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University of Sargodha

BS 7th Semester Examination 2020

Subject: Mathematics **Paper: Fluid Mechanics-I (MATH-425)**

Time Allowed: 2:30 Hours

Maximum Marks: 80

Note: Objective part is compulsory. Attempt any four questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)

(i) Give dimension of density ρ.
(ii) Define Reynolds number.
(iii) Write down three basic system of dimensions.
(iv) Write differential form of equation of continuity.
(v) Write mathematical form of basic pressure field equation.
(vi) Write vector differential form of Navier Stokes equation.
(vii) Define pathlines
(viii) What is difference between incompressible and compressible fluid flow.
(ix) Compute the shear stress in a SAE oil at 20C if $v = 3$m/s and h =2cm.
(x) Give mathematical form of normal and shear forces.
(xi) Define rotational flow.
(xii) Define turbulent flow.
(xiii) What is the difference between control volume and control surface?
(xiv) Define Non-Newtonian fluids.
(xv) Define dilatants.
(xvi) Define stream line.

Subjective Part (4*12)

Q.2. Derive Euler's equation of motion.

Q.3. Derive equation of continuity by applying law of conservation of mass to fluid element.

Q.4. Derive Bernoulli equation by applying Newton's second of motion on the particle along a streamline.

Q.5. The fluid motion is such that the particles of the fluid lies on the surface of a right circular cone whose axis is the Z- axis. (a) Find the equation of continuity. (b) Derive Newton's law of viscosity.

Q.6. A velocity field is given by $v = axi - btyj$, where i, j are unit vectors and $a = 1s^{-1}$ and $b = 1s^{-1}$. Find the equation of streamline at any time t. Plot several streamlines in the first quadrant at $t = 0\ s$, $t = 1\ s$, and $t = 20\ s$.

Q.7. A liquid flows down an inclined plane in a steady , fully developed , laminar flow of thickness h . Simplify the continuity and Navier- Stokes equation equations to model this flow . Obtain expression for liquid's velocity profile, the shear stress distribution.

University of Sargodha

BS 7th Term Examination 2019

Subject: Mathematics **Paper: Fluid Dynamics-I (MATH-425)**

Maximum Marks: 80

Time Allowed: 2:30 Hours

Note: Objective part is compulsory. Attempt any four questions from subjective part.

Objective Part **(Compulsory)**

(16*2)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet.

I. Give dimension of dynamic viscosity μ.
II. Define first law of thermodynamics.
III. Give simple mathematical relation of the shear stress related to velocity gradient.
IV. Write integral form of equation of continuity.
V. Write mathematical form velocity field for laminar and turbulent flow.
VI. Write complete form of Navier Stokes equation.
VII. Define specific gravity.
VIII. Define control surface.
IX. Define streamline.
X. Define vorticity and give its mathematical form
XI. Define law of conservation of mass
XII. Define path line
XIII. Define control volume.
XIV. Give mathematical form of shear stress for non-Newtonian fluid.
XV. What is the difference between Newtonian and non-Newtonian fluid?
XVI. Define hydrostatic pressure

Subjective Part **(4*12)**

Q.2. Derive integral form of linear momentum equation.

Q.3. A velocity field is given by $\vec{V} = axi - btyj$, where $a = 1s^{-1}$ and $b = 1s^{-2}$. Find the equation of streamline at any time t. Plot several streamlines in the first quadrant at $t = 0s, t = 1s, t = 20s$.

Q.4. For a two dimensional flow in the xy plane, the x component of velocity is given by $u = Ax$. Determine a possible y component for incompressible fluid flow. How many y components are possible?

Q.5. Derive Bernoulli's equation for unsteady frictionless flow along a streamline.

Q.6. Consider a fluid particle moving in a general three dimensional flow field may rotate about all three coordinate axes. Derive angular rotation in vector notation of the form $\vec{\omega} = \frac{1}{2} \nabla \times \vec{V}$

Q.7. State and derive second law of thermodynamics

$\frac{d\theta}{dt} = \frac{1}{\theta}$

Flow

University of Sargodha

BS 7th Semester Exam 2019

Subject: Mathematics **Paper: Fluid Dynamics-I / Mechanics (MATH:425)**

Time Allowed: 2:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any four questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each. (2*16)

1: Write types of fluid.
2: Define flow.
3: What is the physical role of density in fluid flow mechanism?
4: Compute the shear stress in a SAE oil at 20C if v = 3m/s and h = 2cm.
5: Define incompressible fluid flow
6: Define strakline
7: Define laminar flow.
8: Define Reynolds number.
9: Define turbulent flow
10: Define stream line
11: Write the statement of Renold transport theorem for arbitrary fixed control volume.
12: What is the physical role of pathline, streakline and streamline?
13: Give basic idea of the relation of equation of continuity and law of conservation of mass
14: Define specific weight
15: Write down Euler's equation of motion.
16: Give the dimension of dynamic viscosity μ.

Subjective Part (4*12)

Q.2. Derive the equation of continuity in differential form.

Q.3. Derive vector differential form of acceleration field of fluid.

Q.4. Discuss translation of a fluid particle in detail.

Q.5. Derive the momentum equation in differential form.

Q.6. A liquid flows down an inclined plane in a steady, fully developed, laminar flow of thickness h. Simplify the continuity and Navier-Stokes equation/equations to model this flow. Obtain expression for liquid's velocity profile, the shear stress distribution the volume flow rate and average velocity.

Q.7. Derive the form of Bernoulli equation for irroational flow.

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Sargodha, Punjab

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University of Sargodha

BS 7th Term Examination 2018

Subject: Mathematics **Paper: Measure Theory (MATH-449)**

Time Allowed: 2:30 Hour **Maximum Ma[rks]**

Note: Objective part is compulsory. Attempt any four questions from subjective part.

Objective Part (Compulsory)

QNo.1 Write short answer of the following questions in 2-3 lines only. (2*16)

(i) Define the outermeasure of a non-empty set.
(ii) If E be any set then write the translate $E+y$ of E.
(iii) State the countable subadditivity of outermeasure.
(iv) Define the Lebesgue measurable set.
(v) Prove that the set of rational numbers is measurable.
(vi) Find the measure of interval (a,b).
(vii) Define σ-algebra of measurable sets.
(viii) State the monotonicity of Lebesgue measure.
(ix) Define a simple function.
(x) What is meant by the property "**almost everywhere**".
(xi) Define the uniform convergence of a sequence of functions.
(xii) State a sufficient condition for composition of two measurable functions is also measurable func[tion].
(xiii) Define the Lebesgue integral of a non-negative measurable function.
(xiv) Let f be a bounded measurable function on a set of finite measure E. Then show that

$$\left|\int_E f\right| \leq \int_E |f|.$$

(xv) State the Monotone Convergence Theorem.
(xvi) Show that f is integrable if and only if both positive and negative parts f^+ and f^- of f are integrable.

Subjective Part (4*12)

Q. No. 2. (a) If $\{E_k\}_{k=1}^{\infty}$ is any collection of sets then prove that

$$m^*\left(\bigcup_{k=1}^{\infty} E_k\right) \leq \sum_{k=1}^{\infty} m^*(E_k).$$

(b) Prove that the union of two measurable sets is also measurable.

Q. No. 3. Prove that every interval is measurable

Q. No. 4. Let f and g be two measurable functions on E that are finite a.e. on E. Then for any α and β, pr[ove] that $\alpha f + \beta g$ and fg are measurable on E.

Q. No. 5. Let f and g be bounded measurable functions on a set of finite measure E. Then for any $\alpha >$ [0] and $\beta > 0$, prove that

(a) $\int_E (\alpha f + \beta g) = \alpha \int_E f + \beta \int_E g.$ **(b)** If $f \leq g$ on E then $\int_E f \leq \int_E g.$

Q. No. 6. Let f and g be two integrable functions over E. Then for any α and β, prove that the functi[on] $\alpha f + \beta g$ is integrable and

$$\int_E (\alpha f + \beta g) = \alpha \int_E f + \beta \int_E g.$$

Q. No. 7. Let $\{f_n\}$ be a sequence of measurable functions on E. Suppose there is a function g that is integrable over E and dominates $\{f_n\}$ on E in the sense that $|f_n| \leq g$ on E for all n. If $\{f_n\} \to f$ pointwi[se] a.e on E then prove that f is integrable over E and

$$\lim_{n\to\infty} \int_E f_n = \int_E f.$$

UNIVERSITY OF SARGODHA
DEPARTMENT OF MATHEMATICS

Class: BS-Mathematics | Course Title: PDEs
Course Code: MATH-405 | Semester: VII
Time Allowed: 2 hours | Maximum Marks: 50

Note: Attempt all questions.

Q No.1: Find the general solution of the following differential equation using Laplace transformation

$$y''(t)+k^2y(t)=f(t)$$

with initial conditions $y'(0)=c_1, y(0)=c_2$.

Q No.2: State and prove "Convolution Theorem for laplace Transform".

Q No.3: Determine the general solution of

$$4u_{xx}+5u_{xy}+u_{yy}+u_x+u_y=2.$$

Q No.4: Transform the equation into canonical form

$$x^2u_{xx}-2xyu_{xy}+y^2u_{yy}=e^x.$$

Q No.5: Using $v=\ln u$ and $v=f(x)+g(y)$, show that the solution of the Cauchy problem

$$y^2u_x^2+x^2u_y^2=(xyu)^2,\quad u(x,0)=e^{x^2}$$

is

$$u(x,y)=\exp\left(x^2+i\frac{\sqrt{3}}{2}y^2\right).$$

$\frac{16-25}{4}=-9/4$

$\frac{4-1}{4}=3/4$

Best of Luck
10-02-2022

$-\frac{9}{4}u_{\xi\eta}=2+\frac{3}{4}u_\eta-3$

$-9u_{\xi\eta}=8+3$

$\frac{16-25}{4}=-\frac{9}{4}$

University Of Sargodha, Sargodha

Department of Mathematics

Final-Term Examination: BS-VII Student's ID (..................)

Paper: MATH-401 (Numerical Analysis-I) (Marks=50) Time: 2 : 00 Hrs

Q1. Solve the following system of linear equations using Croute's Reduction Method.

$$\begin{aligned} 3x + y + 2z &= 3 \\ 2x - 3y - z &= -3 \\ x - 2y + z &= 4 \end{aligned} \tag{1}$$

Q2. Find dominant Eigen value of the matrix A by Power method, find eigen vector too

$$A = \begin{bmatrix} 4 & 1 & -2 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$$

Q3. Construct second Lagrange's Interpolating polynomial for $f(x) = \dfrac{1}{x^2}$ if $x_0 = 2, x_1 = 2.75, x_2 = 4$ and use the polynomial (constructed) to approximate $f(3) = 1/9$

Q4. Following table gives viscosity of oil as a function of x. Use Lagrange formula to the viscosity of oil at temperature $x = 140$.

x	110	130	160	190
f(x)	10.8	8.1	5.5	4.8

Q5. Apply Newton Raphson procedure to find real root of the equation.

$$x^3 + 3x^2 - 1 = 0, \quad [-3, -2]$$

$ind_2 10 =$

$10 \equiv 2^{5}$

Final Term Examination

BS-VII(Regular & Self Support)

Department of Mathematics, University of Sargodha, Sargodha

Subject:Number Theory | Date:08/02/2022

Time Allowed: 02 Hours | Max Marks:50

Q. No. 1 (08 Marks)

Prove that the congruence of degree n

$$f(s) \equiv 0 \pmod{p}$$

has at most n solutions.

Q. No. 2 (06 Marks)

If $ord_m a = t$ and if u is a positive integer, then prove that

$$ord_m(a)^u = \frac{t}{(t,u)},$$

where (t,u) is the greatest common divisor of u and t.

Q. No. 3 (10 Marks)

Find all the primitive roots of 17^2 and solve the following congruence with the help of indices

$$17x^2 \equiv 10 \pmod{29},$$

provided that 2 is the primitive root of 29.

Q. No. 4 (10 Marks)

Prove that all primitive solution of equation $x^2 + y^2 = z^2$ are of the form $x = a^2 - b^2, y = 2ab$ and $z = a^2 + b^2$, where $(a,b) = 1$ and exactly one of the a and b is even.

Q. No. 5 (16 Marks)

State and prove Quadratic reciprocity law and hence evaluate $\left(\frac{2819}{4177}\right)$.

BEST OF LUCK

$n \equiv r^s \pmod{m}$

$s = ind_r n$

$n \equiv r^{ind_r n} \pmod{m}$

$\equiv ind^{10}$

$\equiv 4^{10}$

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University of Sargodha

BS 6th Semester Exam 2019

Subject: Mathematics **Paper: Numerical Analysis (MATH:302)**

Time Allowed: 2:30 Hours **Maximum Mark**

Note: Objective part is compulsory. Attempt any four questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each. (2×16)

1. Discuss Linear and Non-linear equations with example.
2. Explain difference between Newton-Raphson and Secant method.
3. Describe difference between Jacobi and Gauss-Seidel method.
4. Define Lower and upper Triangular matrices.
5. What is the difference between Cholesky's and Doolittle's method.
6. Explain Intermediate value theorem (property).
7. How you define Extrapolation.
8. Describe difference between relative and absolute error.
9. Define Condition Number.
10. Describe Matrix Norm.
11. State Trapezoidal Rule
12. ~~Difference between direct and iterative methods~~
13. What is Numerical Analysis
14. Define positive definite matrix with example
15. Define diagonally dominant matrix
16. What is Crouts Method

Subjective Part. (4 × 12)

Q2. Apply Regula-Falsi method to find root of polynomial

$$2x - 3\sin x - 5 = 0$$

Q3. Find eigen values and corresponding eigen vectors

$$A = \begin{bmatrix} 4 & 1 & -2 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$$

Q4. State and prove Newton-Raphson Method

Q5. Evaluate the following integral using Simpson's $\frac{1}{3}$ Rule, Calculate absolute error too.

$$\int_2^4 \frac{1}{x^2} dx$$

Q6. Apply Jacobi method to approximate the solution (of the following system of linear equations) rounded to at least two significant digits.

$$\begin{cases} 5x_1 - 2x_2 + 3x_3 &= -1 \\ -3x_1 - 9x_2 + x_3 &= 2 \\ 2x_1 - x_2 - 7x_3 &= 3 \end{cases}$$

Q7. Apply Gauus-elimination method.

$$\begin{cases} 2x_1 + x_2 + x_3 &= 7 \\ x_1 + 4x_2 - x_3 &= 6 \\ x_1 + x_2 + x_3 &= 6 \end{cases}$$

MARKS 80

SUPERIOR PRE-UNI EXAM JANUARY 2022

DATE	11.01.2022
TOTAL TIME	02:30 HOURS
ROLL NO	

Q.No.1 Attempt all short questions from the following: 16x2=32

i. Define relative error.
ii. Define transcendental equation.
iii. Explain convergence of N-R method.
iv. Write convergence condition of fixed point method.
v. Use Newton scheme of iteration to find the square root of 12.
vi. Write down the types of errors?
vii. State intermediate property.
viii. Define upper triangular matrix.
ix. What is meant by root of an equation?
x. Define row vector.
xi. What are orthogonal vector?
xii. What is the difference between partial and full pivoting?
xiii. Differentiate between direct and iterative methods and also classify each?
xiv. Why Gauss Seidal method is fast than the Jacobi's method?
xv. Why we use numerical methods to solve the system of linear equations?
xvi. Define orthogonal matrix.

