Symbolic Logic

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Lecture – 07
Propositional Logic: Syntax

Hello. So, today, this is our module 7, and we are starting to learn the Syntax of propositional calculus. So, propositional logic or propositional calculus is going to have a specific Syntax.

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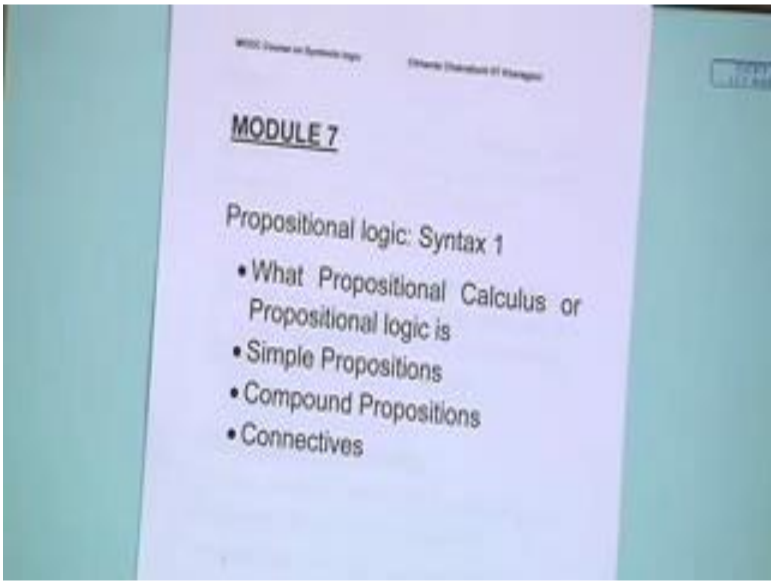

In this module, today's module, what we are going to learn is what this logic is about, what are the units that it works on, and then what are some of the important classifications of propositions that it deals with, and then the idea of the connectives.

So let us start slowly by first understanding what Syntax is. See, I told you, when I introduced what logic is formally, I said formal logic has to have three components: One is Syntax, the other one is semantics and then there has to be the proof theory.

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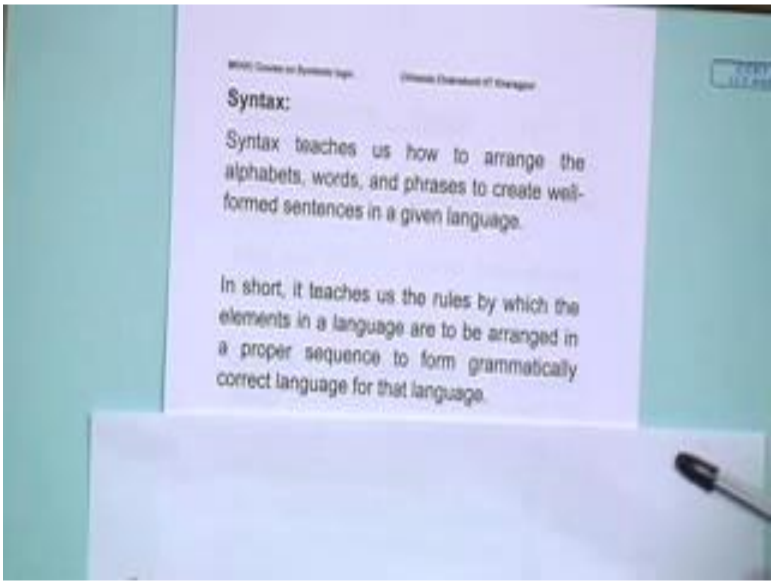

So this is Syntax. What does Syntax do? Syntax is like grammar. It teaches us certain rules. What it teaches us is: How certain alphabets, or how the elements in the language, take any language, how the alphabets, words or phrases in that language have to be arranged? There might be certain sequence how to use. You take example of English for example. We are told that whenever there is a q, it has to be followed by u. So this is a grammar rule. This is a *syntactical* rule, how to have it in the right order. So Syntax teaches us how to create well-formed sentences in a given language, how to arrange the elements of the language in such a way so that there are well-formed sentences. When we say well-formed, what we mean that there are badly formed sentences also. What keeps the badly-formed sentences out of the well-formed is the syntactical structure. Following the rules, and then the arrangement, and then comes out the grammatically correct language for that system.

