University of Sargodha

BS 5th Semester/Term Exam 2021

Subject: Mathematics **Paper: Algebra-II (MATH-309)**

Time Allowed: 02:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)

i. Define quotient ring.
ii. Let a, b and c belongs to an integral domain. If $a \neq 0$ and $ab = ac$ then $b = c$
iii. Define Euclidean domain
iv. State Gauss lemma.
v. Show that $2 - i$ is irreducible element in $Z[i]$
vi. Define GCD in polynomials
vii. Define principal ideal domain.
viii. Define unique factorization domain.
ix. Write the units of Gaussian integers.
x. Find the HCF of $x^2 + 2x$ and $2x + 4$ in $Z[x]$
xi. Let ϕ be a ring homomorphism from R to S. If A is a subring of R, then show that $\phi(A) = \{\phi(a) \mid a \in A\}$ is a subring of S.
xii. Show that intersection of any number of ideals is again an ideal of ring.
xiii. Show that $\langle 3 \rangle$ is maximal ideal of $\mathbb{Z}_{36}$.
xiv. Show that $n\mathbb{Z}$ is an ideal of ring $\mathbb{Z}$ for $n>0$.
xv. If R is a ring with additive identity 0, then show that for any $a, b \in R$, $a(-b) = (-a)b = -(ab)$.
xvi. Define Symmetric Polynomials.

Subjective Part (3*16)

Q.2. (a) Let ϕ be a ring homomorphism from a ring R to S. Then prove that $R/_{Ker\phi} \approx \phi(R)$.

(b) If R is an integral domain, then show that $R[x]$ is an integral domain.

Q.3. (a) Let R be a commutative ring with unity and let A be an ideal of R. Then prove that $R/_A$ is an integral domain if and only if A is prime.

(b) State and prove division algorithm for polynomials.

Q.4. (a) Let R be a commutative ring with identity and P be an ideal in R, then P is prime ideal if and only if R/P is an integral domain.

(b) Prove that every field is PID.

Q.5. (a) Let $R = \left\{\begin{bmatrix} a & -b \\ b & a \end{bmatrix} \mid a, b \in \mathbb{Z}_7\right\}$ with the usual matrix addition and multiplication. Prove that R is commutative ring.

(b) Prove that every principal ideal domain is a unique factorization domain.

Q.6. (a) Prove that every prin $\mathbb{Z}[\sqrt{-5}]=\{a + b\sqrt{-5} | a, b \in\mathbb{Z}\}$ is an integral domain but not a unique factorization domain.

(b) Let R be a principal ideal domain then an element $a \in R$ is irreducible if and only it is Prime.

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BS 5th Semester/Term Exam 2021

Subject: Mathematics **Paper: Topology (Math-301)**

Time Allowed: 02:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)

i. Let $X = \{a, b, c, d, e\}$ and let $A = \{\{a, b, c\}, \{c, d\}, \{d, e\}\}$. Find the topology on X generated by A.

ii. Write the subbase of usual topology on R.

iii. Find interior, exterior, boundary, derived set and closure of set of rational numbers Q.

iv. Give an example why intersection of any number of open set is not an open set.

v. Find accumulation points of Q^c.

vi. Let $X = \{a, b, c\}$, whether the class $\beta = \{\{a, b\}, \{b, c\}\}$ be a base for any topology on X or not?

vii. Show that $(a, b]^c$ is both open and close in upper limit topology.

viii. Define base for topology and write the base for discrete, indiscrete, usual, upper and lower limit topology on R.

ix. Define subbase for topology.

x. Define local base at p and write local base at p in discrete space $X = \{1,2,3\}$.

xi. Define bi-continuous function.

xii. Define Lindelof space with example.

xiii. Write the difference between topological and hereditary property.

xiv. Consider the topology $\tau = \{X, \phi, \{a\}\}$ on $X = \{a, b\}$. Whether (X, τ) is T_1 space or not?

xv. Define product topology.

xvi. Define components.

Subjective Part (3*16)

Q.2. **(a)** Show that in a topological space (X, τ), if $A \subseteq X$, then $\bar{A} = A^o \cup b(A)$.

