B^TAC} = \begin{pmatrix} 2 & 2 \\ 1-3i & 1 \\ -1-5i & -1 \end{pmatrix},\qquad \mathrm{C^{-1}A} = \begin{pmatrix} 0 & -i \\ 1 & -1 \end{pmatrix}$

15.9 $\dfrac{1}{f} = (n-1)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2} + \dfrac{(n-1)d}{nR_1R_2}\right]$

15.13 Area $= \frac{1}{2}\left|\overrightarrow{PQ}\times\overrightarrow{PR}\right| = 7/2$

15.14 $x'' = -x,\ y'' = -y,\quad 180^\circ$ rotation

15.15 $x'' = -y,\ y'' = x$; 90° rotation of vectors or -90° rotation of axes

15.18
1	$(1,1)$
-2	$(0,1)$

15.20
1	$(1,1)$
9	$(1,-1)$

15.22
1	$(1,0,1)$
4	$(0,1,0)$
5	$(1,0,-1)$

15.24
2	$(0,4,3)$
7	$(5,-3,4)$
-3	$(5,3,-4)$

15.27 $3x'^2 - y'^2 - 5z'^2 = 15,\ d = \sqrt{5}$

15.29 $3x'^2 + 6y'^2 - 4z'^2 = 54,\ d = 3$

Chapter 4

1.1	$\partial u/\partial x = 2xy^2/(x^2+y^2)^2$, $\partial u/\partial y = -2x^2y/(x^2+y^2)^2$				
1.3	$\partial z/\partial u = u/(u^2+v^2+w^2)$				
1.4	At $(0,0)$, both $= 0$; at $(-2/3,\ 2/3)$, both $= -4$				
1.7	$2x$	1.9	$2x(1+2\tan^2\theta)$	1.11	$2y$
1.13	$4r^2\tan\theta$	1.15	$r^2\sin 2\theta$	1.17	$4r$
1.19	0	1.21	$-4x\csc^2\theta$	1.23	$2r\sin 2\theta$
1.8′	$-2r^4/x^3$	1.10′	$2y+4y^3/x^2$	1.12′	$2y\sec^2\theta$
1.14′	$2y^2\sec^2\theta\tan\theta$	1.16′	$2r\tan^2\theta$	1.18′	$-2ry^4/(r^2-y^2)^2$
1.20′	$4x(\tan\theta\sec^2\theta)(\tan^2\theta+\sec^2\theta)$				
1.22′	$-8r^3/x^3$	1.24′	$-8y^3/x^3$		

2.1 $y + y^3/6 - x^2y/2 + x^4y/24 - x^2y^3/12 + y^5/120\cdots$
2.3 $x - x^2/2 - xy + x^3/3 + x^2y/2 + xy^2\cdots$
2.5 $1 + xy/2 - x^2y^2/8 + x^3y^3/16 - 5x^4y^4/128\cdots$
2.8 $e^x\cos y = 1 + x + (x^2-y^2)/2 + (x^3-3xy^2)/6\cdots$

4.2	2.5×10^{-13}	4.4	12.2	4.6	9%
4.8	5%	4.10	4.28 nt	4.11	3.95
4.15	8×10^{23}				

5.1	$e^{-y}\sinh t + z\sin t$	5.3	$2r(q^2-p^2)$
5.7	$(1-2b-e^{2a})\cos(a-b)$		

6.2	$y'=1$, $y''=0$	6.3	$y' = 4(\ln 2 - 1)/(2\ln 2 - 1)$
6.5	$2x+11y-24=0$	6.6	$1800/11^3$
6.10	$x+y=0$	6.11	$y''=4$

7.1 $dx/dy = z - y + \tan(y+z)$, $d^2x/dy^2 = \frac{1}{2}\sec^3(y+z) + \frac{1}{2}\sec(y+z) - 2$
7.4 $\partial w/\partial u = -2(rv+s)w$, $\partial w/\partial v = -2(ru+2s)w$
7.7 $(\partial y/\partial\theta)_r = x$, $(\partial y/\partial\theta)_x = r^2/x$, $(\partial\theta/\partial y)_x = x/r^2$
7.8 $\partial x/\partial s = -19/13$, $\partial x/\partial t = -21/13$, $\partial y/\partial s = 24/13$, $\partial y/\partial t = 6/13$
7.10 $\partial x/\partial s = 1/6$, $\partial x/\partial t = 13/6$, $\partial y/\partial s = 7/6$, $\partial y/\partial t = -11/6$
7.13 $(\partial p/\partial q)_m = -p/q$, $(\partial p/\partial q)_a = 1/(a\cos p - 1)$,
$(\partial p/\partial q)_b = 1 - b\sin q$, $(\partial b/\partial a)_p = (\sin p)(b\sin q - 1)/\cos q$,
$(\partial a/\partial q)_m = [q + p(a\cos p - 1)]/(q\sin p)$
7.15 $(\partial x/\partial u)_v = (2yv^2 - x^2)/(2yv + 2xu)$, $(\partial x/\partial u)_y = (x^2u + y^2v)/(y^2 - 2xu^2)$
7.17 $(\partial p/\partial s)_t = -9/7$, $(\partial p/\partial s)_q = 3/2$
7.19 $(\partial x/\partial z)_s = 7/2$, $(\partial x/\partial z)_r = 4$, $(\partial x/\partial z)_y = 3$

8.3	$(-1,2)$ is a minimum point	8.4	$(-1,-2)$ is a saddle point
8.8	$\theta=\pi/3$; bend up 8 cm on each side		
8.9	$l = w = 2h$	8.11	$\theta = 30^\circ$, $x = y\sqrt{3} = z/2$
8.13	$(4/3, 5/3)$	8.16	$m = 5/2$, $b = 1/3$

9.2	$r:l:s = \sqrt{5} : (1+\sqrt{5}) : 3$	9.4	$4/\sqrt{3}$ by $6/\sqrt{3}$ by $10/\sqrt{3}$
9.6	$V = 1/3$	9.8	$(8/13, 12/13)$

9.12 Let legs of right triangle be a and b, height of prism $= h$; then $a = b$, $h = (2-\sqrt{2})a$.

10.2 4, 2
10.4 $d = 1$
10.6 $d = 2$
10.7 $\frac{1}{2}\sqrt{11}$
10.10 (a) $\max T = \frac{1}{2}$, $\min T = -\frac{1}{2}$
(b) $\max T = 1$, $\min T = -\frac{1}{2}$
(c) $\max T = 1$, $\min T = -\frac{1}{2}$
10.12 Largest sum $= 180°$
Smallest sum $= 3 \arccos(1/\sqrt{3}) = 164.2°$
10.13 Largest sum $= 3 \arcsin(1/\sqrt{3}) = 105.8°$, smallest sum $= 90°$

11.1 $z = f(y + 2x) + g(y + 3x)$
11.6 $d^2y/dz^2 + dy/dz - 5y = 0$
11.11 $H = p\dot{q} - L$

12.1 $\frac{1}{2}x^{-1/2} \sin x$
12.3 $dz/dx = -\sin(\cos x) \tan x - \sin(\sin x) \cot x$
12.4 $\frac{1}{2} \sin 2$
12.7 $(\partial u/\partial x)_y = -e^4$, $(\partial u/\partial y)_x = e^4/\ln 2$, $(\partial y/\partial x)_u = \ln 2$
12.10 $dy/dx = (e^x - 1)/x$
12.12 $(2x + 1)/\ln(x + x^2) - 2/\ln(2x)$
12.14 $\pi/(4y^3)$

13.2 (a) and (b) $d = 4/\sqrt{13}$
13.4 $-\csc\theta \cot\theta$
13.5 $-6x$, $2x^2 \tan\theta \sec^2\theta$, $4x \tan\theta \sec^2\theta$
13.9 $dz/dt = 1 + (t/z)(2 - x - y)$, $z \neq 0$
13.10 $[x \ln x - (y^2/x)]x^y$ where $x = r\cos\theta$, $y = r\sin\theta$
13.13 -1
13.14 $(\partial w/\partial x)_y = (\partial f/\partial x)_{s,t} + 2(\partial f/\partial s)_{x,t} + 2(\partial f/\partial t)_{x,s} = f_1 + 2f_2 + 2f_3$
13.18 $\sqrt{26/3}$
13.21 $T(2) = 4$, $T(5) = -5$
13.23 $t \cot t$
13.25 $-e^x/x$
13.29 $dt = 3.9$

Chapter 5

2.1	3	2.3	4	2.5	$\frac{1}{4}e^2 - \frac{5}{12}$	2.7	5/3	2.9	6
2.11	36	2.13	7/4	2.15	3/2	2.17	$\frac{1}{2}\ln 2$	2.19	32
2.21	131/6	2.23	9/8	2.25	3/2	2.27	32/5	2.29	2
2.31	6	2.33	16/3	2.36	1/6	2.37	7/6	2.39	70
2.41	5	2.43	9/2	2.45	$46k/15$	2.47	16/3	2.49	1/3

3.2 (b) $Ml^2/12$ (c) $Ml^2/3$
3.3 (a) $M = 140$ (b) $\bar{x} = 130/21$ (c) $I_m = 6.92M$
(d) $I = 150M/7$
3.5 (a) $Ma^2/3$ (b) $Ma^2/12$ (c) $2Ma^2/3$
3.7 (a) $M = 9$ (b) $(\bar{x}, \bar{y}) = (2, 4/3)$
(c) $I_x = 2M$, $I_y = 9M/2$ (d) $I_m = 13M/18$
3.9 (a) 1/6 (b) $(1/4, 1/4, 1/4)$ (c) $M = 1/24$, $\bar{z} = 2/5$
3.11 (a) $M = (5\sqrt{5} - 1)/6 = 1.7$
(b) $\bar{x} = 0$, $\bar{y} = (313 + 15\sqrt{5})/620 = 0.56$
3.14 $V = 2\pi^2 a^2 b$, $A = 4\pi^2 ab$, where a = radius of revolving circle, and b = distance to axis from center of this circle.
3.15 For area, $(\bar{x}, \bar{y}) = (0, \frac{4}{3}r/\pi)$, for arc, $(\bar{x}, \bar{y}) = (0, 2r/\pi)$

3.18 $s = [3\sqrt{2} + \ln(1 + \sqrt{2}\,)]/2$
3.20 $13\pi/3$
3.21 $s\bar{x} = [51\sqrt{2} - \ln(1 + \sqrt{2}\,)]/32$, $s\bar{y} = 13/6$, s as in Problem 3.18
3.23 $(149/130, 0, 0)$
3.25 I/M has the same numerical value as $\bar{x}$ in Problem 3.21
3.26 $2M/3$ 3.27 $149M/130$ 3.29 2 3.30 $32/5$

4.1 (b) $\bar{x} = \bar{y} = \frac{4}{3}a/\pi$ (c) $I = Ma^2/4$ (e) $\bar{x} = \bar{y} = 2a/\pi$
4.2 (c) $\bar{y} = \frac{4}{3}a/\pi$
(d) $I_x = Ma^2/4$, $I_y = 5Ma^2/4$, $I_z = 3Ma^2/2$
(e) $\bar{y} = 2a/\pi$
(f) $\bar{x} = 6a/5$, $I_x = 48Ma^2/175$, $I_y = 288Ma^2/175$, $I_z = 48Ma^2/25$
(g) $A = (\frac{2}{3}\pi - \frac{1}{2}\sqrt{3}\,)a^2$
4.4 (b) $(0, 0, a/2)$ (c) $2Ma^2/3$ (e) $(0, 0, 3a/8)$
4.5 $7\pi/3$
4.11 12π
4.12 (c) $M = (16\rho/9)(3\pi - 4) = 9.64\rho$
$I = (128\rho/15^2)(15\pi - 26) = 12.02\rho = 1.25M$
4.14 $\pi(1 - e^{-1})/4$ 4.16 $u^2 + v^2$ 4.19 $\pi/4$
4.22 $12(1 + 36\pi^2)^{1/2}$ 4.24 $\rho G\pi a/2$ 4.26 (a) $\frac{7}{5}Ma^2$
4.27 $2\pi ah$ (where h = distance between parallel planes)

