**Symbolic Logic**
**Prof. Chhanda Chakraborti**
**Department of Humanities and Social Sciences**
**Indian Institute of Technology, Kharagpur**

**Lecture - 05**
**Concepts of Validity**
**Soundness**
**Consistency**

Hello, we are starting the module 5 of the NOC course  on Symbolic Logic. We have traveled through some basics of this logic a little bit, in a rather informal sort of way, and I have introduced you to kinds of arguments and that we are going to only focus on deductive arguments and we are going to looking into deductive logic.

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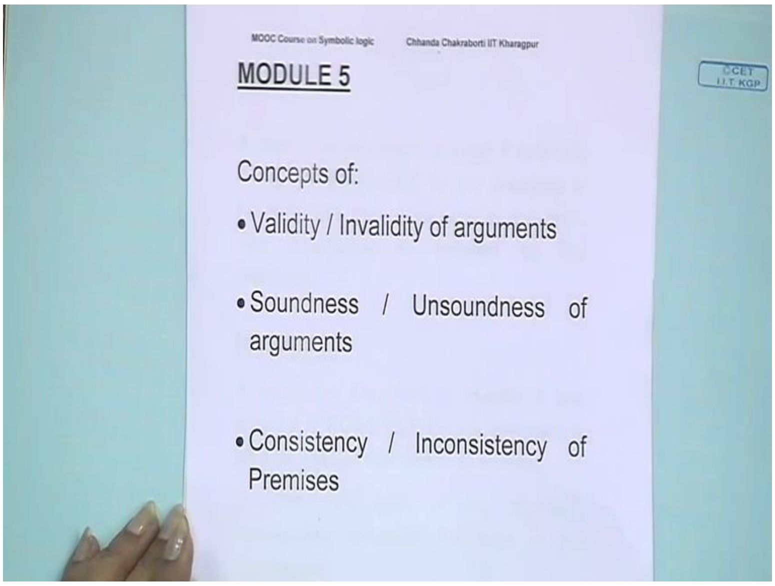


So, today we are going to talk about the concepts - the criterial concepts, the normative concepts, by which we are going to assess the value of the deductive arguments. So, this is going to be the concept of validity and invalidity, soundness, unsoundness, consistency, inconsistency of the premises and so on. So, this is going to be our plan for the module 5.

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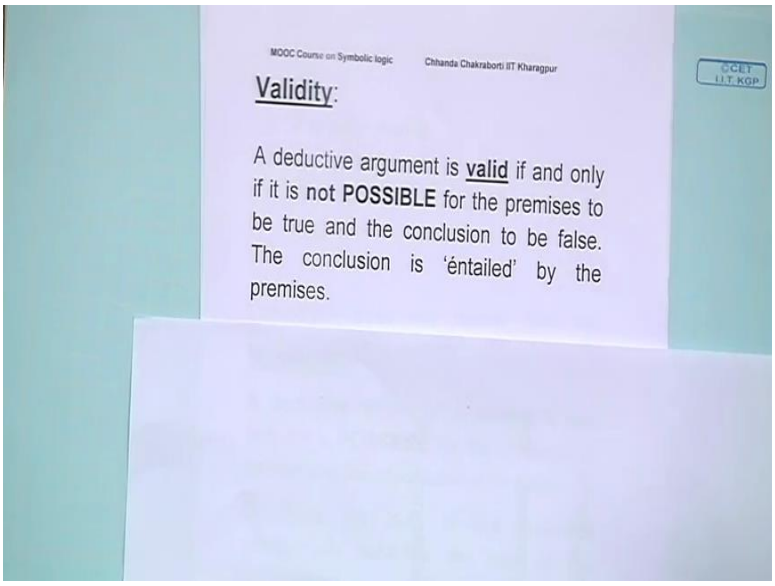


See, we will start with validity. And I am going to define what makes a deductive argument valid in terms of possibility and impossibility. So, let's take a look into the definition of validity.

The way we are going to define it and understand validity is like this: That a deductive argument is valid, *if and only if* it is not *possible* for the premises to be true and the conclusion to be false. I will repeat that. It should not be even possible, so there should not be even a smallest of the smallest possibility when the premises are true, the conclusion to be false. That is when we will call the deductive argument valid. So, elimination of even the smallest of possibility when premises are true, the possibility of conclusion to be false, is completely, completely eliminated. That's when we say deductive argument is valid. And this is when we say the conclusion is *entailed* by the premises. That is what validity is.

Please understand that validity is an *all or none* concept in deductive logic. So, either an argument is valid, or it is what is known as invalid. Invalid means which is not valid. So, if there is even the slightest chance, that when the premises are true, the conclusion can still be false, then the argument is invalid. Only when all such possibilities are eliminated, there is not even the miniscule scope of that happening, only then the argument is going to be called valid.

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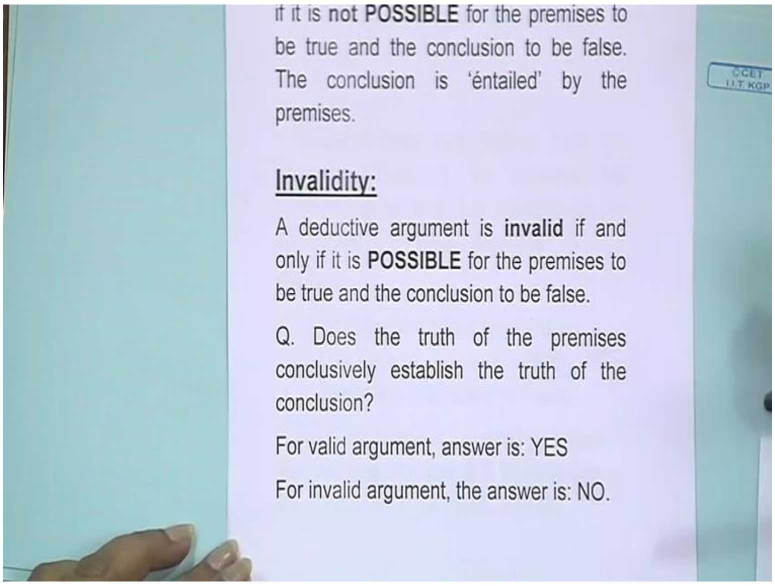


