

The Transportation Model – Formulations

The Transportation Model

The transportation model is a special class of LPPs that deals with transporting(=shipping) a commodity from **sources** (e.g. factories) to **destinations** (e.g. warehouses). The objective is to determine the shipping schedule that minimizes the total shipping cost while satisfying supply and demand limits. We assume that the shipping cost is proportional to the number of units shipped on a given route.

We assume that there are m sources $1, 2, \dots, m$ and n destinations $1, 2, \dots, n$. The cost of shipping one unit from Source i to Destination j is c_{ij} .

We assume that the availability at source i is a_i ($i=1, 2, \dots, m$) and the demand at the destination j is b_j ($j=1, 2, \dots, n$). We make an important assumption: the problem is a **balanced** one. That is

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$$

That is, total availability equals total demand.

We can always meet this condition by introducing a dummy source (if the total demand is more than the total supply) or a dummy destination (if the total supply is more than the total demand).

Let x_{ij} be the amount of commodity to be shipped from the source i to the destination j .

Thus the problem becomes the LPP

Minimize

$$z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^n x_{ij} = a_i \quad (i = 1, 2, \dots, m)$$

$$\sum_{i=1}^m x_{ij} = b_j \quad (j = 1, 2, \dots, n)$$

$$x_{ij} \geq 0$$

Thus there are $m \times n$ decision variables x_{ij} and $m+n$ constraints. Since the sum of the first m constraints equals the sum of the last n constraints (because the problem is a balanced one), one of the constraints is redundant and we can show that the other $m+n-1$ constraints are LI. **Thus any BFS will have only $m+n-1$ nonzero variables.**

Though we can solve the above LPP by Simplex method, we solve it by a special algorithm called the transportation algorithm. We present the data in an $m \times n$ tableau as explained below.

Destination

1

2

.

.

n

Supply

S
o
u
r
c
e

1

2

.

.

m

c_{11}	c_{12}			c_{1n}	a_1
c_{21}	c_{22}			c_{2n}	a_2
c_{m1}	c_{m2}			c_{mn}	a_m
b_1	b_2			b_n	

Demand

Destination

Denver Miami Supply

S
o
u
r
c
e

Los Angeles

80	215	1000
100	108	1300
102	68	1200
0	0	200
2300	1400	

Detroit

New Orleans

Dummy

Demand

We write inside the (i,j) cell the amount to be shipped from source i to destination j. A blank inside a cell indicates no amount was shipped.

Destination

Denver Miami Supply

S
o
u
r
c
e

Los Angeles

80	M	1000
100	108	1300
102	68	1200
200	300	200
2300	1400	

Detroit

New Orleans

Dummy

Demand

Note: M indicates a very "big" positive number.
"infinity".

		Distribution Area		
		1	2	3
Refinery	1	120	180	-
	2	300	100	80
	3	200	250	120

Construct the associated transportation model.

(Solution in the next slide)

S
o
u
r
c
e

Refinery

Demand

Destination
Distribution Area

	1	2	3	Supply
1	12	18	M	6
2	30	10	8	5
3	20	25	12	8
Demand	4	8	7	

The problem is a balanced one. M indicates a very "big" positive number.

The total cost will be 1000*

$$\sum_{i=1}^3 \sum_{j=1}^3 c_{ij} x_{ij}$$

S
o
u
r
c
e

Destination Distribution Area

1 2 3 Dummy Supply

Refinery

1

2

3

Demand

<u>12</u>	<u>18</u>	<u>M</u>	<u>15</u>	6
<u>30</u>	<u>10</u>	<u>8</u>	<u>22</u>	5
<u>20</u>	<u>25</u>	<u>12</u>	<u>0</u>	8
4	8	4	3	

M indicates a very "big" positive number.

The total cost will be 1000*

$$\sum_{i=1}^3 \sum_{j=1}^3 c_{ij} x_{ij}$$

Source
 1
 2
 3
 Demand

Destination Retailer

	1	2	3	4	Dummy Supply	
1	1	2	3	2	0	350
2	2	4	1	2	0	400
3	1	3	5	3	M	250
	150	150	400	100	200	