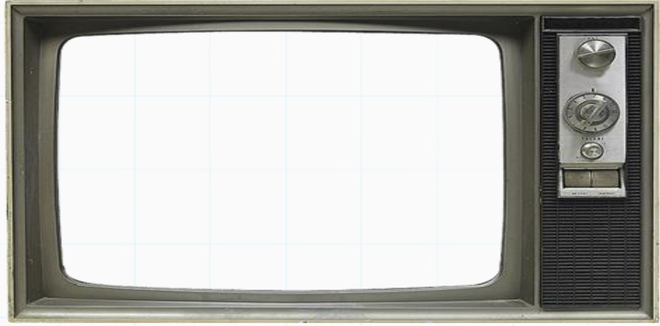
Métodos para PPL



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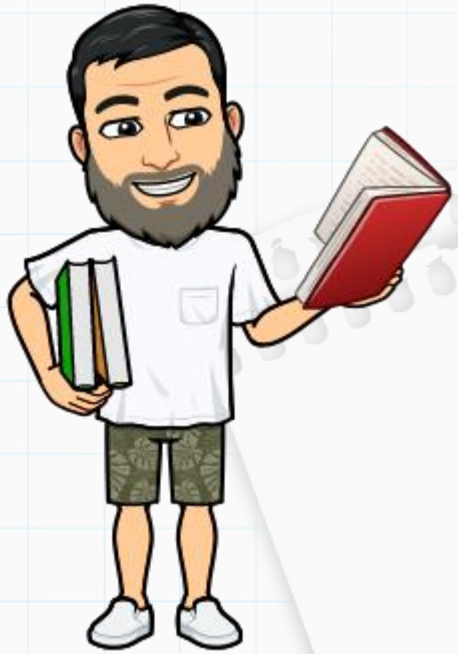
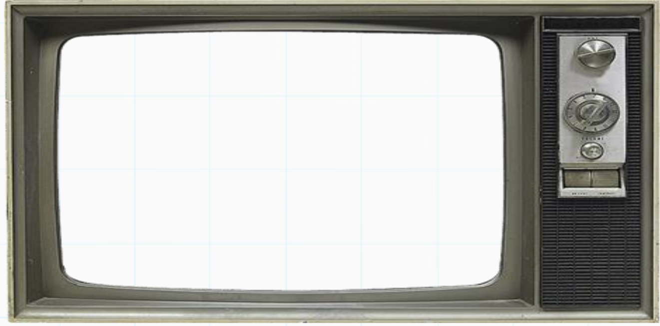


Soluções Básicas



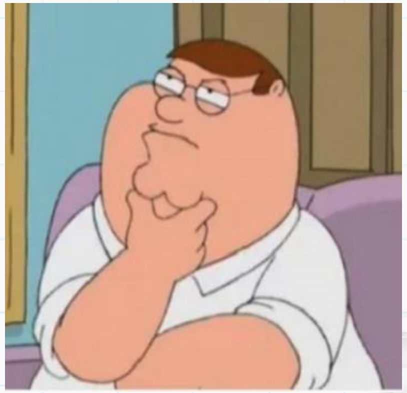
- Considere o PPL:

$$\max c^T x = c^T \begin{pmatrix} x_B \\ x_n \end{pmatrix}$$

$$\text{s.a.} \quad x_B = B^{-1}b - B^{-1}Nx_N$$

$$x_B, x_N \geq 0$$

- Colocamos as restrições em função de xB para termos diretamente o valor de xB ao “zerarmos” xN e chegarmos em um SD
- Que tal colocarmos a F.O. em função de xN ? Porque ?
  - Como xN vai ser “zerada”, vai ficar fácil saber qual o valor da SBV



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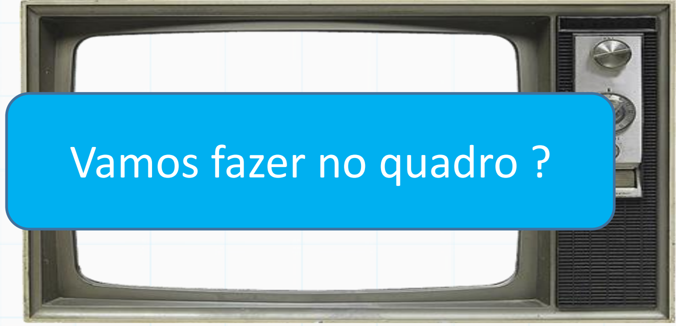

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- Vemos que a f.o. em função de x_N fica:

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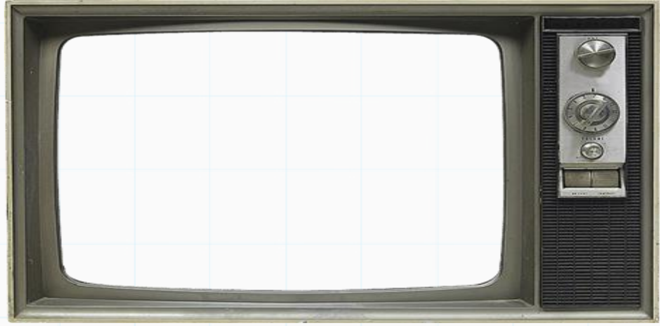

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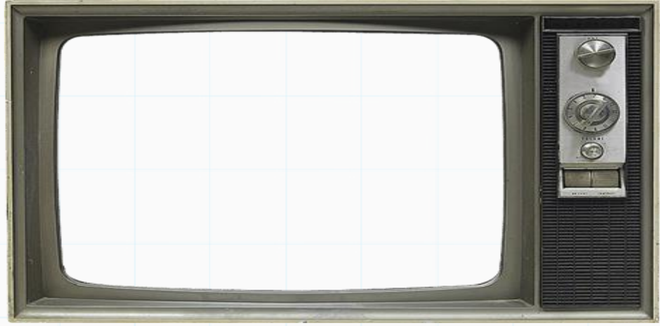

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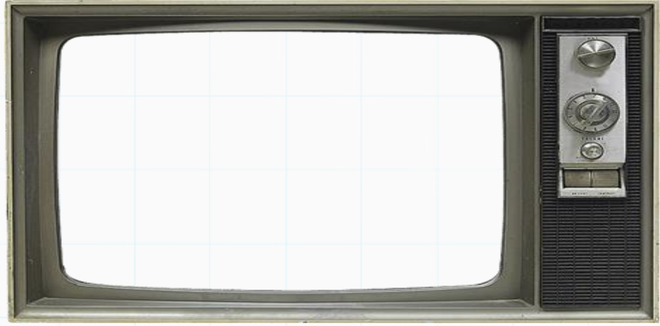

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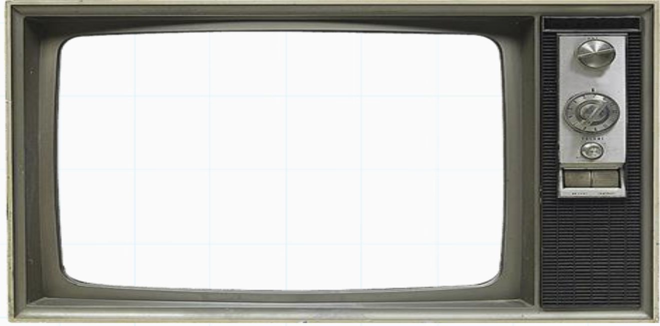
$$\boxed{c_B^T B^{-1}b + (c_N^T - c_B^T B^{-1}N)x_N}$$

Em uma SBV, ao “zerarmos” xN, ficamos com um valor fixo determinado

Soluções Básicas



- Logo reescrevendo temos:

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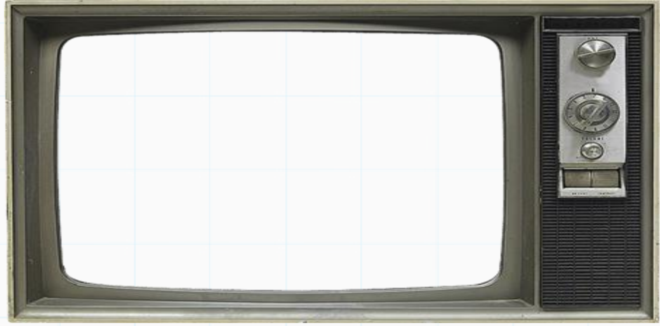

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- Utilizando as seguintes notações para simplificar:

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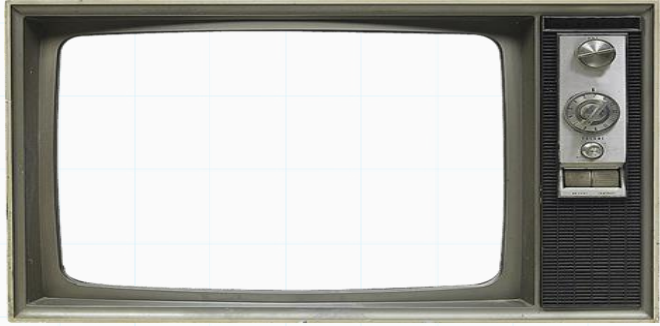

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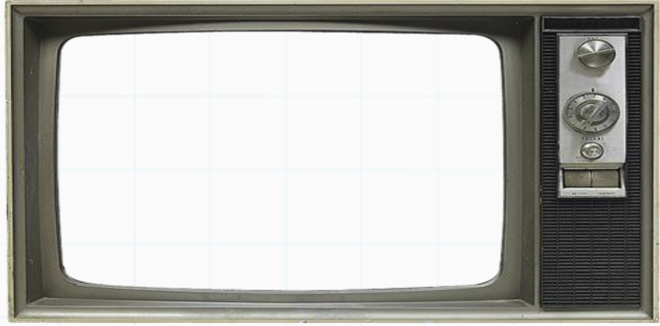

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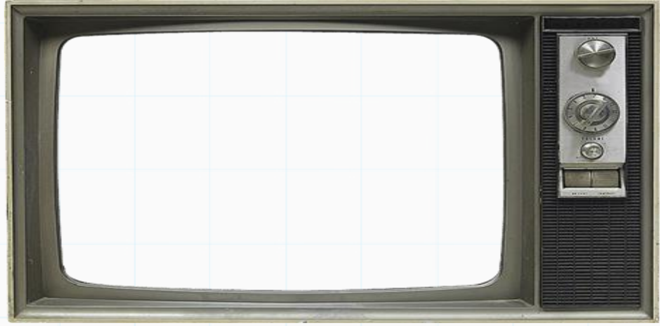

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$[B^{-1} N]_{ij} = y_{ij};\ \ B^{-1} N$ é uma matriz coeficientes das matrizes

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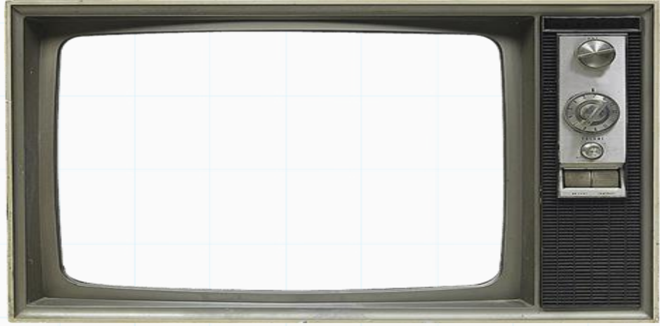

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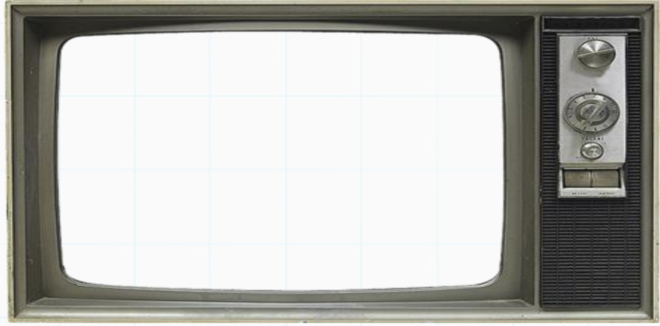
- e vamos considerar também

$A = [a_1, a_2, \ldots, a_n]$

$I_N = \{j \mid a_j \in N\}$ - índice das variáveis não básicas

$I_B = \{j \mid a_j \in B\}$ - índice das variáveis básicas

Soluções Básicas



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$\bar{z} = c_B^T B^{-1} b \in \mathbb{R}$

$c_j - z_j = (c_N^T - c_B^T B^{-1} N)_j$; $(c - z)$ é um vetor

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reescrevendo

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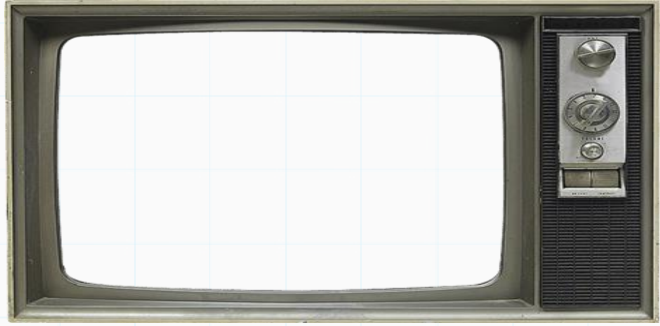

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$$\text{s.a.} \quad x_{B_i} = \bar{x}_{B_i} - \sum_{j \in I_N} y_{ij} x_j, \qquad i = 1, \ldots, m$$
$$x_j \geq 0 \qquad i \in I_N \cup I_B$$

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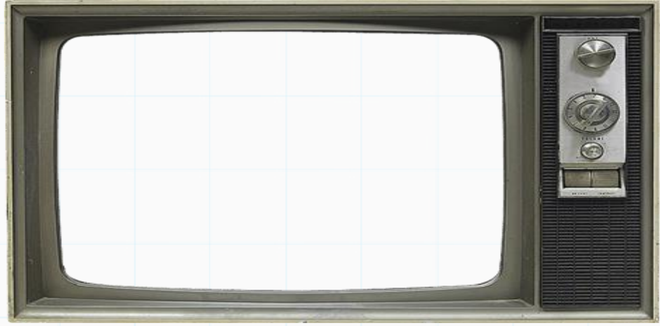
- Chamamos de <u>formato padrão</u> em relação a uma base B

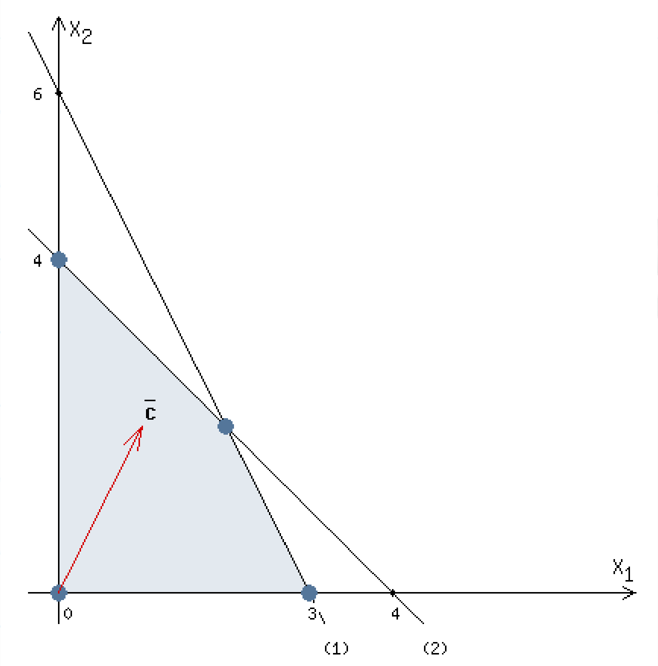
E que sabemos equivale a um vértice do poliedro, ao “zerarmos” xN

Soluções Básicas



- Exemplo:

$$\begin{aligned} \max \quad & x_1 + 2x_2 \\ \text{s.a.} \quad & 2x_1 + x_2 + x_3 = 6 \\ & x_1 + x_2 + x_4 = 4 \\ & x_1,\ x_2,\ x_3,\ x_4 \geq 0 \end{aligned}$$



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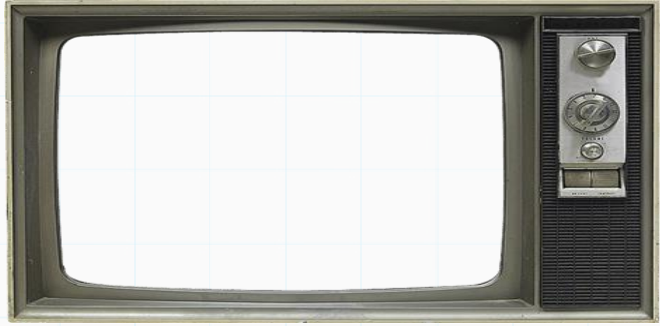

- Exemplo: $\max \; x_1 + 2x_2$

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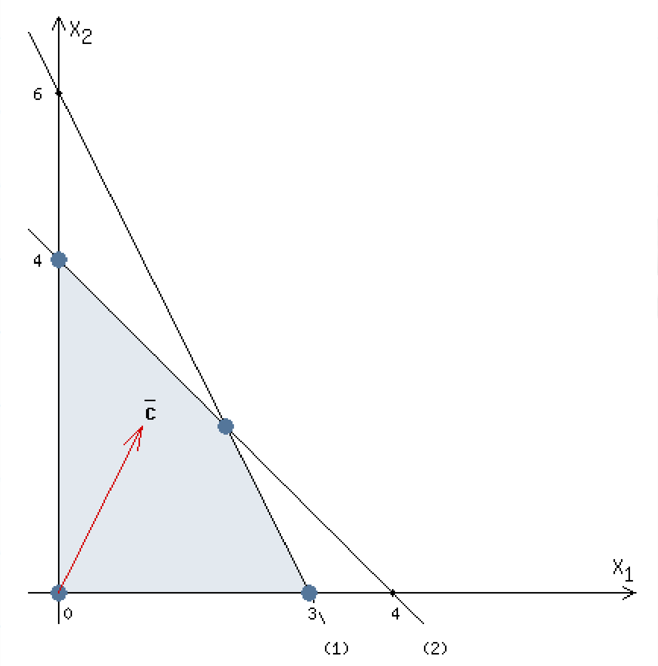
- vamos pegar x_3 e x_4 para compor uma base inversível, logo temos:

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \; N = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, \; c_B = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; c_N = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \; b = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$



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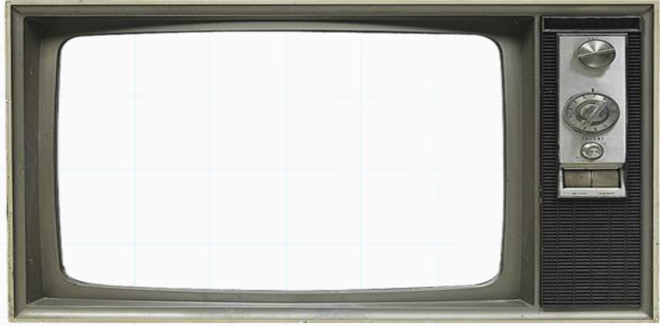

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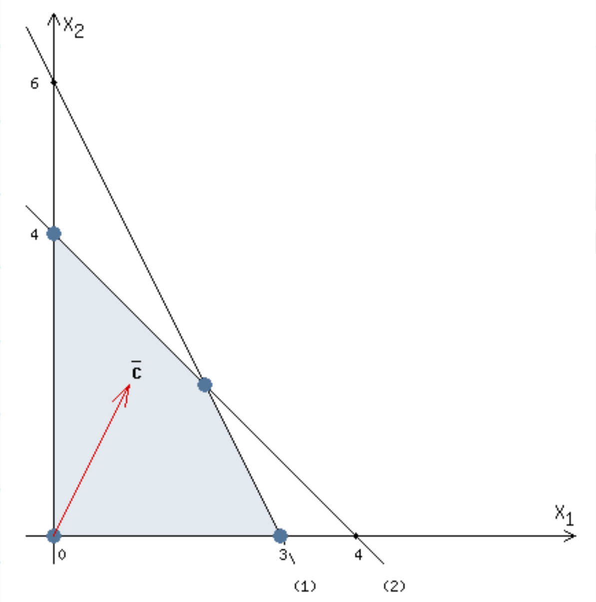

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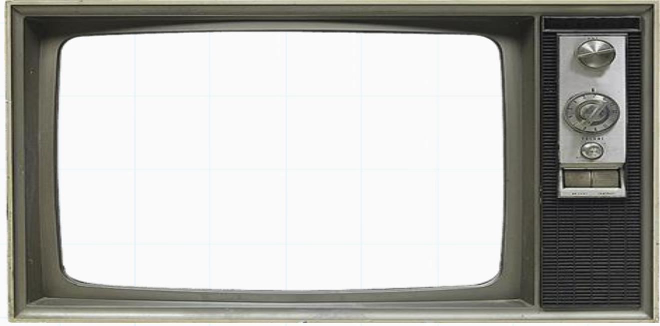

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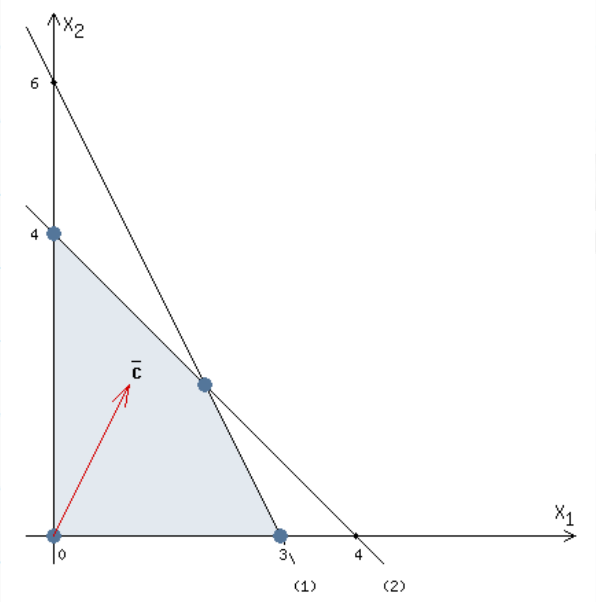

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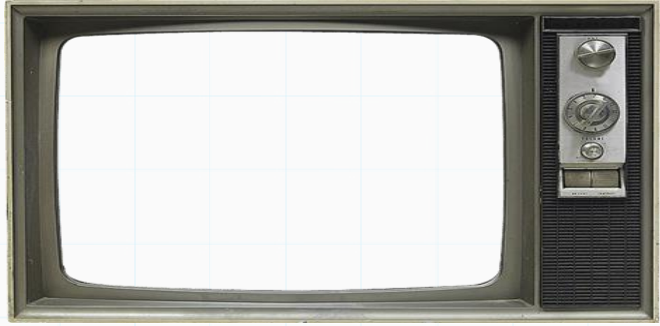

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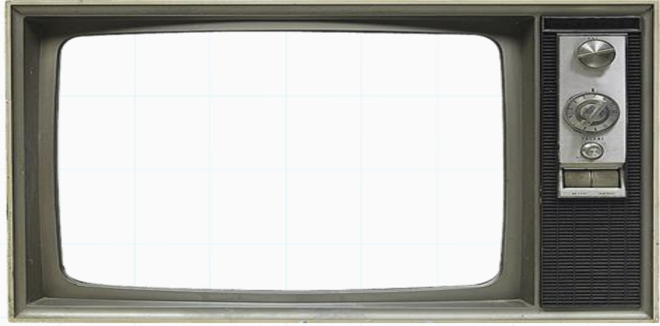

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$$c - z = c_N^T - c_B^T B^{-1} N = [1 \ 2] - [0 \ 0] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = [1 \ 2]$$

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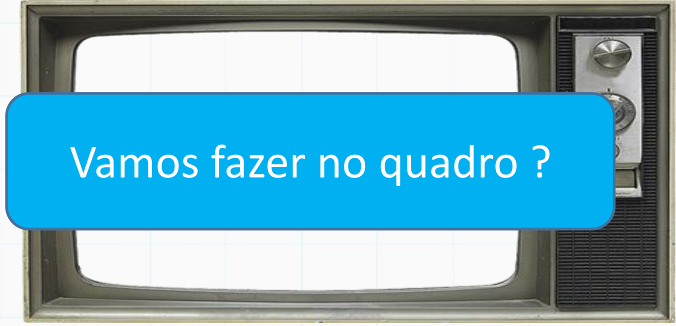

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$$x_j \geq 0 \qquad i \in I_N \cup I_B$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \; N = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, \; c_B = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; c_N = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \; b = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

$$\bar{z} = c_B^T B^{-1} b = [0 \; 0] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \end{bmatrix} = 0$$

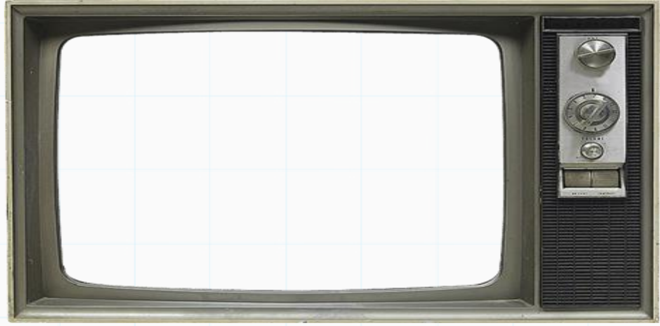
$$\bar{x}_B = B^{-1} b = \begin{bmatrix} 6 \\ 4 \end{bmatrix} \qquad y = B^{-1} N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$c - z = c_N^T - c_B^T B^{-1} N = [1 \; 2] - [0 \; 0] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = [1 \; 2]$$

$\Rightarrow$?

