Symbolic Logic
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Lecture – 27
Indirect Proof
The Format
The Execution of the Indirect Proof

Hello and welcome back to our module 27 of this NOC course in Symbolic Logic. This module 27 we are looking into Limited Scope Assumption Proof and we are going to learn about the Indirect Proof. We are going look into what it is and what the format is, how to do the proof, and so on. In this also you we are going to make use of the 19 rules that you have learnt. So those stay. So that if you have gathered an expertise over the 19 rules, that is going to come very handy for even in this Limited Scope Assumption Proofs. So without further ado, let us proceed to understand what this Indirect Proof is all about.

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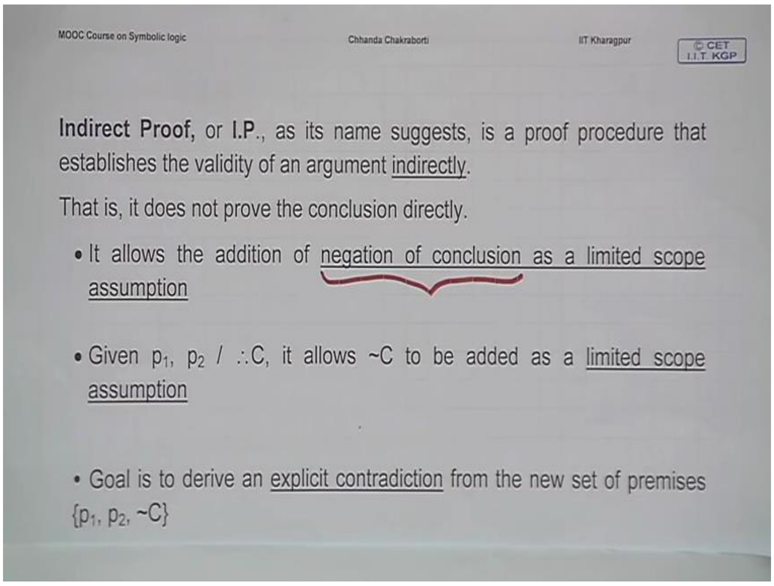

This Indirect Proof, or IP as we are going to call it, it's a proof plus procedure that work indirectly. That's the name, that is why the name is like that. So what it does not do is to prove the conclusion directly. See, in a formal derivation what do you do? You start from the premises and then you slowly derive the conclusion directly. Directly means that the conclusion is what you actually derive out of the premises. Isn't it? With the 19 rules. That is not what is going to happen in Indirect Proof. Rather it is going to go roundabout way to prove the conclusion. What is that roundabout way? Well that is what we are going to learn today.

First of all, note that as a Limited Scope Assumption proof, it will allow you to add a premise, add an assumption as a premise. Right? Limited Scope Assumption I have already explained in the previous module. That the procedures are going to allow you to insert an assumption in your premise base. So there will be some given premises and then there will be some added premise that *you* are going to insert to the premise base. Here in IP what will be permissible to add is *negation of the conclusion given*. So if you have the argument in front of you, instead of starting with the premise and then deriving directly to the conclusion, what do you do? You take the premises, you look at the conclusion, and thenyou add the negation of the conclusion with the premises. Just like we did it in the truth trees, remember? So that would be how this Indirect Proof would work.

You have the premises, with that you add the negation of the conclusion also. As a Limited Scope Assumption. Fine? And then so if you have for example, schematically speaking, here is an argument where you have p_1, p_2 as premises and C is your conclusion given, right? If it were a formal derivation, you would be starting with p_1, p_2 and then try to solve for C. That is not the IP way. What will IP allow you to do? To add tilde C (~C). C is your conclusion. So it will allow you to add negation of that C as a Limited Scope Assumption, right? And then what? You will say, but why are we doing this? What I am supposed to do then? Well, the target is that from this new set, you have a new set now, p_1, p_2 and tilde C, from this new set, you try to derive with the 19 rules that you have already done, explicit contradiction. Anexplicit contradiction. Logical contradiction. Ok? That is your target and the moment, remember the Limited Scope Assumptions are really limited in their scope. So you have a purpose in mind why you are assuming them, the moment you have reached the objective, you have to drop the assumption.