So, we will now going to invalidity, as I said that it is again defined in terms of possibility. So, a deductive argument is invalid if and only if it is *possible* for the premises to be true and the conclusion to be false. Now you will probably start to think what does possibility mean ? Let's think about situations. So, if there is *even one* situation, when you can find the premises to be true and the conclusion to be false, you know the deductive argument has to be invalid. We will give you examples in a second, but let us try to follow the concepts through.

So, if there is a question to settle in terms of validity and invalidity, then what we are asking is that when we know that the premises are true, when we know that the premises are true, or when we *assumed* that the premises are true, does that *conclusively* establish the truth of the conclusion? So, there is no doubt that the conclusion has to be true. Now let's put it in a question sort of form. So, does it happen? If the answer is yes, then you have a valid argument, or for valid argument the answer is going to be always yes. When you know the premises to be true, the conclusion *must* be true. And for invalid argument, the answer is going to be no; which means that even when you know that the premises are true, or when you assumed that the premises are true, there remains *a possibility that the conclusion may still not be true*.

How small the possibility will have be? Doesn't matter. As I said, even the smallest of the chance, if you think that there is a chance remaining, then automatically it falls into the category of invalid arguments.

So, let us just go over what we just said. We are trying to understand what makes a deductive argument *good*, what makes a deductive argument *bad*, and we said that we have introduced two concepts, namely; validity and invalidity. So, valid arguments are what we would call *good* deductive arguments; invalid arguments are what we would call *bad* deductive arguments. And I will just recap that: That validity is that kind of a property of an argument, please note that it is not a property of a statement or stand-alone proposition, it has to be an argument, it has to be a deductive argument, and then what we need to look into is that what happens when the premises are true. In case of valid argument, as we said, it is *guaranteed* that the conclusion is going to be true. But there is no such guarantee when you have an invalid argument.

I think it is time that we take examples and then may be the concepts will fall through. So, let's see what happens.

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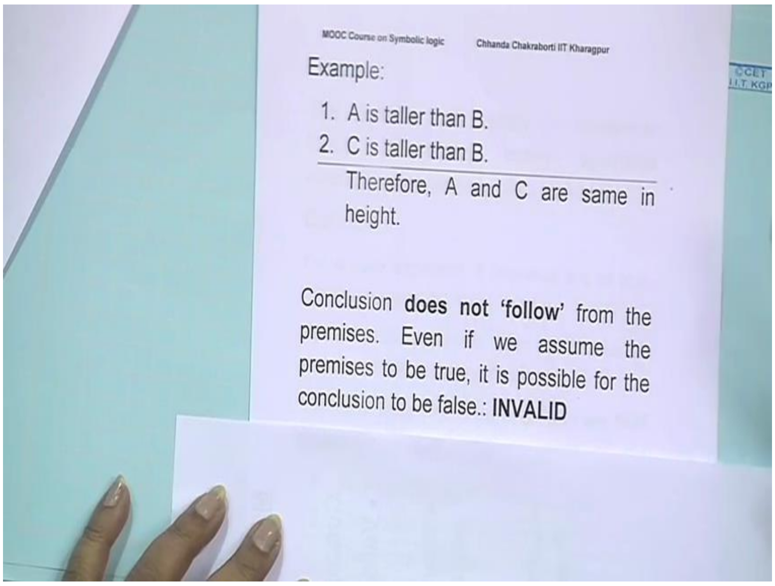


Let us take an example like this one. Suppose the premises are like this: A is taller than B, C is taller than B, and then somebody claims that therefore, A and C are of the same height. Alright? Now we do not know who this A B C are, whether they are persons, whether they are buildings, whether they are trees. Does not matter. Lets assume that it is true that A is indeed taller than B, and let's also assume that C is indeed taller than B. Now the question that you are going to ask here is that: Suppose given the truth of this, and given the truth of this, does that guarantee, does that guard (Refer Time: 06:47) the truth of the conclusion? What do you think? Even when you know that A is taller than B, C is taller than B, does it *follow* that therefore, A and C must be of the same height? The answer is: No. It does not follow. Because there is nothing in the premises that warrants the truth of this. There is no certification of truth for the conclusion.

It is just possible that A is taller than B, but A is also taller than C; or vice versa, C is taller than A. Doesn't matter. But there is no way we can say that it is *conclusively* true that A and C *have* to be of the same height. So, there is a possibility that the conclusion may be false, even when the premises are true; and that is when we say that it has to be an invalid argument. Now why? Because in the deductive sense, remember, what we said that in the deductive argument, it *derives* the conclusion from the truth of the premises. So, the conclusion's truth should follow from the truth of the premises. Now what is happening in here is that the conclusion *does not follow* from the premises. Even when you assume the premises to be true; still it is possible for the conclusion to be false. Hence, this argument is invalid.

So, it is a deductive argument that has gone bad, or that has gone wrong. And if you now notice that the invalidity is about the truth of the conclusion, and how it is conclusively or decisively shown by the truth of the premises, that is what makes this argument invalid. The failure of to do so.