5.1 $\frac{9}{5}\pi\sqrt{30}$ 5.3 $\pi(37^{3/2} - 1)/6$
5.5 8π for each nappe 5.6 4
5.8 $\frac{3}{16}\sqrt{6} + \frac{9}{16}\ln(\sqrt{2} + \sqrt{3}\,)$ 5.9 $\pi\sqrt{2}$
5.12 $M = \frac{1}{6}\sqrt{3}$, $(\bar{x}, \bar{y}, \bar{z}) = (\frac{1}{2}, \frac{1}{4}, \frac{1}{4})$ 5.14 $M = \frac{1}{2}\pi - \frac{4}{3}$
5.16 $\bar{x} = 0$, $\bar{y} = 1$, $\bar{z} = [32/(9\pi)]\sqrt{2/5} = 0.716$

6.2 $45(2 + \sqrt{2}\,)/112$ 6.3 $15\pi/8$
6.4 (a) $\frac{1}{2}MR^2$ (b) $\frac{3}{2}MR^2$ 6.6 (a) $(4\pi - 3\sqrt{3}\,)/6$
6.7 $(8\pi - 3\sqrt{3}\,)(4\pi - 3\sqrt{3}\,)^{-1}M$ 6.8 (b) $27/20$
6.10 (a) $(\bar{x}, \bar{y}) = (\pi/2, \pi/8)$ 6.10 (c) $3M/8$
6.12 $(abc)^2/6$ 6.14 $16a^3/3$
6.15 $I_x = \frac{8}{15}Ma^2$, $I_y = \frac{7}{15}Ma^2$ 6.16 $\bar{x} = \bar{y} = 2a/5$
6.18 $(0, 0, 5h/6)$
6.19 $I_x = I_y = 20Mh^2/21$, $I_z = 10Mh^2/21$, $I_m = 65Mh^2/252$
6.21 $\pi G\rho h(2 - \sqrt{2}\,)$ 6.24 $(0, 0, 2c/3)$
6.26 $\frac{1}{2}\sinh 1$ 6.27 $e^2 - e - 1$

Chapter 6

3.1 $(\mathbf{A} \cdot \mathbf{B})\mathbf{C} = 6\mathbf{C}$, $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = -8$,
$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = -4(\mathbf{i} + 2\mathbf{k})$
3.3 -5
3.6 $\mathbf{v} = (2/\sqrt{6}\,)(\mathbf{A} \times \mathbf{B}) = (2/\sqrt{6}\,)(\mathbf{i} - 7\mathbf{j} - 3\mathbf{k})$,
$\mathbf{r} \times \mathbf{F} = (\mathbf{A} - \mathbf{C}) \times \mathbf{B} = 3\mathbf{i} + 3\mathbf{j} - \mathbf{k}$,
$\mathbf{n} \cdot (\mathbf{r} \times \mathbf{F}) = [(\mathbf{A} - \mathbf{C}) \times \mathbf{B}] \cdot \mathbf{C}/|\mathbf{C}| = 8/\sqrt{26}$
3.7 (a) $11\mathbf{i} + 3\mathbf{j} - 13\mathbf{k}$, (b) 3, (c) 17

3.9 $-9\mathbf{i} - 23\mathbf{j} + \mathbf{k}$, $1/\sqrt{21}$
3.15 $\mathbf{u}_1 \cdot \mathbf{u} = -\mathbf{u}_3 \cdot \mathbf{u}$, $n_1\mathbf{u}_1 \times \mathbf{u} = n_2\mathbf{u}_2 \times \mathbf{u}$
3.17 $\mathbf{a} = (\boldsymbol{\omega} \cdot \mathbf{r})\boldsymbol{\omega} - \omega^2\mathbf{r}$; for $\mathbf{r} \perp \boldsymbol{\omega}$, $\mathbf{a} = -\omega^2\mathbf{r}$, $|\mathbf{a}| = v^2/r$.
3.19 (a) $16\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}$ (b) $8/\sqrt{6}$
3.20 (b) 12

4.2 (a) $t = 2$
(b) $\mathbf{v} = 4\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}$, $|\mathbf{v}| = 2\sqrt{14}$
(c) $(x-4)/4 = (y+4)/(-2) = (z-8)/6$, $2x - y + 3z = 36$
4.5 $|d\mathbf{r}/dt| = \sqrt{2}$; $|d^2\mathbf{r}/dt^2| = 1$; path is a helix.
4.8 $d\mathbf{r}/dt = \mathbf{e}_r(dr/dt) + \mathbf{e}_\theta(r\,d\theta/dt)$;
$d^2\mathbf{r}/dt^2 = \mathbf{e}_r[d^2r/dt^2 - r(d\theta/dt)^2] + \mathbf{e}_\theta[r\,d^2\theta/dt^2 + 2(dr/dt)(d\theta/dt)]$

6.2 $-\mathbf{i}$
6.4 $\pi e/(3\sqrt{5})$
6.6 $6x + 8y - z = 25$, $(x-3)/6 = (y-4)/8 = (z-25)/(-1)$
6.9 (a) $2\mathbf{i} - 2\mathbf{j} - \mathbf{k}$ (b) $5/\sqrt{6}$
(c) $\mathbf{r} = (1,1,1) + (2,-2,-1)t$
6.12 (a) $2\sqrt{5}$, $-2\mathbf{i} + \mathbf{j}$ (b) $3\mathbf{i} + 2\mathbf{j}$ (c) $\sqrt{10}$
6.14 (b) Down, at the rate $11\sqrt{2}$
6.17 $\mathbf{e}_r$ 6.19 $\mathbf{j}$

7.1 $\boldsymbol{\nabla} \cdot \mathbf{r} = 3$, $\boldsymbol{\nabla} \times \mathbf{r} = 0$
7.2 $\boldsymbol{\nabla} \cdot \mathbf{r} = 2$, $\boldsymbol{\nabla} \times \mathbf{r} = 0$
7.4 $\boldsymbol{\nabla} \cdot \mathbf{V} = 0$, $\boldsymbol{\nabla} \times \mathbf{V} = -(\mathbf{i} + \mathbf{j} + \mathbf{k})$
7.6 $\boldsymbol{\nabla} \cdot \mathbf{V} = 5xy$, $\boldsymbol{\nabla} \times \mathbf{V} = \mathbf{i}xz - \mathbf{j}yz + \mathbf{k}(y^2 - x^2)$
7.7 $\boldsymbol{\nabla} \cdot \mathbf{V} = 0$, $\boldsymbol{\nabla} \times \mathbf{V} = \mathbf{i}x - \mathbf{j}y - \mathbf{k}x\cos y$
7.10 0 7.11 $-(x^2+y^2)/(x^2-y^2)^{3/2}$
7.13 $2xy$ 7.14 0
7.16 $2(x^2+y^2+z^2)^{-1}$ 7.19 $2/r$

8.1 $-11/3$ 8.2 (a) -4π (b) -16 (c) -8
8.3 (a) 5/3 (b) 1 (c) 2/3 8.4 (a) 3 (b) 8/3
8.7 (b) 0 (d) 2π 8.8 $yz - x$
8.9 $3xy - x^3yz - z^2$ 8.11 $-y\sin^2 x$
8.14 $-\arcsin xy$ 8.18 (a) $\pi + \pi^2/2$ (b) $\pi^2/2$

9.2 40 9.4 $-3/2$ 9.7 πab
9.8 24π 9.10 -20 9.11 2

10.2 3 10.4 36π 10.5 $4\pi \cdot 5^2$ 10.7 48π 10.9 16π
10.12 $\phi = \begin{cases} 0, & r \le R_1; \\ (k/2\pi\epsilon_0)\ln(R_1/r), & R_1 \le r \le R_2; \\ (k/2\pi\epsilon_0)\ln(R_1/R_2), & r \ge R_2. \end{cases}$

11.2 $2ab^2$ 11.3 0 11.4 -12
11.5 36 11.6 45π 11.7 0
11.10 -6π 11.12 18π 11.15 $-2\pi\sqrt{2}$
11.18 $\mathbf{A} = (xz - yz^2 - y^2/2)\mathbf{i} + (x^2/2 - x^2z + yz^2/2 - yz)\mathbf{j} + \boldsymbol{\nabla}u$, any u
11.20 $\mathbf{A} = \mathbf{i}\sin zx + \mathbf{j}\cos zx + \mathbf{k}e^{zy} + \boldsymbol{\nabla}u$, any u

12.1 $(\sin\theta\cos\theta)\mathbf{C}$

12.7 (a) $9\mathbf{i}+5\mathbf{j}-3\mathbf{k}$ (b) $29/3$

12.9 24

12.11 (a) $\operatorname{grad}\phi=-3y\mathbf{i}-3x\mathbf{j}+2z\mathbf{k}$ (b) $-\sqrt{3}$
(c) $2x+y-2z+2=0$, $\mathbf{r}=(1,2,3)+(2,1,-2)t$

12.13 (a) $6\mathbf{i}-\mathbf{j}-4\mathbf{k}$ (b) $53^{-1/2}(6\mathbf{i}-\mathbf{j}-4\mathbf{k})$ (c) same as (a)
(d) $53^{1/2}$ (e) $53^{1/2}$

12.18 Not conservative (a) $1/2$ (b) $4/3$

12.21 4 12.23 192π 12.25 -18π

12.27 4 12.29 10 12.31 $29/3$

Chapter 7

		Amplitude	Period	Frequency	Velocity Amplitude
2.2		2	$\pi/2$	$2/\pi$	8
2.3		$1/2$	2	$1/2$	$\pi/2$
2.6	$s=6\cos(\pi/8)\sin(2t)$	$6\cos(\pi/8)$ $=5.54$	π	$1/\pi$	$12\cos(\pi/8)$ $=11.1$
2.8		2	4π	$1/(4\pi)$	1
2.10		4	π	$1/\pi$	8
2.11	q	3	$1/60$	60	
	I	360π	$1/60$	60	

2.13 $A=$ maximum value of θ, $\omega=\sqrt{g/l}$

2.16 $t\cong 4.91\cong 281^\circ$

2.19 $A=1$, $T=4$, $f=1/4$, $v=1/4$, $\lambda=1$

2.21 $y=20\sin\frac{1}{2}\pi(x-6t)$, $\partial y/\partial t=-60\pi\cos\frac{1}{2}\pi(x-6t)$

2.23 $y=\sin 880\pi((x/350)-t)$

2.25 $y=10\sin[\pi(x-3\cdot 10^8 t)/250]$

3.6 $\sin(2x+\frac{1}{3}\pi)$

4.5 $\pi^{-1}+\frac{1}{2}$ 4.6 $2/\pi$ 4.8 0

4.11 $1/2$ 4.14 (a) $2\pi/3$ (b) π 4.15 (a) $3/2$

$x\rightarrow$	-2π	$-\pi$	$-\pi/2$	0	$\pi/2$	π	2π
6.2	$1/2$	0	0	$1/2$	$1/2$	0	$1/2$
6.4	-1	0	-1	-1	0	0	-1
6.6	$1/2$	$1/2$	$1/2$	$1/2$	$1/2$	$1/2$	$1/2$
6.8	1	1	$1-\frac{1}{2}\pi$	1	$1+\frac{1}{2}\pi$	1	1
6.10	π	0	$\pi/2$	π	$\pi/2$	0	π

7.1 $f(x) = \dfrac{1}{2} + \dfrac{i}{\pi} \sum_{\substack{-\infty \\ \text{odd } n}}^{\infty} \dfrac{1}{n} e^{inx}$

7.2 $f(x) = \dfrac{1}{4} + \dfrac{1}{2\pi}\Big[(1-i)e^{ix} + (1+i)e^{-ix} - i(e^{2ix} - e^{-2ix}) - \dfrac{1+i}{3}e^{3ix} - \dfrac{1-i}{3}e^{-3ix} + \dfrac{1-i}{5}e^{5ix} + \dfrac{1+i}{5}e^{-5ix} \cdots\Big]$

