Problem Set - 12

AUTUMN 2016

MATHEMATICS-I (MA10001)

November 31, 2016

1. If C is the curve $y = x^3 - 3x^2 + 4x - 1$ joining the points $(1, 1)$ and $(2, 3)$, find the values of $\int_C (12z^2 - 4iz)dz$.
2. Find an upper bound of $|\int_C \frac{e^z}{(z^2+1)}|$ where C is a circle $|z| = 2$ traversed once in anticlockwise direction.
3. Show that $|\int_C \frac{1}{z^2}dz| \leq 2$, where C is the line segment joining $-1 + i$ and $1 + i$.
4. Evaluate the integral
   (i) $\int_C |z|^2 dz$, (ii) $\int_C \frac{1}{z^2}dz$ where C is (a) the line segment with initial point -1 and final point i.
   (b) the arc of the unit circle $Im\ z \geq 0$, with initial point -1 and final point i.
5. Let $f(z) = x^2 + iy^2$. Evaluate $\int_C f(z)dz$, where C is
   (a) the line segment joining 1 and $2 + i$
   (b) the curve $(1 + t) + t^2 i$, $0 \leq t \leq 1$. Are the result same? Justify your answer.
6. Let C be a circle centred at $4+i$ with radius 1, without any calculation explain why $\int_C \frac{dz}{z} = 0$.
7. Let C be the defined parametrically as follows $z(t) = t(1 - t)e^t + [\cos(2\pi t^3)]i$, $0 \leq t \leq 1$. Evaluate $\int_C e^{z^2} dz$.
8. Evaluate $\int_C |z|^2 dz$ around the square with vertices at $(0, 0)$, $(1, 0)$, $(1, 1)$, $(0, 1)$.
9. Evaluate $\int_C \overline{z}^2 dz$ around the circle $|z - 1| = 1$
10. Evaluate $\int_i^{2-i} (3xy + iy^2)dz$, along (i) the straight line joining $z = i$ and $z = 2 - i$, (ii) along the curve $x = 2t - 2$, $y = 1 + t - t^2$.
11. Evaluate $\frac{1}{2\pi i} \int_C \frac{e^z}{z-2}dz$ if C is (i) the circle $|z| = 3$, (ii) the circle $|z| = 1$.
12. Evaluate $\int_C \frac{e^{3z}}{z-\pi i}dz$ if C is the ellipse $|z - 2| + |z + 2| = 6$
13. Evaluate $\int_C \frac{e^{iz}}{z^3}dz$ if C is the circle $|z| = 2$.
14. Evaluate the integral $\int_C \frac{z^2-1}{z^3-z^2+9z-9}dz$ where C is the circle $|z| = 2$.
15. Without computing the integral show that $|\int_C \frac{dz}{z^2+\overline{z}+1}| \leq \frac{3\pi}{10}$ where C is the arc of the circle $|z| = 3$ from $z = 3$ to $z = 3i$ lying on the first quadrant.
16. Show that $\frac{1}{2\pi i} \int_C \frac{e^{zt}}{(z^2+1)}dz = \sin t$ if $t > 0$ and C is $|z| = 3$.
17. Let C be the boundary of the triangle with vertices at the points $0, 3i$ and -4 oriented counterclockwise. Compute the contour integral $\int_C (e^z - \overline{z}dz)$.

18. Find the value of the integral $\int_\gamma \frac{1}{z(z^2+4)}dz$ where γ is the following circle $|z| = 1$ oriented counterclockwise.

19. Evaluate the integral $\int_C \frac{z^2+1}{z(2z-1)}$ where C is the curve $|z| = 1$.