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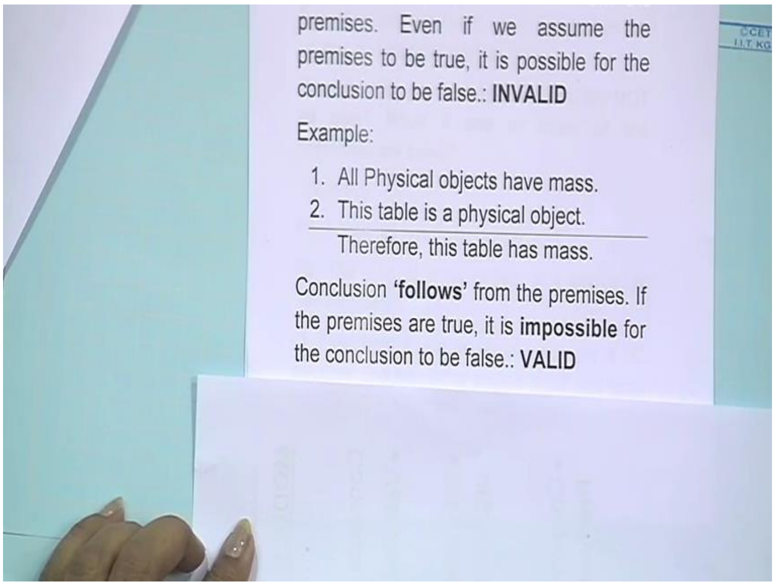


Let’s take another example. This one we have seen earlier. So here is your premises are: “All physical objects have mass; this table is a physical object; therefore, this table has mass”. So, once more we are going to say let’s see this is true, let’s see this is true, combined truth of that. Does this guarantee the truth of the conclusion? So, when this is true, and this is true, what happens to the conclusion? And the answer is: The conclusion follows from the truth of the premises. The truth of the premises are *enough* to guarantee that this has to be the case. If all physical objects have mass, and this happens to be a table, which is a physical object, then it *follows* that this table also has to have mass. And that is why would call this deductive argument *valid;* because it is *impossible* for the conclusion to be false, when the premises are true.

So, with example you can still approach what we have earlier said about validity and invalidity, that this possibility and impossibility will help you to understand what validity and invalidity are. Validity, of course, as I said, is a desirable characteristic in deductive argument; invalidity is not.

Having said that, let us proceed to a slightly subtler discussion. Remember what we have said, is that validity is a requirement that when the premises are true, what happens to the conclusion, and then if it is valid conclusion must be true. And if it is invalid, conclusion could be false - possibly false. This kind of definition of validity, if you have understood it properly, leaves some room open for spurious, very questionable, arguments to be also valid. That is a problem of this definition.

So, once more I will say this. We have a normative definition of validity. Something that we are going to apply to deductive arguments, and it is going to work, to separate good deductive arguments from bad ones. But what I am pointing out is that this definition, in a way, is workable, but it is too wide. It is somewhat loose. Why? Because it allows many spurious deductive arguments also to be, to come out as valid. Such as what? I will try to explain the problem a little bit more.

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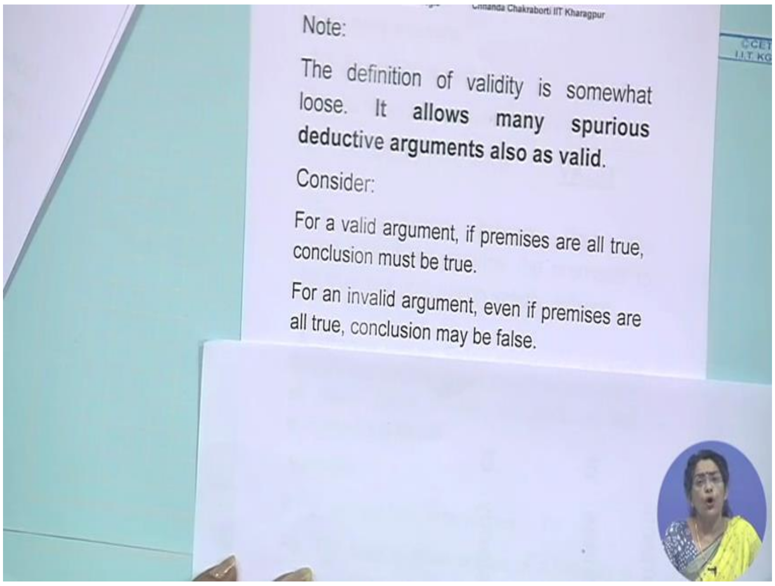


So, I will remind you that we just learnt that for a valid argument, our main concern is what happens when all the promises are true and the conclusion has to be true. For invalidity, again our concern is what if premises are all true, even then the conclusion can be false. So, each time we are looking into the situation when premises are all true. You have number of premises, and all of them are true, then what is happening to the conclusion. That is our concern.