- Como ficaria o modelo na base x3 e x4 ?

Soluções Básicas



$$\max \quad \bar{z} + \sum_{j \in I_N} (c_j - z_j) x_j$$

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- Como ficaria o modelo na base x3 e x4 ?

$$\max \quad 0 + (1)x_1 + (2)x_2$$

$$\text{s.a.} \quad x_3 = 6 - (2x_1 + 1x_2)$$

$$x_4 = 4 - (1x_1 + 1x_2)$$

$$x_1, \; x_2, \; x_3, \; x_4 \geq 0$$

Note que o PPL acima é o mesmo que o original, apenas rearranjado

$$\max \quad x_1 + 2x_2$$

$$\text{s.a.} \quad 2x_1 + x_2 + x_3 = 6$$

$$x_1 + x_2 + x_4 = 4$$

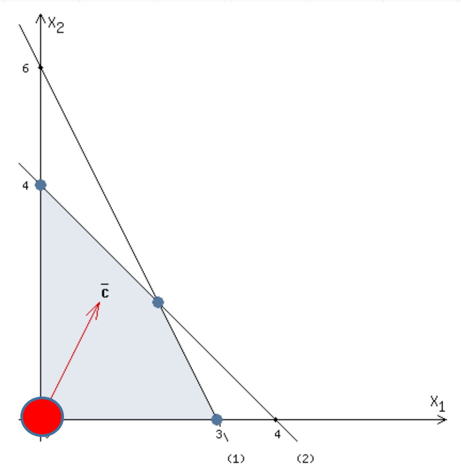
$$x_1, \; x_2, \; x_3, \; x_4 \geq 0$$

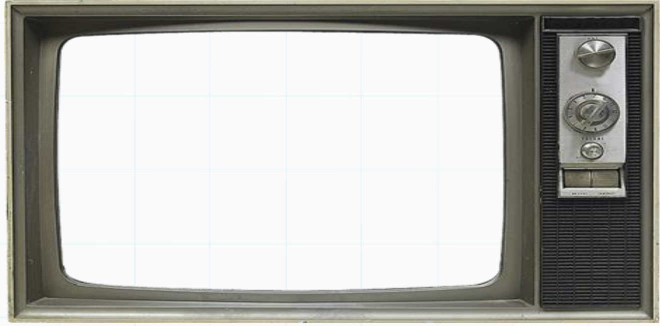
Soluções Básicas

$$\max \quad \bar{z} + \sum_{j \in I_N} (c_j - z_j) x_j$$

$$\text{s.a.} \quad x_{B_i} = \bar{x}_{B_i} - \sum_{j \in I_N} y_{ij} x_j, \qquad i = 1, \dots, m$$

$$x_j \geq 0 \qquad i \in I_N \cup I_B$$





$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \; N = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, \; c_B = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; c_N = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \; b = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

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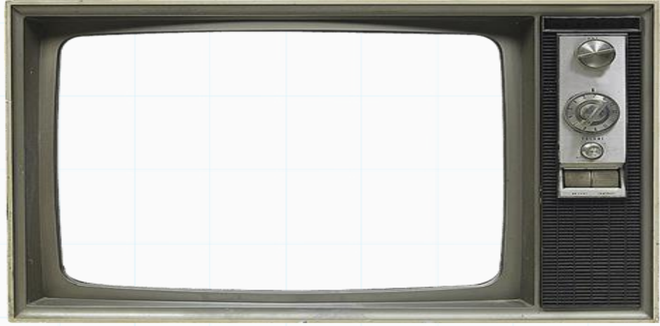
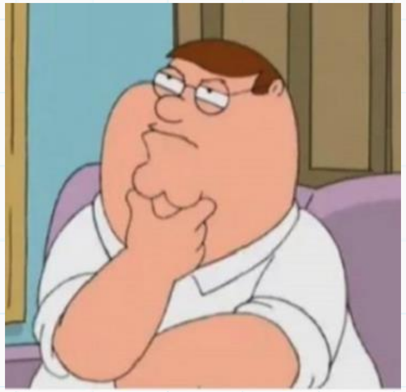
$$x_1, \ x_2, \ x_3, \ x_4 \geq 0$$

Problema no formato da base B, qual é a solução básica em B ?

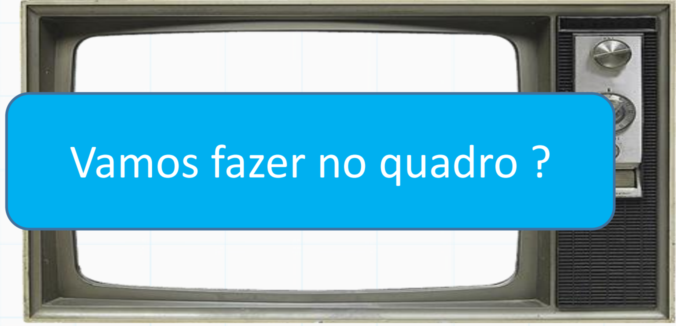
Soluções Básicas



Beleza, já temos como enumerar as bases viáveis, zerar os xN, e chegar em SD onde as soluções são vértices, mas como saber se aquela base (vértice) que estamos vendo é o ótimo sem ter que ver TODOS os vértice ?



Soluções Básicas

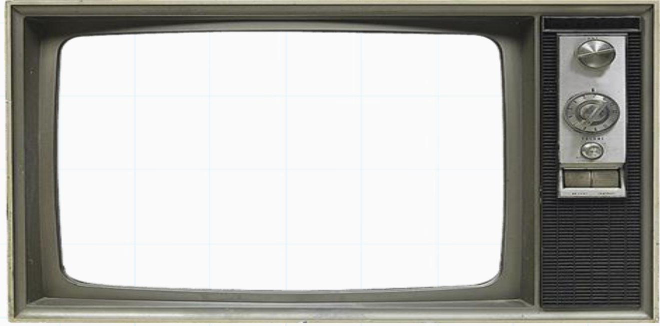


Vamos fazer no quadro ?

Teorema: Se $\bar{x}_B \geq 0$ e $(c_j - z_j) \leq 0$, $\forall j \in I_N$, então a solução x* onde $x_B^* = \bar{x}_B$ e $x_N^* = 0$ será uma solução ótima para o PPL

$$
\begin{array}{lll}
\max & \bar{z} + \sum_{j \in I_N} (c_j - z_j) x_j & \\
\text{s.a.} & x_{B_i} = \bar{x}_{B_i} - \sum_{j \in I_N} y_{ij} x_j, & i = 1, \ldots, m \\
& x_j \geq 0 & i \in I_N \cup I_B
\end{array}
$$

Soluções Básicas



<u>Teorema</u>: Se $\bar{x}_B \geq 0$ e $(c_j - z_j) \leq 0, \forall j \in I_N$, então a solução x^* onde $x_B^* = \bar{x}_B$ e $x_N^* = 0$ será uma solução ótima para o PPL

Vemos que pela função objetivo z = $\bar{z} + \sum_{j \in I_N} \overbrace{(c_j - z_j)}^{\leq 0} \overbrace{x_j}^{\geq 0}$

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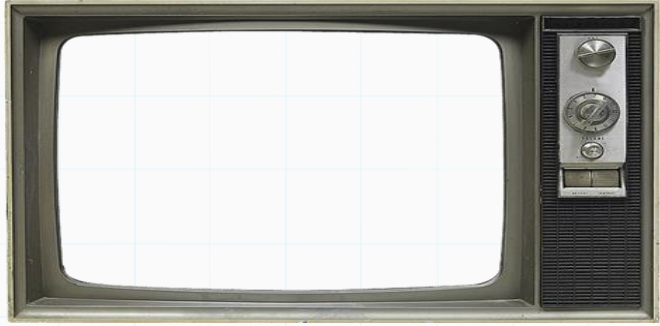
Soluções Básicas



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mas como $c_j - z_j \leq 0$ e $x_j \geq 0$, temos que z $\leq \bar{z} = c_B^T B^{-1} b = c^T x^*$

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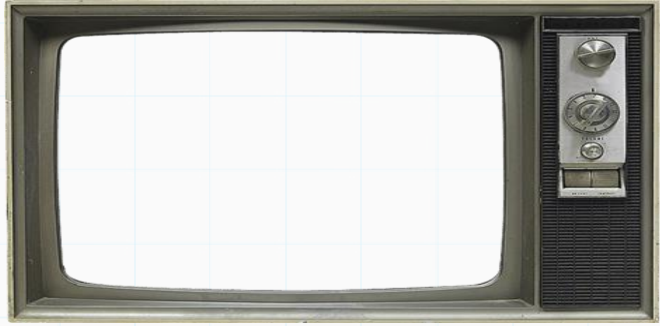
Soluções Básicas



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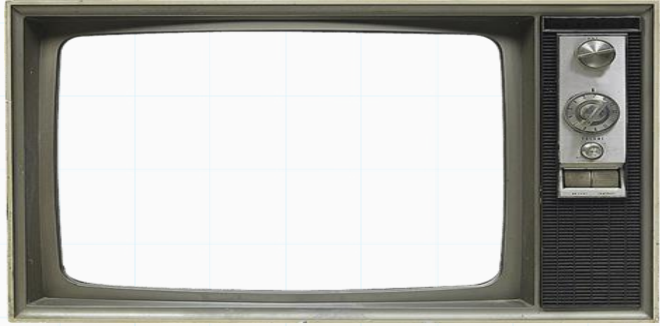
mas como $c_j - z_j \leq 0$ e $x_j \geq 0$, temos que z $\leq \bar{z} = c_B^T B^{-1} b = c^T x^*$

logo, o valor de z nunca ultrapassará cx*, e como x* é uma solução do problema, ela é ótima.

■

$$\begin{aligned} \max \quad & \bar{z} + \sum_{j \in I_N} (c_j - z_j) x_j & \\ \text{s.a.} \quad & x_{B_i} = \bar{x}_{B_i} - \sum_{j \in I_N} y_{ij} x_j, & i = 1, \ldots, m \\ & x_j \geq 0 & i \in I_N \cup I_B \end{aligned}$$

Soluções Básicas



- Voltando ao nosso exemplo anterior na base B={x_3,x_4}:

$$
\begin{aligned}
\max \quad & 0 + (1)x_1 + (2)x_2 \\
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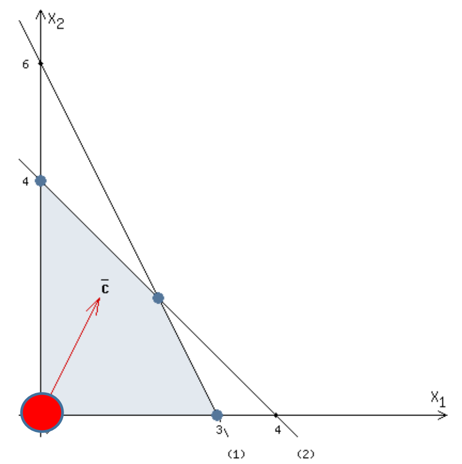
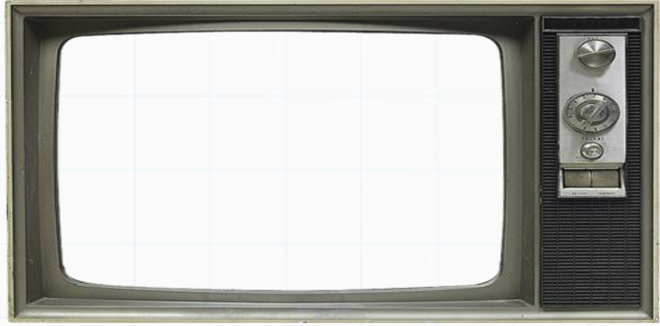


Soluções Básicas



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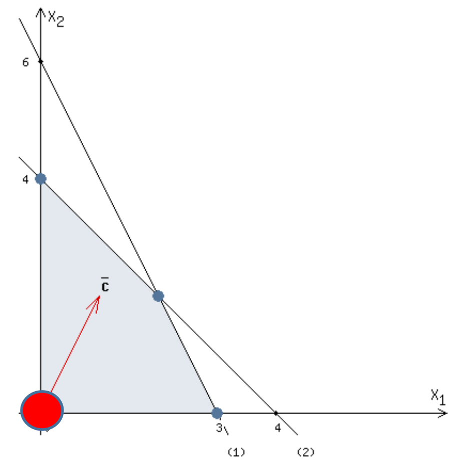
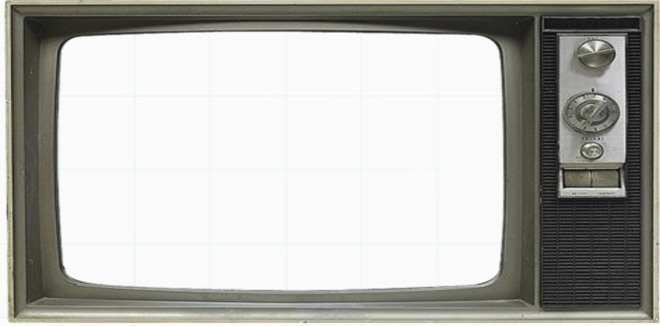
- Como $z-c^T=(1 \;\; 2)$, logo não é ótimo. Vamos considerar agora $B=\{x_2,x_3\}$:



Soluções Básicas



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$$

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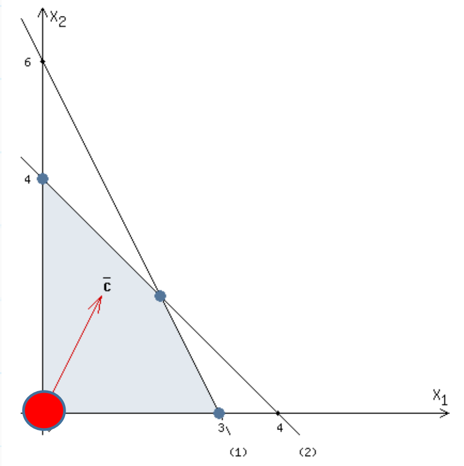
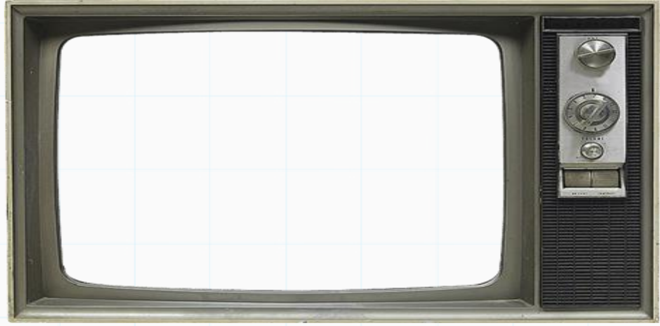


Soluções Básicas



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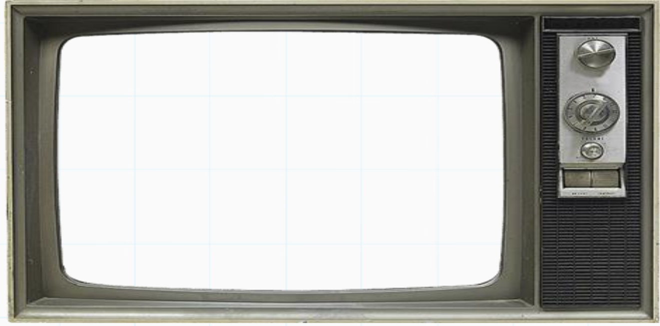
$$B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow B^{-1} = \frac{1}{det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\mathbf{A}^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Soluções Básicas



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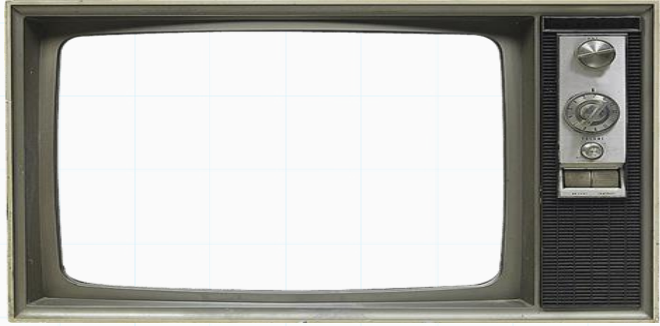
Soluções Básicas



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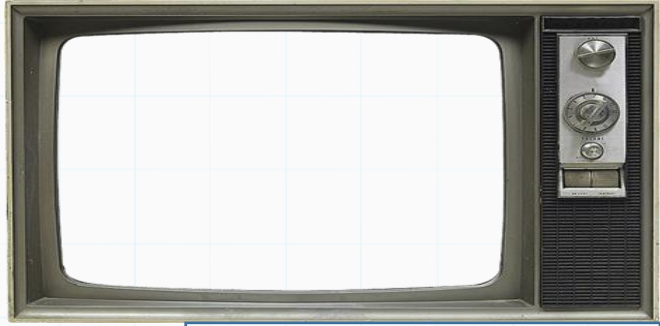
$$\bar{z} = c_B^T B^{-1} b = [2\ 0] \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \end{bmatrix} = 8$$

Soluções Básicas



- Continuando:

$$c - z = c_N^T - c_B^T B^{-1} N = [1\ 0] - [2\ 0] \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} = [-1\ \ -2]$$

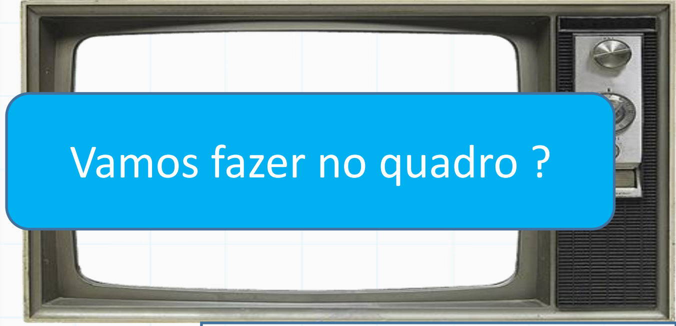
$$c_B = \begin{bmatrix} 2 \\ 0 \end{bmatrix},\ c_N = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$N = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\bar{x}_B = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \quad \bar{z} = 8$$

Soluções Básicas



- Continuando:

$$c - z = c_N^T - c_B^T B^{-1} N = [1\ 0] - [2\ 0] \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} = [-1\ \ -2]$$

$$y = B^{-1} N = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

- Vamos montar o problema nesta base x2 e x3:

?

$$c_B = \begin{bmatrix} 2 \\ 0 \end{bmatrix},\ c_N = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

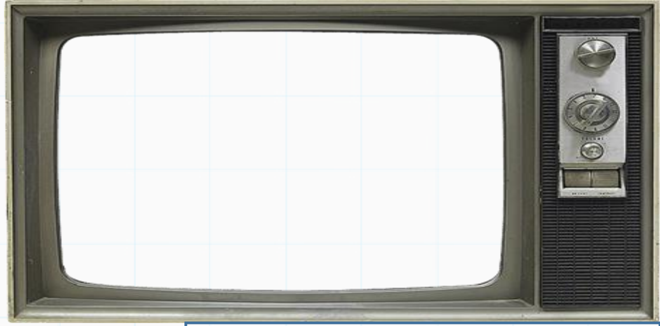
$$N = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\bar{x}_B = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \quad \bar{z} = 8$$

$$\begin{aligned} \max\ & \bar{z} + \sum_{j \in I_N} (c_j - z_j) x_j \\ \text{s.a.}\ & x_{B_i} = \bar{x}_{B_i} - \sum_{j \in I_N} y_{ij} x_j, \\ & x_j \geq 0 \end{aligned}$$

