Hints and Answers of Tutorial Sheet-4, MATHEMATICS-II Spring 2017

1. Use the definition of Hermitian matrix, $A = \bar{A}^T$. $x = 3$, $y = 0$, $z = 3$.

2. (a) Use the definition of an orthogonal matrix and then use inverse and transpose properties.

   (b) Use the definition of an unitary matrix and then use inverse and transpose properties.

3. (a) First use the transpose property $(AB)^T = B^T.A^T$. Then replace I by AA^T and A^TA accordingly.

   (b) First use the transpose property $(AB)^T = B^T.A^T$. Then replace I by AA^T and A^TA accordingly.

4. (a) Take the conjugate to $Ax = \lambda x$. use the symmetric property. Pre-Multiply $Ax = \lambda x$ with $\bar{x}^T$.
   (b), (c), (d) can be done similar to (a).

5. (i)$\lambda = -2 : [-1,\ 0,\ 1]^T$; $\lambda = 3 : [1, -1, 1]^T$; $\lambda = 6 : [1, 2, 1]^T$.
   (ii)$\lambda = -2, -2 : [1,\ 1,\ 0]^T, [-1, 0, 1]^T$; $\lambda = 4 : c[1, 1, 2]^T, c \in \mathbb{R}$.

   (iii) $\lambda = 1 : c[1,\ 1,\ 1]^T$; $\lambda = \frac{-1}{2} + \frac{\sqrt{3}}{2}i :\ \ c[1,\ \frac{-1}{2} + \frac{\sqrt{3}}{2}i,\ \frac{-1}{2} - \frac{\sqrt{3}}{2}i]^T$; $\lambda = \frac{-1}{2} - \frac{\sqrt{3}}{2}i :\ \ c[1,\ \frac{-1}{2} - \frac{\sqrt{3}}{2}i,\ \frac{-1}{2} + \frac{\sqrt{3}}{2}i]^T, c \in \mathbb{R}$.
   (iv)$\lambda = 1 : [1,\ 0,\ i]^T$; $\lambda = 2 : [0, 1, 0]^T$; $\lambda = -3 : [i, 0, 1]^T$.
   (v) $\lambda = -i, -i : [1, 0, -1]^T, [1, -1, 0]^T$; $\lambda = 2i : [1, 1, 1]^T$.

6. Let λ be the eigen value so $Av = \lambda v$. Take conjugate transpose on both sides, we get $v^*A^* = v^*\lambda^*$. Multiply above two equations and use $AA^* = I$.

7. If λ is an eigenvalue then $\lambda = ia$ where a is real number. Then use the fact $|z| = |x + iy| = \sqrt{x^2 + y^2}$.

8. $A^{-1} = \frac{1}{5}\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}$, $\alpha = 1, \beta = 5$.

9. $A^{-1} = \begin{bmatrix} 3 & 1 & \frac{3}{2} \\ \frac{-5}{4} & \frac{-1}{4} & \frac{-3}{4} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4}. \end{bmatrix}$

10. (a) Similar ; (b) Not similar.

11. (a) Use simailarity definition and hence get an invertible matrix P such that $A = PBP^{-1}$. Find the determinant $|A - \lambda I|$. Then use $A = PBP^{-1}$.
    (b) NO. construct an example.

12. (i) Diagonalizable; $P = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 3 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ and $P^{-1}AP = D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$.

(ii) Not diagonalizable. (iii) Not diagonalizable.

(iv) Diagonalizable; $P = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix}$ and $P^{-1}AP = D = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 4 \end{pmatrix}$.

13. The obtained recurrence relation using Cayley-Hamilton theorem is $A^{2i} = iA^2 - (i-1)I$ for $i = 1, 2, ...$ Then by putting $i = 50$ we get : $A^{100} = \begin{bmatrix} 1 & 0 & 0 \\ 50 & 1 & 0 \\ 50 & 0 & 1 \end{bmatrix}$.

14. (a) Let $P = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 4 \\ 1 & 4 & 9 \end{pmatrix}$ and $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$. Then $A = PDP^{-1} = \frac{1}{12} \begin{pmatrix} 30 & -12 & 6 \\ 2 & 4 & 14 \\ -34 & 4 & 38 \end{pmatrix}$.

As $A = PDP^{-1}$, so $A^n = PD^nP^{-1}$. Therefore $D^{500} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{500} & 0 \\ 0 & 0 & 3^{500}. \end{bmatrix}$

(b) Similar to 14(a).