Indian Institute of Technology, Kharagpur

Date...................... FN/AN   Time: 3 Hrs   Full Marks: 50   No. of Students: 10
Supplementary Exam 2013   Deptt: MA/EC/CS/IM/HS/BT/EX/CH
Sub. No. MA 21007   Subject Name: Design and Analysis of Algorithms

Instruction: Answer **all** questions.

1. **[5+2 marks]** (i) Find the optimal matrix ordering that produces the minimum number of multiplications to compute the matrix product $A \times B \times C \times D$ with the following dimensions:

Matrix	Dimension
A	100×1
B	1×100
C	100×1
D	1×100

   (ii) Also determine the minimum and maximum number of multiplications needed to compute the matrix product $A \times B \times C \times D$.

2. **[5+3 marks]** (i) Write Kruskal's algorithm for minimal spanning trees.
   (ii) Find a minimal spanning tree in the following graph G using Kruskal's algorithm.
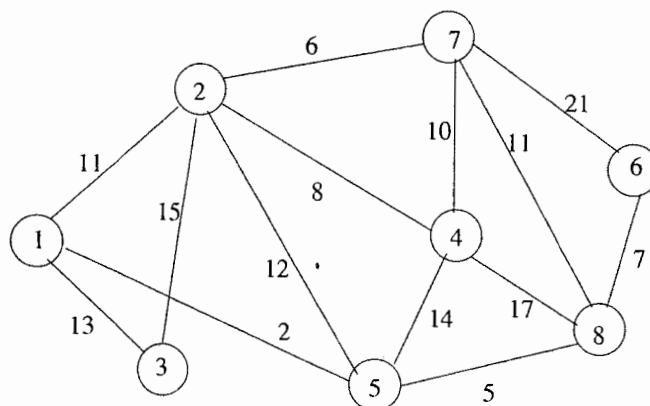
3. **[2+3 marks]** (i) Write down weights for the edges of the following graph, so that Dijkstra's algorithm would not find the correct shortest path from s to t.
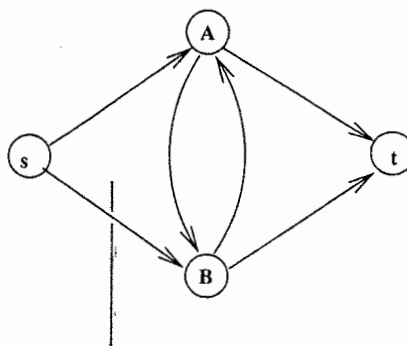
(ii) Which of the shortest path algorithm described in class would be most appropriate for finding paths in the graph of Q3(i) with weights you gave? Explain the working procedure of this algorithm.

4. **[5 marks]** Give an $O(|V| \, |E|)$-time algorithm for computing the transitive closure of a directed graph $G = (V, E)$.

5. **[4+4 marks]** Following is the adjacency matrix of a directed graph $G = (V, E)$. Check if there is a path from vertex a to vertex e using (a) BFS & (b) DFS.

Vertex	a	b	c	d	e	f	g
a	0	1	0	0	0	0	1
b	1	0	1	1	0	0	0
c	0	0	0	1	0	0	0
d	0	0	1	0	0	1	0
e	0	1	0	0	0	1	0
f	0	0	0	0	1	0	1
g	0	0	0	0	0	1	0

6. **[5+5 marks]** (i) Run the Floyd-Warshall algorithm on the following weighted, directed graph. Show the matrix $D^{(k)}$ that results for each iteration of the outer loop.
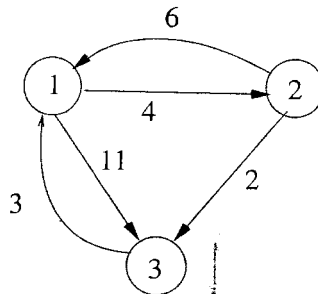
(ii) How can the output of the Floyd-Warshall algorithm be used to detect the presence of a negative-weight cycle?

7. **[5+2 marks]** (i) Write an algorithm for inserting items in a Red-Black tree. What is the computing time of your algorithm?

(ii) Draw a Red-Black tree equivalent to the following 2-3 tree.

------The End ------