Subject: Mathematics   Paper: Ordinary Differential Equations (MATH-6114)

Time Allowed: 02:30 Hours   Maximum Marks: 6

Note: **Objective part is compulsory. Attempt any three questions from subjective part.**

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*12)

i. Write a second order differential equation of a particle moving with constant acceleration.
ii. Write a Wronskian for a second ordinary differential equation.
iii. Define initial conditions of any differential equation.
iv. What is difference between homogenous and non-homogeneous differential equation.
v. Verify that xe^x is a solution of the equation $y''-2y'+y=0$.
vi. Verify that $y^3=x$ is a one-parameter family of the solutions of the differential equation $3y'=1/y^2$
vii. Find the differential equation corresponding to the function $y=c_0+c_1x$.
viii. Find the differential equation corresponding to the function $y=c_1e^{2x}$.
ix. Write the names of three methods of solving first order differential equation.
x. Write the annihilator operator for the function $e^{2x}\sin 2x$.
xi. Define Green functions.
xii. Write Bessel function of second kind.

Subjective Part (3*12)

Q.2. (a) Solve $(x-y)dx+(x+y)dy=0$.

(b) Solve the differential equation $y'+\frac{2}{x^2-1}y=\frac{x+1}{x-1}$.

Q.3. (a) Solve $(x-y+2)dx=(2x+y+1)dy$.

(b) Find the series solution of the differential equation $2xy''+y=0$.

Q.4. (a) Define Cauchy-Euler equation, also derive a formula to solve it.

(b) A mass weighing 24 pounds, attached to the end of a spring, stretches it 4 inches. Initially the mass is released from rest from a point 3 inches above the equilibrium position. Find the equation of motion.

Q.5. (a) Solve the system of linear equations $\begin{aligned}\frac{dx}{dt}-4y=1\\ \frac{dy}{dt}+x=2\end{aligned}$.

(b) Solve the differential equation $y''-2y'+y=xe^x$ by U.C. method by using annihilator approach.

Q.6. (a) Derive a formula variation of parameters for a second order linear, homogeneous ordinary differential equation.

(b) Find power series solution of $y''-xy=0$.

Subject: Mathematics **Paper: Algebra-II (MATH-309/MATH-6116)**

Time Allowed: 02:30 Hours **Maximum Marks: 60**

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*12)

i. Define Zero Ring.

ii. Let a, b and c belongs to an integral domain. If $a \neq 0$ and $ab = ac$ then $b = c$.

iii. Find all units in Ring of Integers.

iv. Prove that the set of even integers is a subring of $\mathbb{Z}$.

v. Show that $2 - i$ is irreducible element in $Z[i]$

vi. Define GCD in polynomials.

vii. Define principal ideal domain.

viii. Define unique factorization domain.

ix. In the ring $\mathbb{Z}_8$, find the nilpotent elements.

x. Define quotient ring.

xi. Let ϕ be a ring homomorphism from R to S. If A is a subring of R, then show that $\phi(A) = \{\phi(a) \mid a \in A\}$ is a subring of S.

xii. Show that intersection of any number of ideals is again an ideal of ring.

Subjective Part (3*12)

Q.2. (a) Let ϕ be a ring homomorphism from a ring R to S. Then prove that $R/_{Ker\phi} \approx \phi(R)$.

(b) If R is an integral domain, then show that $R[x]$ is an integral domain.

Q.3. (a) Let R be a commutative ring with unity and let A be an ideal of R. Then prove that $R/_A$ is an integral domain if and only if A is prime.

(b) Prove that every finite integral domain is a field.

Q.4. (a) Let R be a commutative ring with identity and P be an ideal in R, then P is prime ideal if and only if R/P is an integral domain.

(b) Let R be a ring with unity 1. Then prove that R is division ring if and only if R has no nontrivial left ideals

Q.5. (a) Let $R = \left\{ \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \middle| a, b \in \mathbb{Z}_7 \right\}$ with the usual matrix addition and multiplication. Prove that R is commutative ring.

(b) Let $n \in \mathbb{Z}$ be a fixed positive integer. Then prove that following conditions are equivalent

i. n is prime

ii. $\mathbb{Z}/_{\langle n \rangle}$ is an integral domain

iii. $\mathbb{Z}/_{\langle n \rangle}$ is a field.

Q.6. (a) Let R be a ring. If $Q_R = \{(a_1, a_2, a_3, a_4) \mid a_i \in R\}$ is a ring with respect to following addition and multiplication:

$$(a_1, a_2, a_3, a_4) + (b_1, b_2, b_3, b_4) = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$$

$$(a_1, a_2, a_3, a_4) \cdot (b_1, b_2, b_3, b_4) = (a_1 b_1 - a_2 b_2 - a_3 b_3 - a_4 b_4,\ a_1 b_2 + a_2 b_1 + a_3 b_4 - a_4 b_3,$$
$$a_1 b_3 + a_3 b_1 + a_4 b_2 - a_2 b_4,\ a_1 b_4 + a_2 b_3 - a_3 b_2 + a_4 b_1)$$

Then prove that Q_R is a non-commutative division ring.

