Assignment 4

Transform Calculus (MA20101)

Section 4

To be submitted on or before 14th November (Tuesday, 2017)

Q1) Find the Fourier cosine transform of

$$f(x) = e^{-mx}, \quad m > 0.$$

Hence, show that $\int_0^\infty \frac{\cos pv}{v^2+\beta^2}\,dv = \frac{\pi}{2\beta} e^{-p\beta}$. $(p>0, \beta>0)$

Q2) Solve for $y(x)$, the integral eq^n

$$\int_{-\infty}^{\infty} \frac{y(u)\,du}{(x-u)^2+a^2} = \frac{1}{x^2+b^2}, \quad 0<a<b.$$

Q3) If (i) $f(t) = e^{-t^2}$, find its Fourier transform.

(ii) $f(x) = e^{-a|x|}, -\infty < x < \infty$.

Q4) Derive Parseval's formula —

$$\int_{-\infty}^{\infty} f(t)\, G(it)\, dt = \int_{-\infty}^{\infty} F(it)\, g(t)\, dt,$$

where F & G are the Fourier Transforms of $f(t)$ and $g(t)$ resp^{ly} & all functions are assumed to be well enough behaved.

Q5) Define $f(t) = 1-t^2, -1<t<1$, zero otherwise & $g(t) = e^{-t}, 0 \le t < \infty$, zero otherwise.

Find the Fourier Transforms of each of these functions & hence deduce the value of the integral

$$\int_0^\infty \frac{4e^{-t}}{t^3}\left(t\cosh(t) - \sinh(t)\right) dt$$

by using Parseval's formula (see Q4.).

Further, use Parseval's theorem for Fourier transforms, to evaluate the integral

$$\int_0^\infty \frac{(t\cos t - \sin t)^2}{t^6}\, dt .$$

Q6) Solve the heat eqn $k\dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial u}{\partial t}$, $-\infty < x < \infty$, $t > 0$, subject to

$u(x,0) = f(x)$, where $f(x) = \begin{cases} u_0, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$.

(by Fourier transform).

Q7) The steady-state temperature in a semi-infinite plate is determined from

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < \pi, \quad y > 0$$

$$u(0,y) = 0, \quad u(\pi, y) = e^{-y}, \quad y > 0$$

$$\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0, \quad 0 < x < \pi$$

Solve for $u(x,y)$. (Use Fourier Cosine Transform).

Q8) Using the Fourier cosine transform of e^{-ax} & e^{-bx}, show that

$$\int_0^\infty \frac{d\alpha}{(a^2+\alpha^2)(b^2+\alpha^2)} = \frac{\pi}{2ab(a+b)}, \quad a>0, b>0.$$

Q9) Solve the heat conduction problem described by

$$k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0<x<\infty, \ t>0$$

$$u(0,t) = u_0, \quad t \geq 0 \quad \text{(B. Cond}^n\text{)}$$

$$u(x,0) = 0, \quad 0<x<\infty \quad \text{(I- Cond}^n\text{)}$$

u, & $\frac{\partial u}{\partial x}$ both $\to 0$ as $x \to \infty$.

Q10) Solve the following using the Laplace transform technique:

$$u_t = u_{xx}, \quad 0<x<1, \ t>0$$

$$u(0,t) = 1, \quad u(1,t) = 1, \quad t>0 \quad \text{(B- Cond}^n\text{)}$$

$$u(x,0) = 1+\sin(\pi x), \quad 0<x<1 \quad \text{(I. Cond}^n\text{)}.$$

— X —