Symbolic Logic
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Lecture – 28
Conditional Proof
Preliminary Version
The Format
The Procedure

Hello and welcome back to this module number 28! And we are on the topic of Limited Scope Assumption Procedures. We have already learnt the Indirect Proof.

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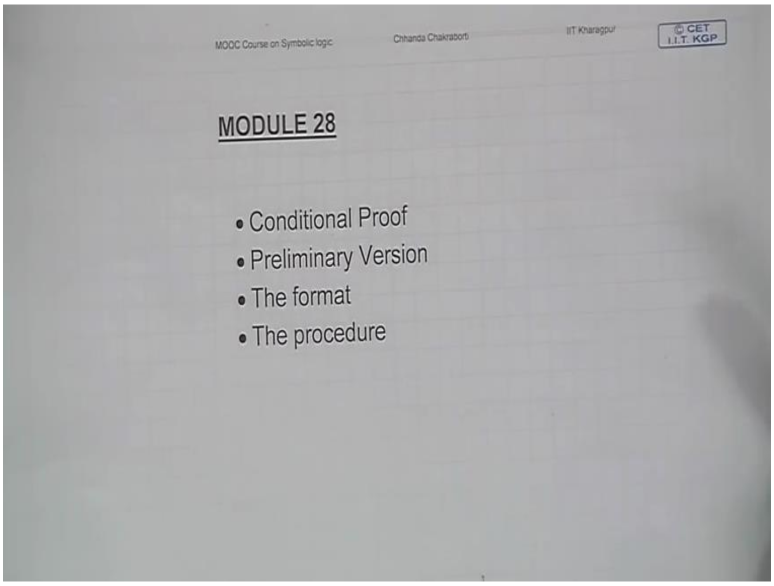

And today we are going to look into this Conditional Proof which is the other Limited Scope Assumption proof procedure, that we are going to learn. Except that I will mention that the Conditional Proof or the CP has more than one version, and today we are going to only look into the preliminary version. Preliminary version which is the elementary. And there will be an advanced version or the Strengthened version that we'll take up in the next module.

So currently we are because we are getting acquainted (Refer Time: 01:06) with proof procedures, so we'll start with the elementary one first. And obviously we need to know if you are learning the new proof procedure, then the format as well as the procedure and so on. So this is going to be the content of our module on number 28.

Before we proceed to the Conditional Proof, let me make one point clear about Indirect Proof.

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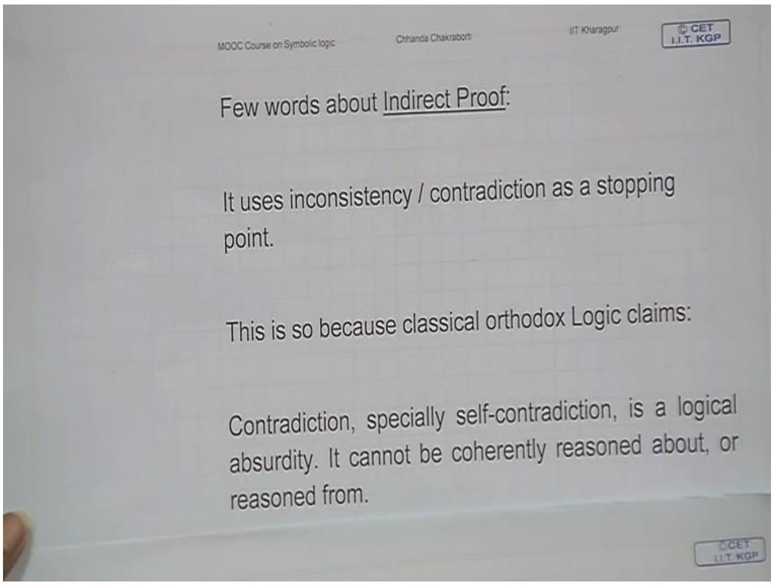

See, I simply said that it sort of stops at an absurdity, where we are taking the absurdity as the self contradiction. Something that is very obvious and of the nature somewhere the proposition and its negation are both being placed as the truth. So it uses, this IP procedure uses inconsistency or self contradiction as a stopping point. Beyond this, the argumentation does not proceed. So that sort of treats (Refer Time: 02:20) this self contradiction or inconsistency as a termination point for logical reasoning. Why is that? Why do we have to stop when we have encountered self contradiction? And the answer is because that is what the assumption of classical orthodox logic is. It claims that, you know, a self contradiction is a rather abominable, unacceptable and logical undesirability. So, once you reach a self contradiction, nothing can be coherently reasoned about or reasoned from. So that is what this whole thing is about, that there is this law of contradiction, the law of non-contradiction in orthodox logic.

