INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Department of Mathematics

Time : 2 hrs. Total Marks : 30.

Mid Semester Exam-Spring, 2019

Subject: MA 20013, Discrete Mathematics

Instruction: "No queries will be entertained during the examination".

Answer all the questions.

(1) Show that $\neg p \longrightarrow (q \longrightarrow r)$ and $q \longrightarrow (p \vee r)$ are logically equivalent. [2]

(2) Express the following statement using mathematical and logical operators, predicate and quantifiers where the domain consists of all integers:

The sum of two negative integers is negative.

[2]

(3) Prove that there is a rational number x and an irrational number y such that x^y is irrational. [2]

(4) Prove by induction that if n is a positive integer then 133 divides $11^{n+1} + 12^{2n-1}$. [2]

(5) Let $S_1, S_2, \ldots$ are disjoint countable sets. Is $S = \cup_{i=1}^{\infty} S_i$ countable? Justify your answer. [2]

(6) Let $f : \mathbb{C} \setminus \{0\} \longrightarrow \mathbb{R}$ defined by $f(z) = |z|$. Define a relation R on $\mathbb{C} \setminus \{0\}$ by aRb if and only if $f(a) = f(b)$. Is this an equivalence relation ? If yes, describe the equivalence classes. [2]

(7) Let $(P, \preceq)$ be a partially ordered set consisting of $mn + 1$ elements. Then show that either there is an antichain consisting of $m + 1$ elements or there is a chain of length $n + 1$ in P. [2]

(8) Draw the Hasse diagram of the following poset

$$(\{\{1\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,4\}, \{3,4\}, \{1,3,4\}, \{2,3,4\}\}, \subseteq).$$

(i) Find the maximal and minimal elements.
(ii) Find the greatest and least element.
(iii) Find all upper bounds and least upper bound of $\{\{2\}, \{4\}\}$.

[4]

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(9) Is the collection of all subgroups of D_4 form a poset with respect to $\subseteq$? If yes then is it a lattice ? Justify your answer. [3]

(10) Let $\sigma = (1\,3)(2\,4\,6) \in S_7$. Find the order of σ. What is the maximum order of an element in S_7. Give an element of S_7 with maximal order. [2]

(11) State Fermat's Little Theorem and using it find the remainder when 3^{40} is divided by 23. [3]

(12) Let $G =< x >$ be the cyclic group of order 10. List all the generators of G and find $|x^{-22}|$. [2]

(13) Show that every finite group of even order contains an element of order 2. [2]