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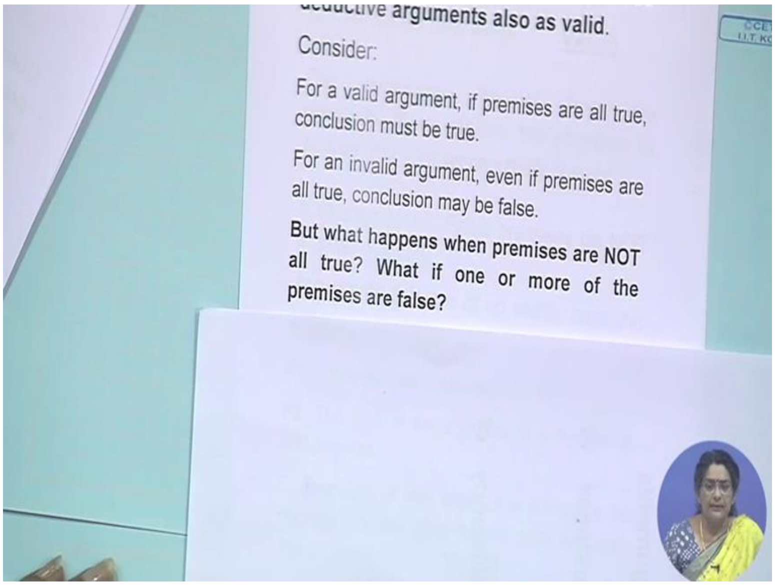


What we are not considering, and which is where the problem sort of shows up, is what happens if the premises are *not all true*. That is, suppose you have a false premise in the premise set, or suppose you have all the premises as false, then what ? Then does this definition work as efficiently? Let us see. And this is the problem where through which we will go into the discussion of soundness.

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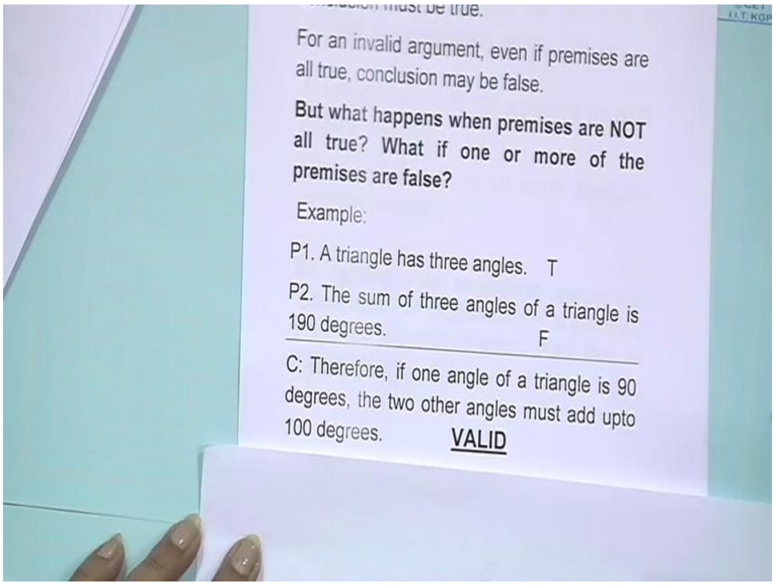


Let me show you an example. Suppose we tell you that here is a premise that: A triangle has three angles. True? And then suppose somebody said that the sum of three angles of a triangle is 190 degrees. OK? That's a premise somebody has given. Now we know that that is false. Now the combination of this is kind of problematic; you have one true premise, one false premise. And suppose given these two, somebody has concluded therefore, if one of the angle of triangle is 90 degrees, the other two angles must add up to 100 degrees. The question that you are asking, first of all, is it a deductive argument? The answer is yes. If we think that the premises are like this, does the conclusion follow? The answer is yes. And if this is true, if this is true, then this also comes out to be true. So, in a way, though we know that this premise is false, but suppose we assumed this to be true, then the conclusion does indeed it follow. So, we have no way but to call it valid. And that is a surprise! Because you think that having a false premise should automatically eliminate this kind of argument to come out to be valid. But there is nothing in the definition of validity, which rules this kind of arguments out. That's what I was trying to point out. Let us take other examples to make this point clearer.

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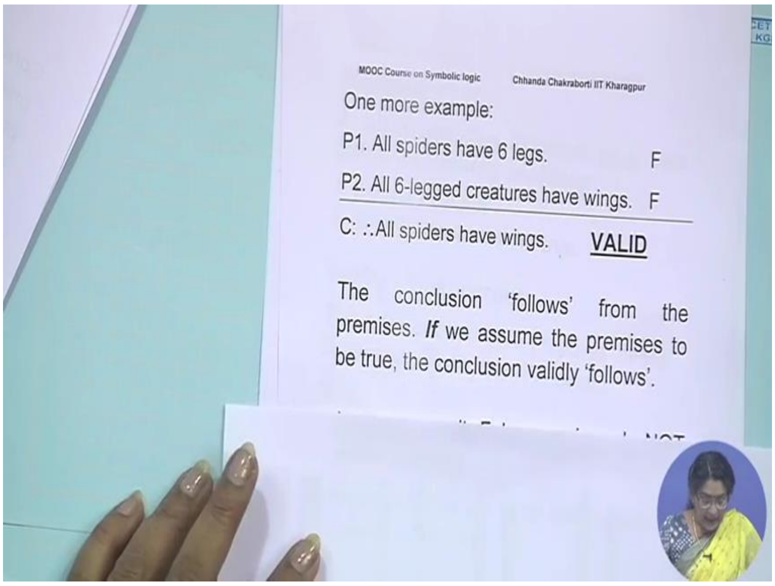


Let us make more radical examples. Suppose we have this argument in front – "All spiders have 6 legs" and that is patentlyfalse. Then somebody says, "All 6 legged creatures have wings": False. So, you have false premise, false premise. The entire premise set is false, and then from these two premises somebody concludes : Therefore, all spiders have wings. The question is we know that the premises are individually false. But given this, and given this, does the conclusion follow? Can we deduce the conclusion? The answer is yes. If this is the case, if this is the case, then it does follow that the conclusion is also entailed; and in that case the argument comes out to be *valid*.

