eg; $\mathcal{L}\{5 \sin t\}$

$F(s) = \dfrac{5}{s^2+1}$,

hence $\mathcal{L}\{5 \sin(t-2)\,u(t-2)\} = e^{-2s} F(s)$

$$= \frac{5e^{-2s}}{(s^2+1)}$$

---

proof of Th-10 :- From the defⁿ of L.T, we have

$$e^{-as} F(s) = e^{-as} \int_0^\infty e^{-s\tau} f(\tau)\, d\tau$$

$$= \int_0^\infty e^{-s(\tau+a)} f(\tau)\, d\tau$$

Let $\tau + a = t$ in the integral, we obtain

$$= \int_{t=a}^{\infty} e^{-st} f(t-a)\,dt$$

$\Rightarrow e^{-as} F(s)$

$$= \int_{t=a}^{\infty} e^{-st} f(t-a)\,dt$$

$\tau + a = t$

$\Rightarrow \tau = t - a$

$\Rightarrow d\tau = dt$

Limits of $\int$

$\tau = 0,\ t = a$

$\tau = \infty,\ t = \infty$

Explain

$$= \int_{0}^{\infty} e^{-st} \underbrace{f(t-a).u(t-a)}\,dt$$

$$= \mathcal{L}\{f(t-a)\,u(t-a)\}$$