Indian Institute of Technology
Department of Mathematics
Spring Mid Semester Examination-2014
Subject Name: Discrete Mathematics
Subject No: MA20013
No. of students: 62 Time: 2 hrs F.M. 30

Instructions: Answer ALL questions. Answer all the parts of the same question TOGETHER.

(1) Answer ALL parts. 2×6=12

(a) Prove or disprove that if a relation on a set A is transitive and irreflexive then it is asymmetric.

(b) List all possible equivalence relations on a set with three elements.

(c) Define relations R_1 and R_2 on the set of integers as (i) $a\ R_1\ b$ if and only if $|a-b| \leq 1$ (ii) $a\ R_2\ b$ if and only if $|a| \leq |b|$. Whether R_1 and R_2 are partial order? Justify.

(d) Whether the posets $(D_{40}, |)$ and $(D_{42}, |)$ are isomorphic? Justify your answer.

(e) Give an example of a finite lattice where at least one element has more than one complement and at least one element has no complement.

(f) Is the lattice $(\mathbb{N}, |)$ distributive? Justify.

(2) Answer ALL parts. 3×6=18

(i) Devise an algorithm to find the smallest equivalence relation containing a given relation. Also verify your algorithm through an example.

(ii) Let (A, R) be a poset, where $A = \{a, b, c, d, e, f, g, h\}$ and matrix of R as given below:

P.T.O.

$$M_R = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Then answer the following: (a) Draw the digraph as well as Hasse diagram of (A, R).
(b) Does (A, R) has a greatest and least element? If so, identify them.
(c) Whether (A, R) is complemented? justify.

(iii) Consider the poset (S, R) where S is the set of all 2×2 Boolean matrices and for elements $A = (a_{ij})$ and $B = (b_{ij})$ of S, A R B if and only if $a_{ij} \leq b_{ij}$, $1 \leq i,\ j \leq 2$. Let H be a subset of S consisting of matrices with exactly two ones. Then find the following:
(a) all upper bounds of H, (b) all lower bounds of H, (c) the least upper bound and greatest lower bound of H if exist.

(iv) Let (S_1, R_1) and (S_2, R_2) be posets and R be a relation on $S_1 \times S_2$ defined as (a_1, a_2) R (b_1, b_2) if and only if $a_1\ R_1\ b_1$ when $a_1 \neq b_1$ and $a_2\ R_2\ b_2$ when $a_1 = b_1$. Prove that ($S_1 \times S_2$,R) is a poset.

(v) Show that in a bounded distributive lattice, the elements which have complements form a sub-lattice.

(vi) Simplify the following for elements x, y, z in a complemented distributive lattice:
(a) $(x \wedge y) \vee (x \wedge y\,')$ (b) $y \wedge (x \vee (x\,' \wedge (y \vee y\,')))$ (c) $(x \wedge y \wedge z) \vee (y \wedge z)$.

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