Soluções Básicas



$$c_B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \quad c_N = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$N = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

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- Continuando:

$$c - z = c_N^T - c_B^T B^{-1} N = [1\ 0] - [2\ 0] \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} = [-1 \ \ -2]$$

$$y = B^{-1} N = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

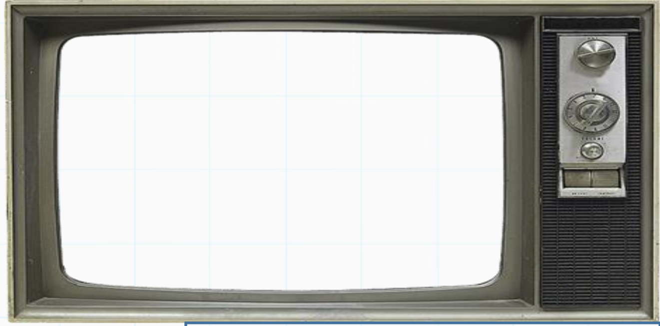
- Vamos montar o problema nesta base x2 e x3:

$$\begin{aligned} \max \quad & \bar{z} + \sum_{j \in I_N} (c_j - z_j) x_j & \\ \text{s.a.} \quad & x_{B_i} = \bar{x}_{B_i} - \sum_{j \in I_N} y_{ij} x_j, & i = 1, \ldots, m \\ & x_j \geq 0 & i \in I_N \cup I_B \end{aligned}$$

$$\Rightarrow$$

$$\begin{aligned} \max \quad & 8 - 1x_1 - 2x_4 \\ \text{s.a.} \quad & x_2 = 4 - (x_1 + x_4) \\ & x_3 = 2 - (x_1 - 1x_4) \\ & x_1,\ x_2,\ x_3,\ x_4 \geq 0 \end{aligned}$$

Soluções Básicas



- Continuando:

$$c - z = c_N^T - c_B^T B^{-1} N = [1\ 0] - [2\ 0] \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} = [-1\ \ -2]$$

$$y = B^{-1} N = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$c_B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \quad c_N = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$N = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\bar{x}_B = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \qquad \bar{z} = 8$$

- Vamos montar o problema nesta base x2 e x3:

$$\begin{aligned} \max \quad & \bar{z} + \sum_{j \in I_N} (c_j - z_j) x_j & \\ \text{s.a.} \quad & x_{B_i} = \bar{x}_{B_i} - \sum_{j \in I_N} y_{ij} x_j, & i = 1, \ldots, m \\ & x_j \geq 0 & i \in I_N \cup I_B \end{aligned}$$

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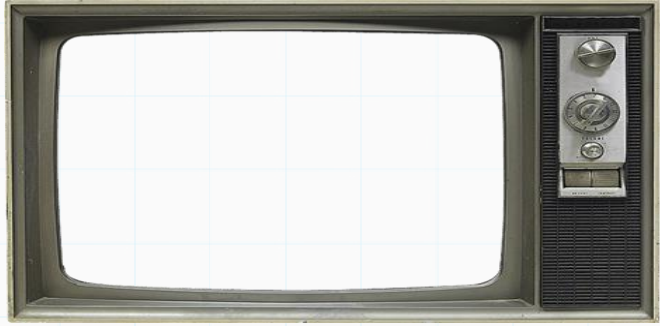
$$\begin{aligned} \max \quad & 8 - 1x_1 - 2x_4 \\ \text{s.a.} \quad & x_2 = 4 - (x_1 + x_4) \\ & x_3 = 2 - (x_1 - 1x_4) \\ & x_1,\ x_2,\ x_3,\ x_4 \geq 0 \end{aligned}$$

- Como $(c_j - z_j) \leq 0, \forall j \in I_N$ então esta é a solução ótima.

Soluções Básicas



- Agora comparando as duas soluções básicas que encontramos, observe que apesar do problema ser expresso em 2 bases diferentes, o problema ainda é o mesmo (apenas rearranjado de forma diferente):

  - Se aplicar a solução básica do PPL baseado em $B_{\{3,4\}}$ no PPL baseado em $B_{\{2,3\}}$, alcançamos que F.O ?
  - Se aplicar a solução básica do PPL baseado em $B_{\{2,3\}}$ no PPL baseado em $B_{\{3,4\}}$, alcançamos que F.O ?

$B_{\{3,4\}}$

$$\begin{aligned} \max \quad & 0 + (1)x_1 + (2)x_2 \\ \text{s.a.} \quad & x_3 = 6 - (2x_1 + 1x_2) \\ & x_4 = 4 - (1x_1 + 1x_2) \\ & x_1,\ x_2,\ x_3,\ x_4 \geq 0 \end{aligned}$$

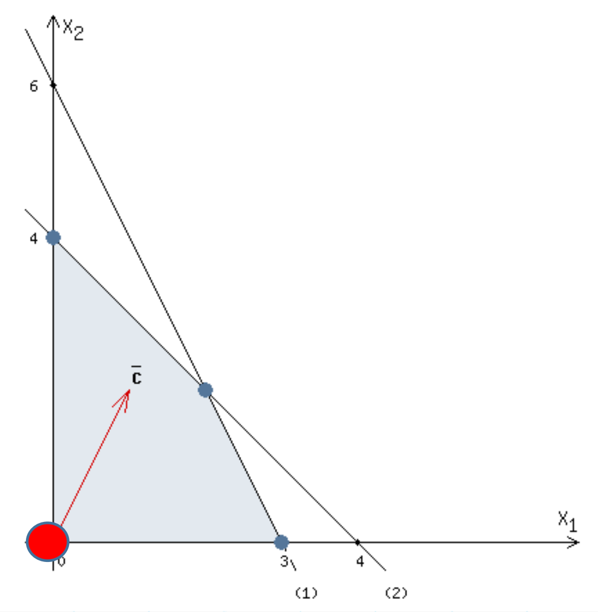
$B_{\{2,3\}}$

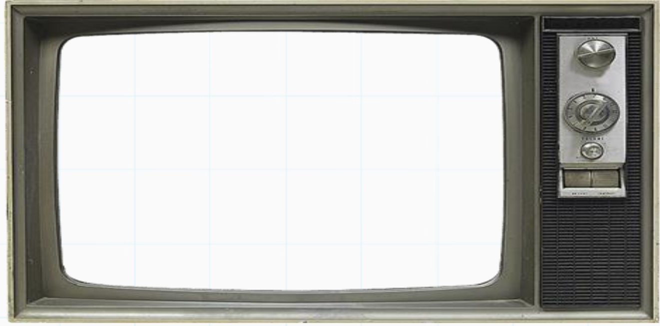
$$\begin{aligned} \max \quad & 8 - 1x_1 - 2x_4 \\ \text{s.a.} \quad & x_2 = 4 - (x_1 + x_4) \\ & x_3 = 2 - (x_1 - 1x_4) \\ & x_1,\ x_2,\ x_3,\ x_4 \geq 0 \end{aligned}$$

Soluções Básicas

Agora vendo as bases no gráfico:

$$\max \quad 0 + (1)x_1 + (2)x_2$$
$$\text{s.a.} \quad x_3 = 6 - (2x_1 + 1x_2)$$
$$x_4 = 4 - (1x_1 + 1x_2)$$
$$x_1,\ x_2,\ x_3,\ x_4 \geq 0$$

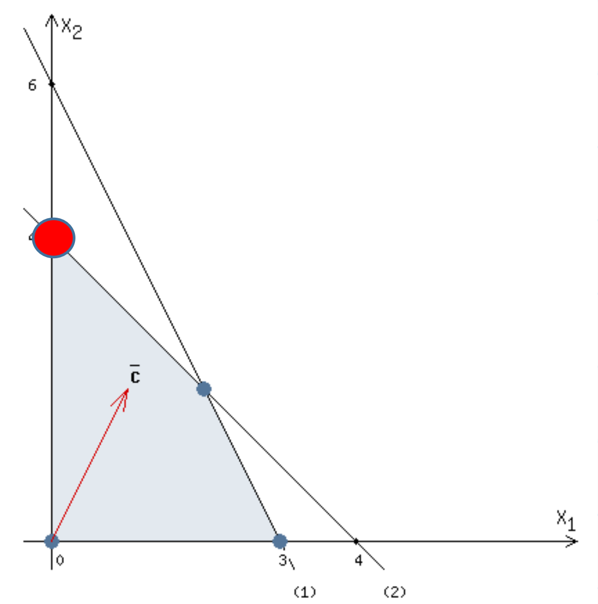




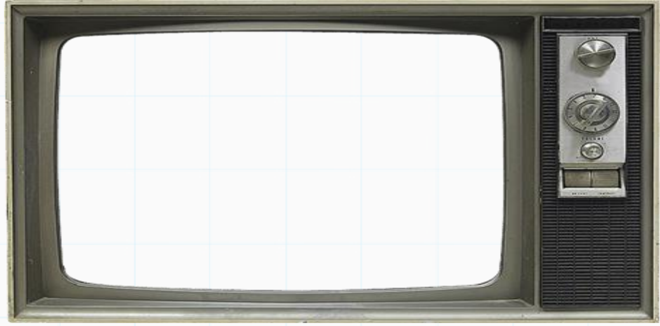
Não ótima

Mas como fazer essa passagem de bases ?

$$\max \quad 8 - 1x_1 - 2x_4$$
$$\text{s.a.} \quad x_2 = 4 - (x_1 + x_4)$$
$$x_3 = 2 - (x_1 - 1x_4)$$
$$x_1,\ x_2,\ x_3,\ x_4 \geq 0$$

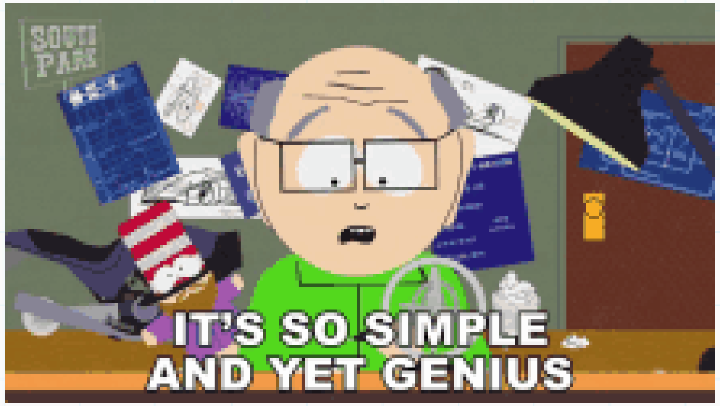


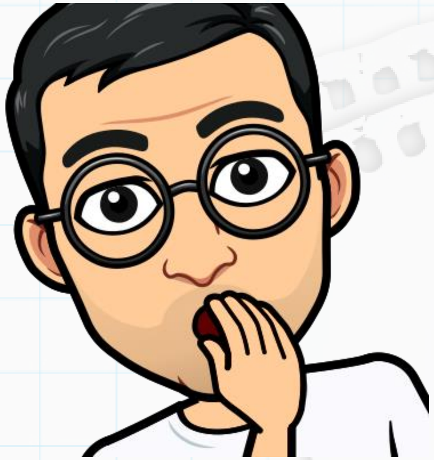
Ótima

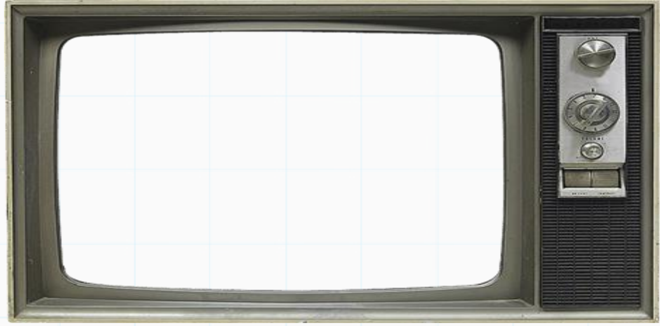


Simplex

Para responder essa pergunta, vamos apresentar o famoso método SIMPLEX.

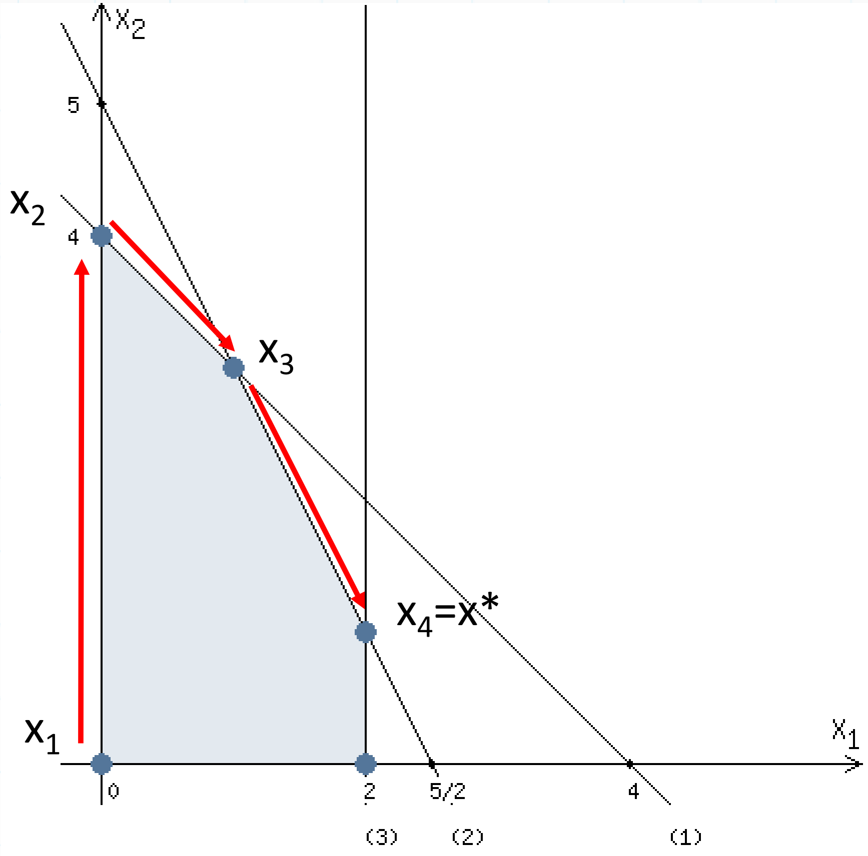






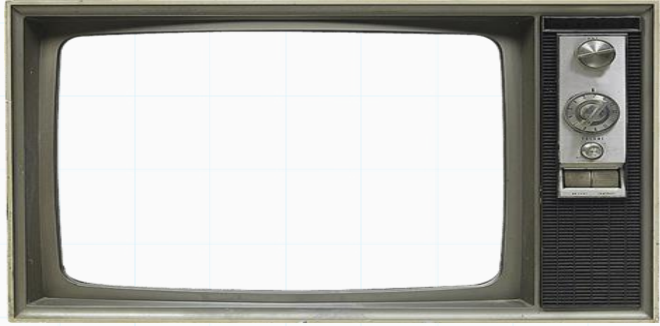
Simplex

Dado um PPL $Ax = b$ e $x \geq 0$, a idéia é partir de uma S.B.V., passar para outra S.B.V. adjacente com f.o. maior ou igual, até atingir o ótimo.



Lembrando que $|S.B.V.| \leq C_n^m$ o método irá convergir (sob certas condições)

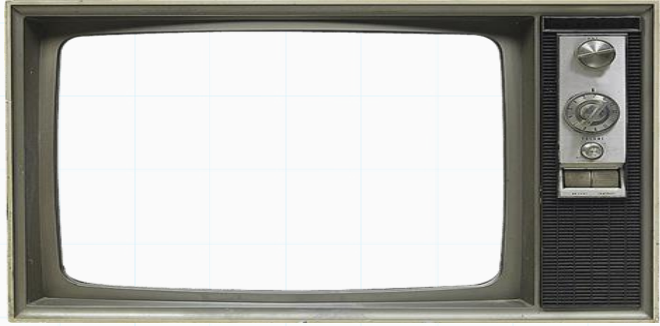
Simplex



Seja:

$$\max\ c^T x$$
$$Ax = b$$
$$x \geq 0$$

Simplex



Seja: $\max\ c^T x$

$$Ax = b$$

$$x \geq 0$$

- <u>Passo 1</u>

  Escolha uma partição $A = [BN]$ onde B_{mxm} é inversível tal que $B^{-1}b \geq 0$, para o PPL:

$$\max\ z = \bar{z} + \sum_{j \in I_N} (c_j - z_j) x_j$$

$$x_{B_i} = \bar{x}_{B_i} - \sum_{j \in I_N} y_{ij} x_j \qquad \forall i = 1 \ldots m$$

$$x_j \geq 0 \qquad \forall j \in (I_B \cup I_N)$$

Onde:

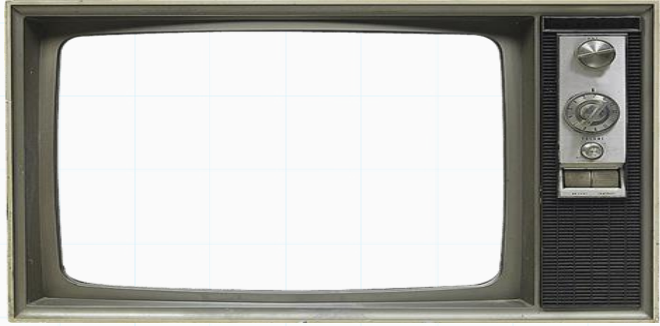
$$\bar{z} = c_B^T B^{-1} b$$

$$(c_j - z_j) = (c_N^T - c_B^T B^{-1} N)_j$$

$$\bar{x}_{B_i} = (B^{-1} b)_i$$

$$y_{ij} = (B^{-1} N)_{ij}$$

Simplex



<u>Passo 2</u>

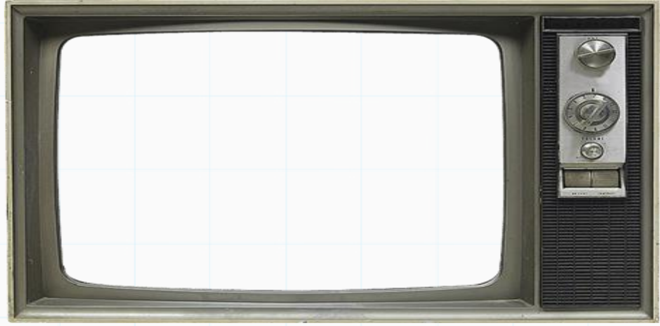
A partir da S.B.V. $x_b = B^{-1}b$ e $x_N = 0$ checar se a solução é ótima

$$\max\ z = \overline{z} + \sum_{j \in I_N} (c_j - z_j)x_j$$

$$x_{B_i} = \overline{x}_{B_i} - \sum_{j \in I_N} y_{ij}x_j$$

$$x_j \geq 0$$

Simplex



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A partir da S.B.V. $x_b = B^{-1}b$ e $x_N = 0$ checar se a solução é ótima

Se $(c_j - z_j) \leq 0, \forall j \in I_N$, então PARE, a solução $\overline{z} = c_B^T B^{-1} b$ é ótima.

Senão, escolher x_k tal que $k \in I_N$ e $(c_j - z_j) > 0$

$$\max\ z = \overline{z} + \sum_{j \in I_N} (c_j - z_j) x_j$$

$$x_{B_i} = \overline{x}_{B_i} - \sum_{j \in I_N} y_{ij} x_j$$

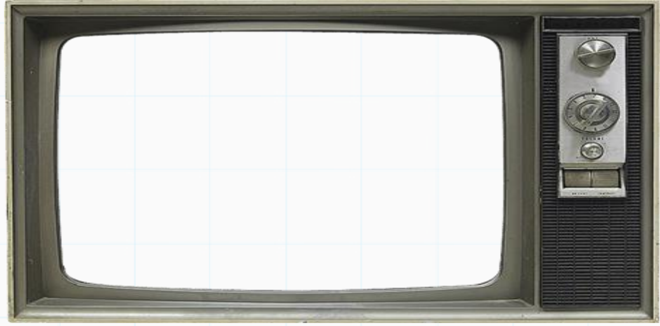
$$x_j \geq 0$$

TROCA DE BASE

- Vai de uma base B1 (vértice 1) para uma base B2 (vértice 2 adjacente a 1)
- Vértices adjacentes no conjunto de soluções, tem bases com diferença de uma coluna

x_k para entrar na base

Simplex



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objetivo Aumentar o valor de x_k, mantendo $x_j = 0, \forall j \in I_N - \{k\}$

Como

$$z = \overline{z} + \overbrace{\boxed{(c_k - z_k)x_k}}^{> 0} + \overbrace{\boxed{\sum_{j \in I_N - \{k\}} (c_j - z_j)x_j}}^{= 0}$$

$$x_k \to z$$

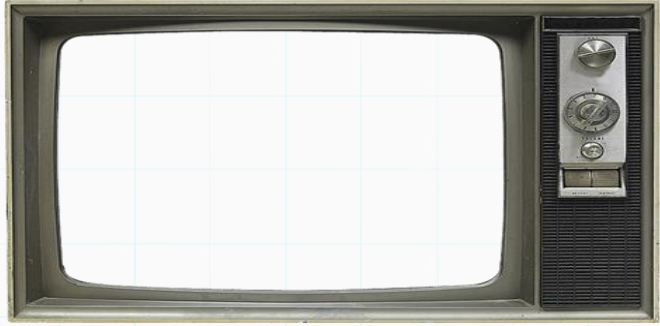
$$\max\ z = \overline{z} + \sum_{j \in I_N} (c_j - z_j)x_j$$

$$x_{B_i} = \overline{x}_{B_i} - \sum_{j \in I_N} y_{ij}x_j$$

$$x_j \geq 0$$

x_k para entrar na base

Simplex



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Como

$$z = \overline{z} + \overset{> 0}{\boxed{(c_k - z_k) x_k}} + \overset{= 0}{\boxed{\sum_{j \in I_N - \{k\}} (c_j - z_j) x_j}}$$

x_k para entrar na base

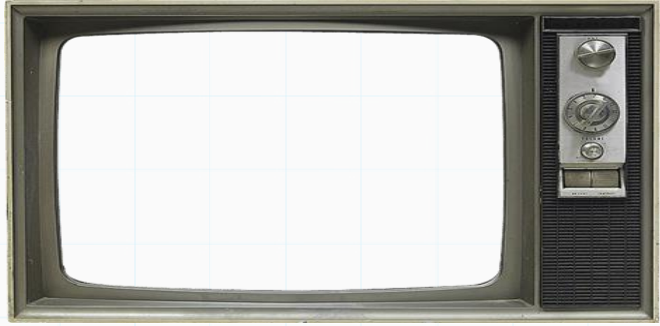
$$x_k \rightarrow z$$

$\nearrow \quad \nearrow$

Mas quanto aumentar x_k ?