So, in this case the goal is this explicit contradiction. Once you have arrived at the explicit contradiction, your job is to drop the original assumption. What is the assumption? Namely, tilde C, right? That was the beginning assumption. You drop it and then you claim therefore C has to follow from the premises. Get it? So this is how indirectly in a roundabout way the proof procedure proves why C has to follow from the given premise set.

So let's see whether how much we understood this or not. Now let me add few more details to this so that our knowledge about IP is somewhat complete.

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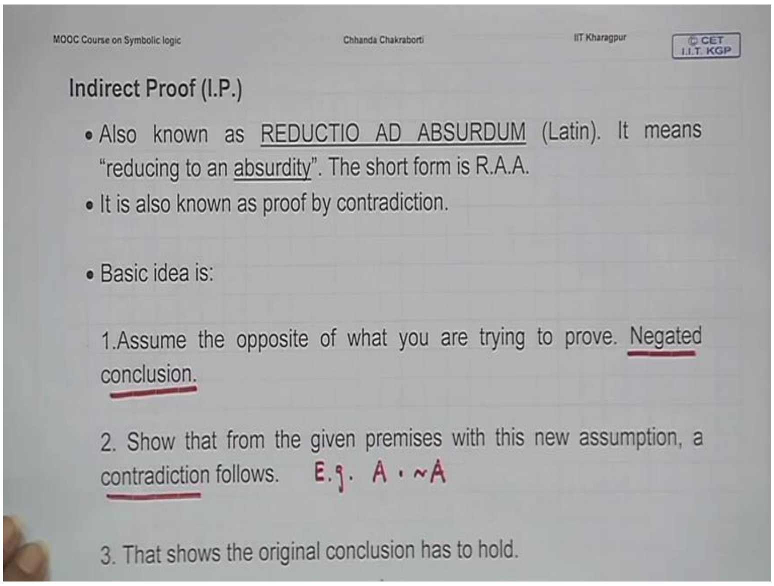

You may have encountered Indirect Proof in another form. When in school where you were doing geometry for example, Euclidean geometry for example, you may have been taught that the theorems can be proven by the *Reductio* method. REDUCTIO AD ABSURDUM. That is a Latin word - REDUCTIO AD ABSURDUM, which means reducing something to an *absurdity*. And the short form for this kind of proof is called R.A.A. And that is precisely what IP is. It's a reduction to an absurdity. You may have said in geometry classes, let's assume that the theorem is not the case. That it does not follow from the axiom (Refer Time: 06:26), then let us see what happens. If you have done that kind of a proof, then this is what, same thing will be also tried out by the IP method.

I have used the word *absurdity*. Remember, I said the IP other name for it is reducing it to an absurdity, REDUCTIO AD ABSURDUM. What is an absurdity? How do we show an absurdity? And the medieval logicians gave us three kind of answers. So, we will take a look into that a little bit.

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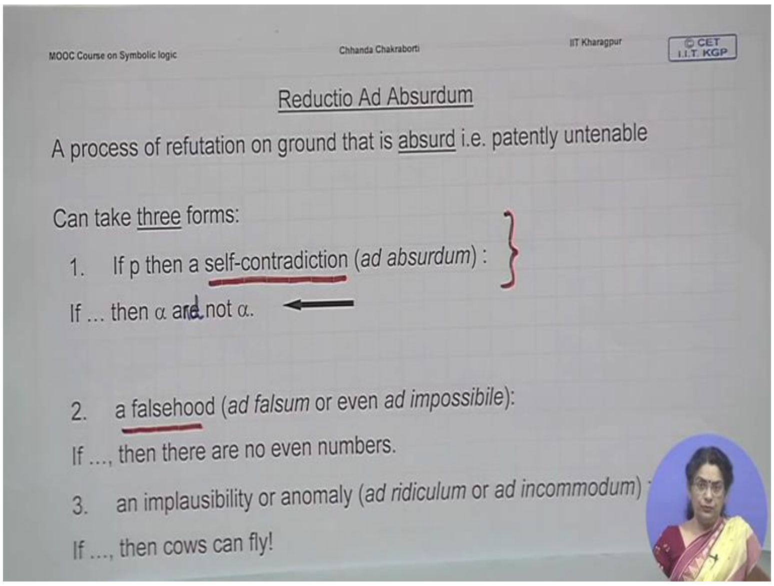

See what they said is that this REDUCTIO AD ABSURDUM procedure is a process of refutation. It's a process of logical refutation where the ground is that what you have shown is something absurd. Absurd in the sense, that it is patently, openly untenable, unacceptable. Ok? But what counts as absurdity? When we can say this is absurd? So that the medieval logicians sort of explained that it can take three forms. Three forms.