(b) Let $f: X \to Y$ be a function from a non-empty set X into a topological space (Y, U). Furthermore, let τ be the collection of inverses of open subsets of Y i.e. $\tau = \{f^{-1}[G]: G \in U\}$, show that τ is a topology on X.

Q.3. **(a)** Let $\mathcal{A}$ be a class of subsets of a non empty set X then show that the topology on X generated by $\mathcal{A}$ is the intersection of all those topologies on X which contain $\mathcal{A}$.

(b) Show that a point p in a topological space (X, τ) is an accumulation point of $A \subset X$ iff each member of some local base β_p at p contains a point of A different from p.

Q.4. **(a)** Show that a function $f: X \to Y$ is continuous iff inverse image of each close subset of Y is closed subset of X.

(b) Show that a function $f: X \to Y$ is continuous iff for every subset $A \subseteq X$ implies $f(\bar{A}) \subseteq \overline{f(A)}$.

Q.5 **(a)** Prove that a set A is connected if it is not the union of two non-empty separated sets.

(b) Prove that continuous images of compact sets are compact.

Q.6 **(a)** Show that if A and B are connected sets and not separated then $A \cup B$ is connected.

(b) Let X be first countable space. Show that a function $f: X \to Y$ is countinuous at p iff it is sequential continuous at p.

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BS 5th Semester/Term Exam 2021

Subject: Mathematics **Paper: Ordinary Differential Equations (MATH-305)**

Time Allowed: 02:30 Hours

Maximum Marks: 80

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)

i. What is a differential equation?

ii. Define solution of an ordinary differential equation.

iii. Determine whether the DE $u\,dv + (v + uv - ve^u)du$ is linear in u.

iv. Write down the domain of definition of $y = \frac{1}{x^2-1}$ as a solution of DE $y' + 2xy^2 = 0$, and as a solution of IVP $y' + 2xy^2 = 0,\ y(0) = -1$.

v. When does a unique solution of an initial value problem exist?

vi. Determine whether the given DE $(2x-1)dx + (3y+7)dy = 0$ is exact.

vii. Define superposition principal.

viii. Find singular points of $(x^2-4)y'' + 3(x-2)y' + 5y = 0$.

ix. Solve $(1+x)dy - ydx = 0$.

x. Write the auxiliary equation, the roots and the corresponding general solution of DE:
$y'' - 4y' + 5y = 0$

xi. What will be the assumed particular solution y_p for $g(x) = 3x^2 - 5\sin 2x + 7xe^x$

xii. Solve $2y'' - 5y' - 3y = 0$.

xiii. Differentiate between a particular solution and a complementary solution.

xiv. What is the order of the given homogeneous function? $M(x,y) = x + \sqrt{xy}$.

xv. Verify $y = \frac{1}{4-x^2}$ is an explicit solution of the DE $y' = 2xy^2$.

xvi. Define separable equation.

Subjective Part (3*16)

Q.2. a) Solve initial value problem $\frac{dy}{dx} = (-2x+y)^2 - 7 \quad , y(0) = 0$

b) Solve the given Bernoulli equation. $t^2\frac{dy}{dt} + y^2 = ty$

Q.3. a) $y = c_1e^x\cos x + c_2e^x\sin x$ is a two parameter family of solution of the DE $y'' - 2y' + 2y = 0$ on $(-\infty, \infty)$. Find the member of family that satisfies the boundary conditions $y(0) = 1,\ y'(\pi) = 0$.

b) Solve the given DE by the method of undetermined coefficients. $y'' + 3y = -48x^2e^{3x}$

Q.4. a) A fossilized bone is found to contain one-thousandth of the C-14 level found in living matter, Estimate the age of the fossil.

b) A thermometer is taken from an inside room to outside, where the air temperature is 5^oF. After 1 minute the thermometer reads 55^oF, and after 5 minutes it reads 30^oF. What is the initial temperature of the inside room?

Q.5. Solve $(x^2+1)y'' + xy' - y = 0$ by using power series method.