You can't rule it as invalid; because what you do not have technically true premises and falsity of the conclusion. Rather, what you have is that even if the premises are false, the truth of the conclusion is entailed by the premises. Strange, but true. So, this is where you see sort of a shortcoming of the definition of validity. You are going to have valid arguments of this kind. You are going to say: What I will do with this argument? Because it is all useless; it's kind of trivial; it's kind of empty because it is all false, but it comes out through the definition as valid. So, this *following from the premises*, especially when you have false premise set, is a problematic sort of a situation. It creates a problematic situation for the definition of validity.

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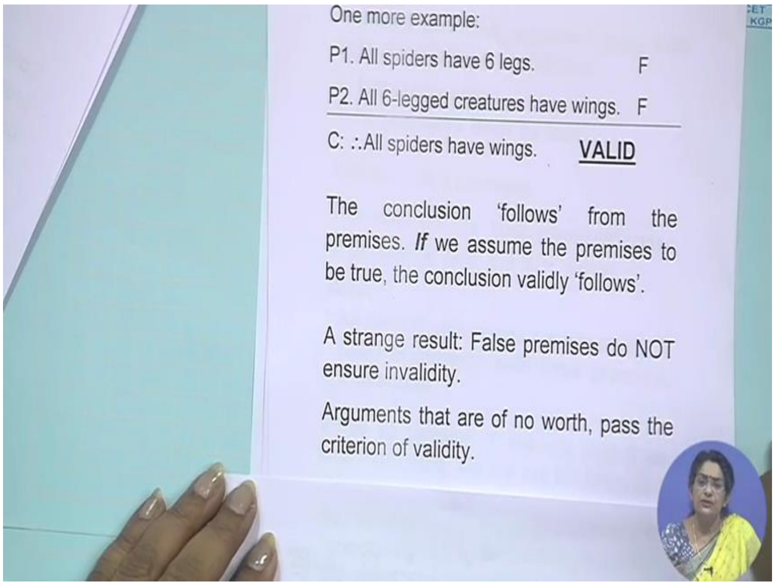


So, strange result is, as you saw, that just because you have a false premise, or just because you have all premises in your premise set, that does not ensure that you have an invalid argument at hand, which should have been the case, but it is not. In fact, as a result, just as you saw, like this argument for example, worthless arguments; arguments that have no value in fact, they are getting to be valid and passable. So, in a way, that tells us there is something wrong with the criterion of validity, the way we have defined it perhaps, we have defined it rather loosely. This was my point that I was trying to make:  That validity is a good concept, but it does not rule out everything that is supposed to rule out. Some of the bad arguments also pass through.

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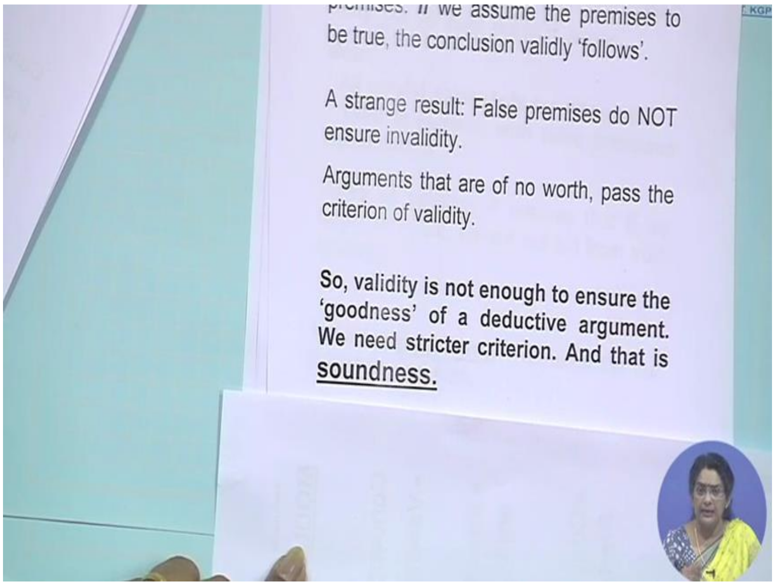


This is why, this is the reason why for deductive arguments, validity is not enough as a criterion. We need something stricter; we need some other notion to supplement the concept of validity, so that this kind of spurious, trivial, worthless arguments can be kept at bay , and that stricter notion is that of *soundness*.

So, our next topic is to look into what *soundnes*s is. We are still talking about deductive arguments. We are still learning what makes deductive arguments good, and I said validity is surely a handy notion, but it is somewhat loose. So, we need to make a stricter notion introducing soundness.

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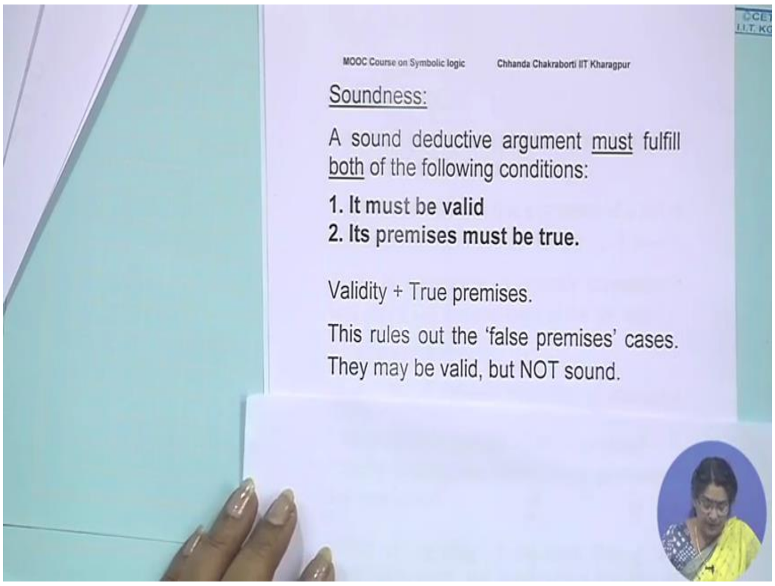


Alright, so, what is soundness? Soundness, for deductive arguments, requires that a deductive argument must fulfill both of these conditions, its a joint condition satisfaction. First of all, the argument has to be valid, in the way we have described it. So first it has to be valid. And then the premises must be true. Alright? So, once more: A sound deductive argument requires that two conditions are satisfied *simultaneously* , at the same time. First of all, it has to be valid, and then the premises must be true. So, the validity keeps out the totally invalid arguments out, and the second condition keeps out the questionable once where you have premises going false and yet the argument is coming out as valid.