Note: **Attempt any three (3) question:** 3x16=48

Q.No.2: Use Regula Falsi Method to find the solution of $2x\cos 2x-(x-2)^2=0$ upto four decimal places?

Q.No.3: a) Show that the N-R method is quadratically convergent?
b) Set up Newton's scheme to find a p-th root of any positive number N?

Q.No.4: Solve the system of equations

$$3x-y+2x=8$$
$$x+2y+3z=5$$
$$2x-2y-2z=2$$

by Crout's reduction method?

Q.No.5: Solve the system of equations

$$6x_1-x_2+4x_3=15$$
$$x_1-7x_2+2x_3=12$$
$$3x_1+5x_2-9x_3=-20$$

By Gauss Seidal method?

$\frac{16-4-6}{3}$

Q.No.6: Using Jacobi's method, find all eigen values and eigen vectors of the Hilbert matrix.

$$A=\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \end{bmatrix}$$

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University of Sargodha

BS 6^{th} Semester Examination 2017

Subject: Mathematics

PDE 2017

(7)

(15)

Numerical 2017

(13)

University of Sargodha

BS 6^{th} Term Examination 2017

Subject: Mathematics **Paper: Numerical Analysis (MATH-302)**

Time Allowed: 2:30 Hours **Maximum Marks: 80**

Objective Part (Compulsory)

Q. No. 1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)

(i) In numerical integration, which method is better than to the other methods, Explain? Newton Raphson is better than Secant

(ii) Explain definite relative error.

(iii) Define transcendental and algebraic equation.

(iv) Explain convergence of Newton Raphson Method.

(v) Express $\Delta^3 y_1$ in terms of the values of the function y.

(vi) Using backward difference prove that $y_{n-3} = (1-\nabla)^3 y_n$.

(vii) Prove that $hD = \log E$, where D, E are differential and Shift operator resp.

(viii) Prove that $\Delta = \frac{\delta^2}{2} + \delta\sqrt{(1+(\delta^2)/2)}$.

(ix) Prove that $\mu E = E\mu$, where μ stands for average operator.

(x) Prove that $hD = \sinh^{-1}(\mu\delta)$.

(xi) What is maximum error formula in 1/3 Simpson's rule?

(xii) What is the convergence condition of Method of iteration to find root? Newton Raphson Method.

(xiii) Show that the operator δ and E commute with one another.

(xiv) Explain truncation error.

(xv) what is the value of x, using backward difference of the following data (1, 3), (2, 6), (4, x)?

(xvi) Geometrically, what is the difference between 1/3 Simpson's rule and 3/8 Simpson rule?

... Partial Differential Equations (MATH-308)

Time Allowed: 2:30 Hours

Maximum Marks: 80

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Numerical 2018

16

University of Sargodha

BS 6th Term Examination 2018

Subject: Mathematics **Paper: Numerical Analysis (MATH- 302)**

Time Allowed: 2:30 Hours

Maximum Marks: 80

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (16*2)

i. Define the term analytical error.

ii. Write the formula of Secant method. NR method

iii. What is criteria for the convergence of iterative method?

iv. Why bisection method always converges?

v. What is difference between Doolittle method and Crout's method?

vi. Find the inverse Matrix of A using L and U method.

vii. Differentiate Jocobi's and Gauss-siedle method.

viii. What is the difference between Lagrange and Newton forward (backward) method?

ix. Prove that $E^{\frac{1}{2}} = \mu + \frac{\delta}{2}$.

x. Show that $\frac{\mu}{\sqrt{1+\frac{\delta^2}{4}}} = 1$.

xi. Define the Direct method and Iterative method.

xii. Define Extrapolation.

xiii. Write the error formula for one-third Simpson's rule.

xiv. Write the second order derivative formula by using forward difference operator.

xv. Explain rounding off error and explain how to reduce it.

xvi. Differentiate eigen value and dominant eigen value.

Subjective Part (3*16)

Q.2. i. Drive Newton-Raphson method and solve $x^6 - \sin x = 10$.

ii. Show that Newton divided difference and Lagrange interpolation formula are identical.

Q.3. Solve $10x+y+z=10$, $x+10y+z=-8$, $x+2y+10z=9$ using Gauss-Siedle method.

Q.4. Find the least Eigen value by power method of $\begin{bmatrix} 8 & 1 & 2 \\ 0 & 10 & -1 \\ 6 & 2 & 15 \end{bmatrix}$

Small A^{-1} Power

$U^{k} = AV^{k-1}$

Q.5. The n^{th} divided differences of n^{th} degree polynomial are constant.

Q.6. From the following data, find the number of student who obtained less than 45 marks by any suitable interpolation technique.

Marks	30-40	40-50	50-60	60-70	70-80
No. of students	31	42	51	35	31

(Subjective Part)

Note: Attempt any three questions.

Q No. 2: Apply Newton-Raphson Method to determine a root of the equation $\cos x = xe^x$ correct to three decimal places by taking $x_0 = 1$. (16)

Q No. 3: Let a function $\chi(x)$ be defined and differentiable in an interval (a, b) with all values $\chi(x) \in [a, b]$, then if there exists a proper fraction α $(0 < \alpha < 1)$ such that $|\chi(x)| \leq \alpha < 1, \forall\ a < x < b$, then the process of iteration described above converges irrespective of the choice of initial approximation $x_0 \in [a, b]$. Moreover, estimate the error of iterative process. (16)

Q No. 4: Construct Newton's divided difference interpolating polynomial using following data

$x =$	1.0	1.3	1.6	1.9	2.2
$f(x) =$	0.7651977	0.6200860	0.4554022	0.2818186	0.1103623

(16)

Q. No. 5: Solve the following system of equations by using Gauss-Seidel method:

$$5x - 2y + z = 4$$
$$7x + y - 5z = 8$$
$$3x + 7y + 4z = 10$$

(16)

Q. No. 6: Find the eigenvalues & corresponding eigenvectors of the matrix

$$\begin{bmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 0 & 4 \end{bmatrix}$$

(16)

BS 6th Semester Examination 2017

Subject: Mathematics **Paper: Numerical Analysis (MATH-302)**

Time Allowed: 2:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any four questions from subjective part.

Objective Part **(Compulsory)**

Q. No.1. Write short answers of the following in 2-3 lines on your answer sheet. (2*16)

i. State intermediate property?
ii. Write two exact methods for solution of methods.
iii. Define upper triangular matrix.
iv. What is eigenvalue of a matrix?
v. Find eigenvalues of $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$

$$\begin{matrix} u_{11} & u_{12} & u_{13} \\ l_{21} & u_{22} & u_{23} \\ l_{31} & 0 & u_{33} \end{matrix}$$

vi. What is meant by roots of equation
vii. Define raw vector.
viii. What are orthogonal vector?
ix. Write the conditions for a system of equations having unique, infinite or no solution.
x. Prove that $\mu\delta = \frac{\Delta E^{-1}}{2} + \frac{\Delta}{2}$.
xi. What is difference between partial pivoting and full pivoting
xii. What do you meant by extrapolation?
xiii. Differentiate between direct and iterative methods and also name them.
xiv. What is geometrical difference between 1/3 simpson and trapezoidal rule of integration
xv. Why Gause siedel method is fast than Jacobi method?
xvi. Prove that $1+\delta^2\mu^2 = \left(1+\frac{\delta^2}{2}\right)^2$.

Bracketing method

Elimination methods

SUBJECTIVE (48)

$AX = B$

$A = 0$

Q. No.2

Solve the system of equations by using by using Gause-Siedel method

$$2x - y + 0z = 7$$
$$-y + 2z = 1$$
$$-x + 2y - z = 1$$

Q. No.3 State and prove Regula-Falsi method.

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Q. No.4

Find the real root of the equation $xe^x - 2 = 0$ correct to two decimal places, using Newton-Raphson method.

Q. No.5

Using Lagrange's interpolation formula, evaluate f(142) from the following of values

x	140	150	160	170	180
f(x)	3.685	4.854	6.302	8.076	10.225
	f_0	f_1	f_2	f_3	f_4

Q. No.6

Evaluate the following by using Trapezoidal rule for five points and also discuss its error

$$\int_1^3 \frac{1}{x^2}\,dx$$

16

$x_n =$

Q. No.7 Find eigenvalue and eigenvector of the matrix

$$A = \begin{bmatrix} 10 & 2 & 1 \\ 2 & 10 & 1 \\ 2 & 1 & 10 \end{bmatrix}$$

In mathematics, the intermediate value property states that if a continuous function f takes values in an interval [a,b] as its domain, takes values f(a) & f(b) at each end of the interval, then it also takes any value b/w f(a) & f(b) at some point within the interval.

(Subjective Part)

Q No. 2: Apply Newton-Raphson Method to determine a root of the equation $\cos x = xe^x$ correct to three decimal places by taking $x_0 = 1$. (16)

Q No. 3: Use Trapezoidal rule with $n = 6$ to estimate

$$\ln 2 = \int_1^2 \frac{1}{x} dx$$

(16)

Q No. 4: Construct Newton's divided difference interpolating polynomial using following data

x =	1.0	1.3	1.6	1.9	2.2
$f(x)$ =	0.7651977	0.6200860	0.4554022	0.2818186	0.1103623

(16)

Q. No. 5: Solve by using LU method:

$$4x + y - z = 2$$
$$x + 3y + 5z = 3$$
$$x - y + z = 3$$

(16)

Q. No. 6: Find the eigenvalues & corresponding eigenvectors of the matrix

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

(16)

University of Sargodha

BS 7th Term/Semester Exam 2021.

Subject: Mathematics **Paper: Number Theory (MATH-403)**

Time Allowed: 2:30 Hours **Maximum Marks: 80**

Note: **Objective part is compulsory. Attempt any three questions from subjective Part.**

Objective Part **(Compulsory)**

Q.1. Write short answers of the following in 2-3 lines each. (2*16)

i. Find G.C.D. and L.C.M. of 273 and 81.
ii. Define Bracket function.
iii. Show that if $ax+by=m$ then $(a,b)|m$.
iv. Define reduced residue system.
v. Define Least common multiple.
vi. Find order of 3 modulo 11.
vii. Define algebraic number field.
viii. Write congruence classes for modulo 11 with remainders 3 and 10.
ix. Define symmetric polynomial.
x. Define units of algebraic number field.
xi. Find the values of x, y and z which satisfy $6x+10y+15z=1$.
xii. Find the Residue of 3^{16} modulo 17.
xiii. Define Linear congruence.
xiv. Define primitive root.
xv. If $na \equiv nb \pmod{m}$ and $(n, m)=1$, then show that $a \equiv b \pmod{m}$.
xvi. Prove that every rational integer is an algebraic integer.

Subjective Part **(3*16)**

Q.2. **(a)** Prove that Mobius function is multiplicative.

(b) Evaluate the Legendre symbol $\left(\frac{503}{773}\right)$.

Q.3. **(a)** State and prove unique factorization theorem.

(b) If $\{a_1, a_2, \ldots, a_{\phi(m)}\}$ is a reduced residue system (mod m) and $(a, m)=1$ then prove that $\{aa_1, aa_2, \ldots, aa_{\phi(m)}\}$ is also a reduced residue system (mod m).

Q.4. **(a)** Prove that an integer α is a root of the congruence $f(x) \equiv 0 \pmod{m}$ if and only if $(x-\alpha)|f(x) \pmod{m}$.

(b) Given that 2 is a primitive root of 9, use indices to solve the following congruence $10x \equiv 8 \pmod{18}$.

Q.5. **(a)** State and prove Chinese Remainder Theorem.

(b) Prove that the set of algebraic numbers forms a field.

Q.6. **(a)** Prove that equation $x^4+y^4=z^4$ has no solution in integers.

(b) Prove that a necessary and sufficient condition that the equation $ax+by=c$ has a solution (x, y) in integers, is that $d \mid c$ where $d=(a, b)$ and this solution is of the form $x=x_0+\frac{b}{d}t,\ y=y_0-\frac{a}{d}t$ where t is an arbitrary integer.

Infinix NOTE 10 PRO

University of Sargodha

BS 7th Term/Semester Exam 2021

Subject: Mathematics **Paper: Partial Differential Equations (MATH:405)**

Time Allowed: 2:30 Hours

Maximum Marks: 80

Note: Objective part is compulsory. Attempt any four questions from subjective part.

Objective Part (Compulsory)

(2*16)

Q.1. Write short answers of the following in 2-3 lines each.

i. Write the difference between linear and non - linear partial differential equation.
ii. Define principle of Superposition.
iii. Obtain PDE $z = x + ax^2y^2 + b$ where a,b are arbitrary constants.
iv. Write down the Physical Meanings of The Drichlet Boundary conditions.
v. Define Canonical Form of first order linear equation.
vi. Write mathematical form of the telegraph equation.
vii. Define specific heat of substance.
viii. Write condition for existence of Fourier transformation.
ix. Prove that $\int_{-\infty}^{\infty}|f(x)|^2\,dx = \int_{-\infty}^{\infty}|F(k)|^2\,dk$
x. Find the Fourier series expansion for the function $f(x) = x + x^2 \quad ; -\pi < x < \pi$
xi. Find Laplace Transformation of the error Function.
xii. If $\mathcal{L}\{f(t)\} = F(s)$ then $\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} F(s)\,ds$
xiii. Find The Laplace transform of an impulse function.
xiv. Let $f'(x)$ be continuous and $f''(x)$ be peicewise continuous in $[0,\pi]$ if $F_s(k)$ is the finite Fourier Sine Transform of $f(x)$ then prove that
$$\mathcal{F}_s\{f''(x)\} = \frac{2k}{\pi}[f(0) - (-1)^k f(\pi)] - k^2 F_s(k)$$
xv. Show that the Hankel transform satisfies the Parseval relation
$$\int_0^{\infty} r f(r)\, g(r)\,dr = \int_0^{\infty} k\tilde{f}(\kappa)\tilde{g}(\kappa)\,dk$$
xvi. Write the Difference between Laplace and Fourier Transform.