This is the reason whenever you are in the domain of classical orthodox logic, this encountering inconsistency or self contradiction will put you to a stop, put you to a halt. But notice that, that may not be always the case, but we'll come there. Let's try to understand what is this all about.

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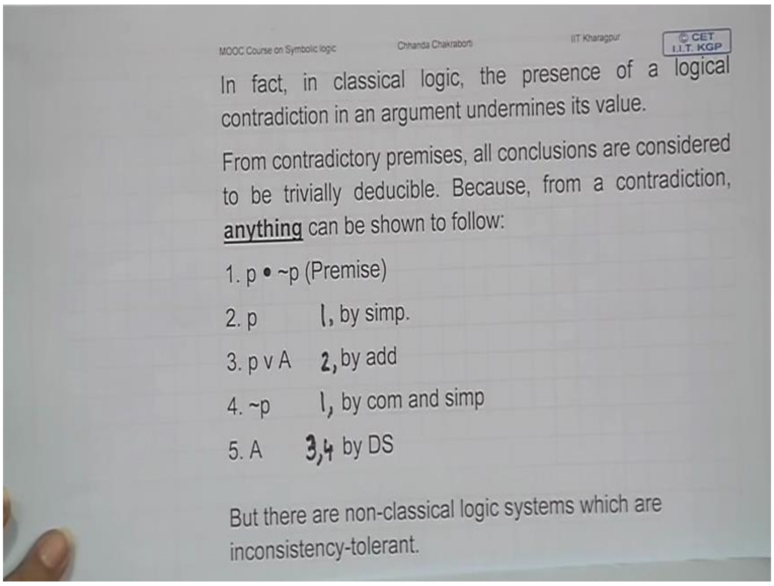

Mainly what happens is that in classical logic, the presence of a logical contradiction in an argument, in a way, completely undermines its value. Completely. It makes it practically trivial and useless. And in fact, in logic, formal logic, classical logic it is said that if you have the premises as contradiction, self contradictory, then all conclusions that are derived from that is to be considered as trivially deducible. Or, in other words, from a contradiction *anything* follows. From a contradiction *anything* follows. And, that claim, we can show it in case you… you can't cross grasp it conceptually, I will show you how this proof works. Like this.

For example, suppose you have the premise like so, p • ~p, which is the patent self contradiction. From that we can simplify p and then we can add A, where A is *any* conclusion of your choice, *anything*. It can be a very complex proposition also, just by addition. See the validity is preserved so far and then we can take not-p out from 1 by commuting and then simplifying. So I have combined these steps with your permission on line 4. On line 1, we switch the position of p and ~p and take ~p out, fine?

But then you put 3 and 4 together and apply DS. What will you get? A. So in a way from a contradictory set of premise, you can show the derivation of any conclusion of your choice. So from a contradiction, anything follows. Which is why it is not really desirable that your premises have inconsistency or your premises should not have any logical self contradiction contained in them.

That is, if you are in the domain of classical logic. If you leave the domain of classical logic and go into some of the newer logic, non-classical logic, there I must tell you there are systems which are inconsistency-tolerant. That is, they can absorb even self contradictions and still the logic system functions. So what logic systems are we talking about? I mean I am just going to mention some names without explaining them, but you are welcome to look it up or if you want, interested, you can, you can further study. Paraconsistent logic is one of them. Dialetheism is, (Refer Time: 06:30) for example, is a system that believes that all contradictions need not be false, you know.

So there are non-classical logical systems. But that is not where we are at. We are at this classical orthodox logical system level, where the self contradiction is a stopping point, alright? I just thought I will mention this because you have learnt Indirect Proof and you if you are asked by somebody: But why do you treat that indirect proof thatif I have reached the self contradiction why do I have to stop? I thought I will give you some theoretical answer behind that.

