University of Sargodha

BS 7th Term Examination 2023

Subject: Mathematics **Paper: Fluid Mechanics-I/ Mechanics (MATH-419 / MAHT-425)**

Time Allowed: 02:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)

i. Define shear stress.
ii. What is the dimension of density?
iii. Write mathematical form of vorticity.
iv. Define integral form of the equation of continuity.
v. Define control volume form of integral equation.
vi. Give mathematical form of Bernoulli equation.
vii. Give mathematical form of the pressure field.
viii. Define irrotational flow.
ix. Define three dimensional flow.
x. Define pathline.
xi. Write the statement of Reynolds transport theorem for arbitrary fixed control volume.
xii. What do you mean by second law of Thermodynamics?
xiii. Write the continuity equation for incompressible and compressible flow.
xiv. Define streak line.
xv. Define stress field.
xvi. Define incompressible flow.

Subjective Part (3*16)

Q.2. Derive equation of pressure field for static fluid.

Q.3. a) Given $= 3yi + 6xj + x^2k$, calculate

i) The angular rotation ii) The vorticity iii) The stress tensor

b) Relate vorticity to velocity and shear s

Q.4. a) State and prove Newton's law of viscosity.

b) What is the difference between the Eulerian and Lagrangian Description?

Q.5. Derive equation of continuity in integral form.

Q.6. Given $V = U\left(1 + \frac{x}{x^2+y^2}\right)i + \frac{Uy}{x^2+y^2}j$

Is the incompressible form of the continuity equation is satisfied? What are the values of x and y for the flow to be stagnant?

------------------------- LK-06-02-4819/4879 -------------------------

University of Sargodha

BS 7th Term Examination 2023

Subject: Mathematics **Paper: Partial Differential Equations (MATH-405)**

Time Allowed: 02:30 Hours **Maximum Marks: 80**

Note: **Objective part is compulsory. Attempt any four questions from subjective part.**

Objective Part **(Compulsory)**

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)

i. Write down the quasi linear partial differential equation with example.
ii. Define Well Posed Problem.
iii. Obtain PDE $z = xy + F(x^2 + y^2)$ where F is arbitrary function.
iv. Write down the physical meanings of the Neumann boundary conditions.
v. Define Standard Form of first order linear equation.
vi. Write mathematical form of the Klein–Gordon equation.
vii. Write wave equation in spherical coordinates with general solution.
viii. Show that $\mathcal{F}\{\mathcal{X}_{[-a,a]}(x)\} = \sqrt{\frac{2}{\pi}}\left(\frac{Sinak}{k}\right)$.
Where $\mathcal{X}_{[-a,a]}(x) = \text{H}(\text{a} - |\text{x}|) = \begin{cases} 1, & |\text{x}| < a \text{ or } -\text{a} < x < a \\ 0, & |\text{x}| > a \end{cases}$
ix. Find the Fourier transformation of unit step function.
x. Find the Fourier series expansion for the function $f(x) = x + x^2$; $-\pi < x < \pi$
xi. Write existence condition for the Laplace transformation.
xii. If $\mathcal{L}\{f(t)\} = F(s)$ then $\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(s)\, ds$
xiii. Find The Laplace transform of an impulse function.
xiv. Let $f'(x)$ be continuous and $f''(x)$ be peicewise continuous in $[0, \pi]$ if $F_s(k)$ is the finite Fourier Sine Transform of $f(x)$ then prove that
$\mathcal{F}_s\{f''(x)\} = \frac{2k}{\pi}[f(0) - (-1)^k f(\pi)] - k^2 F_s(k)$
xv. Show that the Hankel transform satisfies the Parseval relation
$\int_0^\infty r f(r)\, g(r) dr = \int_0^\infty k \tilde{f}(\kappa) \tilde{g}(\kappa)\, dk$
xvi. Write the Difference between Laplace and Fourier Transform.

Subjective Part **(4*12)**

Q.2. Solve the linear equation $yu_x + xu_y = u$ with the Cauchy data $u(x, 0) = x^3$ and $u(0, y) = y^3$

Q.3. Solve the equation $y^2u_x^2 + x^2u_y^2 = (xyu)^2$ with $u(x,0) = 3\exp\left(\frac{x^2}{4}\right)$.

Q.4. State and Prove one dimensional Heat Equation.

