29223

University of Sargodha

BS 5th Term Examination 2022

Subject: Mathematics **Paper: Algebra-II (MATH-309)**

Maximum Marks: 80

Time Allowed: 02:30 Hours

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)

i. Consider the matrix ring $M_2(\mathbb{Z}_2)$. List all units in the ring.
ii. Show that kernel of homomorphism is an ideal of ring R.
iii. Define ring with zero division and find zero divisors of $\mathbb{Z}_6$.
iv. Define module with example.
v. Define prime element of a ring with example.
vi. Define simple ring with example.
vii. Define GCD in polynomials.
viii. Write the units of Gaussian integers.
ix. Show that every field is an integral domain.
x. Let R be a ring with unity 1. The mapping $\alpha: \mathbb{Z} \longrightarrow R$ given by $n \rightarrow n \cdot 1$ is ring homomorphism.
xi. Let ϕ be a ring isomorphism from R onto S. Then show that ϕ^{-1} is an isomorphism from S onto R.
xii. Show that cancellation laws hold in a ring R if R has no zero divisor.
xiii. Let a and b be idempotent in a commutative ring, then show that $a+b-2ab$ is also idempotent.
xiv. Find characteristic of the ring of polynomial $\mathbb{Z}_2[x]$.
xv. Define prime ideal, whether 6Z is prime ideal or not.
xvi. Define Simple Ring.

Subjective Part (3*16)

Q.2. a) Prove that a finite commutative ring R with more than one element and without zero divisors is a field.

b) Let $R = \left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \mid a_i \in \mathbb{Z}\right\}$ and let I be the subset of R consisting of matrices with even entries. Then find factor ring R/I.

Q.3. a) Let R be a commutative ring with identity 1. Let M be an ideal in R, then M is maximal ideal if and only if R/M is a Field.

b) Find all ideals in $\mathbb{Z}$.

Q.4. a) Prove that $\mathbb{Z}[\sqrt{-3}]$ is not a principal ideal domain.

b) Let F be a field, then prove that $F[x]$ is an Euclidean domain.

Q.5. a) Consider the group $(\mathbb{Z},+)$ and the subgroups $\langle 6 \rangle$ and $\langle 3 \rangle$ of $\mathbb{Z}$. Then verify the Third Isomorphism Theorem.

b) Prove that every prime is irreducible in an integral domain.

Q.6. a) Show that the field has no proper ideal.

b) Let f be a ring homomorphism of a ring R to R'. Then Prove that $\text{Ker} f$ is an ideal of R.

University of Sargodha

29223

BS 5th Term Examination 2022

Subject: Mathematics **Paper: Real Analysis-I (MATH-307)**

Time Allowed: 02:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any four questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)

i. In every ordered field, if $x<0$, $y<z$, then $xy>xz$.
ii. State the Archimedian property of real numbers.
iii. What do you meant by monotonic sequence?
iv. What do you meant by ordered field?
v. Define the convergence of a sequence.
vi. Discuss the behavior of the series $\sum_{n=1}^{\infty} 1/n$.
vii. Write the general principle of convergence.
viii. State the least upper bound property of a set.
ix. Define local maximum of a function.
x. State the intermediate value theorem for differentiable functions.
xi. Prove that there exists an irrational number between any two real numbers.
xii. Show that a polynomial is continuous for every value of x.
xiii. If r is a rational and x is irrational, prove that $r+x$ is irrational.
xiv. Prove that every the continuous function is not differentiable.
xv. If $\sum_{n=1}^{\infty} a_n$ converges, then prove that $\lim_{n\to\infty} a_n = 0$.
xvi. If $\lim_{x\to c} f(x)$ exists, then it is unique.

Subjective Part (4*12)

Q.2. Prove that every bounded sequence has a convergent sub-sequence.

Q.3. Prove that there is no rational number p such that $p^2 = 3$.

Q.4. A sequence of real numbers is convergent iff it is a Cauchy sequence.

Q.5. Show that $x_n \to p$ if $y_n \to p$ and $z_n \to p$ as $n \to \infty$ in $y_n \le x_n \le z_n$

Q.6. Check the series $\sum \frac{1}{n}\sin^2\frac{x}{n}$ whether converges or diverges.

Q.7. State and prove Lagrange's Mean Value Theorem.

29223

University of Sargodha

BS 5th Term Examination 2022

Subject: Mathematics **Paper: Topology (MATH-301)**

Time Allowed: 02:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)

i. Let $X = \{a, b, c, d, e\}$ and let $A = \{\{a, b, c\}, \{c, d\}, \{d, e\}\}$. Find the topology on X generated by A.

ii. Find interior and closure of set Q.

iii. Define base for topology and write the base for discrete topological space.

iv. Define bi-continuous function.

v. Consider the topology $\tau = \{X, \phi, \{a\}\}$ on $X = \{a, b\}$. Whether (X, τ) is T_1 space or not?

vi. Define components with example.

vii. Give an example of open set in usual topology on R which is not open interval.

viii. State Heine Boral theorem.

ix. If $X = \{a, b, c, d, e\}$ with topology $\tau_x = \{X, \varphi, \{a\}, \{a, c, d\}, \{a, b\}, \{a, b, e\}, \{a, b, c, d\}\}$ then find (i) Neighborhood system of point d (ii) closure of $\{a\}$.

x. Define convergence of sequence in topological space.

xi. What is closure of any subset of indiscrete topological space?

xii. If $A \subset B$ then $\bar{A} \subset \bar{B}$.

xiii. Write dense subset of discrete topological space.

xiv. Give example of T_1 space which is not Hausdorff space.

xv. Show that every finite T_1 space is discrete space.

xvi. Define countably compact set with example.

