Bm

Indian Institute of Technology, Kharagpur

Date.................... FN/AN Time: 2 Hrs Full Marks: 30 No. of Students: 80
Mid (Spring) Semester 2016-17 Subject Name: Switching and Finite Automata
Sub. No. MA 61002/60036/30006 Deptt: MA/EE/IE/CH/BT

Instruction: Answer all questions. Notations used are as explained in the class.

Question 1 [3 marks]
Either prove each of the following statements or show a counter example.

a) Every non-regular language is infinite.

b) The intersection of any two non-regular languages is non-regular.

c) $a^n b^m$, where the alphabet is $\{a, b\}$ and $n \geq 0, m \geq 0$, is a regular language.

d) The following is an identity, where r, s are regular expressions, and $r = s$ means $L(r) = L(s)$

$$(r + s)^* = r^* + s^*$$

e) The following regular expressions represent the same language, where a, b are letters in an alphabet

$$(a^* b^*)^* = (a + b)^*$$

f) For a language L, if L^* is regular then L is regular, where $\Sigma = \{a, b\}$ is the alphabet.

Question 2 [2 + 1 = 3 marks]

a) Let $\mathcal{M} = \langle Q, \Sigma, \delta, q_1, F\rangle$ be a DFA accepting a regular language L. Suppose $Q = \{q_1, q_2, \ldots, q_n\}$. Define for $i, j > 0, k \geq 0$,

$$R^k_{i,j} \doteq \{x \in \Sigma^* : \widehat{\delta}(q_i, x) = q_j \text{ and } \mathcal{M} \text{ passes through no state } q_l \text{ with } l > k \text{ as it reads } x\}.$$

(i) Express L in terms of the sets $R^k_{i,j}$ with proper justification.

(ii) Assuming that each $R^k_{i,j}$ is regular, suppose the regular expression $r^k_{i,j}$ represents $R^k_{i,j}$ for each i, j, k. Find a regular expression for L.

b) Let r and s be regular expressions. Consider the equation $X = rX + s$, where rX denotes the concatenation of r and X, and $+$ denotes union.

(i) Under the assumption that the set denoted by r does not contain ϵ, find the solution for X.

(ii) What is the solution if $L(r)$ contains ϵ.

——-P.T.O.———

Question 3 [2 + 2 = 4 marks]

a) Differentiate ϵ-NFA and DFA with respect to transition and acceptance.

b) Draw a DFA accepting the following language L over $\Sigma = \{0, 1\}$.

$$L = \{w \in \Sigma^* | w \text{ has the property } P\}$$

where $P \equiv$ Every 00 is followed immediately by a 1.
(For instance, the strings 101, 0010, 0010011001 are in the language, but 0001, 00100 are not.)

Question 4 [$10 \times 2 = 20$ marks]

a) Give a decision algorithm to determine if the set accepted by a DFA is *cofinite* (a set whose complement is finite)

b) Define 2-DFA and the language accepted by it.

c) Devise an algorithm to construct a 2-DFA accepting the language

$$L = (\mathrm{a} + \mathrm{b})^* \mathrm{a} (\mathrm{a} + \mathrm{b})^{\mathrm{n}-1} \mathrm{a} (\mathrm{a} + \mathrm{b})^*$$

with $O(n)$ states.

d) Explain DFA state minimization algorithm.

e) Construct a DFA with reduced states equivalent to the regular expression

$$\mathbf{10 + (0 + 11)0^*1}.$$

f) State and prove the Pumping Lemma for regular languages.

g) Is the following language over $\Sigma = \{a, b, c\}$ regular? Justify your answer.

$$L = \{a^k b^m c^n | k = m \text{ or } m = n \text{ and } k + m + n \geq 2\}$$

h) Use the closure properties of regular languages to show that the following language is not regular.

$$L = \{a^3 b^n c^{n-3} | n > 3\}$$

i) Explain Myhill-Nerode Theorem.

j) Distinguish between Moore machine and Mealy machine.

——–The End——–