Subjective Part (4*12)

Q.2. Show that the general solution of the linear equation
$$(y-z)\,u_x + (z-x)\,u_y + (x-y)\,u_z = 0$$
is $u(x, y, z) = f(x + y + z,\ x^2 + y^2 + z^2)$ where f is an arbitrary function.

Q.3. Solve the initial-value problem $u_x + 2u_y = 0,\ u(0, y) = 4e^{-2y}$.

Q.4. State and Prove one Dimensional Wave Equation.

Q.5. Derive Laplace equation in cylindrical coordinates.

Q.6. Let $f(x)$ and its first derivative vanish as $x \to \infty$. If $F_c(k)$ is the Fourier cosine transform, then prove that $\mathcal{F}_c\{f''(x)\} = -k^2F_c(k) - \sqrt{\frac{2}{\pi}}f'(0)$

Q.7. Consider the motion of a semi-infinite string with an external force $f(t)$ acting on it. One end is kept fixed while the other end is allowed to move freely in the vertical direction. If the string is initially at rest, the motion of the string is governed by
$$u_{tt} = c^2u_{xx} + f(t),\ 0 < x < \infty, t > 0$$
$$u(x, 0) = 0,\ u_t(x, 0) = 0 \text{ and } u(0, t) = 0,\ u_x(x, t) \to 0, \text{ as } x \to \infty$$

Infinix NOTE 10 PRO

University of Sargodha

BS 7th Semester/Term Exam 2021

Subject: Mathematics **Paper: Measure Theory (MATH-449/433)**

Time Allowed: 2:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

(2*16)

Q.1. Write short answers of the following in 2-3 lines each.

i. Define algebra on a set X.
ii. Define measurable function.
iii. Define outer measure.
iv. If A is σ-algebra on a set X, then prove that $\phi, X \in A$.
v. Show that intersection of any number of σ-algebras is a σ-algebra.
vi. Define translation invariant measure.
vii. Define countably additive measure.
viii. Let m be countably additive measure on a σ-algebra M, then if A and B are two sets in A, with $A \subseteq B$, then show that $mA \leq mB$.
ix. Prove that if $m^* A = 0$, then $m^*(A \cup B) = m^* B$.
x. If $m^* E = 0$, then show that E is measurable.
xi. Define positive part of a function
xii. Define Borel sets
xiii. Define F_σ set.
xiv. Define counting measure.
xv. State countably subadditive property of a measure.
xvi. Define sum modulo 1 for $x, y \in [0,1)$.

Subjective Part (3*16)

Q.2. Prove that outer measure of an interval is its length.

Q.3. Show that the interval (a, ∞) is measurable.

Q.4. Let c be a constant and f and g be two measurable real-valued functions defined on the same domain. Then prove that the functions $f+c, cf, f+g, g-f$ and fg are also measurable.

Q.5. Let $<f_n>$ be a sequence of measurable functions defined on a set E of finite measure, and suppose that there is a real number M such that $|f_n(x)| \leq M$ for all n and all x. If $f(x) = \lim f_n(x)$ for each x in E, then prove that

$$\int_E f = \lim \int_E f_n .$$

Q.6. State and prove Fatou's lemma.

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BS 7th Semester/Term Exam 2021

Subject: Mathematics

Time Allowed: 02:30 Hours

Paper: Numerical Analysis-I (MATH-401)

Maximum Marks: 80

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)

i) Define positive deifinite matrix, give example. ii) What is the difference between Doolittle and Cholesky's method. iii) Define Non-linear equation and give examples. iv) How Secant method works, explain v) What is the difference between Gauss-Seidel and Jacobi method. vi) What is an ill-conditioned system. vii) Explain difference between Cholesky's and Crout's method. viii) Explain extrapolation with example ix) How you define Interpolation. x) Descibe advantage of Regula Falsi method over Bisection method of root finding problems. xi) Define Condition Number. xii) Define norm of matrix. xiii) Descibe differnce between relative and absolute error. xiv) What is the difference between direct and iterative methods. xv) What is partial pivoting. xvi) Define dominant eignem value.

Subjective Part. (3×16)

Q2. Solve the following system of equations using Crout's method.

$$\begin{cases} 3x + y + 2z = 3 \\ 2x - 3y - z = -3 \\ x - 2y + z = 4 \end{cases}$$

Q3. Apply Gauss Seidel method to approximate the solution (of the following system of linear equations) by performing three iterations.

$$\begin{cases} 4x_1 + x_2 - x_3 = 5 \\ -x_1 + 3x_2 + x_3 = -4 \\ 2x_1 + 2x_2 + 5x_3 = 1 \end{cases}$$

Q4. Test the matrix for ill-conditioned or not?

$$\begin{bmatrix} 4 & 2 & 6 \\ 3 & 0 & 7 \\ -2 & -1 & -3 \end{bmatrix}$$

Q5. Construct second Lagrange's Interpolating polynomial for

$$f(x) = \frac{1}{x^2}$$

if $x_0 = 2, x_1 = 2.75, x_2 = 4$ and use the polynomial (constructed) to approximate $f(3) = 1/9$

Q6. Apply Newton Raphson procedure to find real root of the equation.

$$x^3 + 3x^2 - 1 = 0, \ [-3, -2]$$

Q.1

...of Sargodha

BS 7th Term/Semester Exam 2021.

Subject: Mathematics

Time Allowed: 2:30 Hours

Paper: Number Theory (MATH-403)

Maximum Marks: 80

Note: Objective part is compulsory. Attempt any three questions from subjective Part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each. (2*16)

i. Find G.C.D. and L.C.M. of 273 and 81.
ii. Define Bracket function.
iii. Show that if $ax+by=m$ then $(a,b)|m$.
iv. Define reduced residue system.
v. Define Least common multiple.
vi. Find order of 3 modulo 11.
vii. Define algebraic number field.
viii. Write congruence classes for modulo 11 with remainders 3 and 10.
ix. Define symmetric polynomial.
x. Define units of algebraic number field.
xi. Find the values of x, y and z which satisfy $6x+10y+15z=1$. $x=1$ $y=1$ $z=-1$
xii. Find the Residue of 3^{16} modulo 17.
xiii. Define Linear congruence.
xiv. Define primitive root.
xv. If $na \equiv nb \pmod{m}$ and $(n, m)=1$, then show that $a \equiv b \pmod{m}$.
xvi. Prove that every rational integer is an algebraic integer.

Subjective Part (3*16)

Q.2. (a) Prove that Mobius function is multiplicative.

(b) Evaluate the Legendre symbol $\left(\frac{503}{773}\right)$.

Q.3. (a) State and prove unique factorization theorem.

(b) If $\{a_1, a_2, \ldots, a_{\phi(m)}\}$ is a reduced residue system (mod m) and $(a, m)=1$ then prove that $\{aa_1, aa_2, \ldots, aa_{\phi(m)}\}$ is also a reduced residue system (mod m).

Q.4. (a) Prove that an integer α is a root of the congruence $f(x) \equiv 0 \pmod{m}$ if and only if $(x-\alpha)|f(x) \pmod{m}$.

(b) Given that 2 is a primitive root of 9, use indices to solve the following congruence $10x \equiv 8 \pmod{18}$.

Q.5. (a) State and prove Chinese Remainder Theorem.

(b) Prove that the set of algebraic numbers forms a field.

Q.6. (a) Prove that equation $x^4 + y^4 = z^4$ has no solution in integers.

(b) Prove that a necessary and sufficient condition that the equation $ax+by=c$ has a solution (x, y) in integers, is that $d \mid c$ where $d=(a, b)$ and this solution is of the form $x = x_0 + \frac{b}{d}t$, $y = y_0 - \frac{a}{d}t$ where t is an arbitrary integer.

University of Sargodha

BS 7th Term/Semester Exam 2021

Subject: Mathematics | **Paper: Partial Differential Equations (MATH:405)**

Maximum Marks: 80 | **Time Allowed: 2:30 Hours**

Note: Objective part is compulsory. Attempt any four questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each. (2*16)

i. Write the difference between linear and non - linear partial differential equation.
ii. Define principle of Superposition.
iii. Obtain PDE $z = x + ax^2y^2 + b$ where a,b are arbitrary constants.
iv. Write down the Physical Meanings of The Drichlet Boundary conditions.
v. Define Canonical Form of first order linear equation.
vi. Write mathematical form of the telegraph equation.
vii. Define specific heat of substance.
viii. Write condition for existence of Fourier transformation.
ix. Prove that $\int_{-\infty}^{\infty} |f(x)|^2\, dx = \int_{-\infty}^{\infty} |F(k)|^2\, dk$
x. Find the Fourier series expansion for the function $f(x) = x + x^2 \;\; ; -\pi < x < \pi$
xi. Find Laplace Transformation of the error Function.
xii. If $\mathcal{L}\{f(t)\} = F(s)$ then $\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} F(s)\, ds$
xiii. Find The Laplace transform of an impulse function.
xiv. Let $f'(x)$ be continuous and $f''(x)$ be peicewise continuous in $[0,\pi]$ if $F_s(k)$ is the finite Fourier Sine Transform of $f(x)$ then prove that
$$\mathcal{F}_s\{f''(x)\} = \frac{2k}{\pi}[f(0) - (-1)^k f(\pi)] - k^2 F_s(k)$$
xv. Show that the Hankel transform satisfies the Parseval relation
$$\int_0^{\infty} r f(r)\, g(r)\, dr = \int_0^{\infty} k \tilde{f}(\kappa) \tilde{g}(\kappa)\, dk$$
xvi. Write the Difference between Laplace and Fourier Transform.

Subjective Part (4*12)

Q.2. Show that the general solution of the linear equation
$$(y - z)\, u_x + (z - x)\, u_y + (x - y)\, u_z = 0$$
is $u(x, y, z) = f(x + y + z, x^2 + y^2 + z^2)$ where f is an arbitrary function.

Q.3. Solve the initial-value problem $u_x + 2u_y = 0$, $u(0, y) = 4e^{-2y}$.

Q.4. State and Prove one Dimensional Wave Equation.

Q.5. Derive Laplace equation in cylindrical coordinates.

Q.6. Let $f(x)$ and its first derivative vanish as $x \to \infty$. If $F_c(k)$ is the Fourier cosine transform, then prove that $\mathcal{F}_c\{f''(x)\} = -k^2 F_c(k) - \sqrt{\frac{2}{\pi}} f'(0)$ $\quad -\sqrt{\frac{2}{\pi}}\, k\, f(0)$

Q.7. Consider the motion of a semi-infinite string with an external force $f(t)$ acting on it. One end is kept fixed while the other end is allowed to move freely in the vertical direction. If the string is initially at rest, the motion of the string is governed by
$$u_{tt} = c^2 u_{xx} + f(t), \; 0 < x < \infty, t > 0$$
$$u(x, 0) = 0,\; u_t(x, 0) = 0 \text{ and } u(0, t) = 0,\; u_x(x, t) \to 0, \text{ as } x \to \infty$$

University of Sargodha

BS 7th Semester/Term Exam 2021

Subject: Mathematics **Paper: Modern Algebra-I (MATH-409/415)**

Time Allowed: 02:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)

i. Define Maximal Ideal.
ii. Define Principal Ideal Domain.
iii. Find the unit elements of $Z[i]$.
iv. Show that the ring of integers is Euclidean Domain.
v. Define Unique Factorization domain.
vi. Define zero divisor.
vii. Define Primitive Polynomial.
viii. Show that $T = \left\{\begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \mid a, b \in Z\right\}$ is a sub ring of $M_2(Z)$.
ix. Prove that intersection of any two subrings of ring R is sub ring.
x. Give the reason that why $z[x]$ is not PID.
xi. Prove that $< 4 >$ is not Prime ideal in the ring of integer.
xii. Define Division ring.
xiii. Prove that Ring of integers is not isomorphic to ring of rationales.
xiv. Define Polynomial Ring.
xv. Define Gussian Integral Domain.
xvi. Define Associate relation between elements of a ring.

Subjective Part (3*16)

Q.2. (a) State and Prove 2nd Theorem of Ring Homomorphism. (8)
(b) If R is integral domain the $R[x]$ is integral domain. (8)

Q.3. (a) State and Prove Division Algorithm in $R[x]$. (8)
(b) Prove that the set $Z[i] = \{a + ib : a, b \in Z\}$ is a subring of set of complex number. (8)

Q.4. (a) Let R be Principal ideal ring and $p \in R$. If p is irreducible then p is prime. (8)
(b) Prove that $Z[i]$ is Euclidean Domain. (8)

Q.5. (a) Prove that $Z[i\sqrt{5}]$ is not UFD. (8)
(b) In a UFD every irreducible element is prime. (8)

Q.6. (a) Prove that every finite Integral Domain is a field. (8)
(b) If $\varphi: R \to R'$ is ring homomarphisem then $ker\varphi$ is ideal of R. (8)

University of Sargodha

BS 7th Term/Semester Exam 2021

Subject: Mathematics **Paper: Partial Differential Equations (MATH:405)**

Maximum Marks: 80 **Time Allowed: 2:30 Hours**

Note: Objective part is compulsory. Attempt any four questions from subjective part.