(b) Prove that every principal ideal domain is a unique factorization domain

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BS 5th Term Examination 2023

Subject: Mathematics **Paper: Real Analysis-I (MATH-307/MATH-6115)**

Time Allowed: 02:30 Hours **Maximum Marks: 60**

Note: Objective part is compulsory. Attempt any four questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*1

i. In every ordered field, if $x<0$, $y<z$, then $xy>xz$.
ii. State the Archimedian property of real numbers.
iii. What do you meant by monotonic sequence?
iv. What do you meant by ordered field?
v. Define the convergence of a sequence.
vi. Discuss the behavior of the series $\sum_{n=1}^{\infty} 1/n$.
vii. Write the general principle of convergence.
viii. State the least upper bound property of a set.
ix. Define local maximum of a function.
x. State the intermediate value theorem for differentiable functions.
xi. Prove that there exists an irrational number between any two real numbers.
xii. Show that a polynomial is continuous for every value of x.

Subjective Part (4*9)

Q.2. Prove that every bounded sequence has a convergent sub-sequence.
Q.3. Prove that there is no rational number p such that $p^2 = 3$.
Q.4. A sequence of real numbers is convergent iff it is a Cauchy sequence.
Q.5. Show that $x_n \to p$ if $y_n \to p$ and $z_n \to p$ as $n \to \infty$ in $y_n \le x_n \le z_n$
Q.6. Check the series $\sum \frac{1}{n} \sin^2 \frac{x}{n}$ whether converges or diverges.
Q.7. State and prove Lagrange's Mean Value Theorem.

University of Sargodha

BS 5th Term Examination 2023

Subject: Mathematics **Paper: Differential Geometry (MATH-6113)**

...e Allowed: 02:30 Hours **Maximum Marks: 6...**

...te: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

... Write short answers of the following in 2-3 lines each on your answer sheet. (2*12)

i. Define space curves, give an example.
ii. What is the condition for a curve to be a Helix?
iii. Show that for any curve $\vec{t}' \cdot \vec{b}' = -\kappa\tau$.
iv. Define the intrinsic equation of a curve.
v. What is Rectifying Plane?
vi. Define the Spherical indicatrix of the tangent to the curve.
vii. Define the torsion of the curve.
viii. Define involutes and evolutes.
ix. Define the tangent line.
x. Write an expression for the first fundamental form.
xi. What is a Geodesic line?
xii. For a curve $\vec{r} = \vec{r}(s)$ prove that $n' = \tau\,\vec{b} + \kappa\,\vec{t}$.

Subjective Part (3*12)

Find the Serret-Frenet formula for $\vec{r}(t) = 2(\cos(t)\,, \sin(t)\,, t)$.

Compute torsion and curvature for the circular helix $r(\theta) = (a\cos(\theta)\,, a\sin(\theta)\,, b\theta)$.

Construct the first fundamental form on the sphere embedded in $\mathbb{R}^3$ where the equation of sphere is $x^2 + y^2 + z^2 = 16$.

Prove that for any curve $\vec{r}(s)$

$$[\vec{r}'', \vec{r}''', \vec{r}''''] = \kappa^3(\kappa\tau' - \kappa'\tau).$$

If tangent and binormal at a point make an angle θ and φ respectively with a fixed direction. Then show that $\frac{\sin\theta}{\sin\varphi}\frac{d\theta}{d\varphi} = \frac{\kappa}{\tau}$.

BS 5th Term Examination 2023

Subject: Mathematics **Paper: Topology (MATH-301/MATH-309/MATH-6112)**

ıe Allowed: 02:30 Hours **Maximum Marks: 60**

e: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

. Write short answers of the following in 2-3 lines each on your answer sheet. (2*12)

i. Write the subbase of usual topology on real line R. 1

ii. Let $X = \{a, b, c\}$, whether the class $\beta = \{\{a, b\}, \{b, c\}\}$ be a base for any topology on X or not? 1

iii. Show that $(a, b]^c$ is both open and close in upper limit topology. 2

iv. Define local base at p and write local base at p in discrete space X. 2

v. Define Lindelof space with example. 2

vi. Write the difference between topological and hereditary property.

vii. Define product topology. 1

viii. If $A \subseteq B$ implies $A' \subseteq B'$. 2

ix. Define finite intersection property with example 2

x. Define separated sets with example. 2

xi. Define nowhere dense set with example.

xii. What is closure of any subset of cofinite topological space. 2

Subjective Part (3*12)

. a) Show that if A and B be subsets of a topological space (X, τ). Then $(A \cup B)' = A' \cup B'$.
b) Let $f: X \to Y$ be a function from a non-empty set X into a topological space (Y, u). Furthermore, let τ be the collection of inverses of open sets of Y i.e $\tau = \{f^{-1}[G]: G \in u\}$. Show that τ is a topology on X.

. a) Show that if A is a subset of topological space (X, τ) and τ_A is relative topology on A. Then A is connected with respect to τ iff A is connected with respect to τ_A on A.
b) Show that a topological space is X is T_1 space iff every singleton is closed.

a) Show that if $\mathcal{A}$ be a class of subsets of a non empty set X then the topology on X generated by $\mathcal{A}$ is the intersection of all those topologies on X which contain $\mathcal{A}$ i.e. $\tau = \cap \tau$.
b) Show that a function $f: X \to Y$ is continuous iff for every subset $A \subseteq X$ implies $f(\bar{A}) \subseteq \overline{f(A)}$.

. a) Show that a set A is connected iff A is not the union of two non-empty separated sets.
b) Prove that continuous images of compact sets are compact.

6. a) Prove that every compact subset of a Hausdorff space is closed.
b) Show that if X is first countable space. Then $f: X \to Y$ is countinuous at p iff it is sequential continuous at p.