7.7 $f(x) = \dfrac{\pi}{4} - \sum_{\substack{-\infty \\ \text{odd } n}}^{\infty} \left(\dfrac{1}{n^2\pi} + \dfrac{i}{2n}\right) e^{inx} + \sum_{\substack{-\infty \\ \text{even } n\neq 0}}^{\infty} \dfrac{i}{2n} e^{inx}$

7.11 $f(x) = \dfrac{1}{\pi} + \dfrac{e^{ix} - e^{-ix}}{4i} - \dfrac{1}{\pi} \sum_{\substack{-\infty \\ \text{even } n\neq 0}}^{\infty} \dfrac{e^{inx}}{n^2 - 1}$

8.2 $f(x) = \dfrac{1}{4} + \dfrac{1}{\pi}\left(\cos\dfrac{\pi x}{l} - \dfrac{1}{3}\cos\dfrac{3\pi x}{l} + \dfrac{1}{5}\cos\dfrac{5\pi x}{l} \cdots\right) + \dfrac{1}{\pi}\left(\sin\dfrac{\pi x}{l} + \dfrac{2}{2}\sin\dfrac{2\pi x}{l} + \dfrac{1}{3}\sin\dfrac{3\pi x}{l} + \dfrac{1}{5}\sin\dfrac{5\pi x}{l} + \dfrac{2}{6}\sin\dfrac{6\pi x}{l} \cdots\right)$

8.6 $f(x) = \dfrac{1}{2} + \dfrac{4}{\pi} \sum \dfrac{1}{n} \sin\dfrac{n\pi x}{l} \quad (n = 2, 6, 10, \cdots)$

8.11 (a) $f(x) = \dfrac{\pi^2}{3} + 4\sum_{1}^{\infty} \dfrac{(-1)^n}{n^2}\cos nx$

(b) $f(x) = \dfrac{4\pi^2}{3} + 2\sum_{-\infty}^{\infty}\left(\dfrac{1}{n^2} + \dfrac{i\pi}{n}\right)e^{inx}, \quad n \neq 0$

8.14 (a) $f(x) = \dfrac{8}{\pi}\sum_{1}^{\infty} \dfrac{n(-1)^{n+1}}{4n^2 - 1}\sin 2n\pi x$

(b) $f(x) = \dfrac{2}{\pi} - \dfrac{4}{\pi}\sum_{1}^{\infty}\dfrac{\cos 2n\pi x}{4n^2 - 1} = -\dfrac{2}{\pi}\sum_{-\infty}^{\infty}\dfrac{1}{4n^2 - 1}e^{2in\pi x}$

8.19 $f(x) = \dfrac{1}{8} - \dfrac{1}{\pi^2}\sum_{\text{odd } n=1}^{\infty}\dfrac{1}{n^2}\cos 2n\pi x + \dfrac{1}{2\pi}\sum_{1}^{\infty}\dfrac{(-1)^{n+1}}{n}\sin 2n\pi x$

8.20 $f(x) = \dfrac{2}{3} - \dfrac{9}{8\pi^2}\left[\cos\dfrac{2\pi x}{3} + \dfrac{1}{2^2}\cos\dfrac{4\pi x}{3} + \dfrac{1}{4^2}\cos\dfrac{8\pi x}{3} + \cdots\right] - \left(\dfrac{3\sqrt{3}}{8\pi^2} + \dfrac{1}{\pi}\right)\sin\dfrac{2\pi x}{3} + \left(\dfrac{3\sqrt{3}}{32\pi^2} - \dfrac{1}{2\pi}\right)\sin\dfrac{4\pi x}{3} - \dfrac{1}{3\pi}\sin\dfrac{6\pi x}{3} - \left(\dfrac{3\sqrt{3}}{128\pi^2} + \dfrac{1}{4\pi}\right)\sin\dfrac{8\pi x}{3} \cdots$

9.2 (a) $\frac{1}{2}\ln|1 - x^2| + \frac{1}{2}\ln|(1-x)/(1+x)|$

9.5 $f(x) = \dfrac{4}{\pi}\sum_{\text{odd } n=1}^{\infty}\dfrac{1}{n}\sin nx$

9.19 $f_c(x) = f_p(x) = \dfrac{2}{\pi} - \dfrac{4}{\pi}\sum_1^\infty \dfrac{(-1)^n \cos 2nx}{4n^2-1}$

$f_s(x) = \dfrac{2}{\pi}\left(\sin x + \sin 3x + \frac{1}{3}\sin 5x + \frac{1}{3}\sin 7x + \frac{1}{5}\sin 9x + \frac{1}{5}\sin 11x \cdots\right)$

9.20 $f_c(x) = \dfrac{1}{3} + \dfrac{4}{\pi^2}\sum_1^\infty \dfrac{(-1)^n}{n^2}\cos n\pi x$

$f_s(x) = \dfrac{2}{\pi}\sum_1^\infty \dfrac{(-1)^{n+1}}{n}\sin n\pi x - \dfrac{8}{\pi^3}\sum_{\text{odd } n=1}^\infty \dfrac{1}{n^3}\sin n\pi x$

$f_p(x) = \dfrac{1}{3} + \dfrac{1}{\pi^2}\sum_1^\infty \dfrac{1}{n^2}\cos 2n\pi x - \dfrac{1}{\pi}\sum_1^\infty \dfrac{1}{n}\sin 2n\pi x$

9.22 $f_c(x) = 15 - \dfrac{20}{\pi}\left(\cos\dfrac{\pi x}{20} - \dfrac{1}{3}\cos\dfrac{3\pi x}{20} + \dfrac{1}{5}\cos\dfrac{5\pi x}{20}\cdots\right)$

$f_s(x) = \dfrac{20}{\pi}\left(3\sin\dfrac{\pi x}{20} - \dfrac{2}{2}\sin\dfrac{2\pi x}{20} + \dfrac{3}{3}\sin\dfrac{3\pi x}{20} + \dfrac{3}{5}\sin\dfrac{5\pi x}{20} - \dfrac{2}{6}\sin\dfrac{6\pi x}{20}\cdots\right)$

$f_p(x) = 15 - \dfrac{20}{\pi}\sum_{\text{odd } n=1}^\infty \dfrac{1}{n}\sin\dfrac{n\pi x}{10}$

9.23 $f(x,0) = \dfrac{8h}{\pi^2}\left(\sin\dfrac{\pi x}{l} - \dfrac{1}{3^2}\sin\dfrac{3\pi x}{l} + \dfrac{1}{5^2}\sin\dfrac{5\pi x}{l}\cdots\right)$

10.1 Relative intensities $= 1:0:0:0:\frac{1}{25}:0:\frac{1}{49}:0:0:0$

10.3 Relative intensities $= 1:25:\frac{1}{9}:0:\frac{1}{25}:\frac{25}{9}:\frac{1}{49}:0:\frac{1}{81}:1$

10.5 $I(t) = \dfrac{5}{\pi}\left[1 - 2\sum_{\text{even } n=2}^\infty \dfrac{1}{n^2-1}\cos 120n\pi t\right] + \dfrac{5}{2}\sin 120\pi t$

10.6 $V(t) = 50 - \dfrac{400}{\pi^2}\sum_{\text{odd } n=1}^\infty \dfrac{1}{n^2}\cos 120n\pi t$

10.7 $I(t) = -\dfrac{20}{\pi}\sum_1^\infty \dfrac{(-1)^n}{n}\sin 120n\pi t$

10.10 $V(t) = 75 - \dfrac{200}{\pi^2}\sum_{\text{odd } n=1}^\infty \dfrac{1}{n^2}\cos 120n\pi t - \dfrac{100}{\pi}\sum_1^\infty \dfrac{1}{n}\sin 120n\pi t$

Relative intensities $= 1.4 : 0.25 : 0.12 : 0.06 : 0.04$

11.5 $\pi^2/8$ 11.7 $\pi^2/6$ 11.9 $\dfrac{\pi^2}{16} - \dfrac{1}{2}$

12.2 $f_s(x) = \dfrac{2}{\pi}\int_0^\infty \dfrac{1-\cos\alpha}{\alpha}\sin\alpha x\,d\alpha$

12.4 $f(x) = \int_{-\infty}^\infty \dfrac{\sin\alpha\pi - \sin(\alpha\pi/2)}{\alpha\pi}e^{i\alpha x}\,d\alpha$

12.6 $f(x) = \int_{-\infty}^\infty \dfrac{\sin\alpha - \alpha\cos\alpha}{i\pi\alpha^2}e^{i\alpha x}\,d\alpha$

12.8 $f(x) = \int_{-\infty}^\infty \dfrac{(i\alpha+1)e^{-i\alpha}-1}{2\pi\alpha^2}e^{i\alpha x}\,d\alpha$

12.10 $f(x) = 2\int_{-\infty}^{\infty} \frac{\alpha a - \sin\alpha a}{i\pi\alpha^2} e^{i\alpha x}\,d\alpha$

12.11 $f(x) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\cos(\alpha\pi/2)}{1-\alpha^2} e^{i\alpha x}\,d\alpha$

12.13 $f_c(x) = \frac{2}{\pi}\int_0^{\infty} \frac{\sin\alpha\pi - \sin(\alpha\pi/2)}{\alpha} \cos\alpha x\,d\alpha$

12.16 $f_c(x) = \frac{2}{\pi}\int_0^{\infty} \frac{\cos(\alpha\pi/2)}{1-\alpha^2} \cos\alpha x\,d\alpha$

12.18 $f_s(x) = \frac{2}{\pi}\int_0^{\infty} \frac{\sin\alpha - \alpha\cos\alpha}{\alpha^2} \sin\alpha x\,d\alpha$

12.19 $f_s(x) = \frac{4}{\pi}\int_0^{\infty} \frac{\alpha a - \sin\alpha a}{\alpha^2} \sin\alpha x\,d\alpha$

12.21 $g(\alpha) = \sigma(2\pi)^{-1/2} e^{-\alpha^2\sigma^2/2}$

12.25 (a) $f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1+e^{-i\alpha\pi}}{1-\alpha^2} e^{i\alpha x}\,d\alpha$

12.28 (a) $f_c(x) = \frac{4}{\pi}\int_0^{\infty} \frac{\cos 3\alpha \sin\alpha}{\alpha} \cos\alpha x\,d\alpha$

(b) $f_s(x) = \frac{4}{\pi}\int_0^{\infty} \frac{\sin 3\alpha \sin\alpha}{\alpha} \sin\alpha x\,d\alpha$

12.30 (a) $f_c(x) = \frac{1}{\pi}\int_0^{\infty} \frac{1-\cos 2\alpha}{\alpha^2} \cos\alpha x\,d\alpha$

(b) $f_s(x) = \frac{1}{\pi}\int_0^{\infty} \frac{2\alpha - \sin 2\alpha}{\alpha^2} \sin\alpha x\,d\alpha$

13.7 $f(x) = \frac{\pi}{2} - \frac{4}{\pi}\sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n^2}\cos nx$

13.8 (b) 1

13.10 (d) -1, $-1/2$, -2, -1

13.14 (a) $f(x) = \frac{1}{3} + \frac{4}{\pi^2}\sum_1^{\infty} \frac{\cos n\pi x}{n^2}$

(b) $\pi^4/90$

13.15 $-\pi/4$

13.23 $\pi^2/8$

Chapter 8

1.5 $x = -A\omega^{-2}\sin\omega t + v_0 t + x_0$

1.7 $x = (c/F)[(m^2c^2 + F^2t^2)^{1/2} - mc]$

2.2 $(1-x^2)^{1/2} + (1-y^2)^{1/2} = C$, $C = \sqrt{3}$

2.3 $\ln y = A(\csc x - \cot x)$, $A = \sqrt{3}$

2.6 $2y^2 + 1 = A(x^2-1)^2$, $A = 1$

2.7 $y^2 = 8 + e^{K-x^2}$, $K = 1$

2.9 $ye^y = ae^x$, $a = 1$

2.13 $y \equiv 1$, $y \equiv -1$, $x \equiv 1$, $x \equiv -1$

2.19 (a) $I/I_0 = e^{-0.5} = 0.6$ for $s = 50$ ft

Half value thickness $= (\ln 2)/\mu = 69.3$ ft

(b) Half life $T = (\ln 2)/\lambda$

2.20 (c) $\tau = RC$, $\tau = L/R$. Corresponding quantities are a, $\lambda = (\ln 2)/T$, μ, $1/\tau$.