One of them is very much obvious, which is a *self contradiction*. So you can go like this: That if p happens, then something absolutely self contradictory follows. So if p then α and not-α (alpha and not-alpha) both follow, right? If this is the case α and not-α both follow, then obviously p cannot be the case. This is one kind of absurdity and this is called self contradiction, which is one of the highest form of absurdity, logical absurdity you can say.

There can be milder versions of absurdity also. For example, which is not self contradiction, but something that is widely known to be false. Clearly false. For example, if p happens then there are no even numbers. If p is taken to be assumed to be true, then something as obliviously false as this statement follows that there are no even numbers. That is not exactly a self contradiction of this kind of form. But that is false, right? So, this is also an absurdity according to the medieval logicians.

The third kind of absurdity is not a self contradiction, not even a patent falsehood, but an implausibility, or something that is anomalouswith the given information, the current information that we have. For example, if I said that if p happens then cows will start to fly. *Cows flying* in this world is an implausibility, or completely in anomaly with whatever we know about cows, about our atmosphere, about gravitation, and so on and so forth, right? So this is sort of a ridiculous level, Ad Ridiculum. So what you have shown is reducing it to a ridiculous nature.

So all three these types were known by them as *absurdity*. Reducing to this to any of this would count as a REDUCTIO AD ABSURDUM proof. But for our purpose we are going to only look into this one; namely, that patent self contradiction will be explicitly shown as derivable and that is what we will call the Reductio proof.

So, the basic idea would be that, you know, we are going to try out the Reductio and our goal would be to show an explicit contradiction to follow from it. That would be the level of absurdity where we would say now we havereached the target of the proof. So where do you start? The answer is by assuming the negation of the conclusion, which happens to be the opposite of what you're trying to prove. Right? The negated conclusion. How do you go about? Well, we start with the new premise base, namely, our given premises plus the negated conclusion. And then what we try to do with the 19 rules is that a self contradiction follows. For example, if you can derive from this new set A • ~A (A and not-A). That's a clear, an open, an explicit contradiction. And then your claim should be, that is the indirect proof, that why the original conclusion has to hold. Because if you negate it, if that does not hold, then a patent contradiction follows, get me? So this is how the Indirect Proof sort of moves. So once you have understood it conceptually, the working out of the proof is not at all a problem.

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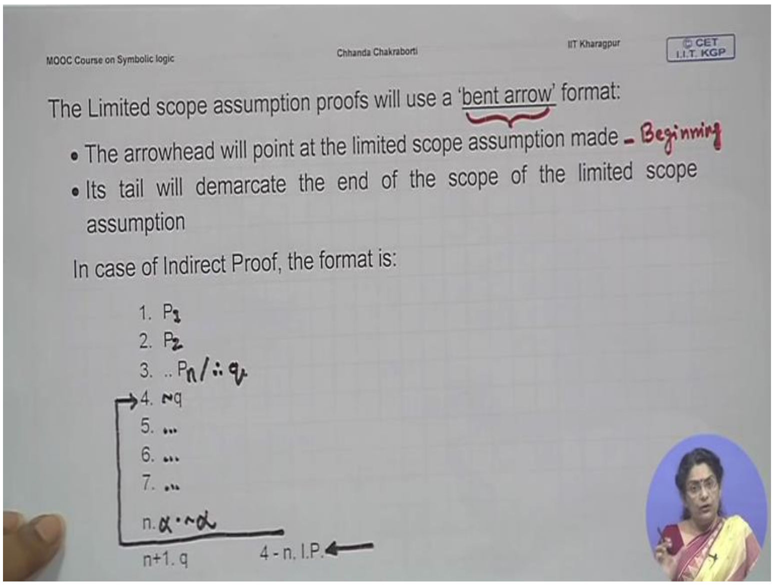

