Tutorial Sheet - 12

SPRING 2017

MATHEMATICS-II (MA10002) *January 2, 2017*

1. Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = x^2\hat{i} + y^3\hat{j}$ and C is the arc of the parabola $y = x^2$ in the plane from $(0,0)$ to $(1,1)$.

2. Let $\vec{F} = y\hat{i} + z\hat{j} + x\hat{k}$. Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where C is the
   (a) curve $x^2 + y^2 = 1, z = 0$,
   (b) triangle in the xy-plane with vertices $(0,0), (2,0)$ and $(2,1)$,
   (c) skew quadrilateral with vertices $(0,0,0), (1,0,0), (1,1,0)$ and $(1,1,1)$.

3. Let $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ then
   (a) show that $\vec{F}$ is a conservative force field,
   (b) find the scalar potential of $\vec{F}$,
   (c) using (b) find the work done in moving an object in this field from $(1,-2,1)$ to $(3,1,4)$.

4. Verify that $\vec{F} = \dfrac{-y\hat{i} + \hat{j}}{x^2 + y^2}, (x,y) \neq (0,0)$ is not a conservative force field and hence find the work done in moving an object in this field from $(1,-2)$ to $(3,1)$.

5. If $\vec{F} = z\hat{i} - x\hat{j} + 3y^2z\hat{k}$, evaluate $\iint_S \vec{F} \cdot \hat{n}\, dS$ where S is the surface of the cylinder $x^2+y^2 = 16$, included in the first octant between $z = 0$ to $z = 5$.

6. If $\vec{F} = y\hat{i} + (x - 2xz)\hat{j} - xy\hat{k}$, evaluate $\iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, dS$, where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy-plane.

7. If $\vec{F} = 4xz\hat{i} + y^2\hat{j} + yz\hat{k}$, evaluate $\iint_S \vec{F} \cdot \hat{n}\, dS$, where S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$.

8. Verify *Green's theorem* in the plane for $\oint_C (xy + y^2)\, dx + x^2\, dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

9. Use *Green's theorem* in a plane to show that $\oint_C (\cos x \sin y - xy)\, dx + \sin x \cos y\, dy = 0$, where C is the circle $x^2 + y^2 = 9$ described in the positive sense.

10. Verify *Gauss's divergence theorem* for the vector function $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ taken over the surface S of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$.

11. Use *Gauss's divergence theorem* to evaluate $\iint_S (x^3\,dy\,dz + x^2y\,dz\,dx + x^2z\,dx\,dy)$, where S is the closed surface bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$.

12. For any closed surface S, bounding a region V, prove that $\oint_S (\nabla \times \vec{F}) \cdot \hat{n}\,dS = 0$, where $\vec{F}$ has continuous first derivative.

13. Verify *Stokes' theorem* for $\vec{F} = (2x - y)\hat{\imath} - yz^2\hat{\jmath} - y^2z\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

14. Use *Stokes' theorem* to evaluate $\oint_C (\cos x\,dx + 2y^2\,dy + z\,dz)$, where C is the curve $x^2 + y^2 = 1, z = 1$.

15. If $\vec{F} = (4xy - 3x^2z^2)\hat{\imath} - 2x\hat{\jmath} - 2x^3z\hat{k}$, then show that $\oint_C \vec{F} \cdot d\vec{r}$ is independent of the curve C joining two given points.

16. Prove that a necessary and sufficient condition that $\oint_C \vec{F} \cdot d\vec{r} = 0$ for every closed curve C is that $\nabla \times \vec{F} = 0$ identically.