Problem Set - 9

AUTUMN 2016

MATHEMATICS-I (MA10001) *September 17, 2016*

1. Show that, the general solution of the differential equation $y'' - 4y' - 5y = 0$ is given by $y(x) = Ce^{5x} + De^{-x}$, where C and D are arbitrary constants.

2. Show that, the general solution of the differential equation $y'' - 4y' + 13y = 18e^{2x} \sin 3x$ is given by $y(x) = e^{2x}(A \cos 3x + B \sin 3x) - 3xe^{2x} \cos 3x$, where A and B are arbitrary constants.

3. Show that, the general solution of the differential equation $y'' + 4y' + 3y = x \sin 2x$ is given by $y(x) = Ae^{-x} + Be^{-3x} - \frac{1}{4225}[65x(8 \cos 2x + \sin 2x) - 18 \cos 2x - 316 \sin 2x]$, where A and B are arbitrary constants.

4. Show that, the general solution of the differential equation $4y'' - 4y' + y = e^{\frac{x}{2}}$ is given by $y(x) = (A + Bx)e^{\frac{x}{2}} + \frac{x^2 e^{\frac{x}{2}}}{8}$, where A and B are arbitrary constants.

5. Show by using the method of variation of parameter that the general solution of the differential equation $y''' - 6y'' + 11y' - 6y = e^{-x}$ is given by $y(x) = C_1 e^x + C_2 e^{2x} + C_3 e^{3x} - \frac{1}{24}e^{-x}$, where C_1 C_2 and C_3 are arbitrary constants.

6. It is given that $y_1 = x$ and $y_2 = \frac{1}{x}$ are two linearly independent solution of the associated homogeneous equation of $x^2 y'' + xy' - y = x$, $x \neq 0$. By using the method of variation of parameter, show that the general solution is given by $y(x) = Cx + D\frac{1}{x} + \frac{x}{2} \ln |x|$, where C, D are arbitrary constants.

7. Show that, the general solution of the equation $y''' - 6y'' + 12y' - 8y = 12e^{2x} + 27e^{-x}$ is given by $y(x) = (Ax^2 + Bx + C)e^{2x} + 2x^3 e^{2x} - e^{-x}$, where A, B and C are arbitrary constants.

8. Show that, the general solution of the Euler-Cauchy equation $x^2 y'' - 5xy' + 13y = 30x^2$ is given by $y(x) = x^3[A \cos(2 \ln x) + B \sin(2 \ln x)] + 6x^2$, where A and B are arbitrary constants.

9. Show that, the general solution of the equation $x^3 y''' + 5x^2 y'' + 5xy' + y = x^2 + \ln x$, $x > 0$ is given by $y(x) = \frac{A}{x} + \frac{1}{\sqrt{x}}[B \cos(\sqrt{3} \ln \frac{x}{2}) + C \sin(\sqrt{3} \ln \frac{x}{2})] + (\frac{x^2}{21} + \ln x - 2)$, where A, B and C are arbitrary constants.

10. Show that, the particular integral of the differential equation $\frac{d^3y}{dx^3} + y = \sin 3x - \cos^2 \frac{x}{2}$ is given by $\frac{1}{730}(\sin 3x + 27 \cos 3x) - \frac{1}{2} - \frac{1}{4}(\cos x - \sin x)$.

11. Show that, the general solution of the differential equation $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = x \sin(\log x)$ is given by $y = x[C \cos(\sqrt{3} \log x) + D \sin(\sqrt{3} \log x)] + \frac{x}{2} \sin(\log x)$,where C and D are arbitrary constants.

12. Show that, the general solution of equation $\frac{d^2y}{dx^2} + 4y = 4x$ is given by $y(x) = A \cos 2x + B \sin 2x + x$, where A and B are arbitrary constants.

13. Show that, the general solution of equation $\frac{d^3y}{dx^3} + 3\,\frac{d^2y}{dx^2} - 4\,y = xe^{-2x}$ is given by $y(x) = C_1e^x + C_2e^{-2x} + C_3xe^{-2x} - \frac{1}{18}(x^3 + x^2)e^{-2x}$, where C_1, C_2 and C_3 are arbitrary constants.

14. Show that, the general solution of equation $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = (e^{2x}+3)^2 + e^{3x}\cosh x$ is given by $y(x) = C_1e^x + C_2e^{3x} + C_3e^{-2x} + \frac{1}{12}e^{4x} - \frac{13}{8}e^{2x} + \frac{3}{2}$, where C_1, C_2 and C_3 are arbitrary constants.

15. Show that, the general solution of equation $\frac{d^3y}{dx^3} - 5\frac{d^2y}{dx^2} + 8\frac{dy}{dx} - 4y = e^{2x} + e^x + 3e^{-x}$ is given by $y(x) = C_1e^x + (C_2 + C_3x)e^{2x} + \frac{1}{2}x^2e^{2x} + xe^x - \frac{1}{6}e^{-x}$, where C_1, C_2 and C_3 are arbitrary constants.

16. Show that, the general solution of equation $\frac{d^2y}{dx^2} - 4y = \sin 2x$ is given by $y(x) = C_1e^{2x} + C_2e^{-2x} - \frac{1}{8}\sin 2x$, where C_1, C_2 are arbitrary constants.

17. Show that, the general solution of equation $3\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 8y = 5\cos x$ is given by $y(x) = C_1e^{\frac{4}{3}x} + C_2e^{-2x} + \frac{1}{25}(2\sin x - 11\cos x)$, where C_1, C_2 are arbitrary constants.

18. Show that, the general solution of equation $\frac{d^4y}{dx^4} - 2\,\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} = x^3$ is given by $y(x) = (C_1 + C_2x) + (C_3 + C_4x)e^x + \frac{x^5}{20} + \frac{x^4}{2} + 3x^3 + 12x^2$, where C_1, C_2, C_3, C_4 are arbitrary constants.

19. Show that, the particular integral of the differential equation $\frac{d^2y}{dx^2} + 4y = \sin 2x$ is given by $-\frac{x}{4}\cos 2x$.

20. Show that, the particular integral of the equation $\frac{d^2y}{dx^2} + 4y = x\sin^2 x$ is given by $\frac{x}{8} - \frac{x^2\sin 2x}{16} - \frac{x\cos 2x}{32}$.

21. Show that, the particular integral of the equation $\frac{d^2y}{dx^2} - y = x^2\cos x$ is $-\frac{1}{2}x^2\cos x + x\sin x + \frac{1}{2}\cos x$.

22. Show that, the particular integral of the equation $\frac{d^3y}{dx^3} + y = e^{\frac{x}{2}}\sin\frac{x\sqrt{3}}{2}$ is given by $-\frac{xe^{\frac{x}{2}}}{6}[\sqrt{3}\cos(\frac{x\sqrt{3}}{2}) + \sin(\frac{x\sqrt{3}}{2})]$.

23. Show that, the general solution of the equation $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-2x}\sin 2x$ is given by $y(x) = C_1e^{-2x} + C_2e^{-3x} - \frac{1}{10}e^{-2x}(\cos 2x + 2\sin 2x)$, where $C_1\,C_2$ are arbitrary constants.

24. Show that, the general solution of the equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 4x$ is given by $y(x) = C_1e^{-x} + C_2e^{2x} + 1 - 2x$, where $C_1\,C_2$ are arbitrary constants.

25. Show that, solution of the equation $\frac{d^2y}{dx^2} - y = 1$, given that $y = 0$ when $x = 0$ and $y \to$ a finite limit when $x \to -\infty$ is given by $y(x) = e^x - 1$.