Boa pergunta

Simplex



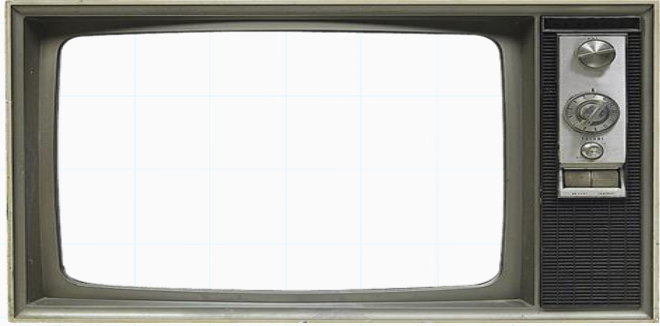
<u>Teste da Razão</u> Determinar o maior aumento em x_k sem ir para a inviabilidade do PPL.

$$\max\ z = \overline{z} + \sum_{j \in I_N} (c_j - z_j) x_j$$

$$x_{B_i} = \overline{x}_{B_i} - \sum_{j \in I_N} y_{ij} x_j$$

$$x_j \geq 0$$

Simplex



Teste da Razão Determinar o maior aumento em x_k sem ir para a inviabilidade do PPL.

$$x_{B_i} = \overline{x}_{B_i} - y_{ik}x_k - \overset{=\,0}{\boxed{\sum_{j \in I_N - \{k\}} y_{ij}x_j}} \qquad \forall i = 1 \ldots m$$

$$\max\ z = \overline{z} + \sum_{j \in I_N} (c_j - z_j)x_j$$

$$x_{B_i} = \overline{x}_{B_i} - \sum_{j \in I_N} y_{ij}x_j$$

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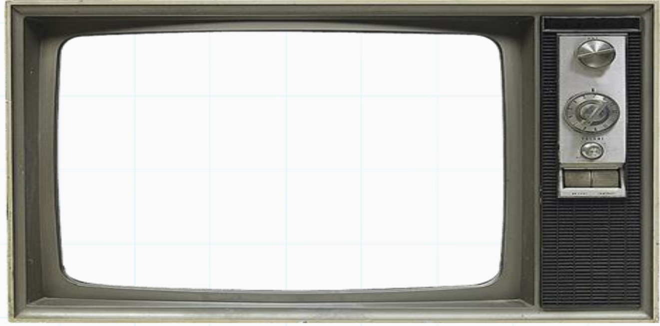

Simplex

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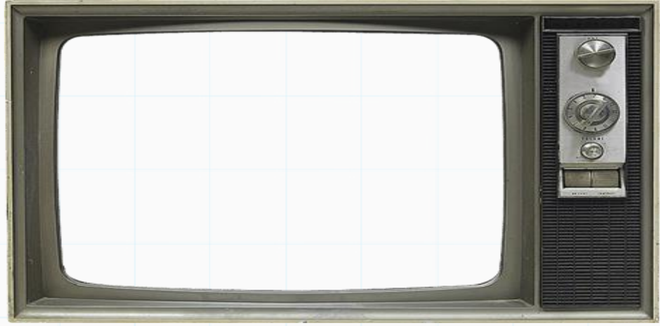

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$$x_{B_i} = \overline{x}_{B_i} - y_{ik}x_k \qquad \forall i = 1...m$$

Seja $L_1 = \{i | y_{ik} > 0\}$

$$\max\ z = \overline{z} + \sum_{j \in I_N} (c_j - z_j)x_j$$

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Simplex

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$$x_j \geq 0$$

Seja $L_1 = \{i | y_{ik} > 0\}$

como $x_{B_i} \geq 0$, temos que

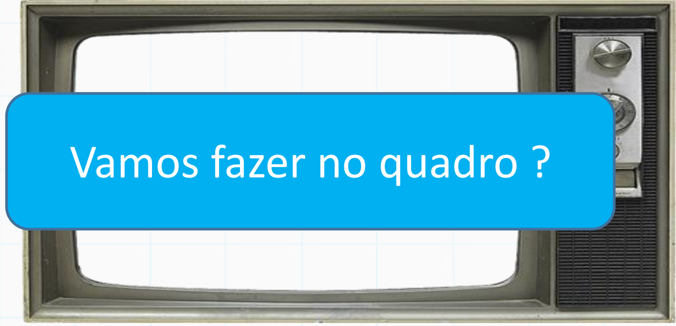
$$\overline{x}_{B_i} - y_{ik}x_k \geq 0$$

$$y_{ik}x_k \leq \overline{x}_{B_i}$$

$$\boxed{x_k \leq \overline{x}_{B_i} / y_{ik}} \qquad \forall i = 1 \ldots m$$

limite superior para x_k

Simplex



Vamos fazer no quadro ?

Teste da Razão Determinar o maior aumento em x_k sem ir para a inviabilidade do PPL.

$$\max\ z = \bar{z} + \sum_{j \in I_N} (c_j - z_j)x_j$$

$$x_{B_i} = \bar{x}_{B_i} - \sum_{j \in I_N} y_{ij}x_j$$

$$x_j \geq 0$$

$$x_{B_i} = \bar{x}_{B_i} - y_{ik}x_k - \overset{= 0}{\boxed{\sum_{j \in I_N - \{k\}} y_{ij}x_j}} \qquad \forall i = 1 \ldots m$$

$$x_{B_i} = \bar{x}_{B_i} - y_{ik}x_k \qquad \forall i = 1 \ldots m$$

Seja $L_1 = \{i | y_{ik} > 0\}$

como $x_{B_i} \geq 0$, temos que

e para y_{ik} <= 0, precisamos analisar ?

$$\bar{x}_{B_i} - y_{ik}x_k \geq 0$$

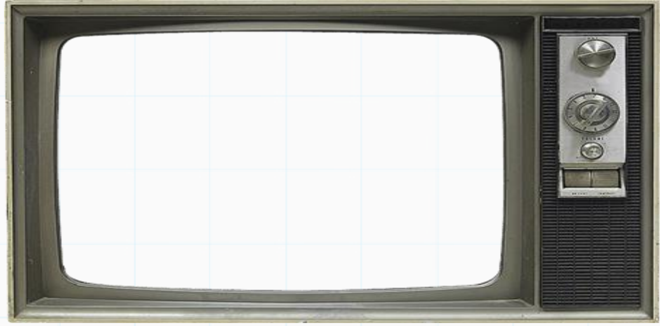
?

$$y_{ik}x_k \leq \bar{x}_{B_i}$$

$$\boxed{x_k \leq \bar{x}_{B_i} / y_{ik}} \qquad \forall i = 1 \ldots m$$

limite superior para x_k

Simplex



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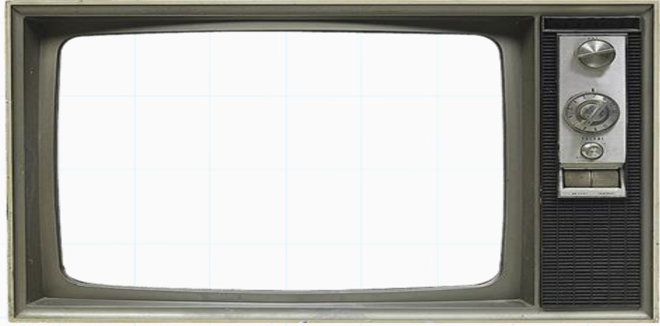
$$x_{B_i} = \overline{x}_{B_i} - \sum_{j \in I_N} y_{ij} x_j$$

$$x_j \geq 0$$

Seja

$$x_k \leq \overline{x}_{B_i} / y_{ik} \quad \Rightarrow \quad \frac{\overline{x}_{B_s}}{y_{sk}} = \min_{i \in L_1} \left\{ \frac{\overline{x}_{B_i}}{y_{ik}} \right\}$$

Simplex



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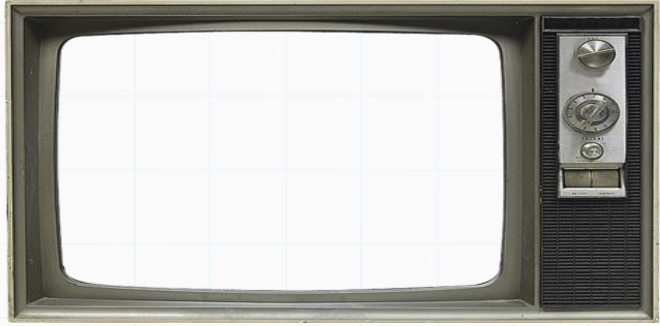
Se $L_1 = \infty$ então PARE (ILIMITADO)

Senão, muda a base:

$$I_B = (I_B \cup \{k\}) - \{B_s\}$$

$$I_N = (I_N \cup \{B_s\}) - \{k\}$$

Simplex



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Se $L_1 = \infty$ então PARE (ILIMITADO)

Senão, muda a base:

$$I_B = (I_B \cup \{k\}) - \{B_s\}$$

$$I_N = (I_N \cup \{B_s\}) - \{k\}$$

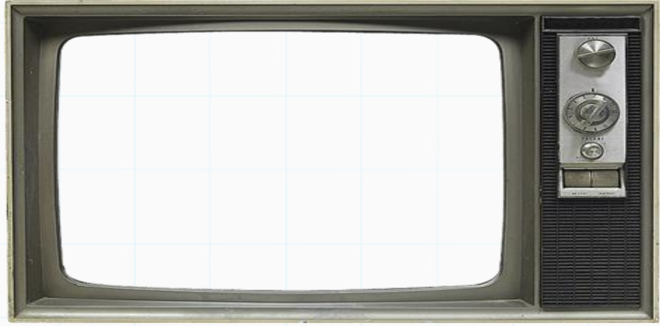
Quando x_k entra na base com valor de $\frac{\bar{x}_{B_s}}{y_{sk}}$

Então a variável x_s vai para zero pois

$$x_{B_i} = \bar{x}_{B_i} - y_{ik} x_k$$

Por isso que a nova matriz B ainda é uma base

Simplex



Teste da Razão Determinar o maior aumento em x_k sem ir para a inviabilidade do PPL.

$$\max\ z = \bar{z} + \sum_{j \in I_N} (c_j - z_j) x_j$$

$$x_{B_i} = \bar{x}_{B_i} - \sum_{j \in I_N} y_{ij} x_j$$

$$x_j \geq 0$$

Seja

$$x_k \leq \bar{x}_{B_i} / y_{ik} \quad \Rightarrow \quad \frac{\bar{x}_{B_s}}{y_{sk}} = \min_{i \in L_1} \{ \frac{\bar{x}_{B_i}}{y_{ik}} \}$$

Se $L_1 = \infty$ então PARE (ILIMITADO)

Senão, muda a base:

$$I_B = (I_B \cup \{k\}) - \{B_s\}$$

$$I_N = (I_N \cup \{B_s\}) - \{k\}$$

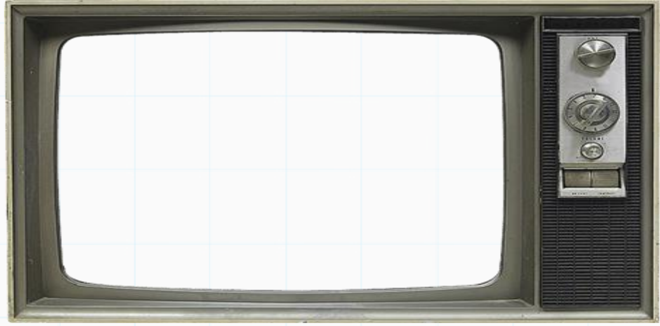
Quando x_k entra na base com valor de $\frac{\bar{x}_{B_s}}{y_{sk}}$

Então a variável x_s vai para zero pois

$$x_{B_i} = \bar{x}_{B_i} - y_{ik} x_k$$

Passo 3 Volte ao Passo 2

Por isso que a nova matriz B ainda é uma base

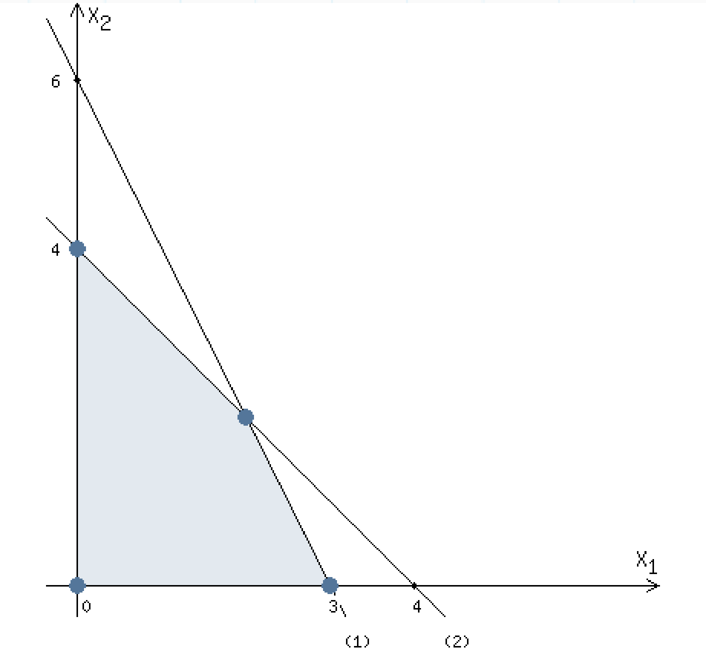


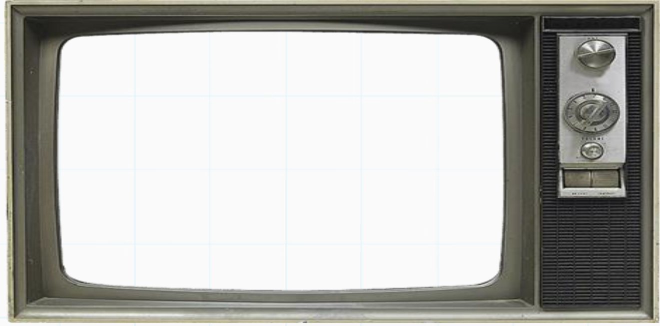
Simplex

Exemplo:

$$\begin{aligned} \max \quad & x_1 + 2x_2 \\ \text{s.a.} \quad & 2x_1 + x_2 \leq 6 \\ & x_1 + x_2 \leq 4 \\ & x_1,\ x_2 \geq 0 \end{aligned}$$

<u>Forma padrão</u>





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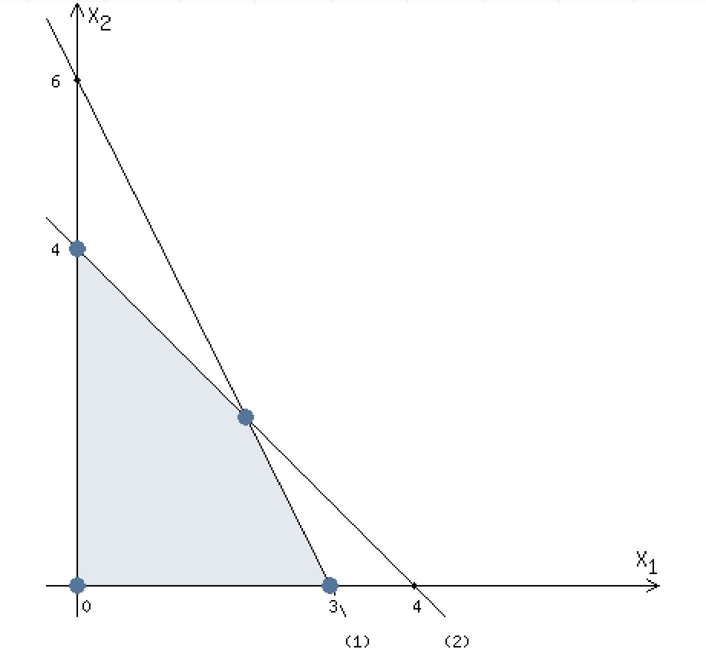

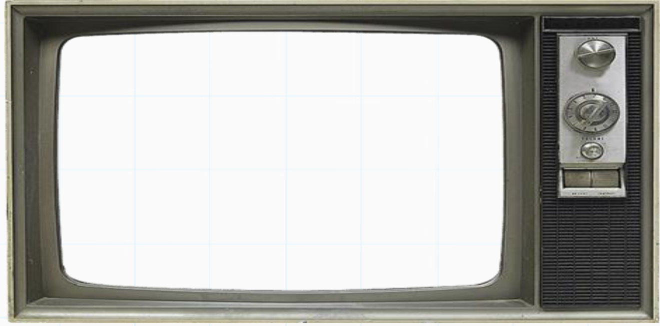

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Vamos partir da base formada pelas variáveis de folga I_B={3,4}

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = B^{-1} \quad N = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \quad b^T = [6\ 4]$$

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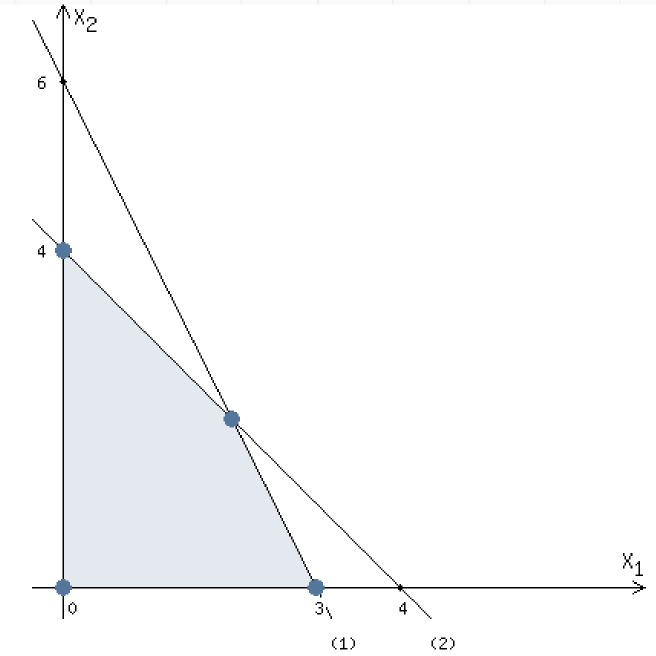

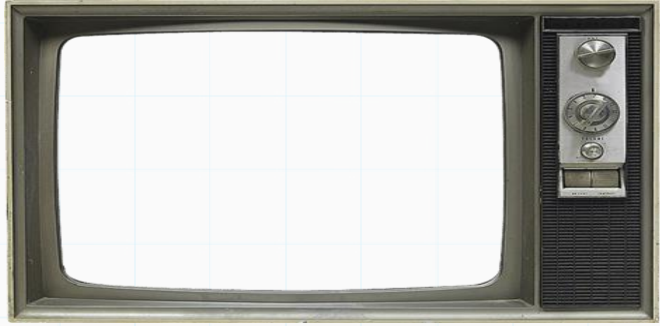

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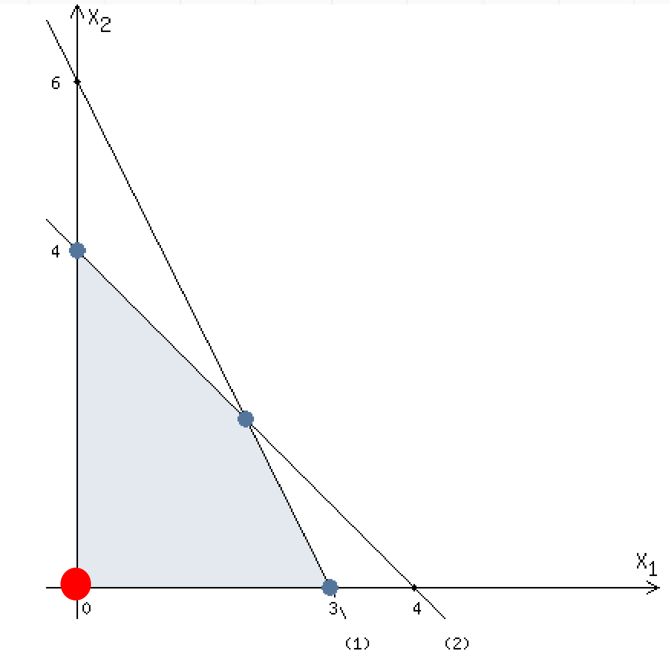