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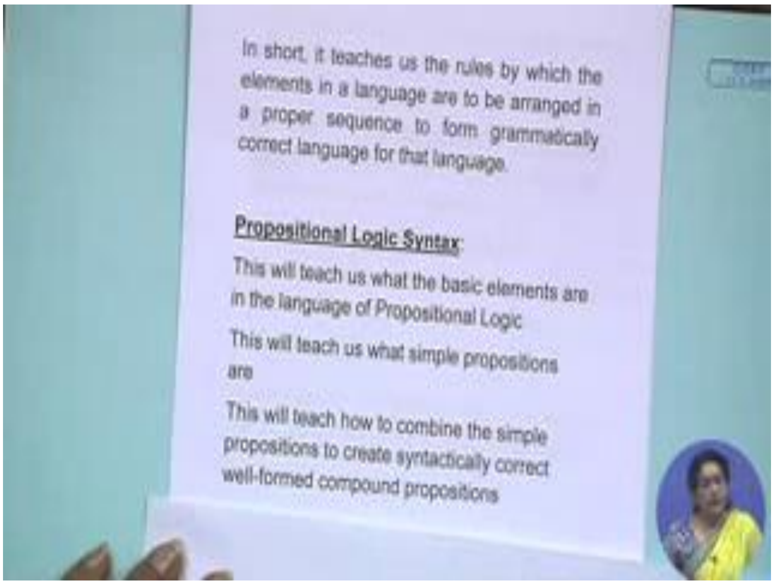

So with that introduction in Syntax, let’s go on to talk about what propositional logic syntax is all about. So, first of all we are going to be introduced in this syntax what are the basic elements in propositional logic. What are the units of operations for us? In that we will be introduced to the idea of *simple propositions*. Earlier I have informally introduced you to the idea of *claims*, and you will find that that idea; the concept of *claims*, is going to be closely, almost the same as simple propositions. And then it will also teach us rules, how to combine the simple propositions to generate compound or complex propositions. So there is going to be an idea of connective also. So this is what you are going to learn in our introductory lecture on syntax of propositional logic.

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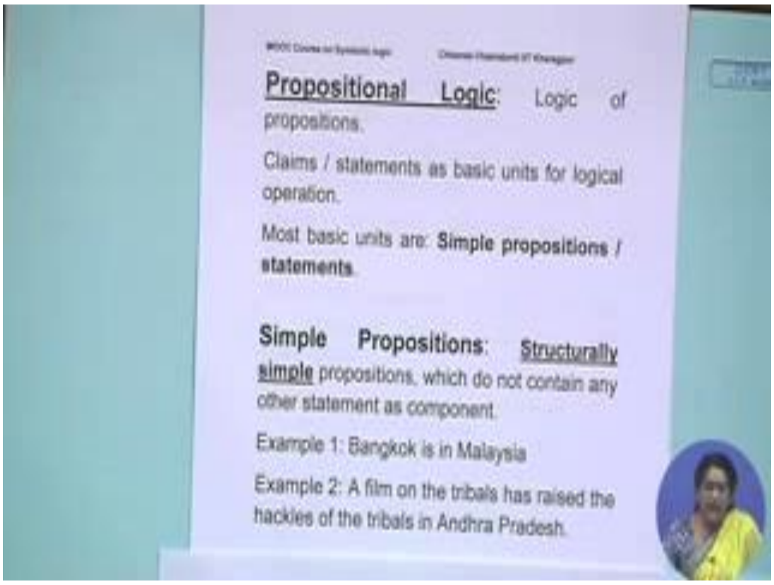

Propositional logic, or Propositional calculus as you have been told, is logic of propositions. I have already introduced to what propositions are. If you have forgotten that, or cannot recall offhand, let me just remind you. Propositions are actually, strictly speaking, are thoughts. When they are articulated, they are sentences, statements. We will not make that subtle distinction. We will synonymously almost use *propositions* with *statements*. So I will intermittently use the term proposition. Sometimes I am going to call them statement also.

