Indian Institute of Technology, Kharagpur

Date..... FN/AN Time: 3 Hrs Full Marks: 50 No. of Students: 65
End (Autumn) Semester 2011-12, Deptt: MA/EC/AE/NA/HS/EX/CH Sub. No. MA 21007
Subject Name: Design and Analysis of Algorithms

Instruction: Answer **all** questions.

Question 1 [$2+2+2+2+2+2=12$ marks]

a) Which data structure has the fastest worst-case insertion time and why: Linked List (LL), Sorted Array (SA), Binary Search Tree (BST)?

b) Which data structure has the fastest worst-case search time and why: Linked List (LL), Sorted Array (SA), Binary Search Tree (BST)?

c) In BIG-O notation how many terms are in the sequence $1, 2, 4, \ldots, \frac{N}{4}, \frac{N}{2}, N$?

d) Write a pseudo-code for finding the k-th largest element in an array of n elements in linear time.

e) Illustrate your algorithm on the following sequence by finding the 3-rd largest element: $13, 14, 15, 16, 17, 12, 11, 10, 9$

f) Explain why the average computing time of your algorithm is linear.

Question 2 [2+2+5+2+1=12 marks]
Consider the following graph G:

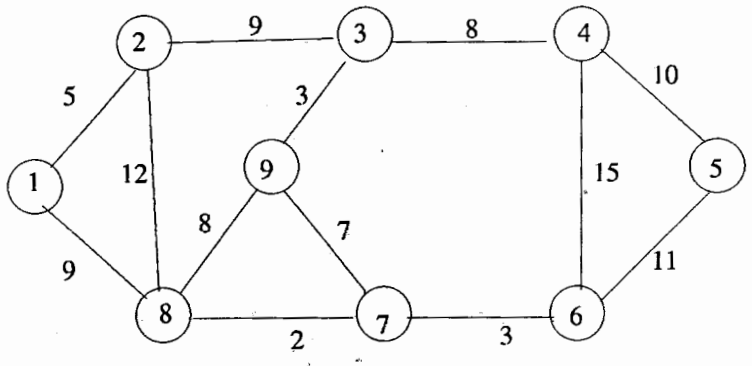


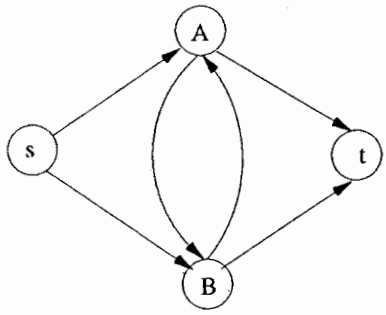
Figure 1:

a) Find a Minimal Spanning Tree in G using Prim's algorithm.

b) Find a Minimal Spanning Tree in G using Kruskal's algorithm.

c) Find the shortest path spanning tree from vertex 1 using Dijkstra's algorithm.

d) Write down weights for the edges of the following graph, so that Dijkstra's algorithm would not find the correct shortest path from s to t.



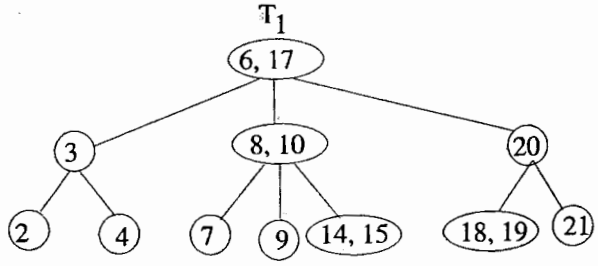
e) Which of the shortest path algorithm described in class would be most appropriate for finding paths in the graph of part (d) with weights you gave?

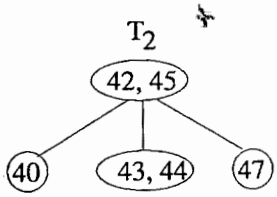
Question 3 [4+4=8 marks]

a) Write the pseudocode for Depth-First-Search (DFS) that finds the spanning tree/forest of an undirected graph and show how it works on the graph of Figure 1?

b) Give an $O(|V| + |E|)$ algorithm that tests whether an undirected graph $G = (V, E)$ is connected. The graph is given in adjacency list representation and has $|V|$ vertices and $|E|$ edges.

Question 4 [$2 \times 5 = 10$ marks]
Consider the following two 2-3 trees T_1 and T_2:





a) Draw the tree after inserting 16 in T_1.

b) Draw the tree after deleting 2 from T_1.

c) Join T_1 and T_2 with new data item 25 to form a single 2-3 tree.

d) Split T_1 in two new trees A and B where all items in A are < 15 and all items in B are > 15.

e) Draw a Red-Black tree equivalent to the 2-3 tree T_1.

Question 5 [4+4=8 marks]

a) Write an algorithm for inserting items in a Red-Black tree. What is the computing time of your algorithm?

b) Start with an empty Red-Black tree and insert the following keys in the given order using your algorithm: 80, 100, 140, 60, 84, 30, 40, 50, 54, 52, 120, 110

——The End——