So, soundness brings in the necessary stricture in the notion of what makes a deductive argument good. Validity is required, but on top of it if you can get true premises, then you have soundness. So, validity is the minimal requirement, and after that what we ask for is, is it also sound? Obviously, sound arguments are more desirable than barely valid arguments. That is what you need to remember.

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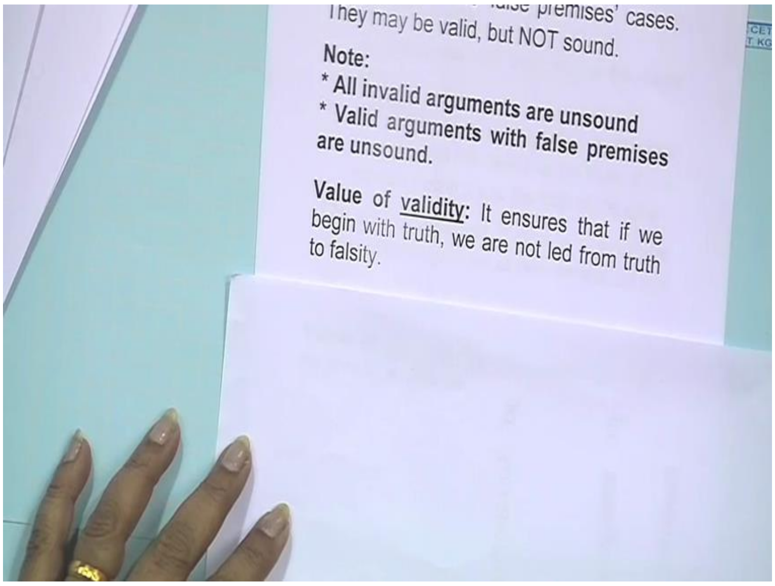


So, false premises situation is ruled out, which are valid, but not sound. Now if you have understood what soundness is, and what validity is, then I will make this kind of a claim also in front of you: That all invalid arguments are necessarily unsound, because it comes out as a requirement for soundness that first of all the argument has to be valid. So, all invalid arguments are necessarily unsound. And then you may have valid arguments with false premises which are valid but unsound. Alright? So, these kinds of arguments you may try constructing your own examples to see that this is how it works. So, just because your validity does not ensure that it is also sound. But once you have invalid argument, you know for sure that argument has to be unsound.

Why are we looking into validity and soundness? Well, here is my suggestion, or here is my sort of insight from my side to you. Why do we need validity? The answer is that it ensures that if we begin with truth, in case of a deductive argument, validity ensures that we are not misled into falsity. Remember? The definition of validity? That if the premise is true, it is never possible that you will land into a false conclusion, if the argument is valid. So, that kind of guarantee. It ensures that if we start with truth, we will not land into falsity.

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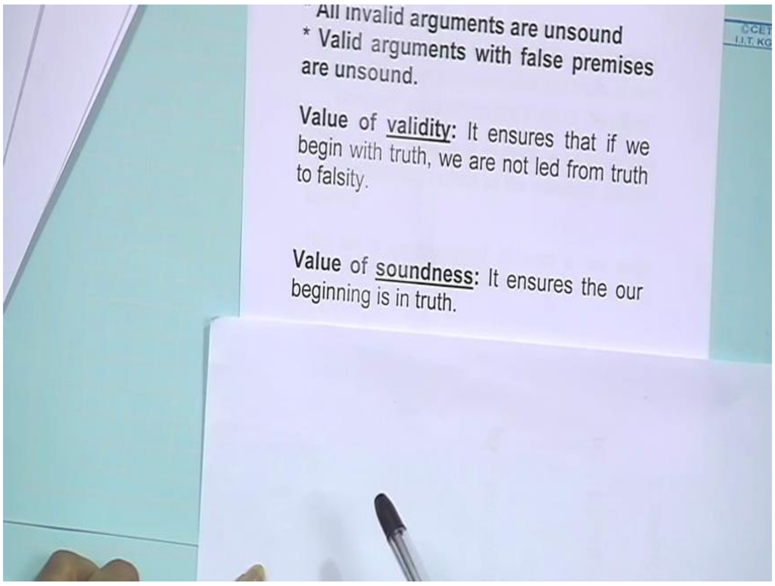


On top of that, why do we need soundness? The answer is that it just ensures that our beginning is indeed in truth. Because you just saw that validity can also happen with false premises. What soundness guarantees that our premises are all true. So, together, these two criteria bring in the required amount of stricture, the required amount of the formal correctness, in deductive argument. So, deductive validity and deductive soundness we have looked into.