2.22 $N = N_0 e^{Kt} - (R/K)(e^{Kt} - 1)$ where N_0 = number of bacteria at $t = 0$, KN = rate of increase, R = removal rate.

2.23 $T = 100[1 - (\ln r)/(\ln 2)]$

2.26 (a) k = weight divided by terminal speed
(b) $t = g^{-1} \cdot$ (terminal speed) $\cdot$ (ln 100); typical terminal speeds are 0.02 to 0.1 cm/sec, so t is of the order of 10^{-4} sec.

2.27 $t = 10(\ln \frac{5}{13})/(\ln \frac{3}{13}) = 6.6$ min

2.29 $t = 100 \ln \frac{9}{4} = 81.1$ min

2.31 $ay = bx$

2.33 $x^2 + ny^2 = C$

2.35 $x(y-1) = C$

3.1 $y = \frac{1}{2}e^x + Ce^{-x}$

3.3 $y = (\frac{1}{2}x^2 + C)e^{-x^2}$

3.6 $y = (x + C)/(x + \sqrt{x^2+1}\,)$

3.8 $y = \frac{1}{2}\ln x + C/\ln x$

3.9 $y(1-x^2)^{1/2} = x^2 + C$

3.11 $y = 2(\sin x - 1) + Ce^{-\sin x}$

3.13 $x = \frac{1}{2}e^y + Ce^{-y}$

3.14 $x = y^{2/3} + Cy^{-1/3}$

3.15 $S = (10^7/2)[(1+3t/10^4) + (1+3t/10^4)^{-1/3}]$, where S = number of pounds of salt, and t is in hours.

3.17 $I = Ae^{-t/(RC)} - V_0\omega C(\sin\omega t - \omega RC\cos\omega t)/(1+\omega^2R^2C^2)$

3.21 $N_n = c_1e^{-\lambda_1 t} + c_2e^{-\lambda_2 t} + \cdots$ where

$$c_1 = \frac{\lambda_1\lambda_2\cdots\lambda_{n-1}N_0}{(\lambda_2-\lambda_1)(\lambda_3-\lambda_1)\ldots(\lambda_n-\lambda_1)},\ c_2 = \frac{\lambda_1\lambda_2\cdots\lambda_{n-1}N_0}{(\lambda_1-\lambda_2)(\lambda_3-\lambda_2)\cdots(\lambda_n-\lambda_2)},$$

etc. (all λ's different)

3.22 $y = x + 1 + Ke^x$

4.1 $y^{1/3} = x - 3 + Ce^{-x/3}$

4.4 $x^2e^{3y} + e^x - \frac{1}{3}y^3 = C$

4.5 $x^2 - y^2 + 2x(y+1) = C$

4.7 $x = y(\ln x + C)$

4.9 $y^2 = Ce^{-x^2/y^2}$

4.11 $\tan\frac{1}{2}(x+y) = x + C$

4.13 $y^2 = -\sin^2 x + C\sin^4 x$

4.16 $y^2 = C(C \pm 2x)$

4.18 $x^2 + (y-k)^2 = k^2$

4.19 $r = Ae^{-\theta}$, $r = Be^{\theta}$

5.1 $y = Ae^x + Be^{-2x}$

5.3 $y = Ae^{3ix} + Be^{-3ix}$ or other forms as in (5.24)

5.5 $y = (Ax+B)e^x$

5.7 $y = Ae^{3x} + Be^{2x}$

5.9 $y = Ae^{2x}\sin(3x+\gamma)$

5.11 $y = (A+Bx)e^{-3x/2}$

5.20 $y = Ae^{-x} + Be^{ix}$

5.22 $y = Ae^x + Be^{-3x} + Ce^{-5x}$

5.24 $y = Ae^{-x} + Be^{x/2}\sin(\frac{1}{2}x\sqrt{3} + \gamma)$

5.26 $y = Ae^{5x} + (Bx + C)e^{-x}$

5.28 $y = e^x(A\sin x + B\cos x) + e^{-x}(C\sin x + D\cos x)$

5.29 $y = (A+Bx)e^{-x} + Ce^{2x} + De^{-2x} + E\sin(2x+\gamma)$

5.35 $T = 2\pi\sqrt{R/g} \cong 85$ min.

6.1 $y = Ae^{2x} + Be^{-2x} - \frac{5}{2}$

6.3 $y = Ae^x + Be^{-2x} + \frac{1}{4}e^{2x}$

6.5 $y = Ae^{ix} + Be^{-ix} + e^x$

6.7 $y = Ae^{-x} + Be^{2x} + xe^{2x}$

6.9 $y = (Ax + B + x^2)e^{-x}$

6.11 $y = e^{-x}(A\sin 3x + B\cos 3x) + 8\sin 4x - 6\cos 4x$

6.13 $y = (Ax+B)e^x - \sin x$

6.15 $y = e^{-6x/5}[A\sin(8x/5) + B\cos(8x/5)] - 5\cos 2x$

6.17 $y = A\sin 4x + B\cos 4x + 2x\sin 4x$

6.18 $y = e^{-x}(A\sin 4x + B\cos 4x) + 2e^{-4x}\cos 5x$

6.20 $y = Ae^{-2x}\sin(2x+\gamma) + 4e^{-x/2}\sin(5x/2)$

6.22 $y = A + Be^{-x/2} + x^2 - 4x$

6.24 $y = (A + Bx + 2x^3)e^{3x}$

6.26 $y = A\sin x + B\cos x - 2x^2\cos x + 2x\sin x$

6.33 $y = A\sin(x+\gamma) + x^3 - 6x - 1 + x\sin x + (3-2x)e^x$

6.34 $y = Ae^{3x} + Be^{2x} + e^x + x$
6.37 $y = (A + Bx)e^x + 2x^2e^x + (3 - x)e^{2x} + x + 1$
6.41 $y = e^{-x}(A\cos x + B\sin x) + \frac{1}{4}\pi$

$$+ \sum_{\text{odd } n=1}^{\infty} [4(n^2 - 2)\cos nx - 8n\sin nx]/[\pi n^2(n^2 + 4)]$$

7.1 (a) $y \equiv 5$ (b) $y = 2/(x + 1)$
(c) $y = \tan(\frac{\pi}{4} - \frac{x}{2}) = \sec x - \tan x$ (d) $y = 2\tanh x$
7.4 $x^2 + (y - b)^2 = a^2$, or $y = C$ 7.11 $x = (1 - 3t)^{1/3}$
7.12 $t = \int_1^x u^2(1 - u^4)^{-1/2}\,du$
7.16 (a) $y = Ax + Bx^{-3}$ 7.16 (c) $y = (A + B\ln x)/x^3$
7.18 $y = Ax + Bx^{-1} + \frac{1}{2}\left(x + x^{-1}\right)\ln x$
7.20 $y = x^2(A + B\ln x) + x^2(\ln x)^3$ 7.22 $y = A\cos\ln x + B\sin\ln x + x$
7.25 $x^{-1} - 1$ 7.27 x^3e^x 7.29 $xe^{1/x}$

8.8 $e^{-2t} - te^{-2t}$ 8.10 $\frac{1}{3}e^t\sin 3t + 2e^t\cos 3t$
8.12 $3\cosh 5t + 2\sinh 5t$ 8.21 $2b(p + a)/[(p + a)^2 + b^2]^2$
8.23 $y = te^{-2t}(\cos t - \sin t)$ 8.25 $e^{-p\pi/2}/(p^2 + 1)$

9.3 $y = e^{-2t}(4t + \frac{1}{2}t^2)$ 9.4 $y = \cos t + \frac{1}{2}(\sin t - t\cos t)$
9.7 $y = 1 - e^{2t}$ 9.9 $y = (t + 2)\sin 4t$
9.11 $y = te^{2t}$ 9.12 $y = \frac{1}{2}(t^2e^{-t} + 3e^t - e^{-t})$
9.13 $y = \sinh 2t$ 9.17 $y = 2$
9.19 $y = e^{2t}$ 9.21 $y = e^{3t} + 2e^{-2t}\sin t$
9.23 $y = \sin t + 2\cos t - 2e^{-t}\cos 2t$ 9.25 $y = (3 + t)e^{-2t}\sin t$
9.27 $\begin{cases} y = t + \frac{1}{4}(1 - e^{4t}) \\ z = \frac{1}{3} + e^{4t} \end{cases}$ 9.28 $\begin{cases} y = t\cos t - 1 \\ z = \cos t + t\sin t \end{cases}$
9.30 $\begin{cases} y = t - \sin 2t \\ z = \cos 2t \end{cases}$ 9.32 $\begin{cases} y = \sin 2t \\ z = \cos 2t - 1 \end{cases}$
9.36 $\arctan(2/3)$ 9.38 $4/5$
9.40 1 9.42 $\pi/4$

10.3 $\frac{1}{2}t\sinh t$
10.5 $[b(b - a)te^{-bt} + a(e^{-bt} - e^{-at})]/(b - a)^2$
10.7 $(a\cosh bt - b\sinh bt - ae^{-at})/(a^2 - b^2)$
10.9 $(2t^2 - 2t + 1 - e^{-2t})/4$
10.12 $(b^2 - a^2)^{-1}(b^{-2}\cos bt - a^{-2}\cos at) + a^{-2}b^{-2}$
10.13 $\frac{1}{2}(e^{-t} + \sin t - \cos t)$
10.15 $\frac{1}{14}e^{3t} + \frac{1}{35}e^{-4t} - \frac{1}{10}e^t$
10.17 $y = \begin{cases} (\cosh at - 1)/a^2, & t > 0 \\ 0, & t < 0 \end{cases}$

11.7 $y = \begin{cases} (t - t_0)e^{-(t-t_0)}, & t > t_0 \\ 0, & t < t_0 \end{cases}$

11.9 $y = \begin{cases} \frac{1}{3}e^{-(t-t_0)}\sin 3(t-t_0), & t > t_0 \\ 0, & t < t_0 \end{cases}$

11.11 $y = \begin{cases} \frac{1}{2}[\sinh(t-t_0) - \sin(t-t_0)], & t > t_0 \\ 0, & t < t_0 \end{cases}$

11.13 (b) $3\delta(x+5) - 4\delta(x-10)$

11.15 (b) 0 (d) $\cosh 1$

11.21 (b) $\phi(|a|)/(2|a|)$ (c) 1/2

11.23 (a) $\delta(x+5)\delta(y-5)\delta(z)$, $\delta(r-5\sqrt{2}\,)\delta(\theta-\frac{3\pi}{4})\delta(z)/r$,
$\delta(r-5\sqrt{2}\,)\delta(\theta-\frac{\pi}{2})\delta(\phi-\frac{3\pi}{4})/(r\sin\theta)$
(c) $\delta(x+2)\delta(y)\delta(z-2\sqrt{3}\,)$, $\delta(r-2)\delta(\theta-\pi)\delta(z-2\sqrt{3}\,)/r$,
$\delta(r-4)\delta(\theta-\frac{\pi}{6})\delta(\phi-\pi)/(r\sin\theta)$

11.25 (c) $G''(x) = \delta(x) + 5\delta'(x)$

12.2 $y = (\sin\omega t - \omega t\cos\omega t)/(2\omega^2)$

12.7 $y = [a(\cosh at - e^{-t}) - \sinh at]/[a(a^2-1)]$

12.11 $y = -\frac{1}{3}\sin 2x$

12.13 $y = \begin{cases} x - \sqrt{2}\,\sin x, & x < \pi/4 \\ \frac{1}{2}\pi - x - \sqrt{2}\,\cos x, & x > \pi/4 \end{cases}$

12.16 $y = -x\ln x - x - x(\ln x)^2/2$

12.18 $y = x^2/2 + x^4/6$

13.1 $y = -\frac{1}{3}x^{-2} + Cx$

13.3 $y = A + Be^{-x}\sin(x+\gamma)$

13.5 $x^2 + y^2 - y\sin^2 x = C$

13.7 $3x^2y^3 + 1 = Ax^3$

13.8 $y = x(A + B\ln x) + \frac{1}{2}x(\ln x)^2$

13.10 $u - \ln u + \ln v + v^{-1} = C$

13.13 $y = Ae^{-2x}\sin(x+\gamma) + e^{3x}$

13.15 $y = (A+Bx)e^{2x} + 3x^2e^{2x}$

13.18 $x = (y+C)e^{-\sin y}$

13.20 $y = Ae^x\sin(2x+\gamma) + x + \frac{2}{5} + e^x(1 - x\cos 2x)$

13.22 $y = (A+Bx)e^{2x} + C\sin(3x+\gamma)$

13.24 $y^2 = ax^2 + b$

13.26 $y = x^2 + x$

13.28 $y^2 + 4(x-1)^2 = 9$

13.30 $y = g/3$, $v = 7g/12$, $a = 5g/12$

13.32 1:23 p.m.

