Indian Institute of Technology Kharagpur QUESTION-CUM-ANSWERSCRIPT	 Stamp/Signature of the Invigilator

END-SEMESTER EXAMINATION	AUTUMN SEMESTER

Roll Number										Section		Name	
Subject Number	M	A	2	0	1	0	1		Subject Name	Transform Calculus			
Department/Centre/School									Additional Sheets				

Important Instructions and Guidelines for Students

1. You must occupy your seat as per the Examination Schedule/Sitting Plan.
2. Do not keep mobile phones or any similar electronic gadgets with you even in the switched off mode.
3. Loose papers, class notes, books or any such materials must not be in your possession; even if they are irrelevant to the subject you are taking examination.
4. Data book, codes, graph papers, relevant standard tables/charts or any other materials are allowed only when instructed by the paper-setter.
5. Use of instrument box, pencil box and non-programmable calculator is allowed during the examination. However, the exchange of these items or any other papers (including question papers) is not permitted.
6. Write on both sides of the answer-script and do not tear off any page. **Use last page(s) of the answer-script for rough work**. Report to the invigilator if the answer-script has torn or distorted page(s).
7. It is your responsibility to ensure that you have signed the Attendance Sheet. Keep your Admit Card/Identity Card on the desk for checking by the invigilator.
8. You may leave the Examination Hall for wash room or for drinking water for a very short period. Record your absence from the Examination Hall in the register provided. Smoking and the consumption of any kind of beverages are strictly prohibited inside the Examination Hall.
9. Do not leave the Examination Hall without submitting your answer-script to the invigilator. **In any case, you are not allowed to take away the answer-script with you**. After the completion of the examination, do not leave your seat until the invigilators collect all the answer-scripts.
10. During the examination, either inside or outside the Examination Hall, gathering information from any kind of sources or exchanging information with others or any such attempt will be treated as '**unfair means**'. Don't adopt unfair means and also don't indulge in unseemly behavior.

Violation of any of the above instructions may lead to severe punishment.

Signature of the Student

To be Filled by the Examiner

Question Number	1	2	3	4	5	6	7	8	9	10	Total
Marks Obtained											

Marks Obtained (in words)	Signature of the Examiner	Signature of the Scrutineer

Instructions :

(i) Answer ALL the questions. Provide answers to all parts of each question together.

(ii) Notation: $\mathcal{L}^{-1}$ and $\mathcal{F}^{-1}$ denote the inverse Laplace and inverse Fourier transforms, respectively.

(iii) **No queries will be entertained during the examination.**

(iv) Marks are indicated in the parenthesis beside each question.

(iv) **No additional sheet will be provided.**

1. (a) Use the Laplace transform technique to solve the initial value problem

$$t\frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} + ty(t) = 0$$

subject to the conditions $y(0) = 1$ and $\frac{dy}{dt} = 0$ at $t = 0$. [3]

(b) **Find** $\mathcal{L}^{-1}\left[\log\left(\frac{s^2+1}{(s-1)^2}\right)\right]$. [3]

(c) Use the convolution theorem to find $\mathcal{L}^{-1}\left(\frac{a^2}{s(s+a)^2}\right)$, where a is a real number. [3]

(d) Solve the following integral equation using Laplace transform technique. [3]

$$\int_0^t f(u)\frac{1}{\sqrt{t-u}}\,du = 1 + 2t - t^2.$$

2. (a) Derive the Fourier series of the function

$$\begin{aligned} f(t) &= mt + c,\ -\pi < t < \pi, \\ f(t) &= f(t + 2\pi),\ t \in \mathbb{R} \end{aligned}$$

where m and c are real constants. [3]

(b) Find the Fourier series of the 2π periodic function

$$f(t) = t^2, \quad -\pi \leq t \leq \pi,$$

and hence find the sum

$$\sum_{n=1}^{\infty} \frac{1}{n^2}.$$

[3]

(c) **Determine the half range Fourier Sine series of the function**

$$f(t) = t^2 + t, \ 0 \le t \le \pi.$$

[3]

(d) i. Let the Fourier transforms of $g(t)$ and $h(t)$ be $G(\omega)$ and $H(\omega)$, respectively. Then prove that

$$\mathcal{F}^{-1}[G(\omega)H(\omega)] = g(t) * h(t)$$

where $*$ denotes the convolution operation. [3]

ii. Show that

$$\int_{-\infty}^{\infty} |F(\omega)|^2 d\omega = \int_{-\infty}^{\infty} |f(t)|^2 dt.$$

[1]

3. (a) The Fourier Cosine transform of $f(t) = \frac{1}{1-t^2}$ is

$$\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

hence the Fourier Sine transform of $\varphi(t) = \frac{t}{1+t^2}$ is

$$\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

[4]

(b) The Fourier Cosine transform of $e^{-t^2/2}$ is [3]

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(c) If

$$\int_0^\infty f(t)\cos(st)\,dt = \begin{cases} 1-s, & 0 \le s \le 1 \\ 0, & s > 1 \end{cases}$$

then

$$f(t) = \text{-----------------------}.$$

[3]

(d) Express the function $f(t) = \begin{cases} 1, & |t| \leq 1 \\ 0, & |t| > 1 \end{cases}$ in the Fourier integral form and hence, find [3]

$$\int_0^\infty \frac{\sin\lambda \cos(\lambda t)}{\lambda}\, d\lambda.$$

4. (a) Solve the following partial differential equation using Laplace transform technique. [5]

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}, \quad 0 < x < \infty, \ t > 0,$$

$$y(x, 0) = 0, \quad \frac{\partial y}{\partial t}(x, 0) = 0 \text{ for all } x,$$

$$y(0, t) = f(t), \text{ and } y(x, t) \text{ is bounded as } x \to \infty.$$

(b) Use the Fourier transform technique to solve the following Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad -\infty < x < \infty, \quad y > 0$$

subject to the conditions

$u(x, 0) = g(x)$; $u(x, y)$ is bounded as $y \to \infty$, $-\infty < x < \infty$.
$u(x, y)$ and $u_x(x, y) \to 0$ as $|x| \to \infty$.

[7]