Objective Part **(Compulsory)**

Q.1. Write short answers of the following in 2-3 lines each. (2*16)

i. Write the difference between linear and non - linear partial differential equation.
ii. Define principle of Superposition.
iii. Obtain PDE $z = x + ax^2y^2 + b$ where a,b are arbitrary constants.
iv. Write down the Physical Meanings of The Drichlet Boundary conditions.
v. Define Canonical Form of first order linear equation.
vi. Write mathematical form of the telegraph equation.
vii. Define specific heat of substance.
viii. Write condition for existence of Fourier transformation.
ix. Prove that $\int_{-\infty}^{\infty}|f(x)|^2\,dx = \int_{-\infty}^{\infty}|F(k)|^2\,dk$
x. Find the Fourier series expansion for the function $f(x) = x + x^2 \quad ; -\pi < x < \pi$
xi. Find Laplace Transformation of the error Function.
xii. If $\mathcal{L}\{f(t)\} = F(s)$ then $\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} F(s)\,ds$
xiii. Find The Laplace transform of an impulse function.
xiv. Let $f'(x)$ be continuous and $f''(x)$ be peicewise continuous in $[0,\pi]$ if $F_s(k)$ is the finite Fourier Sine Transform of $f(x)$ then prove that
$$\mathcal{F}_s\{f''(x)\} = \frac{2k}{\pi}[f(0) - (-1)^k f(\pi)] - k^2 F_s(k)$$
xv. Show that the Hankel transform satisfies the Parseval relation
$$\int_0^{\infty} r f(r)\, g(r) dr = \int_0^{\infty} k \bar{f}(\kappa)\bar{g}(\kappa)\, dk$$
xvi. Write the Difference between Laplace and Fourier Transform.

Subjective Part **(4*12)**

Q.2. Show that the general solution of the linear equation
$$(y - z)\, u_x + (z - x)\, u_y + (x - y)\, u_z = 0$$
is $u(x, y, z) = f(x + y + z, x^2 + y^2 + z^2)$ where f is an arbitrary function.

Q.3. Solve the initial-value problem $u_x + 2u_y = 0$, $u(0, y) = 4e^{-2y}$.

Q.4. State and Prove one Dimensional Wave Equation.

Q.5. Derive Laplace equation in cylindrical coordinates.

Q.6. Let $f(x)$ and its first derivative vanish as $x \to \infty$. If $F_c(k)$ is the Fourier cosine transform, then prove that $\mathcal{F}_C\{f''(x)\} = -k^2 F_c(k) - \sqrt{\frac{2}{\pi}} f'(0)$

Q.7. Consider the motion of a semi-infinite string with an external force $f(t)$ acting on it. One end is kept fixed while the other end is allowed to move freely in the vertical direction. If the string is initially at rest, the motion of the string is governed by
$$u_{tt} = c^2 u_{xx} + f(t), \quad 0 < x < \infty, t > 0$$
$$u(x, 0) = 0,\ u_t(x, 0) = 0 \ \text{ and } \ u(0, t) = 0,\ u_x(x, t) \to 0, \text{ as } x \to \infty$$

University of Sargodha

BS 7th Term/Semester Exam 2021.

Subject: Mathematics **Paper: Number Theory (MATH-403)**

Time Allowed: 2:30 Hours **Maximum Marks: 80**

Note: **Objective part is compulsory. Attempt any three questions from subjective Part.**

Objective Part **(Compulsory)**

Q.1. Write short answers of the following in 2-3 lines each. (2*16)

i. Find G.C.D. and L.C.M. of 273 and 81.
ii. Define Bracket function.
iii. Show that if $ax+by=m$ then $(a,b)|m$.
iv. Define reduced residue system.
v. Define Least common multiple.
vi. Find order of 3 modulo 11.
vii. Define algebraic number field.
viii. Write congruence classes for modulo 11 with remainders 3 and 10.
ix. Define symmetric polynomial.
x. Define units of algebraic number field.
xi. Find the values of x, y and z which satisfy $6x+10y+15z=1$.
xii. Find the Residue of 3^{16} modulo 17.
xiii. Define Linear congruence.
xiv. Define primitive root.
xv. If $na \equiv nb \pmod{m}$ and $(n, m)=1$, then show that $a \equiv b \pmod{m}$.
xvi. Prove that every rational integer is an algebraic integer.

Subjective Part **(3*16)**

Q.2. **(a)** Prove that Mobius function is multiplicative.

(b) Evaluate the Legendre symbol $\left(\frac{503}{773}\right)$.

Q.3. **(a)** State and prove unique factorization theorem.

(b) If $\{a_1, a_2, \ldots, a_{\phi(m)}\}$ is a reduced residue system (mod m) and $(a, m)=1$ then prove that $\{aa_1, aa_2, \ldots, aa_{\phi(m)}\}$ is also a reduced residue system (mod m).

Q.4. **(a)** Prove that an integer α is a root of the congruence $f(x) \equiv 0 \pmod{m}$ if and only if $(x-\alpha)|f(x) \pmod{m}$.

(b) Given that 2 is a primitive root of 9, use indices to solve the following congruence $10x \equiv 8 \pmod{18}$.

Q.5. **(a)** State and prove Chinese Remainder Theorem.

(b) Prove that the set of algebraic numbers forms a field.

Q.6. **(a)** Prove that equation $x^4+y^4=z^4$ has no solution in integers.

(b) Prove that a necessary and sufficient condition that the equation $ax+by=c$ has a solu
(x, y) in integers, is that $d \mid c$ where $d=(a, b)$ and this solution is of the form
$x = x_0 + \frac{b}{d}t$, $y = y_0 - \frac{a}{d}t$ where t is an arbitrary integer.

University of Sargodha

BS 7th Semester/Term Exam 2021

Subject: Mathematics **Paper: Modern Algebra-I (MATH-409/415)**

Time Allowed: 02:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)

i. Define Maximal Ideal.
ii. Define Principal Ideal Domain.
iii. Find the unit elements of $Z[i]$.
iv. Show that the ring of integers is Euclidean Domain.
v. Define Unique Factorization domain.
vi. Define zero divisor.
vii. Define Primitive Polynomial.
viii. Show that $T = \left\{\begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \mid a, b \in Z\right\}$ is a sub ring of $M_2(Z)$.
ix. Prove that intersection of any two subrings of ring R is sub ring.
x. Give the reason that why $z[x]$ is not PID.
xi. Prove that $< 4 >$ is not Prime ideal in the ring of integer.
xii. Define Division ring.
xiii. Prove that Ring of integers is not isomorphic to ring of rationales.
xiv. Define Polynomial Ring.
xv. Define Gussian Integral Domain.
xvi. Define Associate relation between elements of a ring.

Subjective Part (3*16)

Q. 2. (a) State and Prove 2nd Theorem of Ring Homomorphism. (8)
(b) If R is integral domain the $R[x]$ is integral domain. (8)

Q.3. (a) State and Prove Division Algorithm in $R[x]$. (8)
(b) Prove that the set $Z[i] = \{a + ib : a, b \in Z\}$ is a subring of set of complex number. (8)

Q.4. (a) Let R be Principal ideal ring and $p \in R$. If p is irreducible then p is prime. (8)
(b) Prove that $Z[i]$ is Euclidean Domain. (8)

Q.5. (a) Prove that $Z[i\sqrt{5}]$ is not UFD. (8)
(b) In a UFD every irreducible element is prime. (8)

Q.6. (a) Prove that every finite Integral Domain is a field. (8)
(b) If $\varphi: R \to R'$ is ring homomarphisem then $ker\varphi$ is ideal of R. (8)

University of Sargodha

BS 7th Semester/Term Exam 2021

Subject: Mathematics **Paper: Numerical Analysis-I (MATH-401)**

Time Allowed: 02:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)

i) Define positive deifinite matrix, give example. ii) What is the difference between Doolittle and Cholesky's method. iii) Define Non-linear equation and give examples. iv) How Secant method works, explain v) What is the difference between Gauss-Seidel and Jacobi method. vi) What is an ill-conditioned system. vii) Explain difference between Cholesky's and Crout's method. viii) Explain extrapolation with example ix) How you define Interpolation. x) Descibe advantage of Regula Falsi method over Bisection method of root finding problems. xi) Define Condition Number. xii) Define norm of matrix. xiii) Descibe differnce between relative and absolute error. xiv) What is the difference between direct and iterative methods. xv) What is partial pivoting. xvi) Define dominant eignem value.

Subjective Part. (3×16)

Q2. Solve the following system of equations using Crout's method.

$$\begin{cases} 3x + y + 2z = 3 \\ 2x - 3y - z = -3 \\ x - 2y + z = 4 \end{cases}$$

Q3. Apply Gauss Seidel method to approximate the solution (of the following system of linear equations) by performing three iterations.

$$\begin{cases} 4x_1 + x_2 - x_3 = 5 \\ -x_1 + 3x_2 + x_3 = -4 \\ 2x_1 + 2x_2 + 5x_3 = 1 \end{cases}$$

Q4. Test the matrix for ill-conditioned or not?

$$\begin{bmatrix} 4 & 2 & 6 \\ 3 & 0 & 7 \\ -2 & -1 & -3 \end{bmatrix}$$

Q5. Construct second Lagrange's Interpolating polynomial for

$$f(x) = \frac{1}{x^2}$$

if $x_0 = 2, x_1 = 2.75, x_2 = 4$ and use the polynomial (constructed) to approximate $f(3) = 1/9$

Q6. Apply Newton Raphson procedure to find real root of the equation.

$$x^3 + 3x^2 - 1 = 0, \ [-3, -2]$$

Domain lie b/w 0 and 2.

University of Sargodha

BS 6th Term Examination 2017

Subject: Mathematics **Paper: Partial Differential Equation (MATH-308)**

Time Allowed: 2:30 Hours

Maximum Marks: 80

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each. (2*16)

i. Define the canonical form of first order partial differential equations.

ii. Define order, semi linear, linear and non-linear partial differential equation and give at least one example of each.

iii. Define one dimensional wave equation. → $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$

iv. Find the Fourier transform of function $e^{-a|x|}$.

v. If $u(x,t)$ is a displacement of a vibrating string and its end are fixed at $x=-3$ and $x=3$, construct the boundary conditions of at least two types. → ~~P.D.E $u_{tt} = c^2 u_{xx}$~~

vi. Convert $u_t = c^2 u_{xx}$ into ordinary differential equation.

vii. Show that $u(x,t) = f(x-ct) + g(x+ct)$ is a solution of wave equation in one dimension when f and g are any smooth functions.

(5) P.D.E : $u_{tt} = c^2 u_{xx}$, $0<x<L$, $0<t<\infty$
B.C : $u(0,t)=0$, $u(L,t)=0$, $0<t<\infty$
I.C : $u(x,0)=f(x)$, $u_t(x,0)=g(x)$, $0<x<L$

viii. Write the Laplace Transform of a function $f(t) = \operatorname{erfc}\sqrt{t}$.

ix. Find the Laplace Transform of $f'''(t)$.

x. Define Inverse Laplace Transform.

xi. Write the mathematical form of second order Laplace equation.

xii. Give an example of parabolic equation. → the set of points that are equidistance from both the directrix and the focus. $2x^2 - x - 4y + 7 = 0$

xiii. Give an example of hyperbolic equation. (wave equation)

xiv. Find Inverse Laplace Transform of $\frac{1}{s+4}$. ⇒ e^{-4t}

xv. Evaluate $\frac{1}{\pi}\int_{-\pi}^{\pi}(x+x^2)\cos kx\,dx$.

xvi. Define Complex Fourier series. Let the function $f(x,y)$ be defined over the whole complex plane; i.e; $f(x,y)$ is defined for $-\infty<x<\infty$, $-\infty<y<\infty$; Then the fourier transform is defined as $F(k_1,k_2) = \frac{1}{(\sqrt{2\pi})^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)$ ... $F\{f(x,y)\} = \frac{1}{2\pi}\int\int$ ...

Subjective Part (3*16)

Q.2: Find the general solution of $yu_{xx} + 3yu_{xy} + 3u_x = 0,\quad y \neq 0$.

Q. 3: State and Prove the shifting and scaling properties of Fourier Transform of $f(t)$.

Q. 4: State and prove Convolution Theorem of the Fourier Transform.

Q. 5: Show that

(i) $\ell\left[f^{(n)}(t)\right] = s^n \ell\left[f(t)\right] - s^{n-1}f(0) - \dots - sf^{(n-2)}(0) - f^{(n-1)}(0)$

(ii) State and Prove Convolution Theorem of the Laplace Transform

Q. 6: If $f(s)$ and $g(s)$ are the Laplace Transforms of $f(t)$ and $g(t)$ respectively, then

(i) $\ell\left[H(t-a)f(t-a)\right] = e^{-as}\ell\left[f(t)\right]$

(ii) $\ell\left[H(t-a)g(t)\right] = e^{-as}\ell\left[g(t+a)\right]$

Inverse laplace: If $G(s) = L\{g(t)\}$, then the invert transform of $G(s)$ is defined as; $L^{-1}G(s) = g(t)$

University of Sargodha

BS 6th Semester, Final Term Exam 2018

Subject: Mathematics Course: Partial Differential Equations (MATH: 308)

Time Allowed: 2:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

(2*16)

Q. No. 1: Write short answers of the following in 2-3 lines each.

(i) Define order of the partial differential equation.
(ii) Define Neumann boundary conditions.
(iii) Formulate a boundary value problem for heat conduction in a homogeneous rod of constant resection "A" and length "a" insulated laterally and whose ends are perfectly insulated.
(iv) Formulate a boundary value problem for vibration in a string whose ends are fastened to air bearing which are free to move along rod perpendicular to x-axis.
(v) Convert the partial differential equation into ordinary differential equation through suitable substitution. $\frac{\partial u}{\partial t} = K\frac{\partial^2 u}{\partial x^2}$.
(vi) What do you mean by a nonlinear partial differential equation?
(vii) Give an example of a boundary value problem having Dritchlet conditions.
(viii) Write Fourier transform and inverse Fourier transform of a function $f(t)$.
(ix) Find the Fourier transform of

$$f(t) = e^{-bt}; \quad t > 0,$$
$$= 0; \quad t < 0.$$

(x) Find the Fourier transform of the derivative of $f(t)$.
(xi) Convert the equation. $\frac{\partial u}{\partial t} = -C\frac{\partial^4 u}{\partial x^4}$ into ordinary differential equation by fourier transform method.
(xii) Define the basic conditions to find a Laplace transform of the function.
(xiii) Find the Laplace transform of t^n.
(xiv) What is the difference between Laplace and Fourier transformations?
(xv) Find the inverse Laplace transform of $\frac{-2s+6}{s^2+4}$.
(xvi) What do you mean by the Robins boundary conditions?