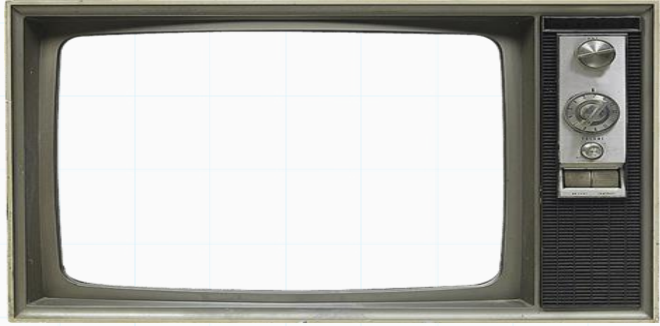

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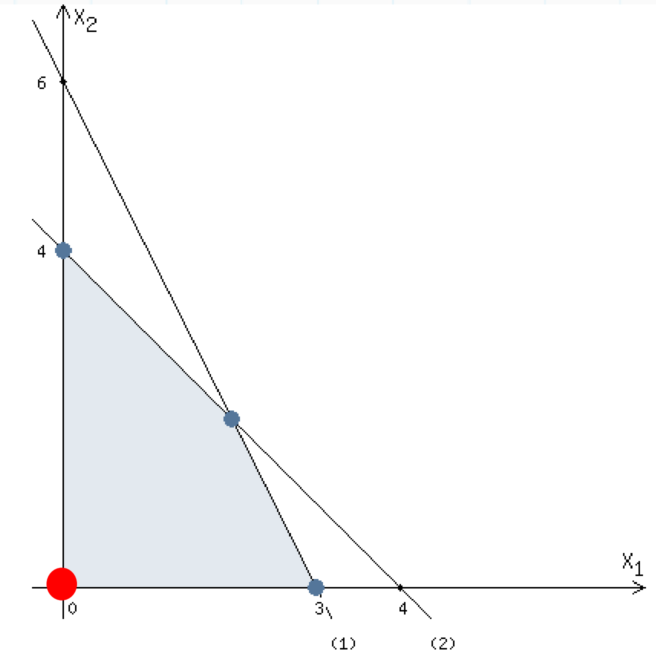
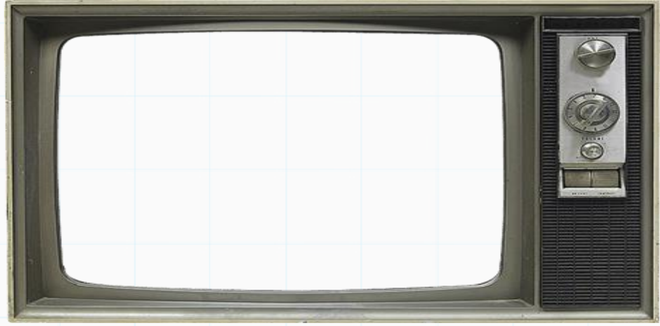


Simplex



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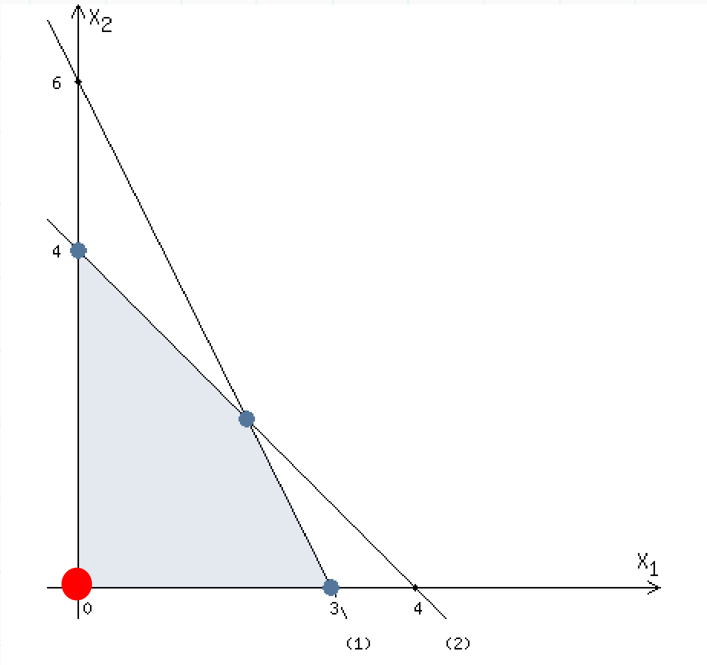

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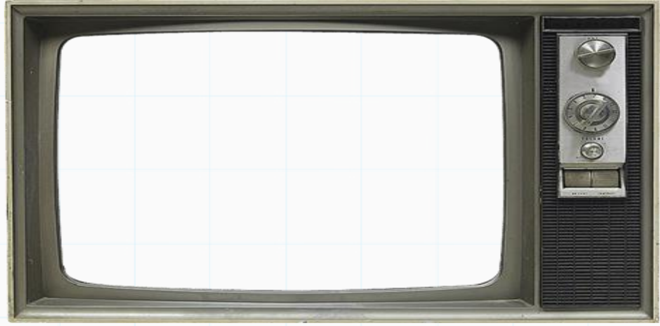

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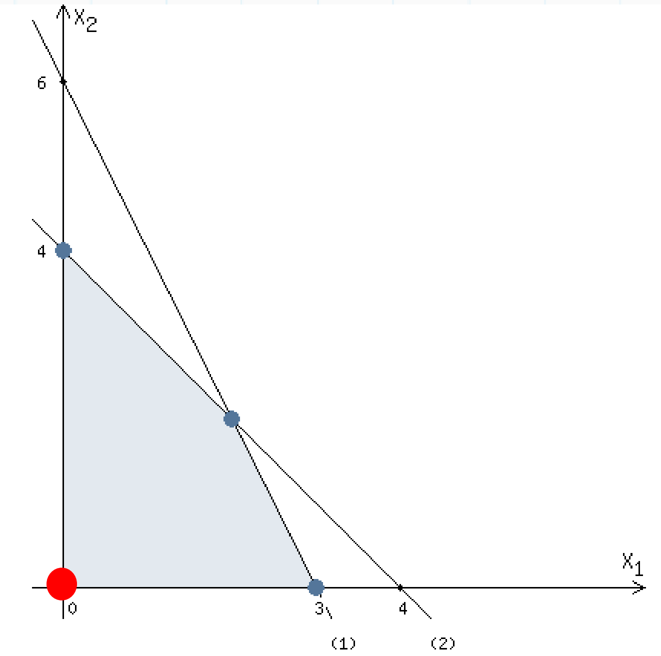
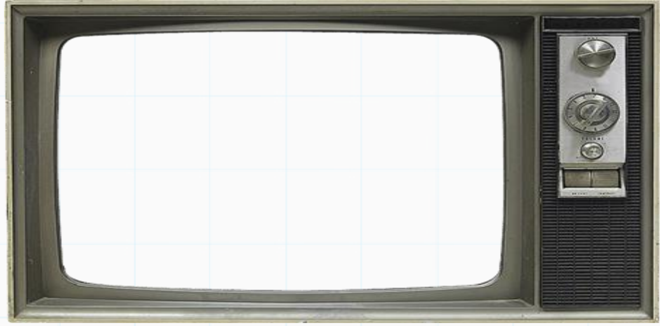
não ótimo

$$y = B^{-1} N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$



Simplex



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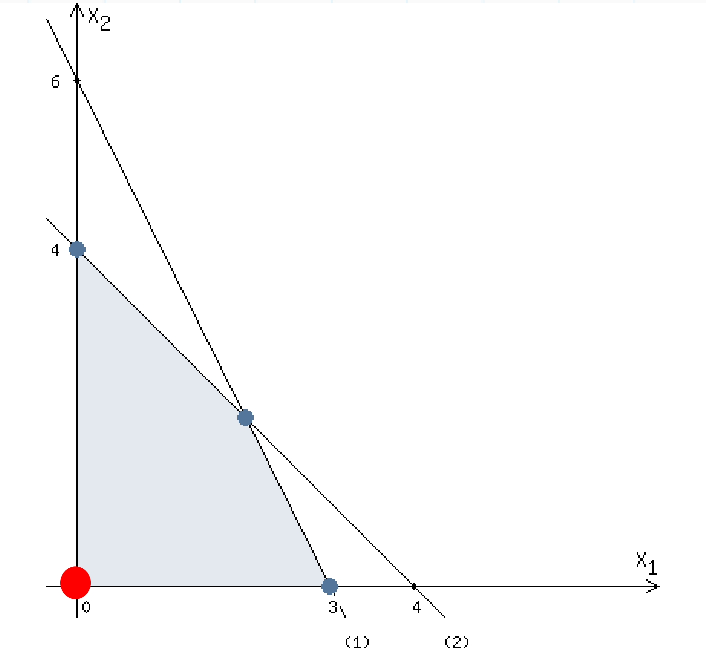
Forma Básica de $I_B=\{3,4\}$:



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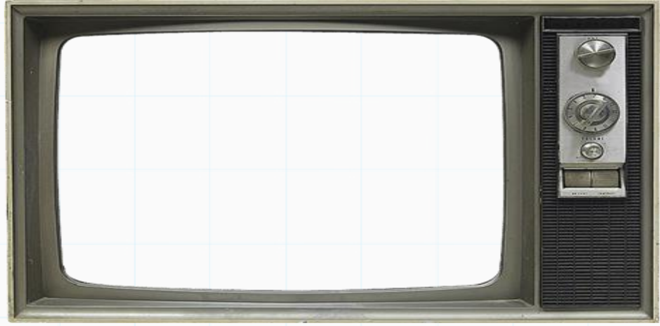

Exemplo: Forma básica

$$\begin{aligned} \max \quad & 0 + x_1 + 2x_2 \\ \text{s.a.} \quad & x_3 = 6 - (2x_1 + x_2) \\ & x_4 = 4 - (x_1 + x_2) \\ & x_1,\ x_2,\ x_3,\ x_4 \geq 0 \end{aligned}$$

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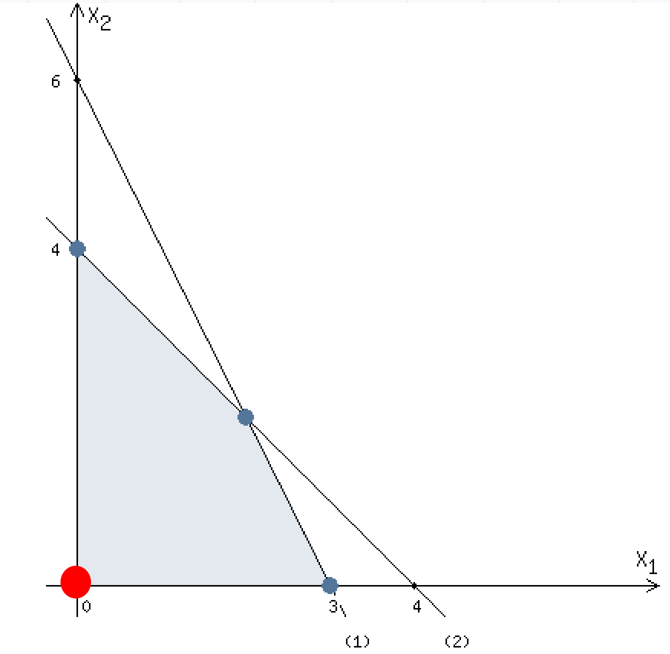

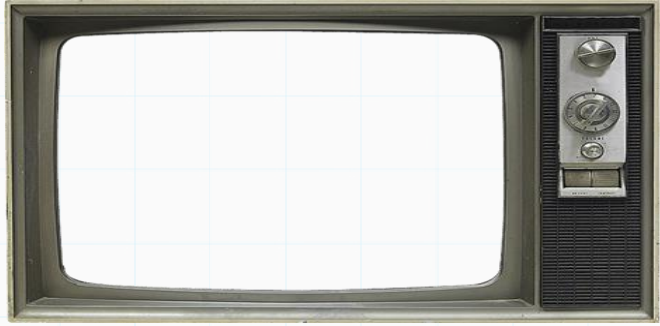

Simplex

Exemplo: Forma básica

aumentar x_1 ou x_2, qual é o melhor ?

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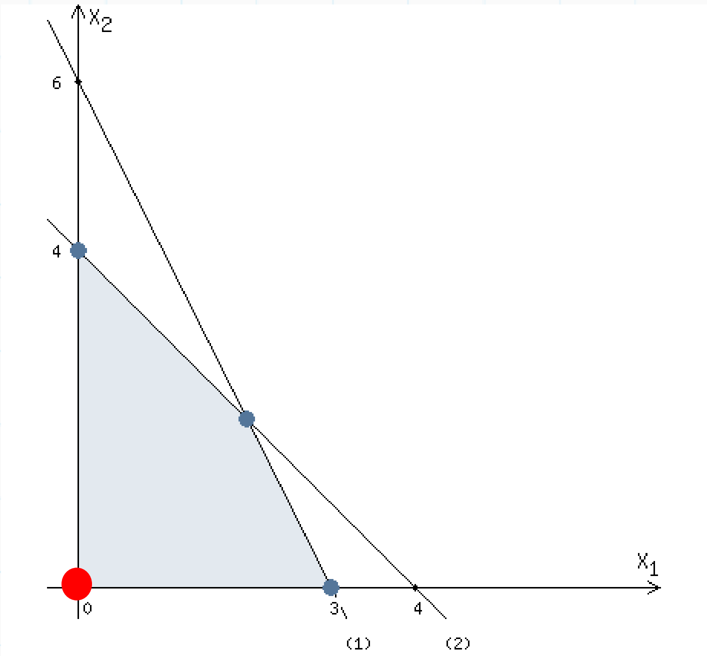
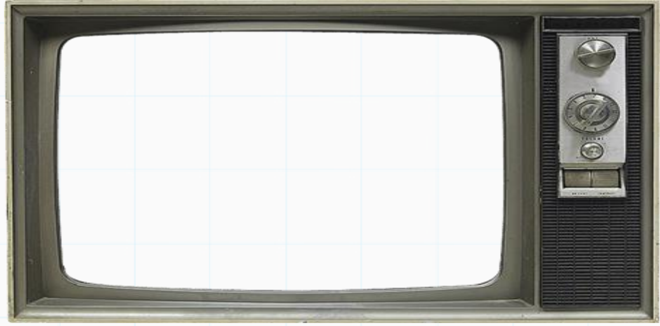
É ótima ? Não pois $c_1 - z_1 = 1 > 0$, vamos aumentar x_1





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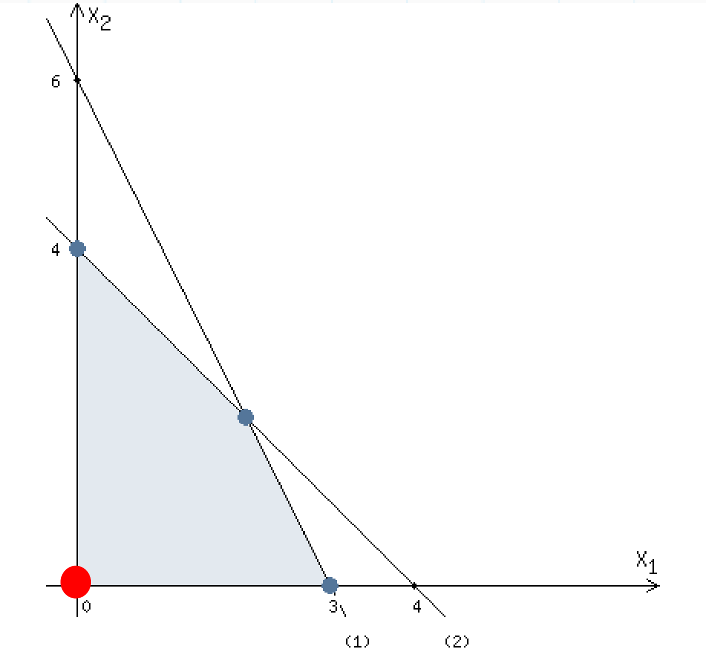
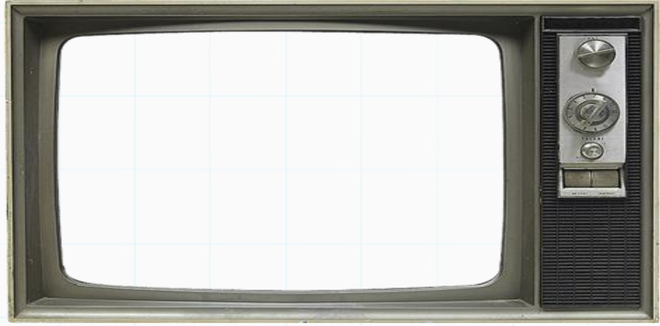
Teste da Razão:





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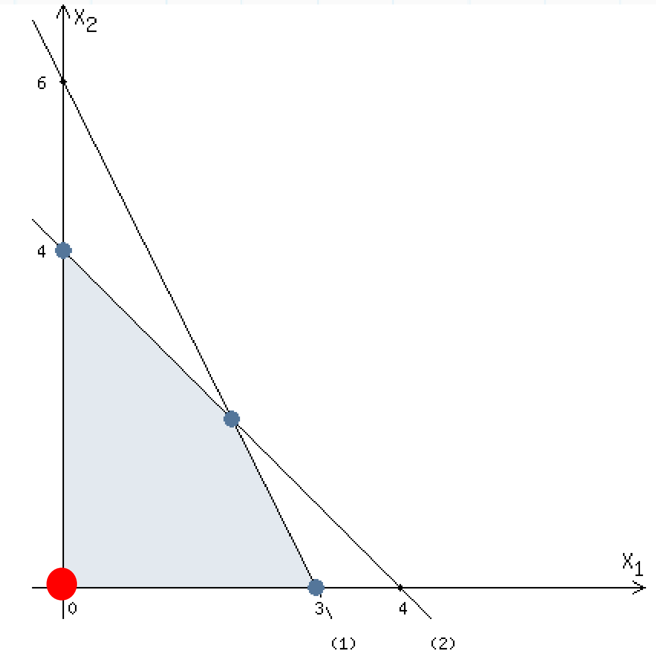
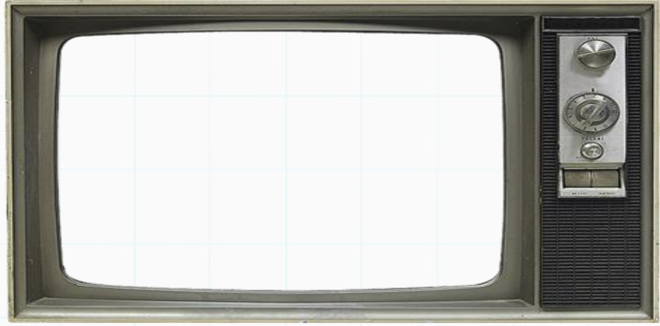
Teste da Razão:

$$x_3:\; x_1 \leq \frac{6}{2} = 3$$
$$x_4:\; x_1 \leq \frac{4}{1} = 4$$

Vemos então que $x_1 \leq 3$ é o menor L.S., logo: $\boxed{x_1 = 3 \text{ e } x_3 = 0}$

$\boxed{x_1 \text{ entra na base e } x_3 \text{ sai,}}$ $I_B=\{1,4\}$





Simplex

Exemplo: Nova base $I_B=\{1,4\}$

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} \quad B^{-1} = \begin{bmatrix} 1/2 & 0 \\ -1/2 & 1 \end{bmatrix} \quad N = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

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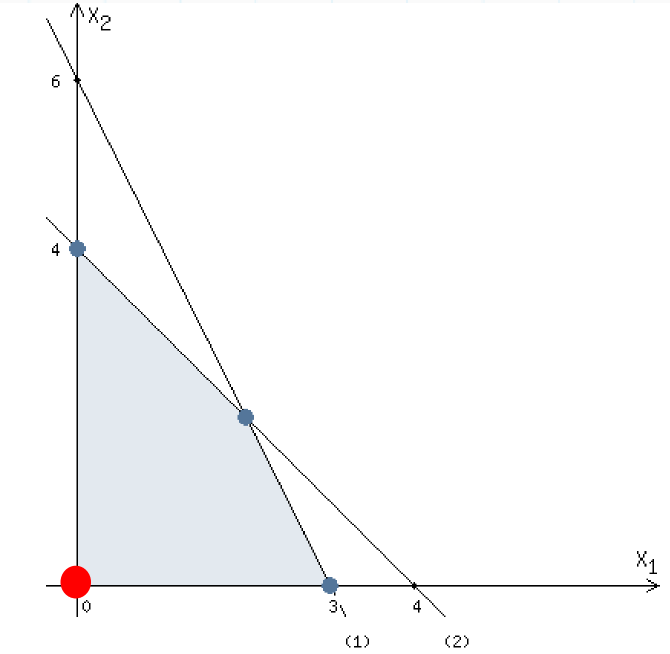

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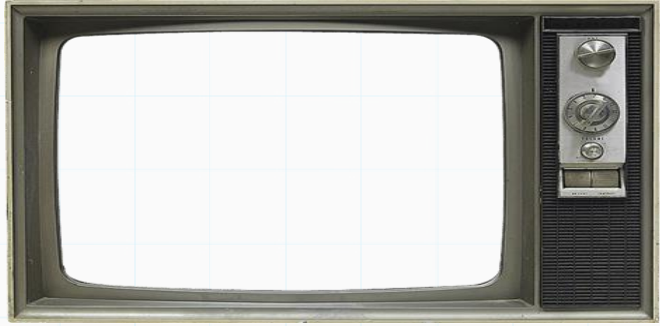

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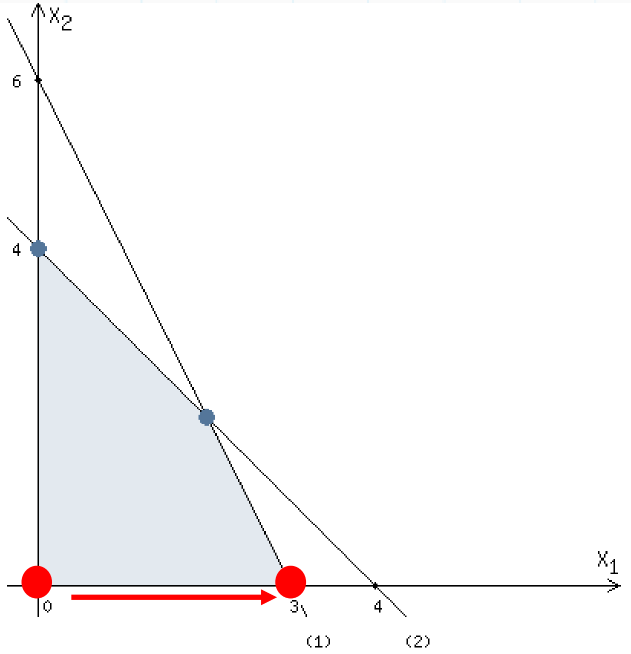
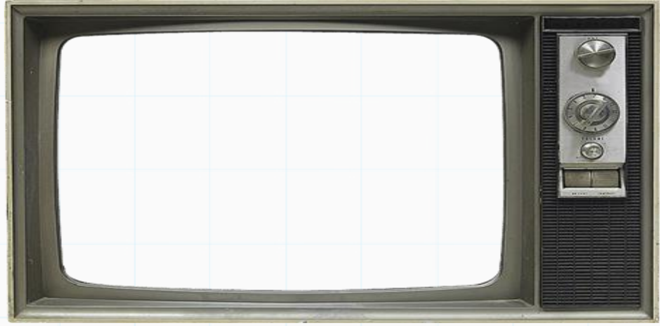
$$x_B, x_N \geq 0$$

$$c_B^T = [1\ 0] \ c_N^T = [2\ 0] \qquad \bar{x}_B = B^{-1} b = \begin{bmatrix} 1/2 & 0 \\ -1/2 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$





