Tutorial Sheet - 10 (Hints and Answer) **SPRING 2017**

MATHEMATICS-II (MA10002) *January 10, 2017*

1. Answer: 9
   Hint: $D = \{0 \le x \le 3; 0 \le y \le -\frac{2x}{3} + 2; 0 \le z \le 6 - 2x - 3y\}$

2. Answer: $\frac{\log 2}{2} - \frac{5}{16}$
   Hint: $D = \{0 \le x \le 1; 0 \le y \le 1 - x; 0 \le z \le 1 - x - y\}$

3(i). Answer: $\frac{4\pi}{2m+3}$
Hint: Change into spherical coordinate and obtain the domain $D = \{0 \le r \le 1; 0 \le \theta \le \pi; 0 \le \phi \le 2\pi\}$

3(ii). Answer: $\frac{27\pi}{2}(2\sqrt{2} - 1)$
Hint: Change into spherical coordinate and obtain the domain $D = \{0 \le r \le \frac{3}{\cos\theta}; 0 \le \theta \le \frac{\pi}{4}; 0 \le \phi \le 2\pi\}$

4(i). Answer: $\frac{128\pi}{3}$
Hint: Change into cylindrical coordinate and obtain the domain $D = \{0 \le r \le 4; 0 \le \theta \le 2\pi; 0 \le z \le 4 - r\}$

4(ii). Answer: 0
Hint: Change into cylindrical coordinate and obtain the domain $D = \{1 \le r \le 2; 0 \le \theta \le 2\pi; 0 \le z \le r\cos\theta + 2\}$

5. Answer: 32.
   Hint: Integrate $\sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}$ over $x^2 + y^2 = 4$.

6. Answer: $\sqrt{3}\pi a^2$.
   Hint: Integrate $\sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}$ over $x^2 + y^2 = a^2$.

7. Answer: $\frac{\pi a^2}{6}(3\sqrt{3} - 1)$.
   Hint: Integrate $\sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}$ over $y^2 = ax$ and $x = a$.

8. Answer: $4\sqrt{2}\pi$.
   Hint: First find the equation of the section in $xy-$plane intersected by two given paraboloids. Then evaluate the integral $\iiint dx\,dy\,dz$ where z varies between the two paraboloids and x, y varies on the section in $xy-$plane.

9. Answer: 16π.
   Hint: Evaluate the integral $\iiint dx\,dy\,dz$ where z varies between the cylinder and the plane and x, y varies on $x^2 + y^2 = 4$.

10. Answer: π.
Hint: First find the equation of the section made by the paraboloid and $z = 0$. Then evaluate the integral $\iiint \, dx\, dy\, dz$ where z varies between the plane and the paraboloid and x, y varies on the section in $x^2 + y^2/4 = 1$.

11. Answer: $19\pi/6$.
Hint: First find the equation of the section made by the sphere and paraboloid and evaluate the integral $\iiint \, dx\, dy\, dz$ accordingly.

12. Ans: $\pi \log[\frac{1}{2}(\sqrt{\alpha} + \sqrt{\beta})]$.
Hint: Differentiate partially w.r.t $\alpha \rightarrow$ Integrate it w.r.t $x \rightarrow$ Integrate the expression w.r.t $\alpha \rightarrow$ Eliminate the arbitrary constant of integration.

13. Ans: $\tan^{-1} \frac{\beta}{\alpha}$
Hint: Differentiate partially wr.t $\beta \rightarrow$Integrate it w.r.t $x \rightarrow$ Integrate the expression w.r.t $\beta \rightarrow$Eliminate the arbitrary constant of integration$\longrightarrow$ Take limit as $\alpha \rightarrow 0$

14. Hint: Differentiate partially w.r.t $\alpha \longrightarrow$Repeated partial derivative w.r.t $\beta \longrightarrow$Integrate it w.r.t $x \longrightarrow\longrightarrow$Integrate the result w.r.t β treating α as constant $\longrightarrow$ Evaluate the arbitrary function $f(\alpha)$ dependent on $\alpha \longrightarrow$ Substitue the value of $f(\alpha)$ and integrate the resulting expression w.r.t α treating β as constant $\longrightarrow$Evaluate the expression for the arbitrary function $g(\beta)$ and substitute it back in the final expression.