Estimat^n of mean of normal population in case of unknown variance.

Let $X_1 X_2 \ldots X_n$ be a r.s. from $N(\mu, \sigma^2)$

σ^2: unknown (both param are unknown)

1) Point estimator $\Longrightarrow \bar{X}$

2) Construct^n of C.I

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

where $S = \sqrt{S^2}$

$S^2 \to$ sample variance

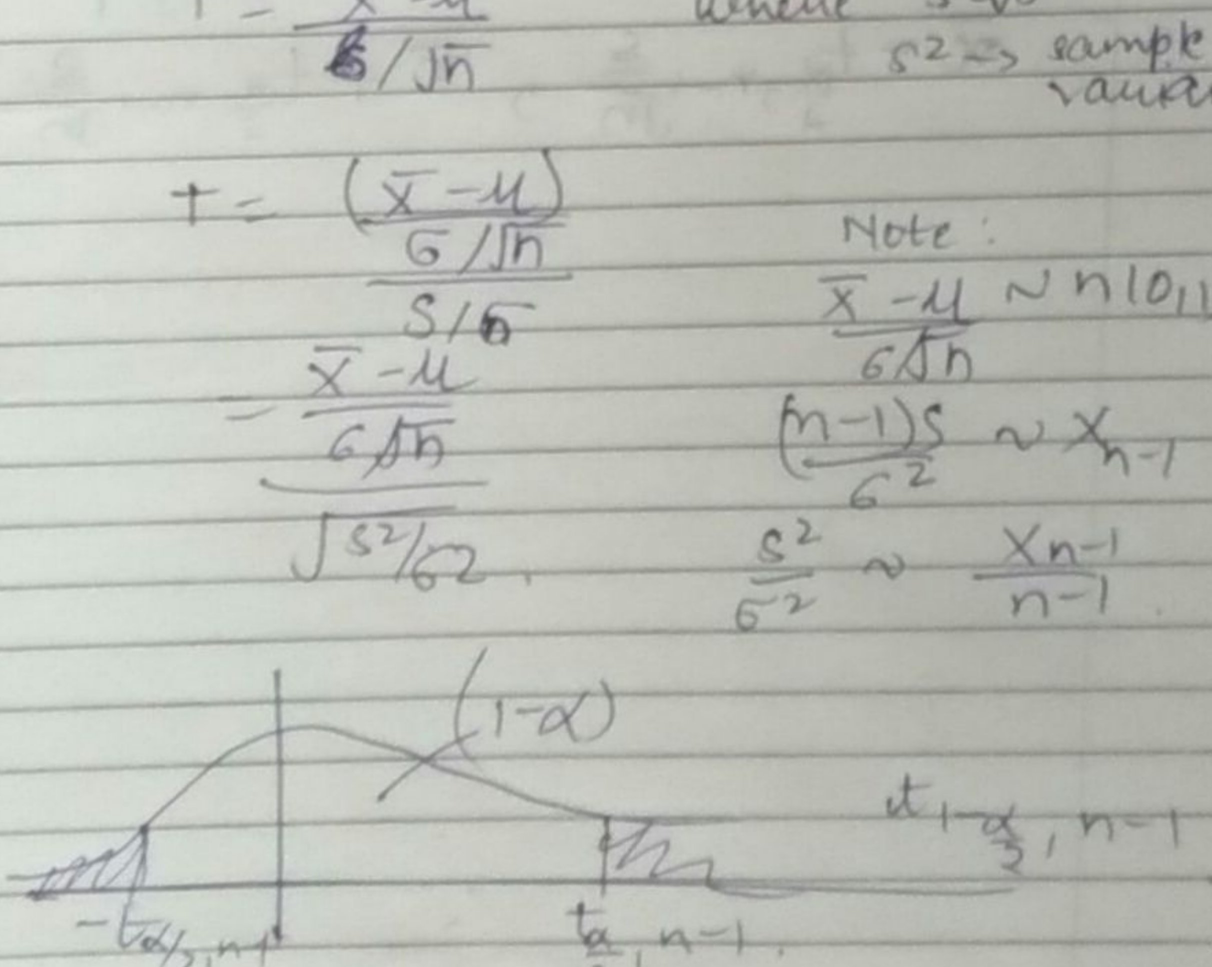
$$t = \frac{\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}}\right)}{S/\sigma} = \frac{\frac{\bar{X} - \mu}{\sigma/\sqrt{n}}}{\sqrt{S^2/\sigma^2}}$$

Note:

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

$$\frac{(n-1)S}{\sigma^2} \sim \chi_{n-1}$$

$$\frac{S^2}{\sigma^2} \sim \frac{\chi_{n-1}}{n-1}$$



= symmetry

$$P\left[-t_{\frac{\alpha}{2},n-1} \leq \frac{\bar{X}-\mu}{s/\sqrt{n}} \leq t_{\frac{\alpha}{2},n-1}\right] = 1-\alpha$$

$$\Rightarrow P\left[\bar{X} - t_{\frac{\alpha}{2},n-1}\frac{S}{\sqrt{n}} \leq \mu \leq \bar{X} + t_{\frac{\alpha}{2},n-1}\frac{S}{\sqrt{n}}\right] = 1-\alpha.$$

$\Rightarrow$ $100(1-\alpha)\%$ CI for μ in case of unknown σ in normal populatn.

$$\bar{x} - t_{\frac{\alpha}{2},n-1}\frac{s}{\sqrt{n}},\ \bar{x} + t_{\frac{\alpha}{2},n-1}\frac{S}{\sqrt{n}}$$

$$P\left[-t_{\frac{\alpha}{2}, n-1} \leq \frac{\bar{X}-\mu}{s/\sqrt{n}} \leq t_{\frac{\alpha}{2}, n-1}\right] = 1-\alpha$$

$$\Rightarrow P\left[\bar{X} - t_{\frac{\alpha}{2}, n-1}\frac{S}{\sqrt{n}} \leq \mu \leq \bar{X} + t_{\frac{\alpha}{2}, n-1}\frac{S}{\sqrt{n}}\right] = 1-\alpha$$

$\Rightarrow$ $100(1-\alpha)\%$ CI for μ in case of unknown σ in normal populatn.

$$\bar{x} - t_{\frac{\alpha}{2}, n-1}\frac{s}{\sqrt{n}}, \quad \bar{x} + t_{\frac{\alpha}{2}, n-1}\frac{S}{\sqrt{n}}$$