13.33 In both (a) and (b), the temperature of the mixture at time t is given by the formula $T_a(1-e^{-kt}) + (n+n')^{-1}(nT_0 + n'T_0')e^{-kt}$.

13.38 $\frac{1}{2}\ln[(a^2+p^2)/p^2]$

13.41 $\frac{1}{4}(\tanh 1 - \operatorname{sech}^2 1) = 0.0854$

13.43 $(\sin at + at\cos at)/(2a)$

13.46 For e^{-x}: $g_s(\alpha) = (2/\pi)^{1/2}\alpha/(1+\alpha^2)$, $g_c(\alpha) = (2/\pi)^{1/2}/(1+\alpha^2)$

13.47 $y = A\sin t + B\cos t + \sin t\ln(\sec t + \tan t) - 1$

Chapter 9

2.1 Parabola

2.2 Circle

2.3 $ax = \sinh(ay+b)$

2.6 $x + a = \frac{4}{3}(y^{1/2} - 2b)(b + y^{1/2})^{1/2}$

3.1 $dx/dy = C/\sqrt{y^3 - C^2}$

3.3 $x^4y'^2 = C^2(1 + x^2y'^2)^3$

3.6 $x = ay^{3/2} - \frac{1}{2}y^2 + b$

3.7 $y = K\sinh(x+C)$

3.9 $\cot\theta = A\cos(\phi-\alpha)$

3.12 $(x-a)^2 + y^2 = C^2$

3.15 $r\cos(\theta+\alpha) = C$ or, in rectangular coordinates, the straight line $x\cos\alpha - y\sin\alpha = C$

3.18 See Problem 3.9

4.6 Cycloid

5.2 $\begin{cases} m(\ddot{r} - r\dot{\theta}^2) = -\partial V/\partial r \\ m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = -(1/r)(\partial V/\partial \theta) \\ m\ddot{z} = -\partial V/\partial z \end{cases}$

Comment: These equations are in the form $m\mathbf{a} = \mathbf{F}$; recall from Chapter 6, equation (6.7), the polar coordinate form for $\mathbf{F} = -\boldsymbol{\nabla} V$.

5.4 $l\ddot{\theta} + g\sin\theta = 0$

5.6 $\begin{cases} a\ddot{\theta} - a\sin\theta\cos\theta\ \dot{\phi}^2 - g\sin\theta = 0 \\ (d/dt)(\sin^2\theta\ \dot{\phi}) = 0 \end{cases}$

5.8 $L = \frac{1}{2}m(2\dot{r}^2 + r^2\dot{\theta}^2) - mgr$

$2\ddot{r} - r\dot{\theta}^2 + g = 0,\ (d/dt)(r^2\dot{\theta}) = 0$

5.11 $L = \frac{1}{2}(m + Ia^{-2})\dot{z}^2 - mgz$

$(ma^2 + I)\ddot{z} + mga^2 = 0$

5.12 $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - \left[\frac{1}{2}k(r - r_0)^2 - mgr\cos\theta\right]$

$\ddot{r} - r\dot{\theta}^2 + \frac{k}{m}(r - r_0) - g\cos\theta = 0, \quad (d/dt)(r^2\dot{\theta}) + gr\sin\theta = 0$

5.14 $L = M\dot{x}^2 + Mgx\sin\alpha, \quad 2M\ddot{x} - Mg\sin\alpha = 0$

5.16 $L = \frac{1}{2}m(l + a\theta)^2\dot{\theta}^2 - mg[a\sin\theta - (l + a\theta)\cos\theta]$

$(l + a\theta)\ddot{\theta} + a\dot{\theta}^2 + g\sin\theta = 0$

5.19 $x = y$ with $\omega = \sqrt{g/l}\,$; $\quad x = -y$ with $\omega = \sqrt{3g/l}$

5.21 $2\ddot{\theta} + \ddot{\phi}\cos(\theta - \phi) + \dot{\phi}^2\sin(\theta - \phi) + \frac{2g}{l}\sin\theta = 0$

$\ddot{\phi} + \ddot{\theta}\cos(\theta - \phi) - \dot{\theta}^2\sin(\theta - \phi) + \frac{g}{l}\sin\phi = 0$

5.23 $\phi = 2\theta$ with $\omega = \sqrt{2g/(3l)}\,$; $\quad \phi = -2\theta$ with $\omega = \sqrt{2g/l}$

6.1 Catenary 6.3 Circular cylinder 6.5 Circle

8.4 $dr/d\theta = Kr\sqrt{r^4 - K^2}$

8.6 $(x - a)^2 + (y + 1)^2 = C^2$

8.8 Intersection of $r = 1 + \cos\theta$ with $z = a + b\sin(\theta/2)$

8.10 Intersection of $y = x^2$ with $az = b[2x\sqrt{4x^2 + 1} + \sinh^{-1} 2x] + c$

8.12 $e^y\cos(x - a) = K$

8.16 Hyperbola: $r^2\cos(2\theta + \alpha) = K$ or $(x^2 - y^2)\cos\alpha - 2xy\sin\alpha = K$

8.17 $K\ln r = \cosh(K\theta + C)$

8.18 Parabola: $(x - y - C)^2 = 4K^2(x + y - K^2)$

8.20 $m(\ddot{r} - r\dot{\theta}^2) + Kr^{-2} = 0$, $r^2\dot{\theta} = \text{const.}$

8.22 $r^{-1}m(r^2\ddot{\theta} + 2r\dot{r}\dot{\theta} - r^2\sin\theta\cos\theta\ \dot{\phi}^2) = -r^{-1}(\partial V/\partial\theta) = F_\theta = ma_\theta$,

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\sin\theta\cos\theta\ \dot{\phi}^2$

8.27 $dr/d\theta = r\sqrt{K^2(1 + \lambda r)^2 - 1}$

Chapter 10

4.6 $I = \begin{pmatrix} 9 & 0 & -3 \\ 0 & 6 & 0 \\ -3 & 0 & 9 \end{pmatrix}$; principal moments: $(6, 6, 12)$; principal axes along the vectors $(1, 0, -1)$ and any two orthogonal vectors in the plane $z = x$, say $(0, 1, 0)$ and $(1, 0, 1)$.

5.6 (a) 3 (c) 2 (e) -1

6.15 (c) vector

8.1
$$h_r = 1, \quad h_\theta = r, \quad h_\phi = r\sin\theta$$
$$d\mathbf{s} = \mathbf{e}_r\,dr + \mathbf{e}_\theta\,r\,d\theta + \mathbf{e}_\phi\,r\sin\theta\,d\phi$$
$$dV = r^2\sin\theta\,dr\,d\theta\,d\phi$$
$$\mathbf{a}_r = \mathbf{i}\sin\theta\cos\phi + \mathbf{j}\sin\theta\sin\phi + \mathbf{k}\cos\theta = \mathbf{e}_r$$
$$\mathbf{a}_\theta = \mathbf{i}r\cos\theta\cos\phi + \mathbf{j}\,r\cos\theta\sin\phi - \mathbf{k}r\sin\theta = r\mathbf{e}_\theta$$
$$\mathbf{a}_\phi = -\mathbf{i}r\sin\theta\sin\phi + \mathbf{j}\,r\sin\theta\cos\phi = r\sin\theta\,\mathbf{e}_\phi$$

8.3
$$d\mathbf{s}/dt = \mathbf{e}_r\dot{r} + \mathbf{e}_\theta\,r\dot{\theta} + \mathbf{e}_\phi\,r\sin\theta\,\dot{\phi}$$
$$\begin{aligned} d^2\mathbf{s}/dt^2 = {} & \mathbf{e}_r(\ddot{r} - r\dot{\theta}^2 - r\sin^2\theta\,\dot{\phi}^2) \\ & + \mathbf{e}_\theta(r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\sin\theta\cos\theta\,\dot{\phi}^2) \\ & + \mathbf{e}_\phi(r\sin\theta\,\ddot{\phi} + 2r\cos\theta\,\dot{\theta}\dot{\phi} + 2\sin\theta\,\dot{r}\dot{\phi}) \end{aligned}$$

8.5 $\mathbf{V} = \mathbf{e}_r\cos\theta - \mathbf{e}_\theta\sin\theta - \mathbf{e}_\phi\,r\sin\theta$

8.6
$$h_u = h_v = (u^2+v^2)^{1/2}, \quad h_z = 1$$
$$d\mathbf{s} = (u^2+v^2)^{1/2}(\mathbf{e}_u\,du + \mathbf{e}_v\,dv) + \mathbf{e}_z\,dz$$
$$dV = (u^2+v^2)\,du\,dv\,dz$$
$$\mathbf{a}_u = \mathbf{i}u + \mathbf{j}v = (u^2+v^2)^{1/2}\mathbf{e}_u$$
$$\mathbf{a}_v = -\mathbf{i}v + \mathbf{j}u = (u^2+v^2)^{1/2}\mathbf{e}_v$$
$$\mathbf{a}_z = \mathbf{k} = \mathbf{e}_z$$

8.9
$$h_u = h_v = a(\cosh u + \cos v)^{-1}$$
$$d\mathbf{s} = a(\cosh u + \cos v)^{-1}(\mathbf{e}_u\,du + \mathbf{e}_v\,dv)$$
$$dA = a^2(\cosh u + \cos v)^{-2}\,du\,dv$$
$$\mathbf{a}_u = (h_u^2/a)[\mathbf{i}(1 + \cos v\cosh u) - \mathbf{j}\sin v\sinh u] = h_u\mathbf{e}_u$$
$$\mathbf{a}_v = (h_v^2/a)[\mathbf{i}\sinh u\sin v + \mathbf{j}(1 + \cos v\cosh u)] = h_v\mathbf{e}_v$$

8.11
$$d\mathbf{s}/dt = (u^2+v^2)^{1/2}(\mathbf{e}_u\dot{u} + \mathbf{e}_v\dot{v}) + \mathbf{e}_z\dot{z}$$
$$\begin{aligned} d^2\mathbf{s}/dt^2 = {} & \mathbf{e}_u(u^2+v^2)^{-1/2}[(u^2+v^2)\ddot{u} + u(\dot{u}^2 - \dot{v}^2) + 2v\dot{u}\dot{v}] \\ & + \mathbf{e}_v(u^2+v^2)^{-1/2}[(u^2+v^2)\ddot{v} + v(\dot{v}^2 - \dot{u}^2) + 2u\dot{u}\dot{v}] + \mathbf{e}_z\ddot{z} \end{aligned}$$

8.14
$$d\mathbf{s}/dt = a(\cosh u + \cos v)^{-1}(\mathbf{e}_u\dot{u} + \mathbf{e}_v\dot{v})$$
$$\begin{aligned} d^2\mathbf{s}/dt^2 = {} & \mathbf{e}_u a(\cosh u + \cos v)^{-2}[(\cosh u + \cos v)\ddot{u} + (\dot{v}^2 - \dot{u}^2)\sinh u + 2\dot{u}\dot{v}\sin v] \\ & + \mathbf{e}_v a(\cosh u + \cos v)^{-2}[(\cosh u + \cos v)\ddot{v} + (\dot{v}^2 - \dot{u}^2)\sin v - 2\dot{u}\dot{v}\sinh u] \end{aligned}$$