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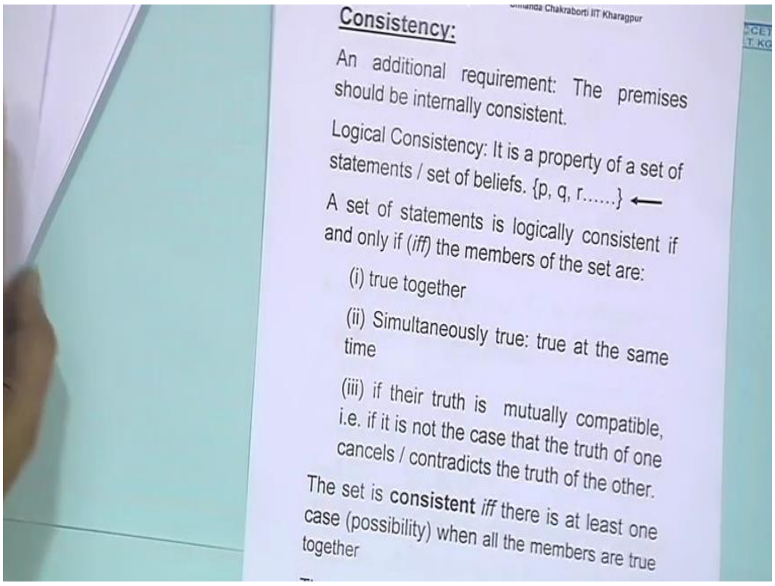

Then, we have just one more consideration for deductive arguments, and that is known as *consistency*. See, so far we have talked about the concepts of validity, invalidity, soundness, in terms of the arguments as a whole. Consistency, on the other hand, is about a set of statements. Consistency, logical consistency is a property of a set of statements, which you can apply in the case of the premises. So, what we are talking about, whether the premise set is internally consistent or not. Let's work the idea through.

This is an additional requirement that the premises should be internally consistent. Otherwise, you know, if you have inconsistent set of premises, as you know we often say that if somebody is inconsistent, there is no value to the…the credibility is undermined. We do not think that the person is saying anything of worth.

So, in a way consistency is a requirement for arguments, and for deductive arguments it is even more required. Why? Because soundness, remember, it only ensures that the premises are true. But you will soon see, consistency ensures that the truth of these premises are not incompatible. So, it's not like that when one premise is true, it cancels the truth of the other premise. So, let's see how we have defined this logical consistency.

First of all, note, as I said, logical consistency is a property of a set of statements, or set of beliefs. So, suppose you have p, q, r: These are propositions or statements, and we are looking into the entire set. There may be many other members and we are asking whether this set is consistent or not. When is it consistent? When is it not consistent? We will give you the definitions in just a moment.

So, a set of statements is going to be logically consistent *if and only if* the members of the set are, one, *true together*. So, individually true is one thing, but *true together*. You are looking into that they are coherently true, or there could be a time reference that at the same moment, the same time slice, they have all come out to be true. This is what we require. Or, the other way to understand is that their truth is mutually compatible. So, it is not the case that when one of them is true, as I was saying, that it cancels or contradicts the truth of some other member in the set. Ok? So, in a way, we call logical consistency a property of a set of statements when either they are true together, or they are simultaneously true, or when their truth is mutually compatible.

Now, in terms of possibility, remember we defined possibility , we defined validity, invalidity in terms of possibility. So, in a way consistency also we can define in terms of possibility like this – that, we call a set logically consistent if and only if (this *iff* stands for if and only if), there is *at least one case* or *at least one possibility*, when all the members are true together. So, if you can find even *one* situation when all the members are true together you can call the set as logically consistent.

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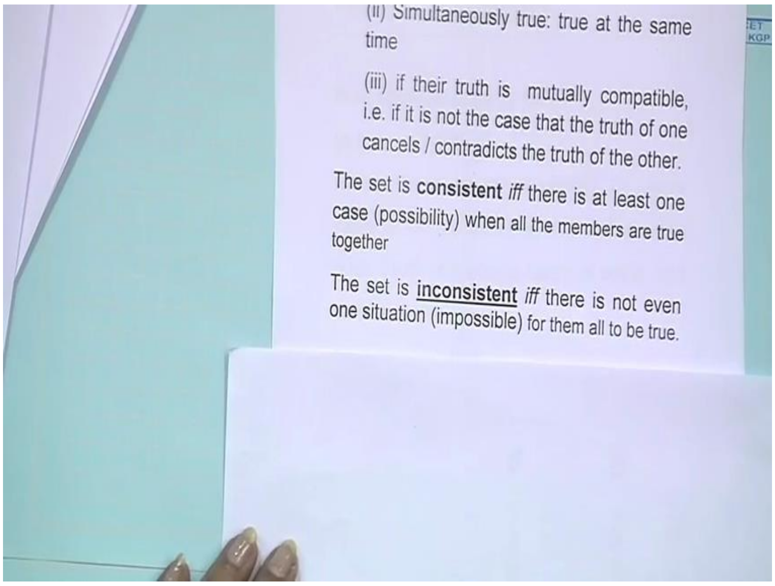


So, now you can imagine what would be inconsistency; namely, when there is *not even one* situation when you can find all of them to be true. So, it should be *impossible* for them to be all true together. Let's follow this concept a little bit, I will give you examples so that it stays, and with this we are going to end this also.  So let us follow this through.

