Assignment-1 : Discrete Maths.
Submission deadline : 15.2.18.

1. Consider the following Sequence :

9, 8, 4, 3, 2, 7, 6, 5, 10, 1.

Find the nos. t_i as defined in the proof of Erdős-Szekeres Theorem; and use these t_i's to find a decreasing Subsequence of at least four terms.

2. A building inspector has 77 days to make his round. He wants to make at least one inspection a day, and has 132 inspections to make. Is there a period of consecutive days in which he makes exactly 21 inspection? Why?

3. There are 3 Slices of olive pizza, 5 Slices of plain pizza, 7 Slices of pepperoni pizza and 8 Slices of anchovy pizza remaining at a pizza party.

(a) How many slices need to be requested to assure that 3 of at least one kind of pizza are received?

(b) How many slices need to be requested to assure that 5 slices of anchovy are received?

4. The "second order" fibonacci sequence is defined by the rule:

$$U_0 = 0, \quad U_1 = 1, \quad U_{n+2} = U_{n+1} + U_n + F_n$$

where F_n is the n-th fibonacci number.

Express U_n in terms of F_n and F_{n+1}

(Hint: Use generating functions).

5. Show that $\sum_k \binom{n}{k} F_t^k F_{t-1}^{n-k} F_{m+k}$ is always a fibonacci number, where F_n is the n-th fibonacci number.

6. What is the generating fun. for the sequence $2, 5, 13, 15, \ldots$ $\left(= \{2^n + 3^n\}\right)$.

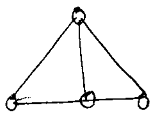
7. Find the generating fun. for $\left\{ \sum_{0 < k < n} \frac{1}{k(n-k)} \right\}$, differentiate it and express the co-efficients in terms of harmonic numbers. ($H_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ is the n-th Harmonic number).

8. The laplace transform of a fun. $f(x)$ is the fun.

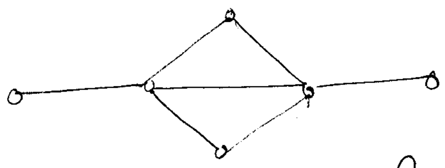
$$Lf(s) = \int_0^\infty e^{-st} f(t)\,dt.$$

Given that $a_0, a_1, a_2, \ldots$ is an infinite sequence having a convergent generating fun, let $f(x)$ be the step fun. $\sum_k a_k [0 \le k \le x]$. Express the Laplace transform of $f(x)$ in terms of the generating fun. G for this sequence.

9. If G is the graph of following figure, express $P(G, x)$ in terms of polynomials $P(I_k, x)$ for various k.



10. Find the chromatic polynomial for the following graph using reduction theorems.



11. A fruit fly is classified as either dominant, hybrid or recessive for eye color. Ten fruit flies are to be chosen for an experiment. In how many different ways can the genotypes (classifications) dominant, hybrid, and recessive be chosen if you are interested only in the number of dominants, number of hybrids, and number of recessives?

12. Suppose that worker a is suitable for jobs 3, 4, 5, worker b is suitable for jobs 2, 3, and worker c is suitable for jobs 1, 5. Also, each worker can be assigned to at most one job, no more than one worker per job, and a worker only gets a job to which he or she is suited. Set up a generating fun. and use it to answer the following questions.

(a) In how many ways can we assign one worker to a job?

(b) In how many ways can we assign two workers to jobs?

(c) In how many ways can we assign three workers to jobs?

13. Professor Jones wants to teach Calculus I or Linear algebra, Professor Smith wants to teach Linear Algebra or Combinatorics, and Professor Green wants to teach Calculus I or Combinatorics. Each professor can be assigned to teach at most one course, with no more than one professor per course, and a professor only gets a course that he or she wants to teach. Set up a generating fun. and use it to answer the following questions.

(a) In how many ways can we assign one professor to a course?

(b) In how many ways can we assign two professors to courses?

(c) In how many ways ~~KO~~ can we assign three professors to courses?

14. Find the number of codewords of length k from an alphabet {a, b, c, d, e} if b occurs an odd number of times.

15. In how many ways can 200 identical terminals be divided among four computer rooms so that each room will have 20 or 40 or 60 or 80 or 100 terminals? (Set up the appropriate generating fu^n, but do not calculate ~~B~~ the answer. Indicate what you are looking for, for example, the co-efficient of x^l)

16. In a computer system overhaul, a bank employee mistakenly deleted records of seven "pin numbers" belonging to seven accounts. After recreating the records, he assigned those pins to the accounts at random. In how many ways could he do this so that at least one pin gets properly assigned.