Subjective Part (3*16)

Q. No. 2: Solve the boundary value problem by separation of variables

Sol (register)

$$\frac{\partial u}{\partial t} = K\frac{\partial^2 u}{\partial x^2},$$

for $u(0,t) = 0 \quad t > 0, \quad u(L,t) = 0 \quad t > 0,$
$u(x,0) = x \quad 0 < x > L.$

Q. No. 3: Solve $r^2\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0$

$u(c,\theta) = 100 \quad 0 < \theta < \pi, \quad u(c,\theta) = 0 \quad 0 < \theta < \pi.$

Q. No. 4: Solve the non homogeneous partial differential equation

$$\frac{\partial u}{\partial t} = K\frac{\partial^2 u}{\partial x^2} + \mathrm{Sin}(x),$$

where $u(0,t) = 500 \quad t > 0, \quad u(\pi,t) = 100 \quad t > 0,$
$u(x,0) = f(x) \quad 0 < x < \pi.$

Q. No. 5: Find out temperature in an infinite rod using Fourier transforms

$$\frac{\partial u}{\partial t} = K\frac{\partial^2 u}{\partial x^2},$$

Subjected to
$u(x,0) = 400K, \quad -4 \leq x \leq 4.$
$0\,K \quad$ elsewhere.

Q. No. 6: By using Laplace transforms, solve the boundary value problem

$$\frac{\partial 2u}{\partial t2} = K\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \quad t > 0$$

Subjected to
$u(0,t) = 0, \quad u(1,t) = 0, \quad t > 0$
$u(x,0) = 0, \quad \frac{\partial u}{\partial t} = \sin\pi x, \quad$ at $t=0$, $0 < x < 1.$

Q.6

University of Sargodha

BS 6th Term Examination 2019

Subject: Mathematics **Paper: Partial Differential Equations (MATH-308)**

Time Allowed: 2:30 Hours

Maximum Marks: 80

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (16*2)

i. What is the difference between particular solution and singular solution?
ii. Define Cauchy Problem for first order partial differential equation.
iii. Define weak solution and explain when a Quasi-linear and non-linear PDE has weak solution.
iv. Write $u_{xx} - 4u_{xy} + 4u_{yy} = e^y$ in canonical form.
v. What is effect if Jacobian vanishes in canonical form of PDE's?
vi. If $u(x,t) = \phi(x+ct) + \psi(x-ct)$ is the solution of wave equation then explain physical representation of ϕ and ψ functions.
vii. Define the Uniqueness Theorem for wave equation.
viii. Why uniform convergence is stronger than both pointwise and mean-square convergence?
ix. Define $\text{sgn}(x)$ function.
x. Write some conditions of applicability of the method of separation of variable.
xi. Give any two examples of time-independent non-homogeneous PDE's.
xii. How Laplace transform is closely related to complex Fourier transform?
xiii. Prove Fourier convolution property that is $f * \sqrt{2\pi}\delta = \sqrt{2\pi}\delta * f$.
xiv. Obtained the zero order Hankel transform of $r^{-1}\exp(-ar)$.
xv. Define Cylindrical wave equation.
xvi. When a function is piecewise smooth on the interval.

Subjective Part (3*16)

Q.2. Determine the integral surface of the equation $x(y^2+u)u_x - y(x^2+u)u_y = (x^2-y^2)u$ with data $x+y=0, u=1$.

Q.3. Use Spherical polar coordinates transform the three dimensional Laplace equation into Spherical form.

Q.4. Find the solution of signaling problem governed problem by the equation $u_{tt} = c^2u_{xx}, x>0, t>0$ with $u(x,0) = u_t(x,0) = 0$, and $u(0,t) = U(t)$.

Q.5. Solve $u_{tt} = c^2u_{xxxx}, -\infty < x < \infty, t > 0$ with $u(x,0) = f(x), u_t(x,0) = 0$ using Laplace transform.

Q.6. Find the temperature distribution in a rod of length l. The faces are insulated, and the initial temperature distribution is given by $x(l-x)$.

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University of Sargodha

BS 6th Semester Exam 2019

Subject: Mathematics **Paper: Partial Differential Equations (MATH:308)**

Time Allowed: 2:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any four questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each. (2*16)

(i)	Define hyperbolic partial differential equation.
(ii)	If $u(x,t)$ is the displacement of a vibrating string and its ends are fixed at $x=0$ and $x=L$. Formulate boundary value problem.
(iii)	Define Neumann Boundary conditions.
(iv)	Write the boundary value problem for a temperature $u(x,t)$ in a latterally insulated rod in which no heat is generated.The temperature at $x=0$ is zero and the end $x=L$ is perfectly insulated, the initial temperature is $100-x$.
(v)	Define orthogonal function.
(vi)	Find the Fourier transform of $f'(t)$.
(vii)	Convert the PDE $c^2U_{xx}=U_{tt}$ into ordinary differential equations..
(viii)	Define Fourier law of heat conduction.
(ix)	Define a well posed mathematical problem.
(x)	Why we need 2nd initial condition to solve wave equation.
(xi)	Find the nature of Heat and Laplace equation.
(xii)	Prove the shifting property for Laplace transform.
(xiii)	Write the formula for Fourier cosine transform pair.
(xiv)	Write the Laplace transform of U_{tt}.
(xv)	Find the Laplace transform of e^{-st}.
(xvi)	Find inverse Laplace transform of $\frac{1}{(s+a)^n}$.

Subjective Part

Q.2	Derive one dimensional heat equation.	12
Q.3	Solve the following Initial Boundary Value Problem by seperation of variables method $$U_t = kU_{xx}, \quad 0 < x < L, \quad t > 0$$ $$U(0,t) = 0$$ $$U(L,t) = 0$$ $$U(x,0) = x.$$	12
Q.4	Find D. Alembert solution of $$U_t = c^2 U_{xx}, \quad 0 < x < a, \quad t > 0$$ $$U(0,t) = 0 = U(a,t)$$ $$U(x,0) = f(x)$$ $$U_t(x,0) = g(x).$$	12
Q. 5	Evaluate $\int_0^\infty \frac{\sin a\alpha}{\alpha} d\alpha$ by first finding the Fourier transform of $$f(x) = \begin{cases} 1, & \lvert x \rvert \leq a \\ 0, & \lvert x \rvert > a \end{cases}$$	12
Q. 6	State and prove differentiation property for nth order derivative of Laplace transform. Also prove that $L[\frac{f(t)}{t}] = \int_s^\infty \bar{f}(s)ds$	12
Q. 7	Solve the potential equaion for the potential $u(x,y)$ in the semi infinite strip $0 < x < c$, $y > 0$ that satisfies the following conditions $$u(0,y) = 0, \quad u_y(x,0) = 0, \quad u_x(c,y) = f(y)$$ by using Fourier sine or cosine tranform method.	12

University of Sargodha

BS 7th Semester Examination 2022

Subject: Mathematics **Paper: Numerical Analysis-I (MATH-401)**

Time Allowed: 2:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q1. Write short answers of the following in 2-3 lines each. (2×16)

i) Define zero of a function, give example. ii) State fixed point theorem. iii) Define fixed point of a function. iv) Descibe differnce between relative and absolute error. v) Define linear equation and give examples. vi) How Regula Falsi method works, explain vii) What is the difference between Gauss-Seidel and Jacobi method. viii) What is an ill-conditioned system. ix) Explain difference between Cholesky's and Crout's method. x) Explain convergence criteria of bisection method. xi) How you define Eigen vector. xii) Descibe advantage of Regula Falsi method over Bisection method. xiii) Define Condition Number. xiv) Define norm of matrix. xv) What is the difference between direct and iterative methods. xvi) What is partial pivoting.

Subjective Part. (3×16)

Q2. Solve the system of linear equations using Doolittle's Method.

$$\begin{aligned} x + y + z &= 5 \\ x + 2y + 2z &= 6 \\ x + 2y + 3z &= 8 \end{aligned}$$

Q3. Apply Jacobi method to approximate the solution by performing three iterations.

$$\begin{cases} 4x_1 + x_2 - x_3 = 5 \\ -x_1 + 3x_2 + x_3 = -4 \\ 2x_1 + 2x_2 + 5x_3 = 1 \end{cases}$$

Q4. Apply power method to find dominant eigen value and corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 4 & 1 & -2 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$$

Q5. Find a real root of the following non-linear equation using fixed point iteration

$$x^4 - 4x^3 + 6x^2 - 2.25 = 0$$

Q6. Apply Newton Raphson procedure to find real root of the equation.

$$x^3 + 3x^2 - 1 = 0, \quad [-3, -2]$$

BMAF16M043

DEPARTMENT OF MATHEMATICS (SU) SARGODHA
FINAL TERM EXAMINATION, FALL 2019

CLASS: BS-VII + MSc-III (R+SS)

Course Title: Advance Group Theory-I | TIME ALLOWED: 2 Hr.

MAX. MARKS: 50 | Course Code: MATH-403(BS) MATH-633 (MSc)

Dated: 30-12-2019

Note: Attempt all questions. Each question carries 10 marks

Q: 1 Answer the following. Each carrying 2 marks.

i. Prove that every characteristic subgroup is normal.

ii. Show that $U(16)$ is a p-group for some prime p.

iii. For $n \geq 5$, prove that any two 3-cycles in A_n are conjugate in A_n.

iv. State whether a group of order 90 and 120 is simple or not?

v. Find the inverse of the following permutations.

(a) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 5 & 3 & 1 & 4 & 7 & 6 \end{pmatrix}$ (b) (2459)(173)(86)

Q: 2 Prove that identity permutation is an even permutation.

Q: 3 Prove that a group G of order 56 has 1 or 8 Sylow 7-subgroups. In latter case, prove that G has a normal Sylow 2-subgroup. Hence deduce that a group of order 56 is not simple.

Q: 4 Find all conjugacy classes of A_4 and further derive all normal subgroups of A_4.

Q: 5 State and Prove Sylow's first theorem.

-----Best of Luck-----

Department of Mathematics, University of Sargodha

Paper: Modern Algebra-I(PPP)

Time Allowed: 2:00hrs. Class: BS VII Max Marks: 50

Q.No.1 Let R be a ring with 1. Prove that $R[x]/<x> \cong R$.

Q.No.2 State and prove 3[rd] isomorphism theorem for rings.

Q.No.3 Every Eucidean domain is a PID.

Q.No.4 Prove that a factorization domain D is a UFD iff every irreducible eleme
of D is a prime element.

Q.No.5 Define ACCP and Prove that every PID D satisfies the ACCP.

University of Sargodha

BS 7th Term/Semester Exam 2021

Subject: Mathematics **Paper: Partial Differential Equations (MATH:405)**

Time Allowed: 2:30 Hours

Maximum Marks: 80

Note: Objective part is compulsory. Attempt any four questions from subjective part.

Objective Part (Compulsory)

(2*16)

Q.1. Write short answers of the following in 2-3 lines each.

i. Write the difference between linear and non - linear partial differential equation.
ii. Define principle of Superposition.
iii. Obtain PDE $z = x + ax^2y^2 + b$ where a,b are arbitrary constants.
iv. Write down the Physical Meanings of The Drichlet Boundary conditions.
v. Define Canonical Form of first order linear equation.
vi. Write mathematical form of the telegraph equation.
vii. Define specific heat of substance.
viii. Write condition for existence of Fourier transformation.
ix. Prove that $\int_{-\infty}^{\infty}|f(x)|^2\,dx = \int_{-\infty}^{\infty}|F(k)|^2\,dk$
x. Find the Fourier series expansion for the function $f(x) = x + x^2 \quad ; -\pi < x < \pi$
xi. Find Laplace Transformation of the error Function.
xii. If $\mathcal{L}\{f(t)\} = F(s)$ then $\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} F(s)\,ds$
xiii. Find The Laplace transform of an impulse function.
xiv. Let $f'(x)$ be continuous and $f''(x)$ be peicewise continuous in $[0,\pi]$ if $F_s(k)$ is the finite Fourier Sine Transform of $f(x)$ then prove that
$$\mathcal{F}_s\{f''(x)\} = \frac{2k}{\pi}[f(0) - (-1)^k f(\pi)] - k^2 F_s(k)$$
xv. Show that the Hankel transform satisfies the Parseval relation
$$\int_0^{\infty} rf(r)\,g(r)dr = \int_0^{\infty} k\bar{f}(\kappa)\bar{g}(\kappa)\,dk$$
xvi. Write the Difference between Laplace and Fourier Transform.

Subjective Part (4*12)

Q.2. Show that the general solution of the linear equation
$$(y-z)\,u_x + (z-x)\,u_y + (x-y)\,u_z = 0$$
is $u(x, y, z) = f(x + y + z, x^2 + y^2 + z^2)$ where f is an arbitrary function.

Q.3. Solve the initial-value problem $u_x + 2u_y = 0$, $u(0, y) = 4e^{-2y}$.

Q.4. State and Prove one Dimensional Wave Equation.

Q.5. Derive Laplace equation in cylindrical coordinates.

Q.6. Let $f(x)$ and its first derivative vanish as $x \to \infty$. If $F_c(k)$ is the Fourier cosine transform, then prove that $\mathcal{F}_c\{f''(x)\} = -k^2F_c(k) - \sqrt{\frac{2}{\pi}}f'(0)$

Q.7. Consider the motion of a semi-infinite string with an external force $f(t)$ acting on it. One end is kept fixed while the other end is allowed to move freely in the vertical direction. If the string is initially at rest, the motion of the string is governed by
$$u_{tt} = c^2u_{xx} + f(t), 0 < x < \infty, t > 0$$
$$u(x, 0) = 0,\ u_t(x, 0) = 0 \text{ and } u(0, t) = 0,\ u_x(x, t) \to 0, \text{ as } x \to \infty$$

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University of Sargodha

BS 6th Term Examination 2016

Subject: Mathematics **Paper: Numerical Analysis (Math: 302)**

Time Allowed: 2:30 Hours Maximum Marks: 80

Objective Part (Compulsory)

Q. No. 1: Write short answers of the following in 2-3 lines each. (2*16)

(i) Define definite relative error.

(ii) Define truncation error.

(iii) Define transcendental equation.

(iv) Explain convergence of Newton Raphson Method.

(v) Express $\Delta^3 y_1$ in terms of the values of the function y.

(vi) Using backward difference prove that $y_{n-3} = (1-\nabla)^3 y_n$.

(vii) Prove that $hD = \log E$, where D, E are differential and Shift operator resp.

(viii) Prove that $\Delta = \frac{\delta^2}{2} + \delta\sqrt{(1+(\delta^2)/2)}$.

(ix) Prove that $\mu E = E\mu$, where μ stands for average operator.

(x) Prove that $hD = \sinh^{-1}(\mu\delta)$.

(xi) What is Simpson's rule and maximum error formula in Simpson's rule?

(xii) What is the convergence condition of Method of iteration to find root?

(xiii) Show that the operator δ and E commute with one another.

