Transportation Problem: Phase-II Solution known as MODI Method: MOdified(MO) DIstribution(DI)

Iterative computations of the Transportation algorithm

After determining the starting BFS by any one of the three methods discussed earlier, we use the following algorithm to determine the optimum solution.

(Use either NWCR or LCM for Phase-I Solution)

Step1: Use the Simplex optimality condition to determine the entering variable as a current non-basic variable that can improve the solution. If the optimality condition is satisfied by all non-basic variables, the current solution is optimal and we stop. Otherwise we go to Step 2.

Step 2. Determine the leaving variable using the Simplex feasibility condition. Change the basis and go to Step 1.

The determination of the entering variable from among the current non-basic variables is done by the **method of multipliers (see Class Note)**.

In the method of multipliers, we associate with each row a dual variable (also called a multiplier) u_i and with each column we associate a dual variable (also called a multiplier) v_j.

Noting that each row corresponds to a constraint and each column corresponds to a constraint we recall from duality theory that

At any simplex iteration ,

$$\begin{pmatrix} \text{Primal z-equation} \\ \text{coefficient of} \\ \text{variable } xij \end{pmatrix} = \begin{pmatrix} \text{Left hand side} \\ \text{of corresponding} \\ \text{dual constraint} \end{pmatrix} - \begin{pmatrix} \text{Right hand side} \\ \text{of corresponding} \\ \text{dual} \end{pmatrix}$$

constraint

That is $$\boxed{"z_{ij} - c_{ij}" = u_i + v_j - c_{ij}}$$

(Verify this by taking m=3 and n=4 !)

Since there are m+n-1 basic variables and since

$$z_{ij} - c_{ij} = 0$$

for all such basic variables, we have m+n-1 equations

$$u_i + v_j = c_{ij}$$

to determine the m+n variables u_i, v_j

We arbitrarily choose one of them and equate to zero and determine the remaining m+n-1 of them. Then we calculate

$$z_{ij} - c_{ij} = u_i + v_j - c_{ij}$$

for all non-basic variables x_{ij}. Then the entering variable is that one for which $u_i + v_j - c_{ij}$ is most positive.

If we consider a maximization type Transportation Problem instead of most positive element, most negative element is selected for the entering variable.

We do this on the transportation tableau itself (and NOT separately) as the following example shows.

Destination

Starting Tableau

Total Cost =48

Source		$v_1=3$	$v_2=7$	$v_3=6$	$v_4=3$	Supply
	$u_1=0$	3 3	7 2	6 0	4 -1	5
	$u_2= -3$	2 -2	4 1	3 1	2 -2	2
	$u_3= 2$	4 1	3 6	8 1	5 2	3
	Demand	3	3	2	2	

Thus x_{32} enters the basis.

Determining the leaving variable

We first construct a **closed loop** that starts and ends at the entering variable cell. The loop consists of connected horizontal and vertical segments only (no diagonals are allowed). Except for the entering variable cell, each vertex (or corner) of the closed loop must correspond to a basic variable cell. The loop can **cross itself** and **bypass one or more basic variables**. The amount θ to be allocated to the entering variable cell is such that it satisfies all the demand and supply restrictions and must be non-negative. Usually

θ is the minimum of the amounts allocated to the basic cells adjacent to the entering variable cell. Having decided about the amount θ to be allocated to the entering cell, for the supply and demand limits to remain satisfied, we must alternate between subtracting and adding the amount θ at the successive corners of the loop. In this process one of the basic variables will drop to zero. In simplex language, we say it leaves the basis. We repeat this process till optimality is reached. We illustrate with a numerical example.

Destination

Starting Tableau

Total Cost =48

Source

	$v_1=3$	$v_2=7$	$v_3=6$	$v_4=3$	Supply
$u_1=0$	3 3	7 2	6 0	4 -1	5
$u_2=-3$	2 -2	4 1	3 1	2 -2	2
$u_3=2$	4 1	3 θ 6	8 1	5 2	3
Demand	3	3	2	2	

Thus x_{32} enters the basis.

Thus θ will become 1 and in the process both the basic variables x_{22} and x_{33} will become simultaneously zero. Since only one of them should leave the basis we make x_{22} leave the basis and keep x_{33} in the basis but with value zero. Thus the transportation cost reduces by 6 (as x_{23} increases by 1) and we say one iteration is over. The resulting new tableau is on the next slide.

Destination

Start of Iteration 2

Total Cost =42

Source

	v_1=3	v_2=7	v_3=12	v_4=9	Supply
u_1=0	3 **3**	7 **2**	6 θ 6	4 5	5
u_2= -9	2 -8	4 -6	3 **2**	2 -2	2
u_3= -4	4 -5	3 **1**	8 **0**	5 **2**	3
Demand	3	3	2	2	

Thus x_{13} enters the basis.

Thus θ will become 0 and x_{32} leaves the basis. Again the BFS is degenerate . But the transportation cost remains the same and we say the second iteration is over. The resulting new tableau is on the next slide.

Destination

Start of Iteration 3

Total Cost =42

Source		$v_1=3$	$v_2=7$	$v_3=6$	$v_4=9$	Supply
	$u_1=0$	3 **3**	7 **2**	6 **0**	4 θ 5	5
	$u_2=-3$	2 -2	4 0	3 **2**	2 4	2
	$u_3=-4$	4 -5	3 **1**	8 -6	5 **2**	3
Demand		3	3	2	2	

Thus x_{14} enters the basis.

Destination

Start of Iteration 4

Total Cost =32

Source

	$v_1=3$	$v_2=7$	$v_3=6$	$v_4=4$	Supply
$u_1=0$	3 **3**	7 **0**	6 **0**	4 **2**	5
$u_2= -3$	2 -2	4 0	3 **2**	2 -1	2
$u_3= -4$	4 -5	3 **3**	8 -6	5 -5	3
Demand	3	3	2	2	

Thus this is the **optimal** tableau. Alt Opt solutions exist.