

Probability and Statistics
Hints/Solutions to Assignment 6

1. $p_{U,V}(0,1) = \frac{1}{12}, p_{U,V}(0,4) = \frac{1}{12}, p_{U,V}(1,1) = \frac{1}{3}, p_{U,V}(1,4) = \frac{1}{2}.$
2. $Z^2 = (X_1 - X_2)^2 + (Y_1 - Y_2)^2. \frac{X_1 - X_2}{\sqrt{2}} \sim N(0,1), \frac{Y_1 - Y_2}{\sqrt{2}} \sim N(0,1).$ So $\frac{Z^2}{2} \sim \chi_2^2.$
Hence Z^2 has a negative exponential distribution with mean 4.
3. $f_{Y_1, Y_2}(y_1, y_2) = \lambda^2 y_2 e^{-\lambda y_2} \cdot \frac{1}{(1+y_1)^2}, y_1 > 0, y_2 > 0.$ The densities of Y_1 and Y_2 can be derived easily now.
4. $f_{Y_1, Y_2}(y_1, y_2) = \frac{1}{2} e^{-y_1/2} \cdot \frac{1}{\pi(1+y_2)^2}, y_1 > 0, y_2 \in \mathbb{R}.$ Clearly Y_1 and Y_2 are independent.
5. Similar. Y and Z are independent. Z and U are independent.
6. Similar. Y_1, Y_2 and Y_3 are independent.
7. $\ln W = 2 + 2 \ln X_1 + 1.5 \ln X_2 + 1.28 \ln X_3 \sim N(17.06, 7.0692).$ Now use the symmetry of normal distribution to get the median of W and values of L and R .
8. $\rho = 0.5, P(-1 < X < 1 | Y = 1) = 0.6772, V(2X + 3Y) = 19, P(-5 < 2X + 3Y < 8) = 0.842.$
9. Use the linearity property of normal distribution. Reqd. Prob = 0.9544.