(xiv) Define Average operator.

(xv) What is the value of x, using forward difference of the following data (1, 2), (2, 5), (4, x)?

(xvi) Geometrically, what is the difference between 1/3 Simpson's rule and 3/8 Simpson rule?

(Subjective Part)

Note: Attempt any three questions.

Q. No. 2: Compute the following by using 1/3 Simpson's Rule with eight intervals.

$$\int_{0.2}^{1.5} e^{-x^2}dx$$

Regular Falsi method

(16)

Q. No. 3: Use Regula-Falsi Method to find the real roots of the equation $3x + \sin x - e^x = 0$ upto five successive approximations between (0, 1). (16)

Q. No. 4: Solve the following system of equations by using Gauss-Seidel method:

$$5x - 2y + z = 4$$
$$7x + y - 5z = 8$$
$$3x + 7y + 4z = 10$$

(16)

Q. No. 5: Find equation of the cubic curve which passes through the points (4, -43), (7, 83), (9, 327) and (12, 1053) using divided difference formula. (16)

Q. No. 6: Find the largest eigenvalue of the matrix $\begin{bmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 0 & 4 \end{bmatrix}$ and the corresponding eigenvector, by power method after fourth iteration starting with the initial vector $v^0 = (0, 0, 1)^T$. (16)

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University of Sargodha

BS 1st Term Examination 2018

Subject: Mathematics   Paper: Number Theory (MATH: 103)   Maximum Marks: 80

e Allowed: 2:30 Hours

ote: Objective Part is compulsory. Attempt any four questions from subjective Part.

Objective Part (Compulsory) (2*16)

Q.1: Write short answers of the following in 2-3 lines each on the answer sheet.

i) Find the argument of $(1-\sqrt{3}i)(1+4i)$.

ii) Express $\frac{-\sqrt{3}+i}{1+2i}$ in polar form.

iii) Prove De Moivre's theorem for $n=2$.

iv) Prove that $Log(-\frac{1}{2}-\frac{3}{2}i)=-\frac{2\pi}{3}i$.

v) Show that $e^{z+2k\pi i}=e^{z}$.

vi) Prove that $\cos^2 z+\sin^2 z=1$ by using exponential function.

vii) Define multiplicative function and give an example.

viii) Find the values of x, y and z which satisfy the equation $6x+10y+15z = 1$.

ix) Write the congruence classes of mod 11 with remainder 7.

x) If (273, 81)=3 then find <273,81>.

xi) Evaluate $\phi(240)$ where ϕ is the Euler's function.

xii) Define reduced residue system.

xiii) Find order of 3 modulo 11.

xiv) Find Quotient and remainder in the division algorithm with divisor 17 and dividend 100.

xv) State the unique factorization theorem".

xvi) Evaluate the following Legender symbols $\left(\frac{-1}{7}\right)$ and $\left(\frac{2}{13}\right)$.

Subjective Part (4*12)

Q.2: Prove that

$$\tan 5\theta=\frac{5\tan\theta-10\tan^3\theta+\tan^5\theta}{1-10\tan^2\theta+5\tan^4\theta}.$$

Q.3: If $\sin(\theta+i\phi)=\cos\alpha+i\sin\alpha$ then Prove that $\cos^2\theta=\pm\sin\alpha$.

Q.4: a) Let $n=p_1^{\alpha_1}p_2^{\alpha_2}...p_r^{\alpha_r}$ be the standard form of n then prove that

$$d(n)=\prod_{i=1}^{r}(\alpha_i+1).$$

Q.4: b) If a, b, and c are integers with $(a,b)=d$ then prove that $(\frac{a}{d},\frac{b}{d})=1$.

Q.5: Find all integral solutions of the following linear Diophantine equation

$$20x+15y=510.$$

Q.6: Prove that number of primes is not finite.

Q.7: Solve the following system of congruences

$$x\equiv 4(mod 5)$$
$$x\equiv 10(mod 14)$$
$$x\equiv 8(mod 11).$$

PDE 2017

University of Sargodha

BS 6th Semester Examination 2017

Subject: Mathematics **Paper: Partial Differential Equations (MATH-308)**

Time Allowed: 2:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part **(Compulsory)**

Q. No.1. Write short answers of the following in 2-3 lines on your answer sheet. (2*16)

1. Define linear and non-linear partial differential equation (PDE). Give an example of each.
2. If $u(x,t)$ is a displacement of a vibrating string and its end are fixed at $x=0$ and $x=L$, find the boundary conditions. $u_x(0,t) = g(t)$ and $u(0,t) = T_0,\ t>0$
3. Convert $u_t = K u_{xx}$ into ordinary differential equation.
4. Show that $u(x,t) = f(x-ct) + g(x+ct)$ is a solution of wave equation in one dimension when f and g are any smooth functions.
5. Define Laplace Transform of a function $f(t)$.
6. Find the Fourier Transform of $f'(t)$.
7. Define Inverse Laplace Transform.
8. Write the mathematical form of telegraph equation.
9. Give an example of parabolic equation.
10. Give an example of hyperbolic equation.
11. Give an example of elliptic equation.
12. Write the heat equation in spherical form.
13. Write the heat equation in cylindrical form.
14. Find Inverse Laplace Transform of $\frac{-2s+6}{s^2+4}$.
15. Evaluate $\frac{1}{\pi}\int_{-\pi}^{\pi}(x+x^2)\cos kx\,dx$.
16. Define Complex Fourier series.

$\frac{1}{\pi}\int_{-\pi}^{\pi}(x+x^2)\cos kx\,dx \Rightarrow \frac{1}{\pi}\left|\frac{x\sin kx}{k}\right|_{-\pi}^{\pi} - \int_{-\pi}$

$\frac{\partial}{\partial t}U_s(k,t) + Kk^2U_s(k,t)\sqrt{\frac{2}{\pi}}$

$u(x,t) =$

$u(0,y) = 0$

$F_c\{u(0,y)\} = F_c(0)$

$U_c(0,k) = 0$

SUBJECTIVE PART

Q.2: Derive one Dimensional heat equation.

Q. 3: Use Fourier Transform to solve $u_{xx} = \frac{1}{K}u_t$ semi-infinite rod with

$$u(0,t) = 0$$

$$u(x,0) = \begin{cases} 1 & 0 < x < \pi \\ 0 & \text{elswhere} \end{cases}$$

Q. 4: Find the Fourier series expansion for the following function

$$f(x) = x + x^2, \quad -\pi < x < \pi$$

Q. 5: Use Laplace Transform to solve the boundary value problem,

$$u_{tt} = u_{xx}, \quad 0 < x < 1,\ t > 0$$

with

$$\begin{cases} u(x,0) = 0, \quad u_t(x,0) = \sin \pi x, & 0 < x < 1 \\ u(0,t) = 0 = u(1,t), & t > 0. \end{cases}$$

Q. 6: Let $f(t)$ be a Piecewise Continuous for $t \geq 0$ and of exponential order. If $f(t)$ is Periodic with Period T, then

$$\mathcal{L}[f(t)] = \frac{1}{1-e^{-sT}}\int_0^T e^{-st} f(t)\,dt$$

8.

University of Sargodha

BS 7th Semester/Term Exam 2021

Subject: Mathematics **Paper: Advanced Group Theory-I (MATH-407)**

e Allowed: 02:30 Hours

Maximum Marks: 80

e: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)

i. Differentiate between inner and outer automorphism.
ii. Define characteristic subgroup.
iii. What is decomposable and indecomposable subgroup?
iv. Define holomorph of a group.
v. What is a transposition? Define even permutation.
vi. What is the length of cycles which generate $A_n, n \geq 3$?
vii. What is Sylow p-group?
viii. Is a group of order 2540 simple? Explain your answer.
ix. Which cyclic group is simple?
x. State second isomorphism theorem.
xi. Define intransitive group.
xii. Define Alternating group.
xiii. Define chief series.
xiv. Define length of an orbit.
xv. Show that the stabilizer is subgroup of the group G.
xvi. What is normal chain condition?

Subjective Part (3*16)

Q.2. a) Prove that $A_n = <(123), (124), \ldots, (12n)>$. (8)
b) Let G be a group with $Z(G)$ as its centre and $I(G)$ the group of inner automorphisms of G. Then show that $G/Z(G) \cong I(G)$. (8)

Q.3. a) State and prove Sylow's third theorem. (8)
b) Prove that a normal subgroup needs not to be a characteristic subgroup. (8)

Q.4. a) State and prove Zassenhaus' lemma. (8)
b) Let $G = B \times A$. Then show that the factor group G/A is isomorphic to B. (8)

Q.5. a) If every Sylow p-subgroup of a finite group G is normal in G then show that G is the direct product of its Sylow p-subgroups. (8)
b) Show that S_3 is a group of permutations of degree 3. Is the group S_3 abelian? If so prove it. (8)

Q.6. a) Prove that the set A_n of all even permutations is normal subgroup of index 2 of S_n and is of order $\frac{1}{2}n!$, where $o(S_n) = n!$. (8)
b) Prove that intersection of two subnormal subgroups of a group is a subnormal subgroup. (8)

University of Sargodha

BS 7^{th} Term Examination 2020.

Subject: Mathematics **Paper: Advance Group Theory-I (MATH: 413)** **Maximum Marks: 80**

Time Allowed: 2:30 Hours

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

1. Write short answers of the following in 2-3 lines each on your answer book. (2*16)

- Prove that every characteristic subgroup is normal.
- Show that $U(16)$ is a p-group for some prime p.
- For $n \geq 5$, prove that any two 3-cycles in A_n are conjugate in A_n.
- State whether a group of order 90 and 120 is simple or not?
- Find the inverse of the following permutations.

  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 5 & 3 & 1 & 4 & 7 & 6 \end{pmatrix}$ (b) (2459)(173)(86)
- Let G be a group of order 12. Find possible number of sylows 2-subgroups of G.
- Let $G = (\mathbb{Z}, +)$, $A = \mathbb{Z}$ and $z \cdot a = z + a \quad \forall z \in G,\ a \in A$ forms a group action G on A. Find Kernel action G on A.
- Prove that group of order 17689 is abelian.
- What is the difference between External Direct Product and Internal Direct Product?
- Let G be a group and $a \in G$. Define $\theta_a : G \to G$ by $\theta_a(b) = aba^{-1}$ for all $b \in G$, then prove th $\alpha \circ \theta_a \circ \alpha^{-1} = \theta_{\alpha(a)} \quad \forall \alpha \in \mathrm{Aut}(G)$.
- Define finite p-groups with an example.
- Let G be a group and G' be a commutator subgroup of G. Prove that G is abelian if and only if $G' = \{e$
- Find the order of (3,10,9) in $\mathbb{Z}_4 \times \mathbb{Z}_{12} \times \mathbb{Z}_{15}$.
- Find the orbits of the permutation $\sigma : \mathbb{Z} \to \mathbb{Z}$, where $(n)\sigma = n+1$.
- Define chief series in groups.

Subjective Part (3*16)

Show that there exists a homomorphism between G and Inn(G). Also find the kernel of that homomorphism.

State and Prove Sylow's first theorem.

Prove that identity permutation is an even permutation.

Show that the group $G = \langle a : a^n = 1 \rangle$ has two composition series which are isomorphic.

Find group of automorphisms of the following group:

·	I	-I	i	-i	[illegible]	[illegible]	k	-k
I	I	-I	i	-i	[illegible]	[illegible]	k	-k
-I	-I	I	-i	i	-j	j	-k	k
i	i	-i	-i	I	k	-k	-j	j
-i	-i	i	I	-I	-k	k	j	-j
j	j	-j	-k	k	-I	I	[illegible]	[illegible]
-j	-j	j	k	-k	I	[illegible]	[illegible]	[illegible]

University of Sargodha

BS 6th Term Examination 2016

Subject: Mathematics **Paper: Partial Differential Equation (MATH: 308)**

Time Allowed: 2:30 Hours **Maximum Marks: 80**

Note: **Objective part is compulsory. Attempt any three questions from subjective part.**

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)

i. Define partial differential equation (PDE). Give an example of each, one and two dimensional PDE.
ii. Define Order of partial differential equation.
iii. Define a well-posed mathematical problem.
iv. Verify that the functions (i) $u(x,y)=x^2-y^2$ (ii) $u(x,y)=e^x \sin y$ are the solutions of the equation $u_{xx}+u_{yy}=0$.
v. Define semi-linear PDE and give at least two examples.
vi. Write the mathematical form of telegraph equation.
vii. Formulate the boundary value problem for a vibrating string of length a, is rigidly fixed at its ends.
viii. Formulate the boundary value problem for the vibrating string if its ends are fastened to air bearings that are free to move on a rod at right angles to the x-axis.
ix. Write the canonical form of parabolic equation.
x. Write the canonical form of hyperbolic equation.
xi. Write the canonical form of elliptic equation.
xii. Write the wave equation in spherical form.
xiii. Write the wave equation in cylindrical form.
xiv. Find the Fourier transform of function $f(ct)$.
xv. Transform $f(t)=Sin\,2t$ using Laplace transform.
xvi. Transform $e^{at}f(t)$ using Laplace transform.

Subjective Part (3*16)

Q.2: Show that the general solution of the linear equation

$$(y-z)u_x+(z-x)u_y+(x-y)u_z=0$$

Is

$$u(x,y,z)=f(x+y+z,x^2+y^2+z^2)$$

where f is an arbitrary function.

Q. 3: Find the characteristic equations and characteristics, and then reduce the equations

$$u_{xx}\mp(Sech^4 x)u_{yy}=0$$

to canonical forms.

Q.4: (a) Find the Fourier series expansion for the following function

$$f(x)=x+x^2,\quad -\pi<x<\pi$$

(b) Obtain the complex Fourier series expansion for the function

$$f(x)=e^x,\quad -\pi<x<\pi$$

Q.5: If f is piecewise smooth in every finite interval, and absolutely integrable on $(-\infty,\infty)$, then

$$\frac{1}{\pi}\int_0^\infty\left(\int_{-\infty}^{\infty}f(t)\cos k(t-x)\,dt\right)dk=\frac{1}{2}\left(f(x+)+f(x-)\right)$$

Q.6: Let $f(t)$ be a Piecewise Continuous for $t\geq 0$ and of exponential order. If $f(t)$ is Periodic with Period T, then

$$\ell[f(t)]=\frac{1}{1-e^{-ST}}\int_0^T e^{-st}f(t)\,dt$$

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Subject : Modren Algebra **Class: Term (VII)**

Total Marks: 80 **Time: 3 hrs**

Note: Question 1 is compulsory. Attempt any (3) from remaining.