Q.6. Use the method of Frobenius to obtain two linearly independent series solution about $x = 0$. Form the general solution. $3xy'' + (2-x)y' - y = 0$

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BS 5th Semester/Term Exam 2021

Subject: Mathematics **Paper: Real Analysis-I (MATH-307)**

Time Allowed: 02:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)
(i) Define order on a set. **(ii)** State least upper bound property. **(iii)** If $x > 0$, then show that $-x < 0$. **(iv)** Show that the sum of a rational and irrational is irrational. **(v)** State Archimedean property of real numbers. **(vi)** Define convergent sequence in metric space. **(vii)** Show that e is irrational. **(viii)** Define local minimum of a function in an interval $[a, b]$. **(ix)** Define monotonic sequence. **(x)** Define limsup and liminf. **(xi)** Prove that $\lim\limits_{n\to\infty} \text{Sin}\frac{1}{n}$ does not exists. **(xii)** Show that limit of a function is unique. **(xiii)** Show that continuity does not implies differentiability. **(xiv)** State intermediate value theorem for differentiable functions. **(xv)** Differentiate between continuity and uniform continuity. **(xvi)** Show that there does not exists any rational whose square is 3.

Subjective Part (3*16)

Q.2. (a) If $x \in R$ and $y \in R$ and $x < y$, then prove that there exists $p \in Q$ such that $x < p < y$.
(b) Show that the differentiability implies continuity.

Q.3. (a) Sate and prove the generalized Mean-value theorem for continuous functions.
(b) Prove that every Cauchy sequence of real numbers has a convergent subsequence.

Q.4. (a) Show that a mapping of a metric space X into a metric space Y is continuous if and only if $f^{-1}(V)$ is open in X for every open set V in Y.
(b) State and prove Hen-Borel theorem.

Q.5. (a) State and prove the Bernoulli's inequality.
(b) If a and c real numbers such that $c > 0$ and f is a function defined on $[-1, 1]$ by

$$f(x) = \begin{cases} x^a Sin(x^{-c}) & ; x \neq 0 \\ 0 & ; x = 0 \end{cases}$$

then discuss the continuity as well as differentiability at $x = 0$.

Q.6. (a) Show that second order derivative exists for the functions $f(x) = \begin{cases} |x^2| & ; x \neq 0 \\ 0 & ; x = 0 \end{cases}$ at $x = 0$.

(b) Let (p_n) be a sequence in a metric space X, then prove the following

i) If (p_n) converges then (p_n) is bounded.

ii) If $E \subseteq X$ and if p is limit point of E ,then there exists a sequence (p_n) in E such that $\lim\limits_{n\to\infty} p_n = p$.

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BS 5th Semester/Term Exam 2021

Subject: Mathematics **Paper: Differential Geometry (MATH-303)**

Time Allowed: 02:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)

1. Define skewed curve.
2. What is circular helix.
3. Define tangent at a point to a curve.
4. What is normal plane.
5. Define principle normal at a point.
6. If γ is position vector of a point on a curve, then prove that $\gamma'' = \kappa \underline{n}$.
7. What is binormal.
8. What is moving trihedron.
9. What is differential geometry?
10. Define a curve in $\mathbb{R}^3$.
11. Write equation of circle in three dimensions.
12. What is curvature of a curve?
13. Define osculating plane.
14. What is circle of curvature
15. Define rectifying plane.
16. What is torsion of a curve?

Subjective part (3*16)

Q.No.2 If tangent and binormal at a point of a curve make angles θ and ϕ respectively with a fixed direction. Show that

$$\frac{\sin\theta}{\sin\phi}\frac{d\theta}{d\phi} = -\frac{\kappa}{\tau}.$$

Q.No.3 Prove that the shortest distance between the principle normals at consecutive points distant ds apart is

$$\frac{\rho}{\sqrt{\rho^2+\sigma^2}}ds.$$

Q.No.4 Prove that a curve is Helix if and only if its curvature and torsion are in a constant ratio.

Q.No.5 Let γ be a regular curve in $\mathbb{R}^3$, then prove that its curvature is

$$\kappa = \frac{\|\ddot{\gamma}\times\dot{\gamma}\|}{\|\dot{\gamma}\|^3}.$$

Q.No.6 If $\gamma(t) = \sigma(u(t), v(t))$ is a unit speed curve on a surface patch σ, then prove that its normal curvature is

$$\kappa_n = L\dot{u}^2 + 2M\dot{u}\dot{v} + N\dot{v}^2.$$