Indian Institute of Technology, Kharagpur

Date: ....-02-2013 FN / AN Time: 2 Hrs Full Marks: 30 No. of Students: 325

Mid Spring Semester, Departs: AE+CH+CY+MA+ME+NA

Subject No: MA20102, 2nd Year B.Tech / M.Sc.

Subject Name: *Numerical Solution of Ordinary and Partial Differential Equations*

Answer all questions

1. (a) Derive Euler method with local truncation error for IVP: $y' = f(x,y), y(x_0) = y_0$. [1]

   (b) For IVP: $y' = x + \sin y, y(0) = 1$ over [0,0.4], find the size of step length h which is sufficient to compute $y(0.2)$ with an error less than 0.05 using Euler's method. [3]

2. (a) Derive Taylor series method of third order with local truncation error for IVP:

$$y' = f(x,y), y(x_0) = y_0.$$ [1]

   (b) Solve the IVP:

$$y' = xu + 1, \quad y(0) = 0$$
$$u' = -xy, \quad u(0) = 1$$

   with h=0.1, and $0 \le x \le 0.2$ using Taylor series method of order three. [5]

3. (a) Derive Backward Euler method with local truncation error for IVP:

$$y' = f(x,y).y(x_0) = y_0.$$ [1]

   (b) Use Backward Euler method for IVP: $y' = -2xy^2, y(0) = 1$ with step h=0.2 over the interval [0,0.4] to compute y(0.2). Use Newton-Raphson method to solve nonlinear equation. Take initial approximation for y(0.2) equal to 1 and perform three iterations. [4]

4. Show that for a consistent linear multistep method, $\rho(1) = 0$ and $\sigma(1) = \rho'(1)$. [2]

5. Define "root condition" for a linear multistep method. Show that the order of the linear multistep method [4]

$$u_{j+1} + (\alpha - 1)\, u_j - \alpha\, u_{j-1} = \frac{h}{4}[(\alpha+3)\, u'_{j+1} + (3\alpha+1)\, u'_{j-1}$$ is THREE if $\alpha = -1$.

   Find the values of α for which the root condition is satisfied.

6. Given $\sigma(\xi) = \frac{1}{12}(23\xi^2 - 16\xi + 5)$, find $\rho(\xi)$ and write the corresponding IMPLICIT linear multistep method. [4]

7. Find $u(0.4)$ correct to 4 decimal places from the IVP: $\frac{du}{dx} = -2u^2, u(0) = 1, h = 0.1$ using the following Predictor – Corrector method: [5]

$$P: u_{j+1} = u_{j-3} + \frac{4h}{3}(2f_j - f_{j-1} + 2f_{j-2}),$$

$$C: u_{j+1} = u_{j-1} + \frac{h}{3}(f_{j+1} + 4f_j + f_{j-1}).$$

   Calculate the starting values using the modified Euler method

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