Simplex

Exemplo: Nova base $I_B=\{1,4\}$

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} \quad B^{-1} = \begin{bmatrix} 1/2 & 0 \\ -1/2 & 1 \end{bmatrix} \quad N = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

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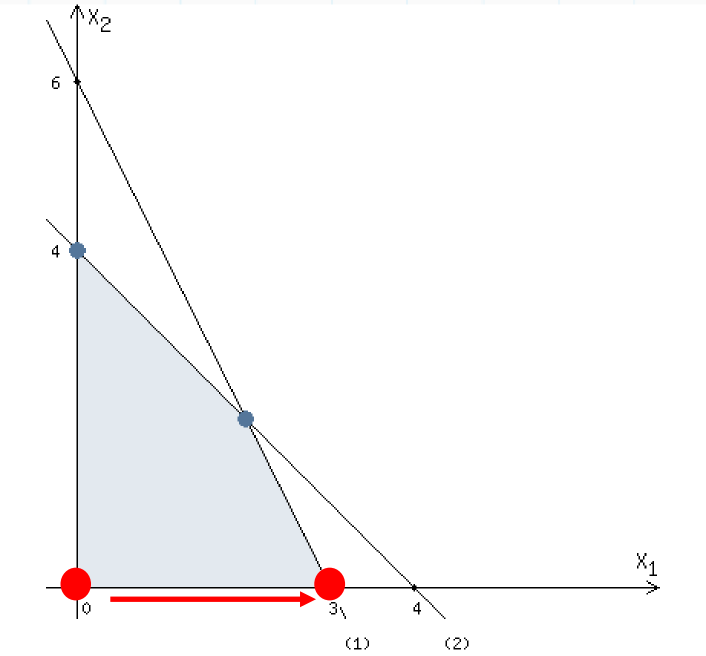

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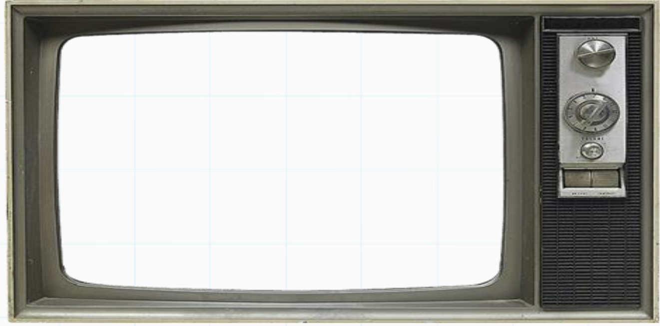

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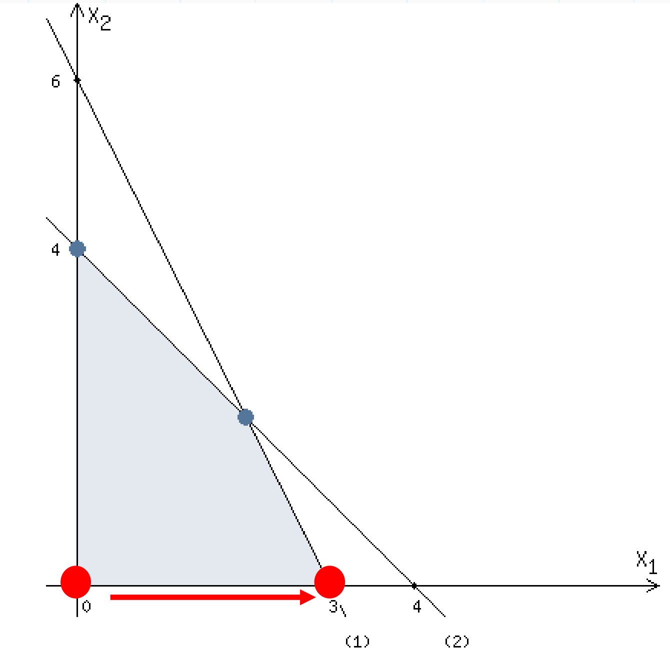
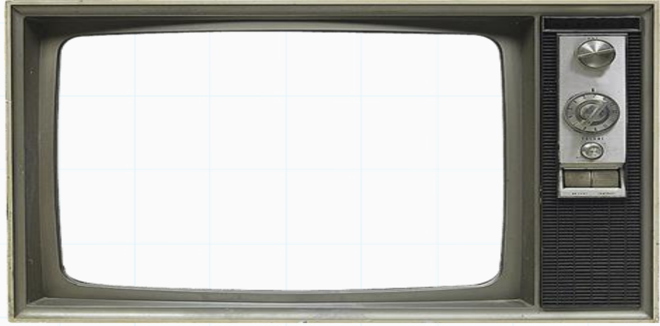


Simplex



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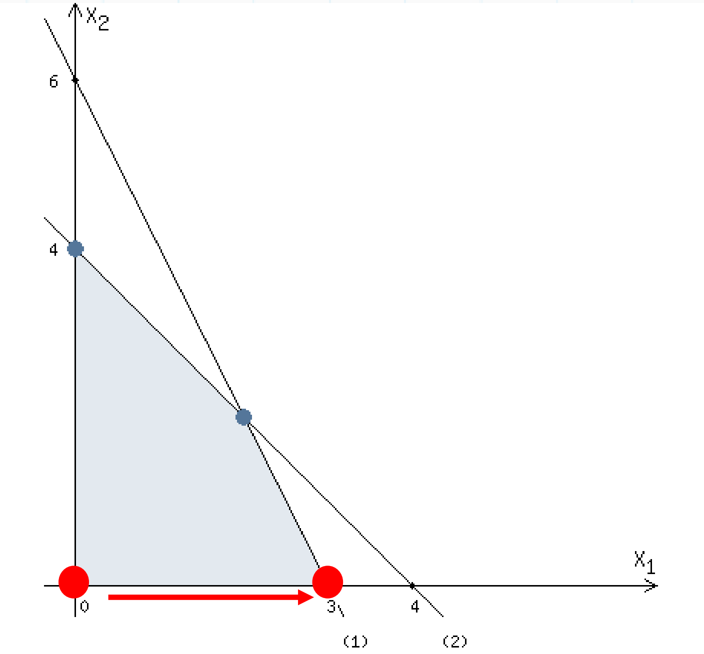

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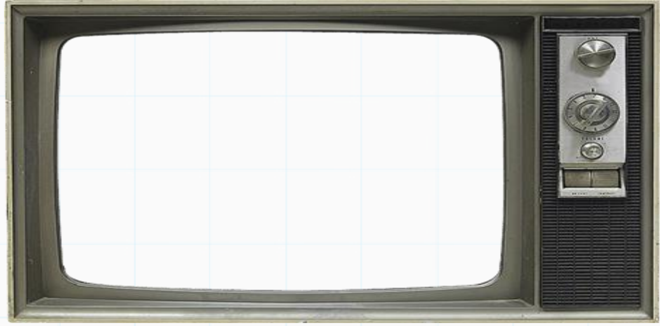

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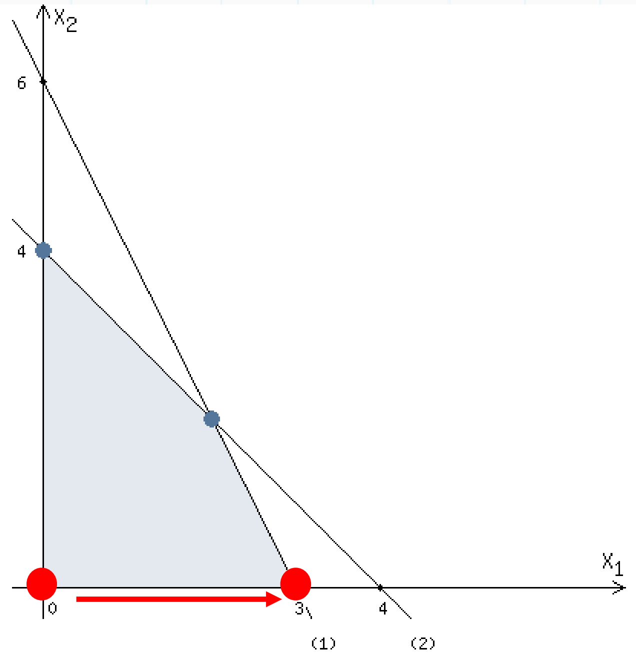
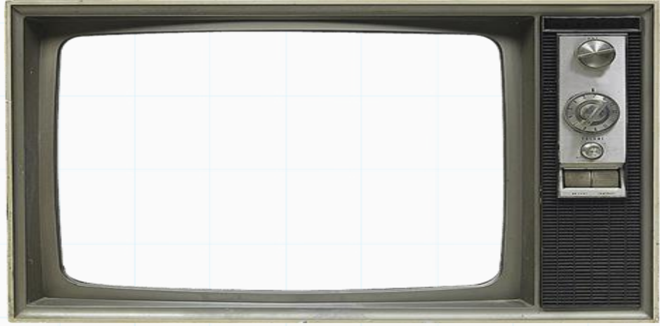


Formato Base para $I_B = \{1,4\}$:



Simplex

Exemplo: Forma básica

$$\max \quad 3 + 3/2x_2 - 1/2x_3$$
$$\text{s.a.} \quad x_1 = 3 - (1/2x_2 + 1/2x_3)$$
$$x_4 = 1 - (1/2x_2 - 1/2x_3)$$
$$x_1,\ x_2,\ x_3,\ x_4 \geq 0$$

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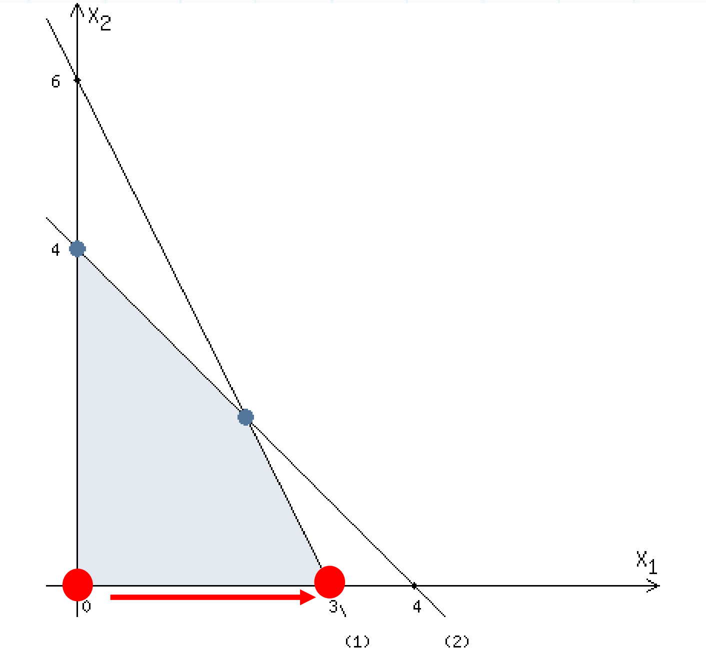
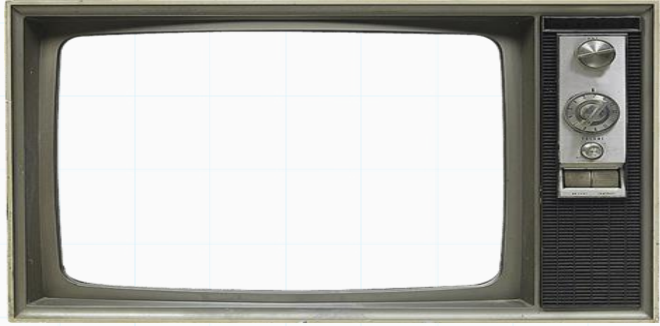


Simplex



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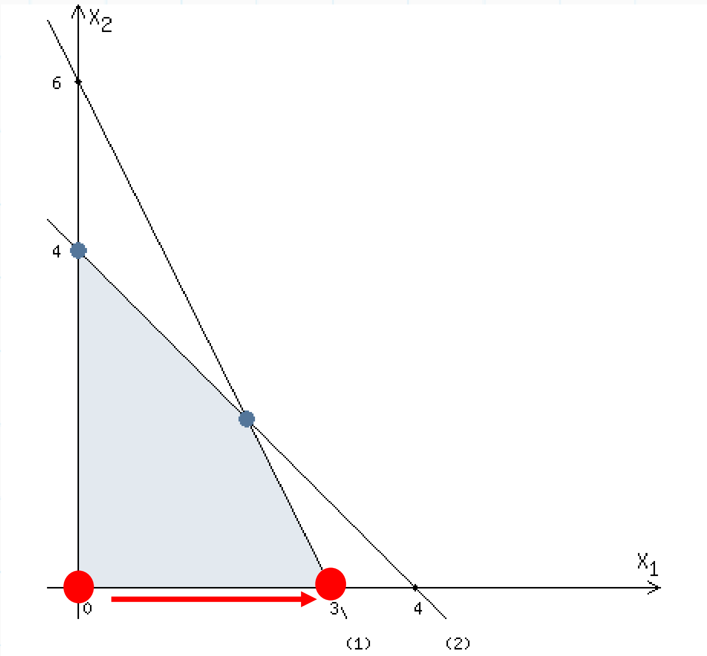
É ótima ? Não pois $c_2 - z_2 = 3/2 > 0$, vamos aumentar x_2



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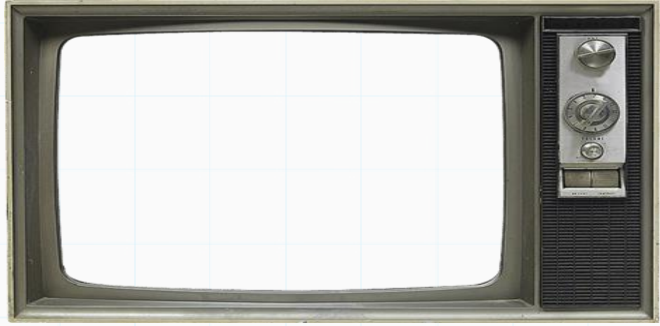

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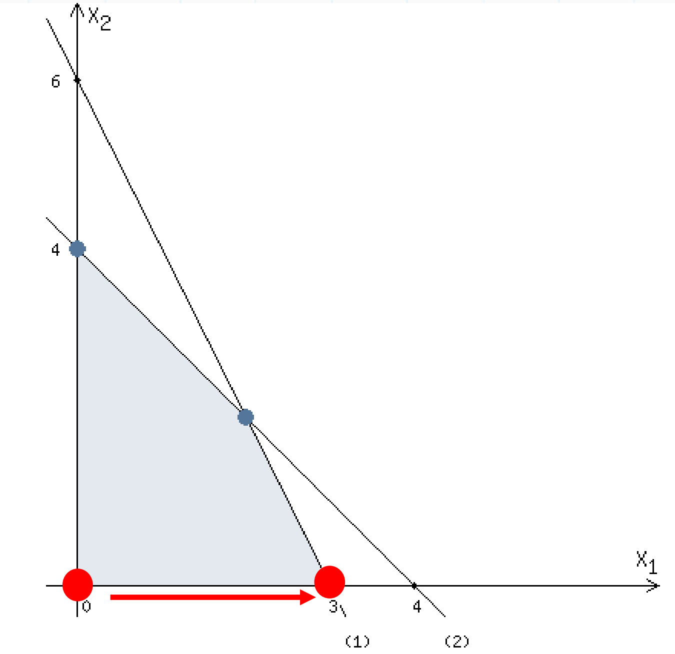
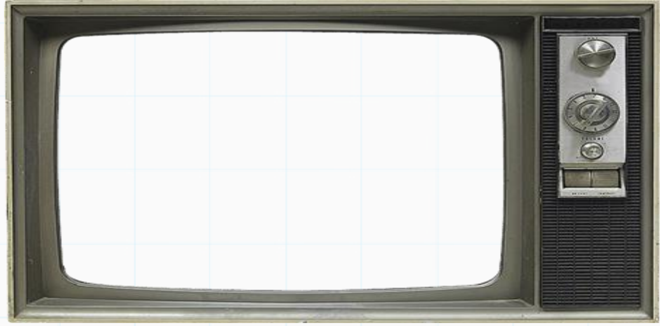
Teste da Razão:



Simplex



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É ótima ? Não pois $c_2 - z_2 = 3/2 > 0$, vamos aumentar x_2

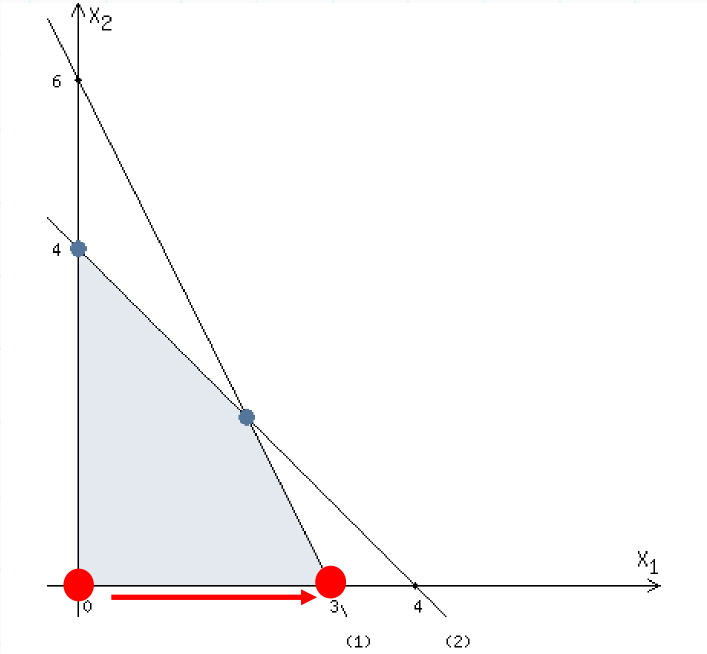
Teste da Razão:

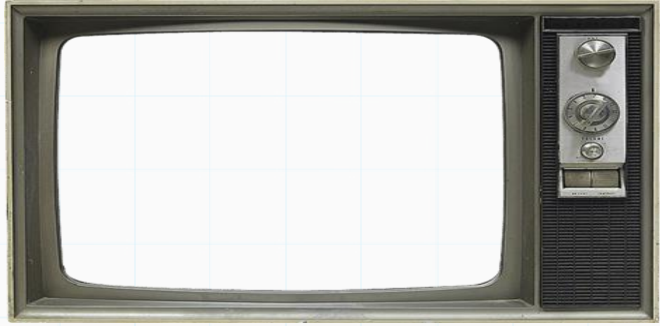
$$x_1:\ x_2 \leq \frac{3}{1/2} = 6$$

$$x_4:\ x_2 \leq \frac{1}{1/2} = 2$$

Vemos então que $x_1 \leq 2$ é o menor L.S., logo: $\boxed{x_2 = 2 \text{ e } x_4 = 0}$

$\boxed{x_2 \text{ entra na base e } x_4 \text{ sai,}}$ $I_B=\{1,2\}$





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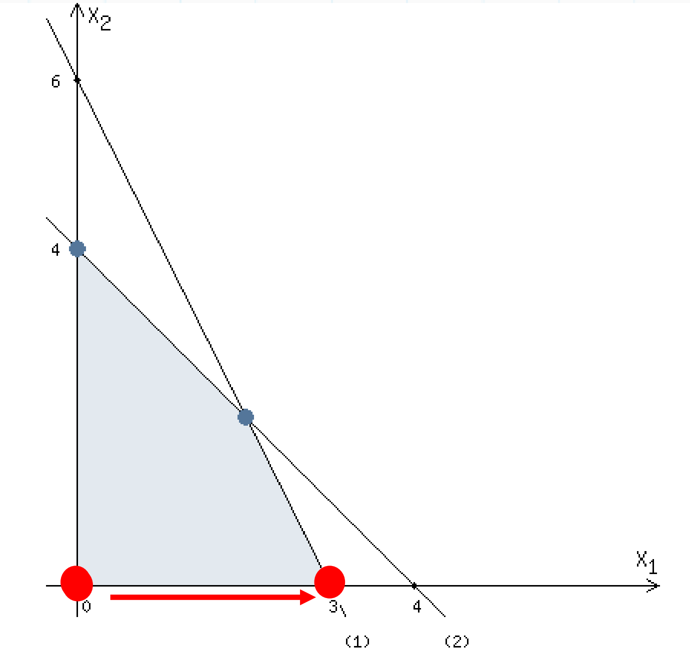

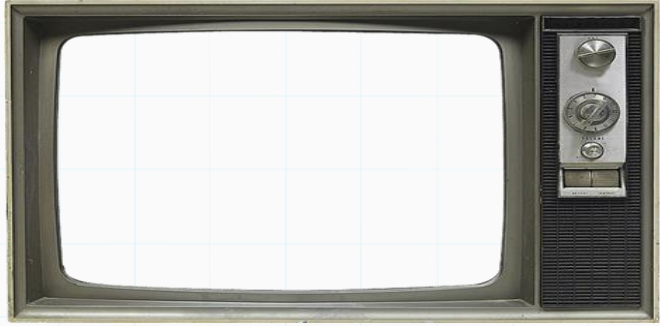

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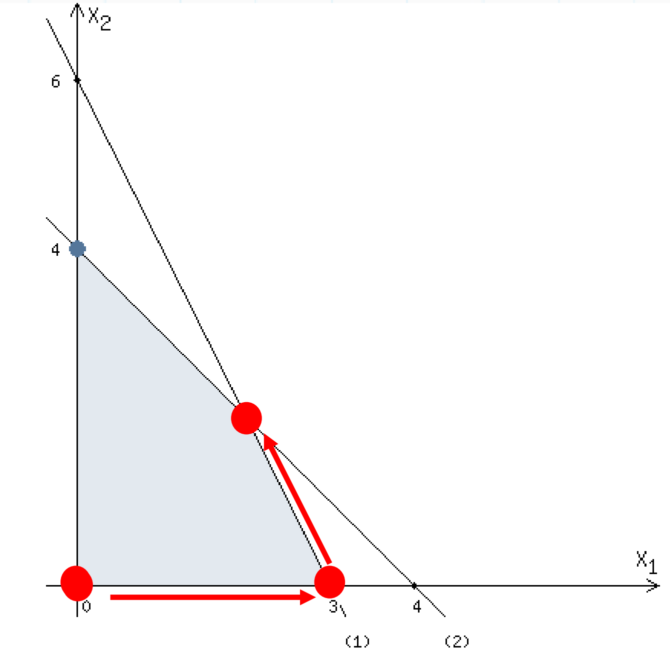