So propositional calculus is about logic, doing logic with propositions. What are the basic units? And this is where you will get to see *propositions* are the basic units, or *statements* are, or the *claims* are. Ok? This is what we deal with here. The most fundamental, if you look at the syntactical structure in propositional logic, then at the very bottom layer, the fundamental layer you will find *simple propositions* or *simple statements*.

What are simple statements? The definition of simple statement is like this: That it is structurally simple. Structurally simple meaning what? That if you look at the proposition, you will not find any other statement as its component. So it is a complete thought, but it does not contain any other proposition or statement in it.

For example, say ‘Bangkok is in Malaysia’. Alright? Now this is a complete thought. But look, complete by itself, there is no other statement to be found in it. And this structural simplicity is what is to be understood as the intended simple propositions in propositional calculus.

You may now recall what we talked about claims. So claims are at the simple proposition level. Let’s take another example. A film on the tribals has raised the hackles of the tribal's in Andhra Pradesh. See, again structurally speaking, it’s one thought and there is no other statement as component of it. Conceptually, it may be complex, but structurally simple. There is only one thought; only one statement in this. And that is what it qualifies it as a simple proposition.

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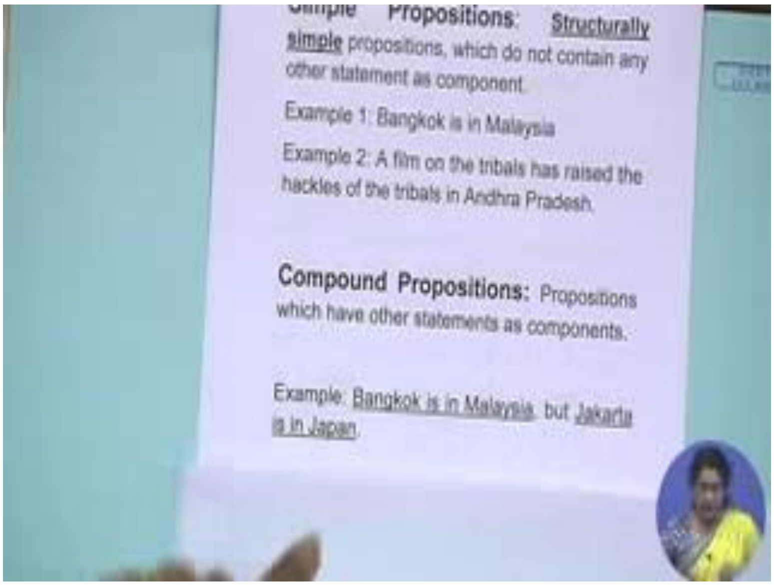

Probably, if you compare it with compound propositions, you will understand the simple proposition’s structural simplicity somewhat better. Compound propositions or complex propositions are those propositions which have other statements as components. So, if you look at this proposition, you will find the presence, the distinct presence of other propositions or statements in it.

For example, take a look: ‘Bangkok is in Malaysia, but Jakarta is in Japan’. Ok? Here is a statement, here is another statement. So, this whole statement contains *two* statements, which is what makes it compound, a compound proposition. So there are simple propositions here, simple proposition number two, and they are the components of this compound proposition. Ok? This is what I was saying. So, whenever you are looking into simple proposition, remember we are talking about structural simplicity. When you are looking into compound compositions, what we are looking at is structural compoundness, so that there are the presence of other components.