Let's talk about the format now. And all Limited Scope of Assumption proofs will follow a bent arrow format, bent arrow, arrow that is bent. How we will show you in a second. What it will do is that you are going to start an arrow where the scope of the assumption starts. And remember what I said that by default the assumption lies within its own scope. So, wherever you are making the assumption that's where the scope of the assumption also starts. Right? Because, the assumption is *included* with in its scope. So your *head of the arrow is going to point at the assumption itself*. That's the beginning of the scope of the assumption. And the tail will demarcate where the scope of the limited scope assumption ends. We will show you where. For example, here your goal is what? Goal is to derive an explicit contradiction. So, when you have reached that contradiction line, you know that you no longer need this assumption. That would be the end of the scope for this assumption.

So the format would be, now you can see visually what I was trying to say here, is like this. Suppose you have a proof like this, where 1, 2, 3 these are, these are all your premises, given premises. p_1, p_2 up to p_n and your conclusion is q, right? So, this is where you stand, you have the q. What is your job? If you are doing the Indirect Proof, the starting point would be not-q. Remember negated conclusion. And this is where the arrowhead is also immediately will show up. Note that there is no justification given. Why not? Because this is an indication that you are starting a Limited Scope Assumption procedure right here. So, that is not-q, that covers it. (Refer Time: 13:34) What is our goal? With this new set, we are going to derive an explicit contradiction. So let's see that let's assume that we are doing that with our 19 rules. We are trying to work out and finally we come to something like α and not-α. . That is your explicit contradiction, right? So, this is where you understand that my job done, we no longer need this not-q. And the way to indicate that in a proof is to close the assumption scope. See ? So your arrowhead starts here, it goes with like this and it goes under this. What does it tell you? That, this whole thing is an *assumption block*. Alright? And this assumption block you refer to it also as a *block*, you close it. When you close it, remember these individual lines are no longer accessible to you. You cannot refer, for example, to 6 anymore, which is within the assumption block. So, it becomes like a closed box, an assumption block.

So all of this is now packed inside and you have put a closure to that. And then in IP what do you do? You repeat the conclusion that is originally given. This is your claim that from this set, this has to follow. Why? Because if it is not, then this kind of contradiction follows, but you have shown that already.

Please note how we are justifying this last line. 4 *through* n. Can you see the hyphen? There is no comma between 4 and n, there is a hyphen, 4 through n, where the beginning is on line 4, the end is on line n, right? So we refer to this whole thing as a block, 4 - n. That block, remember, is no longer accessible, but we refer to it, not two separate lines, but as a block. Here comes the comma, the separator comma from the line numbers; and here is the first time that you say the procedure declaration is IP. So this is telling everybody that look, we derived this by IP method! Ok? Let's take an actual example and maybe you can do this on your own also.

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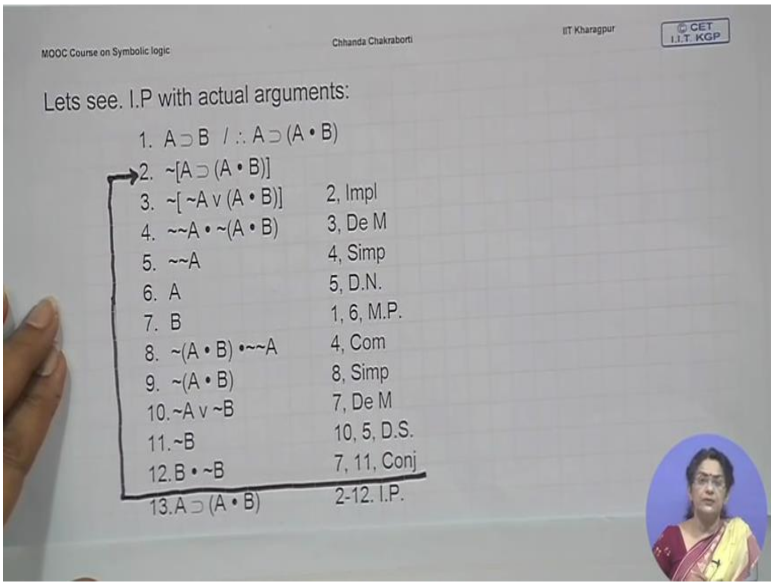