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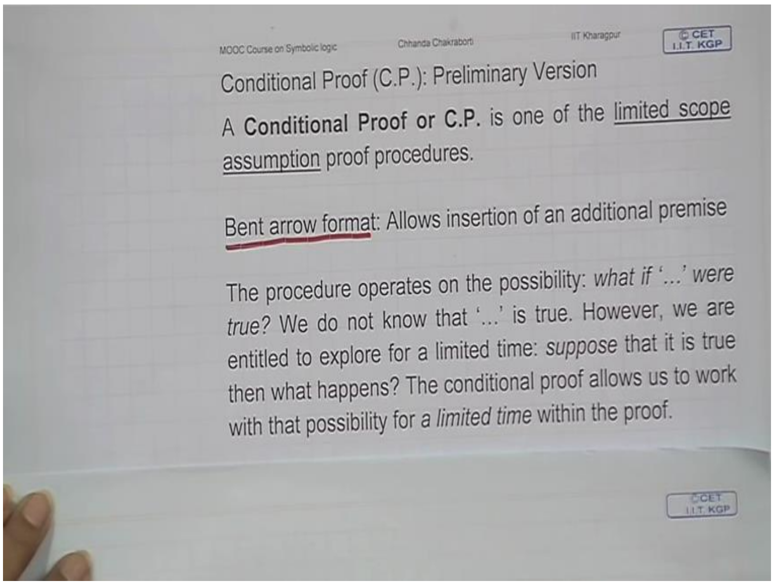

Let's now come to our today's topic, that is your Conditional Proof. But we are going to only look into the preliminary version, right? As I said, elementary version. What it is I have already said it's the Limited Scope Assumption proof. So everything that I have said earlier about the Limited Scope Assumption proof still holds for the conditional proof. For example, the bent arrow format that we have learnt in case of IP or Indirect

Proof still applies. The arrowhead is going to point at the beginning of an assumption and the moment you reach your objective, you are ready to close the assumption or *discharge* the assumption. That's where the assumption must be closed and you know how to do that. The bent arrow format by now you know.

Now what is this Conditional Proof all about? Well, it operates on the possibility of *what if*. So this is not about absurdity. This is simply saying what if this also were the case. Remember, I mean in our common argumentation technique, we often say: For the argument's sake, let us assume that this is true. That does not mean that it is true. All you are saying that let's suppose for the time being that this is also true and then what. So that is exactly what this Conditional Proof is all about. It allows us to add an assumption for limited time and explore what happens, when that assumption holds. Ok? So for a limited time, within the proof you are allowed to add an assumption and see what follows from that.

Having said that, now you will say: But what am I going to assume? But before that let me just assure you. Let's take a look into format.

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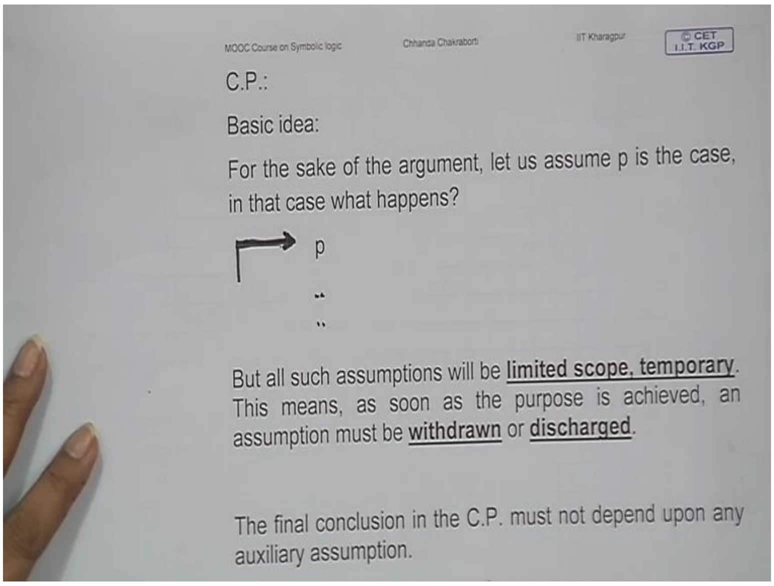

So, as I said you are saying let's assume for example p is the case. The moment you say p is the case, the bent arrow technique will take over. And this is the kind of indication that you need to give. And then, within a limited period and as soon as you reach your target, there is a reason why you said let's assume p. Once you have reached the target, what the target is I will explain later. But once you have reached the target, as before, you need to discharge or withdraw, *close* the assumption. And that is where the lined assumption will close also. So the bent arrow will close down. Final line in a Conditional Proof will not depend upon the… any of the limited scope assumption that we you are going to make.

