MA20104 Probability and Statistics

Problem Set 2

1. Any point in the interval $[0, 1)$ can be represented by its decimal expansion $0.x_1x_2\ldots$. Suppose a point is chosen at random from the interval $[0, 1)$. Let X be the first digit in the decimal expansion representing the point. Compute the density of X.

2. Suppose a box has 6 red balls and 4 black balls. A random sample of size n is selected. Let X denote the number of red balls selected. Compute the density of X if the sampling is (a) without replacement, (b) with replacement.

3. Suppose X is a random variable having density f given by

x	-3	-1	0	1	2	3	5	8
$f(x)$	0.1	0.2	0.15	0.2	0.1	0.15	0.05	0.05

Compute the following probabilities:
(a) X is negative;
(b) X is even;
(c) X takes a value between 1 and 8 inclusive;
(d) $P(X = -3 \mid X \leq 0)$;
(e) $P(X \geq 3 \mid X > 0)$.

4. Suppose X has a geometric distribution with $p = 0.8$. Compute the probabilities of the following events:
(a) $X > 3$;
(b) $4 \leq X \leq 7$ or $X > 9$;
(c) $3 \leq X \leq 5$ or $7 \leq X \leq 10$.

5. Let X be a geometrically distributed random variable having parameter p. Let $Y = X$ if $X < M$ and let $Y = M$ if $X \geq M$; that is, $Y = \text{Min}(X, M)$. Compute the density of Y.

6. Suppose a box has r balls numbered $1, 2, ..., r$. A random sample of size n is selected without replacement. Let Y denote the largest of the numbers drawn and let Z denote the smallest.
(a) Compute the probability $P(Y \leq y)$.
(b) Compute the probability $P(Z \geq z)$.

7. Let X and Y be independent random variables having geometric densities with parameters p_1 and p_2 respectively. Find
(a) $P(X \geq Y)$;
(b) $P(X = Y)$.
(c) the density of $\text{Min}(X, Y)$;
(d) the density of $(X + Y)$.

8. Suppose $2r$ balls are distributed at random into r boxes. Let X_i denote the number of balls in box i.
(a) Find the joint density of $X_1, X_2, \cdots X_r$.
(b) Find the probability that each box contains exactly 2 balls.

9. Use the Poisson approximation to calculate the probability that at most 2 out of 50 given people will have invalid driver's licenses if normally 5% of the people do.

10. Use the Poisson approximation to calculate the probability that a box of 100 fuses has at most 2 defective fuses if 3% of the fuses made are defective.

As additional problems you may try the following problems from the fourth edition of the book "Probability and Statistics in Engineering" by the authors William Hines, Douglas Montgomery, David Goldsman, Connie M. Borror. The list of the problems is as follows:
Chapter 4: 4-1, 4-8, 4-21
Chapter 5: 5-2, 5-4, 5-5, 5-8, 5-12, 5-13, 5-14, 5-16, 5-25, 5-27, 5-30, 5-33, 5-34, 5-37, 5-40