Q.5. Derive Heat equation in cylindrical coordinates.

Q.6. Let f(x) and its first derivative vanish as $x \to \infty$. If $F_s(k)$ is the Fourier cosine transform, then
$$\mathcal{F}_s\{f''(x)\} = \sqrt{\frac{2}{\pi}} kf(0) - k^2 F_s(k)$$

Q.7. Find the temperature distribution in a semi-infinite radiating rod. The temperature is kept constant at $x = 0$, while the other end is kept at zero temperature. If the initial temperature distribution is zero, the problem is governed by
$$u_t = ku_{xx} - hu,\ 0 < x < \infty, t > 0,\ h = \text{constant},$$
$$u(x,0) = 0, u(0,t) = u_0,\ t>0,\ u_0 = \text{constant, where } u(x,t) \to 0, as\ x \to \infty$$

------------------------- LK-01-02-4737-------------------------

University of Sargodha

BS 7th Term Examination 2023

Subject: Mathematics **Paper: Number Theory (MATH-403)**

Maximum Marks: 80

Time Allowed: 02:30 Hours

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

(2*16)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet.

i. Show that every integer a is of the form 3n, 3n+1 or 3n-1.
ii. Define G.C.D. of two integers.
iii. If p is prime such that $p|a^2+b^2$ and $p|a$, then $p|b$.
iv. Find the Mobius function of 500.
v. Define standard form of an integer.
vi. Define number theoretic functions.
vii. Define multiplicative function and give an example.
viii. Find the values of x,y and z which satisfy the equation $6x+10y+15z = 1$.
ix. Find the sum of divisors of 72.
x. If (273, 81)= 3 then find <273,81>.
xi. Find the divisors of p^{α}.
xii. Define reduced residue system.
xiii. Find order of 3 modulo 11.
xiv. Find Quotient and remainder in the division algorithm with divisor 17 and dividend -100.
xv. Differentiate between a Quadratic Residue and Quadratic Non-Residue.
xvi. Evaluate the following Legender symbols (-1/7) and (2/13).

Subjective Part

Q.2. a) Prove that a necessary and sufficient condition that the congruence $ax \equiv b \pmod{m}$ is solvable is that $(a,m)=d$ divides b. If this is so then there are exactly (a,m) solutions modulo m. (10)
b) If $(a,c)=1$, then $(a,bc)=(a,b)$. (06)

Q.3. a) Find all the primitive roots of 17^2. (08)
b) Prove that greatest common divisor of two integers is unique. (08)

Q.4. a) State and prove the fundamental theorem of Arithmatic. (10)
b) Evaluate the following $\left(\frac{182}{271}\right)$ by taking prime factorization of 182. (06)

Q.5. a) State and prove Factor theorem. (08)
b) Prove that Mobius function is multiplicative. (08)

Q.6. a) State and prove Chinese remainder theorem. (08)
b) If order of a modulo m is t and if u is a positive integer, then prove that order of a^u modulo m is t/d, where d is the greatest common divisor of u and t. (08)

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University of Sargodha

BS 7th Term Examination 2023

Subject: Mathematics **Paper: Operation Research (MATH-421 / MATH-429)**

Time Allowed: 02:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)

i. What is optimal solution?
ii. Define linear programming.
iii. Define corner point.
iv. Define pivot element.
v. Graph the feasible solution space of $2x_1 + 3x_2 \leq 6, x_1, x_2 \geq 0$.
vi. What are the slacks variables? Explain with example.
vii. Which requirements impose on LP constraints to develop the simplex method computation?
viii. Convert the constraints $x_1 + 3x_2 \leq 6, 3x_1 + 2x_2 \leq 6, x_1, x_2 \geq 0$, into equations by introducing slacks or surplus variables.
ix. What is optimality condition of simplex method?
x. What is degeneracy in simplex method?
xi. Define artificial variable.
xii. Write the dual problem for the following primal problem

$$\text{Minimize } z = -5x_1 + 2x_2, \text{ subject to } \begin{array}{l} -x_1 + x_2 \leq -2 \\ 2x_1 + 3x_2 \leq 5 \end{array}, x_1, x_2, \geq 0.$$

xiii. What is unbounded solution?
xiv. Define transportation model.
xv. What is dual feasibility condition?
xvi. Write the following OR model in M-Method,

$$\text{Maximize } z = 6x_1 + 2x_2, \text{ subject to } \begin{array}{l} 3x_1 + x_2 = 3 \\ 4x_1 + 3x_2 \geq 6 \\ x_1 + 2x_2 \leq 4 \end{array}, x_1, x_2, \geq 0.$$