Let's take the examples that I showed you earlier that when we say that really there is no proof of this in nineteen rules. So now let us test whether with IP addition can we now show that there exists a proof in our system? So here is the argument A ⊃ B and A ⊃ (A • B). We know that it's valid. I have already, in the earlier module, I have explained why this must be so. Now, we are going to adopt the IP method, therefore our starting point will be what? The negation of the given conclusion. Now you have one, you have two. What is your job? The target is to derive *any* explicit contradiction. It could be A and not-A, could be B and not-B, alright? So that is the target. Now just simply use the 19 rules to derive that. Look at these two, make a plan how you can derive from this something of the kind that will show A and not-A, B and not-B. Something of that. Once you have reached that, you close the assumption and restate that this has to follow. This is our target.

So, there is nothing extraordinary about that. I will just show you how I have derived. You can follow the proof if you want. Like so, and here comes B and ~B. All of these steps are just to get here. All these rules are known to you. These are nothing but your 19 rules. So if you have learnt to do the proof earlier, that will come handy even here as I was trying to say.

This is 12, the line where we have reached B • ~B. Now, what I should do? I should close the assumption, right? That I no longer need this negated conclusion and then line number 13 would be the given conclusion. And once more, what is the justification? From 2 through 12, 2 – 12. Can you see that? 2 - 12, IP. So by IP and with all these lines I have proven why this must be valid conclusion. Get it?

Let's try another example, the one again that sounded like rather strange and we said that there cannot be any proof with the… just with the nineteen rules.

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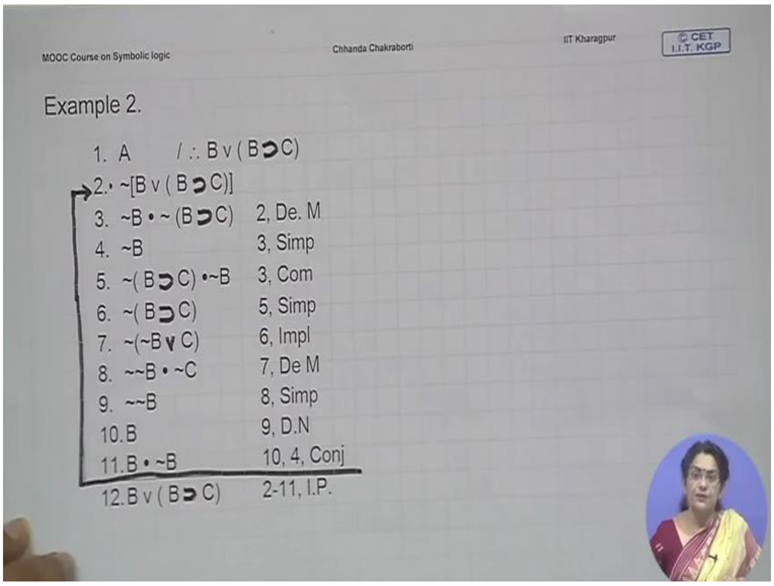

So let us try with the IP, whether now we can show why this argument has to be… this has to be a true conclusion, valid conclusion.

This is the A and we have B ∨ (B ⊃ C). IP will tell you what? That you can assume this. Fine? And now you have 1 and 2, what is your goal? To derive a self contradiction of some kind. So, again your 19 rules will come really helpful, if you know it, and here is how I derived B and not-B. That's an accident, but this is what is derivable from this. You can see in other proofs that there might be other kind of contradictions also, but try to get this kind of literals. So don't go for complex, for example, even if you have B ⊃ C and ~(B ⊃ C), that's not desirable. You try to get it into simple components like this. So, B and ~B, and all these rules are familiar to you. We have really gone through this. So with that then here comes your final assertion. This has to follow, 2 through 11, by IP . Get it ?

So this is our introduction. This was our way to do your Indirect Proof and that has been the content of this module, where we have looked into. You have just learnt how to do one procedure in Limited Scope Assumption and there is more in the next module. But in the meantime try to grasp this Indirect Proof: How to do it. If you know the format, the rest is as usual the 19 rules, right? So that should not be a problem. Ok? Even if there is problem, we are already there, always there. So feel free to try out a little bit, thank you, that's all for this module.

Thank you.