University of Sargodha

BS 7th Semester Examination 2022

Subject: Mathematics **Paper: Partial Differential Equations (MATH-405)**

Time Allowed: 2:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any four questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)

i. Define order of the partial differential equation.
ii. Define linear partial differential equation.
iii. Define Neumann boundary conditions.
iv. Formulate a boundary values problem that governs the heat conduction in a rod of constant cross section $A$and length L that is insulated laterally and its two ends are perfectly insulated.
v. Classify the equation $u_{xx} - 2u_{xy} + u_{yy} = 0$ as parabolic, hyperbolic or elliptic.
vi. Covert the partial differential equation $\partial u/_{\partial t} = k\,\partial^2 u/_{\partial x^2}$ into ordinary differential equation through suitable substitution.
vii. Verify that the function $u(x, y) = x^2 - y^2$is the solution of the equation $u_{xx} + u_{yy} = 0$.
viii. Give an example of a boundary value problem having Dirichlet boundary conditions.
ix. Write Fourier transform and inverse Fourier transform of a function f(t).
x. Find the Fourier transform of the function $f(x) = e^{-ax^2}$
xi. Find the Fourier transform of derivative of $f(t)$.
xii. Convert the equation $\partial^2 u/\partial t^2 = c^2(\partial^4 u/\partial x^4)$ into ordinary differential equation by Fourier transform.
xiii. Define Laplace transform of a function.
xiv. Show that Laplace transform of t^n is $n!/s^{n+1}$.
xv. Find the Laplace transform of $t^2 e^{at}$.
xvi. Find inverse Laplace transform of $1/s(s^2 + 1)$.

Subjective Part (4*12)

Q.2. Derive the one dimensional wave equation.

Q.3. Solve the boundary value problem by the method of separation of variables.

$$\partial u/\partial t = k\ \partial^2 u/\partial x^2 \qquad 0 < x < L,\ 0 < t$$
$$u(0,t) = 0,\quad u(L,t) = 0 \quad 0 < t; \qquad u(x,0) = x \qquad 0 < x < L.$$

Q.4. Find the temperature $u(\rho, \theta)$ in a laterally insulated pie-shaped region of radius c. The temperature satisfies the differential equation

$$\rho^2 u_{\rho\rho} + \rho u_\rho + u_{\theta\theta} = 0$$

and the boundary conditions

$$u(\rho, 0) = 0, \qquad u(\rho, \pi/_6) = 0, \qquad 0 < \rho < c$$
$$u(c, \theta) = 0 \qquad 0 < \theta < \pi/_6$$

Q.5. Solve the nonhomogeneous partial differential equation.

$$\partial u/\partial t = k(\partial^2 u/\partial x^2) + \sin x$$
$$u(0,t) = 500, \quad u(\pi, t) = 100 \quad 0 < t$$
$$u(x, 0) = f(x) \qquad 0 < x < \pi$$

Q.6. Solve the initial boundary value problem for an infinite string using Laplace transform method.

$$\partial^2 u/\partial t^2 = c^2(\partial^2 u/\partial x^2) + f(x,t)$$
$$u(0,t) = 0,$$
$$u(x,0) = (\partial u/\partial t)(x, 0) = 0; \qquad u(x,t) \text{ is always bounded}$$

Q.7. Find out the temperature in an infinite rod using Fourier transform method.

$$1/k\,(\partial u/\partial t) = \partial^2 u/\partial x^2$$
$$u(x,0) = \begin{cases} 400^\circ K & -4 \le x \le 4 \\ 0^\circ K & 4 < |x| < +\infty \end{cases}$$

University of Sargodha

BS 7th Term/Semester Exam 2021

Subject: Mathematics **Paper: Partial Differential Equations (MATH:405)**

Maximum Marks: 80 **Time Allowed: 2:30 Hours**

Note: Objective part is compulsory. Attempt any four questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each. (2*16)

i. Write the difference between linear and non - linear partial differential equation.

ii. Define principle of Superposition.

iii. Obtain PDE $z = x + ax^2y^2 + b$ where a,b are arbitrary constants.

iv. Write down the Physical Meanings of The Drichlet Boundary conditions.

v. Define Canonical Form of first order linear equation.

vi. Write mathematical form of the telegraph equation.

vii. Define specific heat of substance.

viii. Write condition for existence of Fourier transformation.

ix. Prove that $\int_{-\infty}^{\infty} |f(x)|^2\, dx = \int_{-\infty}^{\infty} |F(k)|^2\, dk$

x. Find the Fourier series expansion for the function $f(x) = x + x^2 \;\; ; -\pi < x < \pi$

xi. Find Laplace Transformation of the error Function.

xii. If $\mathcal{L}\{ f(t)\} = F(s)$ then $\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} F(s)\, ds$

xiii. Find The Laplace transform of an impulse function.

xiv. Let $f'(x)$ be continuous and $f''(x)$ be peicewise continuous in $[0,\pi]$ if $F_s(k)$ is the finite Fourier Sine Transform of $f(x)$ then prove that

$$\mathcal{F}_s\{f''(x)\} = \frac{2k}{\pi}[f(0) - (-1)^k f(\pi)] - k^2 F_s(k)$$

xv. Show that the Hankel transform satisfies the Parseval relation

$$\int_0^{\infty} rf(r)\, g(r) dr = \int_0^{\infty} k\tilde{f}(\kappa)\tilde{g}(\kappa)\, dk$$

xvi. Write the Difference between Laplace and Fourier Transform.