17. Find the number of derangements of $\{1,2,3,4,5,6,7,8\}$ in which the first four elements are mapped into:

(a) 1, 2, 3, 4 in some order.

(b) 5, 6, 7, 8 in some order.

18. A codeword from the alphabet $\{0,1,2\}$ is considered legitimate iff no two 0's appear consecutively. Find a recurrence for the number b_n of legitimate codewords of length n.

19. Use the method of characteristic roots to solve the following recurrences

(a) $a_n = -2a_{n-1} - a_{n-2}$, $a_0 = 2$, $a_1 = 2$

(b) $a_n = 9a_{n-2}$, $a_0 = 4$, $a_1 = 2$.

20. Use generating fu^{ns} to solve each of the recurrences in Q(19).

21. Suppose that $\{a_n\}$ satisfies $na_n = 2(a_{n-1} + a_{n-2})$, $n \geq 2$, and $a_0 = e$, $a_1 = 2e$. Let $A(x)$ be the ordinary generating fu^n for $\{a_n\}$. (a) Show that $A'(x) = 2(1+x)A(x)$

(b) Find $A(x)$.

22. If $C_{n+1} = 2nC_n + nC_n + 2$, $n \geq 0$, and $C_0 = 1$, find C_n.

23. Solve simultaneously the recurrences

$$a_{n+1} = a_n + b_n + c_n, \quad n \geq 1$$

$$b_{n+1} = 4^n - c_n, \quad n \geq 1$$

$$c_{n+1} = 4^n - b_n, \quad n \geq 1$$

Subject to the initial conditions $a_1 = b_1 = c_1 = 1$.

24. Let D_n be the no. of derangements of $1, 2, \ldots, n$.
Derive a formula for D_n as follows:

(a) Let $$C_n = \frac{D_n}{n!} - \frac{D_{n-1}}{(n-1)!}$$

Find a recurrence relation for C_{n+1} in terms of C_n.

(b) Solve the recurrence for C_n by iteration.

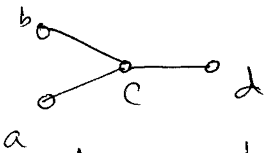
(c) Use the formula for C_n to solve for D_n.

25. Suppose that $A(x)$ is the ordinary generating fun. for the sequence $\{a_n\}$ and $B(x)$ is the ordinary generating fun. for the sequence $\{b_n\}$, and that

$$b_n = a_{n-r}b_0 + a_{n-r-1}b_1 + \cdots + a_0 b_{n-r}, \quad \text{for } n \geq k, \text{ where } k \geq r.$$

Find a relation involving $A(x)$ and $B(x)$.

26. Use the principle of inclusion and exclusion to find the chromatic polynomial of the following graph.



27. Use inclusion and exclusion to find the number of solutions to the eqn.

$$x_1 + x_2 + x_3 = 15$$

in which each x_i is a nonnegative integer and $x_i \leq 15$.

28. Write an expression for the probability that in a sequence of 7 random digits chosen from $0, 1, 2, \ldots, 9$, exactly 2 of the digits will not appear.

29. The names on the files of 10 different job candidates appearing for an interview were unfortunately lost, and a new receptionist placed the names on the files at random. In how many ways could this be done so that exactly 3 candidates' files were labeled properly.

30. Of 100 cars tested at an inspection station, 9 had defective headlights, 8 defective breakes, 7 defective horns, 2 defective windshield wipers, 4 defective headlights and breaks, 3 defective headlights and horns, 2 defective headlights and windshield wipers, 3 defective breaks and horns, none defective breaks and windshild wipers, 1 defective horn & windshield wipers, 1 defective headlights, breaks and horn, 1 defective headlight, horn and windshield wipers, none had any other combinations of defects.

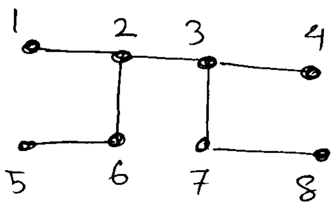
a) How many cars have at least one of the defects in question?

b) How many cars have at least 2 of the defects in question?

c) How many cars have exactly 2 of the defects in question?

31. Find the number of onto functions from a set with 5 elements to a set with 3 elements.

32. Write down the Prüfer code of the following tree:



33. Draw the tree whose Prüfer Code is (1, 7, 5, 7, 7, 1)

34. In checking the work of a proofreader, we look for 5 kinds of misprints in a textbook. In how many ways can we find 12 misprints.

35. In (34), suppose we do not distinguish the types of misprints but we do keep a record of the page on which misprint occurred. In how many different ways can we find 25 misprints in 75 pages.