Important Definitions in Analysis

Name	Definition
Metric space	A metric space is a space (X, d) where X is the set S and d is the metric on X. such that for $x, y, z \in X$ we have 1. $d(x, y)$ is real valued,finite and non-negative 2. $d(x, y) \geq 0$ 3. $d(x, y) = d(y, x)$ 4. $d(x, y) + d(y, z) \geq d(x, z)$
space l^{∞}	Set of all bounded sequences of complex numbers. If $x = (a_1, a_2 \dots a_i \dots)$ then $\forall\ i \in R$ $$\lvert a_i \rvert \leq C_x$$ Where C_x can be a real number defined on X, and the distance metric is defined to be $$d(x, y) = \sup_{i \in N} \lvert x_i - y_i \rvert$$
Space l^p	If $x \in l^p$ and $x = \{a_1, a_2, \dots\}$ then $$\sum^{\infty} \lvert a_i \rvert^p \ converges$$ Distance metric is defined to be $$d(x, y) = \left(\sum_{j}^{\infty} \lvert x_j - y_j \rvert^p \right)^{\frac{1}{p}}$$
Functional Space	Set of all real valued functions $x, y, z \dots$ which are functions of an independent variable t and are defined and continuous on a closed interval $J = [a, b]$. Metric is $$d(x, y) = \max_{t=J} \lvert x(t) - y(t) \rvert$$
Sequence Spaces	This is a space of all (bounded and unbounded) sequence of complex numbers and is defined by $$d(x, y) = \sum_{j=1}^{\infty} \frac{1}{2^j} \frac{\lvert \eta_j - \varepsilon_j \rvert}{1 + \lvert \eta_j - \varepsilon_j \rvert}$$ Where $y = (\varepsilon_j) \forall j = 1\ to\ \infty$ and Where $x = (\eta_j) \forall j = 1\ to\ \infty$
Open set	A subset M of metric space X is said to be open if every ball around point $x \in M$ has atleast 1 element from M except from itself.
Topological space	Topological space is a (X, τ) set X and collection τ of subsets such that it follows T1) $\phi \in \tau, X \in \tau$ T2) The union of any members of τ is also a member of τ T3) Intersection of finitely many members of τ is also a member of τ Then the set τ is topology of X
Dense set	A subset M of X is said to be dense if $\overline{M} = X$ where $\overline{M}$ is the closure of set M
Separable Set	X is said to be separable if it has countable subset which are dense in X. eg R as $\overline{R} = R$

Isometric Mapping	A mapping of T of X into Y is said to be isometric if it preserves distance $d_Y(Tx_1, Tx_2) = d_X(x_1, x_2)$
Isometric Space	A space Y is said to be isometric space with X if ∃ a bijective (1-1,onto) isometry from X to Y.
Homoeomorphic Spaces	T:X->Y Two metric spaces is said to be homoeomorphic spaces if there exists a homeomorphism T st. 1) T is continuous 2) T^{-1} is continuous 3) T is bijective
Convergent sequence	A sequence $x_n \in X$ is said to be convergent if there exists (X,d) a $x \in X$ such that $lim_{n\to\infty} \llbracket d(x, x_n) \rrbracket = 0$
Cauchy Sequence	A sequence $x_n \in (X, d)$ is said to be Cauchy or fundamental if for every $\epsilon > 0$ $\exists N = N(\epsilon)$ st $d(x_M, x_n) < \epsilon \quad \forall m, n > N$
Complete	A space is said to be complete if all Cauchy sequence converges.
Compact	A metric space X is said to be compact if every sequence in X has a convergent subsequence A subset M of X is said to be compact if it is considered as subsequence i.e limit of convergent subsequence lies in M
Norm and Normed Spaces	A normed Space is a space on which norm is defined. A norm is defined to be a real valued fn on X which has following properties $Norm = \|\|X\|\|$ N1) $\|\|X\|\| \geq 0$ N2) $\|\|X\|\| = 0 \; \|\|d\|\|$ N3) $\|\| \alpha x\|\| = \|\alpha\| \; \|\|x\|\|$ N4) $\|\|x + y\|\| \leq \|\|x\|\| + \|\|y\|\|$
Banach Space	A complete normed space is called a Banach Space
Absolute Convergence	Let x_k be a sequence in normed space X if $\|\|x_1\|\| + \|\|x_2\|\| + \cdots$ converges, then the series is called absolute convergence Absolute Convergence ↔ X is complete

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