Assignment - 3

1. Find the Fourier transform of the function

$$f(x) = \begin{cases} 1 + x/a, & -a < x < 0 \\ 1 - x/a, & 0 < x < a \\ 0, & \text{otherwise} \end{cases}$$

2. Find the Fourier transform of

$$f(x) = \begin{cases} 1 - x^2, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$

and hence, find

$$\int_0^\infty \frac{x \cos x - \sin x}{x^3} \cos \frac{x}{2} \, dx$$

3. Find the inverse Fourier transform of

$$F(\omega) = \frac{e^{-|\omega| a}}{\sqrt{2\pi}}$$

4. Find the Fourier cosine transform of $f(x) = \frac{1}{1+x^2}$, hence derive the sine transform of $g(x) = \frac{x}{1+x^2}$.

5. Find the Fourier sine transform of $e^{-|x|}$, hence evaluate

$$\int_0^\infty \frac{x \sin mx}{1+x^2} \, dx$$

6. Solve for $f(x)$ when

$$\int_0^\infty f(x)\cos sx\,dx = \begin{cases} 1-s, & 0 \le s \le 1 \\ 0, & s > 1. \end{cases}$$

and hence, find $\displaystyle\int_0^\infty \frac{\sin^2 t}{t^2}\,dt$.

7. Use Parseval's identity to find

a) $\displaystyle\int_0^\infty \frac{dx}{(1+x^2)^2}$; d) $\displaystyle\int_0^\infty \frac{\sin 3t}{t(t^2+9)}\,dt$

b) $\displaystyle\int_0^\infty \frac{x^2}{(1+x^2)^2}\,dx$; e) $\displaystyle\int_0^\infty \frac{\sin^2 4x}{x^2}\,dx$.

c) $\displaystyle\int_0^\infty \frac{dt}{(a^2+t^2)(b^2+t^2)}$;

8. Find the Fourier sine transform of

a) $f(x) = \dfrac{1}{x(x^2+a^2)}$

b) $f(x) = \begin{cases} \sin x, & 0 < x < a \\ 0, & x > a. \end{cases}$

9. Solve the integral equation

$$\int_0^\infty f(x)\sin tx\,dx = \begin{cases} 1, & 0 \le t \le 1 \\ 2, & 1 \le t \le 2 \\ 0, & t > 2. \end{cases}$$

10. If $F_s[f(x)]$

$= \dfrac{e^{-as}}{s}$

or

$= \dfrac{1}{s}$

Find $f(x)$.

11. Show that

a) $F_s[x f(x)] = -\frac{d}{dw}\left[F_c[f(x)]\right]$

b) $F_c[x f(x)] = \frac{d}{dw}\left[F_s(f(x))\right]$.

and hence, find

$F_c(x e^{-ax})$ & $F_s(x e^{-bx})$

12. find a) $F[e^{-x^2}\sin 3x]$

b) $F[e^{-3(x-2)^2}]$

c) $F[e^{-x^2/4}\cos 2x]$

13. Use Parseval's identity of F_s and F_c of

$$f(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & x > 1 \end{cases}$$

to find

a) $\int_0^\infty \left(\frac{1-\cos x}{x}\right)^2 dx$

b) $\int_0^\infty \frac{\sin^2 x}{x^2}\, dx$

14. Show that

a) $\int_0^\infty \frac{\cos sx}{a^2+s^2}\, ds = \frac{\pi}{2a} e^{-ax}$

b) $\int_0^\infty \frac{s \sin sx}{a^2+s^2}\, ds = \frac{\pi}{2} e^{-ax}$

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