Subjective Part (3*16)

Q.2. a) Show that if (X, τ) is a topological space and $A \subseteq X$ then $\bar{A} = A^o \cup b(A)$.

b) Show that if A is a subset of B, then every limit point of A is also a limit point of B, i.e., $A \subseteq B$ implies $A' \subseteq B'$.

Q.3. a) Let X be a topological space. show that followings are equivalent.

(i) X is disconnected (ii) there exists a non-empty subset of X which is both open and closed.

b) Show that if β is the class of subsets of X, then β is a base for some topology on X if it possesses the following two properties:

i. $X = U\{B: B\epsilon\beta\}$

ii. For any $B, B^*\epsilon\beta$; $B \cap B^*$ is the union of member of β.

Q.4. a) Show that a function $f: X \to Y$ is continuous iff inverse image of each close subset of Y is closed subset of X.

b) Show that a function $f: X \to Y$ is continuous iff for every subset $A \subseteq X$ implies $f(\bar{A}) \subseteq \overline{f(A)}$

Q.5. a) Prove that closed set of compact set is compact.

b) Let A be any subset of a second countable space X. If $\mho$ is an open cover of A, then show that $\mho$ is reducible to a countable cover.

Q.6. a) Show that if A and B are connected sets and not separated then $A \cup B$ is connected.

b) Show that a completely regular space is also regular.

University of Sargodha

BS 5th Term Examination 2022 29223

Subject: Mathematics **Paper: Differential Geometry (MATH-303/MATH-311)**

Time Allowed: 02:30 Hours

Maximum Marks: 80

Note: **Objective part is compulsory. Attempt any three questions from subjective part.**

Objective Part **(Compulsory)**

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)

i. Define osculating plane.
ii. Write is intrinsic equation of a straight line.
iii. What is meant by circle of curvature?
iv. Define rectifying plane.
v. What is plane of curvature?
vi. Define radius of torsion.
vii. Define touching plane.
viii. For a curve $\underline{\gamma} = \underline{\gamma}(s)$, prove that $\underline{n}' = \tau\,\underline{b} - \kappa\,\underline{t}$.
ix. Write an expression for first fundamental form.
x. Find first fundamental form of the surface $\sigma(u,v) = (\cos v, \sin v, u)$.
xi. Define an isometry.
xii. What is curvature of a straight line?
xiii. Write a formula for curvature of a unit speed curve.
xiv. Define conformal map.
xv. Define geodesic.
xvi. Write Frenet-Serret equations.

Subjective Part **(3*16)**

Q.2. Prove that necessary and sufficient condition for a curve to be a plane curve is $[\gamma', \gamma'', \gamma'''] = 0$.

Q.3. Find curvature of spherical indicatrix of unit binormal to curve $\underline{r} = \underline{r}(s)$.

Q.4. Prove that unit speed parametrization of a regular curve $\underline{r} = \underline{r}(t)$ is regular.

Q.5. Find curvature of the circular helix $\underline{r}(t) = (a\cos t, a\sin t, bt)$.

Q.6. If tangent and binormal at a point of a curve make angles θ and ϕ respectively with a fixed direction. Show that

$$\frac{\sin\theta}{\sin\phi}\frac{d\theta}{d\phi} = -\frac{\kappa}{\tau}.$$

29223

University of Sargodha

BS 5th Term Examination 2022

Subject: Mathematics **Paper: Ordinary Differential Equations (MATH-305)**

Time Allowed: 02:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)

i. Show that $y = xe^x$ is solution of $y'' - 2y' + y = 0$.
ii. Define the initial value problem.
iii. Solve $\frac{dy}{dx} = \sin 5x$.
iv. What do you mean by a first order linear differential equation?
v. Define superposition principle for homogeneous equations.
vi. What is the general solution of a non-homogeneous differential equation?
vii. What will be trial particular solution for the function $xe^x \cos bx$?
viii. When does a power series solution exist?
ix. What is an exact differential equation?
x. What is a Strum-Liouville boundary value problem?
xi. What are singular points for a differential equation?
xii. What are self adjoint operators?
xiii. Give an example of Cauchy-Euler equation.
xiv. What is meant by order of a differential equation?
xv. What is a non-trivial solution?
xvi. Find annihilator operator that annihilates the function $4e^{2x} - 10xe^{2x}$.

Subjective part (3*16)

Q.2. a) Solve $(e^{2y} - y\cos xy)dx + (2xe^{2y} - x\cos xy + 2y)dy = 0$ - Pg-66 (book)

b) Solve $x\frac{dy}{dx} = y + \sqrt{x^2 - y^2}$. → Pg 74 Q.10 (Ex 2.5 book)

Q.3. a) Use method of undetermined coefficients to find the solution of $y'' + 4y' - 2y = 2x^2 - 3x + 6$. Pg 140 (Book)

b) Find general solution of $y'' - 4y' + 4y = (x+1)e^{2x}$ using variation of parameter method. ↳ Pg 159 (book).

Q.4. a) Solve the following initial value problem. Pg-67 book

$$\frac{dy}{dx} = \frac{xy^2 - \cos x \sin x}{y(1 - x^2)}, \quad y(0) = 2$$

b) Find the solution of $x^2y'' - xy' + y = \ln x$ - Pg 167 (book)

Q.5. Solve $(x^2 + 1)y'' + xy' - y = 0$ using power series. — Pg 243 (book)

Q.6. Find eigenvalues and eigenfunctions of the following boundary value problem.

$y'' + \lambda y = 0, \quad y'(0) = 0, \quad y'(L) = 0$