9.10 Let $h = h_u = h_v = (u^2+v^2)^{1/2}$ represent the u and v scale factors.
$$\boldsymbol{\nabla}U = h^{-1}\left(\mathbf{e}_u\frac{\partial U}{\partial u} + \mathbf{e}_v\frac{\partial U}{\partial v}\right) + \mathbf{k}\frac{\partial U}{\partial z}$$
$$\boldsymbol{\nabla}\cdot\mathbf{V} = h^{-2}\left[\frac{\partial}{\partial u}(hV_u) + \frac{\partial}{\partial v}(hV_v)\right] + \frac{\partial V_z}{\partial z}$$
$$\nabla^2 U = h^{-2}\left(\frac{\partial^2 U}{\partial u^2} + \frac{\partial^2 U}{\partial v^2}\right) + \frac{\partial^2 U}{\partial z^2}$$
$$\boldsymbol{\nabla}\times\mathbf{V} = \left(h^{-1}\frac{\partial V_z}{\partial v} - \frac{\partial V_v}{\partial z}\right)\mathbf{e}_u + \left(\frac{\partial V_u}{\partial z} - h^{-1}\frac{\partial V_z}{\partial u}\right)\mathbf{e}_v + h^{-2}\left[\frac{\partial}{\partial u}(hV_v) - \frac{\partial}{\partial v}(hV_u)\right]\mathbf{e}_z$$

9.13 Same as 9.10 if $h = a(\cosh u + \cos v)^{-1}$ and terms involving either z derivatives or V_z are omitted. Note, however, that $\nabla \times \mathbf{V}$ has *only* a z component if $\mathbf{V} = \mathbf{e}_u V_u + \mathbf{e}_v V_v$ where V_u and V_v are functions of u and v.

9.15 $h_u = 1$, $h_v = u/\sqrt{1-v^2}$
$\mathbf{e}_u = \mathbf{i}v + \mathbf{j}\sqrt{1-v^2}$, $\mathbf{e}_v = \mathbf{i}\sqrt{1-v^2} - \mathbf{j}v$
$m[\ddot{u} - u\dot{v}^2/(1-v^2)] = -\partial V/\partial u = F_u$
$m[(u\ddot{v} + 2\dot{u}\dot{v})/(1-v^2)^{1/2} + uv\dot{v}^2/(1-v^2)^{3/2}] = -h_v^{-1}\partial V/\partial v = F_v$

9.16 r^{-1}, 0, 0, $r^{-1}\mathbf{e}_z$

9.19 $2\mathbf{e}_\phi$, $\mathbf{e}_r\cos\theta - \mathbf{e}_\theta\sin\theta$, 3

9.21 $2r^{-1}$, 6, $2r^{-4}$, $-k^2 e^{ikr\cos\theta}$

Chapter 11

3.3 $9/10$

3.7 8

3.9 $\Gamma(5/4)$

3.11 1

3.14 $-\Gamma(4/3)$

3.17 $\Gamma(p)$

7.1 $\frac{1}{2}B(\frac{5}{2}, \frac{1}{2}) = 3\pi/16$

7.3 $\frac{1}{3}B(\frac{1}{3}, \frac{1}{2})$

7.5 $B(3,3) = 1/30$

7.7 $\frac{1}{2}B(\frac{1}{4}, \frac{1}{2})$

7.11 $2B(\frac{2}{3}, \frac{4}{3})/B(\frac{1}{3}, \frac{4}{3})$

7.13 $I_y/M = 8B(\frac{4}{3}, \frac{4}{3})/B(\frac{5}{3}, \frac{1}{3})$

8.1 $B(\frac{1}{2}, \frac{1}{4})\sqrt{2l/g} = 7.4163\sqrt{l/g}$
(Compare $2\pi\sqrt{l/g}$)

8.3 $t = \pi\sqrt{a/g}$

10.2 $\Gamma(p,x) \sim x^{p-1}e^{-x}[1 + (p-1)x^{-1} + (p-1)(p-2)x^{-2}\cdots]$

10.5 (a) $E_1(x) = \Gamma(0,x)$

10.6 (b) $\mathrm{Ei}(x)$

11.5 1

12.1
$$K = F(\pi/2, k) = (\pi/2)\{1 + (\tfrac{1}{2})^2k^2 + [(1\cdot 3)/(2\cdot 4)]^2k^4\cdots\}$$
$$E = E(\pi/2, k) = (\pi/2)\{1 - (\tfrac{1}{2})^2k^2 - [1/(2\cdot 4)]^2\cdot 3k^4 - [(1\cdot 3)/(2\cdot 4\cdot 6)]^2\cdot 5k^6\cdots\}$$

Caution: For the following answers, see the warning about elliptic integral notation just after equations (12.3) and in Example 1.

12.5 $E(1/3) \cong 1.526$

12.6 $\frac{1}{3}F(\frac{\pi}{3}, \frac{1}{3}) \cong 0.355$

12.7 $5E(\frac{5\pi}{4}, \frac{1}{5}) \cong 19.46$

12.10 $\frac{1}{2}F(\frac{\pi}{4}, \frac{1}{2}) \cong 0.402$

12.11 $F(\frac{3\pi}{8}, \frac{3}{\sqrt{10}}) + K(\frac{3}{\sqrt{10}}) \cong 4.097$

12.13 $3E(\frac{\pi}{6}, \frac{2}{3}) + 3E(\arcsin\frac{3}{4}, \frac{2}{3}) \cong 3.96$

12.16 $2\sqrt{2}\,E(1/\sqrt{2}) \cong 3.820$

12.23 $T = 8\sqrt{\frac{a}{5g}}\,K(1/\sqrt{5})$; for small vibrations, $T \cong 2\pi\sqrt{\frac{2a}{3g}}$

13.8 $\frac{1}{2}\sqrt{\pi}\,\mathrm{erf}(1)$

13.10 $\sqrt{2}\,K(1/\sqrt{2}) \cong 2.622$

13.11 $\frac{1}{5}F(\arcsin\frac{3}{4}, \frac{4}{5}) \cong 0.1834$

13.13 $-\,\mathrm{sn}\,u\,\mathrm{dn}\,u$

13.15 $\Gamma(7/2) = 15\sqrt{\pi}/8$

13.17 $\frac{1}{2}B(\frac{5}{4}, \frac{7}{4}) = 3\pi\sqrt{2}/64$

13.19 $\frac{1}{2}\sqrt{\pi}\,\mathrm{erfc}\,5$

13.21 $5^4B(\frac{2}{3}, \frac{13}{3}) = (\frac{5}{3})^5(\frac{14\pi}{\sqrt{3}})$

13.24 $-2^{55}\sqrt{\pi}/109!!$

Chapter 12

1.2 $y = a_0 e^{x^3}$ 1.3 $y = a_1 x$
1.7 $y = Ax + Bx^3$ 1.9 $y = a_0(1 - x^2) + a_1 x$

2.4 $Q_0 = \frac{1}{2}\ln\frac{1+x}{1-x}$, $Q_1 = \frac{x}{2}\ln\frac{1+x}{1-x} - 1$

3.3 $(30 - x^2)\sin x + 12x\cos x$ 3.5 $(x^2 - 200x + 9900)e^{-x}$

5.3 $P_0(x) = 1$ $P_4(x) = (35x^4 - 30x^2 + 3)/8$
$P_1(x) = x$ $P_5(x) = (63x^5 - 70x^3 + 15x)/8$
$P_2(x) = (3x^2 - 1)/2$ $P_6(x) = (231x^6 - 315x^4 + 105x^2 - 5)/16$
$P_3(x) = (5x^3 - 3x)/2$
5.9 $2P_2 + P_1$ 5.11 $\frac{2}{5}(P_1 - P_3)$
5.12 $\frac{8}{5}P_4 + 4P_2 - 3P_1 + \frac{12}{5}P_0$

8.2 $N = \sqrt{\frac{2}{5}}$, $\sqrt{\frac{5}{2}}\,P_2(x)$ 8.4 $N = \pi^{1/4}$, $\pi^{-1/4}e^{-x^2/2}$

9.1 $\frac{3}{2}P_1 - \frac{7}{8}P_3 + \frac{11}{16}P_5 \cdots$
9.4 $\frac{1}{8}\pi(3P_1 + \frac{7}{16}P_3 + \frac{11}{64}P_5 \cdots)$
9.6 $P_0 + \frac{3}{8}P_1 - \frac{20}{9}P_2 \cdots$
9.8 $\frac{1}{2}(1-a)P_0 + \frac{3}{4}(1-a^2)P_1 + \frac{5}{4}a(1-a^2)P_2 + \frac{7}{16}(1-a^2)(5a^2-1)P_3 \cdots$
9.11 $\frac{8}{5}P_4 + 4P_2 - 3P_1 + \frac{12}{5}P_0$
9.12 $\frac{2}{5}(P_1 - P_3)$
9.14 $\frac{1}{2}P_0 + \frac{5}{8}P_2 = \frac{3}{16}(5x^2 + 1)$

10.5 $\frac{1}{2}(\sin\theta)(35\cos^3\theta - 15\cos\theta)$

11.2 $y = Ax^{-3} + Bx^3$ 11.4 $y = Ax^{-2} + Bx^3$
11.6 $y = Ae^{-x} + Bx^{2/3}[1 - 3x/5 + (3x)^2/(5\cdot 8) - (3x)^3/(5\cdot 8\cdot 11) + \cdots]$
11.8 $y = A(x^{-1} - 1) + Bx^2(1 - x + 3x^2/5 - 4x^3/15 + 2x^4/21 + \cdots)$
11.10 $y = A[1 + 2x - (2x)^2/2! + (2x)^3/(3\cdot 3!) - (2x)^4/(3\cdot 5\cdot 4!) + \cdots]$
$+ Bx^{3/2}[1 - 2x/5 + (2x)^2/(5\cdot 7\cdot 2!) - (2x)^3/(5\cdot 7\cdot 9\cdot 3!) + \cdots]$
11.11 $y = Ax^{1/6}[1 + 3x^2/2^5 + 3^2x^4/(5\cdot 2^{10}) + \cdots]$
$+ Bx^{-1/6}[x + 3x^3/2^6 + 3^2x^5/(7\cdot 2^{11}) + \cdots]$

16.1 $y = x^{-3/2}Z_{1/2}(x)$ 16.3 $y = x^{-1/2}Z_1(4x^{1/2})$
16.5 $y = xZ_0(2x)$ 16.7 $y = x^{-1}Z_{1/2}(x^2/2)$
16.9 $y = x^{1/3}Z_{2/3}(4\sqrt{x})$ 16.11 $y = x^{-2}Z_2(x)$
16.15 $y = Z_2(5x)$ 16.17 $y = Z_0(3x)$

17.7 (a) $y = x^{1/2}I_1(2x^{1/2})$. Note that the factor i does not need to be included, since *any* multiple of y is a solution.

18.11 1.7 m for steel.

20.1 1/6 20.3 $4/\pi$ 20.5 1/2
20.7 $h_n^{(1)}(x) \sim x^{-1}e^{i[x-(n+1)\pi/2]}$ 20.9 $h_n^{(1)}(ix) \sim -i^{-n}x^{-1}e^{-x}$

21.1 $y = Ax + B\left(x \sinh^{-1} x - \sqrt{x^2+1}\right)$
21.2 $y = A(1+x) + Bxe^{1/x}$
21.5 $y = A(x-1) + B[(x-1)\ln x - 4]$
21.7 $y = A\frac{x}{1-x} + B[\frac{x}{1-x}\ln x + \frac{1+x}{2}]$
21.8 $y = A(x^2+2x) + B[(x^2+2x)\ln x + 1 + 5x - x^3/6 + x^4/72 + \cdots]$

22.4 $H_0(x) = 1$ $\quad H_3(x) = 8x^3 - 12x$
$H_1(x) = 2x$ $\quad H_4(x) = 16x^4 - 48x^2 + 12$
$H_2(x) = 4x^2 - 2$ $\quad H_5(x) = 32x^5 - 160x^3 + 120x$

22.13 $L_0(x) = 1$
$L_1(x) = 1 - x$
$L_2(x) = \frac{1}{2}(2 - 4x + x^2)$
$L_3(x) = \frac{1}{6}(6 - 18x + 9x^2 - x^3)$
$L_4(x) = \frac{1}{24}(24 - 96x + 72x^2 - 16x^3 + x^4)$
$L_5(x) = \frac{1}{120}(120 - 600x + 600x^2 - 200x^3 + 25x^4 - x^5)$

Note: The factor $1/n!$ is omitted in most quantum mechanics books but is included as here in most reference books.