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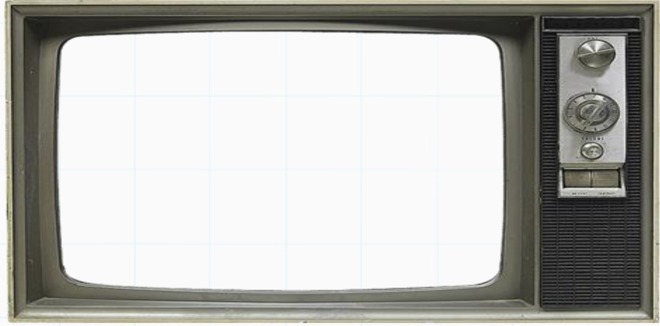

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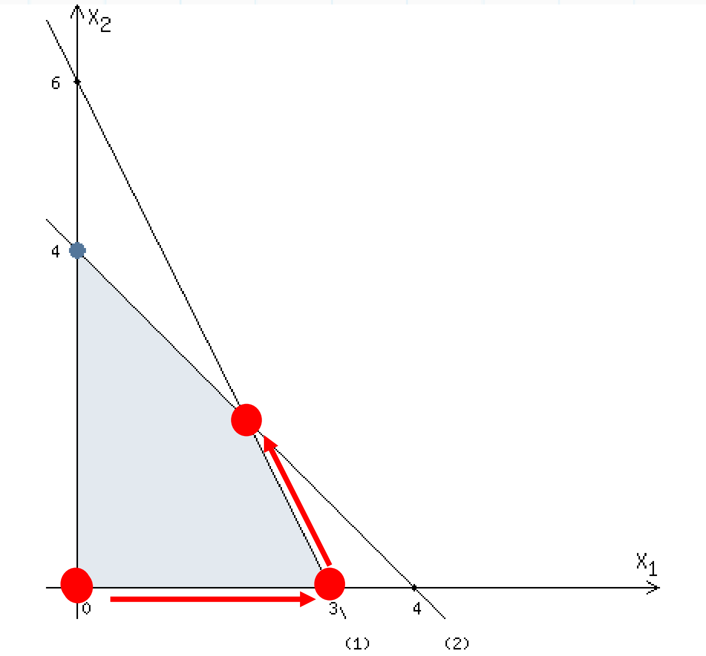

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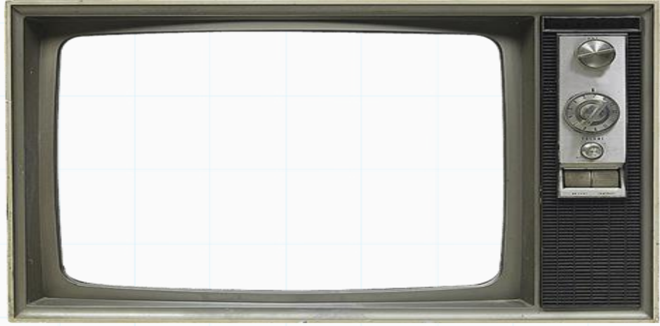

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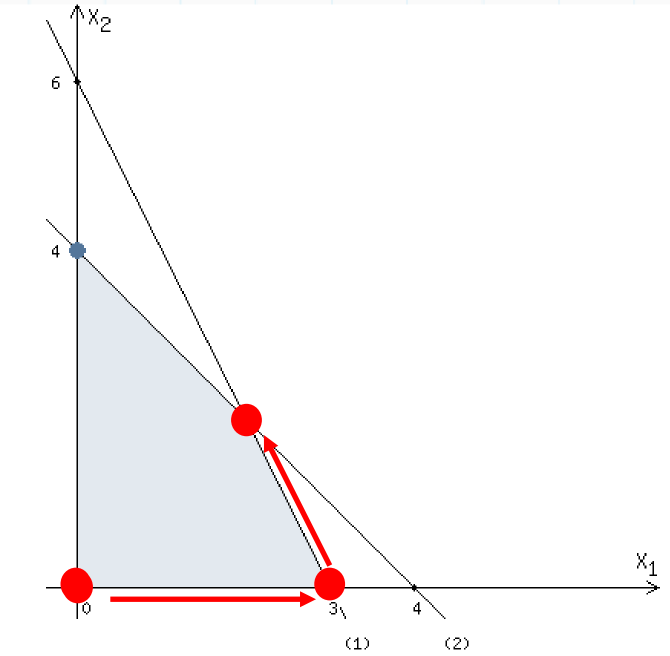

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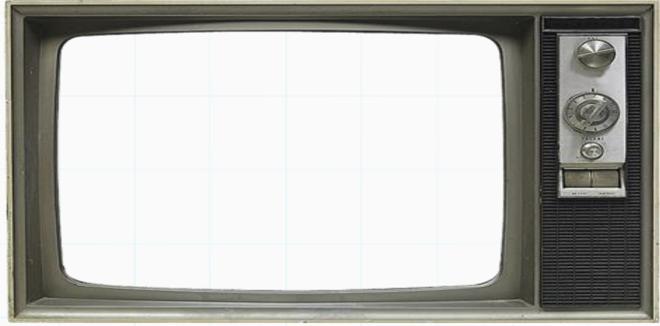

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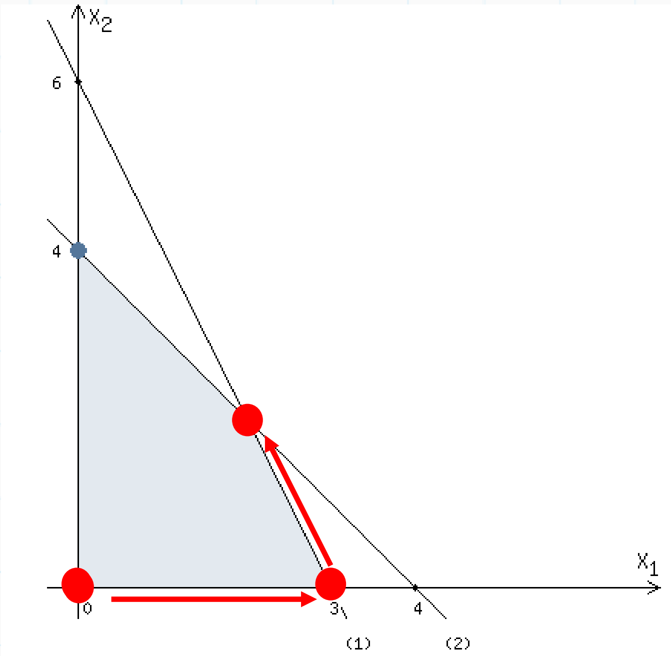
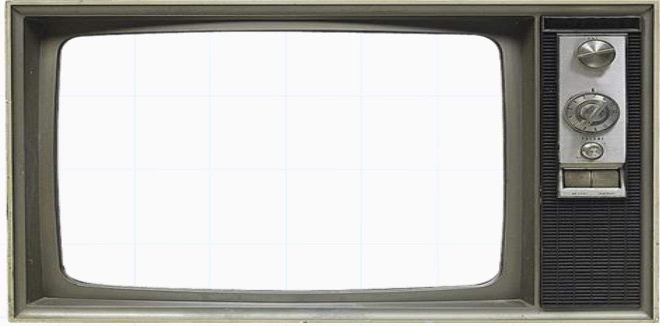


Simplex



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Formato base $I_B=\{1,2\}$:

$$\max c_B^T B^{-1} b + (c_N^T - c_B^T B^{-1} N) x_N$$
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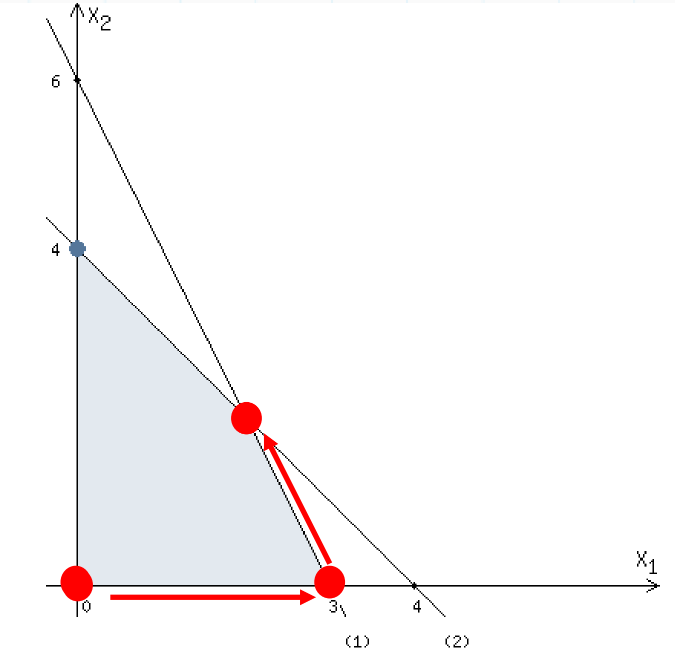

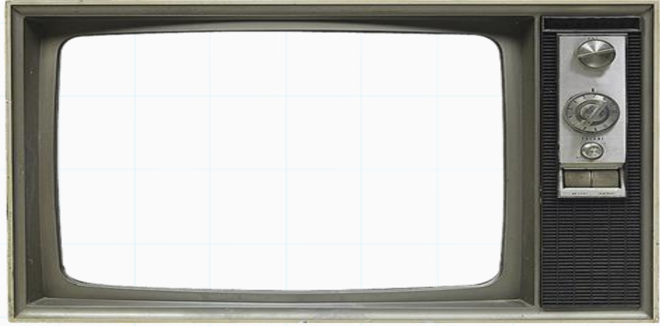

Simplex

Exemplo: <u>Forma básica</u>

$$\begin{aligned} \max\ & 6 + x_3 - 3x_4 \\ \text{s.a.}\ & x_1 = 2 - (1x_3 - 1x_4) \\ & x_2 = 2 - (-1x_3 + 2x_4) \\ & x_1,\ x_2,\ x_3,\ x_4 \geq 0 \end{aligned}$$

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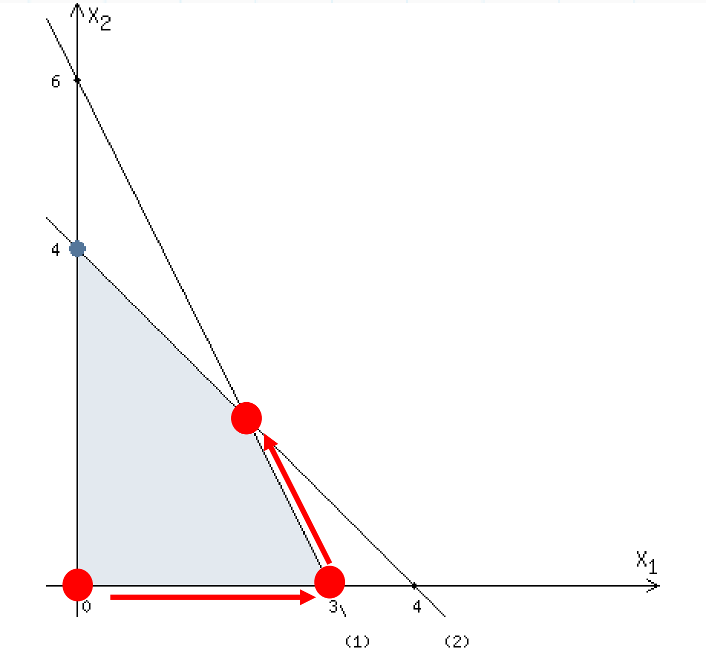

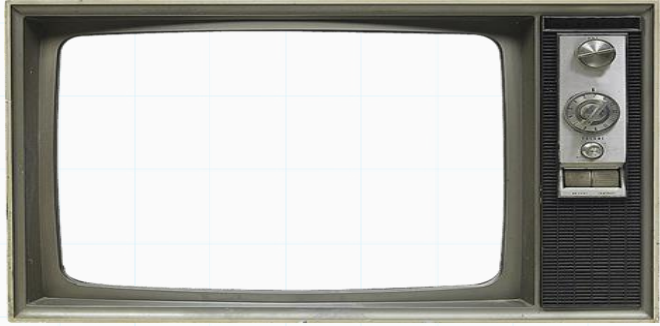

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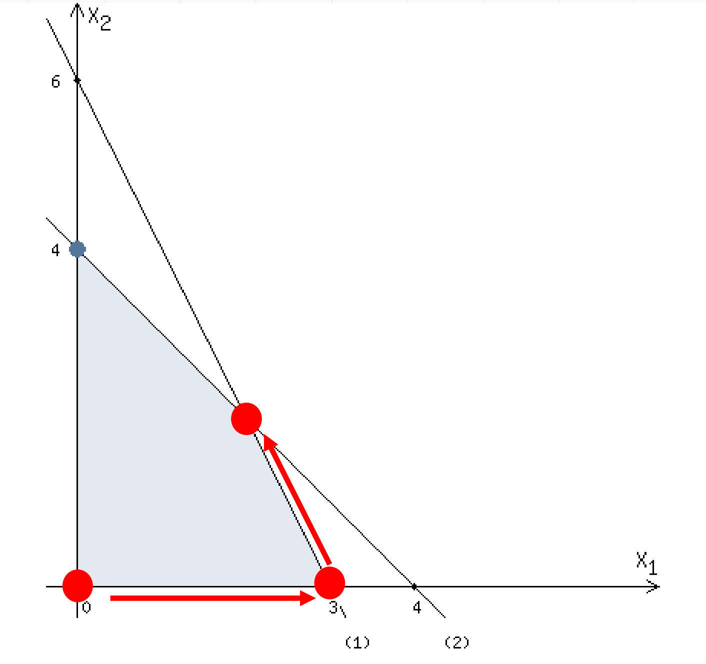

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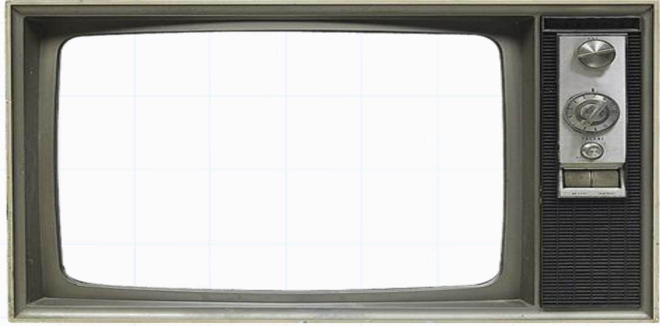

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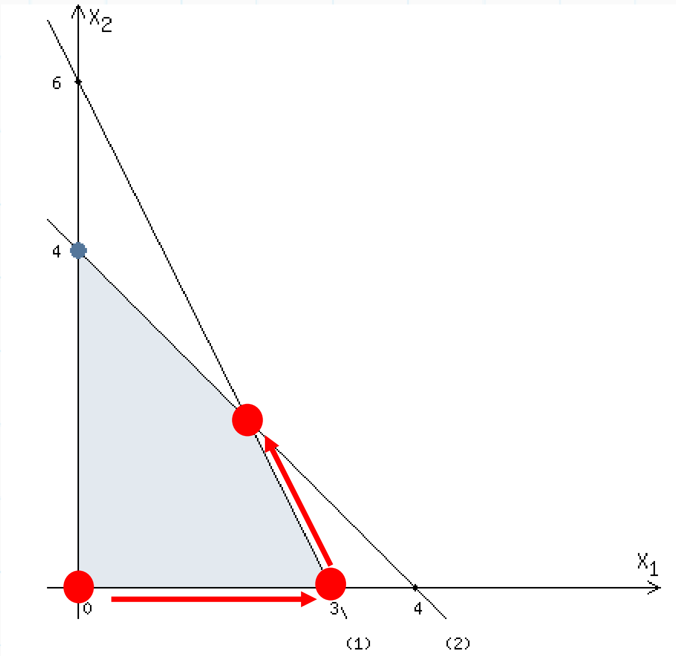

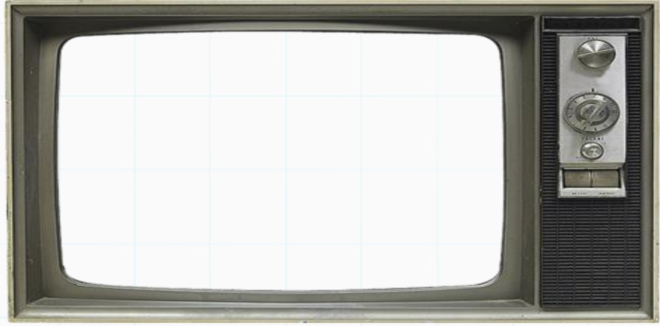

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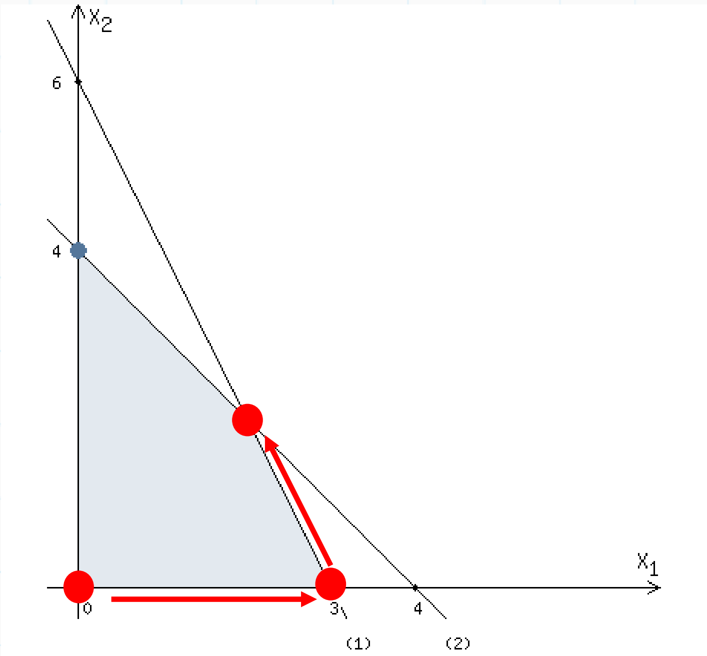
Teste da Razão:

$$x_1:\ x_3 \leq \frac{2}{1} = 2$$

x_2: nos daria apenas um L.I.
$x_3 \geq -2$

Vemos então que $x_3 \leq 2$ é o menor L.S., logo: $\boxed{x_2 = 3 \text{ e } x_4 = 1}$

$\boxed{x_3 \text{ entra na base e } x_1 \text{ sai,}}$ $I_B=\{2,3\}$



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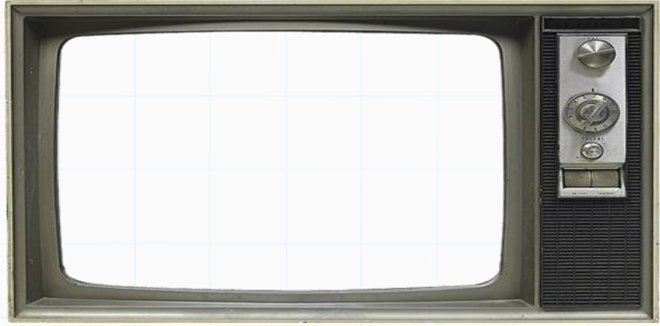

Exemplo: Nova base $I_B=\{2,3\}$

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \quad B^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \quad N = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

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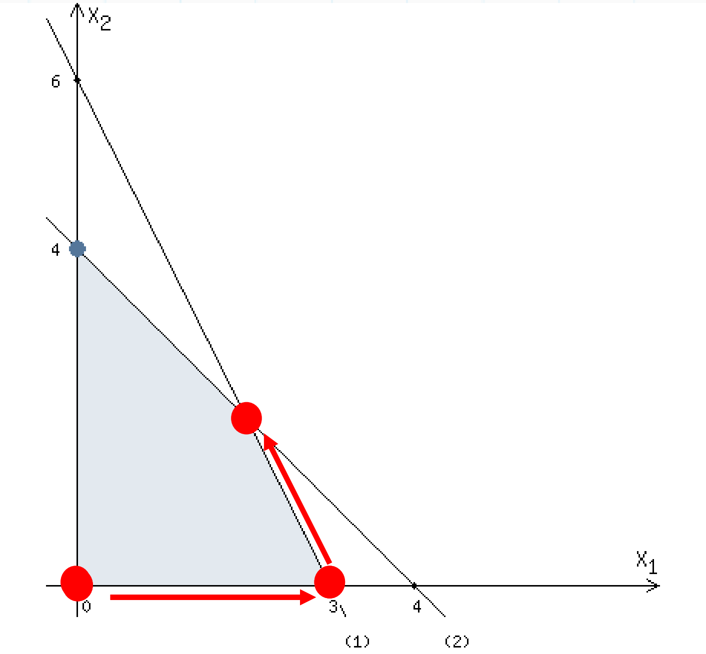

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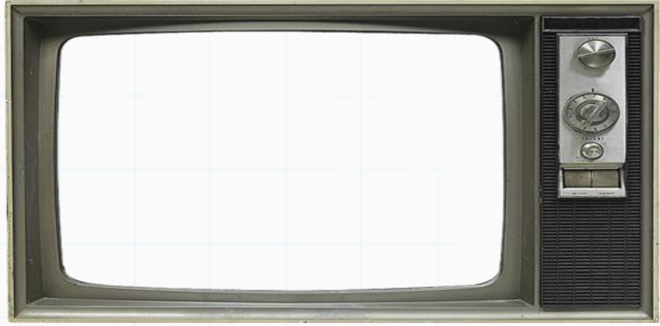