Q1. Answer the following short questions. 16X2 = 32

- **i.** Define Maximal Ideal.
- **ii.** Define Principal Ideal Domain.
- **iii.** Find the unit elemnts of $Z[i]$.
- **iv.** Show that the ring of integers is Euclidean Domain.
- **v.** Define Unique Factorization domain.
- **vi.** Define zero divisor.
- **vii.** Define Primitive Polynomial.
- **viii.** Show that $T = \left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \mid a, b \in Z \right\}$ is a sub ring of $M_2(Z)$.
- **ix.** Prove that intersection of any two subrings of ring R is sub ring.
- **x.** Give the reason that why $z[x]$ is not PID.
- **xi.** Prove that $< 4 >$ is not Prime ideal in the ring of integer.
- **xii.** Define Division ring.
- **xiii.** Prove that Ring of integers is not isomarphic to ring of rationals .
- **xiv.** Define Polynomial Ring.
- **xv.** Define Gussian Integral Domain.
- **xvi.** Define Associate relation between elements of a ring.

Subjective Part: (16x3 = 48)

Q 2. (a) State and Prove 2nd Theorem of Ring Homomarphisem. [8]

(b) If R is integral domain the $R[x]$ is integral domain. [8]

Q 4. (a) State and Prove Division Algorithem in $R[x]$. [8]

(b) Prove that the set $Z[i] = \{a + ib : a, b \in Z\}$ is a subring of set of complex number. [8]

Q 5. (a) Let R be Principal ideal ring and $p \in R$. If p is irreducible then p is prime. [8]

(b) Prove that $Z[i]$ is Euclidean Domain. [8]

Q 6. (a) Prove that $Z[i\sqrt{5}]$ is not UFD. [8]

(b) In a UFD every irreducible element is prime. [8]

Q 7. (a) Prove that every finite Integral Domain is a field. [8]

(b) If $\varphi: R \to R'$ is ring homomarphisem then $ker\varphi$ is ideal of R. [8]

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Final Term Examination
BS-VII(Regular & Self Support)
Department of Mathematics, University of Sargodha, Sargodha

Subject:Number Theory
Time Allowed: 02 Hours

Date:08/02/2022
Max Marks:50

Q. No. 1 (08 Marks)

Prove that the congruence of degree n

$$f(s) \equiv 0 \pmod{p}$$

has at most n solutions.

Q. No. 2 (06 Marks)

If ord_ma =t and if u is a positive integer , then prove that

$$ord_m(a)^u = \frac{t}{(t,u)},$$

where (t,u) is the greatest common divisor of u and t.

Q. No. 3 (10 Marks)

Find all the primitive roots of 17^2 and solve the following congruence with the help of indices

$$17x^2 \equiv 10 \pmod{29},$$

provided that 2 is the primitive root of 29.

Q. No. 4 (10 Marks)

Prove that all primitive solution of equation $x^2+y^2=z^2$ are of the form $x=a^2-b^2, y=2ab$ and $z=a^2+b^2$, where $(a,b)=1$ and exactly one of the a and b is even.

Q. No. 5 (16 Marks)

State and prove Quadratic reciprocity law and hence evaluate $\left(\frac{2819}{4177}\right)$.

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University of Sargodha

BS 6th Semester/Term Exam 2021

Subject: Mathematics **Paper: Partial Differential Equation (MATH-308)**

...me Allowed: 02:30 Hours **Maximum Marks: 80**

...ote: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)

i. Define classification of differential equations.

ii. Define order, linear and non-linear partial differential equation and give at least one example of each.

iii. Define two dimensional wave equation.

iv. Find the Fourier transform of function $f(at)$.

v. If $u(x,t)$ is a displacement of a vibrating string and its end are fixed at $x=a$ and $x=b$, construct the boundary conditions of at least two types.

vi. Convert $u_{tt} = K u_{xx}$ into ordinary differential equation.

vii. Show that $u(x,t) = f(x-ct) + g(x+ct)$ is a solution of wave equation in one dimension when f and g are any smooth functions.

viii. Write the Laplace Transform of a function $f(t) = \operatorname{erfc}\sqrt{t}$.

ix. Find the Fourier Transform of $f'(t)$.

x. Define Inverse Laplace Transform.

xi. Write the mathematical form of telegraph equation.

xii. Give an example of parabolic equation.

xiii. Give an example of hyperbolic equation.

xiv. Find Inverse Laplace Transform of $\frac{c_2}{s+s_2}$.

xv. Evaluate $\frac{1}{\pi}\int_{-\pi}^{\pi}(x+x^2)\cos kx\,dx$.

xvi. Define Complex Fourier series.

Subjective Part (3*16)

Q.2. Derive one Dimensional wave equation.

Q.3. Calculate Fourier sin Transform of $f(x) = xe^{-ax}$.

Q.4. Find the Fourier series of the following function

$$f(x) = \begin{cases} 0 & -2 \le x < 0 \\ 2-x & 0 < x \le 2 \end{cases}$$

Q.5. Solve the problem using Laplace Transform formalism,

$$u_{tt} = a^2 u_{xx} - g,$$

with

$$u(x,0) = 0, \quad u_t(x,0) = 0,$$
$$u(0,t) = 0 = \lim_{x\to\infty} u_x(x,t).$$

Q.6. Find the steady state and transient solutions of the problem,

$$u_{xx} = \frac{1}{k}u_t, \quad 0 < x < a, \quad t > 0,$$

$$\begin{cases} u(0,t) = T_0, & u_x(a,t) = 0 \\ u(x,0) = f(x), & 0 < x < a \end{cases}$$

Also consider the case $f(x) = T_1$.

Mathematics: XIII **Number Theory**

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: Objective part is compulsory. Attempt any four questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each. (2*10)

i. Define LCM and GCD of two positive integers.
ii. If $(b, c) = 1$ then show that $(a,bc)=(a,b)(a,c)$.
iii. Write 24 and 100 as a sum of two prime numbers.
iv. Show that $\sqrt{7}$ is an algebraic number.
v. Define irreducible polynomial.
vi. Define a symmetric polynomial.
vii. What is the difference between algebraic number and algebraic number?
viii. Define norm of an algebraic number.
ix. Define unit in a integral domain.
x. Define discriminant of the ideal.

Subjective Part (4*20)

Q.2. (a) State and prove Euclid's Theorem.
(b) Show that a relation of being congruent modulo a fixed integer is an equivalence relation.

Q.3. (a) State and prove the Fermat's Theorem.
(b) Solve the linear Diophantine equation $1027x + 712y = 1$.

Q.4. (a) State and prove the Chines remainder theorem.
(b) If p and q are distinct odd primes the prove that

$$\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{\frac{1}{2}(p-1)\frac{1}{2}(q-1)}.$$

Q.5. (a) State and prove Eisenstein's irreducibility criterion.
(b) If α is a zero of a monic polynomial with integral coefficients, then prove that α is an algebraic integer.

Q.6. (a) Show that $R[\sqrt{-5}]$ is not a unique factorization domain.
(b) If α and β are elements of $R(\theta)$ then prove that $N_{\alpha\beta} = N_{\alpha}.N_{\beta}$

Q.7. (a) State the Kummer's theorem and the prime number theorem.
(b) Prove that the discriminant of cyclotomic field K_p is $(-1)^{\frac{p-1}{2}}p^{p-2}$.

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Omit Q's b/w 1 and 2.

University of Sargodha

BS 6th Term Examination 2017

Subject: Mathematics **Paper: Partial Differential Equation (MATH-308)**

Time Allowed: 2:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each. (2*16)

i. Define the canonical form of first order partial differential equations.

ii. Define order, semi linear, linear and non-linear partial differential equation and give at least one example of each.

iii. Define one dimensional wave equation. → $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$

iv. Find the Fourier transform of function $e^{-4|x|}$.

v. If $u(x,t)$ is a displacement of a vibrating string and its end are fixed at $x=-3$ and $x=3$, construct the boundary conditions of at least two types.

vi. Convert $u_t = c^2 u_{xx}$ into ordinary differential equation.

vii. Show that $u(x,t) = f(x-ct) + g(x+ct)$ is a solution of wave equation in one dimension when f and g are any smooth functions.

(5) P.D.E : $u_{tt} = c^2 u_{xx}$, $0<x<L$, $0<t<\infty$
B.C : $u(0,t)=0$, $u(L,t)=0$, $0<t<\infty$
I.C : $u(x,0)=f(x)$, $u_t(x,0)=g(x)$, $0<x<L$

viii. Write the Laplace Transform of a function $f(t) = \operatorname{erfc}\sqrt{t}$.

ix. Find the Laplace Transform of $f'''(t)$.

x. Define Inverse Laplace Transform.

xi. Write the mathematical form of second order Laplace equation.

xii. Give an example of parabolic equation. → the set of points that are equidistance from both the directrix and the focus. $2x^2 - x - 4y + 7 = 0$

xiii. Give an example of hyperbolic equation.

xiv. Find Inverse Laplace Transform of $\frac{1}{s+4}$. $\Rightarrow e^{-4t}$

xv. Evaluate $\frac{1}{\pi}\int_{-\pi}^{\pi}(x+x^2)\cos kx\,dx$.

xvi. Define Complex Fourier series. Let the function $f(x,y)$ be defined over the whole complex plane; i.e; $f(x,y)$ is defined for $-\infty<x<\infty$, $-\infty<y<\infty$. Then the fourier transform is defined

Subjective Part (3*16)

$F(k_1,k_2) = \frac{1}{(\sqrt{2\pi})^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)$

$F\{f(x,y)\} = \frac{1}{2\pi}\int\int$

Q.2: Find the general solution of $yu_{xx} + 3yu_{xy} + 3u_x = 0,\quad y \neq 0$.

Q.3: State and Prove the shifting and scaling properties of Fourier Transform of $f(t)$.

Q.4: State and prove Convolution Theorem of the Fourier Transform.

Q.5: Show that

(i) $\ell\left[f^{(n)}(t)\right] = s^n \ell\left[f(t)\right] - s^{n-1}f(0) - \dots - sf^{(n-2)}(0) - f^{(n-1)}(0)$

(ii) State and Prove Convolution Theorem of the Laplace Transform

Q.6: If $f(s)$ and $g(s)$ are the Laplace Transforms of $f(t)$ and $g(t)$ respectively, then

(i) $\ell[H(t-a)f(t-a)] = e^{-as}\ell[f(t)]$

(ii) $\ell[H(t-a)g(t)] = e^{-as}\ell[g(t+a)]$

Inverse laplace:- If $G(s) = L\{g(t)\}$, then the inverse transform of $G(s)$ is defined as; $L^{-1}G(s) = g(t)$

University of Sargodha

BS 6th Term Examination 2018

Subject: Mathematics **Paper: Partial Differential Equation (MATH: 308)**

Time Allowed: 2:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (16*2)

i. Define a partial differential equation (PDE). Give an example of each.

ii. Define a quasi-linear partial differential equation. Give an example.

iii. Define a semi-linear partial differential equation. Give an example.

iv. Verify that the functions (i) $u(x,y)=x^2-y^2$, (ii) $u(x,y)=e^x\sin y$ are the solutions of the equation $u_{xx}+u_{yy}=0$.

v. If $u(x,t)=f(x-ct)-g(x+ct)$, whether it is a solution of wave equation in one dimension when f and g are any smooth functions or not.

vi. Find the Laplace Transform of a function $f(t)=e^{at}\cos bt$.

vii. Find the Fourier Transform of $F_s\left[e^{-ax}\right]$.

viii. Find the inverse Laplace Transform of $f(s)=\frac{s-4}{(s-4)^2+4}$.

ix. Write the mathematical form of Laplace equation.

x. Find $F\left[e^{-ax^2}\right]$.

xi. What is the canonical form of parabolic type equation?

xii. What is the canonical form of elliptic type equation?

xiii. What is the canonical form of hyperbolic type equation?

xiv. Find Inverse Laplace Transform of $\frac{9}{(s-3)^2+9}$.

xv. Evaluate $\frac{1}{\pi}\int_{-\pi}^{\pi} x\sin kx\,dx$.

xvi. Verify that $u(x,y)=Log\left(\sqrt{x^2+y^2}\right)$ satisfies the equation $u_{xx}+u_{yy}=0$.

Subjective Part (3*16)

Q.2. For each of the following, state whether the partial differential equation is linear, quasi-linear or nonlinear. If it is linear, state whether it is homogeneous or non-homogeneous, and gives its order

i.	$u_{xx}+xu_y=y$	ii.	$uu_x-2xyu_y=0$
iii.	$u_x^2+uu_y=1$	iv.	$u_{xxxx}+2u_{xxyy}+u_{yyyy}=0$
v.	$u_{xx}+2u_{xy}+u_{yy}=\sin x$	vi.	$u_{xxx}+u_{xyy}+\log u=0$
vii.	$u_{xx}^2+u_x^2+\sin u=e^y$	viii.	$u_t+uu_x+u_{xxx}=0$

Q.3. Let $f(t)$ and its first derivative vanish as $x\to\infty$. If $F_c(k)$ is the Fourier cosine transform, then

$$F_c\left[f''(x)\right]=-k^2F_c(k)-\sqrt{\frac{2}{\pi}}f'(0)$$

Q.4. Show that the general solution of the linear equation $(y-z)u_x+(z-x)u_y+(x-y)u_z=0$ is $u(x,y,z)=f(x+y+z,x^2+y^2+z^2)$ where f is an arbitrary function.

Q.5. Use Laplace Transform to solve the boundary value problem,

$$u_{tt}=u_{xx},\quad 0<x<1,\ t>0$$

with

$$\begin{cases} u(x,0)=0,\ u_t(x,0)=\sin\pi x, & 0<x<1 \\ u(0,t)=0=u(1,t), & t>0. \end{cases}$$

Q.6. Let $f(t)$ be a Piecewise Continuous for $t\geq 0$ and of exponential order. If $f(t)$ is Periodic with Period T, then

$$\mathcal{L}[f(t)]=\frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt$$

University of Sargodha

BS 7th Semester Exam 2019

Subject: Mathematics **Paper: Advanced Group Theory-I (MATH:413)**

Time Allowed: 2:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each. (2*16)

i. Let G be a group and H be normal in G then show that the action of conjugation $g.h = ghg^{-1}\ \forall\ g \in G,\ h \in H$ is a group action of G.

ii. Prove that every characteristic subgroup is normal.

iii. Let $G = \{1, (132)(465)(78), (132)(465), (123)(456), (123)(456)(78), (78)\}$ be the group acting on set $A = \{1, 2, \ldots, 8\}$. Find all the orbits of the G.

iv. Show that a characteristic subgroup need not to be fully invariant.

v. Express the permutation $[(123)(145)]^{99}$ as a product of disjoint cycles.

vi. State whether a group of order 90 and 120 is simple or not?

vii. For $n \geq 5$, prove that any two 3-cycles in A_n are conjugate in A_n.

viii. Define p-groups with an example.

ix. Let G be a group of order 12. Find possible number of sylows 2-subgroups of G.

x. Find the inverse of the following permutations.