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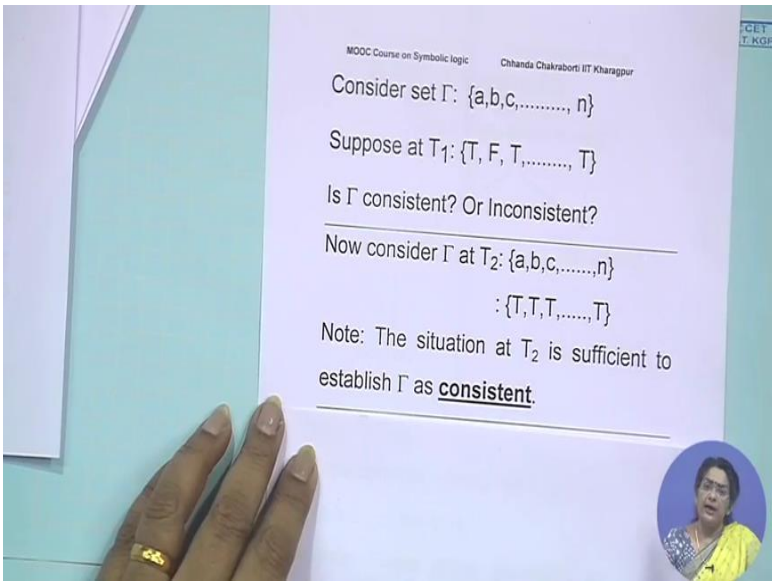


Consider, the set gamma which has these members: a, b, c, you know, up to the $n^{th}$ statements, so many members. Now suppose, we are looking at time slice . So, at T1, or moment T1, we find that the value of a is true, value of b is false, c is true and so on, so forth.

Now, you see this, and you are asking yourself, or I am asking you, is this set consistent or inconsistent? Now you will say that at T1, this seems like it is not consistent. Fine. Let me give you another example. So, let's consider the same set gamma at T2, where you have the same members, but truth-values have changed. At T2 now a is true, b is true, c is true, and the entire set is true; I mean every member is true. So, what happens? What you found is that at T1, gamma was not consistent, but at T2 gamma is consistent. Does that mean that the set is inconsistent? No.  That's what I am trying to explain. Lets see, so you are going to say that at T2 what we have found is sufficient to show that gamma has to be consistent.

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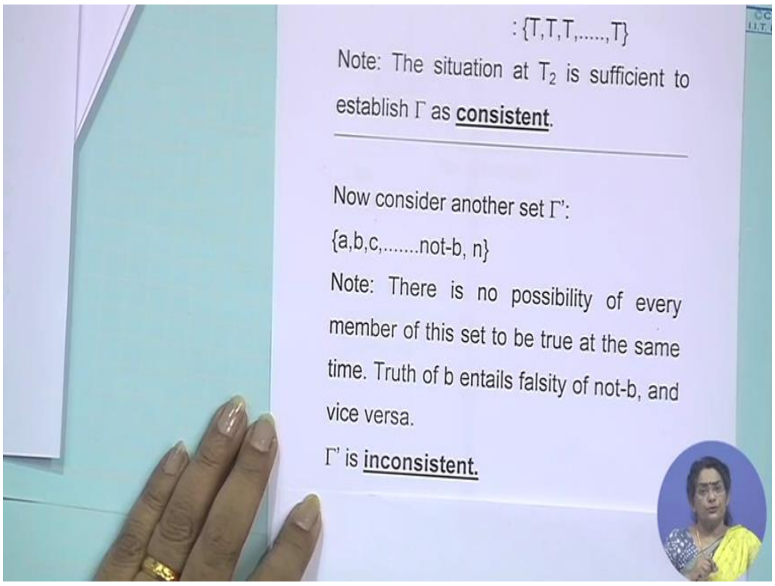


Now, here comes the crux: (Refer Time: 27:39) How to understand inconsistency. Let's consider another set 'gamma dash' where you have a, b, c and one of the member is not-b. Ask yourself, could there be even one moment when every member of the set is going to be true? The answer is no. So, what will happen is that you will never find even *one* moment when every member is going to be true at the same time. Why not? Because b's truth is going to cancel the truth of not-b, and vice versa. And this is what we call an inconsistent set. Alright? So, gamma dash is inconsistent, but gamma was not. Even though in you found in one case that there not all true together, but it does not make the set inconsistent.

So, we are going to finish this module by looking at this sort of a flow chart that for deductive arguments, you know, what we do is that we first check whether the argument is valid or not.

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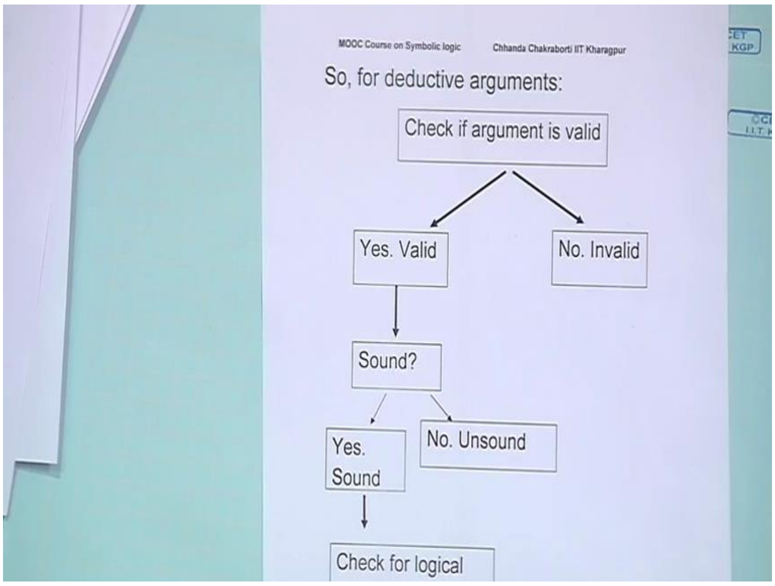


If not, then it is clear it is invalid and these are not desirable kinds. If yes, and it is valid ,then we look further to see whether it is sound. If it is not sound, it can still be valid and you are going to work with them, but these are not the best of the desirable kinds. So, if it is sound, then you have some of the better arguments and still you can see, determine ,whether they have premise set that is logically or internally consistent or not.

With this we are going to end this module, and please work with these concepts so that we can proceed further to the next module.

Thank you so much.