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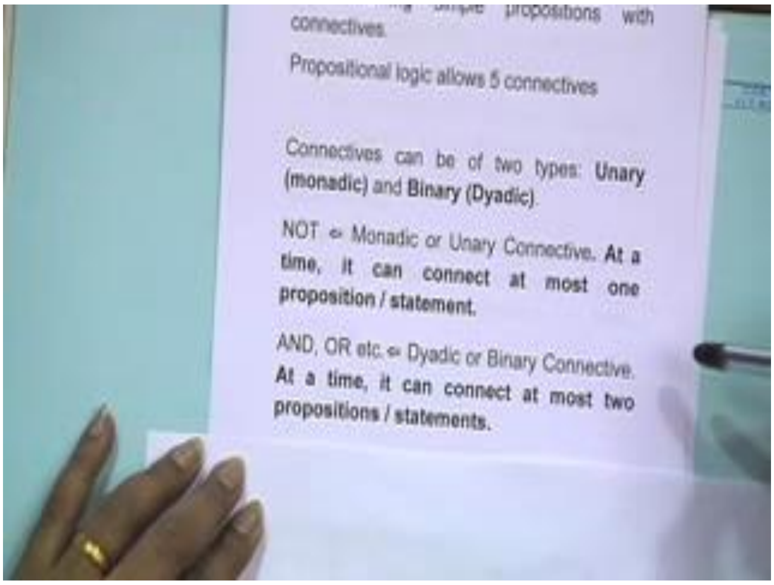

When we are learning the syntax in propositional logic, we cannot overlook the concept of *connectives.* We have learnt simple propositions, we have learnt compound propositions, let's now learn the idea of connectives. What are they? They are basically certain words. They are designated words. So, not any word can qualify as a connective, but certain designated words, whose function is to connect statements or propositions. Let's take example, say 'Bangkok is in Malaysia, *but* Jakarta is in Japan'. It is a compound proposition, but look! How are they connected? They are connected by the presence of this word called 'but'. So here, 'but' is working as a connective. This is, the entire statement is a compound proposition, but this is a connective, and we require them. In fact, you will see, they are ones which help us to combine simple propositions and to create compound propositions out of the simple propositions.

So, syntactical rules will show you how, and we will talk about connectives in details in our next module, but right now what we are learning is that the way in which, in this propositions calculus, you can create compound propositions is with help of connectives. You have to take simple propositions, use connectives and then only compound propositions can be formeccccd.

The propositional logic, we'll soon see, allows 5 connectives. What they are, and what their functions are, what do they mean, etcetera- all those we are going to discuss in the next module. But right now, let's talk about the broad variety or classification of these connectives.

So we understand that the connectives are words. But, not all the words that belong to this category are at par. There are different kinds. One type is called the *Unary* or *Monadic*. The other kind is called *Binary* or *Dyadic*. So there are two kinds of connectives, the Unary and the Binary, or if you want to use these terms, Monadic and Dyadic.

The Unary connectives, as the name suggests, at a time it connect *at most one* proposition or statement. So, remember the job of the connective is to connect. With Unary connective, at most you can pick up only one proposition or statement. So one example of that would be 'not'. You have the proposition and the 'not' sort of attaches itself to it, and you get a compound called 'not' said this. So this is one kind of understanding of connective, which connects to *only one* proposition at a time; as opposed to dyadic or binary connective, which connect at most two proposition or statements at a time. So this requires *two*. This kind of connectives can connect only one. So this is the broad classification that we need to remember as we go along and we will see their functions when we go into details, but this is the classification that you need to remember. Remember that both of these are generating compound statements. Even when you are attaching the Unary connective, you are actually generating compound statements and that is, that is something important to remember. This is also generating compound statements, and these are the only ways in which out of simple propositions you can get compound propositions.

Connectives we are going to revisit ,as I said, in our next module, but in the mean time we are going to pick up one other rather important concepts, syntactical concepts. One of them is called truth-value. All of you have been exposed to numerical values. You know what numbers are, but we are talking about truth-values. Why? Because, as you know, propositions that we are dealing with in propositional logic, propositions are the truth-value bearers. They are the actual truth-values bearers.

