Problem Set - 13

AUTUMN 2016

MATHEMATICS-I (MA10001)

November 07, 2016

1. For the function $f(z) = \frac{1}{z-2}$ determine the Taylor series expansion about each of the given points and determine radius of convergence for each case.
   (a) $z = 0$ (b) $z = -2i$ (c) $z = i$ (d) $z = -1$.

2. Find the Laurent series expansion of the function $f(z) = \frac{1}{z^2-3z+2}$ about the point (a) $z = 1$ (b) $z = 2$ (c) $z = 0$.

3. Expand (a) $\ln\left(\frac{1+z}{1-z}\right)$ (b) $\sinh z$ in Taylor's series about $z = 0$. Determine the radius of convergence for both cases.

4. Find all possible Taylor and Laurent series expansions of the function $f(z) = \frac{1}{(z+1)(z+2)^2}$ about the point $z = 1$.

5. Obtain the first three nonzero terms in the Taylor's series expansion for the following functions about the point $z = 0$
   (a) $\frac{1}{2+e^z}$ (b) $e^{z\cos z}$.

6. Find the principal part for the following Laurent series.
   (a) $\frac{z^2}{z^4-1}$ $(0 < |z-i| < \sqrt{2})$ (b) $\frac{\sin z}{z^4}$ $(|z| > 0)$ (c) $(z-3)\sin(1/z)$ $(|z| > 0)$ (d) $\frac{1-\cos z}{z}$ $(|z| > 0)$.

7. What kind of singularity does the function $f(z) = \frac{1}{e^z-1}$ have at $z = 0$? Find the first three terms of the Laurent series expansion about the point $z = 0$ in the region $0 < |z| < 2\pi$.

8. Expand $e^{\frac{z}{z-2}}$ in a Laurent series about $z = 2$, determine the region of convergence.

9. Find all singularities of the following functions and describe their nature(including ∞).
   (a) $\frac{1}{e^{1/z}+1}$ (b) $\mathrm{cosec}(1/z)$ (c) $e^{z+1/z}$ (d) $\frac{\sin^4 z}{z^4} + \cos(3z)$.

10. Determine and classify all the singularities of the functions(including ∞).
    (a) $\frac{z}{e^{1/z}-1}$ (b) $\cos(z^2 + 1/z^2)$ (c) $\frac{z}{e^z-1}$ (d) $\frac{e^z}{z-\sin z}$

11. Describe the singularity at $z = \infty$ for the following functions.
    (a) $\frac{z^2+10}{e^z}$ (b) $\frac{e^z}{z^2+10}$ (c) $1/z + \sin z$ (d) $\tan z - z$.

12. Given arbitrary distinct three complex numbers z_0, z_1 and z_2, construct a function $f(z)$ having a removable singularity at $z = z_0$, a pole of order k at $z = z_1$, and an essential singularity at $z = z_2$.

13. Determine the order of the pole at $z = 0$ for
    (a) $f(z) = \frac{z}{\sin z - z + z^3/3!}$ (b) $f(z) = \frac{z}{(\sin z - z + z^3/3!)^2}$.