Subjective Part (4*12)

Q.2. Show that the general solution of the linear equation

$$(y - z)\, u_x + (z - x)\, u_y + (x - y)\, u_z = 0$$

is $u(x, y, z) = f(x + y + z, x^2 + y^2 + z^2)$ where f is an arbitrary function.

Q.3. Solve the initial-value problem $u_x + 2u_y = 0$, $u(0, y) = 4e^{-2y}$.

Q.4. State and Prove one Dimensional Wave Equation.

Q.5. Derive Laplace equation in cylindrical coordinates.

Q.6. Let $f(x)$ and its first derivative vanish as $x \to \infty$. If $F_c(k)$ is the Fourier cosine transform, then prove that $\mathcal{F}_C\{f''(x)\} = -k^2 F_c(k) - \sqrt{\frac{2}{\pi}} f'(0)$

Q.7. Consider the motion of a semi-infinite string with an external force $f(t)$ acting on it. One end is kept fixed while the other end is allowed to move freely in the vertical direction. If the string is initially at rest, the motion of the string is governed by

$$u_{tt} = c^2 u_{xx} + f(t), 0 < x < \infty, t > 0$$

$$u(x, 0) = 0,\; u_t(x, 0) = 0 \;\text{ and }\; u(0, t) = 0,\; u_x(x, t) \to 0, \text{ as } x \to \infty$$

University of Sargodha

BS 7th Semester/Term Exam 2021

Subject: Mathematics **Paper: Numerical Analysis-I (MATH-401)**

Time Allowed: 02:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)

i) Define positive deifinite matrix, give example. ii) What is the difference between Doolittle and Cholesky's method. iii) Define Non-linear equation and give examples. iv) How Secant method works, explain v) What is the difference between Gauss-Seidel and Jacobi method. vi) What is an ill-conditioned system. vii) Explain difference between Cholesky's and Crout's method. viii) Explain extrapolation with example ix) How you define Interpolation. x) Descibe advantage of Regula Falsi method over Bisection method of root finding problems. xi) Define Condition Number. xii) Define norm of matrix. xiii) Descibe differnce between relative and absolute error. xiv) What is the difference between direct and iterative methods. xv) What is partial pivoting. xvi) Define dominant eignem value.

Subjective Part. (3×16)

Q2. Solve the following system of equations using Crout's method.

$$\begin{cases} 3x + y + 2z = 3 \\ 2x - 3y - z = -3 \\ x - 2y + z = 4 \end{cases}$$

Q3. Apply Gauss Seidel method to approximate the solution (of the following system of linear equations) by performing three iterations.

$$\begin{cases} 4x_1 + x_2 - x_3 = 5 \\ -x_1 + 3x_2 + x_3 = -4 \\ 2x_1 + 2x_2 + 5x_3 = 1 \end{cases}$$

Q4. Test the matrix for ill-conditioned or not?

$$\begin{bmatrix} 4 & 2 & 6 \\ 3 & 0 & 7 \\ -2 & -1 & -3 \end{bmatrix}$$

Q5. Construct second Lagrange's Interpolating polynomial for

$$f(x) = \frac{1}{x^2}$$

if $x_0 = 2, x_1 = 2.75, x_2 = 4$ and use the polynomial (constructed) to approximate $f(3) = 1/9$

Q6. Apply Newton Raphson procedure to find real root of the equation.

$$x^3 + 3x^2 - 1 = 0, \quad [-3, \ -2]$$

University of Sargodha

BS 7th Semester Examination 2022

Subject: Mathematics **Paper: Numerical Analysis-I (MATH-401)**

Time Allowed: 2:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q1. Write short answers of the following in 2-3 lines each. (2×16)

i) Define zero of a function, give example. ii) State fixed point theorem. iii) Define fixed point of a function. iv) Descibe differnce between relative and absolute error. v) Define linear equation and give examples. vi) How Regula Falsi method works, explain vii) What is the difference between Gauss-Seidel and Jacobi method. viii) What is an ill-conditioned system. ix) Explain difference between Cholesky's and Crout's method. x) Explain convergence criteria of bisection method. xi) How you define Eigen vector. xii) Descibe advantage of Regula Falsi method over Bisection method. xiii) Define Condition Number. xiv) Define norm of matrix. xv) What is the difference between direct and iterative methods. xvi) What is partial pivoting.