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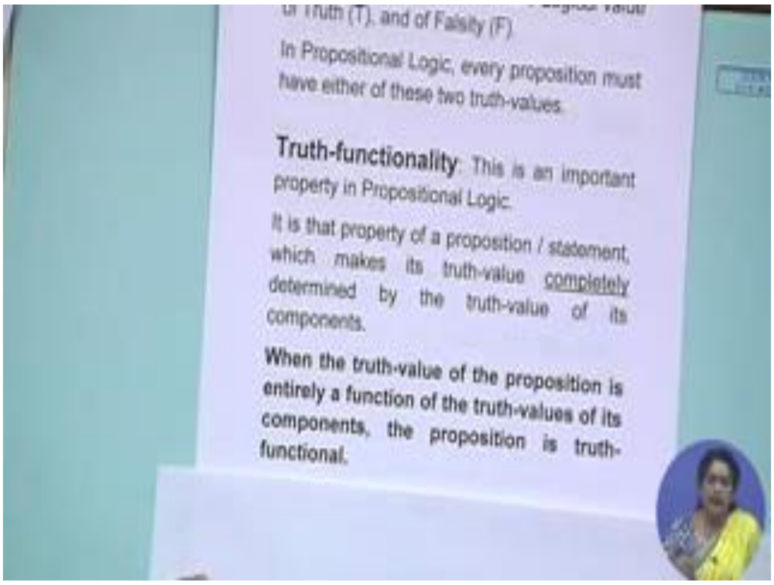

So what are truth-values? Please understand these are not numerical values, but logical values that we call truth-values. What they are is non-numerical values, and in this logic propositional logic allows only two truth values. They are the logical values of truth, which abbreviated form is T, and of falsity the abbreviated form is F. So these are our truth-values and remember propositions bear these values. So propositions are true or false.

It is important to remember that in propositional logic, every proposition must have a truth value, one; and must have either of these truth values. None of them can have both of these truth values; none of them can have neither of the truth values. So if it is a bona-fide syntactically correct proposition, in propositional logic, it has to have a truth value, out of these two choices. Either it is true or it is false. So truth-value, remember, we are going to use this to our understanding of connectives. So remember how to use the truth-values.

Here is the other very important concept that we are going to learn today. That is called *Truth functionality*. Truth functionality. What it is, is a property; it's a property of the propositions or the statements. When can we call a proposition or a statement truth functional? The answer is like this, that when it is such that the truth-value of the proposition or the statement is completely determined by the truth-values of its components. I will repeat that. A proposition or a statement is truth-functional, when its truth-value is completely determined by the truth-value of its components. If you know the truth-value of the components, then you can figure out the truth-value of the whole or the compound proposition itself. In another way, we can say, that when the truth-value of the proposition is entirely a function of the truth-values of its components, we can call the proposition truth-functional.

So, remember the property of truth-functionality is like this, that the truth-value of a truth functional proposition can be known only by knowing the truth-value of its components. You do not need to know anything more. If you know the truth value of the components, you can compute the truth value of the entire whole, and that is what truth-functionality property is. And that is the very important property for propositional logic. You will soon find that the truth-tables or anything that you do actually depends heavily on this property. If you do not have propositions which are truth-functional, you cannot compute the truth-value in the way we usually do. So remember this property. I am going to explain it further.

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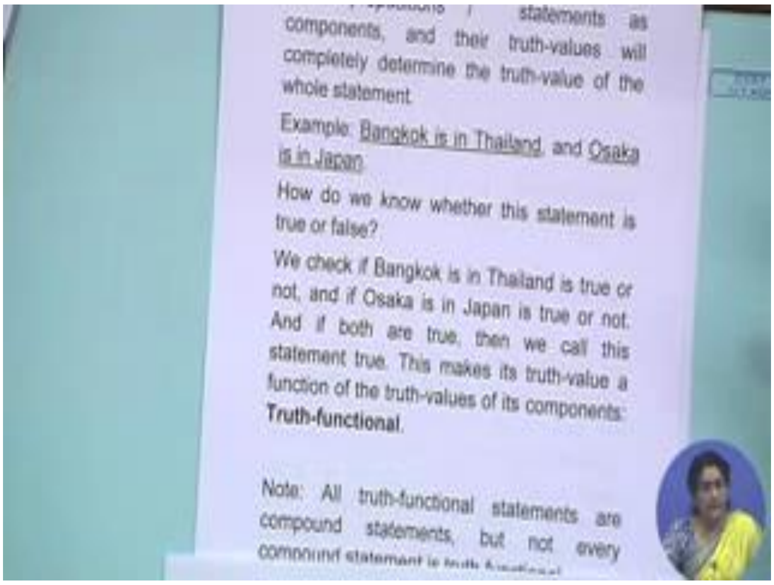