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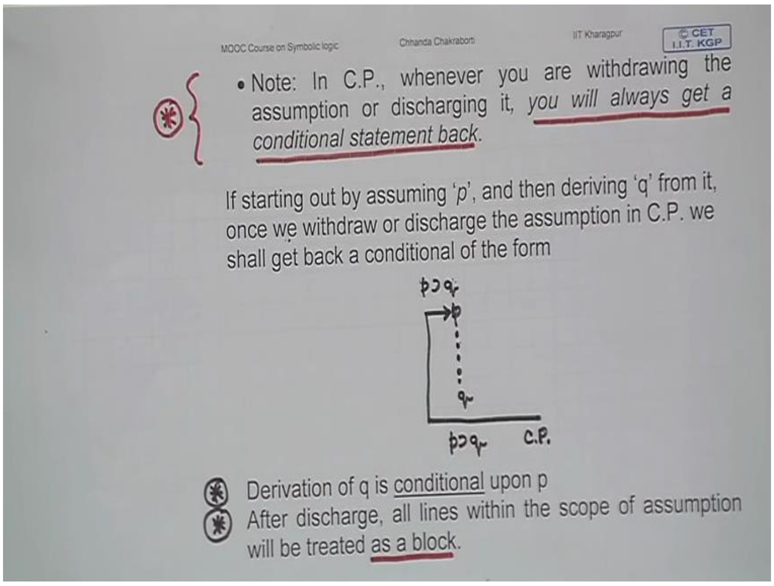

The main and the most important thing in Conditional Proof is this, that in Conditional Proof whenever you are withdrawing an assumption or discharging the assumption, you are going to get back a *conditional statement*. Conditional statement as in ' if p then q' type of statement. So if you said that let us assume that p, and then you derive q from it, and then you said ok, I have reached my goal q, I no longer need this p. So you are going to withdraw or discharge the assumption p. The moment you do that, what are going to get back? In CP you are going to get back a statement like this: $p \supset q$. Ok? In a way, it should make sense. You have said that if p, then sort of q is derivable. So, that is what you deserve in a proof. This line is going to appear in your proof and you better know what to do with this line in your proof. Otherwise, this would be a pointless exercise one after other.

So once more, the difference between IP and the CP is this, that in CP you are saying that let's assume this for some time and you have a certain target in your mind, once you reach the target, you say ok, I do not need it. But when you close the assumption, when you discharge the assumption, what happens? Inside the proof, a conditional statement will appear where the antecedent is going to be your assumption and the consequent is going to be the line that you have derived from it. Based on it. So you are saying that q is conditional upon the assumption of p, and the bent arrow will work like this. This is where the arrowhead will point at, and then through the lines. This is where your 19 rules are going to be operational and you have derived q and then you said I do not need any more. Fine. Then you close it. The moment you close it, CP will ask you to add this line to your proof: if p then q.

So derivation of q, this is your disclosure, that derivation of q is conditional upon the assumption p. And once more I have to remind you, just like in previous IP, everything that is contained within this, where the arrowhead starts and where it ends, is a block. It is treated as an assumption block. An assumption block will not be accessible individually. Which means whatever lines are here, once you are outside of this block, you cannot access those, you cannot refer to that, you cannot use them in the proof anymore. Remember that. So this is once and for all is closed.

So remember in a proof if you are going to need any of these lines later on, you need to have them available to you or get everything done here, so that finally you no longer need this. So this will require some practice on your part to see what is it that I need? So, but in general my point is that this is a closed block and treat it like that. So in justification also it will be treated as a block reference.

