The Chain Rule:

Suppose you are asked to differentiate the function $f(x) = (2x+1)^3$

$$FOIL\ (2x+1)(2x+1)(2x+1) =$$ **TOO MUCH WORK!**

The Chain Rule:

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \times g'(x)$$

* used to differentiate compositions $f \circ g$

*** Alternate form:**

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

EX #1: Find the derivative. $f'(g(x)) \times g'(x)$

a.) $y = (2x+1)^3$

Inner Function

$g = 2x + 1$

$g' = 2$

Outer Function

$f = x^3$

$f' = 3(x)^2$

Always put ( )'s around the *x* of the outer function derivative.

$$\frac{dy}{dx} = f'(g(x)) \times g'(x)$$

$$= 3(\ 2x+1\ )^2 \times 2$$

$$= 6(2x+1)^2$$

b.) $find\ \frac{dy}{dx}\ for\ y = (x^2+1)^3$

Inner Function

$g = x^2 + 1$

$g' = 2x$

Outer Function

$f = x^3$

$f' = 3(x)^2$

$$\frac{dy}{dx} = f'(g(x)) \times g'(x)$$

$$= 3(\ x^2+1\ )^2 \times 2x$$

$$= 6x(x^2+1)^2$$

c.) $h(x) = (3x - 2x^2)^3$

Inner Function **Outer Function**

$g = 3x - 2x^2$ $f = x^3$

$g' = (3 - 4x)$ $f' = 3(x)^2$

$$h'(x) = f'(g(x)) \times g'(x)$$

$$= 3(3x - 2x^2)^2 \times (3 - 4x)$$

$$= 3(3 - 4x)(3x - 2x^2)^2$$

$$= (9 - 12x)(3x - 2x^2)^2$$

d.) $h(x) = \sqrt[3]{(x^2 - 1)^2}$

Inner Function **Outer Function**

$g = x^2 - 1$ $f = \sqrt[3]{x^2} = x^{\frac{2}{3}}$

$g' = 2x$ $f' = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3 \times \sqrt[3]{(x)}}$

$$h'(x) = f'(g(x)) \times g'(x)$$

$$= \frac{2}{3 \times \sqrt[3]{(x^2 - 1)}} \times 2x$$

$$= \frac{4x}{3 \times \sqrt[3]{x^2 - 1}}$$

e.) $h(t) = \dfrac{-7}{(2t-3)^2}$

Inner Function

$g = 2t - 3$

$g' = 2$

Outer Function

$f = \dfrac{-7}{t^2} = -7t^{-2}$

$f' = 14t^{-3} = \dfrac{14}{(t)^3}$

$$h'(t) = f'(g(x)) \times g'(x)$$

$$= \frac{14}{(2t-3)^3} \times 2 = \frac{28}{(2t-3)^3}$$

f.) $y = \dfrac{1}{\sqrt[3]{x^2+x+1}}$

Inner Function

$g = x^2 + x + 1$

$g' = 2x + 1$

Outer Function

$f = \dfrac{1}{\sqrt[3]{x}} = x^{-\frac{1}{3}}$

$f' = -\dfrac{1}{3}x^{-\frac{4}{3}} = -\dfrac{1}{3\times\sqrt[3]{(x)^4}}$

$$\frac{dy}{dx} = f'(g(x)) \times g'(x)$$

$$= -\frac{1}{3\times\sqrt[3]{(x^2+x+1)^4}} \times (2x+1)$$

$$= -\frac{2x+1}{3\times\sqrt[3]{(x^2+x+1)^4}}$$

EX #2: Differentiate.

c.) $h(t) = \left(\frac{t-2}{2t+1}\right)^9$

Chain Rule w/ Quotient Rule

$$h'(t) = f'(g(t)) \cdot g'(t)$$

$$= 9\left(\frac{t-2}{2t+1}\right)^8 \cdot \frac{5}{(2t+1)^2}$$

$$= \frac{9(t-2)^8}{(2t+1)^8} \cdot \frac{5}{(2t+1)^2}$$

$$= \frac{45(t-2)^8}{(2t+1)^{10}}$$

Inner Function

$$g = \frac{t-2}{2t+1}$$

$$g' = \frac{k \cdot j' - j \cdot k'}{k^2}$$

$$g' = \frac{(2t+1) \cdot 1 - (t-2) \cdot 2}{(2t+1)^2}$$

$$= \frac{2t+1-2t+4}{(2t+1)^2} = \frac{5}{(2t+1)^2}$$

Outer Function

$$f = x^9$$

$$f' = 9x^8$$

$$j = t-2$$

$$j' = 1$$

$$k = 2t+1$$

$$k' = 2$$