Roll. No.:..

Name: ....

INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

MID SEMESTER EXAMINATION

Date: 21 -04-2017 AN | Time: 3 hours | Full Marks: 50

Spring Semester: 2016-2017 | Department : Mathematics

Subject No.:MA20104 | Subject Name: Probability & Statistics

Course: B.Tech./M.Sc. 2nd Year (AG, BT, CE, CS, HS, IM, MA, MI, MF, QE, QM, breadth, additional, backlog)

No. of students: 700

Instructions:

1. There are **12** questions in this question paper. Answer all questions. Marks are indicated at the end of each question.
2. Answer to every question should be given in the space provided after that question only on the question paper. Answers at any other place will not be evaluated. After the exam the question paper **MUST** be submitted along with the answer script.
3. Detailed working should be done in the answer script provided. Please write clearly your name and Roll. No. on the answer script also. The submission of question paper and the answer script is **MUST**. If a student does not submit both the question paper and answer script, it will not be evaluated.
4. Statistical tables may be used.

<u>Questions</u>

1. The life X (in hours) of an electronic tube manufactured by a certain process is normally distributed with parameter $\mu = 160$ and standard deviation σ. The value of σ, if the life of the tube is to have probability 0.80 of being between 120 and 200 hours, is __________ 2M
2. A parachute lands on the road joining towns A and B. Suppose the landing point follows a uniform distribution over the length AB. The probability that the ratio of distance of the landing point from A to the distance of the landing point from B is more than 3, is __________ 2M
3. A radioactive source is observed during four time intervals of 6 seconds each. The numbers of particles emitted during each time interval are counted. If the number of

particles emitted follow a Poisson distribution at a rate 0.5 particles per second, find the probability that

(a) in each of the 4 time intervals, 3 or more particles will be emitted ____________

(b) in at least one of the 4 time intervals, 3 or more particles will be emitted__________

(c) in an hour period, there are at most 1850 particles emitted (use normal approximation to Poisson) ________________________________ (1+1+2)

4. In 500 independent calculations a scientist has made 25 errors. If a second scientist cross-checks 7 of these calculations randomly, what is the probability of detecting 2 errors? (Assuming second scientist does not make any errors) ________ (2M)

5. The distribution of service time (in minutes) at a ticket counter is exponential with mean 4 minutes. When John reaches the counter there are two persons ahead (one getting the service and the other one waiting for his/her turn). What is the probability that John will have to wait for at least 6 minutes before his turn comes? ________ (3M)

6. The distribution of IQ of a randomly selected student from a certain college is $N(110,16)$. What is the probability that average IQ of 10 randomly selected students from this college is at least 112? _______________ (2M)

7. A physical quantity is measured 50 times and the average of these measurements is taken as a result. If each measurement has a random error uniformly distributed over $(-1,1)$, what is the probability that the result differs from the actual value by less than 0.1 ? ______________________ (3M)

8. Let the joint probability mass function of the discrete random variables X and Y be given by $p(x,y)=\frac{1}{25}(x^2+y^2)$ if $x=1,2$ and $y=0,1,2$. Then

(a) $E(X)=$ ____________________________

(b) $E(Y)=$ ____________________________

(c) $Var(X)=$ __________________________

(d) $Var(Y)=$ __________________________

(e) $Corr(X,Y)=$ ________________________ (1+1+1+1+2)

9. Let X be a continuous random variable with the density function

$$f(x)=\begin{cases}4x^3, & 0<x<1,\\ 0, & \text{otherwise.}\end{cases}$$

Let $Y=1-3X^2$.

(a) The range of Y is ______________

(b) The density of Y is ________________ (1+1)

10. A point (X,Y) is selected at random uniformly in the region $S=\{(x,y):|x|+|y|\leq 1\}$.

(a) Plot the region S in the two dimensional plane. (2M)

(b) Find the joint density of (X,Y) on S. (1M)

____

(c) Find $P(X^2+Y^2\leq 0.5)$. ________________ (1M)

(d) The marginal density of X for $-1\leq x\leq 1$ is

________________________________ (2M)

(e) $Var\,(X)=$ ________________ (1M)

(f) $P(|X+Y|<0.5)=$ ________________________ (2M)

(g) $P(Y<0.25\,|\,X=0.5)=$ ____________________ (2M)

(h) $Cov\,(X,Y)=$ __________________________ (2M)

11. In order to compare biological values of protein from cow's milk and buffalo's milk at a certain level two independent random samples of milk from **6** cows and **6** buffalos each were taken and the following values were recorded.

Cow's milk (X)	1.81	2.02	1.88	1.61	1.81	1.54
Buffalo's milk (Y)	2.00	1.83	1.86	2.03	2.19	1.88

(a) 95% confidence interval for mean protein content in cow's milk

____________________ (2M)

(b) 90% confidence interval for the variance of protein content in buffalo's milk

________________________ (2M)

(c) difference in the two sample means ____________ (1M)

(d) Pooled Sample Variance = ________________ (1M)

(e) Assuming the values to be normally distributed with equal and unknown variances, find 90% confidence interval for the difference in mean protein contents of buffalo and cow milk. ________________________ (2M)

12. Let $X_1, X_2, \ldots, X_{3n}$ be independent and identically distributed $N(\mu, \sigma^2)$ random variables. Let $Y_1 = \frac{1}{n}\sum_{i=1}^{n} X_i$, $Y_2 = \frac{1}{n}\sum_{i=n+1}^{2n} X_i$, $Y_3 = \frac{1}{n}\sum_{i=2n+1}^{3n} X_i$, $V = \sum_{i=2n+1}^{3n} (X_i - Y_3)^2$.

(a) The distribution of $Y_1 - Y_2$ is ____________________

(b) The distribution of $\frac{V}{\sigma^2}$ is ____________________

(c) The distribution of $\sqrt{\frac{n(n-1)}{2V}}\,(Y_1 - Y_2)$ is ____________________ (3M)