GD

INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date———FN/AN 2 Hrs. Full Marks: 30 No. of Students 58

Mid Autumn Semester 2016-2017 Deptt: MATHEMATICS Sub No: MA 31005

——Yr. B.Tech.(H)/B.Arch.(H)/M.Sc. Sub. Name: Real Analysis

Instruction: Answer all questions, which are of equal values

1. (a) Show that if $y > 0$, there is a natural number $n$ such that $n - 1 \leq y < n$.

   (b) Show that $Q$(set of rational numbers) is dense in $\mathbb{R}$.

2. (a) Let $\{E_n\}$, $n = 1, 2, 3, \cdots$ be a sequence of countable sets, and put $S = \bigcup_{n=1}^{\infty} E_n$. Show that $S$ is countable.

   (b) Let $X$ be an ordered set with the supremum property, and suppose $S$ and $T$ are nonempty subset of $X$ such that $T$ is bounded and $S \subset T$. Show that $S$ is bounded, and that

$$\inf T \leq \inf S \leq \sup S \leq \sup T.$$

3. (a) Define an algebraic number and show that the set of algebraic numbers is countable.

   (b) Show that if $E$ is an infinite subset of a compact set $K$, then $E$ has a limit point in $K$.

4. (a) Show that closed subsets of compact sets are compact.

   (b) Construct a bounded set of real numbers with exactly the following five limit points, 1, 2, 3, 4, 5.

5. (a) State and prove Heine-Borel theorem.

   (b) Give an example of an open covering of the open interval $(0, 3)$ which has no finite sub cover. Is the set $(0, 3)$ compact?

6. (a) Show that a subset $E$ of the real line $\mathbb{R}$ is connected iff it has the following properties: If $x \in E, y \in E$, and $x < z < y$, then $z \in E$.

   (b) Let $(X, d)$ be any metric space and let $M$ be a positive number, then show that $(X, d^*)$ is a metric space with the metric $d^*(x, y) = \frac{Md(x,y)}{1+d(x,y)}$, $\forall x, y \in X$.