Subjective Part (3*16)

Q.2. Solve the problem by graphical method,

$$\text{Minimize } z = 3x_1 + 8x_2, \text{ subject to } \begin{array}{l} x_1 + x_2 \geq 8 \\ 2x_1 - 3x_2 \leq 0 \\ x_1 + 2x_2 \leq 30 \\ 3x_1 - x_2 \geq 0 \\ x_1 \leq 10 \\ x_2 \geq 9 \end{array}, x_1, x_2 \geq 0$$

Q.3. Find optimal solution by simplex method

$$\text{Maximize } z = 2x_1 - 4x_2 + 5x_3 - 6x_4, \text{ subject to } \begin{array}{l} x_1 + 4x_2 - 2x_3 + 8x_4 \leq 2 \\ -x_1 + 2x_2 + 3x_3 + 4x_4 \leq 1 \end{array}, x_1, x_2, x_3, x_4 \geq 0$$

Q.4. Solve by two phase method

$$\text{Minimize } z = 4x_1 + x_2, \text{ subject to } \begin{array}{l} 3x_1 + x_2 = 3 \\ 4x_1 + 3x_2 \geq 6 \\ x_1 + 2x_2 \leq 4 \end{array}, x_1, x_2 \geq 0$$

Q.5. Convert the following LP model into dual problem then solve it

$$\text{Maximize } z = 5x_1 + 12x_2 + 4x_3, \text{ subject to } \begin{array}{l} x_1 + 2x_2 + x_3 \leq 10 \\ 2x_1 - x_2 + 3x_3 = 8 \end{array}, x_1, x_2, x_3 \geq 0$$

Q.6. Cars are shipped from three distribution centers to fire dealers. The shipping cost is based on the mileage between the sources and the destinations and is independent of whether the truck makes the trip with partial or full loads. The table summarizes the mileage between the distribution centers and the dealers together with the monthly supply and demand figures given in number of cars. A full truck load includes 18 cars. The transportation cost per truck mile is $25.

a) Formulate the associated transportation model. b) Determine the optimal shipping schedule.

	Dealer 1	2	3	4	5	Supply
1	100	150	200	140	35	400
2	50	70	60	65	80	200
3	40	90	100	150	130	150
Demand	100	200	150	160	140	

University of Sargodha

BS 7th Term Examination 2023

Subject: Mathematics **Paper: Numerical Analysis-I (MATH-401)**

Time Allowed: 02:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)

i) State intermediate value theorem. ii) Define fixed point of a function. iii) Descibe differnce between relative and absolute error. iv) Define Non-linear equation and give examples. v) How Secant method works. explain vi) What is the difference between Gauss-Seidel and Jacobi method. vii) What is an ill-conditioned system. viii) Explain difference between Cholesky's and Crout's method. ix) Explain order of convergence in Newton Raphson method. x) How you define Eigen value. xi) Descibe advantage of Regula Falsi method over Bisection method of root finding problems. xii) Define Condition Number. xiii) Define norm of matrix. xiv) What is the difference between direct and iterative methods. xv) What is partial pivoting. xvi) Define dominant eignem value.

Subjective Part. (3×16)

Q2. *Solve the following system of equations using Cholesky's method*

$$\begin{cases} 4x + 2y + 2z = 14 \\ 2x + 5y + 3z = 21 \\ 2x + 3y + 6z = 26 \end{cases}$$

Q3. Apply Gauss Seidel method to approximate the solution by performing three iterations.

$$\begin{cases} 4x_1 + x_2 - x_3 = 5 \\ -x_1 + 3x_2 + x_3 = -4 \\ 2x_1 + 2x_2 + 5x_3 = 1 \end{cases}$$

Q4. Apply power method to find dominant eigen value and corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

Q5. Find the error bound for Bisection method. Discuss convergence of Newton-Raphson method with example.

Q6. Apply Regula Falsi procedure to find real root of the equation.

$$f(x) = x^3 + 4x^2 - 10, \quad \text{at} \quad [1,\ 2]$$

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