Point number one. That if you have understood what truth-functionality is, then you will also understand that only compound statements can be truth-functional. Why so? Because, the way we have defined it, and you heard me that truth-functional statements are those statements whose truth-value is determined entirely by the truth-value of its components. So unless you have components, how can you be truth-functional? And which propositions statements have components? Those are the compound statements. So only compound statements can be properly be truth-functional. So that is the first thing to understand. We have understood how compound statements can be generated and the answer was by taking simple propositions and combining them with the use of connectives. Alright?

So, for example say, ‘Bangkok is in Thailand and Osaka is in Japan’. Ok? That is a compound statement. Now if you ask: How do we know whether the statement is true or false? Is this true, the whole compound statement, is it true or false? How do we know that? What is the process? And the answer is that we try to first find out whether Bangkok is in Thailand or not, whether it is true or false, then we try to find out whether Osaka is in Japan or not, and depending on what we find out about their truth-values we decide what the truth-value of the whole thing is.

So that makes this sentence completely truth-functional, because you do not need to know anything more besides the truth-value of the components. Okay? Note, as we said that all truth-functional statements are compound statements. This we have already reiterated. (Refer Time: 18:57). But with that, I am going to now add this further that it is not the case that every compound statement is truth functional. It is not necessary just because you have a compound statement in hand, that it’s going to be truth functional. So this is a special property, and I will give you an example of this to make the point clearer. But this is the property that we need to really look into. In here, what happened ‘Bangkok is in Thailand, Osaka is in Japan’, is that all I needed to know is whether this is true and this is true, in order to figure out about the truth-value of the whole thing.

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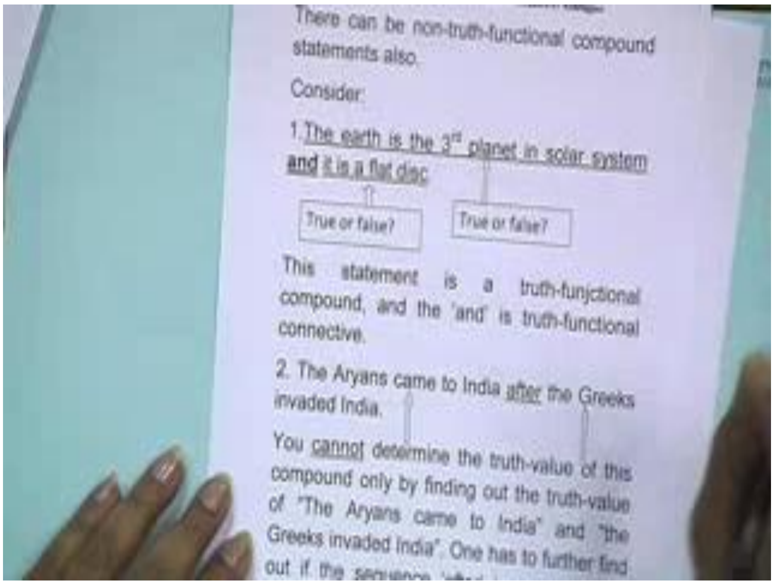

Let us take other example to see the difference so that we can put a contrast here. So what we are trying to understand is that there can be compound statements, which can be non-truth-functional in nature. Ok? Let's move slowly. First let us consider this example. Suppose I tell you 'the earth is a third planet in solar system and it is flat disc'. Okay? It's a flat disc. The question is: How do you know this is true or false? What will you do? And you will probably say: Well, what I will do is find out whether this true or false, this component, then I take this one, whether true or false and we find that this is true, this is false, and somehow we figure out that the whole statement's truth value is so. That's our process, which makes this proposition truth-functional; this makes also the connective truth-functional. Alright?