Subjective Part. (3×16)

Q2. Solve the system of linear equations using Doolittle's Method.

$$\begin{aligned} x + y + z &= 5 \\ x + 2y + 2z &= 6 \\ x + 2y + 3z &= 8 \end{aligned}$$

Q3. Apply Jacobi method to approximate the solution by performing three iterations.

$$\begin{cases} 4x_1 + x_2 - x_3 = 5 \\ -x_1 + 3x_2 + x_3 = -4 \\ 2x_1 + 2x_2 + 5x_3 = 1 \end{cases}$$

Q4. Apply power method to find dominant eigen value and corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 4 & 1 & -2 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$$

Q5. Find a real root of the following non-linear equation using fixed point iteration

$$x^4 - 4x^3 + 6x^2 - 2.25 = 0$$

Q6. Apply Newton Raphson procedure to find real root of the equation.

$$x^3 + 3x^2 - 1 = 0, \quad [-3, -2]$$

University of Sargodha

BS 7th Term/Semester Exam 2021.

Subject: Mathematics **Paper: Number Theory (MATH-403)**

Time Allowed: 2:30 Hours **Maximum Marks: 80**

Note: **Objective part is compulsory. Attempt any three questions from subjective Part.**

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each. (2*16)

i. Find G.C.D. and L.C.M. of 273 and 81.
ii. Define Bracket function.
iii. Show that if $ax+by=m$ then $(a,b)|m$.
iv. Define reduced residue system.
v. Define Least common multiple.
vi. Find order of 3 modulo 11.
vii. Define algebraic number field.
viii. Write congruence classes for modulo 11 with remainders 3 and 10.
ix. Define symmetric polynomial.
x. Define units of algebraic number field.
xi. Find the values of x, y and z which satisfy $6x+10y+15z=1$.
xii. Find the Residue of 3^{16} modulo 17.
xiii. Define Linear congruence.
xiv. Define primitive root.
xv. If $na \equiv nb \pmod{m}$ and $(n, m)=1$, then show that $a \equiv b \pmod{m}$.
xvi. Prove that every rational integer is an algebraic integer.

Subjective Part (3*16)

Q.2. **(a)** Prove that Mobius function is multiplicative.

(b) Evaluate the Legendre symbol $\left(\frac{503}{773}\right)$.

Q.3. **(a)** State and prove unique factorization theorem.

(b) If $\{a_1, a_2, \ldots, a_{\phi(m)}\}$ is a reduced residue system (mod m) and $(a, m)=1$ then prove that $\{aa_1, aa_2, \ldots, aa_{\phi(m)}\}$ is also a reduced residue system (mod m).

Q.4. **(a)** Prove that an integer α is a root of the congruence $f(x) \equiv 0 \pmod{m}$ if and only if $(x-\alpha)|f(x) \pmod{m}$.

(b) Given that 2 is a primitive root of 9, use indices to solve the following congruence $10x \equiv 8 \pmod{18}$.

Q.5. **(a)** State and prove Chinese Remainder Theorem.

(b) Prove that the set of algebraic numbers forms a field.

Q.6. **(a)** Prove that equation $x^4 + y^4 = z^4$ has no solution in integers.

(b) Prove that a necessary and sufficient condition that the equation $ax+by=c$ has a solution (x, y) in integers, is that $d \mid c$ where $d=(a, b)$ and this solution is of the form $x = x_0 + \frac{b}{d}t,\ y = y_0 - \frac{a}{d}t$ where t is an arbitrary integer.

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Subject: Mathematics **Paper: Number Theory (MATH-403)**

Time Allowed: 2:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)

i. Prove that if n is any odd integer then 8 divides (n^2-1).
ii. Define L.C.M. of two integers.
iii. Define Mobius function.
iv. Evaluate $\varphi(\varphi(500)$.
v. Define linear congruence in one variable
vi. Prove transitive property of equivalence relation for congruences.
vii. Define prime number.
viii. Define order of an integer.
ix. Find G.C.D. of 35, 66, 205.
x. Find primitive root of 5.
xi. Find residue classes of modulo 7 for remainder 3 and 6.
xii. Define complete residue system.
xiii. Find sum and number of divisors of 54.
xiv. Find the solution of the congruence $3x \equiv 4$ (mod5).
xv. State Euler's Criterion.
xvi. Evaluate the following Legender symbols (-1/19) and (2/23).

Subjective Part

Q.2. a) State and prove Euclid's theorem. (10)
b) Find integral solutions of $69x+111y=9000$. (06)

Q.3. a) If p is prime number then $(p-1)! \equiv -1$ (mod p). (08)
b) If a and b are non-zero integers and (a,b)=d then (a/d, b/d) =1. (08)

Q.4. a) A composite number n has a prime divisor $\leq \sqrt{n}$. (08)
b) Solve the following congruence with the help of indices. (08)
$17x^2 \equiv 10$ (mod 29), provided that 2 is the primitive root of 29.

Q.5. a) Prove that if z is an integer and y,z > 0 then [[xy]/z]=[x/yz]. (08)
b) Prove that all primitive solution of equation $x^2+y^2=z^2$ are of the form. (08)
$x=a^2-b^2$, $y=2ab$ and $z=a^2+b^2$, where (a,b)=1 and exactly one of the a and b is even.

Q.6. a) State and prove Lagrange's theorem. (08)
b) Evaluate the following (-1457/2389). (08)