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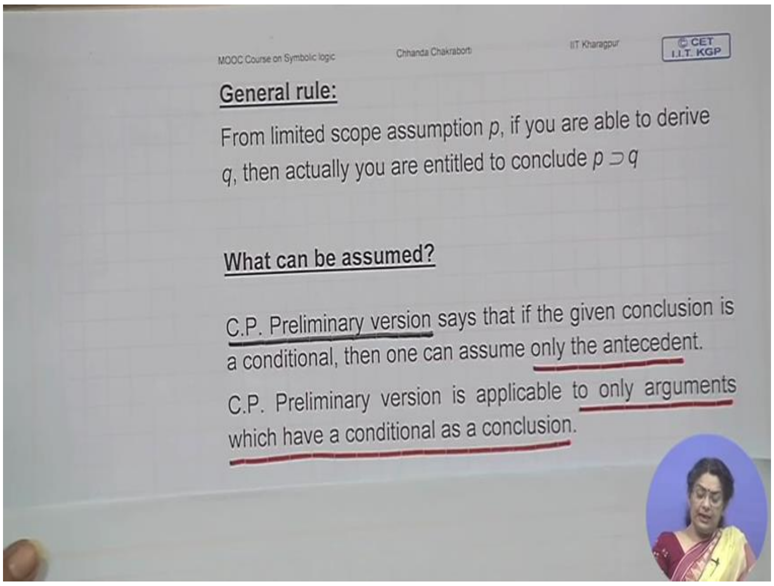

Now, the general rule, therefore, for Conditional Proof is like so that if on the basis of p, you are able to derive q, then you are entitled to conclude ‘ if p then q’ . That is how the Conditional Proof works.

Now comes the question, but so far I understand we have said assume p, assume p, but what exactly is p? What can be assumed in CP? And the answer is that you are going to get in the preliminary version will be different from the advanced or the strengthened version. But we are now learning only the elementary or the preliminary version of the CP. And that preliminary version says this. That given, that you are working with an argument where the conclusion is a conditional statement, given that, then what you can assume is only the *antecedent of that conclusion*. So once more, if you happen to be lucky to have an argument, where the conclusion is something of the form ‘ if p then q’ , then the preliminary version of CP allows you to assume only p and solve for q. Get me? So that would be the target for you. But this is the only kind of assumption that the preliminary version of CP will allow you to add.

So, in a way, therefore, the preliminary version has a certain limitation of application. It applies to only those arguments, which have conclusions which are conditionals. So if your conclusion is of the type $p \vee q$ or $p \bullet q$, then this version of CP will not even be applicable. Right? And you have found the answer what can be assumed. The answer in this version is the *antecedent of the conclusion*, where the conclusion happens to be a conditional. Have I made myself clear in this? Good. So this is how this will work.

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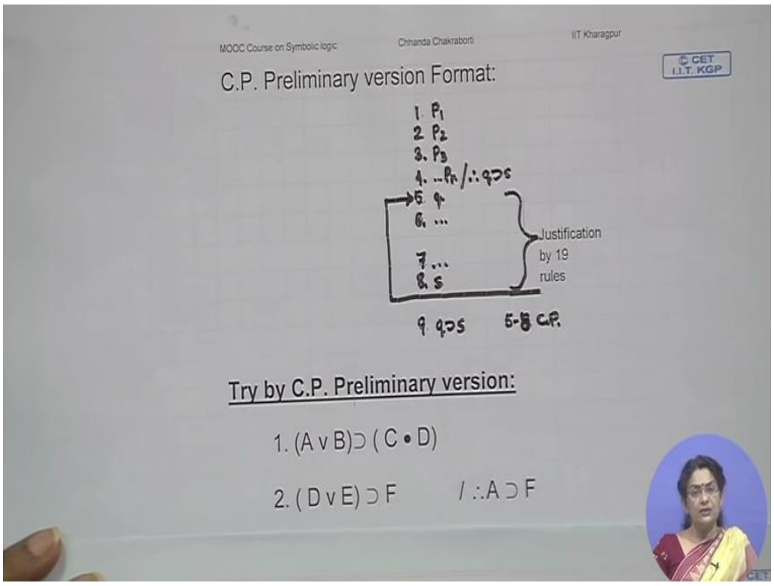

And the format, you know, like as in the case of IP, will be somewhat similar, but you need to sort of get used to the difference with IP also. So here is your proof. Here is your original argument, which happens to have a conclusion which is q ⊃ s. These are all your premises given. The conclusion is q ⊃ s. What preliminary version of CP will allow you is to add q further as your Limited Scope Assumption. Immediately the bent arrow will take over.

From this, what is your target? What are you going to show? The answer is out of all these what we' ll show is that s follows. So we are saying given all these premises and q, let's suppose that q is true, s follows. So that is what we will solve for: s. And this is where your knowledge of 19 rules is going to really again come out and help. Once you have reached this with these 19 rules, what do you say? That I have reached my goal and no longer need this, then you close it like so. And what did I say? The moment you close the assumption, then you are going to get the back what? A conditional of the form q ⊃ s, get it? So, the assumption horseshoe the line that you have derived. This line is going to show up in your proof. Fortunately, this is what you wanted also to derive, right ? This is your conclusion.