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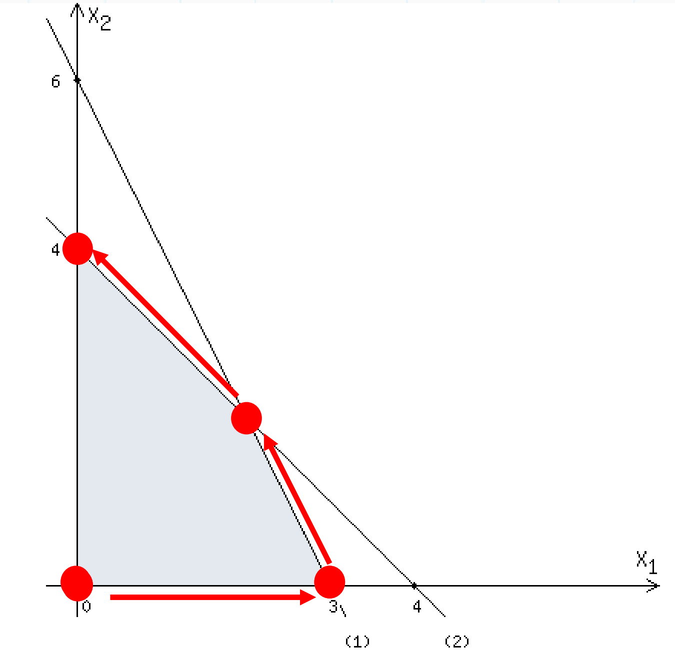

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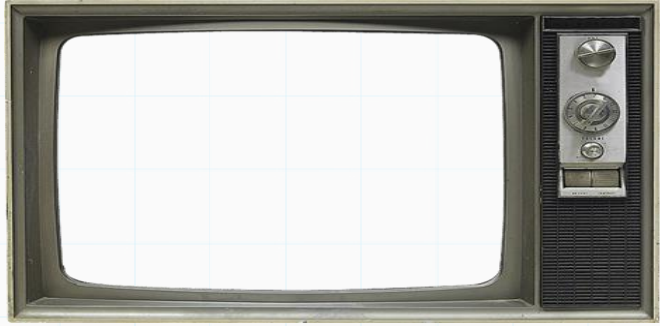

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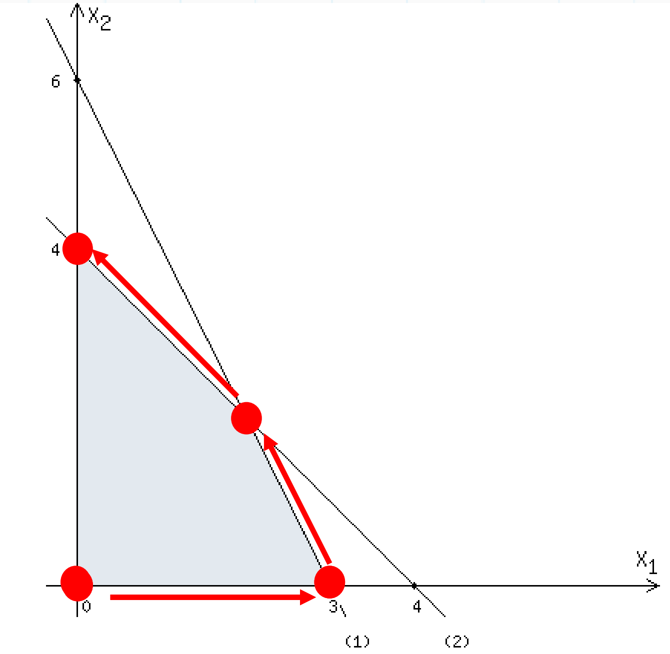

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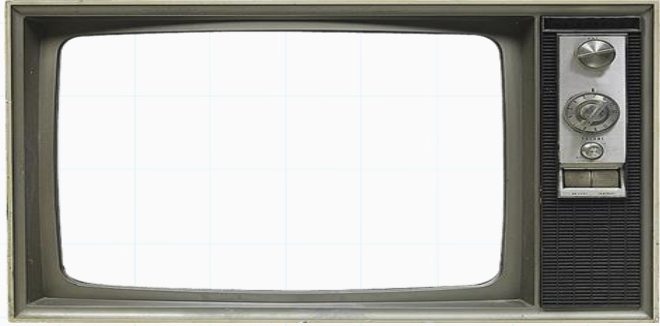

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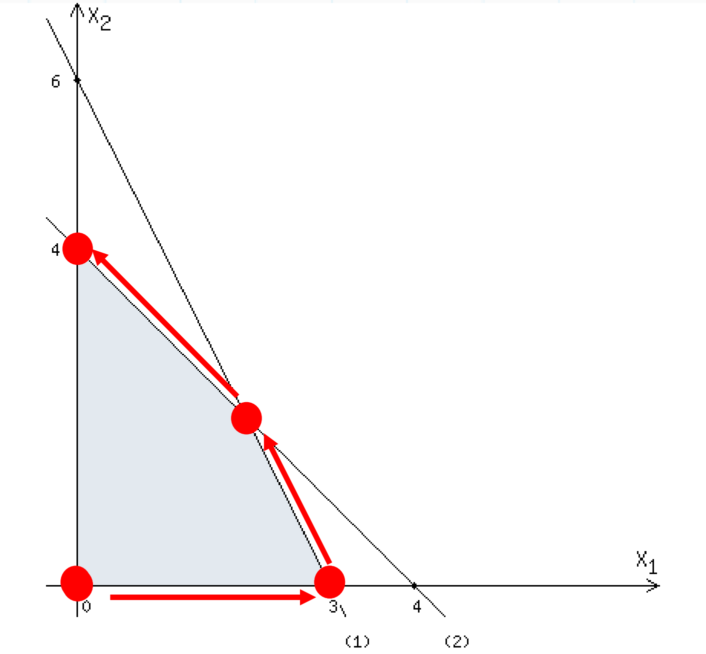

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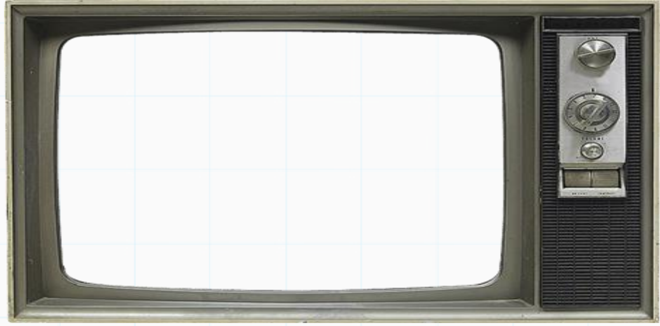

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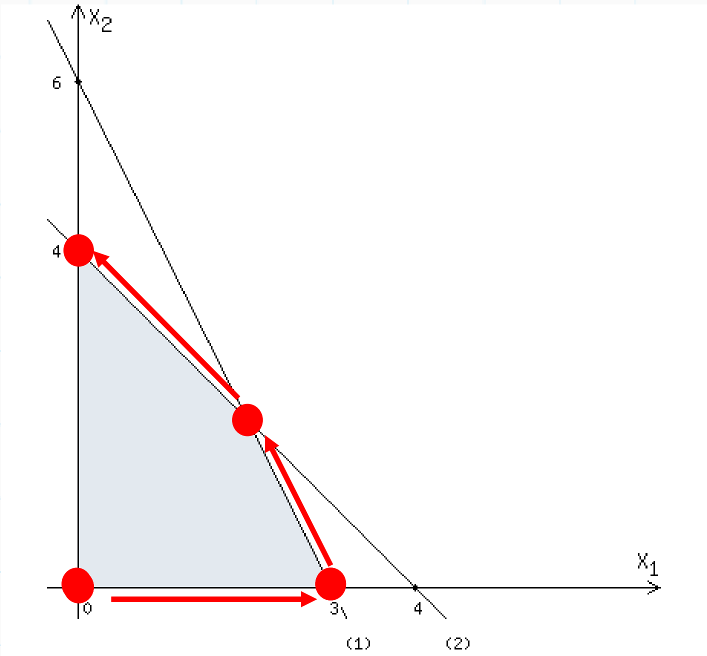

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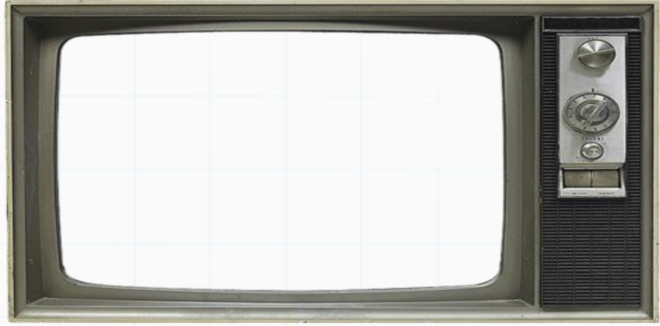

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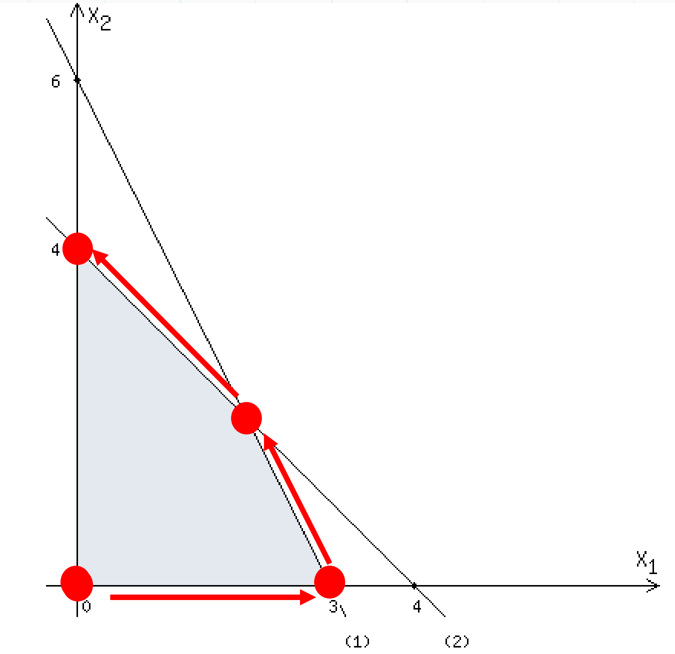

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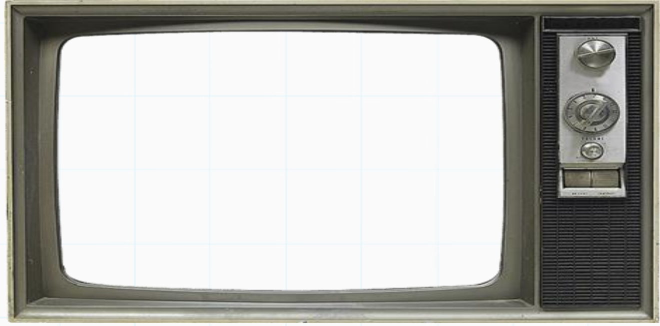

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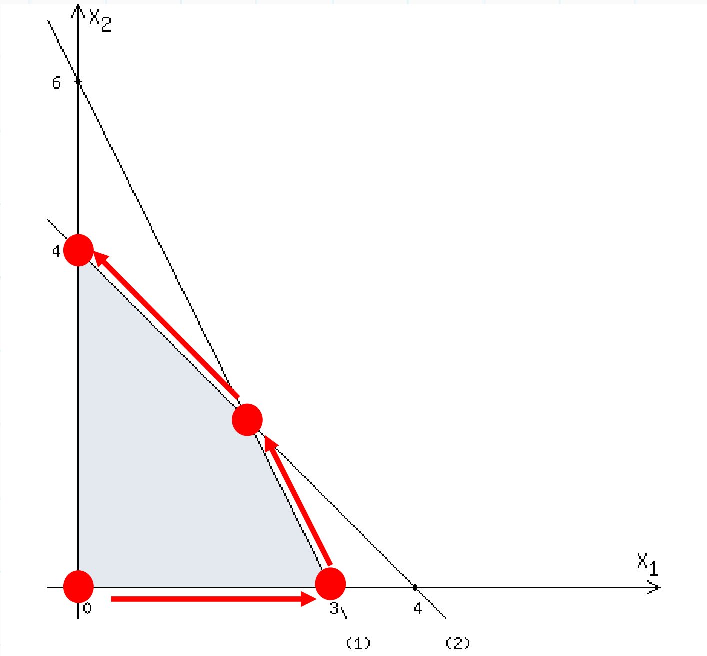

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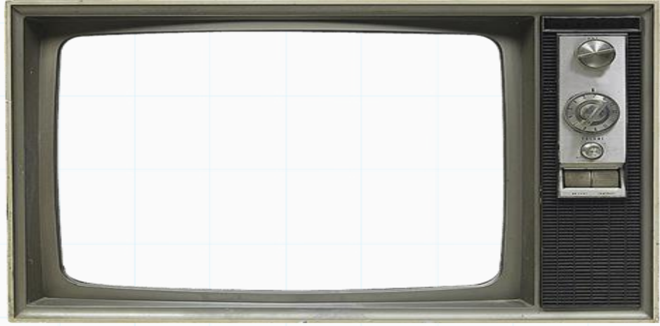

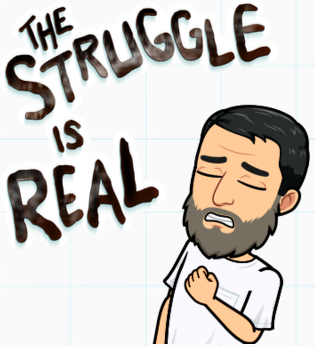

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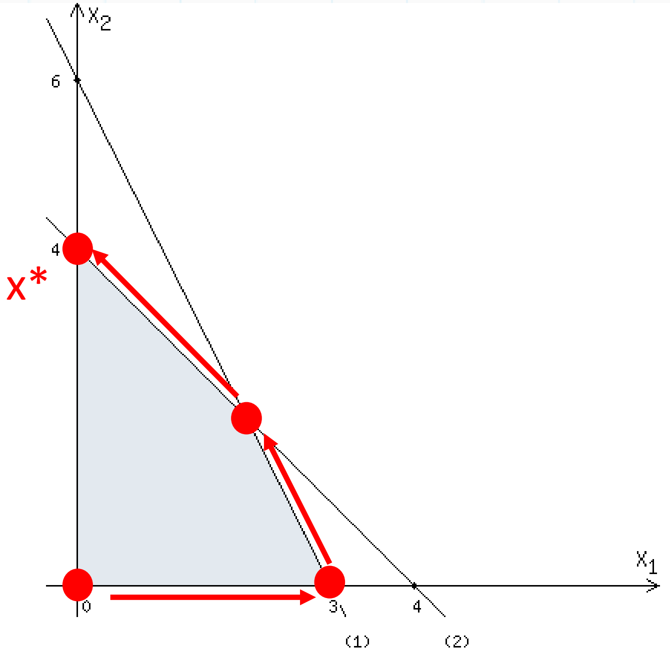

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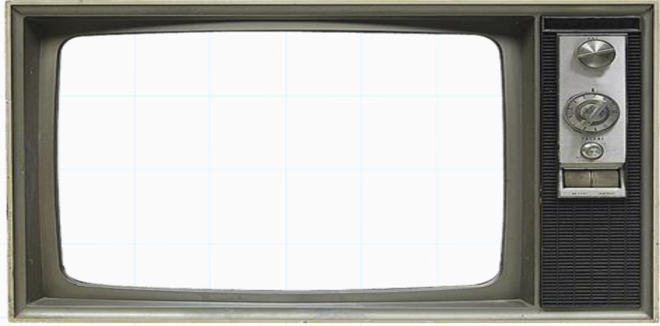

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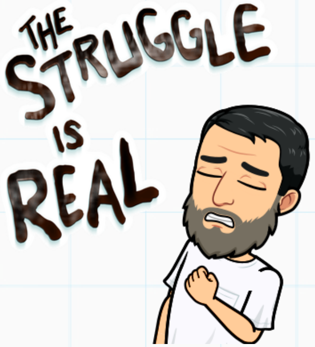
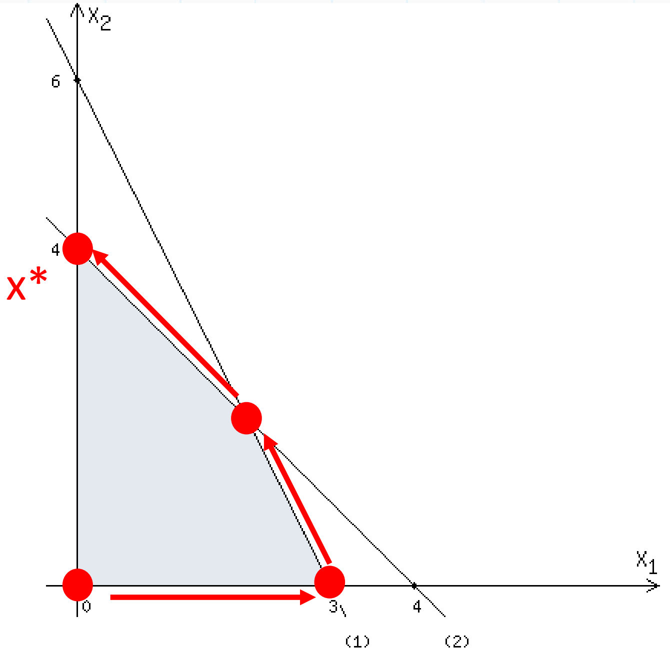
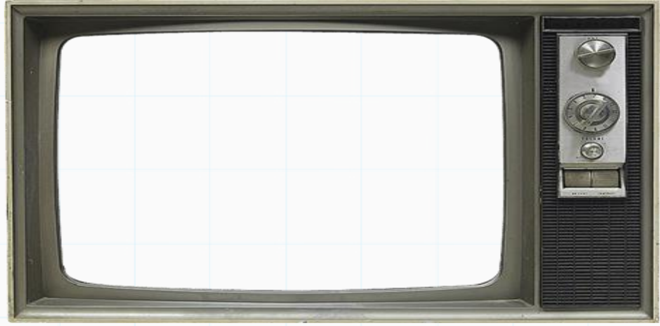


Solução S.B.V. ótima $\boxed{x_1=0,\ x_2=4,\ x_3=2,\ x_4=0, \text{ com } Z=8}$





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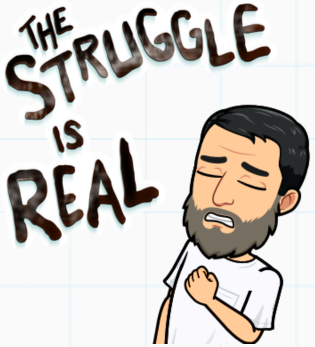

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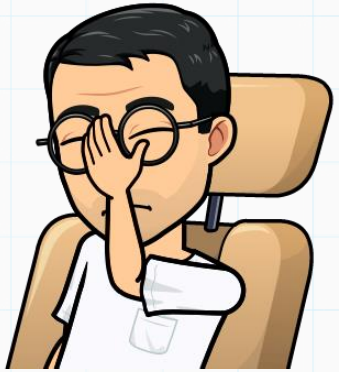
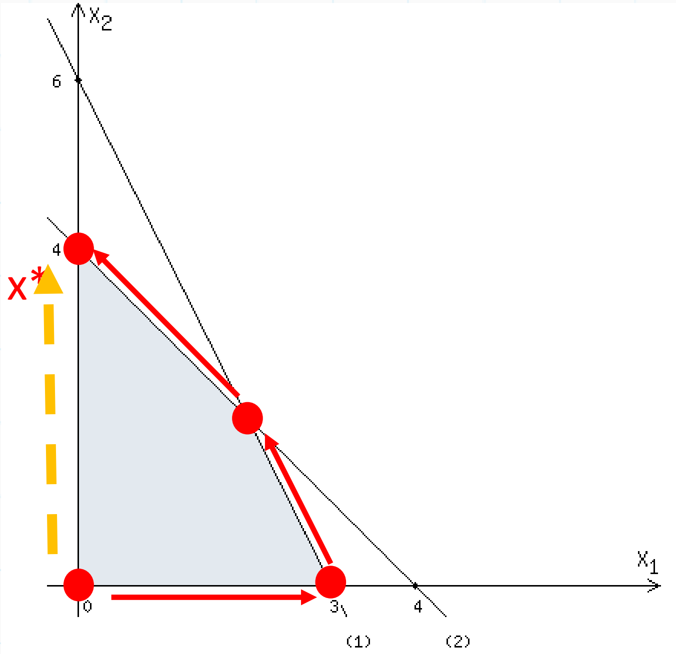
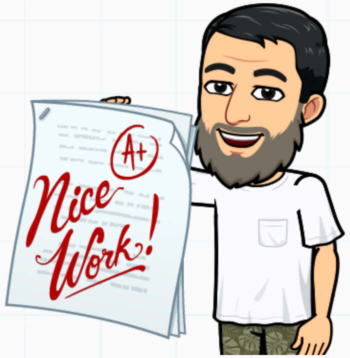
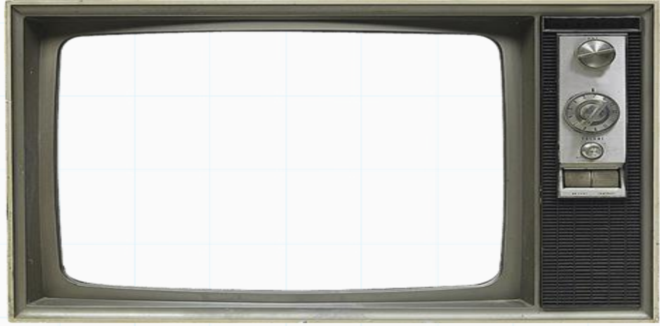


E se na escolha da primeira variável a entrar na base tivesse sido x_2 ao invés de x_1 ?



Exercício





- Dado o PPL abaixo:

$$\max \quad 3x_1 + 2x_2 \qquad (1)$$
$$x_1 + x_2 \leq 4 \qquad (2)$$
$$2x_1 + x_2 \leq 5 \qquad (3)$$
$$x_1, x_2 \geq 0 \qquad (4)$$

1) coloque ele na forma padrão
2) Aplique o método Simplex (a partir da <u>base inicial fornecida pelas variáveis de folga</u>) e encontre a solução ótima. Escolha sempre a variável de entrada na base de maior ganho (de maior custo)
   1) PPL na primeira base
   2) PPL na segunda base
   3) PPL na terceira base

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$$\mathbf{A}^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Até a próxima