$$1 - \cos n\pi = \begin{cases} 2 & \text{for odd } n \\ 0 & \text{for even } n. \end{cases}$$

Hence, the Fourier Co-efficients b_n of our f^n are

$$b_1 = \frac{4k}{\pi}, \quad b_2 = 0, \quad b_3 = \frac{4k}{3\pi},$$

$$b_4 = 0, \quad b_5 = \frac{4k}{5\pi}, \ldots$$

Since the a_n's are zero, the Fourier series of $f(x)$ is

$$\frac{4k}{\pi}\left(\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \ldots\right)$$

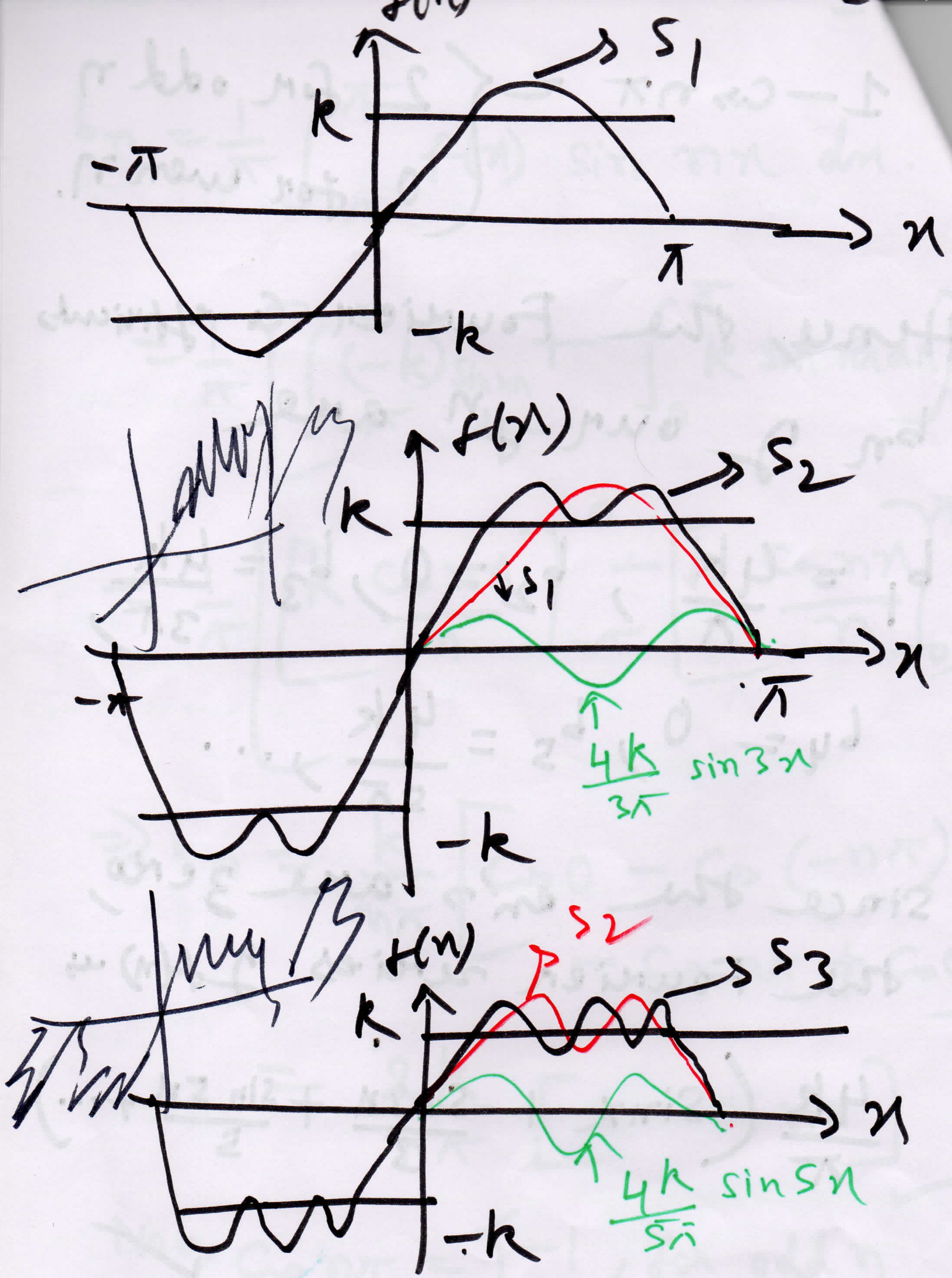

Fig 2

The first 3 partial sums of the corresponding Fourier series.

The partial sums are

$$s_1 = \frac{4k}{\pi}\sin x, \quad s_2 = \frac{4k}{\pi}\left(\sin x + \frac{1}{3}\sin 3x\right)$$

Their graphs in Fig2 seem to indicate that the series is convergent & has the sum $f(x)$, the given fn. etc.

We notice that at $x=0$ & $x=\pm\pi$, the points of discontinuity of $f(x)$, all partial sums have the value zero, the A.M ~~last~~ of the values $(-k)$ & k of our f^n (why?).

Furthermore, assuming $f(x)$ is the sum of this series & setting $x=\pi/2$, we have

1

$$f(\pi/2) = k = \frac{4k}{\pi}\left(1 - \frac{1}{3} + \frac{1}{5} - \cdots\right)$$

$$\Rightarrow \boxed{1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \pi/4}$$

This is a famous result by Leibnitz. obtained in (1673) from geometrical considerations.

This illustrates ~~that the values~~ ~~of~~ various series with constant terms can be obtained by evaluating Fourier series at specific points.