Chapter 13

2.12
$$T = \sum_{\text{odd } n} \frac{400}{n\pi \sinh 3n\pi} \sinh \frac{n\pi}{10}(30-y) \sin \frac{n\pi x}{10} + \sum_{\text{odd } n} \frac{400}{n\pi \sinh(n\pi/3)} \sinh \frac{n\pi}{30}(10-x) \sin \frac{n\pi y}{30}$$

2.14 For $f(x) = x - 5$: $T = -\frac{40}{\pi^2} \sum_{\text{odd } n} \frac{1}{n^2} \cos \frac{n\pi x}{10}\, e^{-n\pi y/10}$
For $f(x) = x$: add 5 to the answer just given.

3.9
$$u = 100 - \frac{400}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} e^{-[(2n+1)\pi\alpha/4]^2 t} \cos\left(\frac{2n+1}{4}\pi x\right)$$

3.11 $E_n = n^2\hbar^2/(2m)$; $\Psi(x,t) = \frac{4}{\pi} \sum_{\text{odd } n} \frac{\sin nx}{n} e^{-iE_n t/\hbar}$

4.8
$$y = \frac{4l}{\pi^2 v}\left[\frac{1}{3} \sin \frac{\pi x}{l} \sin \frac{\pi vt}{l} + \frac{\pi}{16} \sin \frac{2\pi x}{l} \sin \frac{2\pi vt}{l} - \sum_{n=3}^{\infty} \frac{\sin n\pi/2}{n(n^2-4)} \sin \frac{n\pi x}{l} \sin \frac{n\pi vt}{l}\right]$$

4.9 Problem 2: $n = 2$, $\nu = v/l$
Problem 3: $n = 3$, $\nu = \frac{3}{2}v/l$ and $n = 4$, $\nu = 2v/l$ have nearly equal intensity.
Problem 5: $n = 1$, $\nu = \frac{1}{2}v/l$

5.1 (a) $u \cong 9.76$

5.4 $u = 200 \sum_{m=1}^{\infty} \frac{1}{k_m J_1(k_m)} J_0(k_m r/a) e^{-(k_m \alpha/a)^2 t}$, $\quad k_m =$ zeros of J_0

5.10 $u = \frac{6400}{\pi^3} \sum_{\text{odd } n} \sum_{\text{odd } m} \sum_{\text{odd } p} \frac{1}{nmp} \sin \frac{n\pi x}{l} \sin \frac{m\pi y}{l} \sin \frac{p\pi z}{l} e^{-(\alpha\pi/l)^2(n^2+m^2+p^2)t}$

5.11 $R = r^n$, r^{-n}, $n \neq 0$; $R = \ln r$, const., $n = 0$.
$R = r^l$, r^{-l-1}.

5.13 $u = \frac{400}{\pi} \sum_{\text{odd } n} \frac{1}{n} \left(\frac{r}{10}\right)^{4n} \sin 4n\theta$

5.14 $u = \frac{50 \ln r}{\ln 2} + \frac{200}{\pi} \sum_{\text{odd } n} \frac{r^n - r^{-n}}{n(2^n - 2^{-n})} \sin n\theta$

6.5 $z = \frac{64l^4}{\pi^6} \sum_{\text{odd } m} \sum_{\text{odd } n} \frac{1}{n^3 m^3} \sin \frac{n\pi x}{l} \sin \frac{m\pi y}{l} \cos \frac{\pi v(m^2+n^2)^{1/2} t}{l}$

6.8 $\Psi_{mn} = J_n(k_{mn} r) \left\{ \begin{array}{c} \sin n\theta \\ \cos n\theta \end{array} \right\} e^{-iE_{mn}t/\hbar}, \quad E_{mn} = \frac{\hbar^2 k_{mn}^2}{2ma^2}$

7.2 $u = \frac{2}{5} r P_1(\cos\theta) - \frac{2}{5} r^3 P_3(\cos\theta)$
7.5 $u = \frac{1}{2} P_0(\cos\theta) + \frac{5}{8} r^2 P_2(\cos\theta) - \frac{3}{16} r^4 P_4(\cos\theta) \cdots$
7.6 $u = \frac{1}{8}\pi[3r P_1(\cos\theta) + \frac{7}{16} r^3 P_3(\cos\theta) + \frac{11}{64} r^5 P_5(\cos\theta) \cdots]$
7.8 $u = 25[P_0(\cos\theta) + \frac{9}{4} r P_1(\cos\theta) + \frac{15}{8} r^2 P_2(\cos\theta) + \frac{21}{64} r^3 P_3(\cos\theta) \cdots]$
7.10 $u = \frac{1}{15} r^3 P_3^2(\cos\theta)\cos 2\phi - r P_1(\cos\theta)$
7.12 $u = \frac{3}{4} r P_1(\cos\theta) + \frac{7}{24} r^3 P_3(\cos\theta) - \frac{11}{192} r^5 P_5(\cos\theta) \cdots$
7.13 $u = E_0(r - a^3/r^2) P_1(\cos\theta)$

7.15 $u = 100 + \frac{200a}{\pi r} \sum_1^\infty \frac{(-1)^n}{n} \sin \frac{n\pi r}{a} e^{-(\alpha n\pi/a)^2 t}$

$$= 100 + 200 \sum_1^\infty (-1)^n j_0(n\pi r/a) e^{-(\alpha n\pi/a)^2 t}$$

7.19 $\Psi(r,\theta,\phi) = j_l(\beta r) P_l^m(\cos\theta) e^{\pm im\phi} e^{-iEt/\hbar}$, where

$$\beta = \sqrt{2ME/\hbar^2}, \quad \beta a = \text{ zeros of} j_l, \quad E = \frac{\hbar^2}{2Ma^2}(\text{ zeros of } j_l)^2$$

7.20 $\psi_n(x) = e^{-\alpha^2 x^2/2} H_n(\alpha x), \quad \alpha = \sqrt{m\omega/\hbar}$
7.21 Degree of degeneracy of E_n is $C(n+2, n) = (n+2)(n+1)/2$, $n = 0$ to ∞.
7.22 $\Psi(r,\theta,\phi) = R(r) Y_l^m(\theta,\phi)$, $R(r) = r^l e^{-r/(na)} L_{n-l-1}^{2l+1}\left(\frac{2r}{na}\right)$, $E_n = -\frac{Me^4}{2\hbar^2 n^2}$

8.4 Let $K =$ line charge per unit length. Then

$$V = -K \ln(r^2 + a^2 - 2ra\cos\theta) + K \ln a^2 - K \ln R^2$$
$$+ K \ln[r^2 + (R^2/a)^2 - 2(R^2/a) r \cos\theta]$$

8.5 K at $(a, 0)$, $-K$ at $(R^2/a, 0)$

9.2 $u = 200\pi^{-1} \int_0^\infty k^{-2}(1 - \cos 2k) e^{-ky} \cos kx \, dk$
9.7 $u(x,t) = 100 \operatorname{erf}[x/(2\alpha t^{1/2})] - 50 \operatorname{erf}[(x-1)/(2\alpha t^{1/2})] - 50 \operatorname{erf}[(x+1)/(2\alpha t^{1/2})]$

10.3 $T = \frac{1}{4}(2 - y) + \frac{4}{\pi^2} \sum_{\text{odd } n} \frac{1}{n^2 \sinh 2n\pi} \sinh n\pi(2 - y) \cos n\pi x$

10.4 $T = 20 + \dfrac{40}{\pi}\sum_{\text{odd } n}\dfrac{1}{n\sinh(3n\pi/5)}\sinh\dfrac{n\pi y}{5}\sin\dfrac{n\pi x}{5} + \dfrac{40}{\pi}\sum_{\text{odd } n}\dfrac{1}{n\sinh(5n\pi/3)}\sinh\dfrac{n\pi(5-x)}{3}\sin\dfrac{n\pi y}{3}$

10.6 $u = 20 - \dfrac{80}{\pi}\sum_0^\infty \dfrac{(-1)^n}{2n+1}e^{-[(2n+1)\pi\alpha/(2l)]^2 t}\cos\left(\dfrac{2n+1}{2l}\pi x\right)$

10.8 $u = 20 - x - \dfrac{40}{\pi}\sum_{\text{even } n}\dfrac{1}{n}\,e^{-(n\pi\alpha/10)^2 t}\sin\dfrac{n\pi x}{10}$

10.10 $u = \dfrac{1600}{\pi^2}\sum_{\text{odd } n}\sum_{\text{odd } m}\dfrac{1}{nmI_n(3m\pi/20)}I_n\left(\dfrac{m\pi r}{20}\right)\sin n\theta\sin\dfrac{m\pi z}{20}$

10.16 $v\sqrt{5}/(2\pi)$

10.18 ν_{mn}, $n = 3, 6, \cdots$; the lowest frequencies are:
$\nu_{13} = 2.65\,\nu_{10}$, $\nu_{23} = 4.06\,\nu_{10}$, $\nu_{16} = 4.13\,\nu_{10}$, $\nu_{33} = 5.4\,\nu_{10}$

10.20 $\nu = v\lambda_l/(2\pi a)$ where λ_l = zeros of j_l, a = radius of sphere, v = speed of sound

10.22 $u = 1 - \frac{1}{2}rP_1(\cos\theta) + \frac{7}{8}r^3P_3(\cos\theta) - \frac{11}{16}r^5P_5(\cos\theta)\cdots$

10.26 $\nu = [v/(2\pi)][(k_{mn}/a)^2 + \lambda^2]^{1/2}$ where k_{mn} is a zero of J_n

Chapter 14

1.1 $u = x^3 - 3xy^2$, $v = 3x^2y - y^3$

1.3 $u = x$, $v = -y$

1.4 $u = (x^2+y^2)^{1/2}$, $v = 0$

1.7 $u = \cos y\cosh x$, $v = \sin y\sinh x$

1.9 $u = x/(x^2+y^2)$, $v = -y/(x^2+y^2)$

1.11 $u = 3x/[x^2+(y-2)^2]$, $v = (-2x^2-2y^2+5y-2)/[x^2+(y-2)^2]$

1.13 $u = \ln(x^2+y^2)^{1/2}$, $v = 0$

1.17 $u = \cos x\cosh y$, $v = \sin x\sinh y$

1.18 $u = \pm 2^{-1/2}[(x^2+y^2)^{1/2}+x]^{1/2}$, $v = \pm 2^{-1/2}[(x^2+y^2)^{1/2}-x]^{1/2}$,
where the $\pm$ signs are chosen so that uv has the sign of y.

1.19 $u = \ln(x^2+y^2)^{1/2}$, $v = \arctan(y/x)$
[The angle is in the quadrant of the point (x, y).]

