INDIAN INSTITUTE OF TECHNOLOGY
Department of Mathematics
Time : 2 hrs. Full Marks : 30
No. of Students : 65 Mid-Autum, 2015
Subject: MA 31005 Real Analysis

Instructions :Answer ALL the questions. Provide answers to all parts of each question together, otherwise it will be ignored. 'No queries will be entertained during the examination".

1). Prove that the set Q of rational numbers is not a closed set. [3 Marks]

2). Find the derived set and closure of each of the following sets: (i) Q (ii) $R-Q$, (iii) N, (iv) R. [4 Marks]

3). Prove that the interval (a, b), for any $a, b \in R$ with $a < b$ is an open set [3 Marks]

4). Prove that a set A is a neighborhood of a point a if and only if there exists a positive integer n such that $\left(a - \frac{1}{n}, a + \frac{1}{n}\right) \subset S$. [3 Marks]

5). Give one example of each of the following: (i) An infinite set having no limit points (ii) A set with only $\sqrt{2}$ as a limit point (iii) An infinite set having only one limit point. (iv) A set every point of which is a limit point. (v) A set having infinite number of limit points. [5 Marks]

6). Show that for any $a \in R$, $a > 0$, there exists a natural number $n \in N$ such that $a > \frac{1}{n}$. [3 Marks]

7). Prove that the number $\sqrt{3}$ is not a rational number. [3 Marks]

8). If $u > 0$ is any real number and $x < y$, show that there exists a rational number r such that $x < ru < y$. [3 Marks]

9). If $y > 0$, show that there exists $n \in N$ such that $\frac{1}{3^n} < y$. [3 Marks]

**********GOOD LUCK**********