R.M

Indian Institute of Technology
Department of Mathematics
Spring Mid Semester Examination-2016
Subject Name: Discrete Mathematics
Subject No: MA20013
No. of students: 69 Time: 2 hrs F.M. 30

Instructions: Answer ALL questions. Numerals in righthand margin indicate marks. No query on this question paper will be entertained in the examination hall.

(1) Answer ALL parts. $2\times6=12$

(a) Let R be a relation on the set of all integers defined by $x\ R\ y$ if and only if $x \geq y^2$. Whether R is reflexive, irreflexive, symmetric, antisymmetric, asymmetric, and/or transitive?

(b) Let $A = \{a, b, c\}$ and R_1 and R_2 be relations on A defined by: $R_1 = \{(a,a),(a,c),(b,a),(b,b)\}$ and $R_2 = \{(a,b),(c,a),(b,c),(c,c)\}$. Find the matrix representation of $R_2 o R_1$.

(c) Let $A = \{1, 2, \ldots, 7\}$. Determine an equivalence relation R on A with $|R| = r$, $r = 8, 11$, or explain why no such relation exists.

(d) Whether the posets $(D_{40}, |)$ and $(D_{42}, |)$ are isomorphic? Justify your answer.

(e) Prove that every element in a totally ordered set has at the most one immediate predecessor.

(f) Applying the rules of Boolean algebra simplify the following expression:

$$(z' + x).((x.y) + z).(z' + y)$$

(2) Answer ALL parts. $3\times6=18$

(i) Let R be a relation on A. Prove that $(x, y) \in R^n$, $n \geq 1$, if and only if there is a walk of length n from x to y (in the digraph of R).

P.T.O.

(ii) Using Warshall's algorithm find the transitive closure of a relation R on $A = \{a, b, c, d, e\}$, where $R = \{(a, b), (a, c), (a, e), (b, a), (b, c), (c, a), (c, b), (d, a), (e, d)\}$.

(iii) Let (S_1, R_1) and (S_2, R_2) be posets. Define the lexicographical ordering R on $S_1 \times S_2$ and then prove that R is a partial ordering.

(iv) Consider the poset (A, R) where $A = \{1, 2, \ldots, 100\}$ and R is the divisibility relation. Then answer the following:
(a) How many maximal elements does (A, R) have ?
(b) How many minimal elements does (A, R) have ?
(c) Find all lower and upper bounds of $B = \{6, 12, 18, 24\}$. Also find the least upper bound and greatest lower bound of B, if exist.

(v) Consider the poset (A, R), where $A = \{a, b, c, d, e\}$ and $R = \{(a, a), (b, b), (c, c), (d, d), (e, e), (a, b), (a, c), (a, d), (b, c), (b, d), (c, d), (e, c), (e, d)\}$. Then
(a) Draw Hasse diagram of (A, R)
(b) Applying topological sorting find a totally ordered relation on A that contains R.

(vi) Let A be the set of all 2×2 Boolean matrices and R be a relation on A defined as $M \ R \ N$ if and only if $m_{ij} \leq n_{ij}$, $1 \leq i, \ j \leq 2$ (with the convention that $0 < 1$). Is (A, R) a lattice ? Justify.

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