How do you justify? We justify it as a block. So 5 through 8, sorry this is 8, 5 through 8. So, this is 5, this is 8, 5 through 8 and by CP. So no need to have CP appear. This is just an indication that you are starting a Limited Scope Assumption procedure. This is where you disclose the strategy or the proof procedure, alright? This we have seen.

So I think we have said enough now. It's time to apply it and see whether you have learnt. So, can we try by conditional proof preliminary version this little argument? See it calls for the preliminary version of Conditional Proof because the conclusion is actually a conditional $A \supset F$. So you have premise 1, premise 2, what will be your premise 3? Premise 3 will be A. Let's assume that A is true. And what will you solve for? You will solve for F, right? And once you have reached F, you can close it like so. And then you will get back $A \supset F$. Can you do that? And I have the worked out solution as always with me. I will show it to you in a second. But please try it out, try it out on your own with the knowledge of 19 rules and see whether you can derive F from this, using this proof procedure.

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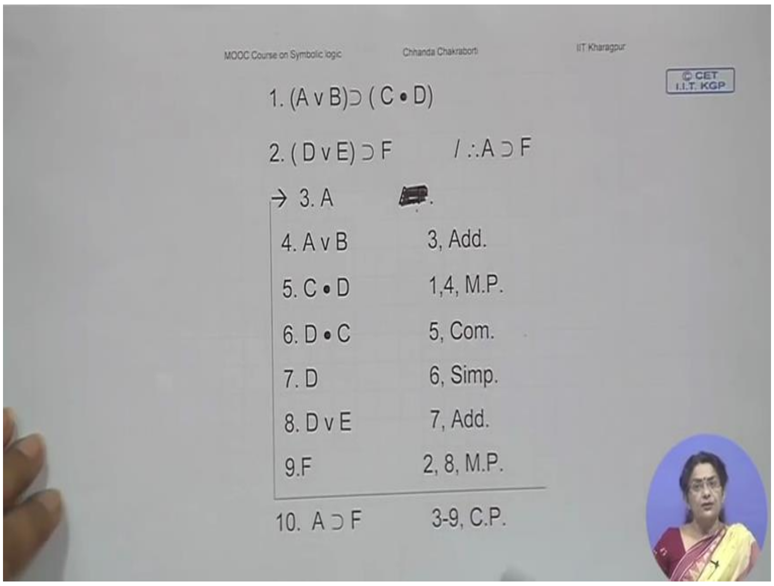

So, this is where I show you the worked out solution. In case you have tried it out, you can work it with me or even check it as it goes along. But this is where you start, right? This is A and this is where the assumption arrowhead sort of starts. Then I do what? Well, this is where your planning is necessary. What you are trying to solve for is F. Here is F. So, if you can get $D \vee E$ somehow, then your job is done. How do you get D

$\vee$ E? Well, here is D, somehow if you can get A $\vee$ B then C $\bullet$ D, and you know there is a rule called Simplification that will allow you to get the D out of C $\bullet$ D. So plugging it altogether, we work out the problem like so. To A, we add B by Addition and then we plug it in with 1 and 4, we get Modus Ponens. And this is what we derive C $\bullet$ D. And this we commute because we want the D out by Simplification, and once we have the D we just add E to it. D $\vee$ E, so that we can have F out. Have I reached my target? The answer is yes. Once I have reached the target what do I do? I close the assumption. So this is like that. And what do I get back? You know what you get back; namely, A $\supset$ F.

How did you obtain it? 3 through 9. See, these are all rules that you already knew and which you have been using so far. So there is nothing new about it. This procedure is the only thing that you have to learn and look at the way the justification goes. That's something new. So 3 through 9, that's a block. That's an assumption block. And we close it and by CP. We mention the proof procedure right here, alright? So this is not too bad. This is our preliminary version of Conditional Proof. We are going to look into the Strengthened version later. But try to absorb the idea of the Conditional Proof. And you will see that you once you get into this, try other proofs that you have tried earlier which have conditionals as conclusions with this preliminary version of CP . And you will see that the proofs have become much easier. Ok? So with that we close this module. We will see you again in the next module.

Thank you, bye-bye.