Department of Mathematics
Indian Institute of Technology Kharagpur
Mid Semester Examination

Date of Exam : 16-09-2016 (FN) Time : 2 Hrs. Full Marks: 60
Subject Name: Regression and Time Series Models Subject No. MA31020/MA41025
Course : B.Tech./M.Sc. (various departments) No. of students: 120

Instructions: Answer all questions. Marks are indicated at the end of each question. Statistical tables may be used.

1. Let $\{Z_t\}$ be *IID* $N(0,\sigma^2)$ noise. Consider the time series $\{X_t\}$ as (i) $X_t = 1+2Z_0$, (ii) $X_t = 1+2Z_t+3Z_{t-2}$, (iii) $X_t = Z_1\cos(\theta t)+Z_2\sin(\theta t)$, where θ is a constant. Check whether the time series $\{X_t\}$ in parts (i), (ii) and (iii) are stationary? [10 marks]
2. (a) Consider an AR(1) process as stationary solution $\{X_t\}$ of the equation $X_t-\phi X_{t-1}=Z_t$, where $\{Z_t\}$ is $WN(0,\sigma^2)$, $|\phi|<1$ and $\{Z_t\}$ is uncorrelated with $\{X_s\}$ for each $s<t$. Find the mean and ACVF of time series $\{X_t\}$.
   (b) Check whether the following ARMA (1, 1) process $X_t+0.5\,X_{t-1}=Z_t+1.2\,Z_{t-1}$, where Z_t is $WN(0,\sigma^2)$, is causal and invertible? [6+4 marks]
3. (a) Discuss the trend elimination by differencing method in absence of seasonality in time series analysis.
   (b) Let $\{Z_t\}$ be *IID* $N(0,\sigma^2)$ noise, define
$$X_t=\begin{cases} Z_t, & \text{if } t \text{ is even} \\ \dfrac{Z_{t-1}^2-1}{\sqrt{2}}, & \text{if } t \text{ is odd} \end{cases}.$$
   Show that $\{X_t\}$ is $WN(0,1)$. [4+6 marks]
4. The chlorine residual (pt/million) Y in a swimming pool at various times (in hours) after being cleaned (X) is as given:

X	2	4	6	8	10	12
Y	1.8	1.5	1.45	1.42	1.38	1.36

   Fit a relation of the form $Y=a\,e^{-bX}$. Find the coefficient of determination? How much model explanation does it give? [7 marks]

5. To illustrate Francis Galton's thesis of regression to the mean, Karl Pearson recorded the heights (y) of 10 randomly chosen sons versus heights (x) of their fathers. The resulting data (in inches) are given in the table. Explain that testing the claim of Galton, that is,

"regression of heights to mean" is equivalent to testing hypothesis $H_0 : \beta_1 \geq 1$ against $H_1 : \beta_1 < 1$ in the simple linear regression model $y = \beta_0 + \beta_1 x + \varepsilon$. Draw a scatter diagram and show that a linear relation is appropriate. Fit the simple linear regression model. Test the hypothesis of Galton's claim at 5% level of significance. What is your conclusion? Find a 95% confidence interval for the average height of sons whose father's heights are 68 inches. Find the coefficient of determination. How much regression does it explain? [8 marks]

x	y	x	y
60	63.6	67	67.1
62	65.2	68	67.4
64	66	70	68.3
65	65.5	72	70.1
66	66.9	74	70

6. A new drug was tested on mice to determine its effectiveness in reducing cancerous tumours. Tests were run on 10 mice, each having a tumour of size 4 grams, by varying the amount of drug used and then determining the resulting reduction in the weight of the tumour. The data were recorded. Draw a scatter diagram and show that a second degree curve will be appropriate to describe the relationship. Use polynomial regression to fit the curve. What is the ideal amount to reduce the size of the tumour? Give a 95% confidence interval for the expected reduction at this point. Also find the coefficient of determination. [8 marks]

x coded amount of drug	y Tumour weight reduction
1	0.50
2	0.90
3	1.20
4	1.35
5	1.50
6	1.60
7	1.53
8	1.38
9	1.21
10	0.65

7. Describe a general Gauss-Markov linear model. What are its assumptions? Derive least squares estimator of the regression vector and show that it is unbiased. Find it dispersion matrix. Find the error sum of squares and an unbiased estimator of σ^2. What is its distribution? Use it to develop test of significance and confidence interval for a linear function $\underline{l}'\underline{\beta}$ of regression vector of the model. [7 marks]