(a) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 5 & 3 & 1 & 4 & 7 & 6 \end{pmatrix}$ (b) $(2459)(173)(86)$

xi. Show by an example that if G is infinite group then the mapping $g \to g^2$ for all $g \in G$ need not to be an automorphism.

xii. Let G be a group and $a \in G$. Define $\theta_a : G \to G$ by $\theta_a(h) = aha^{-1}$ for all $h \in G$, then prove that $\alpha \circ \theta_a \circ \alpha^{-1} = \theta_{\alpha(a)}$ $\forall\ \alpha \in \mathrm{Aut}(G)$.

xiii. What is the difference between External Direct Product and Internal Direct Product?

xiv. Let G be a group. If $H = \{a \in G : a^n = 1\}$ is a subgroup, then show that H is fully invariant.

xv. Find all sylow's 2-subgroups of D_6.

xvi. Let G be a group and G' be a commutator subgroup of G. Prove that G is abelian if and only if $G' = \{e\}$.

Subjective Part (3*16)

Q: 2 (a) Find the sizes of the conjugacy classes of the symmetric group S_5. Hence show that S_5 has only one proper normal subgroup.

(b) Show that A_n is simple for $n > 5$.

Q: 3 (a) Prove that a group G of order 56 has 1 or 8 sylow 7-subgroups. In latter case, prove that G has a normal sylow 2-subgroup.

(b) State & prove 1st Sylow's Theorem.

Q: 4 (a) Let a group G act on a set A. Define stabilizer of $x \in A$ and prove that stabilizer of x is a subgroup of G. Hence show that $|O_x| = |G : G_x|$.

(b) Show that a finite group G has a unique sylow p-subgroup iff it is normal in G

Q: 5 (a) Consider the alternating group A_4. Show that A_4 has no subgroup of order 6.

(b) Show that the order of a permutation is least common multiple of the order of disjoint cycles into whose product is decomposed.

Q:6 (a) Define orbit and stabilizer subgroup of a group and find all the stabilizer subgroups of $G = \{I, (132)(465)(78), (132)(465), (123)(456), (123)(456)(78), (78)\}$.

(b) Prove that the alternating group of degree n is a normal subgroup of S_n and has order $\frac{n!}{2}$.

University of Sargodha

BS 1st Semester Examination 2016

Subject: Mathematics Paper: Number Theory (Math:103)

Time Allowed: 2:30 Hours

Maximum Marks: 80

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1 Write short answers of the following in 2-3 lines on you answer sheet. (2*16)

i. Define G.C.D.

ii. Prove that n is an odd integer, then $8|(n^2-1)$.

iii. If $(b,c)=1$ and $a|c$, then $(a,b)=1$.

iv. If p is prime such that $p|(a^2+b^2)$ and $p|a$, then $p|b$.

v. Define equivalent relation.

vi. If $a\equiv b(\bmod m)$, then $na\equiv nb(\bmod m)$, $\forall n\in z$.

vii. If $a\equiv b(\bmod m_1)$ and $a\equiv b(\bmod m_2)$, then (m_1,m_2), then $a\equiv b(\bmod m_1;m_2)$.

viii. Show that e^z is never zero.

ix. Prove that $\cosh^2 z-\sinh^2 z=1$.

x. If $a|b$ and $b|a$, then $a=\pm b$.

xi. Prove that the greatest common divisor of two integers a,b is unique.

xii. If p is prime and $p|ab$, where $a,b\in z$, then either $p|a$ or $p|b$.

xiii. Find remainder when 3^{21} is divided by 8.

xiv. Find $\phi(500)$.

xv. Find locus of points in the plane satisfying the given condition $|z-2i|\geq 1$.

xvi. Show that $|e^{iz}|=1$.

Subjective Part (3*16)

Q.2. (a) If $Log\sin(x+iy)=u+iv$, then show that $e^{2y}=\frac{\cos(x-v)}{\cos(x+v)}$.

(b) Prove that $\left(\frac{1+\sin x+i\cos x}{1+\sin x-i\cos x}\right)^n=\cos n\left(\frac{\pi}{2}-x\right)+i\sin n\left(\frac{\pi}{2}-x\right)$.

Q.3. (a) If $(a,b)=d$, then $\left(\frac{a}{d},\frac{b}{d}\right)=1$, where $a,b\in z$ and both a,b are not zero.

(b) If $x=\cos\theta+i\sin\theta$, then prove that $x^n-\frac{1}{x^n}=2i\sin\theta$.

Q.4. (a) If $(a,c)=1$, then $(a,bc)=(a,b)$.

(b) Prove that if $n>0, n\in z$, then $14|(3^{4n+2}+5^{2n+1})$.

Q.5. (a) State and prove the Euclid Theorem.

(b) $\phi(m)=m-1$ if and only if m is prime.

Q.6. (a) Prove that $\cos\frac{\pi}{7}-\cos\frac{2\pi}{7}+\cos\frac{3\pi}{7}=\frac{1}{2}$.

(b) If $\sin(A+iB)=x+iy$, then show that $\frac{x^2}{\cosh^2 B}+\frac{y^2}{\sinh^2 B}=1$.

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M.A/M.Sc. Part-II/Composite, 2nd A-Exam 2018

Mathematics: VIII **Numerical Analysis**

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: Objective part is compulsory. Attempt any five questions from subjective part, selecting two questions from Section –I, two questions from Section-II and one question from Section-III.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines on your answer sheet. (10*2)

(i) Explain definite relative error.
(ii) Prove that $hD = \log E$, where D, E are differential and Shift operator resp.
(iii) Explain convergence of Newton Raphson Method.
(iv) Using backward difference prove that $y_{n-3} = (1-\nabla)^3 y_n$.
(v) Prove that $\Delta = \frac{\delta^2}{2} + \delta\sqrt{(1+(\delta^2)/2)}$.
(vi) Prove that $hD = \sinh^{-1}(\mu\delta)$.
(vii) What is the convergence condition of Method of iteration to find root?
(viii) Explain truncation error.
(ix) Geometrically, what is the difference between 1/3 and 3/8 Simpson rule?
(x) What is numerical stability criteria to solve differential equations?

(Subjective Part)

(Section-I)

Q. No. 2: Using Jacobi's method to solve the following system of linear equations:

$$83x + 11y - 4z = 95$$
$$7x + 52y + 13z = 104$$
$$3x + 8y + 29z = 71$$

(16)

Q. No. 3: Let a function $\chi(x)$ be defined and differentiable in an interval (a, b) with all values $\chi(x) \in [a, b]$, then if there exists a proper fraction α $(0 < \alpha < 1)$ such that $|\chi(x)| \le \alpha < 1$, $\forall$ $a < x < b$, then the process of iteration described above converges irrespective of the choice of initial approximation $x_0 \in [a, b]$. Moreover, estimate the error of iterative process. (16)

Q. No. 4: Find the largest eigenvalue of the matrix $\begin{bmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 0 & 4 \end{bmatrix}$ and the corresponding eigenvector, by power method after fourth iteration starting with the initial vector $v^0 = (0, 0, 1)^T$. (16)

(Section-II)

Q. No. 5: Compute the integral

$$\sqrt{\frac{2}{\pi}} \int_0^1 e^{-x^2/2}\,dx$$

using 1/3 Simpson's rule. (16)

Q. No. 6: Use Newton's Divided difference interpolation, find the interpolated value at $x = 4.6$, for the given table

$x =$	1.0	2.00	3.00	4.00	5.00	6.00
$y =$	7.00	13.00	21.00	32.00	48.00	70.00

(16)

Q. No. 7: Estimate $y'(2)$ from the following table.

x =	0	1	2	3	4
y =	6.9897	7.4036	7.7815	8.1281	8.4510

(16)

(Section-III)

Q. No. 8: Using fourth order Runge-Kutta method find the solution of

$$x(dy + dx) = y(dx - dy), \qquad y(0) = 1$$

at $x = 0.1$ and 0.2 by taking h=0.1. (16)

Q. No. 9: Using modified Euler's method, obtain the solution of

$$\frac{dy}{dx} = 1 - y, \qquad y(0) = 0$$

for the range $0 \le x \le 0.2$, by taking $h = 0.1$. (16)

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M.A/M.Sc. Part-II/Composite, 1st A-Exam 2019

Mathematics: VIII **Numerical Analysis**

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: **Objective part is compulsory. Attempt any five questions from subjective part, selecting two questions from Section –I, two questions from Section-II and one question from Section-III.**

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines on your answer sheet. (10*2)

i. Use the Newton's Schemes of iteration to find the square root 12
ii. Define Eigen's values and Eigen's vector of a matrix
iii. Define Central Difference operator
iv. Define Homogeneous Difference Equation.
v. Show that μ and E commute
vi. Prove that $hD = \log E$
vii. Write down the general expression of Euler's method
viii. Solve the Difference equation $y_{k+2} - 4y_{k+1} + 4y_k = 0$
ix. Express $\Delta^2 y_0$ and $\Delta^3 y_0$ in terms of the values of the functions y.
x. Write down the types of errors.

Subjective Part **(16 × 5 = 80 Marks)**

Section-1

Q.2 Find the real root of the equation $x^3 + x^2 - 1 = 0$ by the method of iteration

Q3. Find the dominant Eigen value and corresponding Eigen vector of

$[A] = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$ by power Method after six iteration.

Q.4 By Gauss Seidel Method Solve the system of equation

$20x + y - 2z = 17,\ 3x + 20y - z = -18,\ 2x - 3y + 20z = 25$

Section –II

Q.5 Find $y'(1.10)$ and $y''(1.10)$ from the following data

x	1	1.05	1.10	1.15	1.20	1.25	1.30
y	1	1.0247	1.0488	1.0724	1.0954	1.1180	1.1402

Q.6 Evaluate $\int_0^{\pi} \sin x$ by trapezoidal rule and Simpson's $\frac{1}{3}$ rule by dividing the range of integration into six equal parts.

Q.7 Find the Lagrange's interpolation polynomial for the following table and find $y(5)$

x	1	3	4	6
$y = f(x)$	−3	0	30	132

Section –III

Q.8 Use the Modified Euler's Method to find the numerical solution of $\frac{dy}{dt} = 1 - y = f(t, y)$
$y(0) = 0$ for the $0 \le t \le 0.2$ in steps of 0.1

Q.9 Use the 2nd order Rungi- Kutta Method to find the numerical solution of $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$ at $x = 0.4$ and $h = 0.2$

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M.A/M.Sc. Part-II/Composite Exam 2nd A-2020 & 1st A-2021

Subject: Mathematics   Paper-VIII: Numerical Analysis

Maximum Marks: 100   Time Allowed: 3 Hours

Note: Objective part is compulsory. Attempt five questions from subjective part, selecting two questions from Section-I, two questions from Section-II and one from Section-III

Objective Part (Compulsory)

Q. No. 1: Write short answers of the following questions. (20)

(i) Define absolute and relative error.
(ii) What is the convergence condition for fixed-point iteration.
(iii) What is Zeros of polynomial?
(iv) Explain Finite difference method.
(v) Explain finite difference explicit method with example.
(vi) Explain finite difference implicit method with example.
(vii) Write formula for maximum error in approximating value of $f(x)$ by Simpson's rule.
(viii) Define interpolation.
(ix) Define orthogonal matrix.
(x) Define Schmidt finite difference method.

Subjective Part ($16 \times 5 = 80$ Marks)

Section-I

Q. No. 2: Let a function $\chi(x)$ be defined and differentiable in an interval (a, b) with all values $\chi(x) \in [a, b]$, then if there exists a proper fraction α $(0 < \alpha < 1)$ such that $|\chi'(x)| \leq \alpha < 1, \forall a < x < b$, then the process of iteration described above converges irrespective of the choice of initial approximation $x_0 \in [a, b]$. Moreover, estimate the error of iterative process.

Q. No. 3: Find the dominant eigenvalue and corresponding eigenvector of the matrix

$$\begin{bmatrix} 4 & 1 & 1 & 1 \\ 1 & 3 & -1 & 1 \\ 1 & -1 & 2 & 0 \\ 1 & 1 & 0 & 2 \end{bmatrix},$$

by using $x^{(0)} = (1, -2, 0, 3)^t$

Q. No. 4: Solve the following system of equations by using Gauss-Seidel method:

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

Section-II

Q. No. 5: Suppose that $f(0) = 1$, $f(0.5) = 2.5$, $f(1) = 2$ and $f(0.25) = f(0.75) = \alpha$. Find α if the Composite Trapezoidal rule with $n = 4$ gives the value 1.75 for $\int_0^1 f(x)dx$.

Q. No. 6: Find interpolating polynomial by Newtons Divided Difference Formula and Lagrange's Formula for the following data and hence show that both methods gives raise to same polynomial.

$x =$	1	2	3	5
$f(x) =$	0	7	26	124

Q. No. 7: Use the most accurate three-point formula to determine the missing values in the following

x	$f(x)$	$f'(x)$
8.1	16.94410	
8.3	17.56492	
8.5	18.19056	
8.7	18.82091	

Section-III

Q. No. 8: Use the Rang-Kutta method of order four to solve $y' = te^{3t} - 2y$, $0 \leq t \leq 1$, $y(0) = 0$ with $h = 0.5$.

Q. No. 9: Solve the following heat conduction equation $T_t = T_{xx}$, $0 \leq x \leq 1$, subject to the initial condition at $T = 1$ for $0 \leq x \leq 1$ at $t = 0$, and the boundary conditions $T_x = T$ at $x = 0$ for $t > 0$ and $T_x = -T$ at $x = 1$ for $t > 0$ using an explicit finite difference method, by taking $\Delta x = 0.1$ and $\Delta t = 0.0025$ up to $t = 0.0125$.