INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR
End Semester Exam-Spring, 2018
Department of Mathematics
Time : 3 hrs. Total Marks : 50,
Subject: MA 41002/MA 30002, Modern Algebra

Instruction: "No queries will be entertained during the examination".

Answer all the questions. Throughout all the rings are commutative ring with identity unless specified

(1) State whether the following statements are true or false with justification.

(a) If R is a PID then $R[x]$ is also a PID.

(b) $11x$ is irreducible polynomial in $\mathbb{Z}[x]$.

(c) In an integral domian R every irreducible element is prime element. False, only for UFD's

(d) $\mathbb{Z}_5[x]$ is a Euclidean domain.

(e) $1 + 3i$ is a Gaussian prime. False

[$2 \times 5 = 10$]

(2) Consider the group $G = (\mathbb{Q}/\mathbb{Z}, +)$.

(i) Prove that every element of G has finite order.

(ii) Show that every finite subgroup of G is cyclic. [4]

(3) Let G be a finite group and H, K are normal subgroups of G of order 3 and 5 respectively such that $G = HK$. Show that $G \cong G/H \times G/K$.

[4]

(4) Classify all groups of order 22 with justification. [4]

(5) $G = \mathbb{Z}_8 \times \mathbb{Z}_9 \times \mathbb{Z}_4 \times \mathbb{Z}_{25} \times \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_5 \times \mathbb{Z}_5 \times \mathbb{Z}_7$. Write down the corresponding invariant factor decomposition of G.

[4]

(6) Consider the ring $R = \mathbb{Z}_{256}$.

(i) Show that every element of R is either a unit or a nilpotent element.

(ii) Determine all the maximal ideals of R.

[4]

[PTO]

(7) Let R be the ring of all continuous functions from $\mathbb{R}$ to $\mathbb{R}$.

(i) Is R an integral domain? Justify your answer.

(ii) Let $A = \{f \in R | f(0) \text{ is an even integer}\}$. Is A a subring or an ideal? Justify your answer.

[4]

(8) Is $(1+\sqrt{-5})$ is an irreducible element in $\mathbb{Z}[\sqrt{-5}]$? Is it a prime element? Justify your answer.

[4]

(9) Factorize the polynomial $f(x) = 2x^5+x^4+4x^3+2x^2+2x+1 \in \mathbb{Z}[x]$ into irreducible factors. [2]

(10) State Eisenstein's irreducibility Criterion. Using it justify the irreducibility of $f(x) = x^4+1 \in \mathbb{Z}[x]$.

[3]

(11) Let $\alpha = 27-23i$ and $\beta = 8+i$ be elements in $\mathbb{Z}[i]$. Using the division algorithm of $\mathbb{Z}[i]$ find the values of γ and r in $\mathbb{Z}[i]$ such that $\alpha = \beta\gamma + r$.

[3]

(12) Is $R = \dfrac{\mathbb{Z}[i]}{(3)}$ a field? Justify your answer. Find the number of elements in R.

[4]