Answers-Hints Tutorial Sheet - 11 SPRING 2017

MATHEMATICS-II (MA10002) *January 4, 2017*

1. (a) Hint: Find acceleration $\dfrac{d^2\vec{r}}{dt^2}$, which is constant.
   (b) Hint: Write non-constant vector $\vec{u}$ as $|\vec{u}|^2 = \vec{u}\cdot\vec{u}$ and apply vector differentiation.

2. Answers:
   (a) $\lim\limits_{t\to 1}\vec{r}(t) = \hat{i}+\hat{j}+\hat{k}$.
   (b) $\lim\limits_{t\to 2}\vec{r}(t) = \dfrac{1-e^4}{8}\hat{i}+\hat{j}+16\hat{k}$.
   (c) $\lim\limits_{t\to 1}\vec{r}(t) = \hat{i}+3\hat{j}+e^2\hat{k}$.

3. Answer: $\vec{r}(t) = (1-t)(x_1,y_1,z_1)+t(x_2,y_2,z_2)$, where $0\le t\le 1$.

4. Answers:
   (a) Gradient $= 4\hat{i}+\hat{j}-\hat{k}$ and unit normal vector $= \dfrac{4\hat{i}+\hat{j}-\hat{k}}{3\sqrt{2}}$.
   (b) Gradient $= 2\sqrt{10}\,\hat{i}$ and unit normal vector $= \hat{i}$.
   (c) Gradient $= -2\hat{i}+4\hat{j}+4\hat{k}$ and unit normal vector $= \dfrac{-\hat{i}+2\hat{j}+2\hat{k}}{3}$.

   Hints: For above (a), (b) and (c) use the definition of gradient and unit normal vector.

5. Answers:
   (a) $-\dfrac{1}{\sqrt{5}}$
   (b) $\dfrac{e}{\sqrt{3}}$
   (c) $-\dfrac{1}{16\sqrt{6}}$

   Hints: For above (a), (b) and (c) use the definition of directional derivative.

6. Answer: $-\frac{1}{2}$
   Hint: Compute $\hat{n} = \cos 30^\circ\hat{i}+\sin 30^\circ\hat{j}$ and use the definition of directional derivative.

7. (a) Answer: $4\hat{i}-6\hat{j}+2\hat{k}$ and Maximum magnitude $= 2\sqrt{14}$.
   (b) Answer: $a=6$, $b=24$ and $c=-8$.
   Hints: For above (a) and (b) use the definition of directional derivative.

8. Hint: Use the definition of ∇ operator.

9. Answers:

   (a) $3xy^2z\hat{i} - (y^3z - 4xz\hat{j})$.

   (b) $-x^3y^3z\hat{i} + 2x^2y^4z\hat{j} + 2(x^2z^2 - y)x\hat{k}$.

   (c) $x^2(4yz^2 + 1)$.

   Hints: For above (a), (b) and (c) use the definition of ∇ operator and *curl*.

10. Hint: Use the definition of *curl*, *div* and ∇ operator.

11. Hint: Use the definition and properties of ∇ operator.

12. Answers:
    (a) Conservative vector field.
    (b) Conservative vector field and potential function $= x^2y + xz^3$

    Hints: For above (a) and (b) use the definition of conservative vector field.

13. (a) Answer: $a = 4$, $b = 2$ and $c = -1$, Hint: Use definition of irrotational vector.
    (b) For the constant $a = -2$, Hint: Use definition of solenoidal vector.

14. Answer: 0
    Hint: Use definition of *curl* and *div*.

15. Answer: $\dfrac{-3}{\sqrt{6}}(4 + e^{\pi})$
    Hint: Compute $\nabla\phi \cdot \vec{u}$.

16. Answers: Normal vector $= -\dfrac{1}{\sqrt{2}}\hat{i} - \dfrac{1}{\sqrt{2}}\hat{j} + \hat{k}$ and tangent plane $= x + y - \sqrt{2}z = 0$.
    Hint: $\nabla\phi$ is the normal and use definition to compute tangent plane.