In 2.1–2.23, A = analytic, N = not analytic

2.1	A	2.3	N	2.4	N	2.7	A
2.9	A, $z \neq 0$	2.11	A, $z \neq 2i$	2.13	N	2.17	N
2.18	A, $z \neq 0$	2.19	A, $z \neq 0$	2.23	A, $z \neq 0$		

2.34 $-z - \frac{1}{2}z^2 - \frac{1}{3}z^3\cdots$, $|z| < 1$

2.38 $-\frac{1}{2}i + \frac{1}{4}z + \frac{1}{8}iz^2 - \frac{1}{16}z^3\cdots$, $|z| < 2$

2.42 $z + z^3/3! + z^5/5!\cdots$, all z

2.48	Yes, $z \neq 0$	2.52	No	2.53	Yes, $z \neq 0$
2.54	$-iz$	2.56	$-iz^2/2$	2.59	e^z
2.60	$2\ln z$	2.63	$-i/(1-z)$		

3.1	$\frac{1}{2}+i$	3.3	0	3.5	-1
3.7	$\pi(1-i)/8$	3.9	1	3.12	(a) $\frac{5}{3}(1+2i)$
3.17	(a) 0 (b) $i\pi$	3.19	$16i\pi$	3.23	$72i\pi$

4.4 For $0 < |z| < 1$: $-\frac{1}{4}z^{-1} - \frac{1}{2} - \frac{11}{16}z - \frac{13}{16}z^2\cdots$; $R(0) = -\frac{1}{4}$
For $1 < |z| < 2$: $\cdots + z^{-3} + z^{-2} + \frac{3}{4}z^{-1} + \frac{1}{2} + \frac{5}{16}z + \frac{3}{16}z^2\cdots$
For $|z| > 2$: $z^{-4} + 5z^{-5} + 17z^{-6} + 49z^{-7}\cdots$

4.6	For $0<\|z\|<1$: $z^{-2}-2z^{-1}+3-4z+5z^2\cdots$; $R(0)=-2$
	For $\|z\|>1$: $z^{-4}-2z^{-5}+3z^{-6}\cdots$
4.8	For $\|z\|<1$: $-5+\frac{25}{6}z-\frac{175}{36}z^2\cdots$; $R(0)=0$
	For $1<\|z\|<2$: $-5(\cdots+z^{-3}-z^{-2}+z^{-1}+\frac{1}{6}z+\frac{1}{36}z^2+\frac{7}{216}z^3\cdots)$
	For $2<\|z\|<3$: $\cdots+3z^{-3}+9z^{-2}-3z^{-1}+1-\frac{1}{3}z+\frac{1}{9}z^2-\frac{1}{27}z^3\cdots$
	For $\|z\|>3$: $30(z^{-3}-2z^{-4}+9z^{-5}\cdots)$

4.9	(a) Regular	(b) Pole of order 3
4.10	(b) Pole of order 2	(d) Essential singularity
4.11	(c) Simple pole	(d) Pole of order 3
4.12	(b) Pole of order 2	(d) Pole of order 1

6.1	$z^{-1}-1+z-z^2\cdots$; $R=1$
6.3	$z^{-3}-z^{-1}/3!+z/5!\cdots$; $R=-\frac{1}{6}$
6.5	$\frac{1}{2}e[(z-1)^{-1}+\frac{1}{2}+\frac{1}{4}(z-1)\cdots]$; $R=\frac{1}{2}e$
6.7	$\frac{1}{4}[(z-\frac{1}{2})^{-1}-1+(1-\pi^2/2)(z-\frac{1}{2})\cdots]$; $R=\frac{1}{4}$
6.9	$-[(z-2)^{-1}+1+(z-2)+(z-2)^2\cdots]$; $R=-1$

6.14	$R(-2/3)=1/8$, $R(2)=-1/8$	6.16	$R(0)=-2$, $R(1)=1$
6.18	$R(3i)=\frac{1}{2}-\frac{1}{3}i$	6.19	$R(\pi/2)=1/2$
6.21	$R[\sqrt{2}\,(1+i)]=\sqrt{2}\,(1-i)/16$	6.22	$R(i\pi)=-1$
6.27	$R(\pi/6)=-1/2$	6.28	$R(3i)=-\frac{1}{16}+\frac{1}{24}i$
6.31	$R(0)=9/2$	6.33	$R(\pi)=-1/2$
6.35	$R(i)=0$	6.14′	$\pi i/4$
6.16′	$-2\pi i$	6.18′	0
6.19′	0	6.27′	$-\pi i$
6.28′	$\pi i/4$	6.31′	$9\pi i$
6.33′	0	6.35′	0

7.1	$\pi/6$	7.3	$2\pi/3$
7.5	$\pi/(1-r^2)$	7.7	$\pi/6$
7.9	$2\pi/\|\sin\alpha\|$	7.11	$3\pi/32$
7.13	$\pi/10$	7.15	$\pi e^{-4/3}/12$
7.17	$(\pi/e)(\cos 2+2\sin 2)$	7.19	$\pi e^{-3}/54$
7.23	$\pi/8$	7.24	π
7.26	$-\pi/2$	7.28	$\pi/4$
7.30	$\pi/(2\sqrt{2}\,)$	7.32	$\frac{3}{16}\pi\sqrt{2}$
7.33	$\pi\sqrt{2}/2$	7.36	$-\pi^2\sqrt{2}$
7.39	2	7.41	$(2\pi)^{1/2}/4$

7.45 One negative real, one each in quadrants I and IV

7.48 Two each in quadrants I and IV

7.50 Two each in quadrants II and III

7.52	πi	7.54	$8\pi i$
7.55	$\cosh t\cos t$	7.57	$1+\sin t-\cos t$
7.60	$t+e^{-t}-1$	7.61	$\left(\cosh 2t+2\cosh t\cos t\sqrt{3}\,\right)/3$
7.63	$(\cosh t-\cos t)/2$	7.65	$(\cos 2t+2\sin 2t-e^{-t})/5$

8.3	Regular, $R=-1$	8.5	Regular, $R=-1$
8.7	Simple pole, $R=-2$	8.9	Regular, $R=0$
8.11	Regular, $R=-1$	8.14	$-2\pi i$

9.3 $u = x/(x^2 + y^2)$, $v = -y/(x^2 + y^2)$
9.4 $u = e^x \cos y$, $v = e^x \sin y$
9.7 $u = \sin x \cosh y$, $v = \cos x \sinh y$

10.6 $T = 100y/(x^2 + y^2)$; isothermals $y/(x^2 + y^2) = \text{const.}$; flow lines $x/(x^2 + y^2) = \text{const.}$
10.9 Streamlines $y - y/(x^2 + y^2) = \text{const.}$
10.12 $T = (20/\pi) \arctan[2y/(1 - x^2 - y^2)]$, arc tan between $\pi/2$ and $3\pi/2$
10.14 $\Phi = \frac{1}{2} V_0 \ln\{[(x+1)^2 + y^2]/[(x-1)^2 + y^2]\}$
$\Psi = V_0 \arctan\{2y/[1 - x^2 - y^2]\}$, arc tan between $\pi/2$ and $3\pi/2$
$V_x = 2V_0(1 - x^2 + y^2)/[(1 - x^2 + y^2)^2 + 4x^2y^2]$,
$V_y = -4V_0xy/[(1 - x^2 + y^2)^2 + 4x^2y^2]$

11.2 $-i \ln(1 + z)$ 11.5 $R(i) = \frac{1}{4}(1 - i\sqrt{3})$, $R(-i) = -\frac{1}{2}$
11.8 $R(1/2) = 1/2$ 11.10 -1 11.12 $1/2$
11.14 (a) 2 (b) $-\sin 5$ (c) $1/16$ (d) -2π 11.16 $-\pi/6$
11.18 $\frac{1}{4}\pi e^{-\pi/2}$ 11.20 $\frac{1}{2}\pi(e^{-1} + \sin 1)$
11.29 $\pi^3/8$. Caution: $-\pi^3/8$ is wrong.
11.32 One negative real, one each in quadrants II and III
11.34 Two each in quadrants I and IV, one each in II and III
11.41 $\pi^2/8$

Chapter 15

1.2 3/8, 1/8, 1/4 1.5 1/4, 3/4, 1/3, 1/2
1.6 27/52, 16/52, 15/52 1.8 9/100, 1/10, 3/100, 1/10

2.12 (a) 3/4 (b) 1/5 (c) 2/3 (d) 3/4 (e) 3/7
2.14 (a) 3/4 (b) 25/36 (c) 37, 38, 39, 40
2.17 (a) 3 to 9 with $p(5) = p(7) = 2/9$; others, $p = 1/9$. (c) 1/3

3.4 (a) 8/9, 1/2 (b) 3/5, 1/11, 2/3, 2/3, 6/13 3.5 1/33, 2/9
3.12 (a) 1/49 (b) 68/441 (c) 25/169 (d) 15 times (e) 44/147
3.14 $n > 3.3$, so 4 tries are needed. 3.16 9/23
3.17 (a) 39/80, 5/16, 1/5, 11/16 (b) 374/819 (c) 185/374
3.20 5/7, 2/7, 11/14 3.21 2/3, 1/3

4.1 (a) $P(10, 8)$ (b) $C(10, 8)$ (c) 1/45
4.4 1.98×10^{-3}, 4.95×10^{-4}, 3.05×10^{-4}, 1.39×10^{-5} 4.7 1/26
4.8 1/221, 1/33, 1/17 4.11 0.097, 0.37, 0.67; 13
4.17 MB: 16, FD: 6, BE: 10

5.1 $\mu = 0$, $\sigma = \sqrt{3}$ 5.3 $\mu = 2$, $\sigma = \sqrt{2}$
5.5 $\mu = 1$, $\sigma = \sqrt{7/6}$ 5.7 $\mu = 3(2p - 1)$, $\sigma = 2\sqrt{3p(1-p)}$

6.1 (c) $\bar{x} = 0$, $\sigma = 2^{-1/2}a$ 6.4 $\bar{x} = 0$, $\sigma = (2^{1/2}\alpha)^{-1}$
6.5 $f(t) = \lambda e^{-\lambda t}$, $F(t) = 1 - e^{-\lambda t}$, $\bar{t} = 1/\lambda$, half life $= \bar{t} \ln 2$

6.7 (a) $F(s) = 2[1 - \cos(s/R)]$, $f(s) = (2/R)\sin(s/R)$
(b) $F(s) = [1 - \cos(s/R)]/[1 - \cos(1/R)] \cong s^2$,
$f(s) = R^{-1}[1 - \cos(1/R)]^{-1}\sin(s/R) \cong 2s$

	n	Exactly 7 h	At most 7 h	At least 7 h	Most probable number of h	Expected number of h
7.1	7	0.0078	1	0.0078	3 or 4	7/2
7.2	12	0.193	0.806	0.387	6	6

In the following answers, the first number is the binomial result and the second number is the normal approximation using whole steps at the ends as in Example 2.

8.12 0.03987, 0.03989
8.14 0.9546, 0.9546
8.17 0.0770, 0.0782
8.18 0.372, 0.376
8.20 0.462, 0.455

9.3

Number of particles	0	1	2	3	4	5
Number of intervals	406	812	812	541	271	108

9.5 $P_0 = 0.37$, $P_1 = 0.37$, $P_2 = 0.18$, $P_3 = 0.06$
9.8 3, 10, 3
9.11 Normal: 0.08, Poisson: 0.0729, (binomial: 0.0732)

10.8 $\bar{x} = 5$, $\bar{y} = 1$, $s_x = 0.122$, $s_y = 0.029$,
$\sigma_x = 0.131$, $\sigma_y = 0.030$, $\sigma_{mx} = 0.046$, $\sigma_{my} = 0.0095$,
$r_x = 0.031$, $r_y = 0.0064$,
$\overline{x+y} = 6$ with $r = 0.03$, $\overline{xy} = 5$ with $r = 0.04$,
$\overline{x^3 \sin y} = 105$ with $r = 2.00$, $\overline{\ln x} = 1.61$ with $r = 0.006$
10.10 $\bar{x} = 6$ with $r = 0.062$, $\bar{y} = 3$ with $r = 0.067$,
$\overline{e^y} = 20$ with $r = 1.3$, $\overline{x/y^2} = 0.67$ with $r = 0.03$

11.3 20/47
11.7 $\bar{x} = 1/4$, $\sigma = \sqrt{3}/4$
11.9 (d) $\bar{x} = 1/4$, $\sigma = \sqrt{31}/12$
11.13 30, 60
11.17 $\bar{x} = 2$ with $r = 0.073$, $\bar{y} = 1$ with $r = 0.039$, $\overline{x-y} = 1$ with $r = 0.08$,
$\overline{xy} = 2$ with $r = 0.11$, $\overline{x/y^3} = 2$ with $r = 0.25$