Now, this is a compound and its truth function. Now compare this scenario with say this one. We give you the second example like this: 'The Aryans came to India after the Greeks invaded India'. As you see, it's a compound statement, here is 'the Aryans came to India' and 'the Greeks invaded India' ; those are two components. But the question is : How do we know whether this whole sentence, the compound, is true or false? Are you going to say that first we try to figure out that one: 'Aryans came to India' whether this is true or false? And then we try to figure out whether the 'Greeks invaded India' or not? If you do that, you are in the wrong track,. Why? Because you can't determine the truth-value of the whole thing just by looking at the truth-value of the component. Even if it is true that the Aryans came to India, even if it is true that the Greeks invaded India, that doesn't tell you about the truth-value of the whole sentence. Why not? Because there is an *after*. So you need to know the historical sequence in which these events have taken place. If you need to know the historical sequence even after you know the truth-value of the components, you are dealing with a compound statement that is not truth-functional. Had it been truth functional, all you need to know is the truth-value of the components. That's it. If you need any extra information in order to compute the truth value of the whole thing, then you are dealing with a non-truth-functional compound statement. Ok? So here there is a connective, but notice that it is not a truth-functional connective. There is a compound, but the compound is also not truth-functional.

So I leave it in front of you to understand the property of truth-functionality better, that this is an example of what we would call truth-functional compound. Why? Because all you need is to know what truth-value of the components have and that's how you compute the value of the compound. Not so in this case, where you have something else going on, you need something more. Therefore this is non-truth-functional compound.

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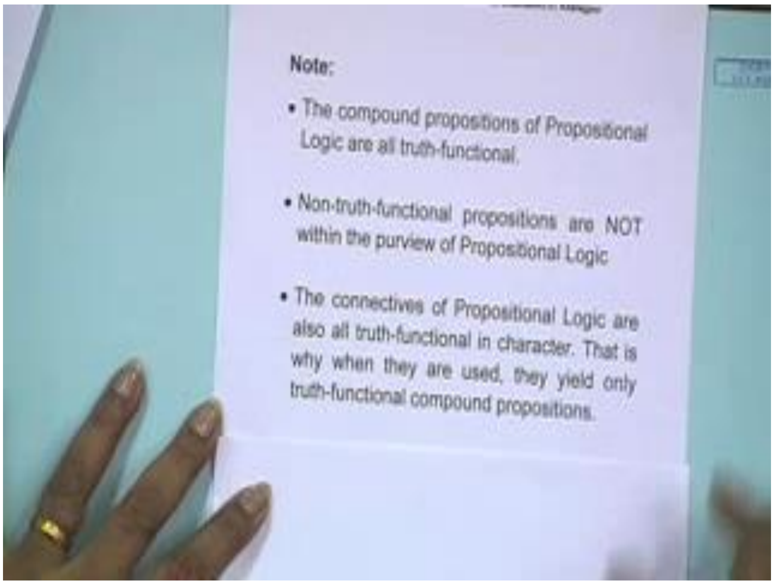

So it's an important property for syntax of propositional logic and remember the reason why we are looking into it as part of the Syntax, is because the compound propositions of propositional logic are all truth-functional. I told you that important logical operations will depend on this property. The procedures that we are going to learn, truth-tables, truth-trees etcetera, they are all dependent on this property that the compound propositions are all truth-functional in character.

If you find non-truth-functional propositions, please remember, they are not within the purview of propositional logic. So if you have non-truth-functional proposition, syntactically they are not acceptable. You are going to have Syntax error, if you are dealing with non-truth-functional propositions in this logic.

And finally, we need to remember, we are going to soon talk about the connectives, that the connectives of propositional logic, I told you there are 5 of them and we are going to introduce you to them in the next module, but they are all truth-functional in character. And that's why when they are used to combine other propositions, they always yield, they always generate truth-functional compound propositions.

With that I am going to end this module, and the next module we are going to talk in details about Connectives and other things.

Thank you.