Question 3 [6 + 3 = 9 marks]

a) For the following DFA:

(i) Give an equivalent minimal DFA. Don't forget to remove inaccessible states.

(ii) Give an equivalent regular expression.

δ	a	b
$\rightarrow 1$	1	3
$*2$	6	3
3	5	7
$*4$	6	1
5	1	7
$*6$	2	7
7	5	3

b) Find an algorithm to test a given DFA $\mathcal{M}$ whether the language it accepts is non-empty. Hence, or otherwise, obtain an algorithm to test, given DFS's $\mathcal{M}_1$ and $\mathcal{M}_2$, whether $L(\mathcal{M}_1) = L(\mathcal{M}_2)$ or not.

Question 4 [2 + 2 + 2 + 2 = 8 marks]

a) State and prove the Pumping Lemma for regular languages.

b) Show that the following language is not regular.

$$\mathcal{L} = \{a^n b^m | 0 < n \leq m\}.$$

c) Write a short note on Myhill-Nerode relation and its applications.

d) Prove or disprove the following:

$$(r + s)^* = ((r)^*(s)^*)^*,$$

here r and s are regular expressions. Here $r = s$ means $L(r) = L(s)$, $L(r)$ means language denoted by r.

——The End——

Indian Institute of Technology, Kharagpur

Date..... FN/AN Time: 2 Hrs Full Marks: 30 No. of Students: 44
Mid (Spring) Semester 2014-15 Subject Name: Switching and Finite Automata
Sub. No. MA 61002/60036/30006 Deptt: MA/ME/EC/EE/IE/MI/CY/BT/MT

Instruction: Answer **all** questions. Notations used are as explained in the class.

Question 1 [2 + 3 = 5 marks]

a) Formally define a non-deterministic finite automation (NFA) and the language accepted by it.

b) Construct a deterministic finite automation (DFA) accepting the following language:

$$\{w \in \{0,1\}^* : w \text{ has neither 00 nor 11 as a substring } \}.$$

Question 2 [2 + 3 + 2 + 1 = 8 marks]

a) Define regular expressions over a given alphabet Σ.

b) Let $\mathcal{M} = \langle Q, \Sigma, \delta, q_1, F \rangle$ be a DFA accepting a regular language L. Suppose $Q = \{q_1, q_2, \ldots, q_n\}$. Define for $i, j > 0, k \geq 0$,

$$R^k_{i,j} = \{x \in \Sigma^* : \widehat{\delta}(q_i, x) = q_j \text{ and } \mathcal{M} \text{ passes through no state } q_l \text{ with } l > k \text{ as it reads } x\}.$$

(i) Express L in terms of the sets $R^k_{i,j}$.

(ii) Assuming that each $R^k_{i,j}$ is regular, suppose the regular expression $r^k_{i,j}$ represents $R^k_{i,j}$ for each i, j, k. Find a regular expression for L.

c) Let r and s be regular expressions. Consider the equation $X = rX + s$, where rX denotes the concatenation of r and X, and + denotes union. Under the assumption that the set denoted by r does not contain ϵ,

(i) find the solution for X and

(ii) prove that it is unique.

d) What is the solution if $L(r)$ contains ϵ in Q2(c)?

——P.T.O.——